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INNOVATION AND GROWTH WITH RICH AND POOR CONSUMERS

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ABSTRACT

This work studies the impact of income inequality on the level of innovative activities in a model where innovations result in quality improvements. In contrast to the standard model of innovations and growth, the equilibrium outcome may be characterized by a situation where not only the quality leader but also producers of worse qualities are on the market. In that case the quality leader sells to the rich, whereas the producer of the second-best quality sells to the poor. In general, we find that a more equal distribution of income is favourable for innovation incentives. This is consistent with empirical evidence suggesting that countries with a more equal distribution of income have grown faster.

1. INTRODUCTION

Is the existence of a rich class necessary to stimulate innovative activities or is it a high purchasing power of the lower classes? According to the former view, high profits accruing from the rich—due to their higher willingness to pay for new goods or better qualities—drive the incentives to conduct R&D. According to the latter, a high purchasing power of the lower classes creates large markets, and consequently high innovation incentives.

It is the aim of this paper to study systematically the impact of income inequality on the level of innovative activities for the case that innovations result in quality improvements. While our set-up resembles that of the standard endogenous growth models of vertical product differentiation (Grossman and Helpman, 1991; Aghion and Howitt, 1992), it differs from those models in two important respects. First, we assume that consumers

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purchase only one unit of an indivisible quality good and the remaining expenditures are spent on a standardized (composite) commodity. This is different from the standard model in which consumers buy the quality goods in amounts of their choice and no standardized good exists. Second, we assume that consumers differ in wealth ownership.

Within this framework we study how income inequality affects the incentive to conduct R&D and thus the rate of economic growth. In the standard models by Aghion and Howitt (1992) and Grossman and Helpman (1991), where the quality good is divisible, income inequality has no impact on research activities because consumers have homothetic preferences. This means that the demand for an innovator's product is independent of inequality. In contrast, in the present model consumers have non-homothetic preferences, which implies that the level of a consumer's income determines his willingness to pay for quality. As a result, the distribution of income determines the prices that innovators can charge and the profits they can earn.

The general equilibrium of the model can be characterized by one of two different regimes. In the first regime, the producer of the top quality sells to both the rich and the poor households. In the second regime, the quality leader sells to the rich, whereas the poor purchase the second-best quality. It is intuitively clear that an equilibrium where a single monopolist serves the entire market, will arise only if the degree of heterogeneity of consumers is rather small. In contrast, when incomes are very unequally distributed, there will be room for the producers of worse qualities to stay on the market and sell their product to the poorer consumers.[1]

The impact of income inequality on the innovation rate is different in the above two regimes. Consider first the monopoly scenario. To conquer the whole market, the quality leader has to set a price such that the poor also can afford the top quality. This price depends only on the poor's willingness to pay for quality and a more equal distribution allows the quality leader to charge a higher price. Hence more equality is favourable for the rate of innovation.

The results are more subtle when the second-best quality is also supplied. In this situation the producer of the second-best quality sets the highest possible price that the poor are willing to pay, given that the third-best quality

[1] The static price equilibrium arising within such a framework has been studied in various papers (Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983) where it is assumed that incomes (or tastes) are uniformly distributed. O'Donoghue *et al.* (1995) extend this approach to study the role of patent policies in a model of cumulative innovations. While using a similar framework to ours, they restrict the analysis to study a partial (industry) equilibrium.

(and all worse qualities) are priced at the lowest possible price, i.e. at marginal cost. The quality leader pursues an analogous policy: given the price of the second-best quality, he will charge the highest price that the rich are willing to pay. A more equal distribution increases the price for the second-best quality, as the poor's willingness to pay for quality increases. On the other hand, how the price of the quality leader is affected is not *a priori* clear. While redistribution decreases the rich's willingness to pay for given prices of the lower qualities, there is also a positive effect since the second-best quality now has a higher price, which leaves scope for a higher price of the top quality.

In general, the results imply that a more equal distribution of income is favourable for innovation incentives. This leads us to two interesting observations. First, while the profits from the rich may be important in quantitative terms, it is for strategic reasons that a high purchasing power of the poor is likely to be more favourable for innovation incentives. Second, the results of our model also shed light on the relationship between income inequality and economic growth.[2] To the extent that quality-improving innovations are the source of economic growth, this relationship between inequality and innovations is consistent with empirical evidence. A number of studies (Alesina and Rodrik, 1994; Persson and Tabellini, 1994; Clarke, 1995) have found that countries with a more equal distribution of income have grown faster. In contrast to many other attempts to explain the evidence, our model stresses the importance of the composition of demand.[3]

There are two recent papers which—independently of the current one—have studied the role of income inequality in the context of innovation-driven endogenous growth. Glass (1995) uses preferences similar to the Grossman and Helpman (1991) model of quality ladders, but assumes that there are two types of households with different tastes about quality. It is because of this assumption that income distribution plays a role. In contrast, in our

[2] The systematic study of how the distribution of income relates to economic growth goes back at least to David Ricardo and has received much attention in the growth literature of the 1950s and 1960s (Kaldor, 1957; Pasinetti, 1962). While the focus of this literature was on the *functional* distribution of income, the recent literature tries to understand the empirically observed correlation between the *personal* distribution of income and growth rates.

