Master Thesis

The Effects of Productivity Growth, Population Growth, and the Rate of Interest on the Distribution of Wealth

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Abstract

In this thesis, Thomas Piketty's book *Capital in the Twenty-First Century* is critically discussed and confronted with macroeconomic growth theory. The thesis gives an overview of Piketty's main theoretical statements and summarises the main strands of criticism raised against his findings. Piketty states that the interest rate exceeding the growth rate of the economy – captured by the famous inequality $r > g$ – is the main driver of wealth inequality. This claim is analysed using standard macroeconomic growth models. Heterogeneity is generated through the introduction of subsistence consumption as done by Bertola, Foellmi and Zweimüller (2006), whose work is extended to include exogenous productivity and population growth. In a Solow model and a one-period lifetime model with bequests it is shown that wage growth is the major equalising force in the economy. While a high interest rate correlates with a longer divergence period, economic growth lengthens the divergence period. In the Ramsey framework, wealth divergence is a rising function of the difference between the interest rate and the productivity growth rate, while the result is independent of population growth.

Contents

Introduction

Ever since the emergence of economics as a field of study for moral philosophers, the distribution of wealth has been of crucial and most highly regarded importance to their studies. In the work of well-known economists such as David Ricardo, Thomas Malthus or Karl Marx, the subject received a lot of attention and their work was dedicated to unveil the deep structure and the driving forces behind the evolution of wealth inequality. However, the emphasis on the study of the distribution of wealth has peaked off in recent decades. The advent of modern economics and the rise of representative agent models has marginalised the role of distributional issues on the research agenda of economic scholars. It has only been lately that the topic of wealth inequality has gotten back into debate. A major contribution, which has received broad attention and stirred a lot of controversy, is Thomas Piketty's *Capital in the Twenty-First Century*. Based on data from several countries over multiple centuries, Piketty illustrates how wealth inequality evolved and how major historical events such as the world wars, the great depression or the Thatcher-Reagan era of market liberalisation affected the distribution of wealth. Piketty's work has been widely received and brought the study of wealth inequality back to the heart of economic analysis.

Even though Piketty's main contribution is of empirical nature, his work is deeply rooted in economic theory. His most well-known hypothesis is the claim that if the rate of return on capital exceeds the growth rate of the economy – expressed in his famous inequality $r > g$ – inequality will increase and the consequences will destabilise the foundations of modern democracies. Thus, he calls $r > g$ the central contradiction of capitalism.

This thesis seeks to provide the reader with a deeper understanding of distributional issues and to critically discuss Piketty's predictions. A major focus will be laid on the implications of $r > g$ and the mechanism behind Piketty's claim that $r > g$ is a crucial driver of wealth inequality. Since economic growth is mainly driven by productivity and population growth, the central concern of this thesis is to shed light on the effects of productivity growth, population growth, and the rate of interest on the distribution of wealth and to reveal the relationship between these variables.

The contribution of this thesis to the field of economics is twofold. On the one hand it offers an insightful overview of Piketty's main theoretical statements and a comprehensive summary of the main strands of criticism raised against him. Along with this, an introduction to Piketty's workhorse model to explain the implications of $r > g$ is provided. On the other hand, the effects of the interest rate and economic growth on the distribution of wealth are discussed in standard macroeconomic growth models. The aim of this analysis is to confront Piketty's claims with economic theory in order to gain a deeper understanding of how the variables of interest affect wealth inequality in widely accepted and commonly used frameworks.

The main tool to generate heterogeneity in standard growth models is through the introduction of subsistence consumption. The study of wealth inequality using such an approach is done extensively in the work of Bertola, Foellmi and Zweimüller (2006). It is their work, which this thesis builds upon. Bertola et al. (2006) analyse wealth inequality in stationary models without exogenous growth. This is where this thesis is tied up to their work. The major contribution is to extend their models to include exogenous productivity and population growth in order to analyse the effects of the interest rate and economic growth on the distribution of wealth.

Beginning with a Solow model, in which individual behaviour is fully specified by a consumption function, it will be shown that wage growth is the major equalising force in a growing economy. While the distribution of wealth is converging in the steady state, there is wealth divergence in the transition period towards the steady state. Even though the convergence result holds independently of r and g, more can be said about their role in the transition period. Wealth is diverging in a state with a low capital stock and a high interest rate. The divergence period takes longer if the capital stock is lower, which is a state in which the interest rate is higher. Thus, a longer divergence period is correlated with a higher interest rate. However, higher economic growth lengthens the divergence period as will be shown in the model. Thus, the findings about the interest rate are in line with Piketty's hypothesis, while the findings about the growth rate of the economy do not coincide with Piketty's claim.

A first step towards a micro-foundation of individual behaviour is made using a one-period lifetime with bequests model. While the savings rate is still given exogenously by the utility function's taste parameter, individual behaviour is based on the agents' optimising behaviour instead of an exogenously given consumption function. The model yields the same results as the Solow model. Thus, it serves as a robustness check.

In order to endogenise the savings rate, the distribution of wealth is analysed in a Ramsey framework with infinitely lived households. While the wealth distribution is persistent in the steady state, it is diverging in the transition period towards the steady state. While being independent of population growth, the model reproduces the statements by Piketty concerning the interest rate and productivity growth: wealth divergence is an increasing function of $r - g_A$, where g_A is the rate of productivity growth.

This thesis is split into two parts and is structured as follows. Part A is concerned with Piketty's *Capital in the Twenty-First Century* and contains three chapters. Chapter 1 provides the reader with an overview of Piketty's main theoretical statements, while in Chapter 2, his major critics are given a voice in a summary of the main strands of criticism. Part A is rounded up in Chapter 3, where Piketty's workhorse model to explain the implications of $r > g$ on the distribution of wealth is presented.

In Part B, Piketty's claims are confronted with standard macroeconomic growth models. Using subsistence consumption and introducing exogenous productivity and population growth, the effects of the interest rate and economic growth are analysed in three chapters. The Solow model with subsistence consumption is presented in Chapter 4. Following this, micro-founded models with a finite and infinite horizon are studied, as done in standard economic theory. While Chapter 5 discusses wealth inequality in a one-period lifetime model with bequests, Chapter 6 focuses on a Ramsey model where agents are infinitely lived. The thesis ends with a conclusion.

Part A: Piketty's Capital in the Twenty-First Century

1 Main Theoretical Statements

1.1 Chapter Overview

This chapter shall provide the reader with an overview of the main theoretical statements of *Capital in the Twenty-First Century*. Even though the book is mainly based on empirical findings, Piketty's work is deeply rooted in economic theory. The main parts containing theoretical concepts are covered in Chapters 1, 5, 6 and 10 of the book. These concepts lay a theoretical foundation for the empirical findings and are therefore fundamental from an economic point of view.

The theoretical statements can be split into three parts, all addressing the increasing influence of wealth ownership.

In the first part, which is covered in Chapters 1 and 5 of the book, the two fundamental laws of capitalism are described. In this part, Piketty predicts an increasing capital/income ratio and therefore an increasing exertion of power over the production factors by capital holders. The topic is covered in Section 1.2.

In Chapter 6 of the book, Piketty describes how an increasing capital/income ratio may result in a higher capital share. This enhances the influence of capital holders even further, since they do not only control a major share of the production factors, but also draw upon a substantial share of national income. This issue is discussed in Section 1.3.

A third force favouring capital ownership is discussed in Chapter 10 of the book and constitutes Piketty's most famous driver of inequality: the rate of return on capital exceeding the growth rate of the economy ($r > g$). The topic is covered in Section 1.4.

Piketty and his co-authors have published several articles on these different topics, which are either addressed to an academic public, or make statements of the book more clear and respond to criticism. Therefore, there is a vast body of literature by Piketty related to the book. In order to guide the interested reader through Piketty's publications, Section 1.5 provides a structured overview of the literature available.

The chapter ends with a summary.

1.2 The Two Fundamental Laws of Capitalism

Piketty (2014a) introduces what he calls the two fundamental laws of capitalism in Chapters 1 and 5 of his book. The first law defines the relationship between three important measures in macroeconomics: the capital share, the return on capital and the capital/income ratio. The second law defines the capital/income ratio as a function of the savings rate and the growth rate of the economy. While the first law is merely an accounting identity and therefore quite uncontroversial, the second law induced more critics to raise their voices. Their objections shall be discussed in the next chapter. The two laws are presented here.

The first fundamental law of capitalism is derived as follows. Let Y denote national income and K the capital stock. Then β is defined as the capital/income ratio:

$$\beta = \frac{K}{Y} \tag{1.1}$$

Let r denote the rate of return on capital. Therefore, rK is capital income. Now define α as the share of capital income in national income:

$$\alpha = \frac{rK}{Y} \tag{1.2}$$

Combining equations (1.1) and (1.2), yields the formula which Piketty calls the first fundamental law of capitalism:

$$\alpha = r \times \beta \tag{1.3}$$

Even though this law is merely an accounting identity, it shows the relationship between the share of capital in income, the rate of return on capital and the capital/income ratio. This relationship will be useful in the discussion of the capital-labour split.

The second fundamental law of capitalism is only presented but not derived in Piketty's book. A more detailed derivation shall be shown here, which is based on more information taken from the book's technical appendix (Piketty, 2014d, p. 28). The formula goes back to the work of Harrod (1939), Domar (1947) and Solow (1956).

Let the growth rate of the economy be denoted by g and let national income at time t, denoted as Y_t, grow at rate g_t. Then the following equation holds:

$$Y_{t+1} = (1 + g_t)Y_t \tag{1.4}$$

Furthermore, define the savings rate s as the ratio of total savings S and national income Y, so that the savings rate at time t is:

$$s_t = \frac{S_t}{Y_t} \tag{1.5}$$

Let the capital stock in a certain time period be defined as the sum of the capital stock in the previous period and total savings in the previous period:

$$K_{t+1} = K_t + S_t \tag{1.6}$$

Now, divide the left hand side of equation (1.6) by Y_{t+1} and the right hand side by $(1 + g_t)Y_t$, which is equivalent according to equation (1.4). This yields the following equation:

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{K_t}{Y_t} \cdot \frac{1}{1 + g_t} + \frac{S_t}{Y_t} \cdot \frac{1}{1 + g_t} \tag{1.7}$$

Next, replace the capital/income ratio by β and S_t/Y_t by the savings rate s_t to get:

$$\beta_{t+1} = \frac{\beta_t + s_t}{1 + g_t} \tag{1.8}$$

As a last step, a steady state condition is introduced. This means that in the steady state, the quantities in equation (1.8) do not change and therefore, $\beta_{t+1} = \beta_t = \beta$, $s_t = s$ and $g_t = g$. Solving the resulting equation for β yields the equation which is presented as the second fundamental law of capitalism:

$$\beta = \frac{s}{g} \tag{1.9}$$

It has to be emphasised that in contrast to the first law, which, as an accounting identity, holds at all times, the second law is an asymptotic law. This means that the law shows a long run tendency of the capital/income ratio depending on the savings rate and the growth rate.

When studying the distribution of wealth, it is important to understand the evolution of the capital/income ratio, since it is a good measure of how much of the economic resources are controlled by capital holders.

Piketty (2014a) emphasises that the Western world has returned to a slow-growth regime, especially due to low demographic growth. Therefore, the capital/income ratio can reach increasingly high levels, enhancing the influence of capital holders. Furthermore, the capital/income ratio is quite sensitive to changes in the growth rate. As an example, he shows that if $s = 12\%$ and $g = 2\%$, then the capital/income ratio $\beta = s/g = 600\%$. This ratio can increase quite heavily for only small changes in the growth rate. In the example, the capital/income reaches levels as high as 800% for $g = 1.5\%$ and 1200% for $g = 1\%$ instead of 2%, meaning that the economy is twice as capital intensive compared to a society with a growth rate of 2%.

Nonetheless, it has to be mentioned already here that this result changes quite drastically if depreciation is included. This issue shall be addressed in the subsequent chapter, where the objections by Piketty's critics shall be presented.

The evolution of the capital/income ratio does also play a crucial role in the next section, where the evolution of the capital-labour split is discussed.

1.3 The Capital-Labour Split

In Chapter 6 of his book, Piketty discusses the evolution of the capital-labour split in a society where the capital/income ratio is growing. The main insight given in the book (Piketty, 2014a) and the book's technical appendix (Piketty, 2014d, pp. 37–39) shall be presented here.

The first fundamental law of capitalism highlighted the relationship between the capital share α, the return on capital r and the capital/income ratio β. The question, which will be addressed in this section, is how the capital share evolves in the growth process. It is sensible to assume that the return on capital is decreasing, if the capital stock is growing. However, it is unclear, whether the return on capital falls more or less than proportionally when the capital/income ratio is growing.

It turns out that the answer to this question depends on the elasticity of substitution between labour and capital. If this elasticity is equal to one, then the increase in β

and the decrease in r cancel each other out, so that the capital share α remains the same. If the elasticity is lower than one, then r falls more than proportionately if β increases, which means that the capital share α falls. Equivalently, if the elasticity of substitution is higher than one, then the capital share α increases if the capital/income ratio β increases, because the return on capital r falls less than proportionately.

