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Inducing Variety: A Theory of Innovation Contests

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Abstract

This paper analyzes the design of innovation contests when the quality of an innovation depends on the research approach, but the best approach is unknown. Inducing a variety of research approaches generates an option value. We show that suitable contests can induce such variety. The buyer-optimal contest is a *bonus tournament*, where suppliers can choose only between a low bid and a high bid. This contest implements the socially optimal variety for a suitable parameter range. Finally, we compare the optimal contest to scoring auctions and fixed-prize tournaments.

Keywords: Contests, tournaments, auctions, diversity, innovation, procurement.
JEL: L14, L22, L23.

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1 Introduction

The use of contests to procure innovations has a long history, and it is becoming ever more popular. Recently, private buyers have awarded the Netflix Prize, the Ansari X Prize, and the InnoCentive prizes. Public agencies have organized, for instance, the DARPA Grand Challenges, the Lunar Lander Challenge and the EU Vaccine Prize.[2] Reflecting the increasing importance of these prizes, a literature on contest design has developed. This literature focuses almost exclusively on how incentives for costly innovation effort can best be provided. However, effort is not the only important requirement for a successful innovation. A case in point is the 2012 EU Vaccine Prize to improve what is known as the cold-chain vaccine technology. The ultimate goal of the prize was to prevent vaccines from spoiling at higher temperatures, which is particularly challenging in developing countries. The rules of the competition contain the following statement:

> "It is important to note that approaches to be taken by the participants in the competition are not prescribed and may include alternate formulations, novel packaging and/or transportation techniques, or significant improvements over existing technologies, amongst others."[3]

This statement explicitly recognizes the fundamental uncertainty of the innovation process: Even when the buyer communicates a well-specified objective (such as finding a way to prevent vaccine spoilage), neither she nor the suppliers will necessarily know the best approach to achieving this goal. This uncertainty about the quality of innovation resulting from a particular approach will only be resolved by the act of innovation itself. The innovator will therefore have to choose between several conceivable approaches without being sure whether they lead to the goal. If innovators pursue different approaches, chances are higher that the best of these approaches yields a particularly valuable (high-quality) innovation. Thus, variety of research approaches has an option value. We therefore ask whether innovation contests can be used to incentivize suppliers to diversify their research approaches so as to generate a high expected value of the innovation.

In addition to the expected value of the innovation, contest design may also affect distribution. A contest that induces diversity may yield a high expected value of the innovation and thereby foster efficiency, but at the same time leave high rents to the suppliers. Thus, the main question of our paper will be: Which contests are optimal for the buyers, when the expected value (reflecting the induced variety of approaches) as well as the expected payments to the suppliers are taken into account? In addition, we address the relation between the buyer's choice and efficiency, asking under which circumstances the optimal contest implements the socially optimal amount of diversity.

[2] See "Innovation: And the winner is…", *The Economist.* Aug 5, 2010.

[3] European Commission (2012), "Prize Competition Rules." August 28, 2012. `http://ec.europa.eu/research/health/pdf/prize-competition-rules_en.pdf` (accessed on April 3, 2015).

The diversity of potential approaches, which is highlighted in the guidelines of the Vaccine Prize cited above, played an important role in many other examples of innovation procurement. First, the often cited Longitude Prize of 1714 for a method to determine a ship's longitude at sea featured two competing approaches.[4] The lunar method was an attempt to use the position of the moon to calculate the position of the ship. The alternative, ultimately successful, approach relied on a clock which accurately kept Greenwich time at sea, thus allowing estimation of longitude by comparison with the local time (measured by the position of the sun). Second, when the Yom Kippur War in 1973 revealed the vulnerability of US aircraft to Soviet-made radar-guided missiles, General Dynamics sought to resolve the issue through electronic countermeasures, while McDonnell Douglas, Northrop, and eventually Lockheed, attempted to build planes with small radar cross-section.[5] Third, the announcement of the 2015 Horizon Prize for better use of antibiotics contains a similar statement as the announcement of the vaccine prize.[6]

Architectural contests are similar to innovation contests. A buyer who thinks about procuring a new building usually does not know what the ideal building would look like, but once she examines the submitted plans, she can choose the one she prefers. Guidelines for architectural competitions explicitly recognize the need for diversity. For example, the Royal Institute of British Architects states: "Competitions enable a wide variety of approaches to be explored simultaneously with a number of designers."[7]

Motivated by these examples, we thus focus on the design of innovation contests, with a view towards the induced variety of research approaches. We consider a setting where both the buyer (the contest designer) and the suppliers (contestants) are aware that there are multiple conceivable approaches to innovation. Furthermore, none of the participants knows the best approach beforehand. However, after the suppliers have followed a particular approach, it is often possible to assess the quality of innovations, for instance, by looking at prototypes or detailed descriptions of research projects. In such settings, can buyers design contests in such a way that suppliers have incentives to provide variety? And will they benefit from doing so?

The existing literature on innovation contests mainly focuses on incentives for costly innovation effort. To our knowledge, we are the first to analyze the optimal design of innovation contests with multiple conceivable research approaches. Our baseline model is chosen to isolate this design problem in a stark way. We assume that there are two homogeneous suppliers who decide whether

[4] See, e.g., Che and Gale (2003) for a discussion of the Longitude Prize.

[5] See Crickmore (2003).

[6] "The rules of the contest specify the targets that need to be met but do not prescribe the methodology or any technical details of the test, thereby giving applicants total freedom to come up with the most promising and effective solution, be it from an established scientist in the field or from an innovative newcomer." European Commission (2015), "Better use of antibiotics." March 24, 2015. http://ec.europa.eu/research/horizonprize/index.cfm?prize=better-use-antibiotics (accessed on April 3, 2015).

[7] See Royal Institute of British Architects (2013), "Design competitions guidance for clients." http://competitions.architecture.com/requestform.aspx (accessed on Apr 3, 2015).

to exert costly research effort and which research approach to choose. In the baseline, the buyer has strong instruments to induce effort: We assume that, once a supplier joins the contest, he cannot shirk. This enables the buyer to use subsidies to ensure that the suppliers exert effort. This assumption allows us to focus on the effects of contest design on the choice of approaches rather than on effort choice.

We model the research approach as a point on the unit interval. The quality of an innovation depends inversely on the distance between the chosen research approach and an ideal approach that is unknown to all parties. The suppliers and the buyer agree about the distribution of this ideal approach, which has a strictly positive, symmetric and single-peaked density. If different suppliers try different approaches, this creates an option value for the buyer who can choose the preferred innovation once uncertainty is resolved. We assume all approaches are equally costly.

In line with the literature on innovation contests, we assume that neither research inputs (approaches) nor research outputs (qualities) are verifiable, because they are both difficult to evaluate and the relation between them is stochastic. The lack of verifiability of research activity precludes any kind of contract that conditions on research inputs or outputs, and it motivates the focus on contests.[8] The notion of contest design that we use was suggested by Che and Gale (2003). The buyer prescribes a possible set of prices and commits herself to paying the price chosen by the supplier from which the innovation is procured. The class of such contests includes fixed-prize tournaments (when the price set is a singleton) as well as scoring auctions (when the price set consists of all non-negative real numbers). Contest design in this setting is the choice of the allowable price set and the subsidies.

The sequence of moves in our model is as follows: After the buyer has communicated the rules of the game (and, in particular, the price set), the suppliers choose whether to enter and, if so, which approach to pursue. Then qualities become common knowledge. After having observed qualities, suppliers choose bids from the price set. Finally, the buyer selects the preferred supplier.

Our main result is that the optimal contest for the buyer is what we call a bonus tournament. In a bonus tournament, the price set is non-convex, consisting of only two elements — a low price and a high ("bonus") price. After qualities have been realized, the suppliers thus can only choose whether to ask for the high price or the low price. The selected supplier will be paid his bid. Anticipating this, the suppliers diversify in the hope that their quality advantage over the competitor will be sufficiently high that they can bid the bonus price and win even so. It will turn out that the amount of diversity implemented in a bonus tournament is determined by the difference between the bonus price and the low price. We show that, with a bonus tournament, the buyer can implement essentially any level of diversity. In particular, a bonus tournament with suitably chosen prices (and possibly a subsidy) implements the socially optimal diversity. However, full rent extraction is not always possible, and the buyer must trade off efficiency against rent extraction. Bonus tournaments

[8] For an extensive discussion see Che and Gale (2003) and Taylor (1995).

are nevertheless optimal for the buyer: They induce any desired level of diversity while minimizing rent extraction. The non-convexity of the price set turns out to be crucial for minimizing rent extraction while maintaining incentives for diversity.

Next, we examine the relation between the optimal contest for the buyer and the socially optimal level of diversity. The bonus tournament just described does not necessarily implement the social optimum, as the buyer may resolve the trade-off between efficiency and rent extraction in favor of the latter. However, the optimal bonus tournament leads to the socially optimal diversity when research costs are relatively high.[9] In this case, the buyer uses the tournament to maximize total expected surplus, and gives a subsidy that is just large enough that the suppliers expect to break even.

We also investigate some other contests that have received attention in the literature, in particular, scoring auctions and fixed-prize tournaments. The social optimum can be implemented with a scoring auction, but in this case the suppliers generally receive higher rents than in a bonus tournament. Fixed-prize tournaments induce no diversity at all. Nevertheless, for low research costs, the buyer prefers the inefficient fixed-prize tournaments to the socially efficient scoring auctions.

We then briefly discuss the robustness of the results to alternative environments. First, we take the possibility into account that agents can shirk by exerting zero effort and producing zero quality. Second, we allow for heterogeneous costs of different research approaches. Third, we consider more general distributions of the ideal approach and more general relationships between quality and the distance to the ideal approach. We also study contests with more than two suppliers and with multiple prizes. We provide conditions under which bonus tournaments still have favorable properties. Moreover, we show that the buyer may benefit from inviting a large number of suppliers, which is a straightforward implication of the option value provided by additional suppliers. We also discuss the option of contracting with a single supplier and the case when suppliers only observe own quality realizations.

Section 2 introduces the model. In Section 3, we derive the optimal mechanism. Section 4 discusses some commonly used mechanisms and compares them with the optimum. Section 5 briefly summarizes some extensions, which are treated in more detail in the online appendix. Section 6 treats related literature. Finally, Section 7 concludes.

2 The Model

Our baseline model derives the optimal contest for a risk-neutral buyer B who needs an innovation that two risk-neutral suppliers ($i \in \{1, 2\}$) can provide. The buyer first designs an innovation contest, the details of which will be discussed below. Facing the contest rules, the suppliers simultaneously decide whether to join the contest. If both suppliers decide to participate, they choose their

[9] Theorem 1 describes the conditions formally.

approaches $v_i \in [0,1]$ simultaneously in the next stage. We apply the convention that $v_1 \leq v_2$ if the ordering of approaches matters. The cost of approach v_i is $C(v_i) \equiv C \geq 0$. Thus all approaches are equally costly, so that, once a supplier has decided to participate in the contest, he cannot influence the research cost anymore. This assumption allows us to study the effects of contest design on the choice of research approaches in isolation and to develop a clear intuition for the results. If neither supplier joins the contest, all players receive their outside option, which is normalized to zero. If only one supplier decides to participate, this results in payoffs of zero for the supplier who does not participate and in non-negative (and otherwise unspecified) payoffs for the buyer and the remaining supplier.[10]

The quality q_i of the resulting innovation depends stochastically on the research approach. Specifically, we assume there is a state $\sigma \in [0,1]$, which is distributed according to $F(\sigma)$ with density $f(\sigma)$, and corresponds to an (ex-post) ideal approach. We maintain the following assumption on how q_i depends on v_i and σ.

Assumption (A1) $q_i = q(v_i, \sigma) \equiv \Psi - b|v_i - \sigma|$ with $b \in (0, \Psi - 2C]$.

Thus, the quality difference between the ideal approach $\hat{\sigma}$ and v_i is proportional to their distance on the unit interval. Note that (A1) implies that $C < (\Psi - b)/2$. This is sufficient to guarantee that any contest generates a non-negative surplus.

We restrict the distribution of the ideal state as follows.

Assumption (A2) *The density function* $f(\sigma)$ *is (i) symmetric:* $f(1/2 - \varepsilon) = f(1/2 + \varepsilon)$ $\forall \varepsilon \in [0, 1/2]$, *(ii) single-peaked:* $f(\sigma) \leq f(\sigma')$ $\forall \sigma < \sigma' < 1/2$, *(iii) has full support:* $f(\sigma) > 0$ $\forall \sigma \in [0,1]$ *and (iv) satisfies* $f'(x) < 2f(0)$ *for all* $x \in [0, 1/2]$.

For each distribution satisfying (A2), the median approach has the highest expected quality ex ante. Furthermore, single-peakedness makes it difficult to induce diversity: As there is less mass on approaches that are further away from the median, contestants who face incentives to produce high expected quality will therefore not want to diversify away from the median without additional incentives. Part (iv) excludes the possibility that some states are much less probable than others; in this sense, it requires that the amount of uncertainty about the ideal approach is sufficiently high.

(A1) and (A2) provide an intuitive and simple way of capturing both the correlation of qualities (two approaches which are closer on the unit interval result in more similar qualities) and their expected quality (the closer an approach is to the median, the higher its expected quality). These assumptions reflect the idea that contestants affect not only the distribution of their own qualities (as in contests with effort choice), but also the level of correlation with the quality of their competitor.

[10] At the end of Online Appendix B.4, we specify the payoffs from single-supplier contracts in more detail and we analyze the related question whether the buyer wants to interact with one supplier rather than organizing a contest.

In this setting, the buyer chooses an innovation contest determining the procedure for selecting and remunerating suppliers. These contests are closely related to those analyzed by Che and Gale (2003), where suppliers choose efforts rather than approaches. In line with these authors, we assume that neither v_i nor q_i is contractible. Contest design consists of choosing a set $\mathcal{P}$ of allowable prices (bids) and subsidies $t \geq 0$. In order to guarantee equilibrium existence in the bidding subgame, we restrict $\mathcal{P}$ to the set of arbitrary finite unions of closed subintervals of $\mathbb{R}^+$. Formally, $\mathcal{P} \in \mathcal{I}(\mathbb{R}^+)$ where $\mathcal{I}(\mathbb{R}^+) \equiv \{\mathcal{S} \subseteq \mathbb{R}^+ : \mathcal{S} = \cup_{k=1}^{\bar{k}} [a_k, b_k]$ or $\mathcal{S} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \cup \left[a_{\bar{k}+1}, \infty\right)$ for $a_k \leq b_k \in \mathbb{R}^+, \bar{k} \in N\}$. We refer to $\{\mathcal{P}, t\}$ as a contest. After the buyer has chosen $\{\mathcal{P}, t\}$, the following procedure is applied:

Period 1: Suppliers simultaneously choose whether to join the contest.
Period 2: They simultaneously select approaches $v_i \in [0, 1]$.
Period 3: The state is realized. All players observe qualities q_1 and q_2.
Period 4: Suppliers simultaneously choose prices $p_i \in \mathcal{P}$.
Period 5: The buyer observes prices; then she selects a supplier $i \in \{1, 2\}$. She pays $p_i + t$ to the selected supplier and t to the other supplier.

Suppose that a supplier i participates in some contest $\{\mathcal{P}, t\}$ and chooses some approach v_i while his competitor chooses an approach v_j. Denote the total expected payoff of supplier i as $\Pi_i^{\mathcal{P}}(v_i, v_j) + t$ and the quality that the buyer receives as $Q(v_1, v_2, \sigma)$, assuming equilibrium play in subgames induced by each possible (v_1, v_2, σ).[11] Then, the buyer's problem is:

$$\max_{\substack{\mathcal{P} \in \mathcal{I}(\mathbb{R}^+),\ t \geq 0, \\ v_1, v_2 \in [0,1]}} E_\sigma \left[Q(v_1, v_2, \sigma)\right] - \Pi_1^{\mathcal{P}}(v_1, v_2) - \Pi_2^{\mathcal{P}}(v_2, v_1) - 2t$$

$$\text{(IC)} \qquad \text{subject to } \Pi_i^{\mathcal{P}}(v_i, v_j) \geq \Pi_i^{\mathcal{P}}\left(v_i', v_j\right), \forall v_i' \in [0, 1]$$

$$\text{(PC)} \qquad \Pi_i^{\mathcal{P}}(v_i, v_j) + t \geq C.$$

Of course, the buyer does not directly choose v_1 and v_2. Rather, these are the approaches that the suppliers choose in equilibrium of a contest designed by the buyer. The set of contests $\{\mathcal{P}, t\}$ over which the buyer optimizes not only includes familiar contests like fixed-prize tournaments and scoring auctions, but also contests with non-convex prize sets, which will turn out to be optimal in this setting.[12] For instance, we have:

1. $\mathcal{P} = \mathbb{R}^+$: an *auction without a price ceiling.*

2. $\mathcal{P} = [0, Z]$: an *auction with a price ceiling* $Z > 0$.

3. $\mathcal{P} = \{A\}$, where $A \geq 0$: a *fixed-prize tournament* (FPT).

[11] For precise notation, see Appendix A.1.1.
[12] Further reasons for using this set of mechanisms are given in Che and Gale (2003, p. 648 and 650).

4. $\mathcal{P} = \{A, a\}$, where $A > a \geq 0$: a *bonus tournament.*

In the first two examples, the buyer allows the suppliers to select bids as in a standard auction after the realization of qualities, without (with) a price ceiling in Example 1 (2). However, the (commonly used) auction terminology is slightly misleading. The rules do not commit the buyer to selecting the supplier as a function of the observed qualities and bids. Instead, the buyer has the discretion to choose the supplier for whom the difference between the monetary value of quality and the bid is maximized. She thus behaves as if she had committed to a scoring rule which weighs prices and qualities in the same way (and the suppliers anticipate this behavior).

In the FPT (Example 3), the buyer does not allow the suppliers to choose a price. The suppliers choose approaches and thereby influence qualities. Once qualities have been realized, the buyer simply selects the higher quality supplier, as she has to pay the prize A no matter which supplier she chooses.

The bonus tournament (Example 4) differs from an FPT in that the buyer proposes two prices, a low price a and a high "bonus" price A at the outset of the game. After the choices of approaches and the realization of quality levels, both suppliers decide whether to ask for a high or a low price. As in Examples 1 and 2, the buyer then chooses the supplier for whom the difference between the quality and the bid is maximized. This implies that, when confronted with a combination of a high bid and a low bid, she will only be prepared to pay the high bid if the quality difference is at least as large as $A - a$.

Note that the suppliers potentially receive two types of payments, namely the revenue from the contest (that is paid only to the successful supplier) and the subsidies paid to both suppliers. For ease of exposition, we sharpen the requirement that qualities are observable by assuming that all players observe v_i and σ, as this allows us to apply subgame perfect equilibrium. It will be obvious that, as long as qualities are observable, the observability of v_i and σ plays no role; as these variables are payoff-relevant only inasmuch as they affect qualities.[13]

We apply the following tie-breaking rules.

(T1) (Preference for quality) If suppliers offer the same surplus ($q_1 - p_1 = q_2 - p_2$), the buyer chooses the higher quality one. If both offer the same surplus ($q_1 - p_1 = q_2 - p_2$) and quality ($q_1 = q_2$), the buyer chooses each supplier with probability 1/2.

(T2) (Preference for winning) If two strategies of the supplier, $(v_i, p_i(\cdot))$ and $(v_i', p_i'(\cdot))$, yield the same expected payoff, the supplier prefers the strategy that maximizes the probability of winning the contest.

[13] It is straightforward to extend the analysis to a Bayesian setting where supplier i does not observe v_j ($j \neq i$) and σ, but only q_j. The subgame perfect equilibria can then be replaced with weakly perfect Bayesian equilibria where the suppliers have degenerate (and correct) beliefs about rival strategies.

