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Central Bank Digital Currency and Bank Intermediation: Medium of Exchange vs. Savings Vehicle

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Abstract

This paper analyzes the effects of a retail central bank digital currency (CBDC) on bank intermediation within a general equilibrium model and provides a quantitative assessment. In the model, banks offer both transaction and savings deposits. This allows to study how a CBDC's impact varies based on its usage as a medium of exchange or as a savings vehicle. When a CBDC is used as a savings vehicle that competes with savings deposits, the level of disintermediation almost doubles compared to a scenario where the CBDC is used solely for transactions and competes only with transaction deposits.

Keywords: *central bank digital currency, bank lending, medium of exchange, savings vehicle, new monetarism, overlapping generations*
JEL codes: E42, E50, E58

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1 Introduction

The concept of a retail central bank digital currency (CBDC) has gathered significant attention. Numerous countries are actively researching the subject and are conducting pilot projects. In some instances, such as Nigeria and certain Caribbean states, a retail CBDC has already been launched. One major concern surrounding the introduction of a retail CBDC is its potential impact on financial stability and the disintermediation of the banking sector. A BIS (2021) report highlights that "*the potential for the introduction of a CBDC to affect financial stability risks arises primarily from a significant substitution away from private money*".

Understanding how the introduction of a CBDC influences bank intermediation and other macroeconomic variables is crucial. The literature generally agrees that a CBDC would act as a substitute for bank deposits, which are a cheap source of funding for banks. Consequently, a potential crowding out of deposits could result in diminished bank intermediation, leading to reduced credit availability or higher credit costs.

To further advance the analysis concerning the impact on banks, we should distinguish the effects based on the CBDC's utilization as a medium of exchange or as a savings vehicle. In the former scenario, the CBDC would mainly compete with short-term transaction deposits, while in the latter case, it would additionally compete with longer-term savings deposits, potentially leading to different effects on bank lending. Panetta (2022), a member of the ECB's Executive Board, expresses a similar perspective, suggesting that the central bank could employ varying remuneration on the CBDC to ensure that "*it is a means of payment that is as attractive as cash*" but "*to reduce the attractiveness of the CBDC as a store of value*".

Thus, the novel contribution of this paper is to consider the different roles of a CBDC and untangle its effects on banks, depending on whether it is used as a medium of exchange or a savings vehicle. The paper addresses the following research questions: What are the impacts of introducing a CBDC on bank intermediation? How do these effects vary depending on a CBDC's use as a medium of exchange or as a savings vehicle? How does an interest rate on CBDC affect these outcomes?

I address the research questions by employing a general equilibrium model that considers agents' needs for both payments and savings. In the model, agents have the option to hold private money with banks or use public money in the form of cash or the newly introduced CBDC. The banks fund themselves through transaction and savings deposits in a perfectly competitive deposit market. Additionally, banks extend loans to entrepreneurs in a loan market where they possess some market power.

I model the introduction of a CBDC as an exogenous shift between private and public money. However, my objective is not to determine the actual number of people who switch between the two forms of money upon the introduction of a CBDC. Instead, my main focus lies in understanding the magnitude of the effects if such a shift indeed occurs, and how it impacts the banking system.

I find that a CBDC has no impact on bank lending when banks hold excess liquidity. However, when liquidity is scarce, a shift from private to public money negatively affects bank lending. This effect is stronger when a CBDC is used not only for payments but also as a savings vehicle.

To quantify the effects on bank lending, I conduct a calibration. In an illustrative example, I calculate the effect on bank lending when 10% of depositors switch to CBDC. If this outflow only affects the medium of exchange, leading people to crowd out from transaction deposits, the result is a decrease in bank lending by 0.4%. In contrast, if the outflow involves solely the savings vehicle, impacting savings deposits, the negative effect on lending increases to 0.7%. The fact that the impact on bank lending nearly doubles highlights the importance of distinguishing between the different types of deposits. In a scenario where there is a simultaneous outflow in both types of deposits, bank lending experiences a reduction of 1.2%.

Lastly, I examine the impact of an interest-bearing CBDC through a policy experiment in which public money pays interest. The findings suggest that a positive interest rate can result in welfare improvements, as CBDC holders receive a more favorable interest rate compared to holding cash. However, if the interest rate on the CBDC becomes excessively high, the positive effect is reversed.

Model

In the theoretical model, I combine the Lagos and Wright (2005) framework with an overlapping generations model based on Wallace (1980). This combination allows me to differentiate between money used for transactions and money utilized as a savings vehicle. The resulting model environment shares similarities with the framework presented in Altermatt (2019).

The model divides time periods into a centralized market (CM) and a decentralized market (DM). It involves four types of agents: consumers, producers, bankers, and entrepreneurs. Consumers live for three subperiods and can only work in the CM when young. After being born, a consumer draws one of two types. The first type is the early consumer, who consumes in the DM, while the second type is the late consumer, who consumes in the CM when old. The types are public information and agents face no uncertainty. I assume that only highly liquid assets can be exchanged in the DM. Thus, the early consumer demands a medium of exchange and the late consumer a savings vehicle.[1]

Entrepreneurs live for one period and have an investment opportunity, but they cannot work when young. Consequently, they must obtain a loan from a banker to finance their investments, which they repay when they are old. This process leads to the endogenous creation of inside money.

Banks issue deposits in a perfectly competitive market. Furthermore, they extend loans to entrepreneurs in the loan market through Cournot competition, which grants them some market power. Additionally, banks must adhere to a minimum reserve requirement constraint on all deposits exchanged after one subperiod in the DM.[2] As a result, all liquid deposits exchanged in the DM can only be partially invested into loans. Consequently, bankers have an incentive to offer two types of deposits: liquid transaction deposits for early consumers and illiquid savings deposits for late consumers, with savings deposits paying a higher interest rate.

[1] The OLG structure is crucial, because in a scenario where the buyer has an infinite lifespan, there exists no need for a savings instrument, as the buyer could simply engage in work again in the CM in period $t+1$.

[2] This aligns with the standard institutional setup where minimum reserve requirements also exclusively apply to liquid deposits. See `https://www.ecfr.gov/on/2021-01-15/title-12/chapter-II/subchapter-A/part-204` or `https://www.federalreserve.gov/monetarypolicy/reservereq.htm`.

Furthermore, two forms of money exist: public money issued by the central bank and private money represented by bank deposits. The measure of agents holding private or public money is exogenous in the model. Subsequently, I model the introduction of a CBDC as an exogenous shift between private and public money.

The rationale behind this is that the introduction of an additional option to hold public money might incentivize certain individuals to transition toward it. However, I do not investigate whether such a transition indeed occurs. Instead, my primary focus lies on comprehending the impacts on bank intermediation and assessing the magnitude of these effects if a shift takes place upon the introduction of a CBDC. Furthermore, this framework facilitates a careful calibration of the model to empirical data, as the simultaneous existence of the two money types remains straightforward even under varying interest rates. Alternatively, I could attempt to microfound the choices regarding the two types of money, but this would come at the expense of sacrificing model tractability and introduce complexity to the calibration process.

I find that the impact of a CBDC on bank lending depends on the bankers' reserve position. If banks hold excess reserves, a deposit outflow reduces their reserve balance but does not affect loan issuance. However, if reserves are scarce and people switch from private to public money, there is a decline in bank lending. Nevertheless, the impact is mitigated by the banks' reaction, which results in higher interest rate on deposits, stimulating the deposit demand and partially offsetting the crowding out effect.

This is especially relevant when there is an outflow in only one type of deposits. For instance, in the case of an exogenous shift in the medium of exchange, there is a decline in transaction deposits on the extensive margin. However, both transaction and savings deposits experience an increase on the intensive margin due to the higher interest rates.

Calibration

To quantify my findings, I perform a calibration of the theoretical model using data from the US economy between 1987 and 2006. This time period is chosen to investigate a scenario without excess reserves. Additionally, I expand the model to incorporate the share of agents holding private and public money as a function of the interest rate spread between the two forms of money.

Deposit interest rate data is obtained from FDIC call reports, while more standard data is sourced from FRED. To calibrate the shift between private and public money in response to changes in interest rates, I utilize county-level variations in deposit levels and interest rates. The data is drawn from various sources, including the FDIC Summary of Deposits, the U.S. Census Bureau and the U.S. Bureau of Economic Analysis.

As an illustrative example, I report a scenario where 10% of the agents holding deposits switch to CBDC. If the shift is solely in the medium of exchange, bank lending decreases by 0.4%. However, if the shift affects the savings vehicle, the effect nearly doubles to 0.7%. When there is a simultaneous crowding out in both types of deposits, bank lending decreases by approximately 1.2%.

Furthermore, I analyze the effects of an interest-bearing CBDC. I find that setting the interest rate on CBDC equal to the Friedman rule results in a decrease in bank lending of approximately 3.0%. Nevertheless, a positive CBDC rate can be welfare improving by offering a more favorable rate on public money. The central bank can maximize welfare by setting the gross real return on CBDC slightly below 1.

Literature

The discourse surrounding the introduction of digital public money traces back to Tobin (1985). In recent years, there has been a significant surge in interest in this topic. My paper contributes to a growing body of literature that explores the implications of CBDC on bank intermediation. It is most closely related to Chiu et al. (2023) who also model an imperfectly competitive banking sector and perform a quantitative analysis. In their model, banks possess some market power in the deposit market, leading to suboptimal deposit holdings. By introducing an interest-bearing CBDC, the central bank creates an alternative to bank deposits, leading to increased competition in the deposit market. Consequently, banks raise the interest rate on deposits, attracting more depositors and boosting bank lending. However, if the interest rate on CBDC becomes excessively high, the banks' interest margin becomes zero, resulting in a decrease in bank lending.

My paper distinguishes from theirs in two key aspects. First, I introduce two types of deposits whose interest rates are both endogenously determined. While they also incorporate time deposits, these deposits have a less significant role and pay a fixed interest rate equal to the Friedman rule. Second, their focus mainly revolves around how the demand for CBDC adjusts concerning changes in the interest rate on CBDC. Furthermore, private and public money only coexist when their returns are equalized. In contrast, my analysis primarily centers on the magnitude of the effect on loan supply resulting from a crowding out of deposits to CBDC.

Keister and Sanches (2023) develop a theoretical model based on Lagos and Wright (2005), featuring a perfectly competitive banking sector in which banks are financially constrained. They examine three types of CBDC introductions: cash-like, bank deposit-like, and universal. A cash-like CBDC has no impact on bank lending, while both the deposit-like and universal CBDC lead to a decrease in bank lending. However, the overall welfare effects remain ambiguous because a CBDC increases the quantity of liquid assets, enabling more efficient levels of exchange.

Andolfatto (2021) examines the impact of an interest-bearing retail CBDC on a monopoly banking sector using an overlapping generations model. He finds that the banks' profit-maximizing lending rate remains unaffected by the introduction of a CBDC, resulting in unchanged bank lending. This outcome is contingent on the assumption that banks can borrow reserves from the central bank at the interest rate on reserves (IOR) when the interest rate on central bank money exceeds the IOR. Similarly, Brunnermeier and Niepelt (2019) establish an equivalence result suggesting that, under specific conditions, a CBDC has no effect on bank lending because banks can borrow from the central bank in response to deposit outflows to CBDC. It is essential to note that the present study does not consider the possibility of borrowing from the central bank and solely analyzes scenarios where borrowing is not feasible.

Assenmacher et al. (2021) develop a general equilibrium model with frictions that require entrepreneurs to borrow both bank deposits and CBDC. They observe that bank lending can increase under certain conditions, such as when there is a high interest rate spread between CBDC and the deposit rate, a tight collateral constraint and high substitutability between the two assets. However, if the substitutability is low, bank lending decreases. Garratt and Zhu (2021) introduce banks with heterogeneous size, where larger banks possess a higher convenience value for consumers and, consequently, more market power compared to smaller banks. Whether CBDC increases or decreases bank lending depends, among other factors, on its own convenience value.

In Barrdear and Kumhof (2016), the authors build a rich DSGE model to investigate the macroeconomic implications of introducing an interest-bearing retail CBDC that competes with privately created bank-issued money. The model comprises four sectors, four lending markets, and various real and nominal rigidities. The main mechanism how a CBDC affects outcomes is that it is only purchasable against government bonds. Using US data for calibration, they find that issuing CBDC equivalent to 30% of GDP could result in an investment gain of 5.28% and a permanent increase in GDP by 3%. Meanwhile, Agur, Ari and Dell'Ariccia (2022) explore the impact of CBDC on the use of cash and deposits, where demand is influenced by varying preferences over anonymity and security. They find that if CBDC closely substitutes bank deposits, it leads to a decrease in bank lending.

Williamson (2022) develops a model with competitive banks and free entry. He then examines the welfare and policy implications of different CBDC designs. Niepelt (2022) explicitly incorporate reserves in a model that considers banks with market power in the deposit market. He finds that optimal interest rates on CBDC and reserves differ and depending on the design choice, a CBDC can raise banks' funding costs by up to 1.5% of GDP.

Whited, Wu and Xiao (2023) introduce a model that distinguishes between retail deposits and wholesale funding. They demonstrate that an interest-bearing CBDC can replace a significant portion of deposits. However, banks can offset the loss in deposits by resorting to wholesale funding, thereby mitigating the negative effect on bank lending. Banet and Lebeau (2022) model the influence of a CBDC on both bank intermediation and financial inclusion. They identify a trade-off that relies on the CBDC interest rate and the fixed cost of using it. If the CBDC is sufficiently affordable, it can enhance financial inclusion without impacting intermediation.

My paper distinguishes itself from the existing literature in several key aspects. To the best of my knowledge, no previous theoretical model has incorporated heterogeneous bank deposits with endogenous interest rates to comprehensively analyze the impact of a CBDC on bank intermediation. Additionally, in the calibration process, I emphasize the significance of the CBDC's usage as either a medium of exchange or a savings vehicle, which yields distinct results. Furthermore, I conduct a detailed examination of the extensive and intensive margin effects on deposits, providing a deeper understanding of the overall effects.

The rest of the paper is organized as follows. Section 2 describes the model and the equilibrium. In Section 3 I discuss the introduction of a CBDC in the theoretical model and calibrate the model to the data in Section 4.

2 Environment

Time is modeled as discrete and continues indefinitely.[3] Each period consists of two consecutive subperiods: a frictionless centralized market (CM) and a frictional decentralized market (DM). Agents discount between periods with discount factor $\beta \in (0, 1)$. There exist two non-storable goods: a CM good x and a DM good y. Both cannot be transferred to the next subperiod.

In each period t, a new generation of consumers - a continuum with measure 1 - is born and lives for three subperiods. Specifically, the consumers are born in the CM in period t, live throughout the DM and die at the end of the subsequent CM in period $t+1$. Among these consumers, there exist two types denoted as $\theta \in \{\theta^m, \theta^s\}$. After birth, a consumer draws a type from a distribution where θ equals θ^m - the early consumer - with probability γ and θ^s - the late consumer - with probability $1-\gamma$. The distribution and realization of θ are common knowledge. Henceforth, I will use the terms θ^m

[3]For a more intuitive description of the model environment and a visual representation of the time line, refer to Appendix E.1.

(θ^s) and early (late) consumer interchangeably.

Both types can only produce when young. In the CM in period t, they produce the general good x at a linear disutility h, where one unit of effort h results in one unit of good x. However, both types do not want to consume in the same period. The early consumer desires to consume in the second subperiod after birth, which is the DM in period t. On the other hand, the late consumer wants to consume in the third subperiod after birth when old, which is the CM in period $t+1$. The lifetime utility W of a consumer with type θ is given by:

$$W^{\theta}(h, x, y) = \begin{cases} -h^{\theta^m} + v(y) & \text{if } \theta = \theta^m \\ -h^{\theta^s} + \beta U(x) & \text{if } \theta = \theta^s \end{cases}$$

I assume that $v'(y) > 0, U'(x) > 0, v''(y) < 0, U''(x) < 0, v'(0) = U'(0) = \infty$, $v'(\infty) = U'(\infty) = 0$ and $-x\frac{U''(x)}{U'(x)} < 1$ for all $x \geq 0$ and $-y\frac{v''(y)}{v'(y)} < 1$ for all $y \geq 0$.[4]

Since goods are non-storable and neither type of consumer can work in the period they want to consume, they face a challenge in finding a way to consume in later periods of life. Since a credit arrangement is infeasible due to the lack of commitment among consumers, there arises a demand for a storable asset that young consumers can acquire by producing and selling good x. I assume that only highly liquid assets can be exchanged in the DM. Consequently, the early consumer θ^m demands a payment vehicle (medium of exchange) m, while the late consumer θ^s demands a savings vehicle s.

Furthermore, there exists a continuum of infinitely-lived producers with measure γ. These producers derive linear utility from consuming x in the CM, but they cannot produce within the same subperiod. Conversely, they do not wish to consume in the DM, yet they can produce the DM good y with linear disutility. The preference of the producers can be described as follows:

$$\sum_{t=0}^{\infty} \beta^t [x_t - y_t].$$

[4]Assuming a coefficient of relative risk aversion smaller than 1 ensures that the agents' money demand increases as the money's return rises.

As good y is non-storable, the producers demand an asset that facilitates acquiring consumption in the CM. Consequently, they sell y to the early consumers in exchange for the payment vehicle. This transaction occurs in the DM through a bilateral trade, which takes place with probability one, and where the early consumer makes a take-it-or-leave-it offer for y.

Moreover, two other types of agents exist: entrepreneurs and bankers. Each period, a unit mass of one-period lived entrepreneurs is born. They have an investment opportunity such that they can invest the CM good x in period t and receive a return $f(x)$ in the subsequent CM in period $t+1$. The production function is represented by $f(x) = Ax^{\eta}$, where $A > 0$ and $\eta \in (0,1)$.[5] Entrepreneurs can neither work nor have an endowment. To acquire good x and invest it, they must borrow from other agents in the economy, giving rise to intermediaries.

There are $B \in \mathbb{N}$ one-period lived, homogeneous bankers, equipped with a costless record-keeping technology. They engage in issuing loans within an imperfectly competitive market, modeled as Cournot competition similar to Chiu et al. (2023) or Altermatt and Wang (2021). Moreover, each banker has the ability to offer heterogeneous deposit accounts with different liquidity characteristics, serving as either a medium of exchange or a savings vehicle for agents. The deposit market operates as a perfectly competitive market, ensuring tractability in the model despite the existence of different types of deposits.

Furthermore, bankers must comply with a minimum reserve requirement mandated by the government. This requirement specifies that bankers are obligated to hold a fraction $\omega \in [0,1]$ of all liquid deposits that can be exchanged in the DM as reserves at the central bank.[6] Only bankers are allowed to hold reserves. The real amount of reserves held by bank b is denoted by e_b, while the overall nominal stock of reserves is denoted by E_b. The central bank has the authority to set an interest rate on reserves (IOR), which serves as the policy rate in the model. The IOR is denoted as i_{e_b}, where i represents the nominal net rate.

[5] This ensures that Proposition 1 holds and banks offer two types of deposit.

[6] The inclusion of only liquid deposits in the reserve requirement aligns with the standard institutional setup, see Footnote 2.

Additionally, the central bank issues money that is available to the general public. The interest rate paid on public money is denoted as i_e. Consumers have the option to hold either private money in the form of bank deposits or public money in the form of cash or, introduced later, CBDC. The distribution of early and late consumers who hold private or public money is exogenous.[7] The real holdings of public money for early and late consumers are denoted by e^m and e^s, respectively.

The total nominal stock of central bank money in period t is denoted by E, and in period $t-1$ as E^-. The stock of central bank money changes each period by $E/E^- = \mu$ through injections of the central bank, which are conducted as lump-sum transfers or taxes to the producers at the beginning of the CM. The amount of CM goods that one unit of central bank money can buy in period t is denoted by ϕ_e. The inflation rate is denoted as $\phi_e/\phi_e^+ - 1 = \pi^+$, where $+$ defines period $t+1$. Furthermore, the government levies a lump-sum tax on the producers. The difference between the transfer from money creation and the tax is denoted as T. The government's consolidated budget constraint is given by:

$$\phi_e[E - E^-] + T = \left[\sum_{b=1}^{B} e_b\right] i_{e_b}/\mu + [e^m + e^s]\, i_e/\mu. \tag{1}$$

Henceforth, I restrict the analysis to a stationary equilibrium and assume that bankers will always honor their promise to pay out central bank money on demand. This implies that $\phi = \phi_d = \phi_\tau = \phi_e$ and $\mu = \phi/\phi^+$. Furthermore, the Fisher equation defines the real return of the assets, which is $(1+i) = \mu R$, where μ is the inflation rate and R represents the gross real interest rate.