To make this more clear, it can be shown using a production function. If the elasticity of substitution between labour and capital is equal to one, the corresponding production function is a Cobb-Douglas production function of the form $F(K, L) = K^a Y^{1-a}$, where a is a technological parameter between zero and one. The marginal product of capital is given by

$$\frac{\partial F(K,L)}{\partial K} = aK^{a-1}L^{1-a} = a\left(\frac{Y}{K}\right) = \frac{a}{\beta} \tag{1.10}$$

If the return on capital is determined by the marginal product of capital, then the capital share α is given by

$$\alpha = r \times \beta = \frac{a}{\beta} \times \beta = a \tag{1.11}$$

and is therefore independent of the capital/income ratio β. In such a case, the capital share is constant and does not change if the capital/income ratio increases.

But Piketty (2014a) argues that historical data show an elasticity of substitution between 1.3 and 1.6 and therefore larger than one. For this case, a more general production function is needed, which allows for different values for the elasticity of substitution. A function satisfying this is the CES production function, which takes the following functional form:

$$F(K, L) = \left[aK^{\frac{\sigma-1}{\sigma}} + (1-a)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1.12}$$

It can be shown that the parameter σ is equal to the elasticity of substitution. Furthermore, it can be shown that the CES function converges to a Cobb-Douglas function if σ converges to one.

For the CES production function, the marginal product is given by

$$\frac{\partial F(K,L)}{\partial K} = a\left(\frac{Y}{K}\right)^{\frac{1}{\sigma}} = a\beta^{-\frac{1}{\sigma}} \tag{1.13}$$

Then again, if the return on capital r is determined by the marginal product of capital, the capital share α is given by

$$\alpha = r \times \beta = a\beta^{\frac{\sigma-1}{\sigma}} \tag{1.14}$$

To see how the capital share changes if the capital/income ratio grows, take the partial derivative with respect to β to obtain

$$\frac{\partial\alpha}{\partial\beta} = \frac{\sigma-1}{\sigma}a\beta^{-\frac{1}{\sigma}} \tag{1.15}$$

which is positive if $\sigma > 1$ and zero if $\sigma = 1$.

Therefore, if the elasticity of substitution is larger than one, the capital share is an increasing function of the capital/income ratio. Whether or not this is the case shall be discussed in the next chapter, giving Piketty's critics a voice.

But if the capital share is increasing in the capital/income ratio, this means that not only a rising share of the production factors, but also a major share of national income is controlled by capital holders. This can have some serious repercussions on the structure of a society.

The third and presumably most well-known statement about wealth inequality is discussed in the next section: the rate of return on capital exceeding the growth rate of the economy ($r > g$). This is another force favouring capital ownership and is identified by Piketty as the main driver of wealth inequality.

1.4 The Main Driver of Wealth Inequality: r > g

In Chapter 10 of his book, Piketty (2014a) presents what is probably his most well-known hypothesis: wealth concentrates, if the rate of return on capital exceeds the growth rate of the economy, expressed in the famous inequality $r > g$.

Piketty explains what he calls a fundamental force of divergence as follows. If the rate of return on capital systematically exceeds the growth rate of the economy, only a fraction of capital income needs to be saved to ensure that capital is growing at the same or even a higher rate as the economy, while the rest can be consumed. As an example, Piketty uses $g = 1\%$ and $r = 5\%$ to show that only one-fifth of capital income needs to be reinvested, while the rest can be consumed and capital will still grow at the same rate as the economy. If more than one-fifth is saved, an individual's capital stock will grow faster than the economy, which leads to a concentration of wealth.

Based on historical data, Piketty shows that the return on capital has indeed exceeded the growth rate of the economy throughout most periods in history. In Figure 1.1, which shows the rate of return before tax and the growth rate of the economy, one can see that without considering tax policies, $r > g$ holds throughout history. Therefore, Piketty considers $r > g$ to be a historical fact.

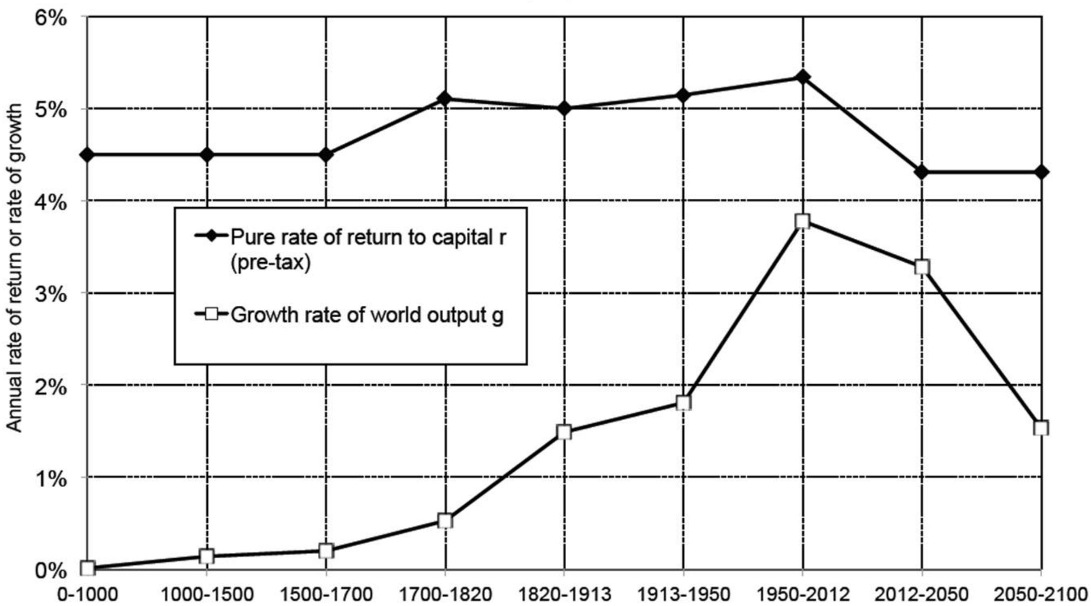

Figure 1.1: Rate of return versus growth rate at the world level, from Antiquity until 2100. Source: Piketty (2014a, p. 354).

Throughout much of human history, economic growth was close to zero, since the structure of the society was mainly based on subsistence agriculture. Therefore, growth was mainly due to demographic growth, which was very low in itself, too. From Figure 1.1 it can be seen that from antiquity until the seventeenth century, the annual growth rate never exceeded 0.1-0.2 percent for long. On the other hand, the return on capital, which was predominantly the rent from land ownership, was steady around 4-5 percent.

The gap between r and g decreased considerably during the twentieth century. Exceptionally high growth rates can be observed especially in the second half of the twentieth century, where the economy grew at a rate of 3.5-4 percent per year. Reasons for this can be found in the rebuilding process which took place after World War II. In addition to that, population growth was exceptionally high ("baby boom").

The twentieth century did also see the emergence of rising taxes on capital. In the time before World War I, taxes on capital were very low and therefore, pre-tax returns on capital can be considered almost equal to after-tax returns. After World War I, the political climate favoured high progressive taxes on capital, which were introduced in various countries.

High growth rates and progressive taxes on capital created a historically unprecedented situation in which the after-tax return on capital was below the growth rate of the economy and thus, $r < g$. Figure 1.2 depicts after-tax rates of return and the growth rates of the economy and neatly illustrates the exceptional period in the twentieth century.

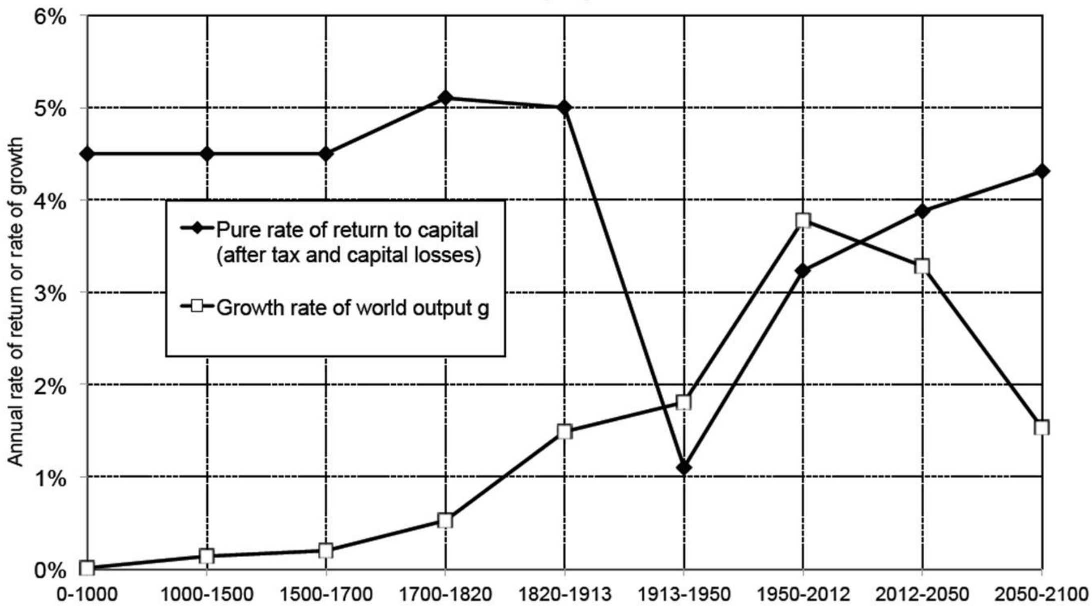

Figure 1.2: After tax rate of return versus growth rate at the world level, from Antiquity until 2100. Source: Piketty (2014a, p. 356).

Nonetheless, Piketty clearly states that with high certainty, this period is going to end. The high growth rates after World War II can be seen as a catch-up process which ends as soon as a country reaches the technological frontier and growth rates decrease. Demographically, especially Western countries show a clear tendency towards low population growth, so that the growth rate of the economy can be assumed to slow down. Piketty estimates that in the absence of shocks, the average growth rate is very likely to return to around 1.5 percent per year, which he already considers a very optimistic outlook.

The gap between r and g widened further, when in the 1980s the ideological climate changed drastically in the heydays of financial globalisation, liberalisation and tax competition. These circumstances lead to falling taxes on capital and in some cases, taxes on capital have almost entirely disappeared.

Against the background of falling growth rates and increasing international tax competition, Piketty argues that it is sensible to assume that in the twenty-first century, the return on capital will exceed the growth rate of the economy and therefore $r > g$ holds. Thus, according to Piketty, there is a strong tendency for wealth to concentrate and wealth inequality to rise inexorably.

1.5 Piketty: Literature Overview

To round up this chapter, an overview of all the publications by Piketty related to the book are provided in this section. It shall serve as a guideline for the interested reader wanting to get more background information on the book.

Capital in the Twenty-First Century (Piketty, 2014a) comes along with a technical appendix (Piketty, 2014d), which contains detailed explanations about all the data and the models behind the main text. The book is accompanied by a website (Piketty, 2014b) from which all the spread sheets, figures and tables can be downloaded.

Piketty and his co-authors have also published several articles on the topics covered in the book. The main article covering the topic of the capital/income ratio can be found in Piketty and Zucman (2014a). Publications concerning models explaining the effects of $r > g$ are available in Piketty (2012) and Piketty and Zucman (2014b). As Piketty proposes different forms of taxation to reduce inequality, he has also published articles on capital and inheritance taxation which can be found in Piketty and Saez (2012, 2013).

Piketty has commented on the book and answered to criticism in several ways. Three important contributions can be found in Piketty (2014c, 2015a, 2015b).

A major part of Piketty's and his co-author's work is the collection of data on income and wealth inequality over multiple centuries and for several different countries. All the data is available online on their published *World Top Incomes Database* (Alvaredo, Atkinson, Piketty, & Saez, 2015).

1.6 Summary

This chapter has provided the reader with a structured overview of the main theoretical statements of Piketty's *Capital in the Twenty-First Century*. Starting with the two fundamental laws of capital, Piketty shows that in the growth process, the capital/income ratio grows to substantially high levels, especially in an environment of low growth. This leads to an increased exertion of influence by capital holders over the production factors. Going on from this, Piketty shows that an increasing capital/income ratio may also result in a higher capital share, thus enabling capital holders to draw upon a higher share of national income. As a third statement, Piketty puts strong emphasis on the fact that if the return on capital exceeds the growth rate of the economy, which he summarises in his famous inequality $r > g$, wealth tends to concentrate at the top. Based on historical data, he suggests that it is sensible to assume that in the twenty-first century $r > g$ holds and therefore, a high wealth concentration has to be expected.

2 Critical Discussion

2.1 Chapter Overview

Piketty's book has stirred a lot of controversy in the economic profession and beyond. This has prompted many critics to raise objections. This chapter aims to give them a voice by providing an overview of the main critique. Criticism towards *Capital in the Twenty-First Centuries* has arisen in several different facets.

The bulk of criticism is aimed towards Piketty's methodological approach and his collection of data. The publicly most noticed critique was published by Giles (2014) in the *Financial Times*. Giles attacks Piketty's data basis and claims that Piketty's data contain a series of errors. Piketty responded to the criticism in an article which can be found in Piketty (2014c). Another point made by several authors such as Dubay and Furth (2014) or Homburg (2014) is that Piketty confuses wealth and capital, using the terms interchangeably instead of distinguishing between capital as a production factor and wealth which might also come in the form of unproductive property. This objection is closely linked with the plea that the recent increase in the capital/income ratio is mainly due to the increase of housing prices (Bonnet, Bono, Chapelle, & Wasmer, 2014; Rognlie, 2014). But since the focus of this thesis is laid on Piketty's main theoretical statements, methodological issues shall not be discussed any further in this chapter.

