Discussion of Harald Uhlig's paper *Macroeconomics and Asset Markets – Some Mutual Implications*

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1 Survey

Great programmatic paper, indicates lines of future research

Gives a measures of our ignorance rather than offering a definite solution

Central Message: labour market frictions are key in understanding asset pricing puzzles.

My comments: reinspecting the mechanism; some simple additional evidence ; implications for predictability of asset returns; possible conclusions for the role of heterogeneity

2 The framework

Puts labour-leisure trade-off center stage:

$$\begin{aligned} \lambda_t &= -\eta_{cc} c_t + \eta_{cl,l} l_t \\ \lambda_t + w_t^f &= \eta_{cl,c} c_t - \eta_{ll} l_t \end{aligned}$$

For asset pricing relation:

$$0 = E_t(\lambda_{t+1} - \lambda_t + r_{t+1})$$

2.1 Parameter restrictions

With

$$\eta_{cl,c} = \frac{u_{cl}(\overline{C},\overline{L})}{u_c(\overline{C},\overline{L})}\ ,\ \eta_{cl,l} = \frac{u_{cl}(\overline{C},\overline{L})}{u_l(\overline{C}.\overline{L})} \text{ and } \overline{WN} = (1-\theta)\overline{Y}$$

we get

$$\frac{\eta_{cl,c}}{\eta_{cl,l}} = \frac{\overline{C}}{\overline{\overline{LW}}} = \frac{\overline{C}\frac{\overline{WN}}{(1-\theta)\overline{\overline{Y}}}}{\overline{\overline{LW}}} = \frac{1-\overline{L}}{\overline{L}}\frac{1-\frac{\overline{X}}{\overline{\overline{Y}}}}{(1-\theta)} = \kappa \approx 0.58$$

Concavity of the utility function requires $\eta_{cc}\eta_{ll} > \eta_{cl,l}\eta_{cl,c}$, so that

$$\eta_{ll} > \kappa\frac{\eta_{cl,l}^2}{\eta_{cc}}$$

This is the key parameter restriction that the model imposes!

2.2 What does that do to pricing?

Standard approach

$$\mathbf{E}_t \left(M_{t+1} R^i_{t+1} \right) = 1$$

with

$$M_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \text{ and } \Lambda_{t+1} = u'(C_{t+1}) = C_{t+1}^{-\gamma}$$

where $\gamma = \eta_{cc}$ is the parameter of RRA.

Standard log-linearized version obtained from

$$\mathbf{E}_t \left(\beta \exp(-\gamma \Delta \widetilde{c}_{t+1})(1 + \widetilde{r}^i_{t+1})\right) = 1$$

which lets us characterize mean

$$\begin{aligned} r_f &= -\log \beta + \gamma E_t \left(\Delta \widetilde{c}_{t+1}\right) - \frac{\gamma^2}{2}\sigma_t^2(\Delta \widetilde{c}_{t+1}) \\ &= -\log \beta - E_t \left(\Delta \widetilde{\lambda}_{t+1}\right) - \frac{1}{2}\sigma_t^2(\Delta \widetilde{\lambda}_{t+1}) \end{aligned}$$

and standard deviation of the stochastic discount factor

$$SR = \frac{E_t(R^i_{t+1} - R_f)}{\sigma_t(R^i_{t+1})} = -\rho_{M,R_i} \frac{\sigma(M_{t+1})}{E(M)} = \frac{\sigma(\beta \exp(\Delta \widetilde{\lambda}_{t+1}))}{E(\beta \exp(\Delta \widetilde{\lambda}_{t+1}))} \approx -\rho \sigma_t(\Delta \widetilde{\lambda}_{t+1})$$

With

$$\Delta\widetilde{\lambda}_{t+1} = \gamma\Delta c_{t+1}$$

a mean consumption growth rate and standard deviation of 1 percent per year, $\rho = -0.2$ and $SR = 0.3 - 0.5$, this implies $\gamma = 150 - 250$.