[7] In the calibration process outlined in Section 4, I endogenize these shares by formulating them as a function of the interest rate spread between private and public money. Alternatively, I could attempt to microfound the choice over the two types of currencies. However, this would result in a loss of model tractability. Since my objective is to analyze the qualitative impacts and assess the magnitude of the effects if a CBDC induces a transition from private to public money, the modeling choice seems reasonable.

2.1 The Banker's Problem

A banker $b \in 1, 2, \ldots, B$ is a one-period-lived agent who issues loans and accepts deposits. The bankers charge interest on loans and pay interest on deposits to the agents who hold the deposits between periods. They can offer two types of deposits: liquid deposits that can be exchanged in the DM and illiquid deposits that can only be transferred in the CM in period $t+1$. Only liquid deposits are subject to the reserve requirement.

Due to the existence of early and late consumers who demand a payment and savings vehicle, bankers are incentivized to offer two types of deposits when faced with a binding reserve requirement constraint: liquid transaction deposits for early consumers and illiquid savings deposits for late consumers, as demonstrated in Proposition 1.

Proposition 1. *Given a binding reserve requirement constraint, a homogeneous banking sector, common knowledge regarding the consumer types* $\theta \in \{\theta^m, \theta^s\}$*, and a production function of the form* $f(x) = Ax^\eta$*, the bankers will offer two types of deposits, liquid transaction deposits* d *and illiquid savings deposits* τ*.*

Proof. A formal proof can be found in Appendix A.1. The intuition behind this result is as follows: A banker that offers only transaction deposits would be holding unnecessary reserves on all deposits of the late consumers. By offering savings deposits that can be invested one-to-one into loans, bankers can increase their profit. Additionally, a banker could opt to offer only illiquid savings deposits to the late consumers and exclude early consumers from any deposit offers. However, in such a scenario, the banker could increase their profit by offering liquid transaction deposits to early consumers and earning additional profit from them. ■

A banker's operations yield a profit denoted as Π_b, which is used to acquire and consume good x in the CM when old before dying. The objective of a banker is to maximize Π_b.

$$\max_{\ell_b, e_b, d_b, \tau_b} \quad \Pi_b = \ell_b R_\ell(\ell) + e_b R_{e_b} - d_b R_d - \tau_b R_\tau$$
$$s.t.\ e_b \geq \omega d_b,$$
$$\ell_b + e_b = d_b + \tau_b,$$

where $\ell = \ell_b + \sum_{b \neq b'} \ell_{b'}$. On the asset side, the bank issues loans ℓ_b with a gross real interest rate $R_\ell(\ell)$ and holds reserves, which earn a gross real interest rate R_{e_b}. On the liability side, the bank offers transaction and savings deposits d_b and τ_b, respectively, on which it pays gross real interest rates R_d and R_τ. The first constraint represents the reserve requirement, while the second constraint corresponds to the balance sheet identity.

By substituting e_b from the balance sheet identity into the maximization problem, I construct a Lagrangian.

$$\begin{aligned} \mathcal{L}(\ell_b, d_b, \tau_b) =& (R_\ell(\ell) - R_{e_b})\ell_b - (R_d + c - R_{e_b})d_b \\ & - (R_\tau + c - R_{e_b})\tau_b + \lambda(d_b(1-\omega) + \tau_b - \ell_b) \end{aligned}$$

Solving it yields the first-order conditions in equations (2)-(4) and the complementary slackness condition in (5).

$$\frac{\partial R_\ell(\ell)}{\partial \ell_b}\ell_b + R_\ell(\ell) - R_{e_b} = \lambda \tag{2}$$
$$R_{e_b} + \lambda(1-\omega) = R_d \tag{3}$$
$$R_{e_b} + \lambda = R_\tau \tag{4}$$
$$\lambda(d_b(1-\omega) + \tau_b - \ell_b) = 0 \tag{5}$$

From equations (2)-(4), the following result can be derived.

Proposition 2. *Given an imperfectly competitive loan market ($B < \infty$), the following relationships between the interest rates on reserve balances R_{e_b}, loans R_ℓ, savings deposits R_τ, and transaction deposits R_d hold. Two cases can be distinguished depending on whether the reserve requirement constraint is binding ($\lambda > 0$) or loose ($\lambda = 0$).*

i. $\lambda > 0 \Rightarrow R_\ell > R_\tau > R_d > R_{e_b}$.

ii. $\lambda = 0 \Rightarrow R_\ell > R_\tau = R_d = R_{e_b}$.

The bank makes a profit $\Pi_b > 0$ *due to the imperfect competition in the loan market, which creates a wedge between the interest rate on loans and the interest rate on deposits.*

Proof. A formal proof is available in Appendix A.2. ■

Given that the Fisher equation holds and the price of deposits is equal to the price of central bank money, Proposition 2 also applies to nominal interest rates i. If the minimum reserve requirement constraint is non-binding, the banker pays the same interest rate on both transaction deposits and savings deposits. This is due to the banker's excess reserves holdings, which make the deposits economically equivalent. Lastly, note that the assumption of a homogeneous banking sector implies that $\ell = B\ell_b$, $d = Bd_b$, and $\tau = B\tau_b$.

2.2 The Consumers' Problem

Next, I analyze the consumers' maximization problem. An early consumer θ^m and a late consumer θ^s have to decide how much to work in the CM when young to acquire the payment vehicle m or the savings vehicle s, respectively. The value function of consumer j with type $\theta \in \{\theta^m, \theta^s\}$ when young is expressed as follows:

$$
\begin{aligned}
W_j^{\theta^m} &= \max_{h_j^{\theta^m}, m_j} \{-h_j^{\theta^m} + v \circ y_j(m_j, i_m, \phi_m)\} \\
&\text{s.t. } m_j = h_j^{\theta^m} \\
W_j^{\theta^s} &= \max_{h_j^{\theta^s}, s_j} \{-h_j^{\theta^s} + \beta U \circ x_j^+(s_j, i_s, \phi_s)\} \\
&\text{s.t. } s_j = h_j^{\theta^s}.
\end{aligned}
$$

h_j^θ represents the amount of work or goods produced by a young consumer of type θ in the CM. For early consumers, $m_j \in \{d_j, e_j^m\}$ represents the real amount of the payment vehicle demanded, while for late consumers, $s_j \in \{\tau_j, e_j^s\}$ represents the chosen real amounts of the savings vehicle. The variables ϕ_m and i_m correspond to the price and nominal interest rate of the payment vehicle, respectively. Similarly, ϕ_s and i_s denote the price and nominal interest rate of the savings vehicle.

In the DM, an early consumer meets a producer with probability one. In the meeting, the early consumer makes a take-it-or-leave-it-offer for the DM good and chooses an offer such that the producer's participation constraint holds. This constraint is defined by $-y_j + \frac{m_j}{\phi_m}(1+i_m)\phi_m^+\beta - T \geq -T$. The producer works with linear disutility to produce good y and sells it to the early consumer in exchange for the payment vehicle m at a price of ϕ_m. The producers earn interest i_m on their m-holdings in the subsequent CM. At this stage, m can be exchanged for ϕ_m^+ units of the CM good, which the producer then consumes with linear utility. The consumption is discounted with β. Furthermore, the tax or transfer T is levied on the producer.

On the other hand, late consumers simply hold onto their savings vehicle s and earn interest i_s on it in the CM in period $t+1$, where s can be exchanged for ϕ^+ units of the CM good. This leads to the following value functions:

$$\begin{aligned}
W_j^{\theta^m} &= \max_{h_j^{\theta m}, m_j} -h_j^{\theta^m} + v\left(\beta\phi_m^+(1+i_m)m_j/\phi_m\right) \\
&\quad \text{s.t. } m_j = h_j^{\theta m} \\
W_j^{\theta^s} &= \max_{h_j^{\theta^s}, s_j} -h_j^{\theta^s} + \beta U\left(\phi_s^+(1+i_s)s_j/\phi_s\right) \\
&\quad \text{s.t. } s_j = h_j^{\theta^s}.
\end{aligned}$$

Solving the maximization problems and rearranging the first-order conditions yields explicit demand functions for m_j and s_j:

$$\begin{aligned}
m_j &= v'^{-1}\left(\frac{\phi_m/\phi_m^+}{\beta(1+i_m)}\right)\frac{\phi_m/\phi_m^+}{\beta(1+i_m)} \\
s_j &= U'^{-1}\left(\frac{\phi_s/\phi_s^+}{\beta(1+i_s)}\right)\frac{\phi_s/\phi_s^+}{1+i_s}.
\end{aligned}$$

To obtain the aggregate demand functions, I need to consider the shares of consumers who are (i) early consumers holding transaction deposits, (ii) early consumers holding central bank money, (iii) late consumers holding savings deposits, and (iv) late consumers holding central bank money. These shares are given by (i) $\gamma\alpha_d$, (ii) $\gamma(1-\alpha_d)$, (iii) $(1-\gamma)\alpha_\tau$, and (iv) $(1-$

$\gamma)(1-\alpha_\tau)$. Applying the Fisher equation, the following demand functions are obtained:

$$d = \gamma\alpha_d \, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d} \tag{6}$$

$$e^m = \gamma(1-\alpha_d)\, v'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{\beta R_e} \tag{7}$$

$$\tau = (1-\gamma)\alpha_\tau \, U'^{-1}\left(\frac{1}{\beta R_\tau}\right)\frac{1}{R_\tau} \tag{8}$$

$$e^s = (1-\gamma)(1-\alpha_\tau)\, U'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{R_e} \tag{9}$$

d denotes the real amount of transaction deposits and e^m the real amount of central bank money that is demanded by early consumers. On the other hand, τ represents the real amount of savings deposits and e^s the real amount of central bank money that is demanded by late consumers.

2.3 The Entrepreneur's Problem

Since entrepreneurs cannot work nor do they have an endowment, they must rely on borrowing from a banker to acquire the CM good x. The loan demanded by entrepreneur n is denoted as ℓ_n in real terms, which corresponds to ℓ_n/ϕ in nominal amounts. The interest rate on this loan is represented by i_ℓ.

Using the deposits credited to their accounts through the loan issuance, entrepreneurs purchase the CM good in period t from the centralized market. They invest the CM good x and, after one period, receive returns of $f(x)$. Part of the return is then sold in the CM during period $t+1$ to acquire bank deposits and pay back the loan. The remaining return is consumed with linear utility before the entrepreneur dies. Therefore, the entrepreneur's maximization problem is as follows:

$$\max_{\ell_n} \beta f\left(\ell_n\right) - \beta\phi^+\ell_n(1+i_\ell)/\phi.$$

Solving the maximization problem yields

$$f'(\ell_n) = \frac{1 + i_\ell}{\phi/\phi^+}.$$

Applying the Fisher equation and considering the homogeneity of entrepreneurs ($\ell = \ell_n$), we obtain the loan demand equation:

$$R_\ell(\ell) = f'(\ell) \tag{10}$$

$R_\ell(\ell)$ is the gross real return on loans.

2.4 Equilibrium

I restrict the results to a stationary equilibrium.

Definition 1. *A stationary and symmetric equilibrium is defined such that the quantity of transaction deposits $d = Bd_b$, the quantity of savings deposits $\tau = B\tau_b$, the quantity of bank loans $\ell = B\ell_b$, the quantity of reserves held by bankers Be_b, the quantities of public money held by the households e^m and e^s, the interest rate on transaction deposit R_d, the interest rate on savings deposit R_τ, and the interest on loans R_ℓ solve*
(1) the money demand equations (6)-(9),
(2) the entrepreneurs' loan demand (10),
(3) the bankers' first-order conditions (2)-(5).

The equilibrium outcomes crucially depend on whether the reserve requirement constraint is binding or not. Thus, the first objective is to gain insights into the parameter space where the constraint is more likely to be binding. To achieve this, I rearrange the bank's first-order condition for loans (2) and obtain a closed-form solution for λ. This expression allows me to analyze the impact of the exogenous variables on the binding or non-binding nature of the reserve requirement, as formulated in Proposition 3.

Proposition 3. *The reserve requirement constraint is more likely to be binding ($\lambda > 0$) in the productivity parameter A, the number of bankers B and the reserve requirement rate ω (i.e., $\frac{\partial \lambda}{\partial A} > 0$, $\frac{\partial \lambda}{\partial B} > 0$, $\frac{\partial \lambda}{\partial \omega} > 0$). It is more likely to be loose ($\lambda = 0$) in the discount factor β, the shares of consumers* *holding deposits* α_d *and* α_τ *and in the interest rate on reserves* R_{e_b} *(i.e.,* $\frac{\partial \lambda}{\partial \beta} < 0$, $\frac{\partial \lambda}{\partial \alpha_d} < 0$, $\frac{\partial \lambda}{\partial \alpha_\tau} < 0$, $\frac{\partial \lambda}{\partial R_{e_b}} < 0$*). The effect of the production function's concavity parameter* η *and the share of early consumers* γ *is ambiguous (i.e.,* $\frac{\partial \lambda}{\partial \eta} \lesseqgtr 0$, $\frac{\partial \lambda}{\partial \gamma} \lesseqgtr 0$*).*

Proof. A formal proof is available in Appendix A.3. ■

A higher A increases the entrepreneur's productivity. Consequently, the equilibrium interest rate on bank loans will rise, incentivizing bankers to provide relatively more loans compared to holding reserves. Thus, it becomes more likely that the reserve requirement constraint will bind. Regarding B, assuming the reserve requirement constraint is non-binding, an increase in the number of bankers leads to a higher total loan amount due to intensified competition. As a result, banks will decrease their excess reserves, increasing the likelihood that the constraint becomes binding. Moreover, it is straightforward that the reserve requirement constraint is more likely to bind if the minimum reserve requirement ratio is higher.

A higher discount factor β implies that agents are more patient and have a greater desire to save. This increases the demand for deposits, resulting in higher ℓ and lower R_ℓ. A lower interest rate on loans makes holding reserves relatively more attractive, decreasing the likelihood of the constraint binding. The same logic applies to α_d and α_τ since a higher share of deposit holders means an increase in deposits. Lastly, a higher interest rate on reserves R_{e_b} makes it more attractive for a bank to hold reserves rather than loans, making it more likely that the bank will hold voluntary reserves.

Lastly, Proposition 4 examines whether the market outcome can achieve the socially optimal allocation.

Proposition 4. *The socially optimal allocation requires that all relevant interest rates equal the Friedman rule, i.e.,*

$$R_e = R_d = R_\tau = R_\ell = 1/\beta.$$

Given the imperfect competition in the loan market, Proposition 2 implies that $R_\ell > R_\tau \geq R_d$*. Consequently, the central bank can only ensure optimality in either the deposit market or the loan market by adjusting the interest* *rate on reserves, but not in both simultaneously. However, the central bank can always set the interest rate on central bank money to the optimal rate, which implies* $R_e = 1/\beta$.

Proof. A formal proof is available in Appendix A.4. ■

3 Introducing CBDC

A central bank digital currency is a potentially interest-bearing form of public money. For now, I assume that the central bank does not pay interest on the CBDC, making it economically equivalent to cash. I model the introduction of a CBDC as an exogenous shift between private and public money.

It is important to emphasize that I do not explicitly model the share of people holding CBDC or determine the optimal CBDC quantity. Instead, my analysis aims to understand the impact of a CBDC on bank intermediation if it triggers a shift between private and public money. The baseline scenario assumes that CBDC leads to a crowding-out effect, with people shifting from private to public money. However, the analysis can be generalized to explore a shift from public to private money. An advantage of this approach is that it allows me to closely match the calibration in Section 4 to the model, as the concurrent coexistence of the two forms of money remains straightforward even when interest rates vary.

In the model, an exogenous shift between private and public money is represented by changes in the shares of early consumers α_d who hold transaction deposits and late consumers α_τ who hold savings deposits. The former reflects a change in the medium of exchange role of money, while the latter represents a shock to the savings role of money. If both transaction and savings deposits are simultaneously crowded out in favor of public money, I denote this as a decrease in α. The early and late consumers' real holdings of public money are still denoted as e^m and e^s respectively, but they now include both cash and CBDC. The effects on bank intermediation depend crucially on whether the constraint is binding or not. Below, I differentiate between these two cases and discuss the effects in detail.

3.1 A Loose Reserve Requirement Constraint

In the case of a loose minimum reserve requirement constraint ($\lambda = 0$), the effects are straightforward and discussed in Proposition 5. The first major result is that a crowding out from private to public money has no effect on bank lending if banks hold excess reserves.

Proposition 5. *If the reserve requirement constraint is loose ($\lambda = 0$), an exogenous shift towards public money (i) has no effect on the deposit interest rates R_d and R_τ, (ii) decreases the amount of deposits held, (iii) increases public money holdings, (iv) has no effect on loans ℓ or on the loan interest rate R_ℓ, (v) reduces the excess reserves held by bankers by the same amount as deposits are withdrawn, (vi) increases total money demand and GDP if the interest rate on deposits is smaller than the interest rate on public money, and (vii) has no effect on the entrepreneurs' and bankers' profit.*

Proof. A formal proof is available in Appendix A.5. ■

Below, I provide the intuition behind the results in Proposition 5. The reasoning applies qualitatively to shifts concerning early consumers, late consumers, or both. Thus, I focus on a shift of early consumers from private to public money, represented by a decrease in α_d.

From Proposition 2, we know that $R_{e_b} = R_d = R_\tau$ if $\lambda = 0$. Consequently, when there is a change in the deposit base, the bank can easily adjust its excess reserve holdings without any impact on profits. The presence of excess reserve holdings indicates that the bank is already holding the optimal loan amount, and thus, there is no incentive for the bank to make any adjustments to it.

Regarding total money demand, it is evident that an outflow of transaction deposits to CBDC will decrease transaction deposits d and increase the amount of public money held as medium of exchange e^m. The magnitude of these changes depends on the relative size of the interest rate of transaction deposits R_d to the interest rate on public money R_e. If $R_d > R_e$, then a consumer who switches from deposit to CBDC will hold a smaller amount of CBDC than the previously held deposits, due to the lower interest rate. As a result, total money demand will decrease.

Concerning GDP (Y), which includes total CM and DM production, the line of argument is similar. If $R_d > R_e$, a consumer switching from deposits to CBDC will lower production in the CM, as their money demand decreases. This effect, in turn, impacts DM production, where the producer produces less due to the early consumer arriving with less money. Output from entrepreneurs remains unchanged, as the total loan amount is not affected. Thus, the overall effect is as follows:

$$\frac{\partial(d+\tau+e^m+e^s)}{\partial\alpha_d}, \frac{\partial Y}{\partial\alpha_d} \begin{cases} > 0 & \text{if } R_d > R_e \\ = 0 & \text{if } R_d = R_e \\ < 0 & \text{if } R_d < R_e \end{cases}$$

The consumption of an individual consumer is directly related to their money demand. Consumption C of an early consumer j who holds either transaction deposits d or public money e^m is defined by:

$$\begin{aligned} C_j^d &= \beta R_d d_j = \beta R_d v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d} = v'^{-1}\left(\frac{1}{\beta R_d}\right) \\ C_j^{e^m} &= \beta R_e e_j^m = \beta R_e v'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{\beta R_e} = v'^{-1}\left(\frac{1}{\beta R_e}\right). \end{aligned}$$

If a consumer switches from deposits to public money, their consumption increases if $R_e > R_d$, and vice versa.

3.2 A Binding Reserve Requirement Constraint

The results for a binding minimum reserve requirement constraint ($\lambda > 0$) are summarized in Proposition 6. The second major finding is that bank lending decreases when there is an exogenous shift from private to public money upon the introduction of a CBDC. Moreover, the crowding out effect is stronger when agents use the CBDC for both payments and savings.

Proposition 6. *If the reserve requirement constraint is binding ($\lambda > 0$), an exogenous shift of early (late) consumers from private to public money (i) increases the interest rates on transaction deposits R_d and on savings deposits R_τ, where R_τ increases by more than R_d, (ii) decreases the demand* *for transaction (savings) deposits on the extensive margin, (iii) increases the demand for transaction and savings deposits on the intensive margin, (iv) increases the demand for public money, (v) decreases the total loan amount ℓ and increases the interest rate on loans R_ℓ, (vi) has an ambiguous effect on the total money demand and GDP, (vii) decreases the profit of the entrepreneurs and the bankers (viii) increases the consumption of consumers holding deposits who do not switch to public money.*

Proof. A formal proof is available in Appendix A.6. ■

Below, I discuss the intuition behind the results in Proposition 6 for an exogenous shift of early consumers from private to public money (α_d decreases). When the reserve requirement constraint is binding, bankers would like to issue more loans, but they are restricted by the binding constraint. In the case of an exogenous shift from private to public money, some early consumers switch from transaction deposits to CBDC, leading to a decrease in the funding available to bankers. As a result, bankers can provide even fewer loans to entrepreneurs.