Another strand of criticism is aimed towards Piketty's normative and political statements. A prominent charge raised against Piketty concerns his attitude towards inequality. Dubay and Furth (2014) accuse Piketty of regarding inequality to be harmful without any proper explanation of why this is the case. Piketty's policy recommendations, though appraised by the political left, have stirred intense reactions by conservative, neoliberal and libertarian exponents. As an example, a response to *Capital in the Twenty-First Century* can be found in Leef (2014), who accuses Piketty of trying to find excuses for state expansion and what Leef calls "legal plunder", a term coined by Piketty's countryman Frédéric Bastiat in the 19th century. Again, to purpose the focus on theory, these concerns shall not be discussed in more detail here.

Therefore, the rest of this chapter is dedicated to summarising the main objections raised towards Piketty's theoretical statements. For this purpose, the chapter is structured in the same way as the previous one. Section 2.2 covers criticism towards the two fundamental laws of capitalism, while Section 2.3 provides an overview of the main issues concerning Piketty's predictions about the capital-labour split. Section 2.4 aims to highlight potential flaws in Piketty's claim that $r > g$ is bound to cause high inequality. The chapter ends with a summary.

2.2 The Two Fundamental Laws of Capitalism

If a relationship between different measures in economics is formulated as a "law", controversy is bound to arise, since economics is no exact science and therefore, "laws" are always vulnerable to criticism. Thus, objections have also been raised against Piketty's two fundamental laws of capitalism.

Since the *first fundamental law of capitalism* is a mere accounting identity, it is less controversial than the second law. But since it is a mathematical necessity rather than a law of capitalism, it can be questioned why Piketty calls it a law. This has been pointed out by Hillinger (2014), who states that it is rather unusual to refer to a pure identity as a law of capitalism. Nonetheless, if one views the first law as more than just an accounting identity and interprets it to be a functional relationship, a critical view might be appropriate. If the first law is to be interpreted as a functional relationship, it defines the capital share as a function of the interest rate and the capital/income ratio, hence $\alpha = f(r, \beta)$. Holcombe (2014) challenges this view by highlighting that the value of capital is defined by what it can earn and not the other way round. Following this logic, the functional relationship should define the capital/income ratio as a function of the interest rate and the capital share, hence $\beta = f(r, \alpha)$. Therefore, according to Holcombe (2014), the first fundamental law of capitalism should rather be formulated as $\beta = \alpha/r$ instead of $\alpha = r \times \beta$.

The *second fundamental law of capitalism* has stirred a lot more controversy. Since Piketty predicts unprecedented levels of the capital/income ratio in a low growth regime, the criticism goes towards the assumptions under which the prediction may hold true. In addition to that, Dubay and Furth (2014) mention that Piketty does not take into consideration that capital accumulation is crucial for wage growth and therefore, a higher capital/income ratio might also lead to increased welfare for individuals owning very little capital and only supplying their labour.

The two major objections against the second law concern Piketty's assumptions about the savings rate and the omission of depreciation in the formulation of the law. These two issues shall be discussed in more detail.

Piketty's second law of capitalism seems to be appealing due to its simplicity and tractability. However, this comes at the cost of very restrictive assumptions upon which the predictive power of the law hinges. A crucial assumption is that the savings rate is independent of both the capital/income ratio and the growth rate of the economy. A closer look into the details of Piketty's second law reveals that a violation of these assumptions might yield much more moderate predictions about the capital/income ratio. As Summers (2014) highlights, it is sensible to assume that the savings rate is a decreasing function of the capital/income ratio. Under this assumption, the increase of the capital/income ratio would imply a falling savings rate, which in turn would decelerate the growth of the capital/income ratio. A second effect, which impacts the capital/income ratio in the same direction is pointed out by Homburg (2014). According to him, one can assume the savings rate to be an increasing function of the economy's growth rate. The impact of this assumption is particularly strong in an environment of low growth, as described by Piketty. If growth is very low, this would imply a low savings rate, which again would decrease the capital/income ratio's growth. Thus, it can be said that under more realistic assumptions about the savings rate, the capital/income ratio does not reach as high levels as predicted by the more simplistic second fundamental law of capitalism formulated by Piketty.

The second and most likely even more severe objection against Piketty's second fundamental law of capitalism is the omission of depreciation in the capital accumulation process. Piketty points out that if the capital/income ratio asymptotically reaches a value $\beta = s/g$, with growth going towards zero the capital/income ratio grows towards infinity. As Krusell and Smith (2014) point out, this would imply a capital accumulation process without bound, which is at odds with standard economic growth models. Therefore, several authors argue that the second law should be extended to include depreciation (Dubay & Furth, 2014; Krusell & Smith, 2014; van Schaik, 2014). If depreciation is included, the law of motion for capital (equation (1.6) on page 7) can be rewritten as follows:

$$K_{t+1} = (1-\delta)K_t + S_t \tag{2.1}$$

where δ is the parameter for linear depreciation of capital.

Following the same steps as in Section 1.2, one arrives at the following modified second law of capitalism:

$$\beta = \frac{s}{g + \delta} \tag{2.2}$$

Even though the difference between the two versions of the law seems to be only minor, the implications for the capital/income ratio in a low growth regime are substantial. While in Piketty's version the capital/income ratio grows without bound, the capital/income ratio is bounded by the value s/δ, since δ is assumed always to be positive.

Summarising the objections raised against Piketty's second fundamental law of capitalism, it can be said that contrary to Piketty's predictions that in a low growth regime capital/income ratios reach unprecedented levels, the capital/income ratio is limited to an upper bound if one assumes the savings rate to be decreasing and capital to depreciate.

2.3 The Capital-Labour Split

In Section 1.3, it has been shown that in a growing economy the capital share might increase or decrease, depending on the elasticity of substitution between labour and capital. If the elasticity is larger than one, this implies a rising capital share and therefore rising inequality, since data show that capital is more unequally distributed than labour income. However, if the elasticity is below one, the capital share decreases and thus, economic growth has an equalising effect.

Piketty estimates the elasticity of substitution to be around 1.3–1.6. This statement is heavily criticised by several authors. As Summers (2014) points out, various empirical researchers have attempted to estimate elasticities of substitution, but there are many methodological difficulties involved. Therefore, estimates vary heavily between different published estimates.

Dubay and Furth (2014) as well as Rognlie (2014) present an overview of different published estimates and state that Piketty's estimate for an elasticity of substitution of 1.3–1.6 lies well outside estimates by most empirical researchers.

This is shown in Figure 2.1. It has to be mentioned that Piketty presents a net elasticity of substitution, while most researchers estimate gross elasticities. Converted from net to gross terms, Piketty's estimate translates into a gross elasticity of around 2.9–3.6, which lies heavily outside most estimates made by other researchers.

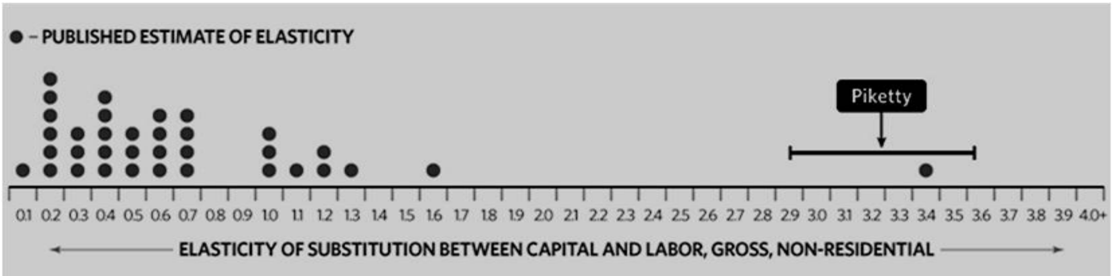

Figure 2.1: Different estimates for the gross elasticity of substitution between labour and capital. Source: Dubay and Furth (2014) based on Chirinko (2008) and Karabarbounis and Neiman (2014).

From the figure it can be seen that the assumption of an elasticity of substitution above one is to be questioned by empirical evidence. It is probably more likely for the elasticity of substitution between labour and capital to be lower and thus, the capital share to be stable or even falling in a growing economy.

2.4 The Main Driver of Wealth Inequality: r > g

As described in the previous chapter, Piketty regards $r > g$ to be the main driver of wealth inequality and the central contradiction of capitalism. If the return on capital exceeds the growth rate of the economy, then wealth grows faster than national income, thus increasing inequality.

It is obvious that this statement hinges upon the assumption that a large enough fraction of capital income is reinvested. This assumption is questioned by several authors, among which Mankiw (2015) and Summers (2014) are two prominent examples. As Summers (2014) states, the Forbes 400 list provides very little evidence that big fortunes are patiently accumulated through perpetual reinvestment of capital income. Out of all the people listed on the 1982 list, only one tenth are still listed in 2012, and according to Gates (2014), about half of the people on the 2014 list are entrepreneurs who did not inherit their wealth out of a perpetually reinvested family fortune.

Mankiw (2015) points out that even if the return on capital exceeds the growth rate of the economy, the $r - g$ gap needs to be much higher than recorded by Piketty in order to amplify inequality. He names three main reasons narrowing down the $r - g$ gap: consumption, procreation and taxation.

This point is illustrated with a short calculation. Assume that the propensity to consume from wealth is around 3 percent, which Mankiw (2015) claims to be a plausible estimate based on both theory and empirical evidence. This implies that with consumption, wealth grows at rate $r-3$. In addition to that, assume that at the passage of wealth from one generation to the next, the fortune is divided among several descendants, which increase in number every generation. Assuming a population growth rate of about 2 percent, this implies that if family wealth grows at rate $r-3$, a descendants wealth grows at rate $r-5$. As a third step, taxation is included. Mankiw (2015) assumes wealth to be taxed in three ways: taxation of capital income, taxation of real estate and inheritance taxation. Translated into a percentage taxation per year, Mankiw (2015) estimates taxation to reduce wealth by about 2 percent a year. Thus, taking all three effects into consideration, Mankiw (2015) estimates the growth rate of wealth to be at around $r-7$. Therefore, the gap between the return on capital and the growth rate of the economy would need to exceed 7 percentage points to amplify wealth inequality.

Even though this is just a back of the envelope calculation rather than an empirical estimate, it shows that the $r-g$ gap needs to be unrealistically high for wealth inequality to increase. Therefore, Piketty's claim about rising inequality due to $r > g$ is based upon very strong assumptions, which might be violated under more realistic conditions.

2.5 Summary

This chapter has provided the reader with an overview of the main objections raised against Piketty's theoretical statements.

Objections against the two fundamental laws of capitalism are mainly addressed against the assumptions behind the second law. Several authors point out that the savings rate being independent of the capital/income ratio and the growth rate of the economy is too strong an assumption. Loosening this assumption would yield much lower capital/income ratios over time. It has also been mentioned that the omission of depreciation in the second fundamental law of capitalism yields an overestimation of future capital/income ratios. By including depreciation, it has been shown that the capital/income ratio has an upper bound.

The capital-labour split is the second subject under discussion. As Piketty describes in his book, the capital share increases over time, if the elasticity of substitution between labour and capital is larger than one. Piketty estimates this parameter to be around 1.3–1.6. This claim is highly controversial. Using meta-studies of different estimates, it has been shown that Piketty's estimate lies well outside most estimates by several other researchers. Therefore, based on empirical research, it is quite unlikely that the elasticity of substitution exceeds unity.

The most frequently discussed topic is Piketty's claim on $r > g$ being the main driver of inequality and the central contradiction of capitalism. Piketty predicts that if $r > g$, then capital grows faster than national income, hence increasing inequality. This prediction is based on the assumption that a large enough fraction of capital income is reinvested and that dynasties get richer by perpetually reinvesting their fortune and passing it over to descending generations. Using the Forbes 400 list and an explanation of how consumption, procreation and taxation reduce wealth, it has been shown that Piketty's assumption about individual investment behaviour is quite likely to be violated in practice.

In the next chapter, the claim that $r > g$ is the main driver of inequality shall be discussed using Piketty's workhorse model to show that wealth inequality is a rising function of $r - g$.

3 Wealth Inequality in Pareto Models

3.1 Chapter Overview

Piketty (2014a) emphasises that the inequality $r > g$ is the main driver of wealth inequality in a capitalistic system. In his book, he explains this giving economic intuition and some examples, but there is a much more complex mechanism behind it. In several articles, Piketty and his co-authors Saez and Zucman have used models which yield a Pareto distribution of wealth. These models neatly illustrate how r and g influence the distribution of wealth and how different tax policies can counteract rising inequality. This chapter seeks to give an overview of the models used by Piketty and his co-authors and to provide the reader with a better understanding of the main mechanism and the economic intuition behind the claim that $r > g$ is the main driver of wealth inequality.

The remainder of this chapter is structured as follows. Section 3.2 motivates the use of Pareto models in the study of wealth distributions. Following this, Section 3.3 provides an overview of the main mechanism used in the articles by Piketty and his co-authors. Since modelling approaches have been around for quite a while and wealth inequality can be explained through the use of various sources of inequality, Section 3.4 provides an overview of different authors' contributions. The chapter ends with a summary.