(T1) and (T2) can be interpreted as second-order lexicographic preference for winning and for higher quality.[14] Finally, we confine our analysis to the case of pure-strategy equilibria.

3 The Optimal Contest for the Buyer

In this section, we characterize the optimal two-supplier contest for the buyer.[15] We start with some auxiliary results. These results characterize the social optimum, and they deal with the pricing subgames.

3.1 Auxiliary Results

We introduce the following terminology which applies when both suppliers participate. For $(v_1, v_2) \in [0,1]^2$, the (expected) *total surplus* is $S_T(v_1, v_2) \equiv E_\sigma[\max\{q(v_1,\sigma), q(v_2,\sigma)\}] - 2C$. The *social optimum* is $(v_1^*, v_2^*) \equiv \arg\max_{(v_1,v_2)\in[0,1]^2} S_T(v_1, v_2)$.

For (v_1, v_2), implemented as an equilibrium of a contest $(\mathcal{P}, t)$, the (expected) *surplus of supplier* i in an equilibrium, $S_i^{(\mathcal{P},t)}(v_1, v_2)$, is the sum of the expected revenue and the subsidies, net of research costs. The (expected) *buyer surplus*, $S_B^{(\mathcal{P},t)}(v_1, v_2)$, is expected quality minus the expected revenues and subsidies of the suppliers. We drop the superscript $(\mathcal{P}, t)$ when there is no danger of confusion.[16]

As the costs of each approach are the same, the social optimum (v_1^*, v_2^*) maximizes the expected maximal quality $E_\sigma[\max\{q(v_1,\sigma), q(v_2,\sigma)\}]$. It is always socially optimal to have at least some diversification. Intuitively, starting from a situation with two identical approaches, changing one of them reduces the minimal distance for some states σ, without increasing it for any other state.

The following result provides a sharper characterization of the social optimum:[17]

Lemma 1 *The unique social optimum with $v_1^* \leq v_2^*$ satisfies $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ and thus $v_2^* = 1 - v_1^*$.*

[14](T1) ensures the existence of equilibria in contests which are similar to Bertrand games with heterogeneous costs (Che and Gale 2003 also impose a tie-breaking rule for similar reasons). (T2) is only necessary in cases where winning results in a prize of 0. Hence, we could dispense with (T2) if we instead assumed that the minimum price the buyer pays out is positive, or alternatively, if winning the contest results in a positive reputational benefit for the winner.

[15]An attentive reader might conjecture that the buyer could implement arbitrary outcomes with a mechanism where he just pays unconditional transfers $t = C$ and sets a singleton prize set $\mathcal{P} = \{0\}$. The suppliers are then indifferent between entering and not entering, and, *in monetary terms,* between all approaches. However, our "preference for winning" assumption (T2) would ensure that such a mechanism would have a unique equilibrium with $v_1 = v_2 = 1/2$. Even if we dispensed with assumption (T2), the equilibrium structure of such a mechanism would not be robust to small changes in the cost of different approaches or to assuming that duplicating an approach is less costly than developing an original one.

[16]For precise definitions of $S_B^{(\mathcal{P},t)}(v_1, v_2)$ and $S_i^{(\mathcal{P},t)}(v_1, v_2)$, we refer the reader to Appendix A.1.1.

[17]The result is similar to the familiar finding that, in a Hotelling model with uniformly distributed consumers and without price competition, firms should optimally spread equally.

Hence v_1^* and v_2^* are symmetric around $1/2$. The socially optimal approaches are fully determined by the distribution F, whereas the level of research costs has no influence on the optimal diversity. We now characterize the equilibria of the pricing subgames, using the following notation.

Notation 1 $\overline{p}(q_i, q_j) \equiv \max\{p \in \mathcal{P} \text{ } s.t. \text{ } p \leq |q_i - q_j| + \underline{P}\}$, *where* $\underline{P}$ *is the minimum of* $\mathcal{P}$.

Thus, for any realization of qualities q_i and q_j, if $q_i > q_j$ then $q_i - \overline{p}(q_i, q_j) \geq q_j - \underline{P}$. Since $q_j - \underline{P}$ is the highest surplus that supplier j can offer to the buyer, $\overline{p}(q_i, q_j)$ is the maximal price from the set $\mathcal{P}$ which guarantees that the supplier with higher quality wins the contest for any price chosen by the supplier with the lower quality. As the next lemma shows, the winning supplier will always set the price at $\overline{p}(q_i, q_j)$. This result relies on the familiar "asymmetric Bertrand" logic that inefficient firms choose minimal prices, whereas an efficient firm's quality advantage translates into a price differential.[18]

Lemma 2 *The subgame of an innovation contest corresponding to* (q_i, q_j) *has an equilibrium such that* $p_i(q_i, q_j) = \overline{p}(q_i, q_j)$ *if* $q_i \geq q_j$ *and* $p_i(q_i, q_j) = \underline{P}$ *if* $q_i < q_j$. *In any equilibrium of any contest,* $p_i(q_i, q_j) = \overline{p}(q_i, q_j)$ *if* $q_i \geq q_j$.

Lemma 2 sharpens the Bertrand logic to account for bounded and/or non-convex price sets: The price differential will only be identical with the quality differential when the corresponding bid of the high-quality supplier is in $\mathcal{P}$. While Lemma 2 uniquely determines the bid of the winning supplier, the equilibrium bid of the losing supplier is not always unique. This is due to the possibly bounded and/or non-convex price sets. Nevertheless, in any subgame equilibrium the losing price will be low enough that the winner cannot profitably deviate upwards. Furthermore, in many cases the loser will uniquely bid $\underline{P}$.[19] We need further notation:

Notation 2 $\Delta q(v_i, v_j) \equiv \max_{\sigma \in [0,1]} |q(v_i, \sigma) - q(v_j, \sigma)| = q(v_i, v_i) - q(v_j, v_i)$ *is the maximum quality difference over* $\sigma \in [0, 1]$ *given* (v_i, v_j).

To understand why $\Delta q(v_i, v_j) = q(v_i, v_i) - q(v_j, v_i)$, note that, for $\sigma \in [0, v_1] \cup [v_2, 1]$ the quality difference between the two approaches is equal to $q(v_i, v_i) - q(v_j, v_i)$, and for $\sigma \in (v_1, v_2)$ it is

[18] The adequacy of pure-strategy equilibria in asymmetric Bertrand games has received some attention, in particular, but not only, because they tend to involve weakly dominated strategies (see Blume 2003 and Kartik 2011). In our setting, these issues are resolved by the appeal to the "preference for quality" (T1) and "preference for winning" (T2). In some of our contests (in particular, in auctions with and without price ceilings), the pure-strategy winning prices can also be obtained using constructions as in Blume (2003) and Kartik (2011), where the low-quality firm mixes over a small interval of prices.

[19] If $\mathcal{P}$ is convex and $\sup \mathcal{P} > \overline{p}(q_i(v_i, \sigma), q_j(v_j, \sigma))$ for all σ, then $p_i(q_i, q_j) = \underline{P}$ for $q_i < q_j$ in every equilibrium. To see this, note that, according to Lemma 2, $p_j = \overline{p}(q_j, q_i) = \underline{P} + q_j - q_i$ in any equilibrium for the high-quality supplier j. If $p_i > \underline{P}$, then j can choose a slightly higher prize, and he still wins. Hence, this is a profitable deviation.

smaller.[20] By Lemma 2, in any subgame the successful supplier chooses the highest available price not exceeding the sum of the quality differential and the minimum bid. We now sharpen this result for subgames following equilibrium choices (v_1, v_2).

Lemma 3 *Let $v_1 \leq v_2$. (i) Any contest $(\mathcal{P}, t)$ which implements (v_1, v_2) satisfies $\Delta q\,(v_1, v_2) + \underline{P} \in \mathcal{P}$. (ii) If $\sigma \in [0, v_1] \cup [v_2, 1]$, the successful supplier bids $p_i\,(q_i, q_j) = \Delta q\,(v_i, v_j) + \underline{P}$.*

Lemma 3 implies that the amount of diversity (optimal or non-optimal) that any contest can implement is limited by the highest price that the contest allows. (i) reflects the intuition that, if $\Delta q\,(v_1, v_2) + \underline{P} \notin \mathcal{P}$, suppliers could increase their chances of winning by small moves towards the approach of the other party, without reducing the price in those cases where they win. (ii) shows that in all states outside the interval (v_1, v_2) the buyer pays a constant price, reflecting the (maximal) quality difference between the two suppliers. Therefore, to implement any (v_1, v_2), a buyer has to pay at least $\Delta q\,(v_1, v_2)\,(F\,(v_1) + 1 - F\,(v_2))$ in expectation to the suppliers.

3.2 Characterizing the Optimum

We now turn to our main results. Before identifying the optimal contest for the buyer, we first show that bonus tournaments can implement a wide range of allocations.

Proposition 1 *Any (v_1, v_2) such that $0 < v_1 \leq 1/2 \leq v_2 < 1$ can be implemented by a bonus tournament with $\mathcal{P} = \{A, 0\}$, where $A = \Delta q(v_1, v_2)$ and $t \geq \max\{C - AF(v_1), C - A(1 - F(v_2)), 0\}$. In particular, the social optimum can be implemented.*

Thus, the buyer can implement any desired diversity in a bonus tournament. Whereas in a standard contest, effort incentives are provided by the spread between winner and loser prizes, incentives for diversity in our model come from the spread between the high winner prize and the low winner prize.

The equilibrium pricing strategies turn out to be $p_1(), p_2()$ such that $p_i\,(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise. Implementation is not unique, as a bonus tournament will generally admit many equilibria. In particular, if $v_i^* < v_j^*$ are equilibrium choices in a bonus tournament, then so are any v_i, v_j such that $|v_i - v_j| = |v_i^* - v_j^*|$ and $v_i \leq 1/2 \leq v_j$.

The supplier only asks for the bonus A when his quality advantage is maximal ($\sigma \in [0, v_1]$ for supplier 1 and $\sigma \in [v_2, 1]$ for supplier 2); otherwise he accepts the low price. Therefore, the buyer pays the lowest price compatible with Lemma 3 for $\sigma \in [0, v_1] \cup [v_2, 1]$. Clearly, the price 0 is also minimal on (v_1, v_2). Thus, non-convexity of the price set $\mathcal{P}$ is a crucial characteristic of optimal contests. If the price set $\mathcal{P}$ included any additional price between 0 and A, this would only increase the payments to the suppliers, without increasing diversity. The bonus tournament is thus

[20]The constancy of the quality differential reflects the linearity of quality in distance (A1).

a flexible instrument with which the buyer can fine-tune diversity with low supplier revenues. This suggests that the optimal contest is in this class. However, this intuition is incomplete, as it does not account for subsidies. We now show that it is nevertheless always optimal for the buyer to use bonus tournaments. However, she will not always implement the social optimum.

Theorem 1 *(i) The buyer optimum can be implemented with a suitable bonus tournament* $(\{A, 0\}, t)$ *in which the suppliers obtain an expected surplus of zero.*
(ii) If $C \geq F(v_1^*)\,\Delta q(v_i^*, v_j^*)$*, the optimal contest for the buyer is a bonus tournament that implements the social optimum, with* $A = \Delta q(v_i^*, v_j^*)$ *and* $t = C - F(v_1^*)\,\Delta q(v_i^*, v_j^*)$*.*
(iii) If $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$*, the optimal contest for the buyer is a bonus tournament that implements suboptimal approaches* $(v_1, v_2) \neq (v_1^*, v_2^*)$*, with* $A = \Delta q(v_1, v_2)$ *and* $t = 0$*.*

Whereas (i) states the optimality of bonus tournaments, (ii) and (iii) specify the details for the two different parameter regions. The participation constraints imply that the total revenue of each supplier has to be at least C, regardless of the approaches implemented. Thus, if the expected revenues needed to implement the socially optimal approaches are below C, then the buyer implements the social optimum and uses subsidies to satisfy the participation constraint. But when these payments are above C, the buyer can reduce payments to the suppliers by implementing inefficient approaches, and it is optimal for her to do so.

It may seem redundant to allow for both $a > 0$ and subsidies t in a bonus tournament. Indeed they can be used as substitutes under certain conditions. Any equilibrium of a bonus tournament $(\{A, 0\}, t)$ with $t > 0$, which is symmetric around $1/2$ (that is, $v_1 + v_2 = 1$), is also an equilibrium of the bonus tournament $(\{A + a, a\}, t - a/2)$ if a is below a certain threshold.[21] If a increases, so does the incentive for suppliers to deviate towards the center and so increase the probability of winning the contest, which now results in (at least) the prize a. When a is too large, this incentive is too strong and such a bonus tournament does not implement any diversity. Thus, a cannot always be used as a substitute for t.[22]

4 Auctions and Fixed Prize Tournaments

In Section 3.2, we characterized the optimal contest. We now study two other types of contests that are discussed in the literature, namely scoring auctions and fixed-prize tournaments.

Proposition 2 *(i) For any subsidy* $t \geq \max\left\{0, C - \int_0^{1/2} \left(q\left(v_1^*, \sigma\right) - q\left(v_2^*, \sigma\right)\right) dF\left(\sigma\right)\right\}$*, the auction mechanism (*$\mathcal{P} = \mathbb{R}^+$*) implements the social optimum. (ii) For any* $A \geq 2C$*, the unique equilibrium*

[21] Using similar arguments as in the proof of Proposition 3 in the online appendix, it can be shown that this threshold is $\min\{2t, F(v_1)/(F(v_2) - 1/2)\}$.

[22] In Online Appendix B.1, we show how $a > 0$ can be useful when shirking is possible.

of an FPT ($\mathcal{P} = \{A\}$) implements $(v_1, v_2) = (1/2, 1/2)$. (iii) Whenever $C < F(v_1^*) \Delta q(v_i^*, v_j^*)$, the buyer prefers the inefficient FPT to the efficient auction.

It is intuitive that auctions implement some diversity: With identical approaches, no supplier would earn a positive revenue. The absence of diversity in an FPT corresponds to the principle of minimum differentiation in the standard model of locational competition with fixed prices (Hotelling 1929) and to the median voter theorem (Downs 1957).[23] As the size of the prize is independent of quality differences in an FPT, the suppliers only maximize the expected winning probability. This implies moving to the center.[24]

As to 2(iii), even though an auction implements the social optimum, it can leave rents to the suppliers. When such rents are high, the buyer prefers to use a suitable FPT. A bonus tournament combines the advantages of FPTs and auctions: It can increase efficiency without the necessity of paying high rents to the suppliers.

Consistent with the logic of Theorem 1(iii), the following result shows that the buyer never resolves the trade-off in favor of efficiency when costs are low.

Corollary 1 *Let $C = 0$. Among all contests where $\mathcal{P}$ is convex, the buyer's surplus is maximal in an FPT with $A = 0$.*

Corollary 1 relies on the fact that higher quality suppliers bid the sum of the quality differential and the minimum $\underline{P}$ when available (Lemma 2). Thus the buyer surplus, the difference between the expected maximal quality and the expected payment, is the difference between the expectation of the minimum quality and the minimum bid. The optimum is an FPT with $A = 0$, because this maximizes the minimum quality and minimizes the minimum bid.

5 Discussion

We now argue that, to some extent, our results are robust to alternative technological assumptions, and we also consider alternative constraints on the allowable mechanisms. All details are in the online appendix.

Inducing Effort. The model implicitly assumes that, once a supplier joins a contest, he cannot shirk by reducing effort.[25] It is therefore possible to focus on implementing diversity in

[23]However, the voting literature has also discussed why parties might differentiate by choosing "polarized platforms" (as in Wittman 1977, 1983). On a broadly related note, the relative weight on accuracy and publicity of forecasts determines whether or not experts want to cluster on the most likely outcome (Laster, Bennett and Geoum 1999).

[24]The result that there is no diversity in an FPT relies on the symmetry of suppliers, in particular, that they share the same belief about the likelihood of success of different approaches. In reality, different suppliers are likely to disagree about which approach is promising and which is not. If this was the case, even an FPT would result in some diversity in equilibrium.

[25]This follows because all research costs are identical.

contests, while shutting down the effects of contest design on effort incentives. This was used above for the result that, when $C \geq F(v_1^*)\,\Delta q(v_i^*, v_j^*)$ (but, by (A1), still low enough that the contest generates a positive surplus), the bonus tournament implementing the social optimum has $a = 0$ and positive subsidies to ensure participation. When suppliers have the option to shirk, the analysis is more subtle, because the buyer can no longer rely on subsidies. We show that as long as $C \leq F(v_2^*)\,\Delta q(v_i^*, v_j^*)$, bonus tournaments are still optimal. If $C \in (F(v_1^*)\,\Delta q(v_i^*, v_j^*), F(v_2^*)\,\Delta q(v_i^*, v_j^*)]$ the optimum is implemented with $a > 0$. Hence, a positive low price can act as a substitute for subsidies in this case. Finally, we show that for any F, there exists some finite $\bar{C}$, such that if $C > \bar{C}$, no contest implements any diversity.

Heterogeneous Costs. We relax the assumption that the cost of developing all approaches is the same. Specifically, we suppose both suppliers have the same symmetric, convex cost function with the minimum at 1/2.[26] Given such a cost function, inducing diversity becomes more costly. Though we cannot prove that the bonus tournament remains optimal, the following results still hold as long as the cost heterogeneity is not above a threshold (specified in the appendix): (1) It is socially optimal to induce variety; (2) FPTs induce no variety; (3) Both bonus tournaments and auctions can induce socially optimal variety; (4) Bonus tournaments do so with lower cost to the buyer. Since choosing the least costly approach is similar to shirking, it is impossible to induce variety when cost heterogeneity is too large (for the same reason as when subsidies are not possible).

Generalized Distributions and Quality Functions. Further, we replace (A1) and (A2) with weaker assumptions. Instead of (A1), we require that quality is a decreasing (but not necessarily linear) function of the distance between the ideal state and the density function. We replace (A2) by requiring that $f(\sigma)$ is symmetric and has full support, but not that it is single peaked and relatively flat, so that (A2)(ii) and (A2)(iv) might be violated. We show that under these assumptions the bonus tournament and the auction mechanism continue to implement the social optimum, whereas there still is no diversity with an FPT. Moreover, we show that a suitable bonus tournament still implements the buyer optimum if costs are above a threshold (specified in the appendix). For lower cost, the buyer strictly prefers a suitable bonus tournament to the FPT, and she prefers to implement the social optimum with a bonus tournament rather than with an auction.

The Number of Suppliers. As will be discussed in Section 6, several papers show that, when contests incentivize effort choice, the optimal number of participants is typically two. In our setting, it may be socially optimal and optimal for the buyer to invite a large number of suppliers. In particular, in a bonus tournament an increase in n not only leads to an increase in the expected

[26] We did not treat the case that the low-cost approaches differ across suppliers. We conjecture that, in such a setting, diversity would even arise in an FPT with two suppliers.

quality (reflecting higher option value), but also to a reduction in supplier rents (reflecting an increase in competition). Specifically, with $n > 3$ suppliers and uniform state distributions the social optimum can still be achieved with a suitable bonus tournament or auction.[27] For costs that are sufficiently large (but still low enough that a bonus tournament can generate positive expected surplus), the optimal n-supplier contest for the buyer is a bonus tournament. While we cannot establish optimality of bonus tournaments for lower costs, we find that: (i) The buyer who wants to implement the social optimum strictly prefers to do so with a bonus tournament rather than with an auction; and (ii) the buyer prefers to implement the social optimum with a bonus tournament over any outcome of an FPT. This holds even though the FPT, in contrast to the stark result for the two-player case, induces some diversity, but less than socially optimal. In addition, we characterize the socially optimal number of suppliers.