However, due to the banks' market power in the loan market, bankers can react to the deposit outflow. This results in higher interest rates on deposits, which attracts deposits on the intensive margin and counteracts the crowding out effect. In fact, both the interest rates on transaction deposits R_d and savings deposits R_τ increase. Thus, both the early consumers who still hold deposits and the late consumers increase their deposit holdings. The increase in R_τ is greater than the increase in R_d because for each additional unit of transaction deposits, a share ω needs to be diverted into reserves, whereas each additional unit of savings deposits can be used one-to-one to provide loans.

Consequently, the total deposit demand decreases in the extensive margin but increases in the intensive margin, resulting in an overall negative but dampened effect on bank lending. As ℓ decreases, the interest rate R_ℓ on loans increases because the marginal product of investment becomes higher for a lower overall loan issuance.

When there is a simultaneous shift in the choice of both the payment and the savings vehicle, we observe outflows from both transaction deposits and savings deposits towards CBDC. Consequently, the reduction in bank lending will be more pronounced compared to the case with only a shift from transaction deposits to CBDC.

The impact on total money demand, whether it increases or decreases, is ambiguous and primarily depends on the relative sizes of the interest rates on deposits and public money R_e. Assume that before the exogenous shift, the interest rate on transaction deposits and public money is equal ($R_e = R_d$). In this case, consumers who switch to CBDC will hold the same amount of CBDC as they previously held in deposits. However, consumers who still hold deposits will increase their deposit holdings due to the higher interest rates on deposits. As a result, total money demand will increase.

If $R_e > R_d$, total money demand will rise even more because consumers switching from deposits to CBDC will demand a higher amount of public money than they previously held in deposits. On the other hand, in a scenario where the interest rate on deposits is higher than the interest rate on public money ($R_d > R_e$), consumers switching from deposits to CBDC will demand a lower amount of public money compared to what they held in deposits before making the switch. However, those who continue to hold deposits will have a higher money demand due to the increased interest rates on deposits. Whether the total money demand increases or decreases depends on which of these two effects is stronger.

The impact of a shift from private to public money on GDP involves two non-linear effects due to the non-linearities in the utility and production functions. First, the output of entrepreneurs decreases as the total amount of loans is reduced. Second, total money demand can either increase or decrease. If total money demand decreases, consumers and producers work less and GDP will inevitably contract. However, an increase in total money demand increases CM and DM production. As a result, the net effect on GDP remains ambiguous and dependent on the relative size of these opposing forces.

When a consumer switches from private to public money, their consumption increases if the interest rate on public money is higher than the interest rate on deposits ($R_e > R_d$). Moreover, consumers who do not switch and continue to hold deposits will also increase their consumption due to the higher interest rate on deposits. The results are summarized in Table 1.

	R_d	R_τ	d (e)	d (i)	τ (e)	τ (i)	ℓ	$e^m + e^s$	C_j^d	C_j^τ
$\alpha_d \downarrow$	$\uparrow$	$\uparrow$	$\downarrow\downarrow$	$\uparrow$	$-$	$\uparrow$	$\downarrow$	$\uparrow\uparrow$	$\uparrow$	$\uparrow$
$\alpha_\tau \downarrow$	$\uparrow$	$\uparrow$	$-$	$\uparrow$	$\downarrow\downarrow$	$\uparrow$	$\downarrow$	$\uparrow\uparrow$	$\uparrow$	$\uparrow$
$\alpha \downarrow$	$\uparrow$	$\uparrow$	$\downarrow\downarrow$	$\uparrow$	$\downarrow\downarrow$	$\uparrow$	$\downarrow\downarrow$	$\uparrow\uparrow$	$\uparrow$	$\uparrow$

Table 1: Effects of an exogenous shift from private to public money upon the introduction of a CBDC. $\alpha_d \downarrow$: CBDC only used as a payment vehicle. $\alpha_\tau \downarrow$: CBDC only used as a savings vehicle. $\alpha \downarrow$: CBDC used as both payment and savings vehicle. R_d: Interest rate on transaction deposits, R_τ: Interest rate on savings deposits, d: Transaction deposits amount, τ: Savings deposits amount, ℓ: Loan amount, $e^m + e^s$: Total amount of public money, C_j^d : Consumption of the consumer holding transaction deposits, C_j^τ: Consumption of the consumer holding savings deposits. (e): Extensive margin effect. (i): Intensive margin effect.

4 Calibration

To quantify the results, I calibrate the model using data from the US economy between 1987 and 2006.[8] This time period is chosen to consider a situation without excess reserves, allowing me to match the model to the scenario where the minimum reserve requirement is binding. As demonstrated in Section 3, in an excess reserve regime, a shift between private and public money has no impact on bank intermediation. However, the objective of this paper is to determine the magnitude of the effects on banks when a crowding out of deposits cannot be absorbed by a reduction in the reserve position. Hence, it is crucial to conduct the calibration using a period with a binding constraint. Throughout Section 4, I assume that the CBDC is non-interest bearing ($i_e = 0$).

Furthermore, I extend the model described in Section 3 in two dimensions. First, I introduce deposit handling costs $c_d \geq 0$ and $c_\tau \geq 0$ per unit of transaction deposits and savings deposits, respectively. Incorporating these handling costs does not alter the analytical outcomes, except that it permits

[8]For a detailed description of the data used in the calibration, refer to Appendix C.

the deposit rates to be lower than the policy rate, which simplifies matching the model to the empirical data.

The second modification relates to the shares of early consumers holding transaction deposits (α_d) and late consumers holding savings deposits (α_τ). In Section 3, I treat these shares as exogenous to maintain model tractability and obtain analytical results. However, recognizing the critical role of these shares in determining the results, I endogenize α_d and α_τ by defining them as functions of the interest rate spread between public money and transaction and savings deposits, respectively.

The rationale behind this endogenization is that an increase in the interest rate spread between private money and public money would lead to a rise in the share of people holding private money (α increases). This relationship is also evident in the data when examining cross-county variation, as I discuss in greater detail below. However, I want to emphasize that this is a reduced form approach, and there may be other contributing factors to the observed relationship that the model does not account for. Nevertheless, considering the central role of the private and public money shares and their interest rates in the model, this extension appears natural and should enhance the calibration's validity.

When endogenizing the shares, most of the results discussed in Section 3 remain qualitatively similar. However, an interesting new channel emerges. As previously mentioned, an exogenous shift from private to public money results in higher deposit interest rates. What is new is that the higher rates trigger some people to switch in the other direction from public to private money, effectively offsetting some of the outflow on the extensive margin. This phenomenon is not observed when the shares are exogenous. I will discuss this aspect in more detail below, but first, I will explain how I match other parameters in the model.

The parameters on the gross real interest rate on cash (R_e), the gross real interest rate on reserves (R_{e_b}) and the minimum reserve requirement ratio (ω) can be matched to their data counterparts. In my model, the interest rate on reserves serves as the policy rate. Therefore, I approximate it using the Federal Funds Rate (FFR) for the calibrated time period and adjust it for inflation expectations measured by the Federal Reserve Bank of Cleveland. The real interest rate on cash is obtained by subtracting inflation expectations from the nominal interest rate of zero for cash. Both parameters are then averaged over the entire time period. The minimum reserve requirement rate is set to 10%, consistent with the US regulation during the specified time period. Additionally, the discount factor (β) is selected to be consistent with the existing literature. Table 2 shows an overview of the directly matched parameters.

Par.	Description	Value	Notes
β	Discount factor	0.98	Consistent with literature
r_{e_b}	Net real IOR	1.7%	FFR - Π_e
r_e	Net real IOC	-3.0%	- Π_e
ω	Min. reserve requirement	10%	US regulation

Table 2: Directly matched parameters. IOR: Interest rate on reserves. IOC: Interest rate on cash. Π_e: Expected inflation rate.

To calibrate the remaining parameters, I require data on cash, deposit and loan quantities, deposit and loan interest rates, and GDP. For bank-related data, I utilize the quarterly FDIC call reports, which provide comprehensive information on bank-level balance sheets and income statements. These reports offer details on transaction and savings deposit amounts, interest expenses, loan amounts, and interest income. To determine total deposit quantities, I aggregate the deposit holdings of all banks (see Figure C.1). For interest rates, I follow Chiu et al. (2023) and calculate nominal interest rates by dividing the interest expense and income by the corresponding deposit and loan amounts (see Figure C.2). To calculate the corresponding real interest rates, I use a time series on inflation expectations provided by the Federal Reserve Bank of Cleveland.

Regarding currency holdings, I assume that all 100$-bills are used as savings, while all smaller denominations are utilized for payments. Furthermore, it is well known that a significant portion of US currency is held abroad. I restrict the analysis to currency held within the US by using estimates from Feige (2012) (see Figure C.1). For GDP, I extract the data from FRED. In the model, GDP is defined as the total production in both the CM (consumers' and entrepreneurs' output) and the DM (producers' output).

The consumers' utility follows the CRRA functional form:

$$v(y) = \frac{y^{1-\sigma_M}}{1-\sigma_M}, \quad U(x) = \frac{x^{1-\sigma_S}}{1-\sigma_S}. \tag{11}$$

Both types of consumers have individual preference parameters σ_M and σ_S because σ serves as the key parameter that links individual deposit and money demand to changes in interest rates, which is essential for the model's results. To estimate σ_M and σ_S, I use the semi-elasticity of transaction deposits and savings deposits demand. In the model, this can be derived from equations (6) and (8) and is given by:

$$\frac{\partial \log(d/GDP)}{\partial R_d} = \frac{1/\sigma_M - 1}{R_d}, \tag{12}$$

$$\frac{\partial \log(\tau/GDP)}{\partial R_\tau} = \frac{1/\sigma_S - 1}{R_\tau}. \tag{13}$$

When calculating equations (12) and (13), I assume that changes in the interest rate on deposits do not impact GDP or the shares of people holding private money. For the sake of determining money demand, this assumption seems reasonable. The objective is to match $\beta_M = \partial \log(d/GDP)/\partial R_d$ and $\beta_S = \partial \log(\tau/GDP)/\partial R_\tau$ with the data and then directly infer σ_M and σ_S using equations (12) and (13).

To achieve this, I utilize both time and cross-sectional variation of county-level data, creating a yearly panel spanning from 2001 to 2006. For each county and each time period, I calculate (i) total deposits using data from the FDIC Summary of Deposits, (ii) the average deposit interest rates using FDIC call report data, and (iii) county GDP by using data from the U.S. Bureau of Economic Analysis (BEA). More detailed explanations on how I derive these variables are provided in Appendix C. Figure C.3 depicts the county-level variation of the parameters of interest. Subsequently, I perform a panel regression incorporating time and county fixed effects to obtain estimates for β_M and β_S. The resulting values for σ_M and σ_S are displayed in Table 3.

Furthermore, I need to specify how the shares of agents holding transaction deposits α_d and savings deposits α_τ relate to the interest rate spread between private and public money. The rate spread is represented as ε^m for the

Par.	Description	Value
σ_M	Utility parameter θ^M-type	0.136
σ_S	Utility parameter θ^S-type	0.162
$\breve{\alpha}_d$	Polynomial Intercept for Medium of Exchange	0.771
$\hat{\alpha}_d$	Polynomial Slope for Medium of Exchange	7.811
$\breve{\alpha}_\tau$	Polynomial Intercept for Savings Vehicle	0.795
$\hat{\alpha}_\tau$	Polynomial Slope for Savings Vehicle	1.332
α_d	Share of θ^m-types holding transaction deposits	0.873
α_τ	Share of θ^s-types holding savings deposits	0.839

Table 3: Externally calibrated parameters.

medium of exchange and ε^s for the savings vehicle. It is defined by:

$$\varepsilon^m = \frac{(1+i_d)}{(1+i_e)} - 1 = R_d/R_e - 1, \quad \varepsilon^s = \frac{(1+i_\tau)}{(1+i_e)} - 1 = R_\tau/R_e - 1. \tag{14}$$

Moreover, expressions for α_d and α_τ can be derived by using equations (6)-(9):

$$\alpha_d = \frac{\frac{d}{e^m}\left(\frac{R_e}{R_d}\right)^{\frac{1-\sigma_M}{\sigma_M}}}{1+\frac{d}{e^m}\left(\frac{R_e}{R_d}\right)^{\frac{1-\sigma_M}{\sigma_M}}}, \quad \alpha_\tau = \frac{\frac{\tau}{e^s}\left(\frac{R_e}{R_\tau}\right)^{\frac{1-\sigma_S}{\sigma_S}}}{1+\frac{d}{e^s}\left(\frac{R_e}{R_\tau}\right)^{\frac{1-\sigma_S}{\sigma_S}}}. \tag{15}$$

To establish a mapping from ε to α, I again utilize time and cross-sectional data at the county level to capture variations in both the shares and the rate spread. For each county in a specific year, I calculate values for α_d, α_τ, ε^m and ε^s by using county-level data on deposits (d and τ), cash holdings (e^m and e^s) and interest rates (R_d, R_τ and R_e).

Subsequently, I fit a first-order polynomial through the data points of a specific year. Thus, the mapping from ε to α is defined by the following polynomials,

$$\alpha_d(\varepsilon^m) = \breve{\alpha}_d + \hat{\alpha}_d\varepsilon^m, \quad \alpha_\tau(\varepsilon^s) = \breve{\alpha}_\tau + \hat{\alpha}_\tau\varepsilon^s, \tag{16}$$

where $\breve{\alpha}$ represents the intercept and $\hat{\alpha}$ the slope parameter. The former determines the level of private money holdings. The latter defines the extent to which agents switch between private and public money in response to changes in the interest rate spread.

Par.	Description	Value	Target	Model	Data
γ	Fraction of early consumers	0.448	Fraction deposits to cash: d/τ	0.75	0.75
B	Number of bankers	30	Rate spread $R_\ell - R_d$	0.07	0.07
η	Production function parameter	0.309	Semi-Elasticity of Loan Demand $\partial(\ell/GDP)/\partial R_\ell$	-0.27	-0.27
A	TFP	2.637	Deposit demand $(d+\tau)/GDP$	0.21	0.23
c_d	Transaction deposit handling cost	0.045	Rate spread $R_d - R_{e_b}$	-0.035	-0.035
c_τ	Savings deposit handling cost	0.027	Rate spread $R_\tau - R_{e_b}$	-0.015	-0.015

Table 4: Jointly calibrated parameters.

I repeat this procedure for each year from 1994 to 2006. Subsequently, I calculate the average slope parameter across these years to estimate the agent's switching sensitivity in the steady state. Furthermore, I determine the intercept such that equations (15) hold when inserting the steady-state values for the asset holdings and interest rates. The resulting parameters are illustrated in Table 3. The larger slope parameter for transaction deposits ($\hat{\alpha}_d$) indicates that agents demonstrate greater sensitivity to changes in interest rates for transaction deposits compared to savings deposits.

Lastly, there remain six parameters to be calibrated: the fraction of early consumers (γ), the total factor productivity (A), the concavity parameter in the production function (η), the number of bankers (B), and the deposit handling costs (c_d and c_τ). These six parameters are jointly calibrated to match six target moments from the data: the fraction of transaction to savings deposits d/τ, the interest rate spreads $R_\ell - R_d$, $R_d - R_e$, and $R_\tau - R_{e_b}$, deposit demand $(d+\tau)/GDP$, and finally the semi-elasticity of loan demand $\partial(\ell/GDP)/\partial R_\ell$. For the loan semi-elasticity, I utilize bank-level variation to establish a panel covering the years 1987-2006. Subsequently, I conduct a panel regression with time and bank fixed effects to determine the semi-elasticity parameter. Additional details are available in Appendix C. Table 4 presents an overview of the parameters and the model fit.

The model closely aligns with the calibration targets, although there is a slight discrepancy in deposit demand, which appears slightly lower in the model compared to the data. Furthermore, I assess the calibration's validity by examining specific moments that were not directly targeted and report the findings in Table 5. The model manages to successfully match these additional moments as well, except for money demand, which is smaller in the model compared to the data. To further explore the robustness of the calibration, I conduct a parameter sensitivity analysis and present the outcomes for a model version featuring imperfect competition in both the loan and deposit markets in Appendix D.

Parameters	Description	Model	Data
e^m/e^s	Fraction cash holdings	0.62	0.64
e^m/d	Fraction cash to deposits (MoE)	0.13	0.14
e^s/τ	Fraction cash to deposits (Savings)	0.16	0.16
R_d	Transaction deposit interest rate	0.98	0.98
R_τ	Savings deposit interest rate	1.002	1.002
R_ℓ	Loan interest rate	1.053	1.052
$(e^m + e^s)/GDP$	Money demand	0.031	0.034

Table 5: Not directly targeted moments.

4.1 Introducing CBDC

As in Section 3, the introduction of a CBDC is modeled as an exogenous shift between private and public money. This shift is represented by changes in the intercept parameters $\breve{\alpha}_d$ and $\breve{\alpha}_\tau$. A reduction in these intercepts indicates that a smaller share of early (late) consumers opt for transaction (savings) deposits and instead hold public money, such as CBDC. For instance, a 10% reduction in $\breve{\alpha}_d$ ($\breve{\alpha}_\tau$) implies that 10% of the agents holding transaction (savings) deposits transition from private to public money. Additionally, I investigate a scenario where individuals switch both the medium of exchange and the savings vehicle, which is represented by a simultaneous shift in the two intercepts.

The calibration's objective is to quantify the impacts of a particular outflow magnitude on bank lending, asset demand, and interest rates. I differentiate between the consequences of an exogenous shift exclusively in the medium of exchange, solely in the savings vehicle, or a simultaneous shift in both. Furthermore, I analyze the impact in terms of the intensive and extensive margin effects on deposits.

In Table 6, I present the results for a 10% outflow from private to public money, as well as a 5% inflow, for comparative purposes. The effects on the endogenous variables exhibit a nearly linear shape for reasonably sized shifts, which is why I focus on these illustrative examples. The first row shows the calibrated equilibrium, with asset levels indexed such that the loan amount in the calibrated equilibrium is set to 100. The observed effects align with the results from Section 3.2. Specifically, following an exogenous shift from private to public money, there is an increase in the interest rates for transaction deposits (i_d), savings deposits (i_τ) and loans (i_ℓ), a decrease in both the quantities of loans and total deposits, and an increase in holdings of public money.

When 10% of agents exclusively transition the medium of exchange from private to public money, transaction deposits experience a decrease of 4.4%, while savings deposits increase by 2.2%. In the case where 10% of agents solely switch the savings vehicle, transaction deposits rise by 7.4%, while savings deposits decline by 6.2%. Lastly, when there is a simultaneous shift in both the medium of exchange and the savings vehicle, transaction deposits increase by 3.1%, whereas savings deposits decrease by 4.0%. This result is attributed to agents reacting more sensitively to changes in the interest rate of transaction deposits compared to savings deposits, allowing banks to mitigate the outflow more effectively through transaction deposits. The interpretation for an inflow from public to private money, as shown in the lower section of Table 6, is inverted but analogous.