3.2 Motivation

As the name indicates, the idea behind these models goes back to the work of Vilfredo Pareto (1896). Data on income and wealth distributions show that, instead of being normally distributed, the distribution is right-tailed and follows a power law, where the fraction of the population with wealth above a certain value is proportional to this value raised to some power as described below:

$$\Pr[\text{Wealth} > y] = y^{-1/\eta} \tag{3.1}$$

Based on Pareto's work, the resulting distributions from such a law are called Pareto distributions. Jones (2014) shows that this distribution can be easily connected to the top income and wealth shares described in Piketty's book (Piketty, 2014a). With such a power law, the fraction of wealth belonging to the top p percentiles equals $(\frac{100}{p})^{\eta-1}$. As an example, if $\eta = 1/2$, the share of wealth belonging to the top 1 percent is $100^{-1/2} = 0.1$. For a higher value of η, for example $\eta = 2/3$, this share equals $100^{-1/3} \approx 0.22$. This means that the parameter η is directly linked to the top shares of wealth, which are higher if η is higher.

Models in which the obtained distribution has a Pareto form have been around for a long time. Early examples can be found in Cantelli (1921) as well as in Wold and Whittle (1957). Mitzenmacher (2004) gives an overview of the history of Pareto models while in Benhabib (2014) a good overview of different Pareto models can be found. Gabaix (2009) provides an excellent overview of the use of power laws in different subdisciplines of economics and finance.

As Jones (2014) indicates, an economic theory of income and wealth inequality needs to address two main questions. First, models need to illustrate which underlying economic mechanisms lead to wealth being Pareto distributed. Secondly, the models need to show which economic forces determine the size of the Pareto coefficient and therefore the level of wealth inequality.

The next section seeks to answer these two questions by highlighting the main mechanism underlying the model used by Piketty and his co-authors to illustrate how r and g affect the distribution of wealth.

3.3 Pareto Models: Main Mechanism

As Piketty (2014a) emphasises in his book, the inequality $r > g$ is a major source of inequality in a system of capitalism. But as Piketty (2015a) states, in standard macroeconomic models, it is not obvious to see the impact of $r > g$ on the wealth distribution. What is needed to generate inequality in the first place is the introduction of extra ingredients into basic models. This is done by the introduction of various types of shocks. A model with shocks shall be introduced in this section.

The idea behind the model is quite straightforward. There are many unequalising forces which come in the form of shocks such as taste shocks, shocks on returns or demographic shocks. In the model, wealth is transmitted from one generation to another and a shock occurs in every generation. Depending on the shock, an individual transmits more or less wealth to the next generation, which again passes on more or less wealth, depending on the size and the sign of the shock they experience. As a result, individuals are rich or poor depending on how their ancestors were affected by the shocks. It can be shown that with such a scheme of shocks, which take a multiplicative form, the equilibrium distribution of wealth follows a power law and has therefore a Pareto shape.

This shall be illustrated in more detail in the following. The model described here can be found in Piketty (2012) and Piketty and Zucman (2014b). Since their model is quite rich and addresses many different questions, only a simplified version along with the economic intuition shall be presented here. The interested reader is encouraged to refer to the sources mentioned above.

The model essentially takes the form of a one-period lifetime model with bequests (OLB) as described in Bertola et al. (2006, pp. 115–122). Piketty (2012) calls the model a wealth-in-the-utility model with finite horizon. This name can be misleading, since the preference to hold wealth is basically a preference to bequeath to the next generation, so there is no direct utility from holding wealth. As Bertola et al. (2006) illustrate, there are two different ways of modelling intergenerational links. One way would be for a parent to directly draw utility from the welfare of their children, so that their utility function is a function of their own consumption and of their children's utility function. Iterating such a utility function forward, the model translates into an infinite horizon model. A Pareto model with an infinite horizon can be found in Nirei (2009). The second way is described as a "warm glow" motive. With such a modelling approach, parents draw direct utility from leaving wealth to their children, so that the size of the bequests enters their utility function directly. This is what is done in Piketty's OLB model.

As the name of the model suggests, individuals live only one period. This is done because the focus lies on the size of wealth saved for bequests and thus, the model abstracts from life-cycle savings. Each individual behaves as to maximise a utility function of consumption and bequests taking a Cobb-Douglas form $U(c, b) = c^{1-s}b^s$, where c denotes consumption and b denotes the size of bequests left to the next generation. Each individual is endowed with bequests from their parents and with income from labour, which is equal for all individuals. The parameter s is a taste parameter and solving a standard maximisation problem, it can be seen that s is equal to an individual's savings rate, since a fraction s of its wealth and income is saved to leave for the next generation. From this, it is straightforward to see that the bequests left to the next generation can be formulated with the following expression:

$$b_{i(t+1)} = s\big[y_{Lit} + (1 + r_t)b_{it}\big] \tag{3.2}$$

where b_{it} denotes bequests for individual i at time t, y_{Lit} denotes labour income of individual i at time t and r_t is the interest rate at time t, which is equal for all individuals.

Up to this point, Piketty follows a standard OLB model without shocks. Now a shock of any sort is introduced. This shock will have a direct impact on the size of bequests transmitted to the next generation. Piketty (2012) uses a shock on the taste parameter s, which introduces heterogeneity into the model. In the most simple form, the shock is binomial, so that the parameter s is equal to $s^* > 0$ with probability p and is zero with probability $1 - p$. This means that only a fraction p of the population leaves bequests to their children.

Following this, the evolution of the distribution of wealth can be described as follows. Assume that in the first generation there is perfect equality. All individuals earn labour income y_L and none of them receives any bequests. A fraction $1 - p$ of the population will consume all their income, leaving nothing to their children, while a fraction p will save a fraction s of their income to bequeath to their children. As a consequence, the second generation is a two-class society, where a fraction p of the population is rich due to having received bequests from their parents and a fraction $1-p$ is poor, because they have not received any bequests. In the second generation, the shock on tastes occurs again. This yields a three-class society in the third generation. A fraction $1 - p$ of the population will belong to the lower class, having received zero bequests. A fraction $(1 - p)p$ belongs to the middle class, having received bequests from their parents who themselves belonged to the poor class. The upper class, consisting of a fraction p^2 of the population, receive bequests from their parents who themselves belonged to the rich class. Continuing this logic, it follows that in the n-th generation there is an n-class society with the poorest class consisting of a fraction $1 - p$ of the population and the richest class consisting of a fraction p^n of the population.

As Piketty (2012) emphasises, the shock introduced into the model could take any other form, yielding a similar evolution of the distribution of wealth, as long as the shock takes a multiplicative form. For example, the taste shock could be multinomial instead of binomial, the shock could be on the labour endowment, the number of children or it could be on the rate of return. The mechanism would always be the same: parents bequeath a different amount of wealth to their children depending on the size and the sign of the shock. An individual in the n-th generation would then be rich or poor depending on the series of shocks affecting their ancestors.

Piketty (2012) shows that as long as the shock takes a multiplicative form, the distribution of wealth converges to a steady state distribution which follows a power law and has therefore a Pareto shape. By introducing exogenous productivity and population growth, one can assess how the distribution of wealth evolves over time. In the steady state, individual labour income y_{Li} grows at rate g, while inherited wealth b_i grows at rate r. From this, it follows that if $r > g$, inherited wealth grows faster than labour income, thus increasing inequality. Piketty's model (Piketty, 2012) gives insight into the evolution of the distribution of wealth and from the model, the following conclusions can be drawn:

(i) Inequality is an increasing function of $r - g$.

(ii) Inequality is a decreasing function of the population growth rate.

(iii) Inequality can be reduced through a tax on capital or a tax on inherited wealth.

The intuition behind these results lies in Piketty's emphasis on the importance of inherited wealth. If r is higher, inherited wealth accumulates faster, increasing inequality. But if economic growth is higher, which means g is higher, inherited wealth has less value relative to new wealth generated in the economy. In addition to that, a higher growth rate of the economy leads to faster wage growth, which has an equalising effect on the distribution of wealth. Moreover, if population growth is higher, inherited wealth is divided up by a higher amount of newborn individuals reducing the concentration of wealth. Therefore, low population growth leads to a higher concentration of wealth. As a remedy, Piketty (2014a) proposes the introduction of a tax on capital to reduce the accumulation of inherited wealth, leading to a more equal distribution of wealth.

Similar models have been introduced by other authors. These models all have in common that they yield a distribution of wealth which has a Pareto shape. The models differ in the type of shocks they introduce. The main sources of inequality introduced into these models shall be presented in the next section.

3.4 Sources of Inequality in Pareto Models

As already mentioned above, Pareto distributions of wealth can be obtained using very different kinds of sources of inequality. A whole range of authors have dedicated their research to modelling frameworks, in which the distribution of wealth follows a power law. As Piketty (2012) points out, the idea behind these models always follows the same logic: there is a series of multiplicative shocks which amplify over time to generate a Pareto shaped distribution. The mathematical formulas behind it stay almost the same and do only change to model richer mechanisms. This section shall outline various contributions to the field.

There are two types of shocks which are used most often: idiosyncratic risk and demographic shocks. It is always assumed that individuals cannot insure against these shocks, so that they are fully exposed to them.

Idiosyncratic risks are modelled in various different forms. The risks include uninsurable investment risks, shocks in labour or capital income or uninsurable endowment risks, where individuals face shocks on the non-accumulated factor. Two contributions have to be pointed out in particular, since they are very close to the contributions by Piketty and his co-authors.

Nirei (2009) models inequality in a Solow model as well as in a Ramsey framework. The shock introduced to generate heterogeneity is an idiosyncratic shock to an individual's asset returns. The author shows that inequality is a decreasing function of the technological growth rate, which is in line with the findings of Piketty (2014a). In addition to that, Nirei (2009) shows that inequality is increasing in the variance of the shock, while redistribution policies financed by income or bequest tax are an effective way to reduce wealth inequality.

A second contribution can be found in the work of Benhabib, Bisin and Zhu (2011). The authors model inequality in an overlapping generations model with intergenerational transmission of wealth. The shock used to generate wealth inequality is a shock on capital and labour income. Their findings indicate that first and foremost, it is idiosyncratic shocks on capital income, which play a crucial role in affecting the distribution of wealth. The authors also show that taxes on capital income and estate taxes as well as institutions favouring social mobility are effective policies to reduce wealth inequality.

Besides the contributions of Nirei (2009) and Benhabib et al. (2011), there are several other authors discussing the impact of idiosyncratic risk on the distribution of wealth. The interested reader shall refer to the contributions by Castañeda, Díaz-Giménez and Ríos-Rull (2003), Bertola et. al (2006, Chapter 9), Fernholz and Fernholz (2012) as well as Fernholz (2015).

The other type of shocks, demographic shocks, is introduced by several authors who use it to model wealth distributions following a power law. Such shocks can be concerning age, as for example the age of parenthood or the age of death, or they can be concerning offspring, as for example the number of children or the rank of children. Three contributions shall be mentioned here.

Stiglitz (1969) discusses the case of primogeniture. In his model, the first born son receives all bequests, while his siblings go away empty-handed. This shock yields a similar wealth distribution as in the model by Piketty (2012). In the first generation, the society is completely egalitarian. After this, there is a two-class society in the second generation, consisting of rich households with a first born son and poor households with second to n-th born sons. Again, as in the model described above, the third generation is a three-class society and the n-th generation is an n-class society. In equilibrium, the distribution of wealth follows a power law with the richest class consisting of households with a first born son whose ancestors were first born sons as well.

Cowell (1998) presents a model in which the number of children varies from family to family as a heterogeneity generating process. Bequests are divided equally between all children, so that children with less siblings inherit higher wealth. In this setting, the richest individuals are single children, whose ancestors were single children as well. The model by Cowell (1998) produces more complex dynamics, since the number of children is finite and hence, there are no individuals whose inheritance is zero. Nonetheless, the model produces a Pareto distribution of wealth in equilibrium.

Jones (2014) presents a model where the age distribution and the accumulation of wealth generate a Pareto distribution of wealth. In his model, individuals face a constant probability of death at each point in time and the population grows according to a given birth-death process. Since individuals have different life spans, they accumulate their wealth over a shorter or longer time period, depending on their date of death. As Jones (2014) shows, such a framework creates a Pareto distribution for wealth, where inequality is a rising function of $r - g$. Thus, Jones (2014) reproduces the results obtained by Piketty (2012).

3.5 Summary

This chapter has provided the reader with an overview of economic models, where the resulting wealth distribution follows a power law and therefore has a Pareto shape. Using such a modelling approach is motivated by the work of Vilfredo Pareto (1896), who discovered that the distribution of wealth is heavy tailed on the right and follows a power law. The resulting distribution is named after him.

Piketty and his co-authors use a one-period lifetime model with bequests (OLB) to model an individual's behaviour. All individuals behave as to maximise utility from consumption and from leaving bequests to their children. Introducing a shock on the level of bequests left to the next generation, heterogeneity is introduced into the model. Piketty (2012) shows that with such a modelling approach, the resulting steady state distribution of wealth has a Pareto shape. The main insight of the model concerns the role of the two variables of interest, r and g. It is shown that inequality is a rising function of $r - g$ and that taxes on inheritance and on capital are effective policies to reduce inequality.

Furthermore, the chapter has given an overview of different sources of inequality introduced into Pareto models. The two main mechanisms are idiosyncratic risk and demographic shocks. The contributions by several authors have been presented along with the economic intuition behind the evolution of the distribution of wealth.