[3] Similar in spirit are Murphy *et al.* (1989), Falkinger (1994) and Zweimüller (2000). These papers study demand composition effects on the incentives to introduce new goods. In contrast, our paper analyses the improvement of the quality of existing goods. Other work has focused either on political issues (see e.g. Perotti, 1993; Saint Paul and Verdier, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994) or has stressed the importance of capital-market imperfections (see e.g. Aghion and Bolton, 1991; Banjeree and Newman, 1993; Galor and Zeira, 1993; Torvik, 1993). Moreover, there is a Scandinavian tradition emphasizing the role of wage differentials for productivity growth and structural change, see e.g. Moene and Wallerstein (1992).

model differences in the willingness to pay for quality do not arise by assuming different tastes but result from the fact that the quality good can only be bought in discrete amounts. Li (1996) has a set-up which is similar to our model but assumes that consumers differ in their labour endowments which are uniformly distributed across households. The assumption of a uniform distribution has the disadvantage that only one dimension of inequality—the range of the distribution—can be studied. Instead, we concentrate the analysis on a discrete distribution with two types of individuals—rich and poor. This allows us to address our initial question in a more meaningful way.

The paper is organized as follows. In section 2, we describe the set-up of the model and the behaviour of consumers and firms, both per period and over time. In section 3, we study the properties of the dynamic equilibrium and consider the effects of changes in inequality. Section 4 is a summary.

2. THE MODEL

2.1 Consumers

Consider an economy where consumers earn a constant wage income w (for a fixed supply of one unit of labour) and interest income θA, with θ as the rate of interest and A as the value of wealth. Current income y at each date is thus given by:

$$y = w + \theta A \tag{2.1}$$

Current income is spent for the consumption of c units of a standardized good with price 1, and for one unit of a quality good with price p, which depends on the index q measuring the quality of the product. Instantaneous utility of a consumer is described by the utility function:

$$u = \ln c + \ln q = \ln(y - p) + \ln q \tag{2.2}$$

For a meaningful solution, $c = y - p > 0$ must hold, only prices fulfilling this property will be considered in the following.

Hence at time τ the intertemporal decision problem of the infinitely-lived household is to maximize

$$\int_{\tau}^{\infty} (\ln c(t) + \ln q(t))\, e^{-\rho(t-\tau)} \mathrm{d}t, \text{s.t. } A(\tau) + \int_{\tau}^{\infty} w e^{-\theta(t)(t-\tau)} \mathrm{d}t \geq \int_{\tau}^{\infty} c(t) e^{-\theta(t)(t-\tau)} \mathrm{d}t$$
$$+ \int_{\tau}^{\infty} p(t, q(t)) e^{-\theta(t)(t-\tau)} \mathrm{d}t$$

where ρ denotes the rate of time preference. The solution of this problem gives us the time path of consumption of the standardized good $c(t)$ and of the quality $q(t)$, given an expected relation $p(t, q(t))$ between quality and price at any point of time t, and an expected time path $\theta(t)$. Note that we have separability in utility, both over time and across goods. For any given time path of expenditures for the quality good, $p(t, q(t))$ that does not exhaust lifetime resources, the optimal path of expenditures for the standardized good fulfils $[\mathrm{d}c(t)/\mathrm{d}t]/c(t) = \theta(t) - \rho$. The optimal path of $q(t)$ depends on the difference between $\theta(t)$ and ρ as well, but also on the time path of $p(t, q(t))$. Since quality choice is discrete (see section 2.2) it is not possible to characterize the solution by a differential equation.

The focus of our analysis is on steady states. These are, as we assume constancy of w over time, characterized by constancy of A and θ, thus of y as well, and of c. Obviously, this can only occur if p is constant as well and if the interest rate θ equals ρ. Hence total expenditures remain constant over time. (However, due to innovations, the quality level q may rise over time, even with a constant price p.) We have

$$c = y - p = w + \theta A - p \tag{2.3}$$

In the next subsection we will discuss the price-setting behaviour of the firms, which determines the steady-state values of c and p and of the quality levels that are chosen by the consumer. We assume in the following that two groups of consumers, the poor (P) and the rich (R), exist, distinguished by wealth, $A_P < A_R$, and, consequently, by income $y_P < y_R$. However, all consumers are identical with respect to preferences and to the wage rate. β denotes the population share of the poor. Index i, i = P, R will be used to identify consumers.

2.2 Technology, prices and market structure

There is a linear technology for the production of the standardized good, with labour as the only input. The market for this good and the labour market are assumed to be competitive, hence the price (normalized to 1) equals marginal costs. As a consequence, unit labour input can be written as $1/w$. We assume that no technical progress exists in the standard good sector, so the wage rate w remains constant over time.

The market for the quality good is non-competitive. At any date t, many different qualities $q_j(t)$, $j = 0, -1, -2, \ldots$ are known and can be supplied on the market, where $j = 0$ denotes the best quality, $j = -1$ the second-best and so on: hence $q_0(t) > q_{-1}(t) > q_{-2}(t) > \ldots$ Successive quality levels differ by a factor $k > 1$: $q_j(t) = k \cdot q_{j-1}(t) = k^2 \cdot q_{j-2}(t)$ and so on.[4] Also the production of quality goods is performed with labour as the only production factor and constant marginal costs. These are written as wa, where a (<1) is the unit labour requirement, the same for all qualities.[5] The lifetime of a successful innovator is uncertain. A firm supplies quality q_j until the next innovation takes place. From that event until a further innovation, this firm supplies quality q_{j-1}, and so on.

In this section we discuss the outcome of the firms' decisions concerning production and prices in the market for the quality good. As is natural in models of vertical product differentiation with fixed quality increments, we assume that firms use prices as strategic variables. They know the shares of groups P and R in the population, the respective incomes y_P, y_R and the preferences, but cannot distinguish individuals by income.

When setting the price of its quality, a firm has to consider two aspects: (1) by lowering the price it may attract a further group of consumers; (2) for a given clientele, it wants to set the price just as high as possible without losing its costumers.