For uniform state distributions, we also consider the option of dealing with only one supplier. In any single-supplier contract, the buyer's expected payoff cannot be greater than the maximum social surplus given a single supplier. Suppose that the buyer obtains this maximum social surplus. We show that if $C \leq F(v_1^*)\,\Delta q(v_1^*, v_2^*) = b/8$, there exists $n \geq 2$ for which the buyer is weakly better off in an n-supplier bonus tournament than when using the optimal single-supplier contract. The preference is strict for $C < b/12$. It turns out that for uniform state distributions, if $C > b/8$, research is so costly that the socially optimal number of suppliers is 1.[28] Thus, whenever it is socially optimal to have two or more suppliers, the buyer is better off holding a bonus tournament than using any single-supplier contract.

Furthermore, there are several reasons why extracting the maximum social surplus might be difficult with a single-supplier contract. In the absence of verifiable information, the buyer cannot write contracts enforcing a particular approach. Thus, even an arbitrarily small amount of cost heterogeneity could induce the supplier to choose a suboptimal approach. Moreover, single-supplier contracts cannot induce costly effort (see Che and Gale, 2003). Thus, bonus tournaments have advantages over single-supplier contracts even when $C > b/8$.

Multiple Prizes. A full analysis of multiple prizes is beyond the scope of this paper. However, we can show that, at least in an FPT, the buyer has nothing to gain from using multiple prizes.[29] For $n > 3$, for any equilibrium in an FPT with two prizes $A_1 > A_2 > 0$, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off. The proof of this result shows that any equilibrium of an FPT with two prizes involves more duplication than the chosen

[27] Though most results also apply to the case $n = 3$, an FPT does not have a pure strategy equilibrium in this case.

[28] Under assumption (A1), $C < (\Psi - b)/2$, so that not inducing research at all is never optimal.

[29] Of course, there may be reasons outside of the model which would make multiple prizes a desirable choice for a contest designer. For example, if suppliers are risk averse, providing multiple prizes may be a way of increasing their expected utility.

equilibrium of an FPT with a single prize, which leads to a lower buyer surplus.[30]

Participation Fees. A buyer who could charge participation fees $e > 0$ would do this only if $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, in which case the optimal fee e^* satisfies $C + e^* = F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, so that she achieves the first-best. If the buyer is limited to setting fees below e^*, she will charge the maximum allowable fee. With or without participation fees, the buyer thus designs the contest so that the suppliers obtain zero expected surplus. Moreover, the bonus tournament is still optimal with participation fees. However, contrary to the case without participation fees, the buyer no longer has to trade off efficiency and rents, so that she induces the optimal diversity.

Knowledge of realized qualities. The assumption that a supplier learns not only his own quality, but also his competitor's, is important for our analysis. The suppliers can learn each other's quality, for example, during testing in the so-called "fly-off" competitions commonly used by the U.S. Air Force when developing new aircraft.[31] Similarly, in architectural competitions submitted designs are commonly made public before the winner is chosen.

Except in the FPT, where the current analysis carries over directly, knowledge of the opponent's quality is important for the pricing behavior of the suppliers. If only own quality was known to each supplier, then the suppliers would have to make inferences about the quality of the opponent from observing their own quality. Because the correlation between qualities is endogeneously determined by the choice of research approaches, this is a non-trivial problem which is beyond the scope of the current paper.

6 Relation to the Literature

This paper contributes to the literature on optimal contest design, especially the design of innovation contests. The existing design literature focuses exclusively on effort incentives. In models of fixed-prize tournaments, Taylor (1995) shows that free entry is undesirable, and Fullerton and McAfee (1999) show that the optimal number of participants is two. Fullerton and McAfee (1999) and Giebe (2014) consider the use of entry auctions in order to select the most efficient contestants. Fullerton,

[30] To interpret the model, one should bear in mind that typical arguments for multiple prizes in contests rely on convexity of effort costs and/or supplier heterogeneity (see Moldovanu and Sela 2001 and more recently Olszewski and Siegel 2018; the latter also consider the effect of risk aversion in addition to convexity of effort costs). Our model does not have these features.

[31] In 1974, the U.S. Air Force held a fly-off contest between two prototypes: General Dynamics' YF-16 and Northrop's YF-17. While the fly-off was ongoing, details from the tests and aircraft characteristics were often made public. For example, an aviation magazine published a detailed description of the two aircraft and their performance in August 1974, five months before the YF-16 won the fly-off (eventually becoming the F-16); see "YF-16 and YF-17: fighters for the future" (D. Godfrey, *Flight International*, August 1, 1974).

Linster, McKee and Slate (2002) find that buyers are better off with auctions than with fixed-prize tournaments. Che and Gale (2003) show that an auction with two suppliers is the optimal contest. Contrary to the previous literature, our paper focuses on the suppliers' choice of research approaches rather than on effort levels. We characterize the optimal two-supplier contests in such settings, highlighting in particular the useful role of bonus tournaments.

Letina (2016) also studies the diversity of approaches to innovation, but the objects of analysis and the employed models are different. He focuses on a market context with anonymous buyers, and he deals with comparative statics rather than optimal design. In particular, the paper finds that a merger decreases the diversity of approaches to innovation.

While we are not aware of any other paper that considers optimal contest design when diversity plays a role, some authors compare contests in related, but different settings. In Ganuza and Hauk (2006), suppliers choose both an approach to innovation and a costly effort.[32] However, these authors focus exclusively on fixed-prize tournaments, while we study the optimal contest design. Erat and Krishnan (2012) analyze a fixed-prize tournament where suppliers can choose from a discrete set of approaches.[33] The authors find that suppliers cluster on approaches delivering the highest quality. This result is related to our result that there is duplication of approaches in the equilibria of fixed-prize tournaments.[34] Schöttner (2008) considers two contestants who influence quality stochastically by exerting effort. She finds that, for large random shocks, the buyer prefers to hold a fixed-prize tournament rather than an auction to avoid the market power of a lucky seller in an auction. This resembles the trade-off underlying our Proposition 2. However, her analysis does not speak to optimal design and the role of bonus tournaments.[35]

Gretschko and Wambach (2016) analyze the design of mechanisms for public procurement when exogenously differentiated suppliers offer different specifications, and the buyer does not know her preferences. The modelling of buyer utility is similar to ours. However, the paper does not deal with the question of inducing variety. Instead the authors ask whether intransparent negotiations

[32]In Ganuza and Pechlivanos (2000), Ganuza (2007) and Kaplan (2012), the buyer has to choose the design or alternatively can reveal information about the preferred design.

[33]See also Terwiesch and Xu (2008) for the effect of number of suppliers when exogeneous random shocks are large. For empirical evidence see Boudreau, Lacetera and Lakhani (2011).

[34]In addition to allowing for alternative contests, our model also considers correlated rather than independent qualities; it is thus meaningful to speak of similar approaches. See also Konrad (2014) for a variant of Erat and Krishnan's model where the first best is restored if the tie-breaking is decided via costly competition (for example lobbying) as opposed to randomly.

[35]More broadly related is Bajari and Tadelis (2001) who study contracting for construction projects. The supplier obtains new information during the contract execution, which allows him to adapt the original approach at some cost. Since the relationship is between a buyer and only one supplier, the question of variety of approaches does not arise. This is also true for the related work by Arve and Martimort (2016) who study risk-sharing considerations in the design of contracts with ex-post adaptation. Additionally, Ding and Wolfstetter (2011) consider a case where a supplier can choose to bypass the contest and negotiate with the buyer directly in an environment where innovation quality is obtained by expending costly effort.

or transparent auctions yield higher social surplus.

Our paper is also related to the literature on innovation contests with exponential-bandit experimentation (see Halac, Kartik and Liu 2017 and references therein). In these models, it is uncertain whether the innovation is feasible. Suppliers participating in the contest expend costly effort to learn the state, and they also learn from their opponents' experimentation. The goal of the contest is to induce experimentation. However, each supplier experiments in the same way. In our model, suppliers are induced to develop different projects.

Like in rank-order tournaments and Tullock contests, but contrary to all-pay auctions, our technology is stochastic, and there are pure-strategy equilibria.[36] However, while the random shocks are i.i.d. in all the papers cited above, in our model they are not only correlated, but the contestants also determine the level of correlation by choosing research projects. The buyer wants to induce diversity of research approaches exactly to reduce correlation in outcomes, which in turn results in the option value discussed before.

Our paper is also related to the literature on policy experimentation. For instance, Callander and Harstad (2015) show that decentralized policy experimentation yields too much diversity. Contrary to our model, they assume that the success probabilities of different experiments are independent, no matter how similar the policies are. This assumption removes the option value of having different experiments, which is central to our model. If there existed an ideal policy (in terms of quality) as in our model, then the option value would have to be traded off against the benefits of convergence emphasized by Callander and Harstad (2015).[37]

7 Conclusions

Our paper investigates how uncertainty about the ideal approach to innovations affects contest design. In our model, it is socially optimal for suppliers to take diverse research approaches, and the social optimum can be obtained with bonus tournaments and auction mechanisms. Inducing diversity of approaches to innovation can give rents to suppliers. To reduce these rents, the buyer may therefore want to induce suboptimal diversification. Our main result is that bonus tournaments are optimal for the buyer. The difference between the bonus and the low price provides incentives for suppliers to diversify, which allows the buyer to fine-tune the amount of diversity induced. At the same time, bonus tournaments minimize the suppliers' power to exploit their quality advantage. The non-convexity of the price set is decisive for this feature. Moreover, we find that for a suitable parameter range the optimal bonus tournament implements the social optimum.

[36] See Baye, Kovenock and de Vries (1996) and Che and Gale (2003) for all-pay auctions, Lazear and Rosen (1981), Fullerton and McAfee (1999), Schöttner (2008) and Giebe (2014) for tournaments; see also the general discussion in Konrad (2009).

[37] See also Bonatti and Rantakari (2016).

Our stylized model has potential implications for the design of innovation contests. In addition to the baseline prize, it might be useful to pay a bonus prize whenever the winner outperforms the second-best contestant by a sufficient margin.[38] Even though we are not aware of such contests being used in practice, bonus prizes would seem easy to implement and would not make the innovation tournaments significantly more complicated than they are today. Bonus prizes would give incentives to contestants to not only win the contest, but to win with a large margin. In the simple setting analyzed in this paper, this incentive would lead to an increase in the diversity of approaches to innovation.

A Appendix

A.1 Basics

In the following, we introduce some notation that we use throughout the appendix. We also formulate the restrictions implied by subgame perfection.

A.1.1 Notation

We consistently use subscripts B for buyers, $i = 1, 2$ for suppliers and T for "total" (buyers plus suppliers). Superscripts such as fpt for fixed-price tournament, bt for bonus tournament or a for auction refer to the contest $\mathcal{P}$ under consideration. We will drop these superscripts whenever there is no danger of confusion.

1. $p_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b,\Psi]^2}$ is a *price strategy function.*[39]

2. $\pi_i(p_i, p_j|\, q_i, q_j)$ is the realized revenue that supplier i earns with prices p_1 and p_2, conditional on qualities q_1 and q_2, assuming that the buyer chooses the i sequentially rationally.

3. $\widehat{\Pi}_i(v_i, v_j, p_i(), p_j())$ is the expectation over $\pi_i(p_i, p_j|\, q_i, q_j)$ when suppliers choose v_1,v_2, $p_1()$ and $p_2()$, where the expectation is taken over all pairs of quality realizations for given (v_1, v_2).

4. $\Pi_i^{\mathcal{P}}(v_i, v_j) = \widehat{\Pi}_i(v_i, v_j, p_i(), p_j())$, where $p_i()$ and $p_j()$ are the subgame equilibria for the contest $\mathcal{P}$ as in Lemma 2, is the *(expected) revenue* of supplier i.

5. $S_i^{\mathcal{P}}(v_i, v_j) = \Pi_i^{\mathcal{P}}(v_i, v_j) + t - C$ is the *(expected) surplus* of supplier i.

6. $Q(v_1, v_2, \sigma)$ is the quality that the buyer obtains, given a realization of σ. Given price functions as in Lemma 2 and sequential rationality of the buyer, in every subgame $Q(v_1, v_2, \sigma) = \max\{q(v_1, \sigma), q(v_2, \sigma)\}$.

[38] This does not require that performance differentials are verifiable; observability of quality suffices: It is in the buyer's own interest to select the high-quality supplier even though he demands the bonus prize.

[39] For sets X and Y, Y^X is the set of all mappings from X to Y.

7. $S_B^{\mathcal{P}}(v_i, v_j) = E_\sigma[\max\{q(v_1,\sigma), q(v_2,\sigma)\}] - \Pi_1^{\mathcal{P}}(v_i,v_j) - \Pi_2^{\mathcal{P}}(v_i,v_j) - 2t$ is the *(expected) surplus* of the buyer.

A.1.2 Subgame-Perfect Equilibrium

A subgame-perfect equilibrium of the innovation contest given by $\mathcal{P}$ consists of supplier strategies $s_i = (v_i, p_i) \in [0,1] \times \mathcal{P}^{[\Psi-b,\Psi]^2}$ and buyer strategies $\nu \in \{1,2\}^{(\mathcal{P}\times[\Psi-b,\Psi])^2}$ such that:

(DC1) ν is sequentially rational. That is, if $\nu = i$ then $q_i - p_i \geq q_j - p_j$.

(DC2) $\pi_i(p_i(q_i,q_j), p_j(q_j,q_i)|\, q_i, q_j) \geq \pi_i(p_i', p_j(q_j,q_i)\,|q_i,q_j)$ for all $p_i' \in \mathcal{P}$,$(q_i,q_j) \in [\Psi-b,\Psi]^2$ (sequential rationality of supplier i)

(DC3) $\widehat{\Pi}_i(v_i, v_j, p_i(), p_j()) \geq \widehat{\Pi}_i(v_i', v_j, \widetilde{p}_i(), p_j())$ for all $v_i' \in [0,1]$ and all $\widetilde{p}_i() \in \mathcal{P}^{[\Psi-b,\Psi]\times[\Psi-b,\Psi]}$ (best-response condition for supplier i).

(PC) $\widehat{\Pi}_i(v_i, v_j, p_i(), p_j(q_j,q_i)) + t \geq C$ (participation constraint for supplier i).

A.1.3 Tie-breaking rules

(T1) (Preference for quality) If $q_i - p_i = q_j - p_j$ and $q_i > q_j$ then $\nu = i$. If $q_i - p_i = q_j - p_j$ and $q_i = q_j$ then $\nu = i$ with probability 1/2 and $\nu = j$ with probability 1/2.

(T2) (Preference for winning) For any two strategies $(v_i, p_i(\cdot))$ and $(v_i', p_i'(\cdot))$ of the supplier i, if $\widehat{\Pi}_i(v_i, v_j, p_i(), p_j()) = \widehat{\Pi}_i(v_i', v_j, p_i'(), p_j())$ and $\Pr(\nu = i|v_i, p_i(\cdot)) > \Pr(\nu = i|v_i', p_i'(\cdot))$, then supplier i prefers $(v_i, p_i(\cdot))$.

A.2 Proofs of Auxiliary Results (Section 3.1)

A.2.1 Proof of Lemma 1

Suppose, without loss of generality, that $v_1 \leq v_2$. The total surplus is

$$S_T(v_1,v_2) = \int_0^1 \max\{q(v_1,\sigma), q(v_2,\sigma)\} dF(\sigma) - 2C =$$

$$\Psi - b\left(\begin{array}{c} \int_0^{v_1} (v_1-\sigma)\, dF(\sigma) + \int_{v_1}^{(v_1+v_2)/2} (\sigma - v_1)\, dF(\sigma) + \\ \int_{(v_1+v_2)/2}^{v_2} (v_2-\sigma)\, dF(\sigma) + \int_{v_2}^{1} (\sigma - v_2)\, dF(\sigma) \end{array}\right) - 2C.$$

This is a continuous function with a compact domain, hence it attains the maximum. Note that

$$\frac{\partial S_T(v_1,v_2)}{\partial v_1} = b\left(-2F(v_1) + F((v_1+v_2)/2)\right) \tag{1}$$

$$\frac{\partial S_T(v_1,v_2)}{\partial v_2} = b\left(1-2F(v_2) + F((v_1+v_2)/2)\right). \tag{2}$$

(1) and (2) imply that there are no boundary optima. To see this, first note that $\partial S_T(0,v_2)/\partial v_1 > 0 \forall v_2 > 0$ and $\partial S_T(v_1,1)/\partial v_2 < 0 \forall v_1 < 1$. Moreover $(v_1,v_2) = (0,0)$ and $(1,1)$ are both dominated by $(1/2,1/2)$. Thus, the optimum must satisfy

$$-2F(v_1) + F((v_1+v_2)/2) = 0 \tag{3}$$

$$1-2F(v_2) + F((v_1+v_2)/2) = 0. \tag{4}$$

Together these conditions imply $F(v_2^*) = 1/2 + F(v_1^*)$.

For $v_1 \in [0,1/2]$, let $g(v_1) = F^{-1}\left(F(v_1) + \frac{1}{2}\right)$. F^{-1} is well-defined because of (A2)(iii). Inserting $v_2 = g(v_1)$ in (3) and (4), the first-order conditions hold for $(v_1,v_2) = (v_1, g(v_1))$ if

$$v_1 = F^{-1}\left(\frac{F((v_1+g(v_1))/2)}{2}\right). \tag{5}$$

(5) has at least one solution $v_1^* \in (0,1/2)$. This holds because both sides of (5) are strictly increasing, and the r.h.s. is positive for $v_1 = 0$ and strictly less than $1/2$ for $v_1 = 1/2$. Now consider $(v_1^*, v_2^*) = (v_1^*, g(v_1^*))$ such that $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Thus $F(v_2^*) = F(v_1^*) + 1/2$. Moreover, symmetry implies $v_1^* + v_2^* = 1$ and thus the r.h.s. of (5) is $F^{-1}(1/4)$, so that the first-order condition holds for (v_1^*, v_2^*).

Before proceeding, we prove one intermediate step.

Lemma 4 *If (A2) is satisfied, then $f(x) < 2f(y)$ for all $x, y \in [0,1]$.*

Proof. First note that $f(1/2) = \int_0^{1/2} f'(x)\,dx + f(0)$. Since by (A2)(iv) $f'(x) < 2f(0)$ for all $x \in [0,1/2]$, it follows that $f(1/2) < \int_0^{1/2} 2f(0)\,dx + f(0) = 2f(0)$. By (A2)(ii) $f(x) \le f(1/2)$ and $f(0) \le f(y)$ for all $x, y \in [0,1]$, the statement in the Lemma follows.□

Finally, consider the Hessian matrix

$$H = b \cdot \begin{bmatrix} -2f(v_1) + \frac{1}{2}f((v_1+v_2)/2) & \frac{1}{2}f((v_1+v_2)/2) \\ \frac{1}{2}f((v_1+v_2)/2) & -2f(v_2) + \frac{1}{2}f((v_1+v_2)/2) \end{bmatrix}.$$

First, H is negative definite at (v_1^*, v_2^*) if and only if $f(1/2) < 2f(v_1^*)$. To see this, note that $f(v_1^*) = f(v_2^*)$ and $f((v_1^*+v_2^*)/2) = f(1/2)$. Hence,

$$-2f(v_1^*) + \frac{1}{2}f((v_1^*+v_2^*)/2) = -2f(v_1^*) + \frac{1}{2}f(1/2) < 0 \Leftrightarrow f(1/2) < 4f(v_1^*).$$

In addition,

$$|H| = b\left[4f(v_1^*)f(v_2^*) - (f(v_1^*) + f(v_2^*))f((v_1^*+v_2^*)/2)\right] = b\left(4f(v_1^*)^2 - 2f(v_1^*)f(1/2)\right).$$

This condition holds if and only if $f(1/2) < 2f(v_1^*)$, which holds by Lemma 4.