Following this overview of Piketty's work, the influence of r and g on the distribution of wealth shall be discussed in different macroeconomic growth models without shocks. This is done in Part B of this thesis. As in Bertola et al. (2006), heterogeneity will be generated by introducing subsistence consumption. Besides two standard growth models, the Solow model and the Ramsey model, Piketty's OLB approach is analysed in a setting without shocks and with subsistence consumption to assess the effect of r and g on the distribution of wealth.

Part B: Wealth Inequality in Macroeconomic Growth Models

4 Wealth Inequality in the Solow Model

4.1 Chapter Overview

As seen in Part A, Piketty's hypothesis stating that $r > g$ leads to higher inequality has stirred a lot of controversy. Therefore, a more detailed analysis of the relationship between the interest rate, economic growth and inequality shall be performed in Part B of this thesis. This shall be done in standard macroeconomic growth models, since they are the most commonly used and are widely accepted in the economic profession. To start with, the analysis shall be done in a Solow growth model. The model goes back to the work of Solow (1956) and Swan (1956) and even though it lacks micro-foundations, it serves the purpose well as a baseline model. A widely used representation of the model can be found in Romer (2012, Chapter 1), of which some aspects are employed in this chapter. The model described here is an extension of the Solow model presented by Bertola et al. (2006, Chapter 2). Their model is extended by introducing exogenous growth in order to addresss the main question about the impact of the interest rate and economic growth on the distribution of wealth.

The main channel through which heterogeneity is generated in the models presented in Part B is through the introduction of subsistence consumption, as done by Bertola et al. (2006). The use of subsistence consumption leads to different savings rates for individuals with different wealth holdings and thus creates heterogeneity.

To avoid confusion, some remarks about notation have do be made here. As Piketty (2014a, p. 593) explains, he uses g to denote the overall growth rate of the economy, which in most economic models is the equivalent of $\dot{Y}/Y$, where Y denotes national income. This notation is different from most macroeconomic models, in which g denotes productivity growth, while n denotes population growth, so that the growth rate of the economy is a composite of the two.[1] To make notation clear and avoid confusion, this thesis slightly deviates form standard notation. Therefore, for the remainder of this thesis, the productivity growth rate shall be denoted by g_A, while g_L shall denote the

[1] In most standard growth models, the growth rate of the economy is the sum of productivity growth and population growth on the balanced growth path.

population growth rate. Wherever g is used without subscript, it shall denote the overall growth rate of the economy.

The remainder of this chapter is structured as follows. Section 4.2 introduces the model and explains the main deviations from the baseline model by Bertola et al. (2006). Since the evolution of the wage rate turns out to be of crucial importance, Section 4.3 is dedicated to provide the reader with more insight about the progression of the wage rate. Section 4.4 describes the model's steady state with a focus on the evolution of the distribution of wealth in the model economy. Following from the insight given in the model, Section 4.5 discusses the role of the interest rate and economic growth and seeks to answer the question of their impact on the distribution of wealth. The chapter ends with a summary.

4.2 The Model

The model described below is taken from Bertola et al. (2006, Chapter 2) and Zweimüller (2013). As an extension, exogenous productivity and population growth is introduced. Deviations from the model are indicated while they are introduced.

The model is characterised by three equations, describing an individual's income, denoted by $y(i)$, an individual's consumption, denoted by $c(i)$, and the law of motion for an individual's wealth, denoted by $k(i)$:

$$y(i) = w + rk(i) \tag{4.1}$$

$$c(i) = c_l w + c_k rk(i) + \tilde{c}k(i) + A\bar{c} \tag{4.2}$$

$$\Delta k(i) = y(i) - c(i) \tag{4.3}$$

Equation (4.1) describes the composition of an individual's income $y(i)$. It is assumed that each individual inelastically supplies one unit of labour and earns wage w. In addition to this, each individual earns capital income $rk(i)$ from their wealth holdings. In Bertola et al. (2006), population size is normalised to one and therefore aggregate labour supply is equal to an individual's labour supply. Since population growth is introduced here, aggregate labour supply is denoted by L and an individual's labour supply is normalised to one.

Equation (4.2) describes an individual's consumption behaviour. Individuals have an exogenous propensity to consume from labour income, denoted by $c_l \in (0, 1)$, and an exogenous propensity to consume from capital income, denoted by $c_k \in (0, 1)$. This is a small deviation from Bertola et al. (2006), since they assume an exogenous propensity to consume from income, regardless of the source of income. This extension is motivated by Zweimüller (2013) to show the consequences of different assumptions about c_l and c_k. In addition to an individual's consumption from income, it is also assumed that an individual consumes a constant fraction $\widetilde{c} \in (0, 1)$ from wealth. This can be interpreted in two ways. As already indicated, it can be interpreted as consumption from wealth, but it can also be interpreted as wealth depreciation. In this model it is assumed that the term $\widetilde{c}$ covers both cases. Therefore, without loss of generality, depreciation is not included explicitly in the model, since the functional form would look the same. As a last part of the consumption function, a constant exogenous subsistence level of consumption, denoted by $\bar{c} > 0$ is introduced. This term is multiplied by the level of productivity A to extend the model by Bertola et al. (2006). Multiplying subsistence consumption by productivity models the case where the level of subsistence consumption is not absolute, but grows at the exogenous rate of productivity growth, denoted by g_A.

Equation (4.3) describes the law of motion of an individual's wealth, which is simply the difference between income and consumption. Plugging equations (4.1) and (4.2) into the law of motion given by equation (4.3), the following law of motion for an individual's wealth $k(i)$ is obtained:

$$\Delta k(i) = (1 - c_l)w + (1 - c_k)rk(i) - \widetilde{c}k(i) - A\bar{c} \tag{4.4}$$

Dividing both sides of equation (4.4) by $k(i)$ yields an equation for the growth rate of an individual's wealth:

$$\frac{\Delta k(i)}{k(i)} = \frac{(1 - c_l)w - A\bar{c}}{k(i)} + (1 - c_k)r - \widetilde{c} \tag{4.5}$$

Using this equation, a statement about the evolution of the distribution of wealth over time can be made. By taking the first derivative of $\frac{\Delta k(i)}{k(i)}$ with respect to $k(i)$, one can see whether wealth grows faster for individuals with higher or lower wealth:

$$\frac{\partial \frac{\Delta k(i)}{k(i)}}{\partial k(i)} = -\frac{(1 - c_l)w - A\bar{c}}{k(i)^2} \tag{4.6}$$

From equation (4.6) it can be seen that the distribution of wealth depends on savings from labour income and subsistence consumption $A\bar{c}$. Three different cases can be distinguished, which are listed in Table 4.1.

Table 4.1: The Distribution of Wealth

Case	Change in growth rate	Distribution of k
$(1-c_l)w < A\bar{c}$	$\frac{\partial \frac{\Delta k(i)}{k(i)}}{\partial k(i)} > 0$	Divergence
$(1-c_l)w = A\bar{c}$	$\frac{\partial \frac{\Delta k(i)}{k(i)}}{\partial k(i)} = 0$	Persistence
$(1-c_l)w > A\bar{c}$	$\frac{\partial \frac{\Delta k(i)}{k(i)}}{\partial k(i)} < 0$	Convergence

Source: Zweimüller (2013)

What is the intuition behind the results presented in Table 4.1? To see the mechanism at work here, one has to distinguish absolute and relative quantities in reference to an individual's wealth. To illustrate this, assume that $(1-c_l)w = A\bar{c}$. This case can be interpreted as a situation where savings from labour income $(1-c_l)w$ are exactly high enough to cover subsistence consumption $A\bar{c}$. Therefore, the only thing which matters for capital accumulation are savings and consumption from wealth $k(i)$. The law of motion for an individual's wealth reduces to the following:

$$\Delta k(i) = (1-c_k)rk(i) - \widetilde{c}k(i) \tag{4.7}$$

Since the parameters c_k and $\widetilde{c}$ as well as the interest rate r are equal for all individuals, everyone saves the same amount of wealth relative to their wealth holdings. This means that the savings rates of all individuals are equal and therefore the wealth distribution is persistent. If savings from labour income exceed subsistence consumption, i.e. $(1-c_l)w > A\bar{c}$, the amount in excess of subsistence consumption goes towards capital accumulation. Here it is important to see that since the parameters are equal for all individuals and since each individual has the same labour endowment, everyone saves an equal quantity in absolute terms. As a consequence of this, individuals with lower wealth holdings $k(i)$ save a higher amount relative to their wealth. Therefore, the savings rate is decreasing in $k(i)$ and the wealth distribution is converging. With the same reasoning, it can be shown that the savings rate is increasing in $k(i)$, if savings from labour income do not cover subsistence consumption, i.e. $(1 - c_l)w < A\bar{c}$. Thus, the distribution of wealth is diverging.

What can be said about the sign of $(1 - c_l)w - A\bar{c}$? In the obvious case of $c_l = 1$ and $\bar{c} > 0$, there are no savings from labour income and therefore, subsistence consumption always exceeds savings from labour income. This would be a case where there is always divergence. But a closer look at such a situation reveals that individuals with low wealth reduce their wealth holdings without bound, while individuals with high wealth increase it without bound. This would yield a situation where there is divergence to the point where some individuals are infinitely rich, while others are infinitely indebted. This situation arises from the mere fact that the model does not include a budget constraint which makes sure that individuals cannot get infinitely indebted. Therefore, it is assumed that c_l and therefore savings from labour income are strictly positive.

If c_l is strictly positive, the distribution of wealth is dependent on the parameters c_l and $\bar{c}$, which are exogenous and do not change over time, and on the wage rate w and the productivity level A. As already described, the productivity level grows exogenously at rate g_A and therefore, subsistence consumption grows at rate g_A. Thus, the interesting question to ask is, whether the wage rate grows faster or slower than subsistence consumption. If the wage rate grows faster than subsistence consumption, this implies that starting from a state of divergence where the wage rate is low, wage growth eventually raises savings from labour income above subsistence consumption. Hence, the economy evolves from a state of divergence into a state of convergence. Therefore, to understand how wealth is distributed in the model economy, it is crucial to understand the evolution of the wage rate. Understanding this will also help to reveal the role of the interest rate and economic growth in the distribution of wealth. This is done in the following sections.

4.3 The Evolution of the Wage Rate

From Table 4.1 it can be seen that the evolution of the distribution of wealth changes, if the wage rate is above a certain threshold, which shall be denoted by $\widetilde{w}$. This threshold is given by the following expression:

$$\widetilde{w} = \frac{A\bar{c}}{1 - c_l} \tag{4.8}$$

As can be seen from the above expression, the threshold level $\widetilde{w}$ grows at rate g_A. The interesting question to ask is, whether starting from a wage rate $w_0 < \widetilde{w}$, the wage rate grows faster than the threshold, so that once the threshold is reached, the wealth distribution changes from being diverging into a state of convergence. This question shall be addressed in this section. It will be shown that in the transition to the steady state, the wage rate grows faster than subsistence consumption, and as soon as the steady state is reached, both the wage rate and subsistence consumption grow at rate g_A.

To address this question, assume that production technology is a standard production function with constant returns to scale and labour-augmenting technology, $F(K, AL)$. The Inada conditions are assumed to be satisfied. The question can be discussed without assuming a particular functional form, so that the results obtained apply to any production function satisfying the above assumptions. With constant returns to scale, the production function can be written in intensive form as $f(k)$ with k being capital per unit of effective labour, so that $k = K/AL$. Observe the distinction between $k(i)$ and k, where the former is individual wealth and the latter is capital per unit of effective labour.

The wage rate w is then defined as the marginal product of labour and can be derived in the following way:

$$\begin{aligned}\frac{\partial F(K, AL)}{\partial L} &= \frac{\partial ALf(k)}{\partial L} \\ &= Af(k) - ALf'(k)\frac{K}{(AL)^2}A \\ &= Af(k) - Af'(k)k \\ &= A[f(k) - f'(k)k]\end{aligned} \tag{4.9}$$

As in the standard Solow model, k grows in the transition period and is constant in the steady state. Therefore, it can be seen from the last line of the above expression that the term in brackets is constant in the steady state, so that the wage rate grows at rate g_A in the steady state.

But how does the wage rate evolve in the transition period? To answer this question, take the logarithmic derivative with respect to time of the expression $A[f(k) - f'(k)k]$ to obtain the wage growth rate. This is done in the following way:

$$\begin{aligned}\frac{\dot{w}}{w} &= \frac{\dot{A}}{A} + \frac{f'(k)\dot{k} - f''(k)\dot{k}k - f'(k)\dot{k}}{f(k) - f'(k)k} \\ &= g_A + \frac{-f''(k)\dot{k}k}{f(k) - f'(k)k}\end{aligned} \tag{4.10}$$

This derivation shows that the wage growth rate is equal to the sum of the productivity growth rate g_A and an expression with a yet unclear sign. If this expression is positive, this implies the wage growth to be exceeding productivity growth. From the Inada conditions it holds that $f''(k) < 0$. In the transition period, both $\dot{k}$ and k are positive, so that the numerator of the expression is positive. The denominator of the expression is positive as well, since it is the wage rate divided by the level of productivity, which are both assumed to be positive. Therefore, from the above expression it can be seen that in the transition period, the wage growth rate exceeds the productivity growth rate, hence $\frac{\dot{w}}{w} > g_A$.

In the steady state, $\dot{k} = 0$ and thus, the above expression confirms the wage rate to grow at rate g_A in the steady state as already stated above.