As a first step it is useful to analyse the decision of the consumers considering two different qualities. The maximum price $\bar{p}_j$ that a consumer is willing to pay for quality j, given that a lower quality $j - m$ is supplied at price p_{j-m}, can be calculated from equations (2.2) and (2.3). Indifference between qualities j and $j - m$ implies that $\ln(y_i - \bar{p}_j) + \ln q_j = \ln(y_i - p_{j-m}) + \ln q_{j-m}$.[6] As all qualities differ by the constant factor k we know that $q_j = k^m q_{j-m}$, for all $j = 0, -1, -2, \ldots, m = 1, 2, \ldots$ Using this in the above equation we get $(y_i - \bar{p}_j)k^m = y_i - p_{j-m}$. Solving for $\bar{p}_j$ yields:

$$\bar{p}_j = y_i\left(\frac{k^m - 1}{k^m}\right) + \frac{p_{j-m}}{k^m} \tag{2.4}$$

[4] The process of how innovations are created is discussed in subsection 2.3.

[5] $a \geq 1$ would mean that a least one unit of labour would be needed to produce one unit of the quality good. As every individual consumes one unit of the latter, no labour force would be left for the standard-good sector.

[6] We use the general convention that the better quality q_j is chosen if, for given p_j and p_{j-m}, buying q_j or q_{j-m} leads to the same utility level.

The most important message of equation (2.4) is that the willingness to pay depends on the income level of consumer i. More precisely, we find that $\bar{p}_j$ is a weighted average of y_i and p_{j-m}. As already mentioned in section 2.1, only prices lower than income are taken into consideration by consumer i, which means $p_{j-m} < y_i$. Together with $k > 1$ (by assumption) this implies that $\bar{p}_j$ will be larger than the price p_{j-m} of the worse quality $j - m$. Obviously, (2.4) can also be used to determine the willingness to pay $\bar{p}_{j-m}$ for the lower quality $j - m$, given a certain price p_j of the higher quality j. We conclude

Lemma 1:

(1) If for prices p_j, p_{j-m} some consumer prefers quality q_j to q_{j-m}, then any richer consumer does the same.
(2) If $p_j \geq wa$ holds for the price of some quality q_j, $j = -1, -2, \ldots$, then for the producer of any higher quality q_{j+m}, $1 \leq m \leq -j$, there exists a price $p_{j+m} > wa$, such that any consumer prefers quality q_{j+m} to q_j.

Proof:

(1) One observes from (2.4) that $\bar{p}_j$ increases with y_i, hence if $p_j \leq \bar{p}_j$ for some consumer i, which makes him prefer quality j to $j - m$, then the same holds for a richer consumer as well.
(2) Follows immediately from the fact that $\bar{p}_{j+m} > p_j$ for any i, as noticed above (use $j + m$ and j instead of j and $j - m$, respectively).

The first part of lemma 1 tells us that, for given prices, there is a unique ranking of the associated qualities by the consumers. The second part states that the producer of a higher quality can always drive out a lower quality, given that the price of the latter cannot be below marginal costs. We can now use lemma 1 to describe the possible equilibrium outcomes, given our assumption that only two groups of consumers, who differ in income, exist.

Lemma 2: In equilibrium

(1) the highest quality is produced (and sold),
(2) at most the two highest qualities q_0 and q_{-1} are actually produced,
(3) the equilibrium price p_{-1} fulfils $wa \leq p_{-1} \leq p^S_{-1}$, where p^S_{-1} denotes the maximum price the q_{-1} producer can set in order to deter q_{-2} from entry (from (2.4) we compute $p^S_{-1} = y_p(k - 1)/k + wa/k$).

Proof: Note first that in equilibrium no firm can make a loss, therefore $p_j \geq wa$ for any quality q_j which is actually produced. Next, as there are only two

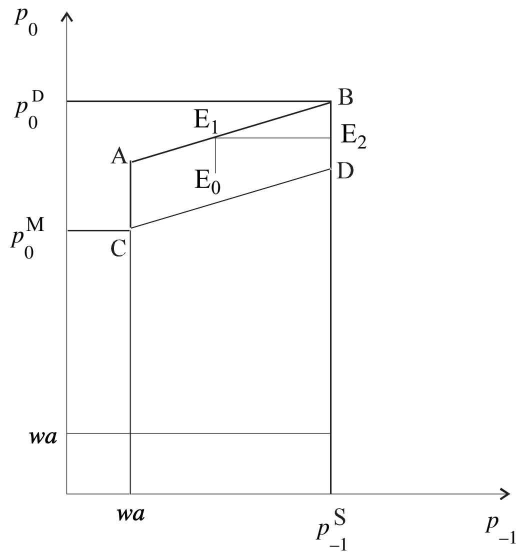

Figure 1. Static equilibrium.

(homogeneous) groups of consumers, at most two qualities can be sold, while one quality will always be sold, as every individual is assumed to buy one unit of the quality good. By Lemma 1 (2), higher qualities drive out lower qualities. Finally, $p_{-1} \leq p^S_{-1}$ follows from the fact that otherwise the q_{-2} producer could profitably enter the market.

Thus, equilibrium prices may pertain to two possible situations: either the top quality q_0 is sold to both groups of consumers, or the top quality q_0 is bought only by the rich group R and the second best quality q_{-1} is bought only by the poor group P.