Moreover, the third major result follows. The impact on bank lending is more pronounced when individuals shift their holdings away from savings deposits compared to an exclusive outflow from transaction deposits. In the case where 10% of agents who hold transaction deposits transition to CBDC, bank lending experiences a reduction of 0.4%. This reduction escalates to 0.7% when there is an outflow solely from savings deposits. Approximately 60% of this amplified reduction can be attributed to the higher proportion

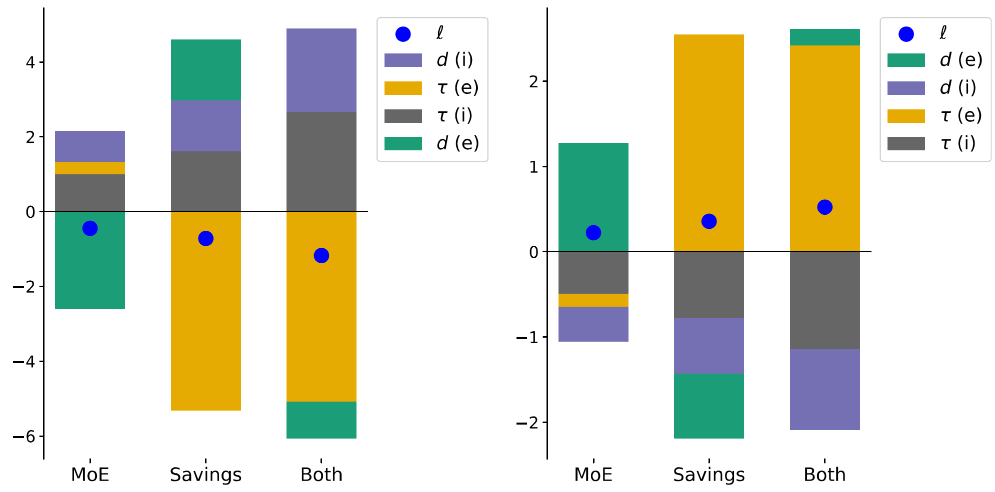
		ℓ	d	τ	e_m	e_s	i_d	i_τ	i_ℓ
Eq		100.0	44.6	59.9	6.0	9.7	1.3	3.3	8.6
-10%	MoE	99.6	42.6	61.2	8.5	9.5	1.6	3.6	8.9
		(-0.4%)	(-4.4%)	(2.2%)	(42.2%)	(-2.7%)			
	Savings	99.3	47.9	56.2	4.2	14.1	1.8	3.8	9.1
		(-0.7%)	(7.4%)	(-6.2%)	(-29.3%)	(45.0%)			
	Both	98.8	46.0	57.5	6.8	13.8	2.1	4.2	9.5
		(-1.2%)	(3.1%)	(-4.0%)	(12.6%)	(42.2%)			
5%	MoE	100.2	45.5	59.2	4.7	9.9	1.2	3.1	8.4
		(0.2%)	(2.2%)	(-1.1%)	(-21.2%)	(1.4%)			
	Savings	100.4	43.0	61.7	6.9	7.5	1.1	3.0	8.3
		(0.4%)	(-3.5%)	(3.0%)	(14.3%)	(-22.5%)			
	Both	100.5	43.7	61.2	5.9	7.6	1.0	2.9	8.2
		(0.5%)	(-1.9%)	(2.1%)	(-1.8%)	(-21.5%)			

Table 6: Impacts of an exogenous shift between private and public money on asset holdings and interest rates. The top row shows the calibrated equilibrium. The asset levels are indexed such that the loan amount equals 100. The middle section depicts a 10% shift from private to public money, while the lower section illustrates a 5% inflow. I depict shifts solely related to the medium of exchange (MoE), solely to the savings vehicles, or both simultaneously. ℓ: Total loan amount, d: Amount of transaction deposits, τ: Amount of savings deposits, e^m: Central bank money held as payment vehicle, e^s: Central bank money held as savings vehicle, i_d: Nominal net interest rate on transaction deposits, i_τ: Nominal net interest rate on savings deposits, i_ℓ: Nominal net interest rate on loans.

of savings deposits on the bank's balance sheet. The remaining 40% can be attributed to the different reactions of agents to changes in the interest rates of the medium of exchange and the savings vehicle, as indicated by the different parameter values of $\hat{\alpha}_d$ and $\hat{\alpha}_\tau$. Finally, if 10% of the agents withdraw from both transaction and savings deposits, bank lending experiences a decrease of 1.2%.

Additionally, I aim to provide a more detailed breakdown of the impact on loans, considering both the extensive and intensive margin effects on deposits. This breakdown is depicted in Figure 1. The scaling is based on the values presented in Table 6, where loans are indexed to 100 in equilibrium.

When 10% of agents who hold deposits switch to CBDC, there is a realized extensive margin outflow of approximately 6.2% observed for transaction deposits (d) when focusing solely on the shift in the medium of exchange. Similarly, there is an approximately 8.6% outflow for savings deposits (τ)



(a) 10% Outflow (b) 5% Inflow

Figure 1: Decomposition of exogenous shift between private and public money in extensive and intensive margin effects. Depicted are an outflow from private to public and an inflow from public to private money. Illustrated are shifts involving only the medium of exchange (MoE), only the savings vehicles, or both. ℓ: Loans, d: Transaction deposits, τ: Savings deposits, (e) = Extensive margin effect, (i) = Intensive margin effect.

when considering a shift in the savings vehicle. The realized extensive margin outflows are smaller than the initial 10% exogenous shift because the higher interest rates on deposits incentivize some agents who hold public money to shift to deposits due to the endogenized α-shares. Notably, the impact is smaller in the case of the medium of exchange, as agents are more responsive to the heightened interest rate on the payment vehicle compared to the savings vehicle. Consequently, the bank can more effectively offset the outflow through higher interest rates for the medium of exchange.

Moreover, there exist counteracting effects to this outflow. In the case of a shift pertaining to the medium of exchange, the heightened interest rates on both transaction and savings deposits incentivize agents to increase their holdings of transaction deposits (d) by 1.9% and savings deposits (τ) by 1.7% on the intensive margin. Furthermore, there exists a small extensive margin effect, where agents switch from public money to savings deposits (τ) due to the increased interest rate. This results in a 0.5% rise in the share of individuals holding savings deposits. A shift in the savings vehicle yields a 3% increase in transaction deposits and a 2.7% increase in savings deposits on the intensive margin. Additionally, the share of agents holding transaction deposits rises by approximately 4.3%.

Finally, in the event that both 10% of agents holding transaction and savings deposits switch to CBDC, the resulting net proportion of agents shifting away from transaction deposits is 1.8%, while for savings deposits, it is 8.1%. However, these effects are partially offset by the intensive margin. Specifically, individual holdings of transaction deposits increase by approximately 5%, whereas individual savings deposit holdings rise by around 4.4%.

4.2 Interest-Bearing CBDC

Furthermore, I examine the implications of an interest-bearing CBDC for bank intermediation.[9] Up until now, I have exclusively considered the scenario where the interest rate on CBDC is equivalent to the interest rate on cash ($i_e = 0$). However, I now extend the analysis to encompass the possibility of the central bank offering an interest rate on CBDC. Given that the model does not differentiate between cash and CBDC and focuses on the total stock of public money, this analysis contemplates an equilibrium in which agents exclusively hold CBDC with the potential for the central bank to offer an interest rate.

Figure 2 illustrates the impact of a change in the CBDC rate on various variables, including deposit and loan interest rates, asset demand, loans, the proportion of agents holding deposits (α_d and α_τ), and welfare. An interest-bearing CBDC enhances the attractiveness of public money in comparison to deposits. This prompts some agents to transition towards public money, leading to a reduction in the shares of agents holding deposits (α_d and α_τ), as illustrated in the lower left graph. However, bankers react to this outflow, which results in higher interest rates on deposits, attracting more deposits and mitigating the crowding out. As shown in the upper right graph, the overall effect on bank lending remains relatively modest, with an interest rate equal to the Friedman rule resulting in a decrease of approximately

[9] In addition to the interest rate on public money, the monetary authority possesses two other policy tools: setting the interest rate on reserves i_{e_b} and choosing the minimum reserve requirement rate ω. An examination of these tools is available in Appendix E.2.

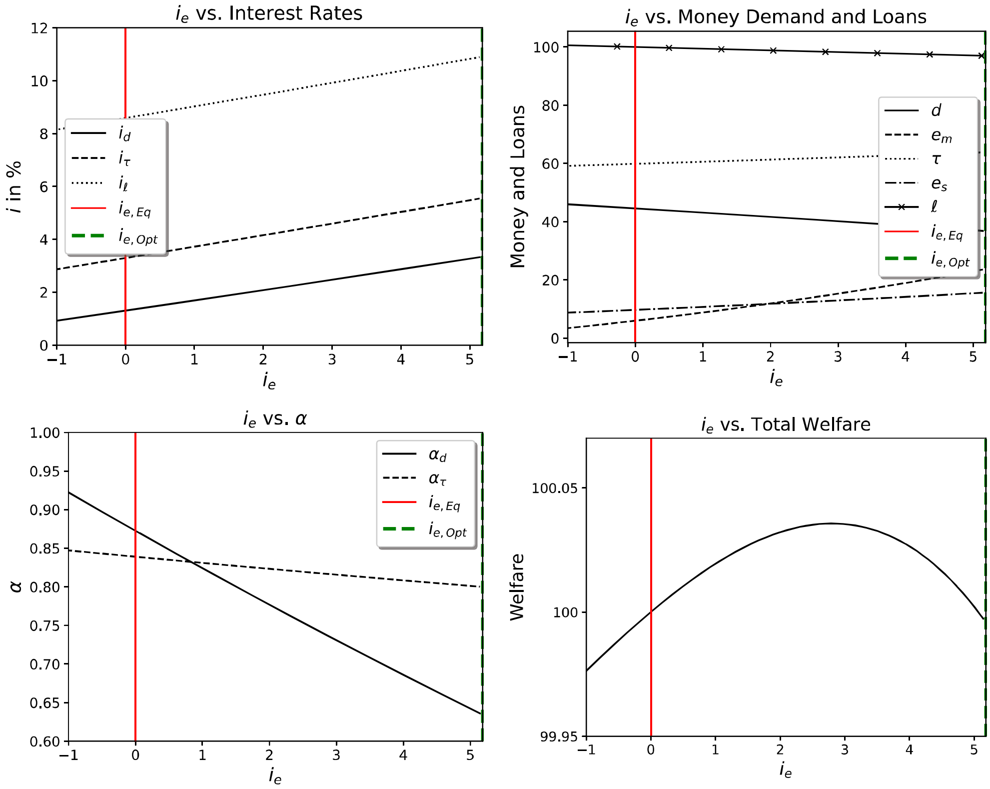


Figure 2: Effects of a change in the interest rate on CBDC on the deposit and loan interest rates, assets and loan quantities, the fraction of agents holding deposits, and total welfare. The loan and assets curves are indexed such that the loan amount in the calibrated equilibrium is equal to 100. Also total welfare is indexed to 100. $i_{e,Eq}$ donates the interest rate at the calibrated equilibrium. $i_{e,Opt}$ is the optimal interest rate on CBDC defined in Proposition 4 ($R_{e,Opt} = 1/\beta$). i_d: Rate on transaction deposits, i_τ: Rate on savings deposits, i_ℓ: Rate on loans, d: Transaction deposits amount, τ: Savings deposits amount, e^m: Central bank money held as payment vehicle, e^s: Central bank money held as savings vehicle, ℓ: Total loan amount, α_d: Fraction of early consumers holding transaction deposits, α_τ: Fraction of late consumers holding savings deposits.

3.0% in bank lending.[10]

In the upper right graph, we see contrasting impacts on transaction deposits and savings deposits. Specifically, as the CBDC rate increases, transaction deposits experience a decline while savings deposits increase. This dynamic arises from the interplay of two opposing effects. On one hand, some con-

[10]The Friedman rule is the optimal interest rate on CBDC, as described in Proposition 4 ($R_{e,Opt} = 1/\beta$, or equivalently, $i_{e,Opt} = 100(\mu/\beta - 1) = 5.18\%$).

sumers shift from deposits to CBDC due to the more attractive CBDC rate. On the other hand, those who continue to hold deposits exhibit heightened demand for deposits on the intensive margin, driven by the concurrent increase in the interest rate on deposits. For early consumers holding transaction deposits, the former effect is stronger, as demonstrated in the bottom left graph. This is attributed to the greater sensitivity of early consumers towards fluctuations in the interest rate spread between private and public money. Conversely, in the case of savings deposits, the latter effect dominates, leading to a scenario where transaction deposits decrease and savings deposits increase in response to an upward adjustment in the interest rate on CBDC.

Lastly, the lower right graph depicts total welfare (W_{Tot}), defined as the cumulative utilities of all agents. The welfare curve follows a bell-shaped pattern, reaching its maximum at a CBDC rate of $i_e = 2.8\%$ ($r_e = -0.3\%$), where welfare is only slightly elevated by 0.04% compared to the calibrated equilibrium.[11] Decomposing total welfare and analyzing the individual agents' utilities offers more insights into the driving forces of this phenomenon. As the loan amount remains relatively stable, the utilities of bankers and entrepreneurs experience only slight declines in response to increases in i_e. Conversely, the effects on consumers and producers are more pronounced. On one hand, consumers benefit from higher interest rates on CBDC and deposits, resulting in a higher utility. On the other hand, the CBDC interest rate is financed through a tax that is convex in the CBDC interest rate and is imposed on producers. Consequently, at higher values of i_e, the producers' disutility outweighs the additional utility gained by consumers. Thus, we can conclude that an interest-bearing CBDC can yield welfare gains, as long as the interest rate is not excessively high.

[11] The Friedman rule does not maximize overall welfare in this context, because the private and public money holdings are modeled with the α-shares. Consequently, the principle of rate-of-return dominance does not apply here and there will always be individuals holding deposits which pay an interest rate below the Friedman rule.

5 Conclusion

This study examines the impact of introducing a central bank digital currency (CBDC) on bank intermediation within a general equilibrium model that considers agents' money needs for both payments and savings. This allows us to analyze how the effects of a CBDC differ depending on its use solely as a medium of exchange or also as a savings instrument.

The introduction of a CBDC is modeled as an exogenous shift between private and public money. I find that a CBDC has no impact on bank lending if banks hold excess reserves. However, if reserves are scarce, a shift from private to public money negatively affects bank lending. This effect is stronger when a CBDC is used not only as a medium of exchange but also as a savings vehicle.

In a calibration, I find that the adverse impact on bank lending nearly doubles when a CBDC is employed as a savings instrument and triples when it serves both payment and savings purposes, in comparison to being exclusively utilized as a medium of exchange. Additionally, I show that an interest-bearing CBDC can enhance welfare. However, this positive effect reverses if the interest rate is set too high.

My findings yield two key policy implications. First, a central bank should design a CBDC exclusively as a payment instrument if it intends to mitigate adverse impacts on bank lending. Secondly, an interest-bearing CBDC can be welfare enhancing by paying holders of public money a more favorable interest rate.

Finally, there are several potential extensions to enhance the depth of analysis in the model presented in this paper. One avenue worth exploring is a thorough microfoundation of the distribution of agents holding private and public money, which could provide insights into the dynamics of transitions between these two forms of currency. Despite this, the approach I take allows for numerous analytical results and a comprehensive understanding of the magnitudes of the impact on bank lending.

Furthermore, in place of the minimum reserve requirement constraint, an alternative framework could be developed that embraces a broader perspective centered around a liquidity coverage ratio – a concept that holds greater relevance in current times. However, within the model, the basis for the existence of two deposit types is rooted in the minimum reserve requirement. Therefore, this constraint is crucial to my analysis. Nonetheless, exploring broader liquidity constraints remains an intriguing avenue for further consideration. Lastly, the current model does not account for the allocation of the central bank's assets. Introducing the possibility of productive investments or lending to banks could open up intriguing avenues for future research.

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Appendix A Proofs

A.1 Proposition 1

Proof. First, I analyze a situation where a banker exclusively offers liquid transaction deposits that can be transferred in the DM. I will then demonstrate that in this scenario, the bank can increase its profit by introducing savings deposits τ.

In this setup, the late consumer will also hold transaction deposits since it is the only type of deposits available. The profit function of bank b when only offering transaction deposits is $\ell_b^* R_\ell(\ell) + e_b^* R_{e_b} - d_b^* R_d$ subject to the reserve requirement constraint $e_b^* = \omega d_b^*$ and the balance sheet identity $\ell_b^* + e_b^* = d_b^*$. $R = 1 + r$ denotes the gross real interest rate. By replacing the two constraints within the profit function, we can obtain a simple expression for the bank's profit Π.

$$\Pi_b^* = \ell_b^* \left(R_\ell(\ell^*) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} R_d \right)$$

I will now demonstrate that this cannot be an equilibrium because the bank can increase its profit by offering a marginal amount of savings deposits, denoted as $\tilde{\tau}$, which will be held by a late consumer switching from transaction to savings deposits.

The new profit function is $\tilde{\ell}_b R_\ell(\ell) + \tilde{e}_b R_{e_b} - \tilde{d}_b R_d - \tilde{\tau}_b R_\tau$. The bank now holds $\tilde{d}_b = d_b^* - \tilde{\tau}_b$ of transaction deposits. The required amount of reserves the bank has to hold is reduced to $\tilde{e}_b = \omega(d_b^* - \tilde{\tau}_b)$ and the amount of loans it can issue is increased to $\tilde{\ell}_b = \ell_b^* + \omega\tilde{\tau}_b$. Furthermore, the new balance sheet identity is $\tilde{\ell}_b + \tilde{e}_b = \tilde{d}_b + \tilde{\tau}_b$. This yields the following profit function.

$$\begin{aligned}\tilde{\Pi}_b =& \ell_b^* \left(R_\ell(\ell^* + \omega\tilde{\tau}_b) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} R_d \right) \\ &+ \tilde{\tau}_b \left(\omega R_\ell(\ell^* + \omega\tilde{\tau}_b) - \omega R_{e_b} + R_d - R_\tau \right)\end{aligned}$$

Next, I conjecture and verify that $\tilde{\Pi}_b > \Pi_b^*$, which means that a bank can increase its profit by offering savings deposits.

$$\begin{aligned}\tilde{\Pi}_b - \Pi_b^* =& \ell_b^* \left(R_\ell(\ell^* + \omega\tilde{\tau}_b) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} R_d \right) \\ &+ \tilde{\tau}_b \left(\omega R_\ell(\ell^* + \omega\tilde{\tau}_b) - \omega R_{e_b} + R_d - R_\tau \right) \\ &- \ell_b^* \left(R_\ell(\ell^*) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} R_d \right) > 0\end{aligned}$$

Rearrange

$$\ell_b^* \left(R_\ell(\ell^* + \omega\tilde{t}_b) - R_\ell(\ell^*) \right) + \tilde{\tau}_b \left(\omega R_\ell(\ell^* + \omega\tilde{\tau}_b) - \omega R_{e_b} + R_d - R_\tau \right) > 0$$

and then substitute the bank's first-order conditions (3) and (4), which determine R_d and R_τ, respectively.

$$\tilde{\Pi}_b - \Pi_b^* = \ell_b^* \left(R_\ell \left(\ell^* + \omega\tilde{\tau}_b \right) - R_\ell(\ell^*) \right) + \omega\tilde{\tau}_b \left(R_\ell(\ell^* + \omega\tilde{\tau}_b) - R_\tau \right) > 0$$

Since $\frac{\partial R_\ell(\ell)}{\partial \ell} = \frac{\partial^2 A\ell^\eta}{\partial \ell^2} < 0$, the first term is negative. Furthermore, the second term is positive since $R_\ell > R_\tau$, as shown in Proposition 2. The first term represents the loss in earnings due to a lower interest rate on all existing loans $\ell^* b$. The second term corresponds to the additional earnings on the spread between $R\ell$ and R_τ that result from the new loans of size $\omega\tilde{\tau}_b$. If the additional earnings from the new loans exceed the loss on all existing loans, the bank can increase its profit by offering savings deposits.

Next, I replace R_τ with the bank's first-order condition (2).

$$\begin{aligned}\ell_b^* \left(R_\ell \left(\ell^* + \omega\tilde{\tau}_b \right) - R_\ell(\ell^*) \right) + \omega\tilde{\tau}_b \left(-\frac{\partial R_\ell(\ell^* + \omega\tilde{\tau}_b)}{\partial \ell_b} (\ell_b^* + \omega\tilde{\tau}_b) \right) > 0 \\ \frac{R_\ell \left(\ell^* + \omega\tilde{\tau}_b \right) - R_\ell(\ell^*)}{\omega\tilde{\tau}_b} \ell_b^* > \frac{\partial R_\ell(\ell^* + \omega\tilde{\tau}_b)}{\partial \ell_b} (\ell_b^* + \omega\tilde{\tau}_b)\end{aligned}$$

Both sides of the inequality correspond to slopes. Given the functional form $f(\ell) = A\ell^\eta$, which yields $R_\ell(\ell) = A\eta\ell^{\eta-1}$ and using Bernoulli's inequality, I can verify that the above expression holds. Without loss of generality, I assume that there is only one bank ($B = 1$) and hence $\ell^* = \ell_b^*$. For the sake of clarity, I denote $a = \ell_b^*$, $b = \ell_b^* + \omega\tilde{\tau}_b$ and I will proceed with a proof by contradiction. Assume that

$$\begin{aligned} a\frac{A\eta(b)^{\eta-1} - A\eta(a)^{\eta-1}}{b-a} &\leq A\eta(\eta-1)b^{\eta-2}b, \\ \frac{a}{b-a}(1-(a/b)^{\eta-1}) &\leq \eta - 1, \\ 1 - \left(\frac{b}{a}\right)^{1-\eta} &\leq (\eta-1)\left(\frac{b}{a}-1\right). \end{aligned}$$

Denote $x = b/a > 1$ and $\alpha = 1-\eta \in (0,1)$.

$$\begin{aligned} 1 - x^\alpha &\leq \alpha(1-x) \\ \frac{1-x^\alpha}{1-x} &\geq \alpha \end{aligned}$$

By Bernoulli's inequality, we have that for $\alpha \in (0,1)$ and $x > 1$

$$\begin{aligned} x^\alpha &< 1 + (x-1)\alpha, \\ -(x-1)\alpha &< 1 - x^\alpha, \\ \alpha &> \frac{1-x^\alpha}{1-x}. \end{aligned}$$

And hence we have

$$\begin{aligned} &\alpha \leq \frac{1-x^\alpha}{1-x} < \alpha, \\ &\alpha < \alpha, \end{aligned}$$

which is a contradiction. Consequently, the scenario where bankers offer only transaction deposits is not an equilibrium, as a bank can enhance its profit by deviating and offering savings deposits.