From this analysis it follows that wages grow at a faster rate than subsistence consumption in the transition period. But since wage growth is equal to productivity growth in the steady state, an important question is yet to be answered: does the wage rate exceed the threshold level $\widetilde{w}$ before the steady state is reached?

Three scenarios are possible. In a first case, it is possible that the wage rate approaches the threshold level in the transition period, but does not reach it before the steady state is reached. This would imply divergence in the steady state and therefore a state of permanent divergence. In a second case, the threshold level is exactly reached and in the steady state the wage rate w is equal to the threshold wage rate $\widetilde{w}$, so that the obtained distribution of wealth is persistent in the steady state. As a third and last case, it is possible that the wage rate exceeds the threshold level in the transition period, and as soon as the steady state is reached, the economy is in a state of convergence. This would imply that the wealth distribution is asymptotically reaching a fully egalitarian state.

As will be demonstrated next, only the third case is possible in a stable steady state. To show this more rigorously, the steady state distribution is analysed in more detail in the next section.

4.4 Wealth Distribution in the Steady State

To derive the distribution of wealth in the steady state, one has to determine the sign of $(1 - c_l)w - A\bar{c}$. This is done by aggregating the individuals' savings behaviour to macroeconomic quantities. It is also in this context, that exogenous productivity and population growth is introduced.

To find the steady state, one has to aggregate the law of motion for an individual's wealth using the Stjelties integral:[2]

$$\begin{aligned} \Delta K &= \int_N \Big[\Delta k(i)\Big] \,\mathrm{d}P(i) \\ &= \int_N \Big[(1-c_l)w + (1-c_k)rk(i) - \widetilde{c}k(i) - A\bar{c}\Big] \,\mathrm{d}P(i) \end{aligned} \tag{4.11}$$

where K denotes aggregate capital. Solving the integral yields the following law of motion for aggregate capital K:[3]

$$\Delta K = (1-c_l)wL + (1-c_k)rK - \widetilde{c}K - AL\bar{c} \tag{4.12}$$

Now, exogenous productivity and population growth is introduced in the same fashion as it is done in Romer (2012, Chapter 1). As before, define $k = K/AL$ as capital per unit of effective labour and let productivity grow at rate g_A and population at rate g_L. From this, it is straightforward to see that the law of motion for k is as follows:

$$\Delta k = \frac{(1-c_l)w - A\bar{c}}{A} + (1-c_k)rk - (g_A + g_L + \widetilde{c})k \tag{4.13}$$

In the steady state, capital per unit of effective labour is stationary and hence $\Delta k = 0$. Setting $\Delta k = 0$ and rearranging terms yields the following steady state condition:

$$\big[(1-c_k)r - (g_A + g_L + \widetilde{c})\big]k = -\frac{(1-c_l)w - A\bar{c}}{A} \tag{4.14}$$

To ensure a stable equilibrium, the first derivative of Δk with respect to k has to be negative. This yields the steady state stability condition:

$$\left.\frac{\partial \Delta k}{\partial k}\right|_{\Delta k = 0} = (1-c_k)r - (g_A + g_L + \widetilde{c}) < 0 \tag{4.15}$$

From this inequality, it can be seen that the left hand side of equation (4.14) has to be

[2] A detailed explanation of the use of the Stjelties integral in this context can be found in Bertola et al. (2006, p. 6).

[3] Observe that since the aggregate labour supply L is equal to the population size N, the result of an integration over unity equals L, i.e. $\int_N 1\mathrm{d}P(i) = N = L$.

negative. Thus, one can conclude that the following must hold in the steady state:

$$(1 - c_l)w - A\bar{c} > 0 \tag{4.16}$$

Therefore, it follows that in the steady state, the distribution of wealth converges. This answers the question posed in the previous section. A consequence of this result is that starting with an initial wage rate $w_0 < \widetilde{w}$,[4] the wealth distribution will evolve over time according to a Kuznets curve: the wealth distribution diverges first and after the threshold wage rate $\widetilde{w}$ is passed, the wealth distribution converges.[5]

The convergence result is independent of both the interest rate and economic growth and therefore, there is no divergence in the steady state, even if $r > g$. Thus, the model does not support the hypothesis stated by Piketty (2014a).

But what is the role of r and g in the transition period, where the wealth distribution can still be diverging? This question shall be addressed in the next section.

4.5 The Role of r and g

In the previous section, it has been shown that the wealth distribution in the model economy follows a Kuznets curve and in the steady state, the distribution is converging.

In this section, the focus is laid on the transition period and on the role of r and g in the divergence phase. Two questions can be posed, which are of interest and can be answered in the model. First, the impact of r and g on the shape of the Kuznets curve shall be discussed. The question here is, whether inequality at the end of the divergence process, and hence the highest point of the Kuznets curve, is influenced by r and g. A second question addresses the time dimension of the divergence process. The question of interest is, whether or not the divergence phase is longer or shorter, depending on r and g. These two questions shall be discussed here.

In order to obtain more specific answers in this section, the questions shall be addressed using a standard Cobb-Douglas production function with labour-augmenting technology: $F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$. In intensive form, the production function takes the form

[4] From equations (4.14) and (4.15) it can also be seen that in the unstable steady state, where $\frac{\partial \Delta k}{\partial k} > 0$, there is divergence. Thus, there exists a capital level k_0 above the unstable steady state, where the wage rate w_0 is below the threshold wage rate $\widetilde{w}$.

[5] The notion of the Kuznets curve goes back to Kuznets (1955). He states that the distribution of wealth follows an inverse U shape. Starting from a low-income agricultural society which is quite egalitarian, an economy evolves towards a high-income manufacturing and service society. In this process, inequality rises until the society as a whole gets richer and introduces welfare programmes and grants broad access to education, which reduces inequality and yields a converging wealth distribution.

$f(k) = k^{\alpha}$, where again, k is capital per unit of effective labour. Using the results obtained in Section 4.3, the wage rate takes the form $w = A[f(k) - f'(k)k] = (1-\alpha)Ak^{\alpha}$.

To answer the question, whether or not r and g have an effect on the shape of the Kuznets curve, the capital stock at which the Kuznets curve reaches its highest point is determined. This is done using the threshold wage rate $\widetilde{w}$. When the highest level of inequality is reached, the wage rate has to be equal to the threshold wage rate and hence

$$\widetilde{w} = \frac{A\bar{c}}{1-c_l} = (1-\alpha)Ak^{\alpha} \tag{4.17}$$

Solving the equation yields the capital stock $\widetilde{k}$ at which inequality is at its maximum:

$$\widetilde{k} = \left(\frac{\bar{c}}{(1-c_l)(1-\alpha)}\right)^{\frac{1}{\alpha}} \tag{4.18}$$

From the above equation it can be seen that $\widetilde{k}$ is fully determined by the exogenous parameters of the model.

Since both productivity growth and population growth are exogenous in the model, one can see from the above equation that the highest point of the Kuznets curve is independent of economic growth. Thus, Piketty's hypothesis regarding the growth rate of the economy is not supported in this context.

The influence of the interest rate on inequality is a bit more delicate to answer, since the interest rate is endogenously determined in the model and it is rather unusual to study the impact of an endogenous variable on the outcome of the model. When looking for a *causal* impact of the interest rate on the Kuznets curve, one can see from the above expression, that the interest rate has no impact on the highest point of the Kuznets curve. Nonetheless, it has to be emphasised that there is a *correlation* between the two and hence, divergence and the interest rate are co-moving. To see why this is the case, the properties of a standard production function have to be taken into account. In standard macroeconomic growth models, the interest rate is defined as the marginal product of capital. In intensive form, this can be written as $r = f'(k)$. Since the Inada conditions state that $f''(k) < 0$, it follows that the interest rate is a decreasing function of the capital stock. Thus, if $k < \widetilde{k}$ and hence the distribution is diverging, the interest rate is high. Therefore, it can be concluded that a low capital stock, which implies a diverging wealth distribution, implies a high interest rate and hence, divergence and a high interest rate are correlated. This is in line with Piketty's hypothesis.

The second question, which shall be posed, is whether or not r and g have an impact on the time dimension of the divergence process. Assuming that the wealth distribution changes according to a Kuznets curve, how long does it take for an economy to get out of the phase of divergence and enter into the phase of convergence?

To analyse this, assume that the economy is in a state with a low capital stock $k_0 < \widetilde{k}$ and the question is how long it takes for the growing capital stock to reach the threshold level $\widetilde{k}$.

The role of the interest rate in the growth process can be answered in a similar way as above. The further away k_0 is from the threshold level $\widetilde{k}$, the longer the divergence process. And since the interest rate $r = f'(k)$ is a decreasing function of k, this means that the interest rate is higher, if k_0 is further away from $\widetilde{k}$. Thus, a high interest rate and a long divergence process are correlated.

To answer the question on what impact economic growth has on the divergence process, one has to analyse the growth rate of the capital stock $\dot{k}/k$, which is obtained by dividing the law of motion of capital given in equation (4.13) on page 38 by the capital stock k:

$$\frac{\dot{k}}{k} = \frac{(1-c_l)w - A\bar{c}}{Ak} + (1-c_k)r - (g_A + g_L + \widetilde{c}) \tag{4.19}$$

In order to obtain an expression which is only dependent on the exogenous parameters and k, the wage rate is replaced by $w = A[f(k) - f'(k)k]$ and the interest rate is replaced by $r = f'(k)$, which yields the following expression:

$$\frac{\dot{k}}{k} = \frac{(1-c_l)[f(k) - f'(k)k] - \bar{c}}{k} + (1-c_k)f'(k) - (g_A + g_L + \widetilde{c}) \tag{4.20}$$

From this equation it can be seen that for any $k_0 < \widetilde{k}$, the growth rate of the capital stock is smaller if either productivity growth g_A or population growth g_L is higher. Since k is defined as $k = K/AL$, one can see that if productivity or the population grows faster, capital per unit of effective labour grows slower. Therefore, it has to be said that both productivity and population growth lengthen the divergence period, which is not in line with Piketty's findings.

In conclusion it can be said that a higher interest rate correlates with divergence and if the capital stock is low and thus the interest rate high, the divergence period lasts longer. This is in favour of Piketty's findings. Moreover, higher productivity and population growth lengthen the divergence period, which does not support Piketty's hypothesis about the impact of economic growth on wealth inequality.

4.6 Summary

In this chapter, the impact of the interest rate and economic growth on the distribution of wealth has been discussed in a Solow model with subsistence consumption. As a baseline, the model by Bertola et al. (2006, Chapter 1) has been used. Their model has been extended to include exogenous productivity and population growth.

The model shows that the distribution of wealth crucially depends on savings from labour income and thus on the wage rate. The distribution evolves according to a Kuznets curve, where in the steady state, there is always convergence. The convergence result holds independently of the interest rate or economic growth. However, in the transition period towards the steady state, there can be divergence if in the initial state, the wage rate is below a certain threshold, which in itself is independent of r and g.

Two analyses have been made to assess the impact of the interest rate and economic growth on wealth inequality.

In a first analysis, it has been shown that the highest point of the Kuznets curve, which is the state in which inequality is at its maximum, is independent of both r and g. Nonetheless, in the period of divergence, the capital stock is low and therefore, the interest rate is high. Thus, divergence and a high interest rate are correlated, which is in line with Piketty's findings.

A second analysis was concerned with the question, whether or not r and g have an impact on the time dimension of the divergence period. It has been shown that the lower the capital stock, the longer the divergence period lasts, and since the interest rate is higher at a lower capital stock, this implies that a higher interest rate correlates with a longer divergence period. In contrast to this finding, which supports Piketty's hypothesis, the findings about the impact of economic growth on the divergence period go in the opposite direction. It has been shown that both higher productivity and population growth lengthen the divergence process. Hence, these results do not coincide with Piketty's findings.

In conclusion, it can be said that the findings concerning the interest rate are in favour of Piketty's claim, since a higher interest rate correlates with a longer divergence period. However, the findings on economic growth are opposite to Piketty's hypothesis, since in the model presented here, both productivity and population growth increase the length of the divergence period.

In the next chapter, the issue will be analysed in a one-period lifetime model with bequests. The model will also be a step towards a micro-foundation of individual consumption behaviour.

5 Wealth Inequality in the One-Period Lifetime Model with Bequests

5.1 Chapter Overview

In the Solow model, the consumption function was given exogenously and therefore the individuals' savings behaviour. A common way to endogenise individual consumption plans is by micro-founding growth models. A model which produces very similar results as the Solow model is the one-period lifetime model with bequests. Even though it is set up quite differently, it will be shown that the resulting capital accumulation function has strong similarities to the one obtained in the Solow model.

Thus, in this chapter, the one-period lifetime model with bequests presented in Piketty (2012) as well as in Piketty and Zucman (2014b) shall be analysed in the context of subsistence consumption. As shown in Chapter 3, Piketty's results follow from a series of shocks introduced into the model. The question addressed in this chapter is how the results change, if there are no shocks but heterogeneity is introduced through subsistence consumption as done in the Solow model.

The model discussed here is a one-period lifetime model with bequests and subsistence consumption as presented in Bertola et al. (2006, Chapter 5). As an extension to the model by Bertola et al. (2006), exogenous productivity and population growth is introduced to study Piketty's hypothesis about r and g in more detail.

The remainder of this chapter is structured as follows. In Section 5.2, the model is presented and the condition under which there is wealth convergence is identified. Section 5.3 provides an analysis of the wealth distribution in the steady state. The chapter ends with a summary.