Figure 1 presents a graphical exposition, where the prices for the top quality and for the second-best quality, p_0 and p_{-1}, are drawn on the axes. Line AB shows the maximum price which the quality leader can charge such that the rich consumers prefer the top quality to the second-best quality, given the price p_{-1} of the second-best quality. The AB-line is determined by equation (2.4) with $j = 0$, $j - m = -1$ and $i = R$). The line CD is analogous to AB, but refers to the P consumers. That is, the line CD describes the maximum price which the quality leader can charge, such that the poor consumers prefer the top quality to the second-best quality, given the price of the latter. These lines allow us to characterize the equilibrium outcome on the market for quality goods. First, the area below and on the line CD implies a structure of prices such that the quality leader gets the entire market. Second, the area between the lines CD and AB (AB included) implies a price structure such that the quality leader sells to the rich, and the producer of the second-best quality sells to the poor. Finally the area above the line AB implies a structure of prices such that all consumers prefer the second-best quality and the top-quality is not actually sold. (We note from Lemma 2 (1) that in equilibrium the highest quality always has a positive market share, so this latter case can be ruled out as an equilibrium outcome.) In addition, prices must be above marginal costs wa, and p_{-1} must be below p^{S}_{-1} to deter quality q_{-2} from entry.

It is intuitively clear that the distribution of income determines the market structure: if all consumers own the same amount of wealth, only the top quality producer is on the market. When the distribution of income is sufficiently unequal, however, there is a scope for the second-best quality to be purchased by the poor group, while the top quality is sold to the rich group. We will come back to this question in section 3.4.

It remains to determine the equilibrium prices in each of the two possible equilibrium outcomes. In order to characterize these prices we first note that the quality leader and the producer of the second-best quality are in their respective positions until the next innovation occurs. While the dynamics of innovation will be described in the next section, we note for now that innovations occur randomly according to a Poisson process with parameter ϕ. When a new innovation occurs, the successful firm becomes the quality leader, the previous quality leader becomes the producer of the second-best quality, and the previous producer of the second-best quality is displaced from the market.[7]

In this set-up the pricing problem can be considered as an infinitely repeated game between the quality leader and the producer of the second-best quality that, at any date t, is likely to end with rate ϕ. Since we are interested only in steady-states, we look for an equilibrium where prices are constant over time.

It is well know that in an infinitely repeated game in general a large set of possible solutions exists, each of which can be supported by appropriate punishment strategies. However, in the case where the quality market is characterized by a monopoly, the equilibrium is unique and described by point C in figure 1. Indeed, given any price p_{-1} charged by the producer of the second-best quality, the quality leader maximizes profits by setting a price p_0 along the line CD. But if p_0 is chosen larger than p^{M}_{0}, then the producer of the

[7] We study situations, where the incumbent does not engage in R&D—see the discussion at the end of section 2.3.

second-best quality can set a price above marginal cost and gain a positive market share. Thus, only point C in figure 1 represents an equilibrium compatible with a monopoly.[8] (In fact, C is a Nash equilibrium of the stage game.) The quality leader charges a price (use (2.4) with $p_{-1} = wa$)

$$p_0 = y_P \frac{k-1}{k} + \frac{wa}{k} \tag{2.5}$$

Unlike above, in the case where both the quality leader and the producer of the second-best quality have a positive market share, no pair of prices represents a Nash equilibrium of the stage game. Indeed, at any point in the interior of ABCD each producer has an incentive to increase his price (and profit), given the price of the other quality. This still holds on line AB (B excluded) for the producer of the second-best quality. Moreover, on line AB (A excluded) this producer has an incentive to charge a slightly lower price and attract the entire market. Altogether, at any point compatible with a duopoly at least one of the producers can do better by changing his price, given the other.

Equilibria of the repeated game can still be established by formulating appropriate punishment strategies, which, however, leaves us with a large set of possible solutions. Fortunately, it is possible to restrict this set to one pair of prices by a simple and plausible general principle: No firm is punished if it changes its price without affecting the other firm's profit.

To see how this principle works, suppose that the currently charged prices are given by Point E_0 in figure 1. Point E_0 is within the area ACDB, which implies that the incumbent firms charge prices such that both have a positive market share (the quality leader sells to the rich, the producer of the second-best quality sells to the poor). Now suppose the quality leader increases his price such that the new price structure is given by point E_1. Point E_1 still corresponds to a situation where the quality leader sells to the rich, whereas the producer of the second-best quality sells to the poor. Hence this increase in the price of the top quality increases profits of the quality leader, but leaves the profits of the producer of the second-best quality unchanged. Similarly, starting from Point E_1 in figure 1, the producer of the second-best quality can raise his price such that the point E_2 describes the new structure of prices.

[8] Hereby we assume that the q_{-1} producer is present as a potential competitor, though he does not sell anything. The q_0-producer loses group P if he increases p_0 above P_0^M, thus he has no incentive to deviate. The q_{-1}-producer is out of the market and can do nothing to change the situation.

This increases the profits of the producer of the second-best quality but leaves the profits of the quality leader unaffected. Repeated application of this argument establishes point B in figure 1 as the only point where (1) both firms have a positive market share and (2) no firm can change its price to increase its profit, without affecting the other firm's profit. In the infinitely repeated game, point B is an equilibrium as it can be supported by appropriate punishment strategies.

To sum up, the above considerations suggest that in a regime where the quality leader sells to the rich and the producer of the second-best quality sells to the poor, point B is the unique equilibrium. The prices that are charged in this equilibrium are given by

$$p_0 = y_{\mathrm{R}} \frac{k-1}{k} + y_{\mathrm{P}} \frac{k-1}{k^2} + \frac{wa}{k^2},\ p_{-1} = y_{\mathrm{P}} \frac{k-1}{k} + \frac{wa}{k} \tag{2.6}$$

So far we have not been specific under which conditions only the quality leader is on the market and when both firms have a positive market share. In section 3.4 we will analyse which regime will occur depending on the parameters of the model. For the moment we only note that the quality leader plays the decisive role, whereas the producer of the second-best can only adapt.