Second, H is negative definite $\forall (v_1, v_2)$ if $f(1/2) < 2f(0)$. To see this, note that $f(v)$ is minimized at $v = 0$ and maximized at $v = 1/2$. Hence, $f(1/2) < 2f(0) < 4f(0)$ implies

$$-2f(v_i) + \frac{1}{2} f\left(\frac{v_1 + v_2}{2}\right) \leq -2f(0) + \frac{1}{2} f\left(\frac{1}{2}\right) < 0 \ \forall i \in \{1, 2\}.$$

and

$$|H| = b\left[f(v_1)\left(2f(v_2) - f\left(\frac{v_1 + v_2}{2}\right)\right) + f(v_2)\left(2f(v_1) - f\left(\frac{v_1 + v_2}{2}\right)\right)\right] > 0.$$

Therefore, $f(1/2) < 2f(0)$, which holds by Lemma 4, is a sufficient condition for (v_1^*, v_2^*) to be the unique global optimum.

A.2.2 Proof of Lemma 2

Consider the equilibrium for the subgame defined by (v_1, v_2, σ) and the resulting quality vector (q_1, q_2).

Step 1: *Pricing subgame for $q_1 = q_2$.*

If $q_1 = q_2$, the standard Bertrand logic implies that $(\overline{p}(q_1, q_2), \overline{p}(q_1, q_2)) = (\underline{P}, \underline{P})$ is the unique equilibrium.

Step 2: *Pricing subgame for $q_i > q_j$.*

Clearly, if $q_i > q_j$ the suggested strategy profile is a subgame equilibrium. To see that i must bid $\overline{p}(q_i, q_j)$ in equilibrium, first suppose $p_i > \overline{p}(q_i, q_j)$. If $p_i > p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier j wins. By setting $p_i = \overline{p}(q_i, q_j) \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i can ensure that he wins, which is a profitable deviation by (T2). If $p_i > \overline{p}(q_i, q_j)$ and $p_i \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i wins. By setting $p_j = \underline{P}$, supplier j can profitably deviate. If $p_i < \overline{p}(q_i, q_j)$, supplier i can deviate upwards to $\overline{p}(q_i, q_j)$. He then still wins by (T1), and revenues are higher.

A.2.3 Proof of Lemma 3

(i) The result is trivial for $v_1 = v_2$. For $v_1 < v_2$, we show that supplier 1 can profitably deviate to some $v_1' > v_1$ if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$.

Step 1: *If $v_1 < v_2$, then after any deviation to $v_1' \in (v_1, v_2)$ the probability that supplier 1 wins strictly increases.*

Before the deviation, supplier 1 has higher quality (and therefore wins) whenever $\sigma < (v_1 + v_2)/2$. Thus, before the deviation, the probability that supplier 1 wins is $F((v_1 + v_2)/2)$. Using the same argument, the probability that supplier 1 wins after the deviation is $F((v_1' + v_2)/2) > F((v_1 + v_2)/2)$, since $v_1' > v_1$. Step 1 thus follows.

Step 2: *There exists a deviation $v_1' \in (v_1, v_2)$ such that after this deviation, supplier 1 wins and receives a weakly higher price than before deviation for all $\sigma < (v_1 + v_2)/2$.*

First, note that for any $\sigma \in [0, v_1]$ the quality difference between the two suppliers, that is $q(v_1, \sigma) -$

$q(v_2, \sigma)$, is constant. Then, by Lemma 2, supplier 1 receives the same price in all those states of the world, which is given by $\bar{p}(q(v_1, v_1), q(v_2, v_1))$. In all states $\sigma \in (v_1, (v_1+v_2)/2]$, supplier 1 wins with a price that is weakly lower than $\bar{p}(q(v_1, v_1), q(v_2, v_1))$, because the quality difference is smaller than the maximal quality difference. Since $\Delta q\,(v_1, v_2) + \underline{P} \notin \mathcal{P}$, it must be that $\bar{p}(q(v_1, v_1), q(v_2, v_1)) < \Delta q\,(v_1, v_2) + \underline{P}$. By continuity, there exists some $v_1' \in (v_1, v_2)$ such that $\bar{p}(q(v_1, v_1), q(v_2, v_1)) \leq \Delta q\,(v_1', v_2) + \underline{P}$. Consider a deviation to such v_1'. For any $\sigma \in [0, v_1']$, supplier 1 receives the price $\bar{p}(q(v_1, v_1), q(v_2, v_1))$. For any $\sigma \in (v_1', (v_1 + v_2)/2]$, we have $q(v_1', \sigma) > q(v_1, \sigma)$. Since $q(v_2, \sigma)$ is unchanged, the quality difference in those states of the world increases, and by Lemma 2 the price that supplier 1 receives is at least as high as before the deviation.
Combining Steps 1 and 2, v_1' is a profitable deviation by (T2), which proves the claim.
(ii) follows directly from Lemmas 2 and 3(i).

A.3 Proofs of Main Optimality Results (Section 3.2)

A.3.1 Proof of Proposition 1

Let $A = \Delta q(v_1, v_2)$ for some (v_1, v_2). We will show that, in the bonus tournament with $\mathcal{P} = \{A, 0\}$ and subsidies t, the strategy profiles $(v_1, v_2, p_1\,(), p_2\,())$ such that $p_i\,(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise, form an equilibrium.

Sequential rationality of $p_i\,()$ follows from Lemma 2. We now show that $(v_1, p_1\,())$is a best response of supplier 1 to $(v_2, p_2\,())$; the argument for supplier 2 is analogous. For $A = 0$, only $(v_1, v_2) = (1/2, 1/2)$ satisfies the above conditions. The proof of Lemma 5 below shows that, in this case, (v_1, v_2) can be implemented with a fixed-prize tournament with $A = 0$. If $v_1 < v_2$, $\Delta q\,(v_1, v_2) > 0$, and the probability that supplier 1 wins with a positive prize is $F\,(v_1)$. We will consider three possible types of deviations: (i) deviating to $v_1' < v_1$, (ii) deviating to $v_1'' \in (v_1, \widetilde{v})$ where $\widetilde{v} = \min\{2v_2 - v_1, 1\}$, and (iii) deviations to $v_1''' \geq \widetilde{v}$. Note that if $\widetilde{v} = 2v_2 - v_1 < 1$, then the distance between v_1 and v_2 is exactly the same as the distance between v_2 and $\widetilde{v}$. Thus, deviations of type (i) and (iii) increase the distance between the chosen projects, while deviations of type (ii) decrease the distance between the chosen projects. Next we show that none of the deviations are profitable.

Deviating to $v_1' < v_1$ is not profitable, because the winning probability falls to $F\,(\widehat{v}_1)$, with $\widehat{v}_1 < v_1$ implicitly defined by $q\,(v_1', \widehat{v}_1) - q\,(v_2, \widehat{v}_1) = \Delta q\,(v_1, v_2)$, and the prize does not rise. It is not profitable to deviate to $v_1'' \in (v_1, \widetilde{v})$, since for such deviations, $\Delta q\,(v_1'', v_2) < \Delta q\,(\widetilde{v}, v_2) \leq \Delta q\,(v_1, v_2)$, so that the probability of winning a positive prize is 0. Finally, if $\widetilde{v} < 1$, deviating to $v_1''' \in [\widetilde{v}, 1]$ is not profitable. To see this, note that $\widetilde{v} = 2v_2 - v_1$ which implies $1 - \widetilde{v} = 1 - 2v_2 + v_1 \leq v_1$ since $v_2 \geq 1/2$. This implies, by symmetry of the state distribution, that $F\,(v_1) \geq 1 - F\,(\widetilde{v}) \geq 1 - F\,(v_1''')$. Thus, v_1''' is not a profitable deviation. By analogous arguments, there are no profitable deviations for supplier 2.

Finally, the expected surplus of the suppliers are $S_1 = AF(v_1) + t - C$ and $S_2 = A(1 - F(v_2)) + t - C$. Since $t \geq \max\{C - AF(v_1), C - A(1 - F(v_2)), 0\}$, it is immediate that $S_1 \geq 0$ and $S_2 \geq 0$.

By Lemma 1, the social optimal satisfies $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Clearly, it must be that $0 < v_1^* \leq 1/2 \leq v_2^* < 1$, and the social optimum can be implemented.

A.3.2 Proof of Theorem 1

The buyer optimally chooses $(v_1, v_2, p_1, p_2, \mathcal{P}, t) \in [0,1]^2 \times \left(\mathcal{P}^{[\Psi - b, \Psi]^2}\right)^2 \times \mathcal{I}(\mathbb{R}^+) \times [0, +\infty)$ so as to maximize

$$S_T(v_1, v_2) - \widehat{\Pi}_1(v_1, v_2, p_1(), p_2()) - \widehat{\Pi}_2(v_1, v_2, p_1(), p_2()) - 2t$$

such that, for all $i \in \{1, 2\}$ and $j \neq i$, (DC1)-(DC3) hold and PC holds for $i = 1, 2$.

(i) The statement follows from three main lemmas. Lemma 6 shows that allocations maximizing buyer surplus satisfy the conditions of Proposition 1 and can thus be implemented by a bonus tournament. Lemma 7 shows that implementation requires lower expected transfers than any alternative; hence buyer surplus is maximal. Finally, Lemma 8 shows that the suppliers optimally break even on expectation. Before proving these three lemmas, we prove a preliminary result about the unique equilibrium in an FPT, which is then used in the proof of Lemma 6.

Lemma 5 *In any FPT ($\mathcal{P} = \{A\}$ for $A \geq 2C$), the unique equilibrium is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

Proof. First, we show that the suggested (v_1, v_2) emerges as an equilibrium. Let v_j be such that $F(v_j) = 1/2$. Since f is everywhere positive, such a v_j is unique. Now if supplier $i \in \{1, 2\}$ plays $v_i = v_j$, his revenue is $\Pi_i(v_i, v_j) = A/2$. For any $v_i < v_j$ the revenue is $\Pi_i(v_i, v_j) = AF((v_i + v_j)/2) < A/2$. Similarly, for any $v_i > v_j$ the revenue is $\Pi_i(v_i, v_j) = A(1 - F((v_i + v_j)/2)) < A/2$. Thus, $v_i = v_j$ is an equilibrium. Second, $v_i' = v_j'$ is an equilibrium only if $F(v_j') = 1/2$. Suppose not. Then, a supplier i can profitably deviate to v_i such that $F(v_i) = 1/2$, since his revenue will be $\Pi_i(v_i, v_j) > A/2$. Third, $v_i \neq v_j$ is never an equilibrium. Suppose it was. Let $v_1 < v_2$. Then, the revenue of supplier 1 is $\Pi_1(v_1, v_2) = AF((v_1 + v_2)/2)$, while deviating to $(v_1 + v_2)/2$ leads to a revenue of $AF((v_1 + 3v_2)/4) > AF((v_1 + v_2)/2)$. ■

Lemma 6 *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium of a contest that maximizes buyer surplus, then $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$.*

We prove this lemma in two steps.

Step 1: *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium where w.l.o.g. $v_1^B \leq v_2^B$, then $v_1^B \leq 1/2 \leq v_2^B$.*

Proof. We will show that $v_1 \leq 1/2 \leq v_2$ must hold in any contest equilibrium. Suppose, to the contrary, that $v_1 \leq v_2 < 1/2$. The case that $1/2 < v_1 \leq v_2$ follows analogously. Let p_1, p_2 be the associated pricing strategies. Then, the expected revenue of supplier 1 is $\Pi_1(v_1, v_2) =$

$\int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. Consider the deviation $v_1' = 2v_2 - v_1 < 1$ with the same pricing function. Supplier 1 now wins whenever $\sigma > (v_2 + v_1')/2$. We can write the expected revenue as $\Pi_1(v_1', v_2) = \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma) + \int_{2v_2}^{1} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. Clearly, $(v_1 + v_2)/2 = 2v_2 - (v_1' + v_2)/2$. Moreover, there exists a bijective mapping $[0, (v_1 + v_2)/2] \to [(v_1' + v_2)/2, 2v_2]$; $\sigma' \mapsto \sigma''$ where $\sigma'' = 2v_2 - \sigma'$. Observe that $q(v_1, \sigma') = \Psi - b|v_1 - \sigma'| = \Psi - b|v_1 - 2v_2 + \sigma''| = \Psi - b|\sigma'' - v_1'| = q(v_1', \sigma'')$ and similarly $q(v_2, \sigma') = q(v_2, \sigma'')$. Thus, a property of this mapping is that $q(v_1, \sigma') - q(v_2, \sigma') = q(v_1', \sigma'') - q(v_2, \sigma'')$ and (by single-peakedness) $f(\sigma') \leq f(\sigma'')$. In a state where quality difference is the same, the winning price is also the same, so that $\int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma) \leq \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. As a result, $\Pi_1(v_1, v_2) \leq \Pi_1(v_1', v_2)$ and v_1' leads to strictly higher probability of winning, hence v_1' is a profitable deviation.[40] Thus, $v_1 \leq 1/2 \leq v_2$ must hold in any equilibrium; in particular, therefore $v_1^B \leq 1/2 \leq v_2^B$.

Step 2: *If* $(v_1^B, v_2^B, p_1^B, p_2^B)$ *is an equilibrium maximizing buyer surplus, then* $0 < v_i^B < 1$ *for* $i \in \{1, 2\}$.

Proof. By Step 1, we know that $v_1 \leq 1/2 \leq v_2$. Suppose $v_1^B = 0$ and $v_2^B = 1$. We will distinguish two cases, $C = 0$ and $C > 0$. First suppose $C = 0$. By single-peakedness (A2), $v_1 = v_2 = 1/2$ results in weakly higher total surplus than (v_1^B, v_2^B). As the allocation $(v_1, v_2) = (1/2, 1/2)$ can be implemented with an FPT and $A = 2C$ by Lemma 5, the buyer would be strictly better off than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$ where the suppliers earn positive surplus. Finally, observe that $v_1^B = 0$ and $v_2^B = 1$ cannot be implemented so that the suppliers earn zero surplus, as the suppliers could increase their probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. Next suppose $C > 0$. There exists some small ε such that $S_T(v_1^B = 0, v_2^B = 1) < S_T(\varepsilon, 1 - \varepsilon)$ and $F(\varepsilon)\Delta q(\varepsilon, 1 - \varepsilon) < C$. But then a bonus tournament with subsidy $t' = C - F(\varepsilon)\Delta q(\varepsilon, 1 - \varepsilon)$, and $\mathcal{P} = \{\Delta q(\varepsilon, 1 - \varepsilon), 0\}$ implements $(\varepsilon, 1 - \varepsilon)$, achieves higher total surplus, and the supplier surplus not higher than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$. Hence, the buyer surplus is higher, which is a contradiction.

Next suppose $v_1 = 0$ and $v_2 < 1$ (the case that $v_1 > 0$ and $v_2 = 1$ follows analogously). By Lemma 2, the revenue is $\Pi_1(0, v_2) = \int_0^{\frac{v_2}{2}} \bar{p}(q_1(0, \sigma), q_2(v_2, \sigma))\, dF(\sigma)$ for supplier 1 and $\Pi_2(v_2, 0) = \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(v_2, \sigma), q_1(0, \sigma))\, dF(\sigma) + \int_{v_2}^{1} \bar{p}(q_2(v_2, \sigma), q_1(0, \sigma))\, dF(\sigma)$ for supplier 2. Moreover, it must be $\Pi_1(0, v_2) > 0$, because otherwise supplier 1 could increase his probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. Single-peakedness (A2) implies

$$\int_0^{\frac{v_2}{2}} \bar{p}(q_1(v_1, \sigma), q_2(v_2, \sigma))\, dF(\sigma) \leq \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(v_2, \sigma), q_1(0, \sigma))\, dF(\sigma).$$

Suppose that this equilibrium is implemented with transfers t such that $t + \Pi_1(0, v_2) \geq C$. This implies $t + \Pi_2(v_2, 0) > C$. Further, using (1), $dS_T(v_1^B, v_2^B)/dv_1^B\big|_{v_1^B = 0} = bF(v_2/2) > 0$, so that there exists some $\bar{\varepsilon} > 0$ such that $S_T(\varepsilon, v_2^B) > S_T(0, v_2^B)$ for every $\varepsilon \in (0, \bar{\varepsilon})$. Fix ε such that

[40] Given the tie-breaking rule T2, this is even true for $p = 0$.

$F(\varepsilon)\Delta q(\varepsilon, v_2) \leq \Pi_1(0, v_2)$ and $F(\varepsilon) < 1 - F(v_2)$. Let $t' = t + \Pi_1(0, v_2) - F(\varepsilon)\Delta q(\varepsilon, v_2)$. Now consider a bonus tournament with subsidy t' and $\mathcal{P} = \{\Delta q(\varepsilon, v_2), 0\}$. By Proposition 1, this bonus tournament will implement (ε, v_2) if the participation constraint is met. This condition holds for both suppliers, because $t' + (1 - F(v_2))\Delta q(\varepsilon, v_2) > t' + F(\varepsilon)\Delta q(\varepsilon, v_2) \geq C$. Compared to the original situation with $v_1 = 0$ and $v_2 < 1$, the rent of supplier 1 is unchanged, but the rent of supplier 2 decreases since $\int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(v_2, \sigma), q_1(0, \sigma))\, dF(\sigma) + t > t'$ and $\int_{v_2}^{1} \bar{p}(q_2(v_2, \sigma), q_1(0, \sigma))\, dF(\sigma) > (1 - F(v_2))\Delta q(\varepsilon, v_2)$. Since the total surplus increases and the suppliers' surplus decreases, the buyer's surplus must increase. Therefore, the bonus tournament that implements (ε, v_2) increases the buyer surplus, which is a contradiction.□

Lemma 7 *If $\left(v_1^B, v_2^B, p_1^B, p_2^B\right)$ is an equilibrium of a contest maximizing buyer surplus, then it can be implemented by a contest with $\mathcal{P} = \{A, 0\}$.*

Proof. From Proposition 1 and Lemma 6, we know that the bonus tournament with $A = \Delta q\left(v_1^B, v_2^B\right)$ implements $\left(v_1^B, v_2^B\right)$. It remains to be shown that the buyer cannot implement $\left(v_1^B, v_2^B\right)$ with lower expected total transfers with any other contest. First, suppose that $v_1^B + v_2^B = 1$. By Lemmas 2 and 3, in any contest that implements $\left(v_1^B, v_2^B\right)$ the price paid by the buyer is exactly $\Delta q(v_1^B, v_2^B) + \underline{P}$ if $\sigma \in [0, v_1^B] \cup [v_2^B, 1]$ and it is at least 0 if $\sigma \in \left(v_1^B, v_2^B\right)$. Thus, if $\Delta q(v_1^B, v_2^B)F(v_1^B) > C$, a bonus tournament implements $\left(v_1^B, v_2^B\right)$ with the lowest possible expected total transfers. If $\Delta q(v_1^B, v_2^B)F(v_1^B) \leq C$, a bonus tournament with an appropriate t implements $\left(v_1^B, v_2^B\right)$ with zero expected supplier surplus. Next, consider an arbitrary contest implementing $\left(v_1^B, v_2^B\right)$ with $v_1^B + v_2^B < 1$ with subsidy t (the case $v_1^B + v_2^B > 1$ is analogous). The surplus of supplier 1 is then $S_1 = \Delta q(v_1^B, v_2^B)F(v_1^B) + \int_{v_1^B}^{\frac{v_1^B + v_2^B}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma) + t - C$, and for supplier 2 it is $S_2 = \Delta q(v_1^B, v_2^B)(1 - F(v_2^B)) + \int_{\frac{v_1^B + v_2^B}{2}}^{v_2} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma) + t - C$. By similar arguments as in Lemma 6, $\Delta q(v_1^B, v_2^B)F(v_1^B) < \Delta q(v_1^B, v_2^B)(1 - F(v_2^B))$ and $\int_{v_1^B}^{\frac{v_1^B + v_2^B}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma) \leq \int_{\frac{v_1^B + v_2^B}{2}}^{v_2} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. Now consider a bonus tournament with $\mathcal{P} = \{\Delta q\left(v_1^B, v_2^B\right), 0\}$ and $t' = \int_{v_1^B}^{\frac{v_1^B + v_2^B}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma) + t$. The surplus of supplier 1 now becomes $S_1' = S_1$ by construction. On the other hand, $S_2' \leq S_2$, but $S_2' > S_1'$. Thus, the proposed bonus tournament implements $\left(v_1^B, v_2^B\right)$ with lowest possible net supplier surplus, which implies that the buyer surplus is maximized.□

Lemma 8 *In the buyer optimum, the suppliers obtain zero expected surplus.*

Proof. The proof follows from the three steps below.
Step 1: *In an optimal contest $v_1^B + v_2^B = 1$.*

Consider any (v_1, v_2) such that $v_1 + v_2 < 1$. We show that $(v_1, v_2) \neq \left(v_1^B, v_2^B\right)$; the case $v_1 + v_2 > 1$ follows analogously. By Lemma 7, the optimal outcome can be implemented by some $\mathcal{P} = \{A, 0\}$

and $t \geq 0$. The equilibrium values of p_i in this contest are zero if and only if $\sigma \in (v_1, v_2)$. Hence, the participation constraint for supplier 1 implies that $F(v_1) A + t \geq C$; thus $v_1 + v_2 < 1$ implies $(1 - F(v_2)) A + t > C$. Now suppose the buyer implements $(v_1 + \varepsilon, v_2 + \varepsilon)$, where ε is sufficiently small. We know that $(v_1 + \varepsilon, v_2 + \varepsilon)$ can also be implemented with $\mathcal{P} = \{A, 0\}$. Thus, we can write the buyer surplus as

$$S_B(\varepsilon) = S_T(v_1 + \varepsilon, v_2 + \varepsilon) - F(v_1 + \varepsilon) A - (1 - F(v_2 + \varepsilon)) A - 2t + 2C$$

for $\varepsilon \geq 0$. Thus $dS_B(\varepsilon)/d\varepsilon = dS_T(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon - Af(v_1 + \varepsilon) + Af(v_2 + \varepsilon)$.