Now, with non-separabilities

$$\Delta\widetilde{\lambda}_{t+1} = -\eta_{cc}\Delta\widetilde{c}_{t+1} + \eta_{cl,l}\Delta\widetilde{l}_{t+1}$$

and

$$\begin{aligned} SR &= -\rho_{\lambda,r}\sigma_t(\Delta\widetilde{\lambda}_{t+1}) = -\frac{cov(-\eta_{cc}\Delta c + \eta_{cl,l}\Delta l, r_t)}{\sigma(r^i)} \\ &= \rho_{c,r}\eta_{cc}\sigma(\Delta c) - \rho_{l,r}\eta_{cl,l}\sigma(\Delta l) \end{aligned}$$

So, bringing down η_{cc} only possible if the second term on the RHS is positive. Since, $\rho_{l,r} < 0$, this implies $\eta_{cl,l} > 0$ –> leisure and consumption are complements.

- this complementarity is a key reason why the non-separability *per se* cannot solve the equity premium puzzle: bringing down η_{cc} lowers the self insurance motive for consumption (i.e. the motive to work more to keep consumption from falling). Absent other smoothing mechanisms (e.g. because of high adjustment costs) that induces a negative correlation between c and n and a positive one between c and l. The complementarity between c and l just reinforces these counterfactual correlations.

- For a given degree of complementarity, lower η_{cc} just implies higher η_{ll}:

$$\eta_{ll} > \kappa \frac{\eta_{cl,l}^2}{\eta_{cc}}$$

So the smoothness moves from consumption into hours worked (and leisure), which is again counterfactual.

- And the increased variability in c does not even buy us anything: it's offset by the decline in η_{cc}, so that SR remains lowish.

2.3 What works?

Wage rigidity:

$$\begin{aligned}\widetilde{w}_t &= z_t + \phi_{nn}(k_{t-1} - n_t) \\ \lambda_t + w_t^f &= \eta_{cl,c}c_t - \eta_{ll}l_t\end{aligned}$$

and

$$\widetilde{w}_t = \mu w_t^f + (1-\mu)\widetilde{w}_{t-1}$$

Introducing higher than efficient wages, people stand ready to work more. This leaves hours demand-determined and effectively offsets the self insurance (or complementarity) effect discussed earlier by making leisure sufficiently volatile without having to induce additional smoothness in consumption.

3 Bottom line

- Results suggest that RBC-models do badly on pricing assets because they do badly in describing labour market facts –> Harald's results big step forward, because that suggests that fixing the standard growth model at the right place (labour markets) may just mean to fix it all around.

- The impact of the labour market specification on asset pricing properties is pervasive: Paper shows that endogenizing labour supply actually messes up the asset pricing implications of limited participation models (and possibly also of other heterogeneous agent models?).

4 Comments

- What's priced: some simple β-evidence.

- The hours-consumption correlation and expected returns.

- Non-separabilities again: could they still help fix things?

4.1 Some simple β-evidence

$$E_t(R^e_{t+1} - R_f) = \frac{cov(R^e_{t+1}, M_{t+1})}{var(M_{t+1})}\left(-\frac{var(M)}{E(M)}\right) = \beta_{e,M}\boldsymbol{\lambda}_M$$

and in the standard model

$$\boldsymbol{\lambda}_M = \gamma var(\Delta c)$$

Now in the model with endogenous labour supply

$$\boldsymbol{\lambda}_M = f(var(\Delta c), var(\Delta l), \rho(\Delta c, \Delta l)$$

Which suggest an approximate factor model of the form:

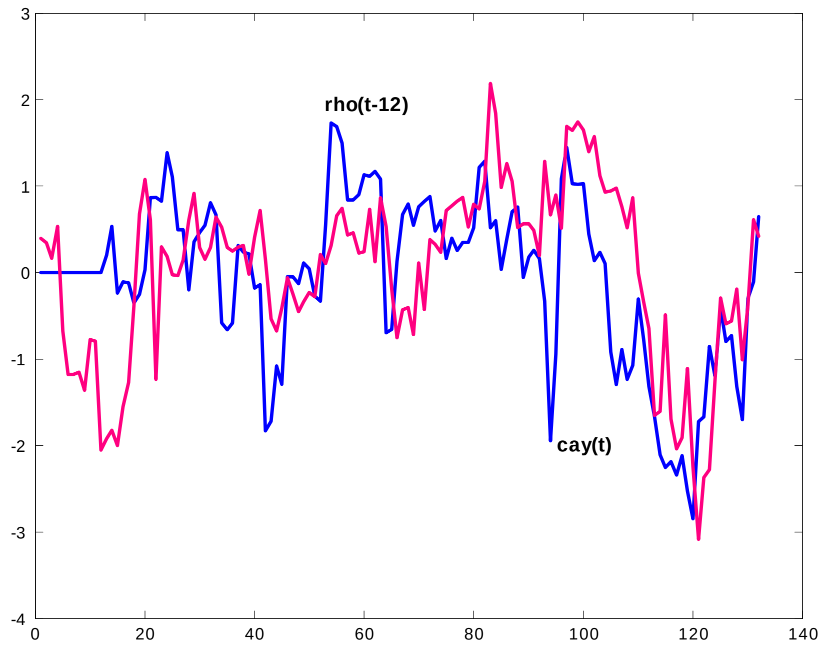
$$E_t(R^e_{t+1} - R_f) = \beta_{e,c}\boldsymbol{\lambda}_C + \beta_{e,l}\boldsymbol{\lambda}_l$$