Moreover, a bank could choose to offer only savings deposits τ, which means that no θ^m-type holds bank deposits because they cannot be used for payment in the DM. I will now demonstrate, that a bank can increase its profit in this scenario by also offering transaction deposits d. A bank b's profit function when only offering savings deposit τ is

$$\Pi_b^* = \ell_b^* R_\ell(\ell^*) - \ell_b^* R_\tau,$$

where I use the balance sheet identity $\ell_b^* = \tau_b^*$. A bank that deviates and offers an additional marginal amount of transaction deposits denoted as $\tilde{d}_b$ at the market rate R_d has the profit function: $\tilde{\ell}_b R_\ell(\tilde{\ell}) + \tilde{e}_b R_{e_b} - \tau_b^* R_\tau - \tilde{d}_b R_d$. Additionally, we have $\tilde{\ell}_b = \ell_b^* + (1-\omega)\tilde{d}_b$, $\tilde{e}_b = \omega \tilde{d}_b$ and the balance sheet identity $\tilde{\ell}_b + \tilde{e}_b = \tau_b^* + \tilde{d}_b$. The rearranged profit function of the bank is then:

$$\begin{aligned}\tilde{\Pi}_b =& \ell_b^*[R_\ell(\ell^* + (1-\omega)\tilde{d}_b) - R_\tau] \\ &+ \tilde{d}_b[(1-\omega)R_\ell(\ell^* + (1-\omega)\tilde{d}_b) + \omega R_{e_b} - R_d].\end{aligned}$$

Again, I conjecture and verify that $\tilde{\Pi}_b - \Pi_b^* > 0$.

$$\begin{aligned}\tilde{\Pi}_b - \Pi_b^* &> 0, \\ \ell_b^*[R_\ell(\ell^* + (1-\omega)\tilde{d}_b) - R_\ell(\ell^*)] & \\ +\tilde{d}_b[(1-\omega)R_\ell(\ell^* + (1-\omega)\tilde{d}_b) + \omega R_{e_b} - R_d] &> 0, \\ \ell_b^*[R_\ell(\ell^* + (1-\omega)\tilde{d}_b) - R_\ell(\ell^*)] & \\ +\tilde{d}_b(1-\omega)[R_\ell(\ell^* + (1-\omega)\tilde{d}_b) - R_\tau] &> 0.\end{aligned}$$

In the last step, I use the bank's first-order conditions (3) and (4). The resulting inequality is similar to the inequality in the scenario where the bank offers only transaction deposits. It has a similar interpretation as well. The first term is negative and represents the loss on all existing loans because the higher total loan amount yields a lower interest rate. The second term is positive and depicts the gain on the net revenue from the additionally issued loans. Rearranging and using the bank's FOCs yields

$$\frac{R_\ell(\ell^* + (1-\omega)\tilde{d}_b) - R_\ell(\ell^*)}{(1-\omega)\tilde{d}_b}\ell_b^* > \frac{\partial R_\ell(\ell^* + (1-\omega)\tilde{d}_b)}{\partial \ell_b}(\ell_b^* + (1-\omega)\tilde{d}_b)$$

This inequality has the same form as in the scenario where the bank offers only transaction deposits. Once again, using Bernoulli's inequality confirms that, given the assumption on the functional form of the production function, the inequality holds. ■

A.2 Proposition 2

Proof. From the bank's first-order equation (2), we have

$$R_\ell = R_{e_b} + \lambda - \frac{\partial R_\ell(\ell)}{\partial \ell_b}\ell_b.$$

Given the functional form of the production function $f(\ell) = A\ell^\eta$ and the definition of the loan rate from the entrepreneur's problem $R_\ell(\ell) = \eta A \ell^{\eta-1}$, it follows that

$$\frac{\partial R_\ell(\ell)}{\partial \ell_b} = (\eta - 1)\eta A(\ell_b + \sum_{b \neq b'} \ell_{b'})^{\eta-2} < 0$$

since $\eta \in (0, 1)$. Thus, we have $R_\ell > R_{e_b}$ $\forall \lambda \geq 0$. Next, combine the bank's first-order conditions (2) and (4) to get

$$R_\ell = R_\tau - \frac{\partial R_\ell(\ell)}{\partial \ell_b}\ell_b.$$

Because $\dfrac{\partial R_\ell(\ell)}{\partial \ell_b} < 0$ it follows that $R_\ell > R_\tau$.

Given $\omega \in (0, 1)$, it is straightforward from equations (3) and (4), i.e.,

$$R_{e_b} + \lambda(1 - \omega) = R_d,$$
$$R_{e_b} + \lambda = R_\tau,$$

that $R_\tau > R_d > R_{e_b}$ if the reserve requirement constraint is binding ($\lambda > 0$) and that $R_\tau = R_d = R_{e_b}$ if the reserve requirement constraint is non-binding ($\lambda = 0$). ■

A.3 Proposition 3

Proof. First, I will demonstrate how to derive a closed-form solution for λ.

I start from the bank's first-order condition for loans (2) and substitute the entrepreneurs' loan demand $R_\ell(\ell) = f'(\ell)$, the functional form of the production function $f(\ell) = A\ell^\eta$, and $\ell = \ell_b + \sum \ell_b'$.

$$\begin{aligned}
\lambda =& \frac{\partial R_\ell(\ell)}{\partial \ell_b}\ell_b + R_\ell(\ell) - R_{e_b} \\
=& \frac{\partial \eta A(\ell_b + \sum \ell_b')^{\eta-1}}{\partial \ell_b}\ell_b + \eta A\ell^{\eta-1} - R_{e_b} \\
=& (\eta-1)\eta A(\ell_b + \sum \ell_b')^{\eta-2}\ell_b + \eta A\ell^{\eta-1} - R_{e_b} \\
=& (\eta-1)\eta A\ell^{\eta-2}\ell_b + \eta A\ell^{\eta-1} - R_{e_b}
\end{aligned}$$

Since the equilibrium is symmetric, we have $\ell = B\ell_b$ such that:

$$\begin{aligned}
\lambda &= (\eta-1)\eta A\ell^{\eta-2}\ell/B + \eta A\ell^{\eta-1} - R_{e_b} \\
\lambda &= \eta A\ell^{\eta-1}\left(\frac{\eta-1}{B} + 1\right) - R_{e_b}
\end{aligned}$$

Next, I aim to obtain a closed-form expression for ℓ. To do this, I utilize the reserve requirement constraint as defined in the complementary slackness condition in equation (5) and consider the fact that the equilibrium is symmetric.

$$\begin{aligned}
\ell_b &= d_b(1-\omega) + \tau_b \\
\ell &= d(1-\omega) + \tau.
\end{aligned}$$

Then, I use the deposit demand equations (6) and (8) to get

$$\ell = \gamma\alpha_d\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d}(1-\omega) + (1-\gamma)\alpha_\tau\, U'^{-1}\left(\frac{1}{\beta R_\tau}\right)\frac{1}{R_\tau}.$$

Using the bank's FOCs (3) and (4) yields

$$\ell = \gamma\alpha_d\, v'^{-1}\left(\frac{1}{\beta(R_{e_b}+\lambda(1-\omega))}\right)\frac{1}{\beta(R_{e_b}+\lambda(1-\omega))}(1-\omega) + (1-\gamma)\alpha_\tau\, U'^{-1}\left(\frac{1}{\beta(R_{e_b}+\lambda)}\right)\frac{1}{(R_{e_b}+\lambda)}$$

Lastly, I substitute this into the expression above and obtain a closed-form solution for λ.

$$\lambda = \eta A\Big[\gamma\alpha_d\, v'^{-1}\left(\frac{1}{\beta(R_{e_b}+\lambda(1-\omega))}\right)\frac{1}{\beta(R_{e_b}+\lambda(1-\omega))}(1-\omega) + (1-\gamma)\alpha_\tau\, U'^{-1}\left(\frac{1}{\beta(R_{e_b}+\lambda)}\right)\frac{1}{(R_{e_b}+\lambda)}\Big]^{\eta-1}\left(\frac{\eta-1}{B}+1\right) - R_{e_b}$$

Without loss of generality, I use the following functional form of the utility function:

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}.$$

The resulting expression for λ is as follows:

$$\lambda = \eta A\Big(\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\,(R_{e_b}+\lambda(1-\omega))^{\frac{1-\sigma}{\sigma}}\,(1-\omega) + (1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\,(R_{e_b}+\lambda)^{\frac{1-\sigma}{\sigma}}\Big)^{\eta-1}\cdot\left(\frac{\eta-1}{B}+1\right) - R_{e_b}.$$

By employing implicit differentiation, I can calculate the partial derivatives of this expression with respect to all exogenous variables.

Productivity parameter A: $\frac{\partial\lambda}{\partial A}$.

$$\lambda' = \underbrace{\eta\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b} + (1-\omega)\lambda(A)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega) + (1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b} + \lambda(A)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-1}\left(\frac{\eta-1}{B}+1\right)}_{\Phi_0>0}$$
$$+\underbrace{\eta A(\eta-1)\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b} + (1-\omega)\lambda(A)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega) + (1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b} + \lambda(A)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-2}\left(\frac{\eta-1}{B}+1\right)}_{\Phi_1<0}$$
$$\cdot\left[\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b} + (1-\omega)\lambda(A)\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)^2}_{\Phi_3>0}\lambda' + \underbrace{\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b} + \lambda(A)\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_4>0}\lambda'\right]$$

Note that $\eta - 1 < 0$ and $(\frac{\eta-1}{B} + 1) > 0$.

$$\lambda' = \Phi_0 + \Phi_1(\Phi_3\lambda' + \Phi_4\lambda')$$
$$\lambda' = \frac{\overbrace{\Phi_0}^{>0}}{\underbrace{1 - \Phi_1\Phi_3 - \Phi_1\Phi_4}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial A} > 0$$

Number of bankers B: $\frac{\partial \lambda}{\partial B}$.

$$\lambda' = \underbrace{\eta A\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(B)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(B)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-1}\left(\frac{1-\eta}{B^2}\right)}_{\Phi_0>0}$$
$$+\underbrace{\eta A\left(\frac{\eta-1}{B}+1\right)(\eta-1)\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(B)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(B)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-2}}_{\Phi_1<0}$$
$$\cdot\left[\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(B)\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)^2}_{\Phi_3>0}\lambda'+\underbrace{\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\Big(R_{e_b}+\lambda(B)\Big)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_4>0}\lambda'\right]$$

Note that $\frac{1-\eta}{B^2}>0$.

$$\lambda' = \Phi_0+\Phi_1(\Phi_3\lambda'+\Phi_4\lambda')$$
$$\lambda' = \frac{\overbrace{\Phi_0}^{>0}}{\underbrace{1-\Phi_1\Phi_3-\Phi_1\Phi_4}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial B}>0$$

Discount factor β: $\frac{\partial\lambda}{\partial\beta}$.

$$\lambda' = \underbrace{\eta(\eta-1)A\left(\frac{\eta-1}{B}+1\right)\left[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\beta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\beta)\right)^{\frac{1-\sigma}{\sigma}}\right]^{\eta-2}}_{\Phi_0<0}$$
$$\cdot\left[\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-2\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\beta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)}_{\Phi_1>0}+\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\beta)\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)^2}_{\Phi_2>0}\lambda'\right.$$
$$\left.+\underbrace{\frac{1}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+\lambda(\beta)\right)^{\frac{1-\sigma}{\sigma}}}_{\Phi_3>0}+\underbrace{\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\beta)\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_4>0}\lambda'\right]$$

$$\lambda' = \Phi_0(\Phi_1+\Phi_2\lambda'+\Phi_3+\Phi_4\lambda')$$
$$\lambda' = \frac{\overbrace{\Phi_0\Phi_1+\Phi_0\Phi_3}^{<0}}{\underbrace{1-\Phi_0\Phi_2-\Phi_0\Phi_4}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial\beta}<0$$

Concavity parameter in production function η: $\frac{\partial\lambda}{\partial\eta}$.

$$\lambda' = A\frac{d}{d\eta}\left(\frac{\eta^2}{B}-\frac{\eta}{B}+\eta\right)\left[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}\right]^{\eta-1}$$

$$\lambda' = \left(\frac{2\eta}{B}-\frac{1}{B}+1\right)\underbrace{A\left[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}\right]^{\eta-1}}_{\Phi_0>0}$$

$$+A\left(\frac{\eta^2}{B}-\frac{\eta}{B}+\eta\right)\underbrace{\frac{d}{d\eta}e^{(\eta-1)log\left(\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}\right)}}_{\Phi_1}$$

Only last part:

$$\Phi_1 = \underbrace{\left(\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}\right)^{\eta-1}}_{\Psi_0>0}$$

$$\cdot\left[\underbrace{\log\left(\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}\right)}_{\Psi_1\lesseqgtr 0}\right.$$

$$\cdot\underbrace{(\eta-1)\frac{1}{\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-\sigma}{\sigma}}}}_{\Psi_2<0}$$

$$\left.\cdot\,\lambda'\underbrace{\left(\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\eta)\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)^2+\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\eta)\right)^{\frac{1-2\sigma}{\sigma}}\right)}_{\Psi_3>0}\right]$$

Combine everything to get

$$\lambda' = \left(\frac{2\eta}{B} - \frac{1}{B} + 1\right)\Phi_0 + A\left(\frac{\eta^2}{B} - \frac{\eta}{B} + \eta\right)\lambda'\Psi_0\Psi_1\Psi_2\Psi_3$$

$$\lambda' = \frac{\overbrace{\left(\frac{2\eta}{B} - \frac{1}{B} + 1\right)}^{>0}\overbrace{\Phi_0}^{>0}}{1 - A\underbrace{\left(\frac{\eta^2}{B} - \frac{\eta}{B} + \eta\right)}_{>0}\underbrace{\Psi_0}_{>0}\underbrace{\Psi_1}_{\lesseqgtr 0}\underbrace{\Psi_2}_{<0}\underbrace{\Psi_3}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial\eta} \lesseqgtr 0$$

Share of θ^m-types γ: $\frac{\partial\lambda}{\partial\gamma}$.

$$\lambda' = \underbrace{\left(\frac{\eta-1}{B}+1\right)(\eta-1)\eta A\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\gamma)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\gamma)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-2}}_{\Phi_0<0}$$
$$\cdot\Big[\underbrace{\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\gamma)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)}_{\Phi_1>0}+\underbrace{\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\gamma)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)^2}_{\Phi_2>0}\lambda'$$
$$-\underbrace{\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\gamma)\right)^{\frac{1-\sigma}{\sigma}}}_{\Phi_3>0}+\underbrace{(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\gamma)\right)^{\frac{1-\sigma}{\sigma}}}_{\Phi_4>0}\lambda'\Big]$$

$$\lambda' = \Phi_0[\Phi_1+\Phi_2\lambda'-\Phi_3+\Phi_4\lambda']$$

$$\lambda' = \frac{\overbrace{\Phi_0}^{<0}\big[\overbrace{\Phi_1}^{>0}-\overbrace{\Phi_3}^{>0}\big]}{1-\underbrace{\Phi_0}_{<0}\big[\underbrace{\Phi_2}_{>0}+\underbrace{\Phi_4}_{>0}\big]} \Rightarrow \frac{\partial\lambda}{\partial\gamma}\lesseqgtr 0$$

Reserve requirement ω: $\frac{\partial\lambda}{\partial\omega}$.

$$\lambda' = \underbrace{(\eta-1)\left(\frac{\eta-1}{B}+1\right)\eta A\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\omega)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\omega)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-2}}_{\Phi_0<0}$$
$$\cdot\Big[\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\omega)\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_1>0}[(1-\omega)\lambda'-\lambda](1-\omega)-\underbrace{\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\omega)\right)^{\frac{1-\sigma}{\sigma}}}_{\Phi_2>0}$$
$$+\underbrace{\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\omega)\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_3>0}\lambda'\Big]$$

$$\lambda' = \Phi_0\left(\Phi_1[(1-\omega)\lambda'-\lambda](1-\omega)-\Phi_2+\Phi_3\lambda'\right)$$
$$\lambda' = \underbrace{\Phi_0\Phi_1(1-\omega)^2}_{\Phi_4<0}\lambda' - \underbrace{\Phi_0\Phi_1(1-\omega)\lambda}_{\Phi_5<0} - \underbrace{\Phi_0\Phi_2}_{\Phi_6<0} + \underbrace{\Phi_0\Phi_3}_{\Phi_7<0}\lambda'$$
$$\lambda' = \left(\Phi_4\lambda'-\Phi_5-\Phi_6+\Phi_7\lambda'\right)$$
$$\lambda' = \frac{\overbrace{-\Phi_5-\Phi_6}^{>0}}{\underbrace{1-\Phi_4-\Phi_7}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial\omega}>0$$

Share of θ^m-types holding deposits α_d: $\frac{\partial\lambda}{\partial\alpha_d}$.

$$\lambda' = \underbrace{\eta A\left(\frac{\eta-1}{B}+1\right)(\eta-1)\Big[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\alpha_d)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\alpha_d)\right)^{\frac{1-\sigma}{\sigma}}\Big]^{\eta-2}}_{\Phi_0<0}$$

$$\cdot\Bigg[\underbrace{\gamma\beta^{\frac{1-\sigma}{\sigma}}(1-\omega)\left(R_{e_b}+(1-\omega)\lambda(\alpha_d)\right)^{\frac{1-\sigma}{\sigma}}}_{\Phi_1>0}+\underbrace{\gamma\beta^{\frac{1-\sigma}{\sigma}}(1-\omega)\alpha_d\frac{1-\sigma}{\sigma}\left(R_{e_b}+(1-\omega)\lambda(\alpha_d)\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)}_{\Phi_2>0}\lambda'$$

$$+\underbrace{\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\alpha_d)\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_3>0}\lambda'\Bigg]$$

$$\lambda' = \Phi_0(\Phi_1+\Phi_2\lambda'+\Phi_3\lambda')$$

$$\lambda' = \frac{\overbrace{\Phi_0\Phi_1}^{<0}}{\underbrace{1-\Phi_0(\Phi_2+\Phi_3)}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial\alpha_d}<0$$

Share of θ^s-types holding deposits α_τ: $\frac{\partial\lambda}{\partial\alpha_\tau}$.

$$\lambda' = \underbrace{\eta A\left(\frac{\eta-1}{B}+1\right)(\eta-1)\left[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\alpha_\tau)\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\alpha_\tau)\right)^{\frac{1-\sigma}{\sigma}}\right]^{\eta-2}}_{\Phi_0<0}$$

$$\cdot\left[\underbrace{(1-\gamma)\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(\alpha_\tau)\right)^{\frac{1-\sigma}{\sigma}}}_{\Phi_1>0}+\underbrace{(1-\gamma)\beta^{\frac{1}{\sigma}}\alpha_\tau\frac{1-\sigma}{\sigma}\left(R_{e_b}+\lambda(\alpha_\tau)\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_2>0}\lambda'\right.$$

$$\left.+\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(\alpha_\tau)\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)}_{\Phi_3>0}\lambda'\right]$$

$$\lambda' = \Phi_0(\Phi_1+\Phi_2\lambda'+\Phi_3\lambda')$$

$$\lambda' = \frac{\overbrace{\Phi_0\Phi_1}^{<0}}{\underbrace{1-\Phi_0(\Phi_2+\Phi_3)}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial\alpha_\tau}<0$$

Interest rate on reserves R_{e_b}: $\frac{\partial \lambda}{\partial R_{e_b}}$.

$$\lambda' = \overbrace{\eta A \left(\frac{\eta-1}{B}+1\right)(\eta-1)\left[\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(R_{e_b})\right)^{\frac{1-\sigma}{\sigma}}(1-\omega)+(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(R_{e_b})\right)^{\frac{1-\sigma}{\sigma}}\right]^{\eta-2}}^{\Phi_0<0}$$
$$\cdot\left[\underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\,\beta^{\frac{1-\sigma}{\sigma}}\left(R_{e_b}+(1-\omega)\lambda(R_{e_b})\right)^{\frac{1-2\sigma}{\sigma}}(1-\omega)}_{\Phi_1>0}(1+(1-\omega)\lambda')\right.$$
$$\left.+\underbrace{\frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}}\left(R_{e_b}+\lambda(R_{e_b})\right)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_2>0}(1+\lambda')\right]-1$$

$$\lambda' = \Phi_0[\Phi_1(1+(1-\omega)\lambda')+\Phi_2(1+\lambda')]-1$$
$$\lambda' = \Phi_0\Phi_1+\Phi_0\Phi_1(1-\omega)\lambda'+\Phi_0\Phi_2+\Phi_0\Phi_2\lambda'-1$$
$$\lambda' = -\frac{\overbrace{1-\Phi_0\Phi_1-\Phi_0\Phi_2}^{>0}}{\underbrace{1-\Phi_0\Phi_1(1-\omega)-\Phi_0\Phi_2}_{>0}} \Rightarrow \frac{\partial\lambda}{\partial R_{e_b}} < -1$$

A.4 Proposition 4

Proof. To find the socially optimal allocation, I begin by solving the planner's problem, which involves maximizing the utility of a representative generation. I hereby ignore the initial old. Moreover, the bankers are not relevant for the planner's problem. I assume that the planner weights the utilities of different agents equally.