Since $v_1 + v_2 < 1$, single-peakedness and symmetry (A2) imply $f(v_1 + \varepsilon) < f(v_2 + \varepsilon)$. Thus $dS_B(\varepsilon)/d\varepsilon > dS_T(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon$. We will show that $dS_T(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon > 0$; because $F(v_1 + \varepsilon) A + t > C$ and (for sufficiently small ε) $(1 - F(v_2)) A + t \geq C$, the buyer will thus be better off implementing $(v_1 + \varepsilon, v_2 + \varepsilon)$ than (v_1, v_2). Maximizing total surplus is equivalent to minimizing the expected distance

$$\begin{aligned} D(v_1 + \varepsilon, v_2 + \varepsilon) = &\int_0^{v_1 + \varepsilon} (v_1 + \varepsilon - \sigma) f(\sigma) d\sigma + \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2} + \varepsilon} (\sigma - v_1 - \varepsilon) f(\sigma) d\sigma \\ &+ \int_{\frac{v_1 + v_2}{2} + \varepsilon}^{v_2 + \varepsilon} (v_2 + \varepsilon - \sigma) f(\sigma) d\sigma + \int_{v_2 + \varepsilon}^{1} (\sigma - v_2 - \varepsilon) f(\sigma) d\sigma. \end{aligned}$$

From this we obtain

$$\begin{aligned} \frac{dD(v_1 + \varepsilon, v_2 + \varepsilon)}{d\varepsilon} &= \int_0^{v_1 + \varepsilon} f(\sigma) d\sigma - \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2} + \varepsilon} f(\sigma) d\sigma + \int_{\frac{v_1 + v_2}{2} + \varepsilon}^{v_2 + \varepsilon} f(\sigma) d\sigma - \int_{v_2 + \varepsilon}^{1} f(\sigma) d\sigma \\ &= 2F(v_1 + \varepsilon) + 2(F(v_2 + \varepsilon)) - 2F\left(\frac{v_1 + v_2}{2} + \varepsilon\right) - 1. \end{aligned}$$

We will show that this expression is negative for $v_1 + v_2 < 1$ and sufficiently small ε. To see this, fix any v_2 such that $1/2 \leq v_2 < 1$. Note that $h(v_1, v_2) \equiv dD(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon|_{\varepsilon = 0} = 0$ for $v_1 = 1 - v_2$. Furthermore $\partial h / \partial v_1 = 2f(v_1) - f((v_1 + v_2)/2) > 0$, where the last inequality follows by Lemma 4. Thus, $v_1 + v_2 < 1$ implies $2F(v_1) + 2(F(v_2)) - 2F((v_1 + v_2)/2) - 1 < 0$ and thus $dD(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon < 0$ for small enough ε. This in turn implies that $S_T(v_1 + \varepsilon, v_2 + \varepsilon)$ increases in ε so that buyer surplus also increases in ε.

Step 2: *The buyer surplus when implementing any $(v_1, 1 - v_1)$ with a bonus tournament and fixed t is strictly convex in v_1.*

The buyer surplus when implementing $(v_1, 1 - v_1)$ with fixed t can be expressed as

$$\begin{aligned} S_B(v_1, 1 - v_1) = &\, 2\int_0^{v_1} (\Psi - b(v_1 - \sigma)) dF(\sigma) + 2\int_{v_1}^{1/2} (\Psi - b(\sigma - v_1)) dF(\sigma) \\ &- 2F(v_1)\Delta q(v_1, 1 - v_1) - 2t \\ = &\, 2\left[\int_0^{v_1} (\Psi - b(1 - v_1 - \sigma)) dF(\sigma) + \int_{v_1}^{1/2} (\Psi - b(\sigma - v_1)) dF(\sigma)\right] - 2t. \end{aligned}$$

Straightforward calculations show that $\partial^2 S_B(v_1, 1-v_1)/\partial v_1^2 = 2b(2f(v_1) + 2v_1 f'(v_1) - f'(v_1)) \geq 2b(2f(v_1) - f'(v_1)) > 0$, where the last inequality follows from (A2)(iv).

Step 3: *In the buyer optimum, suppliers earn zero expected surplus.*

From Proposition 1 and Step 1 we know that the buyer optimum can be implemented by a suitable bonus contest $\mathcal{P} = \{\Delta q(v_1^B, 1-v_1^B), 0\}$ and some transfer t. This implies that the suppliers have symmetric payoffs. Suppose, in contradiction to the statement above, that the suppliers have a positive expected surplus. If $t > 0$, the buyer can increase her surplus by marginally reducing t. Hence, it must be that $t = 0$. Then, the supplier payoff is $F(v_1^B)\Delta q(v_1^B, 1-v_1^B) - C > 0$ and thus $\Delta q(v_1^B, 1-v_1^B) > 0$. Thus $v_1^B < 1/2 < 1-v_1^B$. By Step 2 in the proof of Lemma 6 we know that $v_1^B > 0$. Hence, $v_1^B \in (0, 1/2)$. Since $F(v_1^B)\Delta q(v_1^B, 1-v_1^B) - C$ is a continuous function, then there exists $\varepsilon > 0$, such that $F(v_1^B + \varepsilon)\Delta q(v_1^B + \varepsilon, 1-v_1^B - \varepsilon) - C \geq 0$ and $F(v_1^B - \varepsilon)\Delta q(v_1^B - \varepsilon, 1-v_1^B + \varepsilon) - C \geq 0$. But since $S_B(v_1, 1-v_1)$ is strictly convex by Step 2, than either $S_B(v_1^B, 1-v_1^B) < S_B(v_1^B + \varepsilon, 1-v_1^B - \varepsilon)$ or $S_B(v_1^B, 1-v_1^B) < S_B(v_1^B - \varepsilon, 1-v_1^B + \varepsilon)$, a contradiction. Hence, suppliers earn zero expected surplus.□

(ii) Suppose $C \geq F(v_1^*)\Delta q(v_1^*, v_2^*)$. From Proposition 1 we know that for the proposed $\mathcal{P} = \{A, 0\}$, (v_1^*, v_2^*) emerges in equilibrium; and the proof of this result also gives the pricing strategies p_1 and p_2. For $t = C - F(v_1^*)\Delta q(v_1^*, v_2^*)$, the buyer surplus in the proposed equilibrium is $S_T(v_1^*, v_2^*)$. This is the highest surplus that the buyer can achieve without violating the suppliers' participation constraints.

(iii) Suppose $C < F(v_1^*)\Delta q(v_1^*, v_2^*)$. By Lemma 3, the minimum supplier revenue in any contest implementing (v_1^*, v_2^*) is $F(v_1^*)\Delta q(v_1^*, v_2^*)$. Thus, in any such contest the suppliers would earn a positive expected surplus. By Part (i) this is suboptimal.

A.4 Proofs on Auctions and Tournaments (Section 4)

A.4.1 Proof of Proposition 2

(i) By Lemma 2, the unique equilibrium of the pricing subgame induced by q_1 and q_2 is $p_i = \max\{q_i - q_j, 0\}$ for $i, j \in \{1, 2\}$; $j \neq i$. Suppose that an auction does not implement the social optimum (v_1^*, v_2^*). Then, for some i, there exists $\bar{v}_i \neq v_i^*$ such that $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$. Let $\Theta_i(v_i, v_j) = \{\sigma \in [0,1] | \, q(v_i, \sigma) \geq q(v_j, \sigma)\}$ and $\Theta_{-i}(v_i, v_j) = [0,1] \setminus \Theta_i(v_i, v_j)$. Thus $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$ if and only if

$$\int_{\Theta_i(\bar{v}_i, v_j^*)} \left(q(\bar{v}_i, \sigma) - q(v_j^*, \sigma)\right) dF(\sigma) > \int_{\Theta_i(v_i^*, v_j^*)} \left(q(v_i^*, \sigma) - q(v_j^*, \sigma)\right) dF(\sigma),$$

or equivalently

$$\int_{\Theta_i\left(\bar{v}_i,v_j^*\right)} \left(q\left(\bar{v}_i,\sigma\right) - q\left(v_j^*,\sigma\right)\right) dF\left(\sigma\right) + \int_0^1 q\left(v_j^*,\sigma\right) dF\left(\sigma\right) >$$
$$\int_{\Theta_i\left(v_i^*,v_j^*\right)} \left(q\left(v_i^*,\sigma\right) - q\left(v_j^*,\sigma\right)\right) dF\left(\sigma\right) + \int_0^1 q\left(v_j^*,\sigma\right) dF\left(\sigma\right)$$

Splitting $[0,1]$ into $\Theta_i\left(\bar{v}_i,v_j^*\right)$ and $\Theta_{-i}\left(\bar{v}_i,v_j^*\right)$ on the left-hand side and into $\Theta_i\left(v_i^*,v_j^*\right)$ and $\Theta_{-i}\left(v_i^*,v_j^*\right)$ on the right-hand side and simplifying, this is equivalent with

$$\int_{\Theta_i\left(\bar{v}_i,v_j^*\right)} q\left(\bar{v}_i,\sigma\right) dF\left(\sigma\right) + \int_{\Theta_{-i}\left(\bar{v}_i,v_j^*\right)} q\left(v_j^*,\sigma\right) dF\left(\sigma\right) >$$
$$\int_{\Theta_i\left(v_i^*,v_j^*\right)} q\left(v_i^*,\sigma\right) dF\left(\sigma\right) + \int_{\Theta_{-i}\left(v_i^*,v_j^*\right)} q\left(v_j^*,\sigma\right) dF\left(\sigma\right).$$

and thus $\int_0^1 \max\{q(\bar{v}_i,\sigma), q(v_j^*,\sigma)\} dF\left(\sigma\right) > \int_0^1 \max\{q(v_i^*,\sigma), q(v_j^*,\sigma)\} dF\left(\sigma\right)$, contradicting optimality of (v_1^*, v_2^*).

(ii) This follows from Lemma 5.

(iii) Using Proposition 2(ii), any FPT such that the supplier breaks even has a unique equilibrium with $(v_1, v_2) = (1/2, 1/2)$. For $A = 2C$ and $t = 0$, the participation constraint of the suppliers binds. Hence, this contest gives the highest buyer surplus within the class of FPTs, namely

$$\begin{aligned} S_B^{fpt} &= \int_0^{1/2} \left(\Psi - b\left(\frac{1}{2} - \sigma\right)\right) f\left(\sigma\right) d\sigma + \int_{1/2}^1 \left(\Psi - b\left(\sigma - \frac{1}{2}\right)\right) f\left(\sigma\right) d\sigma - 2C \\ &= \Psi + \int_0^{1/2} b\sigma f\left(\sigma\right) d\sigma - \int_{1/2}^1 b\sigma f\left(\sigma\right) d\sigma - 2C. \end{aligned}$$

By Lemma 2, in an auction the winning supplier bids exactly the quality difference. Hence, the revenue of supplier 1 (supplier 2 follows by symmetry) is

$$\begin{aligned} \Pi_1^a &= F\left(v_1^*\right) \Delta q\left(v_1^*, v_2^*\right) + \int_{v_1^*}^{1/2} \left(q\left(v_1^*,\sigma\right) - q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma \\ &= \frac{b\left(v_2^* - v_1^*\right)}{4} + \int_{v_1^*}^{1/2} \left(q\left(v_1^*,\sigma\right) - q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma. \end{aligned}$$

Thus whenever $C < b\left(v_2^* - v_1^*\right)/4$, the participation constraint of the suppliers is satisfied even with $t = 0$. Further, in any state of the world, the buyer's payoff is equal to the quality of the losing supplier. Then, the buyer surplus in an auction with $t = 0$ is

$$\begin{aligned} S_B^a &= \int_0^{1/2} \left(\Psi - b\left(v_2^* - \sigma\right)\right) f\left(\sigma\right) d\sigma + \int_{1/2}^1 \left(\Psi - b\left(\sigma - v_1^*\right)\right) f\left(\sigma\right) d\sigma \\ &= \Psi + \int_0^{1/2} b\sigma f\left(\sigma\right) d\sigma - \int_{1/2}^1 b\sigma f\left(\sigma\right) d\sigma - \frac{bv_2^*}{2} + \frac{bv_1^*}{2} \end{aligned}$$

Thus $S_B^{fpt} - S_B^a > 0$ holds if $bv_2^*/2 - bv_1^*/2 - 2C > 0$ or equivalently $b\left(v_2^* - v_1^*\right)/4 > C$.

When $b\left(v_2^* - v_1^*\right)/4 < C < \Pi_1^a$, the participation constraint is still satisfied with $t = 0$, but the buyer prefers the FPT to the auction. When $\Pi_1^a < C$, the participation constraint in the auction is satisfied only with sufficiently large positive subsidies. In this case, the buyer implements the social optimum by using an auction with $t = C - \Pi_1^a$ with zero supplier surplus. Obviously this outperforms the inefficient FPT.

A.4.2 Proof of Corollary 1

Denote the minimum allowable price with $\underline{P}$. If $v_1 \neq v_2$ in equilibrium, by Proposition 2(ii), the contest is not an FPT. Suppose that $v_1 < v_2$. Since $\mathcal{P}$ is convex, by Lemmas 2 and 3, the buyer pays $q_i - q_j + \underline{P}$ to the supplier with $q_i \geq q_j$ in equilibrium. Thus, for any σ, the buyer surplus is $\min\{q_1, q_2\} - \underline{P}$. Hence, the surplus of a buyer who induces $v_1 < v_2$ with $\underline{P}$ is $S_B\left(v_1, v_2\right) = \int_0^{(v_1+v_2)/2} q_2\left(v_2, \sigma\right) dF(\sigma) + \int_{(v_1+v_2)/2}^1 q_1\left(v_1, \sigma\right) dF(\sigma) - \underline{P}$. Thus

$$\frac{dS_B}{dv_1} = \int_{\frac{v_1+v_2}{2}}^1 \frac{\partial q_1}{\partial v_1} dF(\sigma) > 0; \; \frac{dS_B}{dv_2} = \int_0^{\frac{v_1+v_2}{2}} \frac{\partial q_2}{\partial v_2} dF(\sigma) < 0.$$

Thus, the buyer surplus is maximal for $v_1 = v_2$ and $\underline{P} = 0$. Given $v_1 = v_2$, the buyer surplus is maximal for $v_1 = v_2 = 1/2$, the unique equilibrium of an FPT with A arbitrarily close to 0. Given (T2), it is an equilibrium for $A = 0$.

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B Online Appendix: Not for Print

In this online appendix we provide formal results for the extensions discussed in Section 5.

B.1 Inducing Effort

In the baseline model, a supplier i who entered the contests had to exert effort. In this section, we allow the suppliers to shirk. We model this by assuming that the supplier not only chooses an approach v_i, but also an effort level $e_i \in \{0,1\}$. No matter which approach is chosen, the cost of shirking is zero and the cost of exerting effort is $C > 0$. Shirking produces zero quality. When effort is exerted, the quality is determined as in the baseline model. As in the main model, we assume that the value of the innovation is large enough, so that production with two agents is profitable (formally, $\Psi > b + 2C$). One major difference to the model of Section 2 is that subsidies cannot be used to induce agents to produce output, as a supplier could simply collect the subsidy and then shirk. Hence, the buyer will not use subsidies, and has to rely on expected revenues from the contest to induce the suppliers to exert effort.

The next result examines the optimal contests which induce both diversity and effort.