Suggests regressions of the form

$$R^e_{t+1} - R_f = \beta_c \Delta c + \beta_l \Delta l + ...$$

	I	II	III	IV	V	VI	VII
Δc	*3.20*	**4.02**	*3.20*	**4.29**	**4.31**	**5.05**	**5.17**
	(1.76)	(2.0)	(1.75)	(2.32)	(2.37)	(2.72)	(2.82)
Δhrs		-0.93					
		-0.95					
$\Delta \sigma_{hrs,t}$				**11.08**		*8.75*	**10.34**
				(2.23)		(1.76)	(2.07)
$\Delta \sigma_{c,t}$					**26.99**	**23.67**	**29.63**
					(2.80)	(2.43)	(2.91)
$\Delta \rho_{ch,t}$							-0.08
							(-1.82)

4.1.1 The hours-consumption correlation and expected returns



When self-insurance works well, agents require lower expected returns. Composition risk in consumption-leisure seems priced!

4.2 Non-separability may amplify effect of heterogeneity

Paper convincingly shows the key role of labour markets for asset pricing.

But empirical results above show that rejection of frictionless model with non-separabilities between consumption and leisure is possibly too quick.

Consumption growth heterogeneity often hard to rationalize, even in incomplete markets. But possibly more plausible to consider heterogeneity in leisure.

Example: Write

$$M_{t+1}^k = \beta \frac{\Lambda_{t+1}^k}{\Lambda_t^k} = \beta \exp(\Delta \lambda_{t+1}^k) = \beta \exp(-\gamma \Delta c_{t+1}^k) \exp(\eta \Delta l_{t+1}^k)$$

Then define

$$\overline{M}_{t+1} = \frac{1}{K} \sum M^k_{t+1} = \frac{1}{K} \sum \beta \exp(-\gamma \Delta c^k_{t+1}) \exp(\eta \Delta l^k_{t+1})$$

and expand $\overline{M}_{t+1}$ around $\overline{\Delta c}_{t+1} = \frac{1}{K} \sum \Delta c^k_{t+1}$ and $\overline{\Delta l}_{t+1} = \frac{1}{K} \sum \Delta l^k_{t+1}$to obtain:

$$\begin{aligned} \overline{M}_{t+1} \;\; &\approx \;\; \beta \exp(-\gamma \overline{\Delta c}_{t+1}) \exp(\eta \overline{\Delta l_{t+1}}) \\ & \left\{ 1 + \frac{1}{2} \left[\gamma^2 var_{K,t}(\Delta c^k_{t+1}) - 2\gamma\eta cov_{K,t}(\Delta c^k_{t+1}, \Delta l^k_{t+1}) + \eta^2 var_{K,t}(\Delta l^k_{t} \right. \right. \end{aligned}$$

With $\eta^2 = \frac{\gamma \eta_{ll}}{\kappa}$, we get a potentially very powerful amplification mechanism. E.g. $\gamma = \eta_{ll} = 4$, with $k = 0.5$ we get $\eta^2 = 32$!!

5 Conclusion

Non-separabilities in leisure cannot fix the asset pricing properties of the standard model. Frictions in labour markets are needed.

My comment: consumption-leisure composition risk seems priced. Could it be that x-sectional heterogeneity matters?

Heterogeneity in leisure likely to be a good candidate for heterogeneity to affect asset prices: relatively easy to explain how agents get stuck with leisure heterogeneity in equilibrium. But of course, that again points to labour market frictions and underpins the point of the paper.