2.3 Research and innovation

Profit-seeking entrepreneurs engage in R&D to improve the quality good. A research success enables a firm to produce a quality which is k (>1)-times better than the current top quality. Innovations are random and arrive according to a Poisson process with parameter ϕ, which reflects the research intensity in the economy. For the representative research firm the research intensity ϕ is a choice variable: employing ϕF workers produces research intensity ϕ, so the flow of R&D costs is $w\phi F$, where F is a positive constant that is inversely related to the efficiency of the R&D technology. The flow of expected profits is ϕB, where B denotes the value of an innovation and ϕ is the rate at which an innovation occurs. B is the present value of profits of a successful researcher. The subsequent life-cycle of such a firm can be divided into several 'periods', where a period is defined as the random interval between two successive innovations. We will denote by period 0 the interval that starts after an innovation has occurred and ends with the next innovation. During period 0 the firm is the quality leader. Period 1 is the interval between the first and the second subsequent innovation, during which the firm produces the second-best quality. Since the third-best producer will never have positive demand, further periods are not relevant. Denoting ϕ_e as the expected intensity of future research activities and Π_0 and Π_1 as the profits in periods 0 and 1, respectively, B may be calculated as $\Pi_0/(\phi_e + \theta) + \phi_e\Pi_1/(\phi_e + \theta)^2$.[9]

The objective function of the representative research firm may be written as $\phi B - \phi wF$. Since there is costless access to R&D activities, in equilibrium $B \leq wF$ must hold, with equality for $\phi > 0$. Otherwise entering R&D would still be profitable. Moreover, in equilibrium expectations must be satisfied. All future generations have the same problem as previous researchers, so we must have $\phi = \phi_e$. The innovation equilibrium condition $wF = B$ can then be written as

$$wF = \frac{\Pi_0}{\phi+\theta} + \frac{\phi}{(\phi+\theta)^2}\Pi_1 \qquad \text{for } \phi > 0 \tag{2.7}$$

The form of the right-hand-side of (2.7) depends on the particular regime which we have discussed in section 2.2. To make precise what the level of profits are in the respective regimes, we denote by Π^i the profit flow from serving the market for group i, i = P, R. We denote by p^{P} the price paid by the poor, which is either p_0 in the case of a monopoly or p_{-1} in the case of a duopoly. Recall that the rich always buy the top quality with price p_0.

In a monopoly situation the quality leader sells to both the rich and the poor, and the producer of the second-best quality makes no profit. Hence we have:

$$\Pi_0 = \Pi^{\mathrm{R}} + \Pi^{\mathrm{P}},\ \Pi_1 = 0,\ p^{\mathrm{P}} = p_0 \tag{2.8}$$

When the producer of the second-best is also on the market we have:

$$\Pi_0 = \Pi^{\mathrm{R}},\ \Pi_1 = \Pi^{\mathrm{P}},\ p^{\mathrm{P}} = p_{-1} \tag{2.9}$$

For later use we note

$$\Pi^{\mathrm{R}} = (1-\beta)L(p_0 - wa) \tag{2.10}$$

$$\Pi^{\mathrm{P}} = \beta L(p^{\mathrm{P}} - wa) \tag{2.11}$$

where L is the population size.

[9] The value of an innovation B is the expected present value of the profit flow following an innovation, given by $\int_\tau^\infty e^{-\theta(s-\tau)} E\Pi(s)\mathrm{d}s$, where the expected profit at date s, $E\Pi(s) = [\Pi_0 + \phi_e(s - \tau)\Pi_1]e_e^{-\phi(s-\tau)}$. Using this expression to evaluate the above integral yields the expression in the text.

Finally, one observes that it is only for a successful competitor that the gain from R&D equals the value of a new innovation. If the incumbent firm successfully innovates in the monopoly regime, it gains a profit flow from the new top quality, but loses the profit flow from the previous innovation which is driven out of the market. As these profits are equally large in the steady state, the incumbent has nothing to gain from R&D and hence does not engage in it. By analogous arguments, the incumbent also has no incentive to engage in R&D in the duopoly regime.

3. INEQUALITY AND THE RATE OF INNOVATION

3.1 The distribution of assets and the allocation of resources

It is assumed that profits after an innovation constitute the unique source of aggregate wealth, denoted by v. The distribution of v among households is given by the respective population shares of poor and rich consumers (which we have defined above as β and $1 - \beta$); and by the relative wealth positions of the poor and the rich. Denote by $d < 1$ the ratio of assets owned by a poor household relative to economy-wide per-capita wealth, i.e. $d = A_P/(v/L)$. Given the wealth position of the poor d and their population share β, the wealth position of the rich, d_R, is determined by $d_R = (1 - \beta d)/(1 - \beta)$. With these definitions, βd and $1 - \beta d$ are the respective shares of groups P and R in aggregate wealth.

Figure 2 shows the resulting Lorenz-curve. The Lorenz-curve is piecewise linear, with slopes d (= tan α) and $(1 - \beta d)/(1 - \beta)$ along 0A and AB, respectively. From inspection of the Lorenz-curve it is evident that increasing the wealth position of the poor d, while holding their population share β constant, shifts the Lorenz-curve closer towards the diagonal. An increase in the poor's population share β, holding their wealth position d constant, shifts A to A′, that is, a larger population share of group P leads to more inequality. This is because, for given d, relative wealth of a rich household $d_R = (1 - \beta d)/(1 - \beta)$ becomes larger.