Proposition 3 *Suppose (A1) and (A2) hold and shirking is possible. Denote the social optimum with* v_1^*, v_2^*.
(i) If $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$*, the optimal contest for the buyer with both suppliers producing is a bonus tournament with prizes* $\mathcal{P} = \{A, 0\}$*, which implements suboptimal diversity.*
(ii) If $F(v_1^*)\,\Delta q(v_i^*, v_j^*) \leq C \leq F(v_2^*)\,\Delta q(v_i^*, v_j^*)$*, the optimal contest for the buyer with both suppliers producing is a bonus tournament that implements the socially optimal diversity. The prizes are* $\mathcal{P} = \{A, a\}$*, where* $A = 2C + \Delta q(v_i^*, v_j^*)/2$ *and* $a = 2C - \Delta q(v_i^*, v_j^*)/2$.
(iii) For any F*, there exists* $\bar{C}$ *such that if* $C > \bar{C}$*, no contest can implement strict diversity.*

Proof. (i) Follows directly from Theorem 1(iii).
(ii) As $F(v_1^*)\,\Delta q(v_i^*, v_j^*) \leq C$ by assumption, both prizes are weakly positive. For supplier 1, the expected revenue of following the candidate equilibrium is $\Pi_1(v_1^*, v_2^*) = F(v_1^*)\,A + (1/2 - F(v_1^*))\,a$. Inserting the values of A and a and $F(v_1^*) = 1/4$ by Lemma 1, $\Pi_1(v_1^*, v_2^*) = C$. By symmetry, the expected surplus of both suppliers is zero. Thus, the suggested allocation maximizes total surplus, with full rent appropriation by the buyer. It thus suffices to show that $\{A, a\}$ implements (v_1^*, v_2^*). Consider supplier 1. First, consider any feasible deviation to the right of v_2^*, that is a deviation to $\widetilde{v}_1 = v_2^* + \varepsilon$, for any $0 < \varepsilon \leq 1 - v_2^*$. Denote the pricing function with p_1. Supplier 1's revenue is then $\Pi_1(\widetilde{v}_1, v_2^*) = \int_{(\widetilde{v}_1 + v_2^*)/2}^{1} p_1(q_1(\sigma), q_2(\sigma))\,dF(\sigma)$. Since $v_2^* > 1/2$, an alternative deviation to $v_1' = v_2^* - \varepsilon$ and the same pricing function pricing function p_1 is also feasible. In this case, the expected revenue can be written as $\Pi_1(v_1', v_2^*) = \int_0^{2v_2^*-1} p_1(q_1(\sigma), q_2(\sigma))\,dF(\sigma) + \int_{2v_2^*-1}^{(v_1'+v_2^*)/2} p_1(q_1(\sigma), q_2(\sigma))\,dF(\sigma)$. Analogously to the proof of Theorem 1, there exists a bijective

mapping $[(\widetilde{v}_1 + v_2^*)/2, 1] \to [2v_2^* - 1, (v_1' + v_2^*)/2]$; $\sigma' \mapsto \sigma''$ where $\sigma'' = 2v_2^* - \sigma'$ with the property that $q(\widetilde{v}_1, \sigma') - q(v_2^*, \sigma') = q(v_1', \sigma'') - q(v_2^*, \sigma'')$ and (by single-peakedness) $f(\sigma') \leq f(\sigma'')$. In a state where quality difference is the same, the winning price is also the same. Therefore,

$$\int_{\frac{\widetilde{v}_1 + v_2^*}{2}}^{1} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma) \leq \int_{2v_2^*-1}^{\frac{v_1'+v_2^*}{2}} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma).$$

As a result, $\Pi_1(\widetilde{v}_1, v_2^*) \leq \Pi_1(v_1', v_2^*)$ and v_1' leads to strictly higher probability of winning, hence v_1' is a deviation that is preferable to $\widetilde{v}_1$. Thus, it is sufficient to show that no profitable deviation of the type $v_1' \leq v_2^*$ exists. A deviation to $v_1' < v_1^*$ cannot increase expected supplier revenue, as the probability of winning decreases and the price charged in any state of the world does not increase. Third, the only remaining case is a deviation to $v_1' \in (v_1^*, v_2^*]$. The expected revenue can be written as $\Pi_1(v_1', v_2^*) = aF((v_1' + v_2^*)/2)$. This is clearly increasing in v_1' and the revenue of supplier 1 is at most $\Pi_1(v_1', v_2^*) = aF(v_2^*)$. The expected revenue of following the candidate equilibrium is $\Pi_1(v_1^*, v_2^*) = F(v_1^*)A + (1/2 - F(v_1^*))a$. Thus there is no profitable deviation to values just below v_2^* if and only if $F(v_1^*)A + (1/2 - F(v_1^*))a \geq aF(v_2^*)$. Inserting the values of A and a and $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ shows that (v_1^*, v_2^*) is an equilibrium.
(iii) The proof shows that the expected revenues in any contest implementing $v_1 \neq v_2$ are bounded. Thus, if costs are higher than this bound, no contest can implement any $v_1 \neq v_2$.

Suppose that a contest implements some $v_1 \neq v_2$, where $v_1, v_2 \in [0,1]$. Denote with $\Pi_i(v_i, v_j)$ the expected revenue of supplier i. Suppose (w.l.o.g.) that $\Pi_1(v_1, v_2) \leq \Pi_2(v_2, v_1)$. By Lemma 2, supplier 1 wins the contest if $q_1(v_1, \sigma) \geq q_2(v_2, \sigma)$ and obtains the price $p_1(v_1, v_2, \sigma) \leq q_1(v_1, \sigma) - q_2(v_2, \sigma) + \underline{P}$. But then, in any contest implementing v_1, v_2, we have

$$\begin{aligned}(6)\qquad \Pi_1(v_1, v_2) &\leq F(v_1)(\Delta q(v_1, v_2) + \underline{P}) + \int_{v_1}^{\frac{v_1+v_2}{2}} (q_1(v_1, \sigma) - q_2(v_2, \sigma) + \underline{P})\, dF(\sigma) \\ &= F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) dF(\sigma) + F\left(\frac{v_1+v_2}{2}\right)\underline{P} \equiv \overline{\Pi}_1(v_1, v_2, \underline{P})\end{aligned}$$

where $\overline{\Pi}_1(v_1, v_2, \underline{P})$ is the upper bound on the supplier's revenue, given that a contest implements v_1, v_2 and the lowest feasible price is $\underline{P}$. Observe that by deviating to $v_1' = v_2 - \varepsilon$, supplier 1 wins the contest whenever $\sigma < (v_1' + v_2)/2 = v_2 - \varepsilon/2$. That is, the probability that supplier 1 wins is $F(v_2 - \varepsilon/2)$. Since the minimum prize that the winner obtains is $\underline{P}$, supplier 1 can achieve an expected payoff which is arbitrarily close to $F(v_2)\underline{P}$. Since the contest implements (v_1, v_2) there can be no profitable deviations, so that $\Pi_1(v_1, v_2) \geq F(v_2)\underline{P}$. This inequality implies

$$F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma + F\left(\frac{v_1+v_2}{2}\right)\underline{P} \geq F(v_2)\underline{P} \Leftrightarrow$$

$$\frac{F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} \geq \underline{P}$$

Using this upper bound for $\underline{P}$, we obtain that for any $\underline{P}$

$$\begin{aligned}\overline{\Pi}_1 (v_1, v_2, \underline{P}) &\leq F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma)dF(\sigma) \\ &+ F\left(\frac{v_1+v_2}{2}\right) \frac{F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma)dF(\sigma)}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} \\ &= \left(F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma)dF(\sigma)\right) \frac{F(v_2)}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)}\end{aligned}$$

To show that the RHS of the expression is finite for any v_1 and v_2, it suffices to show that the last expression converges to a finite value as $v_1 \to v_2$. As $v_1 \to v_2$, the denominator $F(v_2) - F\left(\frac{v_1+v_2}{2}\right)$ and the numerator both converge to 0. Both are differentiable with respect to v_1 on the interval $(0, v_2)$ and $\frac{d}{dv_1}\left(F(v_2) - F\left(\frac{v_1+v_2}{2}\right)\right) \neq 0$ for $v_1 < v_2$. Hence, we can use L'Hôpitals Rule to evaluate the RHS as $v_1 \to v_2$. Standard calculations show that

$$\lim_{v_1 \to v_2} \left(F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma)f(\sigma)\,d\sigma\right) \frac{F(v_2)}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} = \frac{bF(v_2)^2}{f(v_2)\frac{1}{2}}.$$

which is finite for any v_2. Thus $\Pi_1(v_1, v_2)$ is bounded. Then there exists $\bar{C}$ such that $\bar{C} > \Pi_1(v_1, v_2)$ for any contest and any $v_1 \neq v_2$, where $v_1, v_2 \in [0, 1]$, so that no supplier can break even in expectation in a contest without subsidies. ■

Part (i) of Proposition 3 is a direct implication of Theorem 1(iii), which shows that, even when subsidies are available, a bonus tournament without subsides is optimal if $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$. In the baseline model of Section 2, when $C \geq F(v_1^*)\,\Delta q(v_i^*, v_j^*)$ the buyer implements the socially optimal diversity, and uses subsidies to make sure that the suppliers break even. However, in the current setting subsidies are not feasible. Proposition 3(ii) shows that the buyer can actually use the low price in the bonus tournament as an (imperfect) substitute for the subsidies. A positive low price a acts as a subsidy because it increases expected revenues of the suppliers. It is imperfect because, as we know from Lemma 3, $A - a = \Delta q(v_i^*, v_j^*)$ in any bonus tournament that implements the social optimum. Thus as a grows, the relative difference between the bonus and the low price $((A-a)/A)$ decreases; as a result, it becomes more attractive to deviate and increase the probability of winning the low price (at the cost of never winning the bonus price). This can be shown to imply that there is a bound to the revenue that can be given to the suppliers in a bonus tournament, while implementing the socially optimal diversity — which is given by $F(v_2^*)\,\Delta q(v_i^*, v_j^*)$. More generally, as the proof of 3(iii) shows, there is a bound on the revenue that can be given to suppliers in *any* contest implementing any (strict) diversity of research approaches. Since subsidies are not feasible, when costs are higher then revenues, suppliers would not participate in the contest.

B.2 Heterogeneous Costs

In the baseline model, all approaches are equally costly, that is, $C(v) = C$ for all $v \in [0,1]$. In this section, we consider heterogeneous costs. Of course, there are many ways in which costs might be different. We will consider one specific case. Suppose that $C : [0,1] \to \mathbb{R}_+$, where

$$\text{(C1)} \qquad C(1/2) > 0, C'(1/2) = 0, C''(v) > 0 \forall v \in [0,1], C(1/2 - \varepsilon) = C(1/2 + \varepsilon) \forall \varepsilon \in [0, 1/2].$$

In words, the cost function is symmetric and convex, with a minimum at 1/2. Furthermore, denoting the socially optimal approaches as v_1^* and v_2^*, assume

$$\text{(C2)} \qquad F\left(v_1^*\right) \Delta q(v_1^*, v_2^*) \geq C(0) - C(1/2).$$

Thus, we assume that the heterogeneity of costs is not too large.

Proposition 4 *Suppose (A1), (A2), (C1) and (C2) hold. Denote the social optimum with v_1^*, v_2^*.*
(i) The social optimum is symmetric around 1/2 ($v_1^* = 1 - v_2^*$) and less variety is induced than with homogeneous costs: $F(v_1^*) > 1/4$ and $F(v_2^*) < 3/4$.
(ii) A bonus tournament can implement the social optimum.
(iii) FPTs implement no variety in any PSE.
(iv) Auctions can implement the social optimum.
(v) The buyer (weakly) prefers to implement the social optimum with a bonus tournament than with an auction. The preference is strict if $C(v_1^*) < \Pi^a$, where Π^a is the expected revenue from the auction.

Proof. (i) Assume w.l.o.g. that $v_1 \leq v_2$. The total surplus is

$$S_T(v_1, v_2) = E_\sigma[\max\{q(v_1, \sigma), q(v_2, \sigma)\}] - C(v_1) - C(v_2).$$

The same argument as in the proof of Lemma 1 guarantees the existence of the socially optimal choices v_1^*, v_2^*. Furthermore, the necessary conditions for the social optimum are

$$\frac{\partial S_T(v_1, v_2)}{\partial v_1} = b\left(-2F(v_1) + F((v_1 + v_2)/2)\right) - C'(v_1) = 0 \tag{7}$$

$$\frac{\partial S_T(v_1, v_2)}{\partial v_2} = b\left(1 - 2F(v_2) + F((v_1 + v_2)/2)\right) - C'(v_2) = 0. \tag{8}$$

The remainder of the proof proceeds in three steps.

Step 1: *Full duplication is never optimal.*
Suppose given any $v_1 = v_2 \leq 1/2$ (the case that $v_1 = v_2 > 1/2$ is analogous). Then the necessary condition $\partial S_T(v_1, v_2)/\partial v_2 = 0$ implies $b(1 - F(v_2)) = C'(v_2)$. This is never satisfied as $b(1 - F(v_2)) > 0$ and $C'(v_2) \leq 0$ for all $v_2 \leq 1/2$.

Step 2: *If v_1 and v_2 are socially optimal, then $v_1 = 1 - v_2$.*
Suppose to the contrary that $v_1 + v_2 < 1$ (the opposite case is again analogous). Consider the effects on the total surplus by moving both v_1 and v_2 to the right by some small $\varepsilon > 0$. The total surplus can be expressed as $S_T(v_1 + \varepsilon, v_2 + \varepsilon) = E_\sigma[\max\{q(v_1 + \varepsilon, \sigma), q(v_2 + \varepsilon, \sigma)\}] - C(v_1 + \varepsilon) - C(v_2 + \varepsilon)$. That is, expected highest quality net of research costs. We know from Step 1 in Lemma 8 that $dE_\sigma[\max\{q(v_1 + \varepsilon, \sigma), q(v_2 + \varepsilon, \sigma)\}]/d\varepsilon > 0$ when ε is small. Hence, it is sufficient to show that $d(-C(v_1 + \varepsilon) - C(v_2 + \varepsilon))/d\varepsilon > 0$. For this, it is sufficient that $-C'(v_1) - C'(v_2) > 0$. This follows from symmetry and convexity of C.

Step 3: $F(v_1^*) > 1/4$.
By Step 2, $F((v_1^* + v_2^*)/2) = 1/2$, so that (7) becomes $b\left(-2F(v_1^*) + 1/2\right) = C'(v_1^*)$. Since $C'(v_1^*) < 0$ for all $v_1^* \in [0, 1/2)$, this implies $-2F(v_1^*) + 1/2 < 0$, or $F(v_1^*) > 1/4$ (similarly $F(v_2^*) < 3/4$).
(ii) Let $A = \Delta q(v_1^*, v_2^*)$. Then a bonus tournament with $\mathcal{P} = (A, 0)$ and appropriate subsidies implements the social optimum. As in the proof of Proposition 1, we will consider three possible types

of deviations: (i) deviating to $v_1' < v_1^*$, (ii) deviating to $v_1'' \in (v_1^*, \widetilde{v})$ where $\widetilde{v} = \min\{2v_2^* - v_1^*, 1\}$, and (iii) deviations to $v_1''' \geq \widetilde{v}$. In the proof of Proposition 1, we have shown that deviations of type (i) and (iii) decrease the probability of winning the prize A. Since $v_1' < v_1^*$ and $v_1''' > v_2^*$, then $C(v_1') > C(v_1^*)$ and $C(v_1''') > C(v_1^*)$. Hence, these deviations are not profitable. Furthermore, we have shown that all deviations of type (ii) only lead to the prize 0. Hence, the optimal deviation of type (ii) is to 1/2, and the payoff to supplier 1 is $-C(1/2)$. The payoff of following the candidate equilibrium is $F\left(v_1^*\right)\Delta q(v_1^*, v_2^*) - C(v_1^*)$. Thus, no profitable deviation exists if $F\left(v_1^*\right)\Delta q(v_1^*, v_2^*) - C(v_1^*) \geq -C(1/2)$ or $F\left(v_1^*\right)\Delta q(v_1^*, v_2^*) \geq C(v_1^*) - C(1/2)$. By (C2) $F\left(v_1^*\right)\Delta q(v_1^*, v_2^*) \geq C(0) - C(1/2)$ and $C(0) > C(v_1^*)$, thus the inequality is satisfied and no profitable deviation exists.

(iii) Suppose that an FPT with some prize $A \geq 0$ implements some $v_1 < v_2$. Then, if $C(v_1) = C(v_2)$, it must be that $v_1 = 1 - v_2$. Hence each supplier wins with probability 1/2. A supplier who deviates to $v = 1/2$ will have lower costs and higher probability of winning, hence this is a profitable deviation. Next, suppose that $C(v_1) > C(v_2)$ (the opposite case follows analogously). The expected payoff of supplier 1 is $F((v_1 + v_2)/2)A - C(v_1)$. By convexity of C, any deviation to $v_1' \in (v_1, v_2)$ leads to lower costs and higher probability of winning, hence v_1' is a profitable deviation. From this it follows directly that in any PSE of a FPT, no variety is implemented.

(iv) The proof is analogous to the proof of Proposition 2 and is omitted.

(v) The proof is analogous to the proof of Proposition 5(iii) below, when the homogeneous C is substituted with $C(v_1^*) = C(v_2^*)$, where the equality follows from Step 2 above and symmetry of $C(v)$. ■

B.3 Generalized Distributions and Quality Functions

We generalize the assumptions of Section 2 as follows:

Assumption (A1)' $q_i(v_i, \sigma) = \Psi - \delta(|v_i - \sigma|)$, where $\delta(|v_i - \sigma|)$ *is increasing and continuous and* $\delta(1) \in (0, \Psi - 2C]$.

Assumption (A2)' *The density function* $f(\sigma)$ *is (i) symmetric and (ii) has full support:* $f(\sigma) > 0$ $\forall \sigma \in [0,1]$.

Thus, we relax the requirements that the distribution be single-peaked and relatively flat and that the distance function be linear. We allow for individual subsidies t_1 and t_2.

Lemmas 2, 3 and Proposition 1 also hold under the relaxed assumptions (A1)' and (A2)'. The proofs are analogous and are therefore omitted here. As a result, the main contests that we previously dealt with have the same properties as before:

Lemma 9 *In the baseline model, replace (A1) and (A2) with (A1)' and (A2)'. Then, (i) the bonus tournament* ($\mathcal{P} = \{\Delta q(v_1^*, v_2^*), 0\}$) *and the auction mechanism* ($\mathcal{P} = \mathbb{R}^+$) *implement the social optimum. Moreover, (ii) in any FPT* ($\mathcal{P} = \{A\}$ *for* $A \geq 2C$)*, the unique equilibrium is such that*

$v_1 = v_2$ *and* $F(v_i) = 1/2$ *for* $i = 1, 2$.

Proof. (i) The proof of the result on auctions is the same as the proof of Proposition 2(i) above. By the generalized Proposition 1, the social optimum can be implemented with a bonus tournament if $0 < v_1^* \leq 1/2 \leq v_2^* < 1$. Thus, we only need to show that the social optimum always satisfies these conditions. Therefore, first consider any $v_1 = 0$ ($v_2 = 1$ is analogous). Clearly, $\left.\frac{\partial S_T(v_1,v_2)}{\partial v_1}\right|_{v_1=0} > 0$. Hence, in the social optimum $v_1^* > 0$. Next, consider (v_1, v_2) such that $v_1 \leq v_2 < 1/2$ (the case $1/2 < v_1 \leq v_2$ is analogous). Supplier 2 offers higher quality than supplier 1 in the interval $\left[\frac{v_1+v_2}{2}, 1\right]$. We can write the total surplus from this interval as

$$S_T(v_1, v_2)|_{\sigma \geq \frac{v_1+v_2}{2}} =$$

$$\Psi\left(1 - F\left(\frac{v_1 + v_2}{2}\right)\right) - \int_{\frac{v_1+v_2}{2}}^{v_2} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma - \int_{v_2}^{1/2} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma$$

$$- \int_{1/2}^{1-v_2} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma - \int_{1-v_2}^{1-\frac{v_1+v_2}{2}} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma - \int_{1-\frac{v_1+v_2}{2}}^{1} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma - 2C$$

Now suppose supplier 2 chooses $v_2' = 1 - v_2$ instead of v_2 Symmetry of $f(\sigma)$ implies that

$$\int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma = \int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}} \delta(|v_2' - \sigma|) f(\sigma)\, d\sigma.$$

As the highest available quality determines the total surplus, it follows $S_T(v_1, v_2)|_{\frac{v_1+v_2}{2} \leq \sigma \leq 1 - \frac{v_1+v_2}{2}} \leq S_T(v_1, v_2')|_{\frac{v_1+v_2}{2} \leq \sigma \leq 1 - \frac{v_1+v_2}{2}}$. Observe that $\int_{1-\frac{v_1+v_2}{2}}^{1} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma > \int_{1-\frac{v_1+v_2}{2}}^{1} \delta(|v_2' - \sigma|) f(\sigma)\, d\sigma$, because δ is increasing. Thus $S_T(v_1, v_2)|_{\sigma \geq \frac{v_1+v_2}{2}} < S_T(v_1, v_2')|_{\sigma \geq \frac{v_1+v_2}{2}}$. For $\sigma < \frac{v_1+v_2}{2}$, the highest quality always comes from v_1. Thus $S_T(v_1, v_2)|_{\sigma < \frac{v_1+v_2}{2}} = S_T(v_1, v_2')|_{\sigma < \frac{v_1+v_2}{2}}$. Thus, we obtain $S_T(v_1, v_2) < S_T(v_1, v_2')$. Thus, there can be no social optimum with $v_1^* \leq v_2^* < 1/2$.
(ii) *The unique equilibrium in an FPT is such that* $v_1 = v_2$ *and* $F(v_i) = 1/2$ *for* $i = 1, 2$.
This statement follows directly from the proof of Lemma 5, which holds under the more general assumptions as well. ■

We are not able to prove the optimality of bonus tournaments in general when assumptions on distributions and quality functions are relaxed. However, as the next result shows, when C is higher than the expected revenue of each supplier in the bonus tournament implementing the social optimum (while by (A1)' costs are still sufficiently low that it is socially optimal for production to take place), then a bonus tournament is optimal. Furthermore, we can compare bonus tournaments to auctions and FPTs and we can show that bonus tournaments generally perform better (from the buyer's perspective) than the other two institutions.