The planner maximizes the utility of a representative generation g that is born in period t.

$$\begin{aligned} V_t^g = {} & \gamma[-h_{j,t}^{\theta m} + v(y_{j,t}^{\theta m})] + (1-\gamma)[-h_{j,t}^{\theta s} + \beta U(x_{j,t+1}^{\theta s})] \\ & + \gamma[x_{j,t}^s - y_{j,t}^s] + \beta x_{j,t+1}^c \end{aligned}$$

The first term reflects the utility of the early consumers with mass γ, who are working when young in the CM of period t and consume good y in the DM of period t. The second term is the utility of the late consumers with mass $(1-\gamma)$ who are working when young in the CM of period t and consume when old in period $t+1$. The third term is the utility of the producers who consume x in the CM and work in the DM to produce y. The last term reflects the consumption of the entrepreneurs who consume x^c with linear utility in period $t+1$.

The DM consumption of the early consumers has to be financed by direct transfers from the producers. The CM good x is produced by both consumers and by the entrepreneurs. It is consumed by the late consumers, producers, old entrepreneurs and used by entrepreneurs for the investment denoted by x^e. This yields the following market clearing conditions.

$$\gamma y_{j,t}^s \geq \gamma y_{j,t}^{\theta m} \qquad (\mu_{1,t})$$

$$\gamma h_{j,t}^{\theta m} + (1-\gamma)h_{j,t}^{\theta s} + f(x_{j,t-1}^e) \geq (1-\gamma)x_{j,t}^{\theta s} + x_{j,t}^e + \gamma x_{j,t}^s + x_{j,t}^c \qquad (\mu_{2,t}).$$

The social planner's problem can be defined by the following Lagrangian

$$
\begin{aligned}
&\mathcal{L}(h^{\theta m}_{j,t}, y^{\theta m}_{j,t}, h^{\theta_s}_{j,t}, x^s_{j,t}, y^s_{j,t}, x^{\theta_s}_{j,t+1}, x^p_{j,t+1}, x^e_{j,t}) = \\
&\quad h^{\theta m}_{j,t} + v(y^{\theta m}_{j,t}) - h^{\theta_s}_{j,t} + x^s_{j,t} - y^s_{j,t} + \beta U(x^{\theta_s}_{j,t+1}) + \beta x^c_{j,t+1} \\
&\quad + \mu_{1,t}[y^s_{j,t} - y^{\theta m}_{j,t}] \\
&\quad + \mu_{2,t}[\gamma h^{\theta m}_{j,t} + (1-\gamma)h^{\theta_s}_{j,t} + f(x^e_{j,t-1}) - (1-\gamma)x^{\theta_s}_{j,t} - x^e_{j,t} - \gamma x^s_{j,t} - x^c_{j,t}] \\
&\quad + \mu_{2,t+1}\big[\gamma h^{\theta m}_{j,t+1} + (1-\gamma)h^{\theta_s}_{j,t+1} + f(x^e_{j,t}) \\
&\quad - (1-\gamma)x^{\theta_s}_{j,t+1} - x^e_{j,t+1} - \gamma x^s_{j,t+1} - x^c_{j,t+1}\big]
\end{aligned}
$$

which yields the optimality conditions

$$
\begin{aligned}
v'(y^{\theta m}_j) &= 1, \\
U'(x^{\theta_s}_j) &= 1, \\
f'(x^e) &= 1/\beta.
\end{aligned}
$$

Next, I compare this to the market outcomes. For the early consumer we know that $y^{\theta m}_j = \beta R_m m_j$ and hence

$$
v'(\beta R_m m_j) = v'\left(\beta R_m v'^{-1}\left(\frac{1}{\beta R_m}\right)\frac{1}{\beta R_m}\right) = 1
$$

which only holds if $R_d = R_e = 1/\beta$. Analogously, for the late consumer we know that $x^{\theta_s}_j = R_s s_j$ and hence

$$
U'(R_s s_j) = U'\left(R_s U'^{-1}\left(\frac{1}{\beta R_s}\right)\frac{1}{R_s}\right) = 1
$$

which again only holds if $R_\tau = R_e = 1/\beta$.

In the loan market, we know from the planner's problem that optimality requires $f'(x) = 1/\beta$ and since $f'(x) = R_\ell$, this implies $R_\ell = 1/\beta$. Thus, the socially optimal allocation can only be reached at the Friedman rule, i.e.,

$$
R_e = R_d = R_\tau = R_\ell = 1/\beta
$$

It is important to emphasize that achieving the socially optimal allocation is not feasible in a market outcome when there is imperfect competition in the loan market ($B < \infty$). In this case, we have $R_\ell > R_\tau \geq R_d$ (as shown in

Proposition 2). However, the monetary authority can still target the optimal allocation either in the loan market or in the deposit market.

To find the interest rate on reserves that leads to the optimal allocation in the loan market, substitute $R_\ell = 1/\beta$ into the equation for R_ℓ derived in Appendix B.1:

$$R_{e_b} = \left(1 - \frac{1-\eta}{B}\right)/\beta - \lambda.$$

In that case, we have $R_d < 1/\beta$ and $R_\tau < 1/\beta$, indicating that the interest rates on deposits are too low to achieve optimality in the deposit market.

To achieve the optimal allocation in the deposit market, the central bank can eliminate the minimum reserve requirement constraint by setting $\omega = 0$, and then set the interest rate on reserves as follows:

$$R_{e_b} = R_d = R_\tau = 1/\beta.$$

However, we then have $R_\ell > 1/\beta$.

For central bank money demand, the monetary authority can always achieve the optimal allocation by implementing the Friedman rule, which is given by:

$$R_e = 1/\beta.$$

Lastly, in the case of perfect competition in the loan market, the loan interest rate would equal:

$$R_\ell = \lim_{B\to\infty} \frac{R_{e_b} + \lambda}{1 - \frac{1-\eta}{B}} = R_{e_b} + \lambda.$$

Thus, the monetary authority could achieve optimality by abolishing the minimum reserve requirement and setting $R_e = R_{e_b} = 1/\beta$. ■

A.5 Proposition 5

Proof. The minimum reserve requirement constraint is loose ($\lambda = 0$). An exogenous shift from private money to public money is represented by a decrease in α_d or α_τ. I present the proof for a change in α_d. The proof for α_τ or α is analogous.

(i) By differentiating equations (3) and (4) with respect to α_d, we obtain:

$$\frac{\partial R_d}{\partial \alpha_d} = \frac{\partial R_\tau}{\partial \alpha_d} = \frac{\partial R_{e_b}}{\partial \alpha_d} = 0.$$

This shows that there is no effect on the deposit interest rates when α_d changes.

(ii)-(iii) From equations (6)-(9), I can derive

$$\begin{aligned}
\frac{\partial d}{\partial \alpha_d} &= \gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d} > 0 \\
\frac{\partial e^m}{\partial \alpha_d} &= -\gamma\, v'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{\beta R_e} < 0 \\
\frac{\partial \tau}{\partial \alpha_d} = \frac{\partial e^s}{\partial \alpha_d} &= 0,
\end{aligned}$$

which shows that d decreases, e^m increases, and τ and e^s are unaffected upon a decrease in α_d.

(iv) A change in α_d has neither an effect on ℓ nor on R_ℓ. This can be demonstrated by using the closed-form solutions for ℓ and R_ℓ derived in Appendix B.1:

$$\begin{aligned}
\frac{\partial \ell}{\partial \alpha_d} &= \frac{\partial\left(\left[\frac{A\eta(1-\frac{1-\eta}{B})}{R_{e_b}}\right]^{1/(1-\eta)}\right)}{\partial \alpha_d} = 0 \\
\frac{\partial R_\ell}{\partial \alpha_d} &= \frac{\partial\left(\frac{R_{e_b}}{1-\frac{1-\eta}{B}}\right)}{\partial \alpha_d} = 0.
\end{aligned}$$

(v) Total reserves are defined by $Be_b = d(\alpha_d) + \tau - \ell$. Upon a change in α_d, only d is affected, while τ and ℓ remain unaffected. Hence, an outflow of deposits reduces reserves one-by-one.

(vi) Total money demand is defined by $d + \tau + e^m + e^s$. As shown in (ii) and (iii), a change in α_d only affects d and e^m. The overall effect is

$$\frac{\partial d}{\partial \alpha_d} + \frac{\partial e^m}{\partial \alpha_d} = \gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d} - \gamma\, v'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{\beta R_e} \lesseqgtr 0$$

and it follows that

$$\frac{\partial d}{\partial \alpha_d} + \frac{\partial e^m}{\partial \alpha_d} \begin{cases} > 0 & \text{if } R_d > R_e \\ = 0 & \text{if } R_d = R_e \\ < 0 & \text{if } R_d < R_e \end{cases}$$

Thus, upon a decrease in α_d, total money demand increases if $R_e > R_d$.

GDP is defined as the sum of CM and DM production.

$$Y = A\ell^\eta + d + e^m + \tau + e^s + \beta(dR_d + e^m R_e)$$

It follows that

$$\begin{aligned} \frac{\partial Y}{\partial \alpha_d} =& \gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d} - \gamma\, v'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{\beta R_e} \\ &+ \beta\left[\gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta} - \gamma\, v'^{-1}\left(\frac{1}{\beta R_e}\right)\frac{1}{\beta}\right] \end{aligned}$$

and thus

$$\frac{\partial Y}{\partial \alpha_d} \begin{cases} > 0 & \text{if } R_d > R_e \\ = 0 & \text{if } R_d = R_e \\ < 0 & \text{if } R_d < R_e \end{cases}$$

(vii) The entrepreneurs' profit is defined by

$$\Pi^e = A\ell^\eta - \ell R_\ell.$$

Using result (iv), it is straightforward that $\frac{\partial \Pi^e}{\partial \alpha_d} = 0$. The bankers' profit is defined by

$$\Pi^b = \ell R_\ell + B e_b R_{e_b} - dR_d - \tau R_\tau.$$

Since the change in d directly translates to reserves, and given that $R_{e_b} = R_d$, we have:

$$\begin{aligned}\frac{\partial \Pi^b}{\partial \alpha_d} &= \left[\gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d}\right] R_{e_b} - \left[\gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d}\right] R_d \\ &= \left[\gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d}\right](R_{e_b} - R_d) \\ &= \left[\gamma\, v'^{-1}\left(\frac{1}{\beta R_d}\right)\frac{1}{\beta R_d}\right](R_d - R_d) = 0.\end{aligned}$$

■

A.6 Proposition 6

Proof. The minimum reserve requirement constraint is binding ($\lambda > 0$). An exogenous shift from private money to public money is represented by a decrease in α_d or α_τ. I present the proof for a change in α_d. The proof for α_τ or α is analogous. From Proposition 3, it follows that $\partial\lambda/\partial\alpha_d < 0$.

(i) To determine the effect on R_d and R_τ, we can use equations (3) and (4).

$$\begin{aligned}\frac{\partial R_d}{\partial \alpha_d} &= \frac{\partial(R_{e_b} + (1-\omega)\lambda(\alpha_d))}{\partial \alpha_d} = (1-\omega)\frac{\partial \lambda}{\partial \alpha_d} < 0 \\ \frac{\partial R_\tau}{\partial \alpha_d} &= \frac{\partial(R_{e_b} + \lambda(\alpha_d))}{\partial \alpha_d} = \frac{\partial \lambda}{\partial \alpha_d} < 0\end{aligned}$$

Note that the effect on R_τ is stronger than on R_d due to the presence of the $(1-\omega)$ term in the equation.

(ii) Next, consider the extensive margin effect on transaction deposits d using equation (6):

$$\frac{\partial d}{\partial \alpha_d} = \frac{\partial\left(\gamma \alpha_d d_j\right)}{\partial \alpha_d} = \gamma d_j > 0$$

(iii) For the intensive margin effects on d and τ, consider the individual deposit demand for a single consumer j using equations (6) and (8). Without loss of generality, assume $v(x) = U(x) = \frac{x^{1-\sigma}}{1-\sigma}$.

$$\frac{\partial d_j}{\partial \alpha_d} = \frac{\partial \left((\beta R_d(\alpha_d))^{\frac{1-\sigma}{\sigma}} \right)}{\partial \alpha_d} = \frac{1-\sigma}{\sigma} (\beta R_d)^{\frac{1-2\sigma}{\sigma}} \frac{\partial R_d}{\partial \alpha_d} < 0$$

$$\frac{\partial \tau_j}{\partial \alpha_d} = \frac{\partial \left(\beta^{\frac{1}{\sigma}} R_\tau(\alpha_d)^{\frac{1-\sigma}{\sigma}} \right)}{\partial \alpha_d} = \frac{1-\sigma}{\sigma} \beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-2\sigma}{\sigma}} \frac{\partial R_d}{\partial \alpha_d} < 0$$

This holds due to the assumption that $-x\frac{U''(x)}{U'(x)} < 1$ for all $x \geq 0$ and $-y\frac{v''(y)}{v'(y)} < 1$ for all $y \geq 0$, which translates to $\sigma < 1$ for this specific utility function.

(iv) From equations (7) and (9), we can observe that the demand for public money increases when there is a decrease in α_d.

$$\frac{\partial e^m}{\partial \alpha_d} = -\gamma\, v'^{-1}\left(\frac{1}{\beta R_e}\right) \frac{1}{\beta R_e} < 0$$

$$\frac{\partial e^s}{\partial \alpha_d} = 0$$

(v) By using the closed-form expressions derived in Appendix B.1, I can demonstrate that the total loan amount ℓ decreases and the interest rate on loans R_ℓ increases when α_d decreases.

$$\ell(\alpha_d) = \left[\frac{R_{e_b} + \lambda(\alpha_d)}{A\eta(1 - \frac{1-\eta}{B})} \right]^{-1/(1-\eta)}$$

$$\frac{\partial \ell(\alpha_d)}{\partial \alpha_d} = -1/(1-\eta) \left[\frac{R_{e_b} + \lambda(\alpha_d)}{A\eta(1 - \frac{1-\eta}{B})} \right]^{-\eta/(1-\eta)} \frac{1}{A\eta(1 - \frac{1-\eta}{B})} \frac{\partial \lambda}{\partial \alpha_d} > 0$$

$$R_\ell = \frac{R_{e_b} + \lambda(\alpha_d)}{1 - \frac{1-\eta}{B}}$$

$$\frac{\partial R_\ell(\alpha_d)}{\partial \alpha_d} = \frac{1}{1 - \frac{1-\eta}{B}} \frac{\partial \lambda}{\partial \alpha_d} < 0$$

(vi) The effect on total money demand $d+\tau+e^m+e^s$ consists of the following individual effects.

$$\begin{aligned}
\frac{\partial d}{\partial \alpha_d} &= \underbrace{\gamma\left(\beta R_d\right)^{\frac{1-\sigma}{\sigma}}}_{>0} + \underbrace{\frac{1-\sigma}{\sigma}\gamma\alpha_d\left(\beta R_d\right)^{\frac{1-2\sigma}{\sigma}}\beta\frac{\partial R_d}{\partial \alpha_d}}_{<0} \gtreqless 0 \\
\frac{\partial \tau}{\partial \alpha_d} &= \frac{1-\sigma}{\sigma}(1-\gamma)\alpha_\tau\,\beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-2\sigma}{\sigma}}\frac{\partial R_\tau}{\partial \alpha_d} < 0 \\
\frac{\partial e^m}{\partial \alpha_d} &= -\gamma(\beta R_e)^{\frac{1-\sigma}{\sigma}} < 0 \\
\frac{\partial e^s}{\partial \alpha_d} &= 0
\end{aligned}$$

Thus, the effect of α_d on total money demand depends on the specific values of the parameters.

As defined above GDP is

$$Y = A\ell^\eta + d + e^m + \tau + e^s + \beta(dR_d + e^m R_e)$$

and thus

$$\frac{\partial Y}{\partial \alpha_d} = \eta A\ell^{\eta-1}\underbrace{\frac{\partial \ell}{\partial \alpha_d}}_{>0} + \underbrace{\frac{\partial d}{\partial \alpha_d}}_{\gtreqless 0} + \underbrace{\frac{\partial e^m}{\partial \alpha_d}}_{<0} + \underbrace{\frac{\partial \tau}{\partial \alpha_d}}_{<0} + \beta\left[\underbrace{\frac{\partial d}{\partial \alpha_d}}_{\gtreqless 0} R_d + d\underbrace{\frac{\partial R_d}{\partial \alpha_d}}_{<0} + \underbrace{\frac{\partial e^m}{\partial \alpha_d}}_{<0} R_e\right] \gtreqless 0.$$

Hence, also the effect of α_d on GDP is ambiguous.

(vii) For the entrepreneurs' profit Π^e, we have

$$\begin{aligned}
\Pi^e &= A\ell^\eta - \ell R_\ell \\
\frac{\partial \Pi^e}{\partial \alpha_d} &= \eta A\ell^{\eta-1}\frac{\partial \ell}{\partial \alpha_d} - \frac{\partial \ell}{\partial \alpha_d}R_\ell - \ell\frac{\partial R_\ell}{\partial \alpha_d}. \\
&= \underbrace{\frac{\partial \ell}{\partial \alpha_d}\big(\underbrace{\eta A\ell^{\eta-1}}_{R_\ell} - R_\ell\big)}_{=0} - \ell\underbrace{\frac{\partial R_\ell}{\partial \alpha_d}}_{<0} > 0.
\end{aligned}$$

Due to the Cournot competition in the loan market, where entrepreneurs and bankers "share" the profit, an increase in the entrepreneurs' and bankers' profits occurs simultaneously.

(viii) The individual consumption of the different consumer types is defined by

$$
\begin{aligned}
C_d =& \beta R_d d_j = \beta R_d (\beta R_d)^{\frac{1-\sigma}{\sigma}} = (\beta R_d)^{\frac{1}{\sigma}} \\
C_{e^m} =& \beta R_e e_j^m = \beta R_e (\beta R_e)^{\frac{1-\sigma}{\sigma}} = (\beta R_e)^{\frac{1}{\sigma}} \\
C_\tau =& \beta R_d d_j = R_\tau \beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-\sigma}{\sigma}} = (\beta R_\tau)^{\frac{1}{\sigma}} \\
C_{e^s} =& \beta R_e e_j^s = R_e \beta^{\frac{1}{\sigma}} R_e^{\frac{1-\sigma}{\sigma}} = (\beta R_e)^{\frac{1}{\sigma}}
\end{aligned}
$$

It follows

$$
\begin{aligned}
\frac{\partial C_d}{\partial \alpha_d} &= \frac{1}{\sigma} (\beta R_d)^{\frac{1-\sigma}{\sigma}} \frac{\partial R_d}{\partial \alpha_d} < 0 \\
\frac{\partial C_\tau}{\partial \alpha_d} &= \frac{1}{\sigma} (\beta R_\tau)^{\frac{1-\sigma}{\sigma}} \frac{\partial R_\tau}{\partial \alpha_d} < 0 \\
\frac{\partial C_{e^m}}{\partial \alpha_d} = \frac{\partial C_{e^s}}{\partial \alpha_d} &= 0
\end{aligned}
$$

■

Appendix B Derivations

B.1 Closed-Form Solutions for ℓ and R_ℓ

As demonstrated in the proof of Proposition 3, I can derive

$$
R_{e_b} + \lambda = \eta A \ell^{\eta-1} \left(\frac{\eta - 1}{B} + 1 \right).
$$

from the bank's first-order condition for loans (2). Solving for ℓ yields:

$$
\begin{aligned}
\ell^{1-\eta} &= \frac{\eta A \left(\frac{\eta-1}{B} + 1 \right)}{R_{e_b} + \lambda}, \\
\ell &= \left(\frac{\eta A \left(1 - \frac{1-\eta}{B} \right)}{R_{e_b} + \lambda} \right)^{\frac{1}{1-\eta}}.
\end{aligned}
$$

To get an expression for R_ℓ, use that $R_\ell = \eta A \ell^{\eta-1}$ and substitute it into the expression above:

$$R_{e_b} + \lambda = R_\ell \left(\frac{\eta - 1}{B} + 1 \right)$$
$$R_\ell = \frac{R_{e_b} + \lambda}{1 - \frac{1-\eta}{B}}$$

Appendix C Data

I use data from five distinct sources for the calibration process. (1) FRED to obtain standard time series such as GDP, inflation, and data on cash holdings. (2) FDIC call reports to gather bank level data on interest rates and total deposits. (3) FDIC Summary of Deposits data to get county level deposit data. (4) U.S. Census Bureau data for county level population. (5) U.S. Bureau of Economic Analysis (BEA) data to obtain county level GDP data.