By assumption, each household supplies one unit of labour, so the total supply is equal to L.[10] How L is allocated among sectors depends (1) on labour

[10] The assumption that each household earns the same labour income is, of course, a simplification. Note however, that the way that inequality affects innovation incentives does not depend on the particular source of income inequality. Hence we would get equal results by studying a population with unequal labour endowments (for instance, due to persistent differences in innate abilities).

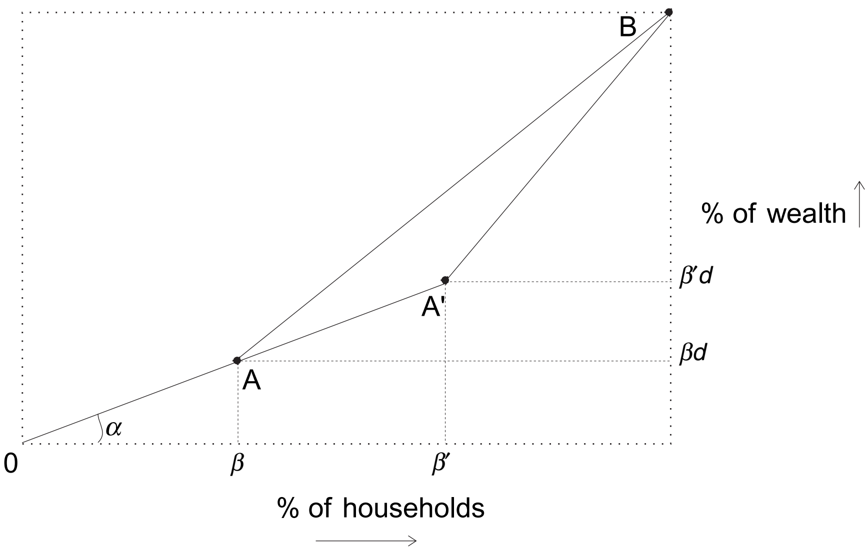

Figure 2. Lorenz curve.

demand for research, ϕF, (2) on employment in the quality sector, aL (recall that every individual buys one unit of the quality good and that the unit labour requirement to produce the quality good is a) and (3) on labour demand for the standard good, which is given by $(1/w)[\beta c_P L + (1 - \beta)c_R L]$ (recall that the unit labour input to produce this good is $1/w$, c_p and c_R denote consumption of the strandard good by the poor and the rich respectively). Equilibrium requires that labour supply equals the demand for labour, that is

$$L = \phi F + aL + \frac{L}{w}(\beta c_P + (1-\beta)c_R) \tag{3.1}$$

It turns out to be convenient to rewrite the expression for employment in the standard–good sector in terms of aggregate flow profits $\Pi_0 + \Pi_1$. To do this we first express per-capita wealth of the two groups as

$$A_P = \frac{dv}{L} \qquad A_R = \frac{(1-\beta d)v}{(1-\beta)L} \tag{3.2}$$

Combining (2.3), (3.1) and (3.2) yields:

$$wL = w\phi F + waL + L\left[\beta\left(w + \theta d\frac{v}{L} - p^P\right) + (1-\beta)\left(w + \theta\frac{(1-\beta d)v}{(1-\beta)L} - p_0\right)\right] \tag{3.3}$$

Further manipulations of equation (3.3) lead to[11]

$$\phi wF = \Pi_0 + \Pi_1 - \theta v \tag{3.4}$$

Substituting the expressions for the prices of the quality goods (2.5) and (2.6), together with the budget constraint (2.1) and the resource balance (3.2) into the expressions for the profits that accrue from selling, respectively, to the rich and the poor (2.10) and (2.11), allows us to rewrite the expressions Π_0 and Π_1 for aggregate flow profits in terms of the endogenous variable v. In the regime where only the quality leader is on the market, this gives:

$$\Pi_0 = \frac{k-1}{k}[Lw(1-a) + d\theta v] \tag{3.5}$$

$$\Pi_1 = 0 \tag{3.6}$$

When the producer of the second-best quality also has a positive market share, we get:

$$\Pi_0 = \frac{k^2-1}{k^2}(1-\beta)Lw(1-a) + \frac{k-1}{k}(1-\beta d)\theta v + \frac{k-1}{k^2}(1-\beta)d\theta v \tag{3.7}$$

$$\Pi_1 = \frac{k^2-1}{k^2}\beta Lw(1-a) + \frac{k-1}{k}\beta d\theta v \tag{3.8}$$

$$\frac{k-1}{k}$$

3.2 The steady-state equilibrium

The general equilibrium is now defined by equation (2.7)—the innovation equilibrium, equation (3.4)—the full employment condition, and either (3.5), (3.6) or (3.7), (3.8)—the equilibrium relations for aggregate flow profits in a situation where, respectively, only the quality leader is on the market, and where the producer of the second-best quality also has a positive market share. These are four equations with four unknowns: ϕ, v, Π_0 and Π_1.

Π_0 and Π_1 (which are linear in v and independent of ϕ) can be substituted into the innovation equilibrium condition (2.7) and the full employment condition (3.4). As a result, the innovation equilibrium condition im-

[11] Split waL in (3.3) into $(1-\beta)waL + \beta waL$, observe (2.10), (2.11) and note from (2.8), (2.9) that we can always write $\Pi_0 + \Pi_1 = \Pi^R + \Pi^P$.