Proposition 5 *In the baseline model, replace (A1) and (A2) with (A1)' and (A2)'. Denote with* (v_1^*, v_2^*) *the social optimum. (i) If* $C \geq \max\{F(v_1^*)\,\Delta q(v_1^*, v_2^*),\ (1 - F(v_2^*))\,\Delta q(v_1^*, v_2^*)\}$ *then a suitable bonus tournament is an optimal contest for the buyer. (ii) The buyer strictly prefers a* *suitable bonus tournament to the FPT whenever* $C > 0$*. (iii) The buyer prefers to implement the social optimum with a bonus tournament rather than with an auction. The preference is strict if* $C < \max\{\Pi_1^a, \Pi_2^a\}$*, where* Π_i^a *is the expected revenue from the auction.*

Proof. (i) By Lemma 9(i) the bonus tournament with $(\Delta q(v_1^*, v_2^*), 0)$ implements the social optimum with appropriate subsidies. Let the subsidies be $t_1 = C - F(v_1^*)\Delta q(v_1^*, v_2^*)$ and $t_2 = C - (1 - F(v_2^*))\Delta q(v_1^*, v_2^*)$. $C \geq \max\{F(v_1^*)\Delta q(v_1^*, v_2^*),\ (1 - F(v_2^*))\Delta q(v_1^*, v_2^*)\}$ implies $t_1, t_2 \geq 0$, and the participation constraint of both suppliers bind. Thus, the bonus tournament implements the social optimum and extracts all surplus from the suppliers and hence it is the optimal contest for the buyer.

(ii) By Lemma 9(ii), the FPT uniquely implements $v_1 = v_2 = 1/2$ and $F(v_i) = 1/2$ for $i = 1, 2$. Thus there exists $\varepsilon > 0$, such that

$$F(1/2 - \varepsilon)\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon) = (1 - F(1/2 - \varepsilon))\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon) < C.$$

Then, by the generalized version of Proposition 1, a bonus tournament with prices

$$\mathcal{P} = \{\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon), 0\}$$

and transfers

$$t_1 = t_2 = C - F(1/2 - \varepsilon)\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon)$$

implements $(v_1, v_2) = (1/2 - \varepsilon, 1/2 + \varepsilon)$. This yields strictly greater total surplus, with weakly lower supplier surplus than any FPT. Hence, buyer surplus is strictly greater in such a bonus tournament than in any FPT.

(iii) By Lemma 9(i), both the auction and the bonus tournament implement the social optimum with appropriate subsidies. The revenue of the suppliers in the auction are

$$\Pi_1^a = F(v_1^*)\Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma)\, d\sigma$$
$$\Pi_2^a = (1 - F(v_2^*))\Delta q(v_1^*, v_2^*) + \int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma)\, d\sigma$$

while the revenues in the bonus tournament $(\Delta q(v_1^*, v_2^*), 0)$ are

$$\Pi_1^{bt} = F(v_1^*)\Delta q(v_1^*, v_2^*)$$
$$\Pi_2^{bt} = (1 - F(v_2^*))\Delta q(v_1^*, v_2^*).$$

Suppose first that $\max\{\Pi_1^a, \Pi_2^a\} \geq C$. Let $t_1^a, t_2^a \geq 0$ be the minimum subsidies needed for the suppliers to be willing to participate in the auction. Then, given subsidies

$$t_1^{bt} = t_1^a + \int_{v_1^*}^{\frac{v_1^*+v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma)\, d\sigma$$
$$t_2^{bt} = t_2^a + \int_{\frac{v_1^*+v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma)\, d\sigma$$

both suppliers are willing to participate in the bonus tournament. Furthermore, since both the auction and the bonus tournament implement the social optimum and have the same total transfers to the suppliers ($\Pi_1^j + \Pi_1^j + t_1^j + t_2^j$, for $j = a, bt$), the buyer is indifferent between the auction and the bonus tournament. However, suppose $C < \max\{\Pi_1^a, \Pi_2^a\}$ and suppose w.l.o.g. that $\max\{\Pi_1^a, \Pi_2^a\} = \Pi_1^a$. Because $\Pi_1^a = \Pi_1^{bt} + t_1^{bt}$, there exists $\varepsilon > 0$ such that $\Pi_1^{bt} + t_1^{bt} - \varepsilon \geq C$. Then, a bonus tournament with $(\Delta q(v_1^*, v_2^*), 0)$ with subsidies $\hat{t}_1^{bt} = t_1^{bt} - \varepsilon$ and $\hat{t}_2^{bt} = t_2^{bt}$ still implements the social optimum but with strictly lower total transfers than the auction with the minimum subsidies. Thus, the buyer strictly prefers this bonus tournament to the auction with any subsidies. ■

B.4 The Number of Suppliers

For simplicity, we assume that the distribution of ideal states $f(\sigma)$ is uniform. Otherwise the model corresponds to the baseline case of Section 2. In this framework, we can characterize the social optimum and the equilibria of the main contests previously discussed. Though most results also apply to the case $n = 3$, an FPT does not have a pure strategy equilibrium in this case.[41] To allow for simple formulations, we confine ourselves to $n > 3$. Finally, in this extension, suppliers that choose v_1 and v_n could ex ante be in a more favorable position than others. This is because, depending on the contest the buyer chooses, they could be facing less competition then other suppliers (in the states of the world $\sigma \leq v_1$ and $\sigma \geq v_n$ respectively). For this reason, the minimum subsidy needed to satisfy the participation constraints will generally be different across suppliers. To focus on the incentives of suppliers to diversify, we will allow the buyer to offer different subsidies to different suppliers. That is, for each supplier i, the buyer will offer a subsidy t_i.

Lemma 10 *Suppose there are $n > 3$ suppliers, (A1) holds and $f(\sigma)$ is uniform.*
(i) The social optimum is $(v_1^*, ..., v_n^*) = (1/2n, 3/2n, 5/2n, ..., (2n-1)/2n)$.
(ii) The social optimum can be implemented with a suitable bonus tournament or with an auction.
(iii) In any equilibrium of an FPT with n suppliers, there is duplication, and the amount of diversity (defined as the distance between the highest and lowest approach) is inefficiently low. As n increases, the difference between the socially optimal diversity and the minimal diversity in any FPT equilibrium converges to zero.

Proof. (i) Arguing as for two suppliers, $v_i^* \neq v_j^*$ for all $i \neq j \in \{1, ..., n\}$. Thus

$$S_T(\mathbf{v}) = \int_0^{\frac{v_1+v_2}{2}} q_1(v_1, \sigma)\, d\sigma + \sum_{k=2}^{n-1} \int_{\frac{v_{k-1}+v_k}{2}}^{\frac{v_k+v_{k+1}}{2}} q_k(v_k, \sigma)\, d\sigma + \int_{\frac{v_{n-1}+v_n}{2}}^{1} q_n(v_n, \sigma)\, d\sigma - nC$$

The maximum of this function exists and it obviously does not involve corner solutions. Hence, it

[41] This has been observed for the equivalent Hotelling model with fixed prizes by Eaton and Lipsey (1975); see Shaked (1982) for a calculation of the mixed-strategy equilibrium.

is given by the first order conditions

(9)
$$\frac{\partial S_T(\mathbf{v})}{\partial v_1} = -bv_1 + b\frac{v_2 - v_1}{2} = 0$$

(10)
$$\frac{\partial S_T(\mathbf{v})}{\partial v_k} = -b\frac{v_k - v_{k-1}}{2} + b\frac{v_{k+1} - v_k}{2} = 0$$

$$\text{for } k \in \{2, ..., n-1\}$$

(11)
$$\frac{\partial S_T(\mathbf{v})}{\partial v_n} = -b\frac{v_n - v_{n-1}}{2} + b(1 - v_n) = 0$$

(10) can be rearranged to give $v_k - v_{k-1} = v_{k+1} - v_k \equiv \Delta^v$ for $k = 2, ..., n-1$. (9) and (11) give $v_1 = 1 - v_n = \Delta^v/2$. Inserting these equations into $v_1 + (v_2 - v_1) + ... + (v_n - v_{n-1}) + (1 - v_n) = 1$ gives $\Delta^v = \frac{1}{n}$. Thus, $v_1 = \frac{1}{2n}$ and $v_k = \frac{1}{2n} + \frac{k-1}{n} = \frac{2k-1}{2n}$ for $k \in \{2, ..., n\}$.

(ii) To ensure participation set $t_i = C$ for all i. The proof of the result on auctions is analogous to the proof of Proposition 2(i) above. Consider the bonus tournament with $A = b/n$. If suppliers $1, ..., n$ choose $v_1^*, v_2^*, ..., v_n^*$, then suppliers $2, .., n-1$ receive no revenues, but they break even because of the subsidy. There are no feasible deviations for which they can earn a positive price. Consider supplier 1 (supplier n is analogous): His surplus is $\frac{1}{2n}\left(\frac{b}{n}\right) + C - C = \frac{b}{2n^2}$. Deviating to $v_1 < v_1^*$ would reduce the probability of winning the prize, with no compensating benefits. Deviating to $v_1 > v_1^*$ would mean that supplier 1 would only win the low prize 0. This is clearly not profitable.

(iii) Let $\mathbf{v} = [v_1, ..., v_n]$ be the vector of approaches, ordered so that $v_1 \leq ... \leq v_n$. Step 1 shows that there is duplication in any FPT. In Step 2-5, we show that diversity is less than socially optimal in the FPT. In Step 6, we consider the effect of increasing n.

Step 1: *In any equilibrium of the FPT, $v_1 = v_2$ and $v_{n-1} = v_n$. This implies that there are at most $n-2$ active approaches.*

Suppose $v_1 < v_2$. Then the revenue of supplier 1 is $A\frac{v_1+v_2}{2}$. For $v_1' = v_1 + \varepsilon$, $\varepsilon > 0$, such that $v_1' < v_2$, the revenue is $A\frac{v_1'+v_2}{2} > A\frac{v_1+v_2}{2}$. A similar argument holds for $v_{n-1} < v_n$.

Step 2: *For any supplier i, let $P^i_{\sigma<v_i}$ ($P^i_{\sigma>v_i}$) be the probability that supplier i wins and, in addition, $\sigma < v_i$ ($\sigma > v_i$). Let $P^i = P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$. If for suppliers i and j there exist $k \neq i$ and $l \neq j$ such that $v_i = v_k$ and $v_j = v_l$, then $P^i_{\sigma<v_i} = P^i_{\sigma>v_i} = P^j_{\sigma<v_j} = P^j_{\sigma>v_j}$ in any equilibrium.*

Suppose first that $P^i_{\sigma<v_i} \neq P^i_{\sigma>v_i}$ for some supplier i using the same approach as another one. Suppose that $P^i_{\sigma<v_i} > P^i_{\sigma>v_i}$ (the opposite case is analogous). Then, a deviation to $v_i - \varepsilon$ for some sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P^i_{\sigma<v_i} > P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$,[42] which is a profitable deviation. Next, suppose that $P^i_{\sigma>v_i} < P^j_{\sigma<v_j}$(the opposite case is analogous). Then, a deviation of supplier i to $v_j - \varepsilon$ for sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P^j_{\sigma<v_j} > P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$, which is a profitable deviation.

Step 3: *In any equilibrium of an FPT with n suppliers, $P^c \equiv P^1 = P^2 = P^{n-1} = P^n \geq \frac{1}{2(n-2)}$.*

By Step 1, all extreme approaches are duplicate. The three equalities thus follow from Step 2.

[42]The winning probability is approximately $2P^i_{\sigma<v_i}$ if $v_i = \min\{v_1, ..., v_n\}$.

Suppose that the inequality does not hold. Then $P^1 + P^2 + P^{n-1} + P^n < \frac{2}{n-2}$ which in turn implies that $\sum_{j=3}^{n-2} P^j \geq \frac{n-4}{n-2}$. But then there exist at least one $k \in \{3, ..., n-2\}$ such that $P^k \geq \frac{1}{n-2}$. By deviating to v_k, each supplier $1, 2, n-1$ or n would win with a probability of at least $\frac{1}{2(n-2)}$, which would be a profitable deviation.

Step 4: *Any equilibrium of an FPT with n suppliers satisfies $v_n - v_1 \leq \frac{n-3}{n-2}$.*

Suppose not. As $\frac{2(n-2)-1}{2(n-2)} - \frac{1}{2(n-2)} = \frac{n-3}{n-2}$, there exists an equilibrium of an FPT such that either $v_n > \frac{2(n-2)-1}{2(n-2)}$ or $v_1 < \frac{1}{2(n-2)}$ or both. If $v_n > \frac{2(n-2)-1}{2(n-2)}$, then Steps 1 and 2 imply $P^n < \frac{1}{2(n-2)}$, which is impossible by Step 3. If $v_1 < \frac{1}{2(n-2)}$, then $P^1 < \frac{1}{2(n-2)}$ by Steps 1 and 2, which is again impossible by Step 3.

Step 5: *The diversity in an FPT is lower than socially optimal.*

By (i), the socially optimal diversity is $\frac{n-1}{n}$. By Step 4, the diversity in an FPT is at most $\frac{n-3}{n-2} < \frac{n-1}{n}$.

Step 6: *The difference between the FPT and the social optimum converges to zero as the number of suppliers increases.*

By Step 3, we know that each supplier $1, 2, n-1, n$ wins with probability P^c. Then in any equilibrium of an FPT, there exists a supplier j such that $P^j \leq \frac{1-4P^c}{n-4}$. A deviation to $v_1 - \varepsilon$ would result in a probability of winning approximately P^c. Thus, a necessary condition for an equilibrium is that $P^c \leq \frac{1-4P^c}{n-4}$, which implies that $P^c \leq 1/n$ and consequently $v_1 \leq 1/n$ and $v_n \geq (n-1)/n$. Then, $v_n - v_1 \geq \frac{n-2}{n}$ in any equilibrium of an FPT. By (i), the socially optimal diversity is $(n-1)/n$, so the difference between the socially optimal diversity and diversity in any equilibrium of an FPT is at most $\frac{n-1}{n} - \frac{n-2}{n} = 1/n$. Thus, the difference converges to zero as n increases. ■

Lemma 10 shows that there is no duplication in the social optimum, and the approaches are evenly spread. The buyer can implement the social optimum with a bonus tournament or an auction. An FPT will lead to duplication of research approaches and suboptimal diversity.

Part (iii) of Lemma 10 is closely related to familiar results for locational competition (Eaton and Lipsey, 1975). We nevertheless state it here for completeness because we are interested in the comparison between the different institutions.

We now provide sufficient conditions for equilibria in the FPT. In particular, we find that equilibria with the maximal number of approaches $(n-2)$ consistent with Lemma 10 exist.

Lemma 11 *An outcome with k active approaches $(r_1, ..., r_k)$ can be supported in an equilibrium of an FPT if the following conditions both hold:*
(a) $k \in \{\underline{k}, ..., \bar{k}\}$, where $\bar{k} = n-2$ and $\underline{k} = n/2$ if n is even and $\underline{k} = (n+1)/2$ if n is odd;
(b) $(r_1, ..., r_k) = (1/2k, 3/2k, 5/2k, ..., (2k-1)/2k)$.
Two suppliers choose the extreme approaches r_1 and r_k and each of the intermediate approaches $r_2, \ldots, r_{k-1}$ is chosen by one or two suppliers.

Proof. Step 1: *Suppose n is even and $k = n/2$. Then any choice of $r_1, ..., r_k$ as stated in part (b) of the lemma can be supported as an equilibrium.*

In the suggested equilibria, the active approaches are equidistant. Also, $r_1 = 1/n$ and $r_{n/2} = 1-1/n$. For any $1 < m < n/2$, $r_m - r_{m-1} = 2/n$, any of the active approaches offers the highest quality with probability $1/k = 2/n$. Now suppose each approach $r_1, ..., r_k$ is chosen by exactly two suppliers. Then each supplier has a revenue of $\Pi_i = A/n$. Deviating to any other active approach leads to payoff of $2A/3n$; hence it is not profitable. A deviation to $[0, r_1)$ or $(r_{n/2}, 1]$ results in a winning probability strictly lower than $1/n$, so this is not a profitable deviation either. Finally, consider a deviation to $v \in (r_{m-1}, r_m)$, $m \in \{2, ..., n/2\}$. The deviating supplier wins if and only if σ is in the set $[\frac{v+r_{m-1}}{2}, \frac{v+r_m}{2}]$, so that the winning probability is $1/n$ and this is also not a profitable deviation.

Step 2: *Now suppose n is even or odd and $k > n/2$. Then any choice of $r_1, ..., r_k$ as stated in part (b) of the lemma is an equilibrium.*

Arguing as in Step 1, any of the active approaches offers the highest quality with probability $1/k$. Suppose two suppliers choose r_1 and r_k, respectively. Moreover, suppose that each of the approaches $r_2, ..., r_{k-1}$ is chosen by one or two suppliers. Thus, if there are two suppliers using an approach, each of them wins with probability $1/2k$, and if there is only one supplier using this approach, he wins with probability $1/k$. Consider a supplier who wins with probability $1/2k$. By the same argument as in Step 1, if he deviates to $[0, r_1)$ or $(r_k, 1]$, he wins with probability strictly lower than $1/2k$. Deviating to any approach in some interval (r_l, r_{l+1}); $l \in \{1, ..., k-1\}$, he wins with probability of at most $1/2k$; hence such a deviation is not profitable either. If he deviates to any active approach, he wins with a probability of at most $1/2k$. Thus, such suppliers do not have profitable deviations. Finally consider a deviation by a supplier who is the only one to choose some r_m, where $1 < m < k$. Any deviation to $[0, r_{m-1}]$ or $[r_{m+1}, 1]$ leads to strictly lower revenues, by the same argument as above. For any approach $v \in (r_{m-1}, r_{m+1})$, he wins whenever $\sigma \in [\frac{v+r_{m-1}}{2}, \frac{v+r_{m+1}}{2}]$, so that the winning probability is $\frac{v+r_{m+1}}{2} - \frac{v+r_{m-1}}{2} = \frac{r_{m+1}-r_{m-1}}{2} = 1/k$. Hence, this is not a profitable deviation either. ■

Proposition 6 *Suppose there are $n > 3$ suppliers, (A1) holds and $f(\sigma)$ is uniform.*
(i) If $C \geq b/2n^2$, a suitable bonus tournament is an optimal contest for the buyer.
(ii) If $C < b/2n^2$ the buyer strictly prefers to implement the social optimum with a bonus tournament rather than with an auction.
(iii) The buyer prefers the social optimum implemented with a bonus tournament over any outcome of an FPT whenever $C \geq b(n-4)/4n^2(n-2)$.