I provide a detailed discussion of each data source and the specific data derived from them below. To compute steady-state values, I utilize data spanning the period from 1987 to 2006. However, for certain parameters I use a shorter time frame due to data limitations. These instances are specified below.

FRED
From FRED, I gather data on total GDP (Y), the federal funds rate (R_{e_b}), cash holdings, and 1-year inflation expectations (see Table C.1). Expected inflation is utilized for computing real interest rates. Regarding cash holdings, I assume that all 100$-bills are used for savings, whereas smaller denominations are used for payments. I utilize FRED's denomination-specific series to compute e^m and e^s. However, these series are only available starting from 1993. To overcome this limitation, I combine data from another series related to the currency component of M1, which dates back to 1975, with information from Feige (2012, Figure 2), illustrating the percentage of currency by denomination since 1964. Additionally, considering that a

significant portion of US currency is held abroad, I focus on currency held within the US for my analysis. To account for this, I incorporate estimates from Feige (2012) regarding currency holdings abroad.

Variable	**Mnemonic**
GDP	GDP
Federal Funds Rate	DFF
Expected Inflation	EXPINF1YR
Currency component of M1	WCURRNS
Currency denominations ($\{X\}$\$)	CURRVAL$\{X\}$

Table C.1: FRED data.

FDIC Call Reports

To obtain historical data on deposits and loans and to compute the corresponding interest rates, I rely on FDIC call report data. This dataset is available on a quarterly basis and includes balance sheets and income statements at the bank level. Historical quarterly data for all banks can be downloaded in bulk from the following link: `https://www.fdic.gov/foia/ris/`. The specific time series utilized are listed in Table C.2.

To derive data on overall transaction deposits, total savings deposits, and aggregate loans, I aggregate the values from all banks within a given quarter. The category of savings deposits encompasses both regular savings deposits and money market deposit accounts (MMDAs). Regarding loans, I focus on total loans within domestic offices. The aggregated time series of deposits and cash holdings are illustrated in Figure C.1. The steady-state values used in the calibration for transaction deposits (d), savings deposits (τ), and loans (ℓ) are computed by averaging the corresponding time series.

To compute the corresponding interest rates, I utilize data on interest expenses for transaction and savings deposits, as well as interest income for loans. These interest expenses and earnings are reported in a year-to-date format. Consequently, I first recalculate quarterly expenses and earnings for all banks over the specified time period. Subsequently, I divide the series of expenses and income by their corresponding deposit and loan amounts, respectively, to obtain an approximation of the quarterly interest rates.

The dataset contains outliers both on the higher and lower ends. As a result,

Variable	Source	Mnemonic
Transaction Deposit Expense	FDIC	ETRANDEP; RIAD4508
Transaction Deposit Amount	FDIC	TRN; RCON2215
Savings Deposit Expense	FDIC	ESAVDP; RIAD0093
Savings Deposit Amount	FDIC	AVSAVDP; RCONB563
Loan Income	FDIC	ILN; RIAD4010
Loan Amount	FDIC	AVLN; RCON3360

Table C.2: FDIC call report data. The first set of FDIC mnemonics (e.g. ETRANDEP) are the ones used in the bulk download data. The second set of FDIC mnemonics are the ones that are used in the call reports (e.g. RIAD4508). A mapping can be found here: `https://www7.fdic.gov/DICT/app/templates/Index.html#!/Main`

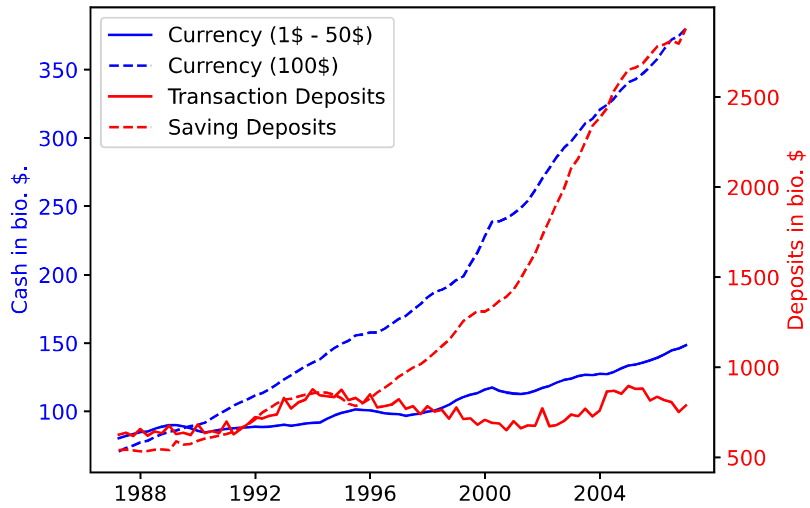


Figure C.1: Quarterly data on currency and deposit holdings.

for the deposit interest rates, I exclude the top 10% and bottom 10% of the respective values. For loan interest rates, following the approach of Chiu et al. (2023), I only consider the bottom 25% of bank-level interest rates, since I do not model risky loans. Additionally, due to inconsistent outliers, I once again exclude the bottom 10% of observations.

The calculated net nominal interest rates are illustrated in Figure C.2. To derive the corresponding real values (R_d, R_τ, and R_ℓ) used in the calibration, I subtract inflation expectations from the nominal rates and then calculate the averages over the entire time period.

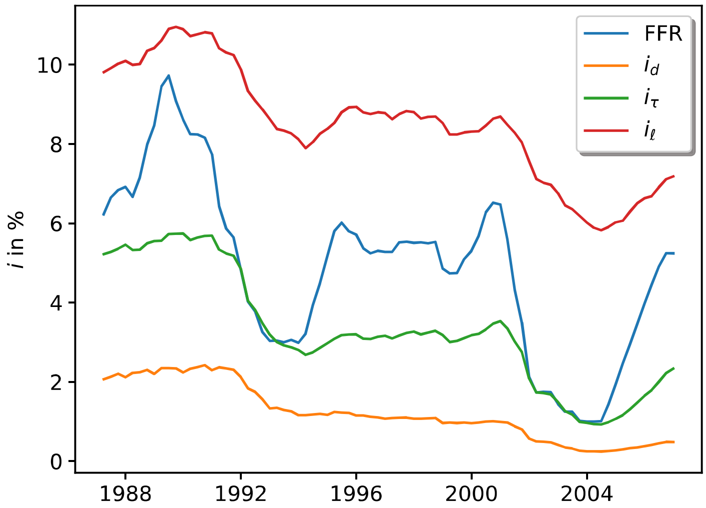


Figure C.2: Quarterly interest rate data using FDIC call report data.
FFR: Federal funds rate; i_d: Interest rate on transaction deposits; i_τ: Interest rate on savings deposits; i_ℓ: Interest rate on loans.

Furthermore, I use FDIC call report data for the computation of the loan elasticity. For this purpose, I construct a panel encompassing all banks featured in the reports during the period 1987-2006. I exclude banks with incomplete data and exclusively consider those that have remained operational throughout the entire time span to ensure a balanced panel. I further remove the top and bottom 2% of banks with the highest and lowest loan rates to account for outliers. Subsequently, I conduct a panel regression model incorporating time and bank fixed effects, formulated as follows:

$$\log(\ell/GDP)_{it} = \beta_\ell R_\ell + \alpha_i + \lambda_t + u_{it} \tag{17}$$

The parameter β_ℓ serves as an estimate of the semi-elasticity of loan demand. Note that the observed data provides loan amounts for individual banks, while the model focuses on the aggregate national loans. To address this discrepancy, I scale the parameters to account for total loans. The outcomes of the regression are presented in Table C.3.

County level data

To estimate the parameters of the utility function (σ) and the functions $\alpha(\varepsilon)$, I leverage county-level variations. While I extensively discussed the parameter computation in Section 4, I now elaborate more on the preparation of

	Dependent variable:		
	$\log(d/GDP)$	$\log(\tau/GDP)$	$\log(\ell/GDP)$
R_d	6.485^{***}		
	(0.981)		
R_τ		5.210^{***}	
		(0.572)	
R_ℓ			-0.268^{***}
			(0.0385)
const	-8.199^{***}	-6.544^{***}	0.3148^{***}
	(0.9625)	(0.568)	(0.0408)
Tot. Obs.	$16'518$	$16'518$	$249'360$
Time periods	6	6	80
Entities	$2'753$	$2'753$	$3'117$

Note: $^{*}p < 0.1;^{**} p < 0.05;^{***} p < 0.01$

Table C.3: Estimates for semi-elasticities on transaction deposit demand, savings deposit demand and loan demand. The standard errors are reported in brackets. The estimates for the deposits are based on annual data and county level variation, the one for loans on quarterly data and bank level variation.

input data used for calibration. This involves collecting county-level data on (i) deposit holdings d and τ, (ii) cash holdings e^m and e^s, (iii) interest rates R_d, R_τ, R_e, and (iv) GDP.

(i) For the computation of county-level deposit holdings, I rely on data from the FDIC Summary of Deposits (SOD), which is released annually and accessible for download from: `https://www7.fdic.gov/sod/dynaDownload.asp?`. This dataset provides a comprehensive breakdown of a bank's branches, including their locations along with corresponding county and state codes. I use these codes to derive the counties' FIPS (Federal Information Processing Standard) codes, which allows to match the data with other datasets. Aggregating the deposits across all branches within a particular county yields the total deposits for that county. However, note that the SOD data does not differentiate between transaction and savings deposits. To estimate county-level transaction and savings deposits, I take the county aggregates and apply the national shares of transaction and savings deposits derived from the FDIC call report data mentioned earlier.

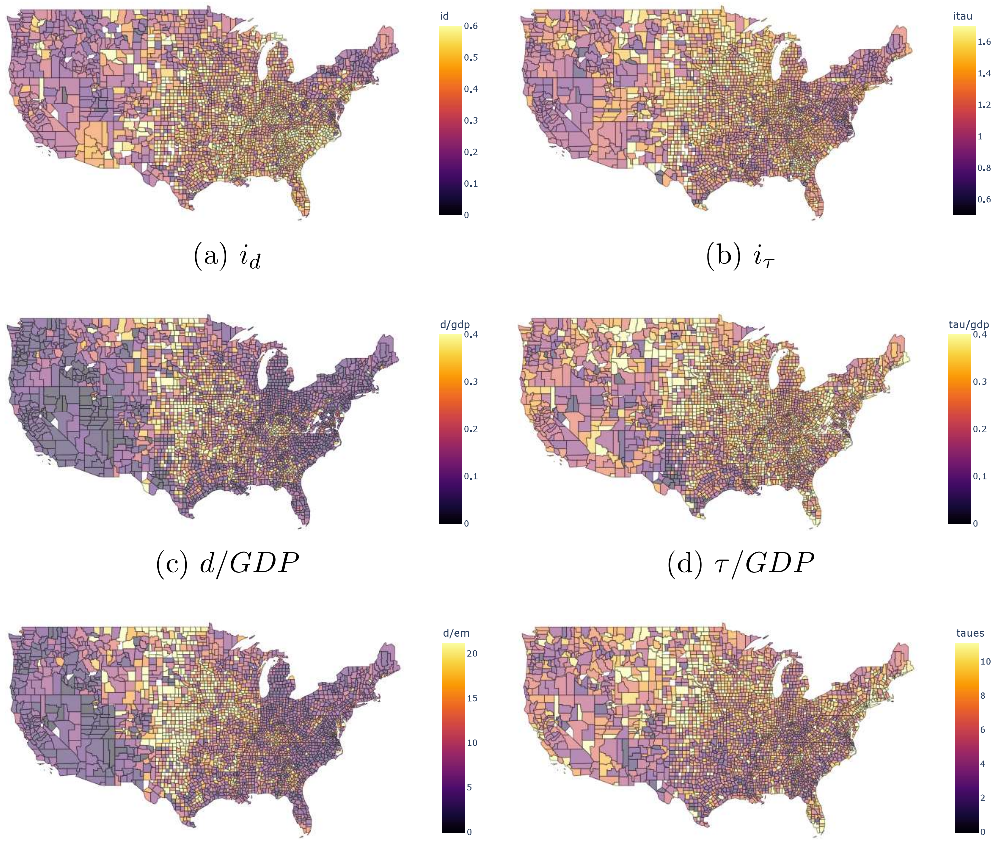
(ii) Regarding cash holdings, county-level data is not available. To estimate cash holdings at the county level, I use the aggregate country-level amounts mentioned earlier. By utilizing population data, I calculate a per-person estimate and then extrapolate the total county holdings based on the county's population. To access county-level population data, I utilize the U.S. Census Bureau's API and match counties using the FIPS codes.

(iii) Calculating the average county-level interest rate presents challenges due to the absence of interest expense data at that level of granularity. To obtain an estimate, I assume a uniform interest rate across the entire country for each bank. This assumption allows me to utilize the interest rate data from FDIC call reports. While this is a strong assumption, it can be justified by the composition of the US banking system, which features numerous regional and community banks introducing variation at the county level. I compute the county-level interest rate by averaging all branches' interest rates within a county. Figures C.3a and C.3b provide visual evidence of substantial variations across counties. Lastly, the interest rate on cash is uniform across all counties, denoted as $R_e = 1/\Pi_e$, where Π_e represents inflation expectations.

(iv) I collect county-level GDP data from the U.S. Bureau of Economic Analysis (BEA). Time series data dating back to 2001 can be accessed here: `https://apps.bea.gov/regional/histdata/releases/1221lagdp/index.cfm`. I utilize FIPS codes to match counties.

Using this data, I can compute values for σ and derive functions for $\alpha(\varepsilon)$, as detailed in Section 4. To estimate σ, I need to determine the semi-elasticity of transaction deposits and savings deposits demand. Utilizing the aforementioned dataset, I construct an annual panel spanning from 2001 to 2006. This panel encompasses information on transaction deposits demand d/GDP, savings deposits demand τ/GDP, and the corresponding interest rates R_d and R_τ. To gain insights into county-level variation, I visualize the data for the year 2005 in Figure C.3. I then proceed to estimate the following two panel regression models:

$$\log(d/GDP)_{it} = \beta_M R_d + \alpha_i + \lambda_t + u_{it}$$
$$\log(\tau/GDP)_{it} = \beta_S R_\tau + \alpha_i + \lambda_t + u_{it}$$



(a) i_d (b) i_τ

(c) d/GDP (d) τ/GDP

(e) d/e^m (f) τ/e^s

Figure C.3: County variation for parameters of interest. Only one year (2005) is depicted for illustration purposes. 0.2% of the counties are excluded because of outliers.

β_M and β_S represent the estimated semi-elasticities of transaction and savings deposit demand, respectively. The outcomes of the estimation are shown in Table C.3. Utilizing equations (12) and (13), along with the elasticity estimates and the steady-state interest rate values for transaction deposits (R_d) and savings deposits (R_τ), I compute values for σ_M and σ_S:

$$\sigma_M = \frac{1}{\beta_M R_d + 1} = \frac{1}{6.484 \cdot 0.982 + 1} = 0.136$$
$$\sigma_S = \frac{1}{\beta_S R_\tau + 1} = \frac{1}{5.210 \cdot 1.002 + 1} = 0.16$$

Appendix D Robustness of the Calibration

To analyze the robustness of the calibration, I conduct a parameter sensitivity analysis and present the model outcomes for a version incorporating imperfect competition in both the loan and deposit markets.

D.1 Sensitivity of the Parameters

In this section, I present an analysis of how the key parameters influence the targeted moments (listed in Table 4), as well as the indirectly targeted moments (listed in Table 5). This approach is akin to the methodology utilized by Elenev, Landvoigt and Nieuwerburgh (2021), which is based on Andrews, Gentzkow and Shapiro (2017). The sensitivity analysis is demonstrated in Table D.4, where I display the elasticities of the target moments in blue and the validation moments in red, following a 1% increase in each key parameter. I report the six parameters used in the joint calibration and the utility function parameter σ.

As expected, there is a strong positive correlation between the fraction of θ^m-types (γ) and the proportion of agents holding a payment vehicle or a savings vehicle (d/τ and e^m/e^s). Similarly, it is unsurprising that an increase in competition (B increases) leads to a decrease in the loan interest rate, thereby diminishing the spread $R_\ell - R_d$. The deposit handling cost significantly impacts the interest rate spread between deposit rates and the

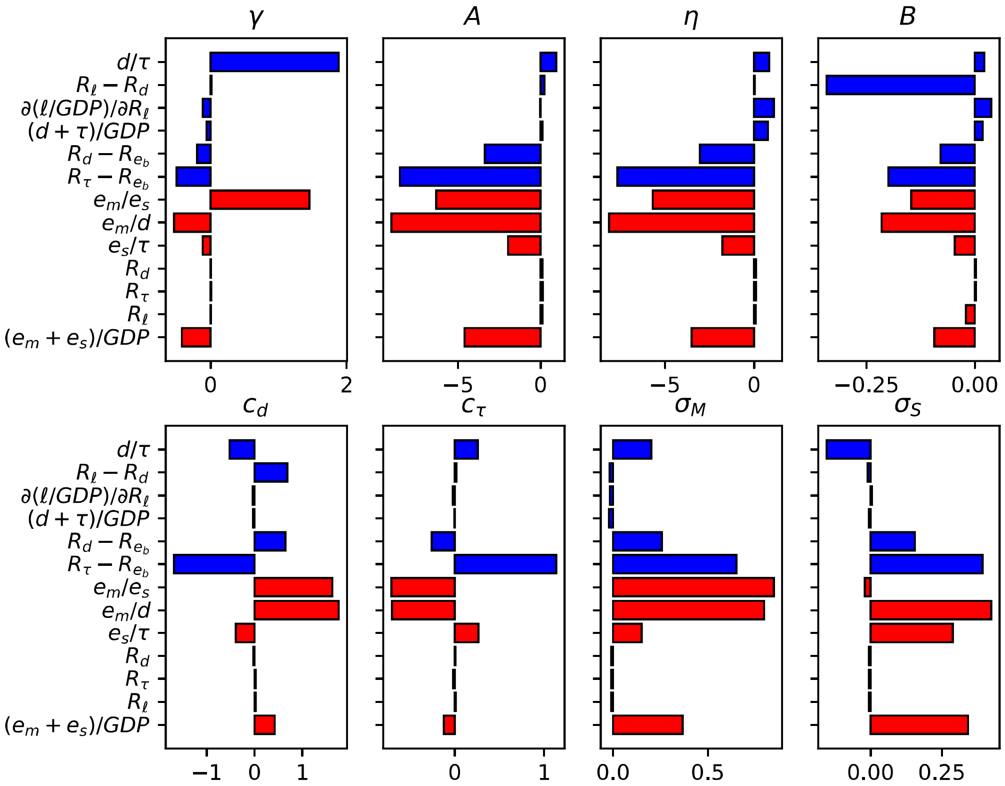


Figure D.4: Parameter sensitivity analysis. Reported are the elasticities of the moments with respect to a 1% increase in a parameter. Blue: Targeted moments from Table 4. Red: Validation moments from Table 5.

interest on reserves (IOR). Lastly, a higher utility parameter σ boosts the demand for assets, resulting in a positive correlation between σ_M and the shares d/τ and e^m/e^s, while σ_S exhibits a negative correlation with them.