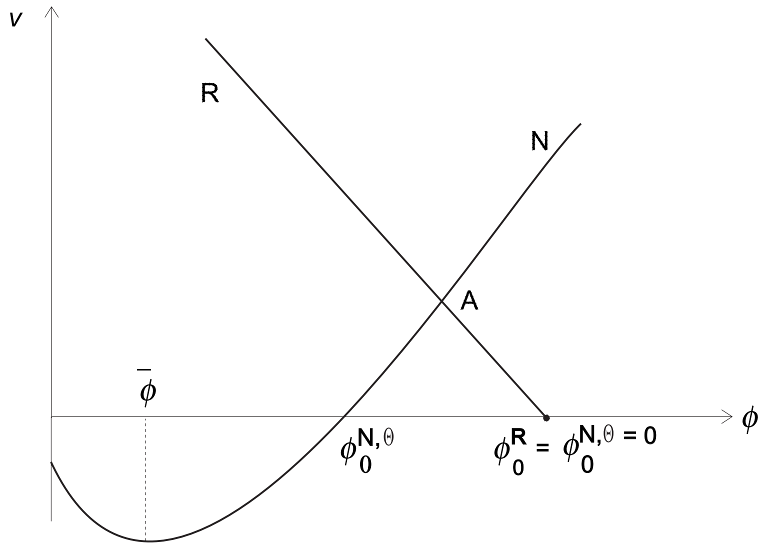

Figure 3. General equilibrium.

plicitly defines a function $v = \varphi^N(\phi)$, the 'N-locus' in figure 3. Analogously, the resulting resource balance condition leads to the function denoted $v = \varphi^R(\phi)$, the 'R-line' in figure 3.

Lemma 3:

(1) In both price regimes the function $v = \varphi^R(\phi)$ (the R-line) is linear in ϕ and has a negative slope.
(2) In the regime where the quality leader serves the entire market, the function $v = \varphi^N(\phi)$ (the N-line) has a positive slope.
(3) In the regime where the quality leader sells to the rich and the producer of the second-best quality sells to the poor, there exists some $\bar{\phi} \geq 0$ such that the slope $\partial\varphi^N/\partial\phi$ is positive for $\phi > \bar{\phi}$ and non-positive for $\phi \leq \bar{\phi}$. $\bar{\phi} = 0$ if $\Pi_1 \leq \Pi_0$, otherwise $\bar{\phi}$ approaches zero with $\theta \to 0$.

Proof: See appendix.

Lemma 3 summarizes the trade-off between R&D intensity ϕ and the aggregate wealth level v under the condition that consumers have optimally chosen consumption, and that incumbent firms charge equilibrium prices. The N-locus shows all combinations between ϕ and v such that the expected value of an innovation is equal to its costs. The R-line represents (ϕ, v)-values that guarantee full employment in the economy. Intuitively, the slopes can be derived by the following considerations.

The N-curve is characterized by the condition that the R&D costs equal the value of an innovation, i.e. $wF = B$. The left-hand side, the R&D costs wF, is independent of the research intensity ϕ and aggregate wealth level v, hence the right-hand side, the value of an innovation B remains constant along the N-curve, for varying values of ϕ and v. B is the present value of the resulting profit flow once a research firm has had success. B depends both on the aggregate wealth level v and the research intensity ϕ. B is increasing in v, irrespective of the particular price regime, because a higher v increases the willingness to pay for quality, so incumbent firms can charge higher prices. Since demand is fixed, higher prices increase flow profits, Π_0 and Π_1.

How the value of an innovation B depends on the research intensity ϕ is not *a priori* clear. In a monopoly regime, B is unambiguously decreasing in ϕ, because a higher steady-state research intensity is associated with more creative destruction: the quality leader has a shorter expected lifetime since subsequent innovations will on average occur more quickly. In the duopoly regime there is an additional effect of ϕ on B which works in the opposite direction. An increase in ϕ means that the profit-flow resulting from being the second-best will accrue earlier, which increases B. Lemma 3 says that if ϕ is large enough (larger than some critical value $\bar{\phi}$), the former effect always dominates and the value of an innovation B is decreasing in the research intensity ϕ. In sum: we know that the innovation equilibrium requires B (= wF) to be constant. Thus, an increase in ϕ, which lowers B, has to be accompanied by an increase in v, which increases B, in order to offset this. As a result, the N-curve must be positively sloped.

The R-line is characterized by the equality of labour supply and labour demand. Labour supply is fixed and given by L, whereas labour demand is increasing in both the research intensity ϕ and aggregate wealth v: A higher ϕ is associated with more demand for labour in R&D. A higher v also raises labour demand, as wealthier consumers have larger consumption expenditures. Since prices in the standard-good sector are fixed, higher expenditures for this good are equivalent to more production and therefore to a higher demand for labour in the standard-good sector. The demand for labour in the quality-good sector, however, is not affected, because labour demand in that sector is always equal to aL, as each consumer buys one unit of the quality good and the unit labour requirement for the production is a. This trade-off is captured by the negative slope of the R-line: with a fully employed labour force, more research activities (a higher ϕ) is only possible by a reduction of output in the standardized sector (due to a lower v).

Proposition 1: There exists a unique general equilibrium with positive ϕ^* and v^*, provided that the rate of time preference is sufficiently small.

Proof: See appendix.