5.2 The Model

As the name of the model suggests, agents only live for one period and are connected through a flow of bequests. In contrast to models with multiple period lifetimes, the model abstracts from life-cycle savings and focuses on the flow of inheritance.

In the model by Piketty (2012), agents behave as to maximise a Cobb-Douglas utility function of consumption and bequests, thus acting with a “warm glow” motive. The same utility function is used here with the extension of subsistence consumption $\bar{c}$, which again is multiplied by the level of productivity A. In this way, subsistence consumption grows with the productivity level, ensuring that subsistence consumption does not become negligible in a growing economy. The agents are linked together through a flow of inheritance, forming dynasties. Therefore, the smallest unit of interest in this model is the dynasty. Assume that the size of a dynasty is the total size of population, denoted by L, divided by the number of dynasties. Without loss of generality, the number of dynasties is normalised to one, so that the size of the dynasty is equal to the population size L. Subsistence consumption $A\bar{c}$ is assumed to be per capita and thus, a dynasty’s subsistence consumption is $AL\bar{c}$. The model is set in discrete time.

The model starts with a dynasty’s utility function, which is given by the following form, as explained above:

$$U(c_t, b_{t+1}) = (1 - s)\log(c_t - A_t L_t \bar{c}) + s \log b_{t+1} \tag{5.1}$$

Each dynasty supplies L units of labour at rate w and is endowed with bequests received from their ancestors. Bequests yield interest r and depreciate at rate δ. Thus, a dynasty’s budget constraint can be formulated as follows:

$$c_t + b_{t+1} = w_t L_t + (1 + r_t - \delta) b_t \tag{5.2}$$

Solving a standard maximisation problem, an expression for bequests passed on to the next generation is obtained:

$$b_{t+1} = s\Big[w_t L_t + (1 + r_t - \delta) b_t - A_t L_t \bar{c}\Big] \tag{5.3}$$

As can be seen, the above equation has a strong resemblance to the individual’s capital accumulation function obtained in the Solow model (see equation (4.4) on page 33). A fixed fraction s from labour income and capital income, i.e. returns from bequests, is saved and there is depreciation, which can also be interpreted as consumption from capital, and subsistence consumption. In contrast to the Solow model, this function is a result of a dynasty’s optimisation behaviour and thus a micro-foundation of the Solow model.

Dividing equation (5.3) by b_t yields the growth rate of bequests within a dynasty:

$$\frac{b_{t+1}}{b_t} = \frac{s(w_t L_t - A_t L_t \bar{c})}{b_t} + s(1 + r_t - \delta) \tag{5.4}$$

To see whether the wealth distribution is converging or diverging, the growth rate of bequests is analysed. By taking the first derivative of $\frac{b_{t+1}}{b_t}$ with respect to b_t, one can assess whether bequests grow faster in dynasties with higher bequests. Taking the first derivative results in the following expression:

$$\frac{\partial \frac{b_{t+1}}{b_t}}{\partial b_t} = -\frac{s(w_t L_t - A_t L_t \bar{c})}{b_t^2} \tag{5.5}$$

Thus, if $w_t L_t - A_t L_t \bar{c} > 0$ and hence $w_t > A_t \bar{c}$, the distribution of wealth is converging.

This result is very similar to the results obtained in the Solow model. Wage growth is the equalising force in the model and as soon as the wage rate is high enough to cover subsistence consumption, poorer households catch up and the distribution of wealth is converging.

Therefore, the results obtained in the Solow model do also apply here. Starting from a low wage rate $w_0 < A_t \bar{c}$, the wage rate grows at a higher rate than subsistence consumption. As soon as the steady state is reached, both the wage rate and subsistence consumption grow at the rate of productivity growth and hence, in the steady state, the difference between the wage rate and subsistence consumption persists. Thus, it is crucial to analyse the evolution of the distribution of wealth in the steady state in order to understand wealth inequality in the model economy. This is done in the next section.

5.3 Wealth Distribution in the Steady State

To obtain the distribution of wealth in the steady state, the expression for individual bequests passed on to the next generation (see equation (5.3)) is aggregated in the same manner as in the Solow model. Define the aggregate capital stock, denoted by K, as the sum of all bequests made in one period. The aggregation yields the following expression for the capital stock:

$$K_{t+1} = s\Big[w_t L_t + (1 + r_t - \delta) K_t - A_t L_t \bar{c}\Big] \tag{5.6}$$

Again, one can see the similarities with the capital accumulation function in the Solow model. As done there, the capital accumulation function can be rewritten in intensive form. In order to obtain this, divide equation (5.6) by $A_{t+1}L_{t+1}$ and let capital per unit of effective labour be denoted by k. Exogenous growth is introduced such that $\frac{A_{t+1}}{A_t} = (1+g_A)$ and $\frac{L_{t+1}}{L_t} = (1+g_L)$. The resulting expression for capital per unit of effective labour is formulated as follows:

$$k_{t+1} = \frac{s(w_t - A_t\bar{c})}{(1+g_A)(1+g_L)A_t} + \frac{s(1+r_t-\delta)}{(1+g_A)(1+g_L)}k_t \tag{5.7}$$

This equation is divided by k_t to obtain the growth rate of capital per unit of effective labour:

$$\frac{k_{t+1}}{k_t} = \frac{s(w_t - A_t\bar{c})}{(1+g_A)(1+g_L)A_t k_t} + \frac{s(1+r_t-\delta)}{(1+g_A)(1+g_L)} \tag{5.8}$$

In the steady state, capital per unit of effective labour is constant, hence $\frac{k_{t+1}}{k_t} = 1$. To obtain a stable steady state, the first derivative of $\frac{k_{t+1}}{k_t}$ with respect to k_t has to be negative in the steady state, thus yielding the following stability condition:

$$\left.\frac{\partial \frac{k_{t+1}}{k_t}}{\partial k_t}\right|_{\frac{k_{t+1}}{k_t}=1} = -\frac{s(w_t - A_t\bar{c})}{(1+g_A)(1+g_L)A_t k_t^2} < 0 \tag{5.9}$$

From this equation it follows that in the steady state $w_t > A_t\bar{c}$ and thus, the distribution of wealth is converging.[1]

This result is identical with the results obtained in the Solow model and therefore, the statements made about the effects of r and g on the distribution of wealth do also hold in this setting. The convergence result holds independently of both r and g and there is no causal effect on the highest point of the Kuznets curve. However, a high interest rate is correlated with a low capital stock and thus, in the phase of divergence the interest rate is high. Therefore, Piketty's hypothesis is supported insofar as there is a correlation between wealth divergence and a high interest rate.

[1] From equation (5.9) it can also be seen that in the unstable steady state where $\frac{\partial \frac{k_{t+1}}{k_t}}{\partial k_t} > 0$, there is divergence. Therefore, there exists a capital level k_0 above the unstable steady state where the wage rate w_0 is low enough for the distribution of wealth to diverge, so that the distribution of wealth in this model follows a Kuznets curve.

In addition to that, the results obtained on the impact of r and g on the length of the divergence period hold in this model as well. The further away the capital stock is from the threshold capital stock above which the distribution of wealth is converging, the higher the interest rate and the longer the divergence period. Therefore, a higher interest rate and a longer divergence period go hand in hand, a statement coinciding with Piketty's findings. However, both higher productivity and population growth lengthen the period of divergence, which is not in line with Piketty.

5.4 Summary

In this chapter, the impact of r and g on the distribution of wealth has been analysed in a one-period lifetime model with bequests and subsistence consumption. Agents live only one period and are linked through bequests to form dynasties. The model has shown that the distribution of wealth depends on the wage rate, which is endogenously determined in the model. As long as the wage rate is below subsistence consumption, the distribution of wealth is diverging. In the transition period, wages grow at a higher rate than subsistence consumption and before the steady state is reached, the wage rate exceeds subsistence consumption, so that the distribution of wealth is converging in the steady state.

The convergence result holds independently of r and g, but in the period of divergence, the capital stock is low, which implies a higher interest rate. Thus, divergence and a higher interest rate are correlated.

The length of the divergence period is longer, if the capital stock is lower. In such a case the interest rate is higher, so that a higher interest rate implies a longer divergence period. However, both productivity and population growth lengthen the divergence period.

Therefore, it has to be concluded that the results concerning the interest rate and divergence are in line with Piketty's findings, whereas the findings concerning economic growth and divergence stand in contradiction to Piketty's claim.

As has been shown, this model was a first attempt for a micro-foundation of individual consumption plans. Nonetheless, the savings rate was given exogenously by the savings taste parameter of the utility function. In the next chapter, a model shall be presented in which the savings rate is endogenised.

6 Wealth Inequality in the Ramsey Model

6.1 Chapter Overview

While in the Solow model the individuals' behaviour was completely determined by the exogenously given consumption function, their behaviour was based on optimising behaviour in the one-period lifetime model with bequests. Therefore, the OLB model was a first step towards endogenising the agents' behaviour and can thus be seen as a micro-foundation of the Solow model. Nonetheless, an individual's savings rate was given by the taste parameter s in its utility function and was therefore given exogenously in the model.

This chapter aims to address this issue by endogenising the savings rate and thus seeks to explain the distribution of wealth without assuming a given savings rate. One of the earliest and most widely used attempts to do so is based on the work of Ramsey (1928), Cass (1965) and Koopmans (1965), who formulate a growth model with an infinitely lived representative agent, who maximises utility subject to an intertemporal budget constraint. An excellent textbook representation of the model is given by Romer (2012, Chapter 2) from which many ideas are used here.

As in the previous chapters, the main question posed is whether or not the interst rate and economic growth have an impact on the distribution of wealth. As in the Solow and OLB model, heterogeneous agents are introduced by means of a subsistence consumption parameter $\bar{c}$. The model by Bertola, Foellmi and Zweimüller (2006, Chapter 3) and Zweimüller (2013), who formulate such a model without growth, shall be presented and used as a baseline. Afterwards, the model shall be extended by introducing exogenous productivity and population growth.

The remainder of this chapter is structured as follows. Section 6.2 gives an overview of the baseline model with heterogeneous agents by Bertola et. al (2006, Chapter 3). In Section 6.3, the model is extended by exogenous productivity and population growth. Section 6.4 discusses the role of r and g. The chapter ends with a summary.

6.2 The Model without Growth

The model by Bertola et al. (2006) and Zweimüller (2013) is set up as an infinite horizon model with discrete time. In contrast to the standard model described in Romer (2012, Chapter 2), preferences include a parameter for subsistence consumption $\bar{c}$. The particular utility function used is a generalised Stone-Geary utility function, which takes the following form:

$$u(c) = \frac{(c - \bar{c})^{1-\sigma}}{1 - \sigma} \tag{6.1}$$

For the special case $\bar{c} = 0$, the function simplifies to a standard CRRA utility function used in Romer (2012). Using L'Hôpital's rule, it can be shown that the utility function simplifies to $u(c) = \log(c - \bar{c})$ if $\sigma = 1$. It is assumed that $\bar{c} > 0$.

By means of micro-founding individual behaviour through a utility function, the agents' savings behaviour can be explained much more profoundly. As shown in Zweimüller (2013), people become relative less risk averse with increasing income, if their preferences are described in the functional form given above. Therefore, poor individuals have a low intertemporal elasticity of substitution, which means that their consumption growth does respond less pronounced to variations in the interest rate. As pointed out by Bertola et al. (2006), poorer individuals choose a flatter consumption path in order to maintain their subsistence consumption. The mechanism works through the individuals' consumption-smoothing motive.

A main finding of this type of models is that the individuals' consumption decisions are based on permanent rather than current income and wealth. As shown in Bertola et al. (2006), consumption is a linear function of lifetime wealth with generalised Stone-Geary preferences. Thus, a comparison of consumption paths between poor and rich individuals suffices to make a conclusive statement about the individuals' savings rates and therefore about wealth inequality.

The main conclusion of the model by Bertola et al. (2006) is that as long as the interest rate is higher than the time preference parameter, rich individuals save more than poor individuals and therefore, the wealth distribution diverges. In a model without exogenous growth, this is true as long as the economy is in a transition phase towards the steady state. In the transition phase there is growth, but growth is due to the process of capital accumulation and therefore endogenous. As soon as the steady state is reached, the economy does not grow anymore and the distribution of wealth is persistent.

To find out whether or not this statement still holds if exogenous productivity and population growth is introduced, the model shall be extended to include these properties in the next section. This will serve as a basis to discuss the role of r and g in the distribution of wealth.

6.3 The Model with Exogenous Growth

As the baseline model by Bertola et al. (2006, Chapter 3), the extension presented here features positive subsistence consumption $\bar{c}$ in order to create heterogeneity. For the purpose of better tractability, the model is set up in continuous time as opposed to the model by Bertola et al. (2006), in which time is discrete. Therefore, the model is closer to the Ramsey model presented in Romer (2012, Chapter 2), which is also set up in continuous time. Thus, it can be said that the model presented in this section is built up as in Romer (2012), but includes Stone-Geary preferences as in Bertola et al. (2006). As in the Solow model, exogenous productivity growth is denoted by g_A and exogenous population growth is denoted by g_L to avoid confusion. Confusion might arise, because Piketty (2014a) uses g to denote the overall growth rate of the economy, while Romer (2012) uses g to denote productivity growth. Hence the small deviation in notation from Romer (2012).