D.2 Imperfect Competition Model

In the presence of imperfect competition in both the loan and deposit markets, the equilibrium equations are:

$$
\begin{aligned}
R_d &= R_{e_b} + \lambda(1-\omega) - \frac{\partial R_d(d)}{\partial d_b} d_b \\
R_\tau &= R_{e_b} + \lambda - \frac{\partial R_\tau(\tau)}{\partial \tau_b} \tau_b \\
d &= \gamma \alpha_d(R_d)\ (\beta R_d)^{\frac{1-\sigma}{\sigma}} \\
e^m &= \gamma(1-\alpha_d(R_d))\ (\beta R_e)^{\frac{1-\sigma}{\sigma}} \\
\tau &= (1-\gamma)\alpha_\tau(R_\tau)\, \beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-\sigma}{\sigma}} \\
e^s &= (1-\gamma)(1-\alpha_\tau(R_\tau))\, \beta^{\frac{1}{\sigma}} R_e^{\frac{1-\sigma}{\sigma}} \\
\ell &= \left[\frac{A\eta(1-\frac{1-\eta}{B})}{R_{e_b}+\lambda} \right]^{1/(1-\eta)} \\
0 &= \lambda(d(1-\omega) + \tau - \ell)
\end{aligned}
$$

The only changes are found within the equilibrium equations for the deposit interest rates R_d and R_τ. In this version of the model, I have excluded deposit handling costs. The calibration process remains consistent with the explanation in Section 4, but I now calibrate three distinct parameters to reflect competition in the loan market, the transaction deposit market, and the savings deposit market.

Table D.4 replicates the information presented in Table 6, aiming to analyze the disparities in bank intermediation, asset demand, and interest rates in contrast to the model featuring perfect competition within the deposit market. The observed effects are very similar in terms of magnitude. However, the distinction between an exogenous shift exclusively in the savings vehicle and a shift focused solely on the medium of exchange is even more pronounced in this model version.

Table D.5 presents a comparison between the two models under the scenario of an interest-bearing CBDC. Specifically, it illustrates the transition from a 0% interest rate on public money to the Friedman rule, which is $i_e = 5.18\%$. In the model with imperfect competition, the adverse impact on loans is

		ℓ	d	τ	e_m	e_s	i_d	i_τ	i_ℓ
Eq		100.0	44.3	59.6	6.1	9.7	1.3	3.3	8.5
-10%	MoE	99.7	42.9	60.6	8.4	9.5	1.6	3.5	8.8
		(-0.3%)	(-3.2%)	(1.6%)	(36.7%)	(-2.0%)			
	Savings	99.2	47.2	56.4	4.6	14.0	1.7	3.9	9.1
		(-0.8%)	(6.5%)	(-5.5%)	(-25.2%)	(44.0%)			
	Both	98.9	45.8	57.3	6.8	13.8	2.1	4.1	9.4
		(-1.1%)	(3.4%)	(-3.9%)	(11.0%)	(41.9%)			
5%	MoE	100.2	45.0	59.2	5.0	9.8	1.1	3.1	8.4
		(0.2%)	(1.5%)	(-0.8%)	(-18.5%)	(1.0%)			
	Savings	100.4	42.9	61.2	6.9	7.6	1.1	2.9	8.2
		(0.4%)	(-3.1%)	(2.6%)	(12.5%)	(-22.0%)			
	Both	100.4	42.9	61.2	6.9	7.6	1.1	2.9	8.2
		(0.4%)	(-3.1%)	(2.6%)	(12.4%)	(-22.0%)			

Table D.4: Imperfect competition in the loan market and deposit market. Replication of Table 6.

less pronounced. Additionally, in the presence of imperfect competition in the deposit market, banks exhibit a more substantial increase in the interest rate for transaction deposits while showing a comparatively smaller increase in the interest rate for savings deposits, as compared to the model featuring perfect competition in the deposit market.

	$i_{e,Eq}$ (0%) $\to$ $i_{e,Opt}$ (5.18%)	
	Perf. Comp.	Imperf. Comp.
ℓ	-3.0%	-2.6%
d	-17.3%	-12.4%
τ	$+6.6\%$	$+4.4\%$
e^m	$+293\%$	$+263\%$
e^s	$+60.8\%$	$+64.1\%$
i_d	$1.3\% \to 3.3\%$	$1.3\% \to 3.6\%$
i_τ	$3.3\% \to 5.6\%$	$3.3\% \to 5.2\%$
i_ℓ	$8.6\% \to 10.9\%$	$8.6\% \to 10.5\%$

Table D.5: Difference in effect sizes for an interest-bearing CBDC with an interest rate equal to the Friedman rule.

Appendix E Misc

E.1 Model Timeline

I here provide a more intuitive description of the model environment. The basic timeline is illustrated in Figure E.5. Time is discrete and continues forever $(\ldots, t-1, t, t+1, \ldots)$. Agents discount between time periods with the discount factor β. Within each time period, two subperiods exist: a centralized market (CM) and a decentralized market (DM), reflecting the framework outlined in Lagos and Wright (2005). The CM constitutes a centralized, Walrasian market wherein all agents engage in trading. The DM, on the other hand, is a decentralized market where exclusively bilateral trades occur.

Moreover, the model integrates generational dynamics akin to the overlapping generations model. Generation t is born at the onset of time period t in the CM and dies at the end of the CM in period $t+1$. The economy entails two distinct goods: the CM good denoted as x, and the DM good represented as y. Both goods are perishable which implies that they cannot be stored and deferred for consumption to subsequent subperiods or periods.

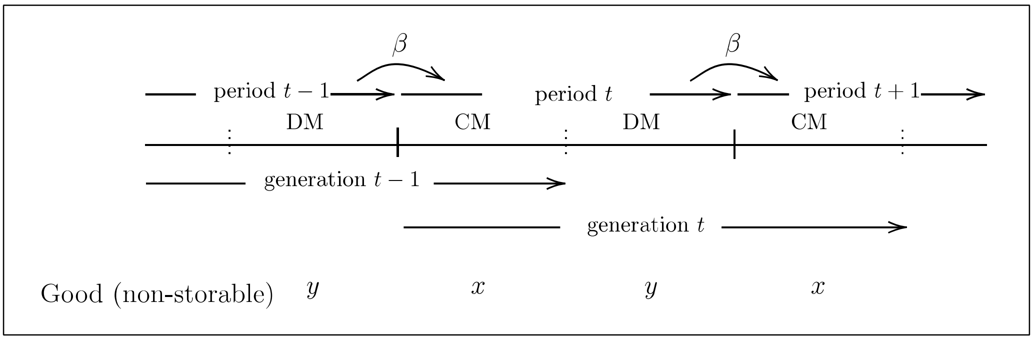


Figure E.5: Basic Timeline.

The model entails four distinct agent types, as depicted in Figure E.6. There are consumers who are divided into early consumers and late consumers. Both types of consumers can work and produce the good x when young in the CM. However, they wish to consume at a later stage in life. Specifically,

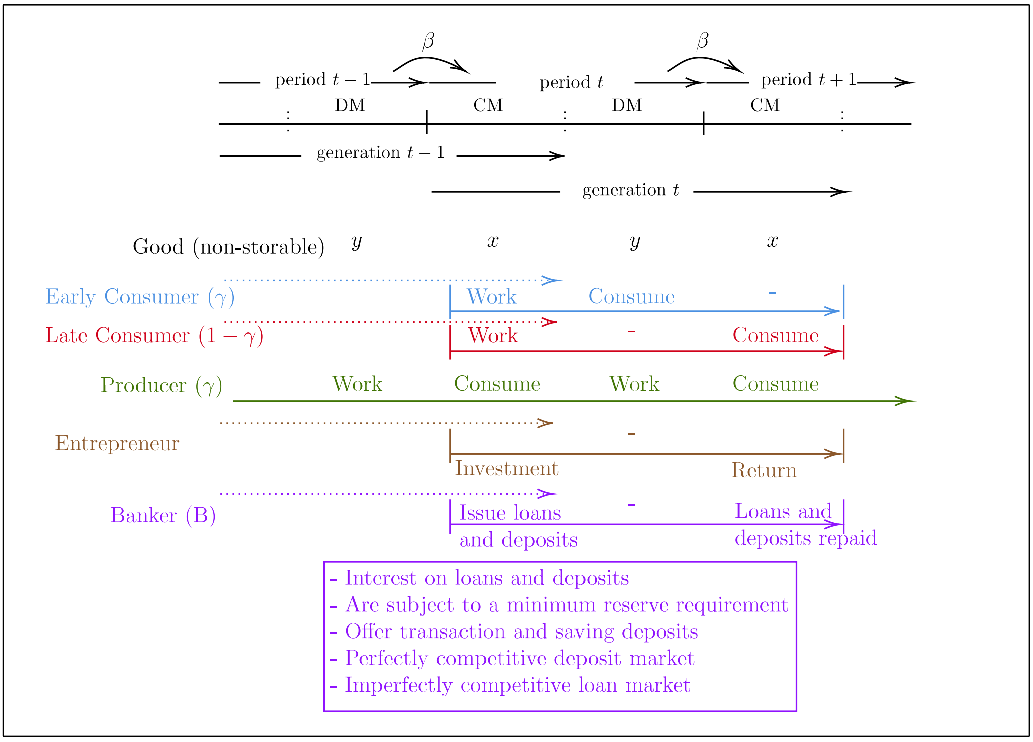


Figure E.6: Timeline with an overview of the different agents.

the early consumer wants to consume one subperiod later in the DM, while the late consumer intends to do so in the CM in period $t+1$.

Given the perishable nature of the goods, consumers need alternative means to defer consumption to later periods. Consequently, the early consumer engages in work, selling good x, and receiving payment in the form of a payment vehicle (central bank money or liquid transaction deposits). In contrast, the late consumer obtains a savings vehicle (central bank money or illiquid savings deposits). The early consumer can subsequently use the payment vehicle to acquire goods in the DM, while the late consumer utilizes the savings vehicle to purchase goods in the CM in period $t+1$.

Moreover, there are producers who live indefinitely. They can only work in the DM and produce good y, but they want to consume in the CM. As y is not storable, the producers have a demand for a payment vehicle, which they can obtain in the DM from the early consumer in return for selling good y.

Entrepreneurs live for one period and possess an investment opportunity, but they lack an initial endowment and are unable to engage in work. Consequently, they rely on obtaining a loan from a bank. When a loan is granted to an entrepreneur, the bank deposits the corresponding amount into the entrepreneur's bank account. The entrepreneur can subsequently utilize these funds to purchase the good x within the centralized market and invest it. After one period, returns are generated. The entrepreneur then sells a fraction of the returns during the CM in period $t+1$, acquiring bank deposits to repay the loan. The remaining portion of the returns is used for consumption.

Bankers also live for one period. The banker issues loans when young, whereby deposits are created endogenously. The bankers pay an interest on deposits and charge an interest rate on loans. Additionally, they have to comply with a minimum reserve requirement constraint on all deposits that are exchanged after one subperiod in the DM. As a consequence, all liquid deposits that are exchanged in the DM can only be partly invested into loans. Thus, the bankers have an incentive to offer two types of deposits: liquid transaction deposits and illiquid savings deposits. The former ones will be held by early consumers who want to consume in the DM, and the latter ones by late consumers who want to consume in the CM when old. The banking sector is perfectly competitive in the deposit market and imperfectly competitive in the loan market, which is modeled as Cournot competition.

E.2 Monetary Policy

In this section, I will explore the impacts of altering the minimum reserve requirement and adjusting the interest rate on reserves.

Figure E.7 illustrates the implications of modifying the minimum reserve requirement (ω). In the top left panel, we can observe that an elevated ω leads to higher interest rates on savings deposits and loans. The impact on the transaction deposit interest rate follows a bell-shaped pattern. As the reserve requirement rises, a larger proportion of transaction deposits must be held as reserves, leading to reduced bank lending. Banks respond to this decline in lending, which raises deposit interest rates, attracting more

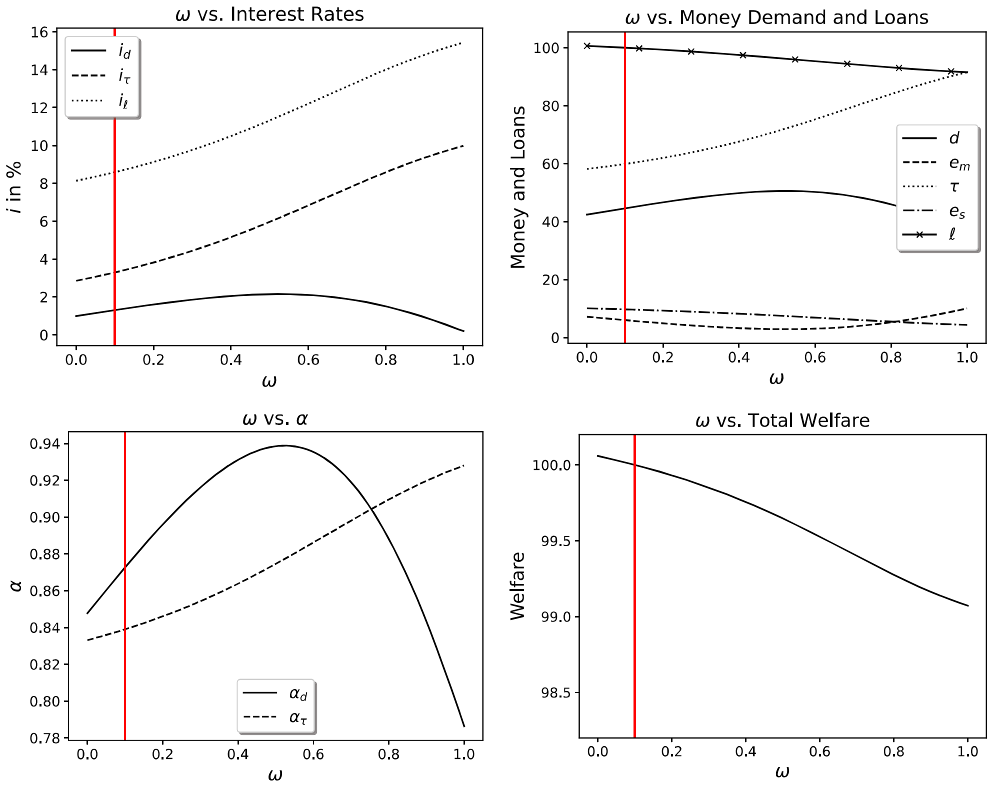


Figure E.7: Effect of a change in the minimum reserve requirement ω on the deposit interest rates, money demand, loans, the fraction of consumers holding deposits and total welfare. The red vertical line represents the calibrated equilibrium. The curves for the money demand are indexed such that the loan amount in the calibrated equilibrium is equal to 100. Also total welfare is indexed to 100. i_d: Rate on transaction deposits, i_τ: Rate on savings deposits, i_ℓ: Rate on loans, d: Transaction deposits amount, τ: Savings deposits amount, e^m: Central bank money held as payment vehicle, e^s: Central bank money held as savings vehicle, ℓ: Total loan amount, α_d: Fraction of early consumers holding transaction deposits, α_τ: Fraction of late consumers holding savings deposits.

deposits. However, if ω becomes excessively large ($\omega \geq 0.52$), it becomes optimal for banks to attract additional funds through savings deposits rather than transaction deposits. Consequently, the interest rate curve for savings deposits becomes steeper, and the interest rate on transaction deposits decreases for $\omega > 0.52$.

The top right graph indicates that bankers attract more deposits due to the elevated interest rates. However, despite the expansion of a banker's balance sheet, bank lending experiences a decline as a greater fraction of deposits must be held as reserves. Moving to the bottom left graph, we observe that a larger proportion of consumers desire to hold deposits with higher ω due to the increase in interest rates. Nevertheless, this pattern only holds true if $\omega \leq 0.52$. Beyond this threshold, the interest rate on transaction deposits decreases, leading to a decrease in the fraction of early consumers demanding transaction deposits.

The bottom right graph illustrates total welfare. Two effects come into play. On one hand, consumers benefit from a higher ω as bankers respond with higher interest rates. On the other hand, both bankers' and entrepreneurs' utilities diminish. Ultimately, the latter effect outweighs the former, resulting in a decrease in the overall welfare (W_{Tot}) as ω increases.

The impact of altering the interest rate on reserves (i_{e_b}) is depicted in Figure E.8. When i_{e_b} is decreased from the calibrated equilibrium, there is minimal influence on the interest rates for savings deposits and loans. This can be attributed to the fact that a reduction in i_{e_b} results in a nearly equivalent increase in the Lagrangian parameter λ associated with the reserve requirement constraint.

When the interest rate on reserves (i_{e_b}) is increased in comparison to the calibrated equilibrium, the minimum reserve requirement constraint becomes loose ($\lambda = 0$) at approximately $i_{e_b} = 6.02\%$. Beyond this threshold, a further rise in i_{e_b} translates directly to corresponding increases in deposit and loan rates and a decrease in bank lending.

Furthermore, the impact on welfare is adverse when the interest rate is set at such a high level. The increased interest rate expense is imposed on the producer in the form of a tax, resulting in a greater disutility for the producer. Despite the potential benefits enjoyed by consumers as a result of higher interest rates, the negative consequences outweigh these positives, leading to an overall negative effect on total welfare.

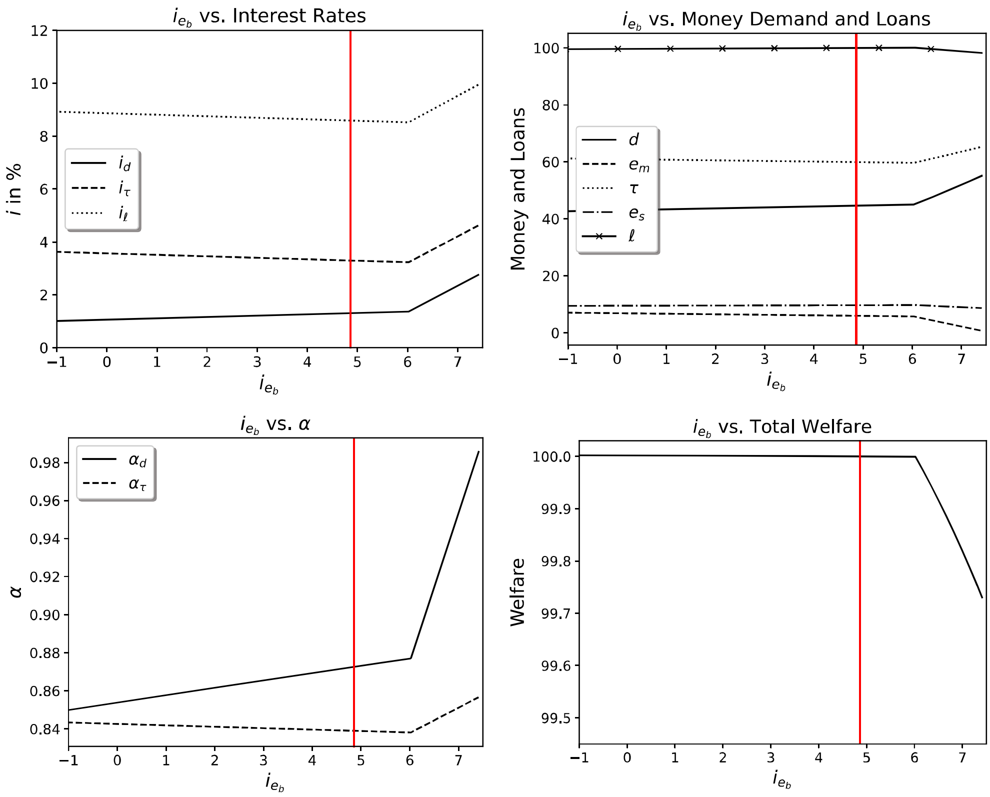


Figure E.8: Effect of change in the interest rate on reserves i_{e_b} on deposit interest rates, money demand, loans, the fraction of agents holding deposits and welfare. The red vertical line represents the calibrated equilibrium. The curves for money demand are indexed such that the loan amount in the calibrated equilibrium is equal to 100. Also total welfare is indexed to 100. i_d: Rate on transaction deposits, i_τ: Rate on savings deposits, i_ℓ: Rate on loans, d: Transaction deposits amount, τ: Savings deposits amount, e^m: Central bank money held as payment vehicle, e^s: Central bank money held as savings vehicle, ℓ: Total loan amount, α_d: Fraction of early consumers holding transaction deposits, α_τ: Fraction of late consumers holding savings deposits.