Proof. (i) Arguing as in Proposition 1, the bonus tournament $(b/n, 0)$ implements the social optimum. Thus, to prove the optimality of the bonus tournament it is sufficient to show that the buyer can extract all surplus from the suppliers. Let $t_2 = \cdots = t_{n-1} = C$ and $t_1 = t_n = C - b/(2n^2)$. Suppliers $i \in \{2, \ldots n-2\}$ win the bonus price with probability zero, hence their revenue is zero, and their participation constraint binds. Suppliers 1 and n win the bonus price with probability $1/2n$, and their expected revenue is $\Pi_1 = \Pi_n = (1/2n)(b/n) = b/2n^2$. Thus, their participation

constraint binds as well, and hence the bonus tournament implements the optimum for the buyer.
(ii) In an auction, the conditional transfers to suppliers 1 and n differ from those for the remaining suppliers. The revenue of supplier 1 is

$$\Pi_1 = \frac{b}{2n^2} + \int_{1/2n}^{2/2n} \left(\Psi - b \left(\sigma - \frac{1}{2n} \right) - \left(\Psi - b \left(\frac{3}{2n} - \sigma \right) \right) \right) d\sigma = \frac{3b}{4n^2}$$

For supplier 2 it is

$$\Pi_2 = 2 \int_{2/2n}^{3/2n} \left(\Psi - b \left(\frac{3}{2n} - \sigma \right) - \left(\Psi - b \left(\sigma - \frac{1}{2n} \right) \right) \right) d\sigma = \frac{2b}{4n^2}$$

By symmetry, $\Pi_1 = \Pi_n$ and $\Pi_2 = \Pi_i$ for all $i \in \{2, ...n-1\}$. As $\Pi_1 > \Pi_2 = b/(2n^2) > C$, the participation constraint of all suppliers is satisfied without any subsidies. Then, the buyer optimally sets $t_i = 0$ for all i in the auction. The total transfers of the buyer to the suppliers are thus $\Sigma_{i=1}^{n} \Pi_i = (n-2)\Pi_2 + 2\Pi_1 = (n-2)\frac{b}{2n^2} + \frac{3b}{2n^2} = (n+1)\frac{b}{2n^2}$.
Now consider a bonus tournament with $(b/n, 0)$, $t_2 = \cdots = t_{n-1} = C$ and $t_1 = t_n = 0$. As argued before, this bonus tournament implements the social optimum if participation constraints are met. For suppliers 2 to $n-2$ the subsidy ensures participation. For suppliers 1 and n the expected revenue is $\Pi_1 = \Pi_n = b/(2n^2) > C$. Hence, participation is ensured. Thus the total transfers of the buyer to the suppliers are $\Sigma_{i=1}^{n}(\Pi_i + t_i) = \Pi_1 + \Pi_n + \Sigma_{i=2}^{n-1} t_i = \frac{b}{n^2} + (n-2)C$. The buyer will strictly prefer the bonus tournament to the auction if and only if $(n+1)\, b/\, 2n^2 > b/\, n^2 + (n-2)C$, or equivalently, $(n-1)\, b/\, 2n^2 > (n-2)C$. This always holds since $b/(2n^2) > C$.
(iii) According to the proof of Lemma 10(iii), an FPT can implement at most $n-2$ different approaches. By Lemma 11, an FPT implementing $n-2$ approaches exists. The FPT implementing maximum diversity (hence maximizing total surplus) thus implements $k = n-2$ with $A = 0$ and $t_i = C$ for all i. The participation constraint of all suppliers binds, so that this is the best outcome for the buyer that is obtainable as an FPT equilibrium. In the FPT, the buyer has expected costs from suboptimal quality of $\frac{b}{4(n-2)}$. Moreover, she pays subsidies nC. Now consider the bonus tournament implementing the socially optimal outcome, as above. If $C \geq b/(2n^2)$, the bonus tournament extracts the entire surplus like the FPT, but implements a strictly more efficient outcome. Hence, the buyer payoff is strictly higher. Next, suppose that $C < b/(2n^2)$, and let the subsidies be as in (ii). Then, the buyer has expected costs from suboptimal quality of $\frac{b}{4n}$, pays expected bonus prizes $\frac{b}{n^2}$ and subsidies $(n-2)C$; together these costs amount to $\frac{b}{4n} + \frac{b}{n^2} + (n-2)C$. Thus, the buyer is better off in the bonus tournament if $\frac{b}{4n} + \frac{b}{n^2} + (n-2)C < \frac{b}{4(n-2)} + nC$. This is equivalent to the condition in the proposition. ■

For the case $n = 2$, we know that a bonus tournament is an optimal contest for the buyer for any C. Furthermore, we know that when C is high (while still low enough that it is optimal that the expected social surplus is non-negative), the buyer will implement the social optimum and extract the entire surplus. However, as C becomes lower, the buyer trades off efficiency for surplus extraction and implements less diversity than socially optimal. Proposition 6(i) is analogous to the first of these results for $n = 2$: Namely, when C is sufficiently high (but low enough that total social surplus is positive), the buyer can still use bonus tournaments to implement the social optimum and extract the surplus from the suppliers. For the case of lower C, we cannot characterize the optimal contest, as the tradeoff between surplus extraction and efficiency becomes more complicated. We therefore confine ourselves to the comparison of auctions and FPTs with a bonus tournament which implements the social optimum.

Proposition 6(ii) shows that the buyer still prefers to implement the social optimum with a bonus tournament rather than with an auction even when C is arbitrarily low. The intuition is the same as for $n = 2$, as a suitable bonus tournament requires lower ex post payments than an auction.

Proposition 6(iii) compares the bonus tournament implementing the social optimum with the (inefficient) FPTs. It says that when $C \geq b(n-4)/4n^2(n-2)$, the buyer prefers bonus tournaments to FPTs, without claiming that bonus tournaments are optimal contests. For $C < b(n-4)/4n^2(n-2)$, an FPT with $n-2$ active approaches outperforms a bonus tournament implementing the social optimum. Intuitively, to implement the social optimum the buyer has to give positive surplus to the suppliers, while an appropriately designed FPT can extract all surplus from the suppliers. However, $b(n-4)/4n^2(n-2)$ is equal to zero for $n = 4$ and it approaches zero as $n \to \infty$, so the parameter range in which FPTs outperform bonus tournaments is restricted.

Lemma 10 has another simple but important implication: It may be socially optimal to invite a large number of suppliers. This differs from the case of contests that merely influence the suppliers' efforts: Several papers show that, in those settings, the optimal number of participants is typically two.

Corollary 2 *Suppose research costs are $C > 0$,(A1) holds and $f(\sigma)$ is uniform. Define $n_-(C) = \max\left\{n \in \mathbb{N} | \, 2 \leq n \leq \sqrt{b} \Big/ 2\sqrt{C}\right\}$ and $n_+(C) = n_-(C) + 1$. Auctions or bonus tournament with $n_-(C)$ or $n_+(C)$ suppliers maximize total surplus in the set of all contests with an arbitrary number of suppliers.*

Proof. According to Lemma 10(i), the social optimum is given by the choices $v_k^* = (2k-1)/2n$ ($k \in \{1, ..., n\}$). The average quality in the social optimum is thus $\Psi - b/4n$. Therefore the total surplus is $\Psi - b/4n - nC$. The maximum of this expression in $\mathbb{R}^+$ is $n = \sqrt{b}/2\sqrt{C}$. By concavity of the objective function, the optimal choice of $n \in \mathbb{N}$ is thus given by $n_-(C)$ or $n_+(C)$. According to Lemma 10(ii), the social optimum for any given number of suppliers can be implemented with an auction. ■

Corollary 2 describes the number of suppliers that optimally balances the gains from higher expected quality against the losses from higher research costs. The result implies that the optimal number of suppliers increases in b and decreases in C. While the corollary is stated for the socially optimal contest, it is simple to show that the buyer can also often benefit from inviting more than two suppliers and that the comparative statics are similar. In particular, in a bonus tournament an increase in n leads not only to an increase in the expected quality (reflecting higher option value), but also to a reduction in rents that suppliers 1 and n obtain (reflecting an increase in competition).

B.4.1 Dealing with a Single Supplier

We now consider the alternative of dealing with a single supplier. The buyer has to pay a quality-independent transfer C to compensate the supplier for his research costs. Moreover, she would want such a supplier to choose the approach which maximizes the expected quality (given that there is only one supplier), namely $1/2$. Without verifiable quality information, the buyer cannot write contracts enforcing a particular approach. As the supplier is indifferent between all approaches (as long as there is no cost heterogeneity), any choice of an approach is an equilibrium response of the supplier. We therefore remain agnostic about the probability with which the supplier chooses a particular approach and we denote with η the probability measure describing this choice.

Proposition 7 *Suppose that (A1) holds, $C > 0$ and $f(\sigma)$ is uniform. (i) If $C \leq b/8$ then there exists a bonus tournament with $n \geq 2$ suppliers that the buyer prefers to any single supplier contract. If $C < b/12$ the preference is strict. (ii) If $C \leq b/8$ and $\eta(1/2) \neq 1$, then the buyer strictly prefers the optimal bonus contest with two suppliers to the single-supplier contract.*

Proof. (i) The proof will immediately follow from Steps 1-5 below.
Step 1: *If $n \geq 3$ and $C \geq b/2n^2$, there exists a bonus contest implementing the socially optimal choice of approaches for that n such that the buyer extracts the entire surplus.*
For $n > 3$, Step 1 follows from the proof of Proposition 6. The proof for $n = 3$ is analogous and omitted here.
Step 2: *The surplus $\bar{S}_B^n$ that the buyer obtains in this contest is larger than $\bar{S}_B^1$, the maximal buyer payoff that the buyer obtains in any single supplier contract, if and only if $C < b/4n$.*
To see this, note that the expected buyer surplus in such a bonus contest with n suppliers and uniform $f(\sigma)$ is $\bar{S}_B^n = \Psi - b/4n - Cn$. The maximum buyer payoff in any single supplier contract is implemented with the socially optimal research project by extracting all the surplus, which is $\bar{S}_B^1 = \Psi - b/4 - C$. Simple calculations show that $\bar{S}_B^n > \bar{S}_B^1$ if and only if $C < b/4n$.
Step 3: *For $C \in (0, b/12)$, there exists $n \geq 3$ such that $b/2n^2 \leq C < b/4n$.*
This will follow from two observations. First, $b/2n^2 < b/4(n+1) < b/4n$ for any $n \geq 3$. Second, for any $C \in (0, b/12)$, there exists some $n \geq 3$ such that $b/2n^2 \leq C$. Let $\bar{n} \equiv \bar{n}(b, C)$ be the smallest n satisfying these conditions. If $\bar{n} = 3$, then $b/2\bar{n}^2 \leq C < b/12 = b/4\bar{n}$. If $\bar{n} > 3$, then by construction $b/2(\bar{n}-1)^2 > C$. Since $b/4\bar{n} > b/2(\bar{n}-1)^2$, we obtain $b/2\bar{n}^2 \leq C < b/4\bar{n}$.
Step 4: For $C \leq b/8$, *the buyer is indifferent between the optimal two-supplier contest and the optimal single-supplier contract.*
Suppose that $C \leq b/8$. By Theorem 1(i), in the optimal two-supplier bonus contest the buyer

implements v_1 and $v_2 = 1 - v_1$ such that $t = 0$ and $F(v_1)\Delta q(v_1, v_2) = C$. Then, the buyer surplus in the optimal two-supplier bonus contest is

$$S_B^2 = \int_0^1 \max\{q(v_1\,\sigma), q(v_2\,\sigma)\}\, dF(\sigma) - 2C$$
$$= \Psi - \frac{b}{4} + bv_1 - 2bv_1^2 - 2C = \Psi - \frac{b}{4} - C = \bar{S}_B^1,$$

where the second equality follows by symmetry and the assumption that F is uniform, and the third equality follows from $F(v_1)\Delta q(v_1, v_2) = bv_1 - 2bv_1^2 = C$. Thus, whenever $C \leq b/8$, the buyer is indifferent between the optimal bonus contest and the maximum surplus that can possibly be achieved with a single supplier.

Step 5: *Result (i) follows.*

For $C \in (0, b/12)$, Steps 1-3 imply that there exists an $n \geq 3$ and an n-player contest such that the buyer obtains strictly higher payoffs than in the optimal single-supplier contract. For $C \in [b/12, b/8)$, the buyer can do at least as well with the optimal two-player contest as with any single supplier contract by Step 4.

(ii) By Step 4, for all $C \leq b/8$ the buyer is indifferent between the optimal two-supplier bonus contest and the first-best surplus which can be attained with a single supplier. If $\eta(1/2) \neq 1$ then the buyer surplus with a single supplier is strictly lower than the first-best surplus. Hence, the buyer strictly prefers the optimal two-supplier bonus contest to the single-supplier contract. ■

B.5 Multiple Prizes

We now consider the case of multiple prizes. The setup is the same as in Section B.4.

Lemma 12 *In the model with $n > 3$ players of Section B.4, suppose that the buyer uses a uniform subsidy $t = C$. For any equilibrium in an FPT with two prizes $A_1 > A_2 > 0$, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off.*

For the proof of this result, recall that $f(\sigma)$ is uniform. For notational convenience, suppose that $v_1 \leq v_2 \leq \cdots \leq v_n$. We first provide an intermediate result.

Lemma 13 *If $v_1, v_2, \ldots, v_n$ is an equilibrium of an FPT with two prizes, then $v_1 = v_2 = v_3$ and $v_{n-2} = v_{n-1} = v_n$.*

Proof. We will prove that $v_1 = v_2 = v_3$. The other claim follows by an analogous argument.

Step 1: $v_1 = v_2$.

Suppose not. Then $v_1 < v_2$. Thus, the revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1 + v_2}{2} A_1 + \frac{v_3 - v_2}{2} A_2.$$

Therefore, a deviation to any $v_1' \in (v_1, v_2)$ increases the probability of winning the first prize, while not affecting the probability of winning the second prize. Hence, it is profitable.

Step 2: $v_1 = v_2 < v_3 = v_4$ *cannot be an equilibrium.*

Denote with $P^{i,1}_{\sigma<v_i}$ the probability that supplier i wins the first prize when $\sigma < v_i$. Analogously define the probabilities of winning when the state is greater than the chosen approach and the probabilities of winning the second prize. By random tie breaking we have $P^{1,1}_{\sigma<v_1} = P^{2,1}_{\sigma<v_2} = P^{1,2}_{\sigma<v_1} = P^{2,2}_{\sigma<v_2}$ and $P^{1,1}_{\sigma>v_1} = P^{2,1}_{\sigma>v_2} = P^{1,2}_{\sigma>v_1} = P^{2,2}_{\sigma>v_2}$. We will show that $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$. Suppose that this was not true. First, suppose $P^{1,1}_{\sigma<v_1} > P^{1,1}_{\sigma>v_1}$. Then, there exist $\varepsilon, \varepsilon', \varepsilon'' > 0$ arbitrarily small such that a deviation $v_1' = v_1 - \varepsilon$ leads to revenues

$$\Pi_1\left(v_1', v_{-1}\right) = 2\left(P^{1,1}_{\sigma<v_1} - \varepsilon'\right)A_1 + 2\left(P^{1,1}_{\sigma>v_1} - \varepsilon''\right)A_2.$$

For sufficiently small ε this constitutes a profitable deviation. The case $P^{1,1}_{\sigma<v_1} < P^{1,1}_{\sigma>v_1}$ follows by an analogous argument, but the incentives to deviate are even stronger.

Now suppose that $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$ and $v_1 = v_2 < v_3 = v_4$. We will show that this cannot be an equilibrium. In the proposed equilibrium $P^{1,1}_{\sigma<v_1} = v_1/2$ and $P^{1,1}_{\sigma<v_1} + P^{1,1}_{\sigma>v_1} = P^{1,2}_{\sigma<v_1} + P^{1,2}_{\sigma>v_1} = v_1$. Hence, the expected revenue is $\Pi_1(v_1, v_{-1}) = v_1A_1 + v_1A_2$. For any deviation $v_1' \in (v_2, v_3)$ the probability of winning the first prize is

$$\frac{v_1' + v_3}{2} - \frac{v_1' + v_2}{2} = \frac{v_3 - v_2}{2} = v_1$$

where the last equality follows from $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$. Using $v_3 = v_4$, the probability of winning the second prize is $(v_2 + v_1')/2 > v_1$. It follows that $\Pi_1(v_1', v_{-1}) > \Pi_1(v_1, v_{-1})$.

Step 3: $v_1 = v_2 < v_3 < v_4$ *cannot be an equilibrium.*

The revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1}{2}A_1 + \frac{v_3 - v_1}{4}A_1 + \frac{v_3 + v_1}{4}A_2 + \frac{v_4 - v_3}{4}A_2. \tag{12}$$

Consider a deviation to $v_1' \in (v_1, v_3)$. The revenue is

$$\Pi_1\left(v_1', v_{-1}\right) = \frac{v_3 - v_1}{2}A_1 + \frac{v_1' + v_1}{2}A_2 + \frac{v_4 - v_3}{2}A_2.$$

If $\Pi_1(v_1', v_{-1}) > \Pi_1(v_1, v_{-1})$, then this is a profitable deviation. If $\Pi_1(v_1', v_{-1}) \leq \Pi_1(v_1, v_{-1})$ is equivalent with

$$\frac{v_1}{2}A_1 - \frac{v_3 - v_1}{4}A_1 + \frac{v_3 - v_1}{4}A_2 - \frac{v_1'}{2}A_2 - \frac{v_4 - v_3}{4}A_2 \geq 0 \tag{13}$$

But consider in that case a deviation to $v_1'' = v_1 - \varepsilon$ for small positive ε. The expected revenue is

$$\Pi_1\left(v_1'', v_{-1}\right) = \frac{v_1'' + v_1}{2}A_1 + \frac{v_3 - v_1}{2}A_2$$

and $\lim_{\varepsilon\to 0}\Pi_1\left(v_1'', v_{-1}\right) = v_1 A_1 + \frac{v_3-v_1}{2}A_2$. Together with (12), this implies

$$\lim_{\varepsilon\to 0}\Pi_1\left(v_1'', v_{-1}\right) - \Pi_1\left(v_1, v_{-1}\right) = \frac{v_1}{2}A_1 + \frac{v_3-v_1}{4}A_2 - \frac{v_3-v_1}{4}A_1 - \frac{v_1}{2}A_2 - \frac{v_4-v_3}{4}A_2.$$

Since $v_1' > v_1$, (13) implies $\lim_{\varepsilon\to 0}\Pi_1\left(v_1'', v_{-1}\right) - \Pi_1\left(v_1, v_{-1}\right) > 0$. Hence, there always exists $\varepsilon > 0$ small enough such that $\Pi_1\left(v_1'', v_{-1}\right) - \Pi_1\left(v_1, v_{-1}\right) > 0$. ■

The lemma implies that the maximal number of active approaches in an FPT with two prizes is $n-4$. By Lemma 11 an FPT with a single prize implements an equilibrium with $n-2$ active approaches. By Lemma 10(ii), it is possible to implement the socially optimal allocation with $n-2$ approaches in an FPT with a single prize. Implementing this equilibrium in a single-prize FPT, where the prize size is the sum of the two prizes in an FPT with two prizes, strictly increases the total payoff. On the other hand, the payoff of the suppliers remains the same, as the total size of the fixed prize remains the same. Hence, the expected buyer payoff strictly increases. This gives Lemma 12.

References for the Online Appendix

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