The model setup starts with the description of individual behaviour. Assume that there is a fixed number of households H in the economy with $L(t)/H$ members each, where $L(t)$ denotes the population size at date t. Each household divides its income between consumption and saving at each point in time. Households behave as to maximise their lifetime utility given by the following functional form:

$$U = \int_{t=0}^{\infty} e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt \tag{6.2}$$

where $C(t)$ is per capita consumption and $u(\cdot)$ is the instantaneous utility function. The discount rate is denoted by ρ. Assume that the instantaneous utility function is given by a log-function with subsistence consumption: $u(C(t)) = \log\left[C(t) - A(t)\bar{c}\right]$. As in the previous models, subsistence consumption is multiplied by the productivity level $A(t)$ to prevent subsistence consumption from becoming negligible in a growing economy.

Exogenous growth is included as follows. Assume that the population size grows at a constant rate g_L, so that $L(t) = L(0)e^{g_L t}$. To include productivity growth, define $c(t)$ as consumption per unit of effective labour, so that $c(t) = C(t)/A(t)$. Let productivity grow at rate g_A.

Using these specifications, lifetime utility can be rewritten as follows:

$$U = \frac{L(0)}{H} \int_{t=0}^{\infty} e^{(g_L - \rho)t} \log \left[A(t)c(t) - A(t)\bar{c} \right] dt \tag{6.3}$$

As a next step, the households' budget constraint is described. Each household takes the wage rate $W(t)$ and the interest rate $r(t)$ as given.[1] The budget constraint states that the present value of lifetime consumption cannot exceed the present value of initial wealth and lifetime labour income. To calculate the present value, one has to discount back with the interest rate. To account for variations in the interest rate over time, let $R(t)$ be defined as $\int_{\tau=0}^{t} r(\tau)d\tau$. Let $K(0)$ denote the initial capital stock, which is assumed to be equally distributed among all households, so that $K(0)/H$ is equal to a household's initial wealth. The intertemporal budget constraint can then be written as follows:

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} W(t) \frac{L(t)}{H} dt \tag{6.4}$$

To include exogenous growth, all the quantities are diverted into quantities per unit of effective labour. To do this, define $w(t)$ as the wage rate per unit of effective labour, so that $w(t) = W(t)/A(t)$, and define $k(t)$ as capital per unit of effective labour, hence $k(t) = K(t)/A(t)L(t)$. As before, consumption per unit of effective labour is denoted by $c(t)$. The budget constraint can then be rewritten as follows:

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) \frac{A(t)L(t)}{H} dt \leq k(0) \frac{A(0)L(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w(t) \frac{A(t)L(t)}{H} dt \tag{6.5}$$

Using the fact that $A(t)L(t) = A(0)L(0)e^{(g_L+g_A)t}$ and dividing both sides of the budget constraint by $A(0)L(0)/H$ yields the budget constraint in units of effective labour:

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(g_L+g_A)t} dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(g_L+g_A)t} dt \tag{6.6}$$

As in Romer (2012), it is assumed that households must satisfy a no-Ponzi-game condition.

[1] Observe that unlike in the previous models, the wage rate is denoted by a capital letter here. The lower case letter $w(t)$ will be used afterwards to define the wage per unit of effective labour.

Using equations (6.3) and (6.6), the household's maximisation problem can be formulated by setting up the Lagrangian:

$$\begin{aligned}\mathcal{L} &= \frac{L(0)}{H}\int_{t=0}^{\infty} e^{(g_L-\rho)t}\log\left[A(t)c(t)-A(t)\bar{c}\right]dt \\ &\quad + \lambda\left[k(0)+\int_{t=0}^{\infty} e^{-R(t)}w(t)e^{(g_L+g_A)t}dt-\int_{t=0}^{\infty} e^{-R(t)}c(t)e^{(g_L+g_A)t}dt\right]\end{aligned} \tag{6.7}$$

Optimising over $c(t)$ yields the following first-order condition:

$$\frac{L(0)}{H}e^{(g_L-\rho)t}\left[A(t)c(t)-A(t)\bar{c}\right]^{-1}A(t)=\lambda e^{-R(t)}e^{(g_L+g_A)t} \tag{6.8}$$

Now, logs are taken on both sides of equation (6.8) to yield:

$$\log\frac{L(0)}{H}+(g_L-\rho)t-\log\left[A(t)c(t)-A(t)\bar{c}\right]+\log A(t)=\log\lambda-R(t)+(g_L+g_A)t \tag{6.9}$$

Taking the first derivative with respect to time on both sides of this equation, the following is obtained:

$$g_L-\rho-\frac{\dot{A}(t)c(t)+A(t)\dot{c}(t)-\dot{A}(t)\bar{c}}{A(t)c(t)-A(t)\bar{c}}+\frac{\dot{A}(t)}{A(t)}=-r(t)+g_L+g_A \tag{6.10}$$

Using the fact that $\dot{A}(t)/A(t)=g_A$ and thus $\dot{A}(t)=A(t)g_A$, the equation can be rewritten as follows:

$$g_L+g_A-\rho-\frac{A(t)g_Ac(t)+A(t)\dot{c}(t)-A(t)g_A\bar{c}}{A(t)c(t)-A(t)\bar{c}}=-r(t)+g_L+g_A \tag{6.11}$$

This equation can be simplified to yield:

$$\rho\left[A(t)c(t)-A(t)\bar{c}\right]+A(t)g_Ac(t)+A(t)\dot{c}(t)-A(t)g_A\bar{c}=r(t)\left[A(t)c(t)-A(t)\bar{c}\right] \tag{6.12}$$

Dividing both sides of the equation by $A(t)c(t)$, the following equation is obtained:

$$\rho - \frac{\rho\bar{c}}{c(t)} + g_A + \frac{\dot{c}(t)}{c(t)} - \frac{g_A\bar{c}}{c(t)} = r(t) - \frac{r(t)\bar{c}}{c(t)} \tag{6.13}$$

Rearranging terms yields the Euler equation for a Ramsey model with subsistence consumption:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - g_A - \rho - \frac{(r(t) - g_A - \rho)\bar{c}}{c(t)} \tag{6.14}$$

The Euler equation obtained above serves as a basis to discuss the influence of the interest rate and economic growth on the distribution of wealth. This is done in the next section.

6.4 The Role of r and g

What can be seen from the Euler equation obtained in equation (6.14), is that as in the standard Ramsey model, the economy moves towards a steady state in which $\frac{\dot{c}(t)}{c(t)} = 0$ and thus, the standard Euler equation $r(t) - g_A = \rho$ holds. Therefore, the $r - g_A$ gap is given exogenously by the parameter of time preference ρ.

In order to analyse what the distribution of wealth looks like, one can observe the evolution of consumption per capita, as justified above. Since per capita consumption is given by $C(t) = A(t)c(t)$, the following Euler equation for per capita consumption holds:

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{A}(t)}{A(t)} + \frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \frac{(r(t) - g_A - \rho)A(t)\bar{c}}{C(t)} \tag{6.15}$$

To see whether consumption grows faster for individuals who have a higher consumption level and are therefore wealthier, one can take the first derivative of $\frac{\dot{C}(t)}{C(t)}$ with respect to $C(t)$ to obtain the following:

$$\frac{\partial \frac{\dot{C}(t)}{C(t)}}{\partial C(t)} = \frac{(r(t) - g_A - \rho)A(t)\bar{c}}{C(t)^2} \tag{6.16}$$

From this expression, it can be seen that in the steady state, the derivative is zero and thus, the distribution of wealth is persistent. Therefore, any wealth distribution can be an equilibrium distribution of wealth in the Ramsey model.

But what happens in the transition period towards the steady state? As can be seen from the expression above, the distribution of wealth is diverging in the transition period and the level of divergence is an increasing function of $r - g_A$. Thus, even though the distribution of wealth is independent of population growth, the distribution of wealth diverges faster if $r - g_A$ is larger, which is in favour of Piketty's hypothesis. Therefore, it can be concluded that even though Piketty's prediction about the distribution of wealth does not hold in the steady state, it does hold in the transition period in the model economy obtained in this type of model.

This conclusion becomes particularly interesting in a small open economy, where the economy is in a state of low growth, but investors can still earn high returns if they invest in foreign assets. In such a situation, the interest rate is higher than the economy's equilibrium rate, so that the distribution of wealth can be diverging.

Thus, in the Ramsey model with growing subsistence consumption, Piketty's prediction can be reproduced even without shocks.

6.5 Summary

In this chapter, the effects of the interest rate and economic growth on the distribution of wealth have been examined in a Ramsey model with growing subsistence consumption. Using the model by Bertola et al. (2006) as a baseline, their model has been extended to include exogenous productivity and population growth.

The model has shown that even though in the steady state there is no divergence, the distribution of wealth is diverging in the transition period towards the steady state. From the Euler equation it follows that divergence is an increasing function of $r - g_A$ in the transition period, where g_A denotes the growth rate of productivity.

Thus, even though independent of population growth, Piketty's prediction about the consequences of a higher gap between the interest rate and economic growth was reproduced in the model, at least for the transition period. In the steady state, the distribution of wealth is persistent.

Conclusion

In this thesis, Thomas Piketty's book *Capital in the Twenty-First Century* has been critically discussed and confronted with macroeconomic growth theory.

The first part of the thesis provided the reader with an overview of Piketty's work. A summary of his main theoretical statements has been provided along with the main strands of criticism raised against his findings. Piketty predicts the capital/income ratio to increase to substantially high levels in a low growth regime. This gives capital owners a large share of control over the economy's production factors. In addition to that, Piketty predicts the capital share to increase, if the elasticity of substitution between labour and capital is above one, which he estimates to be the case. Thus, capital owners do not only control a high share of the production factors, but do also draw upon an increasing share of national income. However, Piketty's critics point out that he omits capital depreciation, and taking it into account would lead to much lower captial/income ratios even in a low growth regime. In addition to that, a meta-study of estimated elasticities of substitution shows that Piketty's estimate lies well outside most estimates, which suggest the elasticity of substitution to be below one.

His most famous claim is that wealth inequality increases, if the return to capital exceeds the growth rate of the economy, which he summarises in his well-known inequality $r > g$. Based on data from several centuries, he predicts $r > g$ to hold true in the twenty-first century and thus expects wealth inequality to rise. Piketty demonstrates the implications of $r > g$ in a one-period lifetime model with bequests, where shocks occur on the level of bequests. Such a model produces a Pareto distribution of wealth in which inequality is a rising function of $r - g$. Piketty is criticised to make unrealistic assumptions about individuals' savings behaviour and his critics point out that the $r - g$ gap needs to be unrealistically high in order to produce increasing inequality.

The second part of the thesis confronted Piketty's claim about $r > g$ with standard macroeconomic growth theory. Using a Solow growth model, a one-period lifetime with bequests model and a Ramsey framework, his predictions have been analysed. The author's contribution to the field of economics is based on the work of Bertola, Foellmi and Zweimüller (2006), who analyse wealth inequality in macroeconomic growth models with subsistence consumption. The introduction of subsistence consumption yields different savings rates for individuals with different wealth holdings and thus generates heterogeneity. In this thesis, the models by Bertola et al. (2006) have been extended to include exogenous productivity and population growth.

In the Solow model, it has been shown that wage growth is the major equalising force in the economy. While the distribution of wealth is converging in the steady state, there is divergence in the transition period towards the steady state. Even though the convergence result holds independently of r and g, the two variables play a role in the transition period. The distribution of wealth is diverging in a state where the capital stock is low and the interest rate is high. The lower the capital stock and therefore the higher the interest rate, the longer the divergence period. Thus, a longer divergence period and a higher interest rate are correlated. This finding is in favour of Piketty's predictions. However, higher productivity and population growth lengthen the divergence period, which is a finding not in line with Piketty's predictions.

The one-period lifetime model with bequests served as a micro-foundation of the Solow model, in which individual behaviour was fully specified by a given consumption function. Based on the agents' optimising behaviour, the model confirms the findings made in the Solow model and serves therefore as a robustness check.

In the Ramsey model, the savings rate was endogenised as opposed to the one-period lifetime model with bequests, where it was given by the utility function's exogenous taste parameter. It has been shown that richer individuals choose a steeper consumption path, because they become relative less risk averse with increasing income. The model did not confirm Piketty's findings in the steady state, where the distribution of wealth is persistent. However, in the transition period, the distribution of wealth is diverging and is an increasing function of $r - g_A$, where g_A denotes the growth rate of productivity. Therefore, even though independent of population growth, Piketty's findings are reproduced in the Ramsey model, at least in the transition period.

This thesis leaves scope for future research. Since the models analysed suggest wealth to be diverging in the transition period towards the steady state and not in the steady state itself, a deeper study of the economy's evolution in the transition period could prove interesting and yield more insight about the distribution of wealth. In addition to that, potential shocks which keep the economy out of the steady state could be analysed along with the agents' behaviour if these shocks are to be anticipated. Moreover, potential effects of policy interventions in periods of wealth divergence are worth to be studied.

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Declaration of Authorship

I hereby declare that the enclosed master thesis entitled

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has been composed by myself, and describes my own work, unless otherwise acknowledged in the text. Any thoughts, quotations and models which were inferred from other sources are clearly marked as such and are cited according to established academic citation rules.

This thesis has not been and will not be submitted in the current or in a substantially similar version for any other degree or the obtaining of ECTS points at the University of Zurich or any other institution of higher education and has not been published elsewhere.

Zurich, April 2015 Janosch Weiss