To see the intuition behind the proof of Proposition 1 we look at the innovation equilibrium condition (2.7) when the interest rate θ (which is equal to the rate of time preference) goes to zero. In that case the research intensity ϕ approaches the limit $(\Pi_0 + \Pi_1)/wF$, which is strictly larger than zero, since $wF > 0$ and since $\Pi_0 > 0$. This means that ϕ^* is strictly positive for small θ. To see why in any equilibrium the aggregate value v^* of wealth is strictly positive, note that v consists of the total value of all incumbent quality producers. Since the value of the top-quality firm is always wF (namely the value of an innovation, see (2.7)), aggregate wealth is at least wF, which is strictly positive. We note further that the producer of the second-best quality has a value of $\Pi_1/(\phi + \theta)$. Evidently, in an equilibrium where the quality leader serves the entire market we have $v^* = wF$, as $\Pi_1 = 0$ and the producer of the second-best quality producer has zero value. In an equilibrium where the producer of the second-best quality also has a positive market share, aggregate wealth v is strictly larger than wF, since $\Pi_1 > 0$, i.e. the second-quality producer has positive value.

Before we turn to the relation between inequality and the rate of innovation, we briefly note the effect of the productivity parameters. One observes from formula (3.10) below that in an equilibrium where the quality leader serves the whole market, the innovation rate is independent of the wage rate w (labour productivity in the standard-good sector). On the other hand, it is straightforward to show, by implicit differentiation of the two equilibrium conditions (2.7), (3.4), that in the other regime an increase in w increases ϕ^*. That is, in the latter case, rising productivity in the standard-good sector allows higher prices for the quality goods and, thus, leaves more resources for innovation. Concerning the effect of an increase in F and a, respectively (i.e. lower efficiency in innovation and in the production of the quality goods, respectively), on the equilibrium rate of innovation, one finds immediately that it is negative, as one expects: higher required labour input, either in innovating or in producing the quality good means that lower resources are devoted to the process of innovation.

As to the role of w, there are three different effects at work. First, a higher wage reflects higher productivity in the standardized sector. This creates the potential for more resources to be employed in the R&D sector. Second, a higher wage implies higher incomes so that consumers have a higher willingness to pay for quality goods. Finally, a higher wage has a direct effect both on the cost of an innovation and on the mark-ups that innovators can charge. The first two effects tend to raise the equilibrium innovation rate, whereas the last effect tends to reduce it. In the monopoly regime, the first two effects and the last effect exactly offset each other, so that a higher w does not affect the equilibrium innovation rate. In the duopoly regime, however, the increase in the willingness to pay affects both the willingnesses to pay for both the top quality and the second-best quality. Hence in the duopoly regime, the positive effect of a higher willingness to pay for quality is stronger. As a result, the first two effects dominate the latter effect and a higher w increases the equilibrium innovation rate.

3.3 Inequality and the rate of innovation

We now come to discuss the central topic of the paper, namely how inequality in the distribution of assets influences the rate of innovation. It turns out that the answer to this question depends on the particular price regime. We start with the simpler case where the quality leader is a monopolist, i.e. he is the unique supplier of the quality good. We then turn to the case where the market for the quality good is characterized by a situation where the quality leader sells to the rich and the producer of the second-best quality sells to the poor consumers. Subsection 3.4 returns to the question of how inequality is related to the occurrence of a particular regime. We generally assume that θ is sufficiently small, so that an equilibrium with $\phi^* > 0$ exists.

3.3.1 The quality leader serves the entire market

Proposition 2: In a situation where all consumers buy the top quality:

(1) Redistribution from the rich to the poor (an increase of the wealth position of the poor d) increases the equilibrium rate of innovation.
(2) The equilibrium innovation rate is independent of population shares.

Proof: When only the quality leader is on the market, the value of wealth is trivially determined by the value of the recent innovator. As mentioned above, the present value of the innovator's profit flow is equal to its costs, so

$$v^* = wF \tag{3.9}$$

Substituting this into the resource balance condition (3.4), and using the expressions for the flow profits (3.5) and (3.6) yields an explicit expression for ϕ^*, the equilibrium innovation rate:

$$\phi^* = \frac{k-1}{k}\left[\frac{L(1-a)}{F} + d\theta\right] - \theta \tag{3.10}$$

From this equation it is obvious that ϕ^* increases in d, but is independent of β.

The intuitive reason for this result is that redistributing wealth from the rich to the poor increases the price an innovator can charge for his product. In an equilibrium where the top quality is purchased by all households, the poor are the 'critical' consumers meaning that prices depend on the willingness to pay of this group. If the poor are richer, they are willing to pay a higher price for the quality good. This means that innovators can charge higher prices and earn larger profits which increases the incentive to innovate.

Equation (3.10) also reveals that the population parameter β does not have an impact on the innovation rate. This is because all households buy the top quality at the same price. Therefore, the level of demand for the innovator is fixed and profits are independent of the relative sizes of rich and poor.

Changes in the inequality parameters β and d are associated with movements of the Lorenz-curve, as indicated at the beginning of section 3.1. Figure 4 illustrates the shifts of the Lorenz-curve associated with those changes of β and d which lead to a higher ϕ^*: those changes induce movements of point A into the shaded area (line 0C excluded). Changes in distribution parameters such that point A moves to the area 0A0′ are associated with lower inequality, provided that α increases (remember that $d = \tan \alpha$). However, movements of A to the area AC0′ lead a new Lorenz-curve which crosses the original one: we cannot say whether the new distribution is more or less equal than the original one.

The impact of the remaining parameters is straightforward, and qualitatively identical to the results of Grossman and Helpman (1991): a larger step-size of innovations k increases the incentive to innovate, since a new innovation provides relatively higher qualities, for which consumers will pay more. A larger size of the market L, a lower interest rate θ, a lower set-up cost for innovations F, and more productivity in the production of the quality good (a lower a), lead for obvious reasons to a higher ϕ^*.

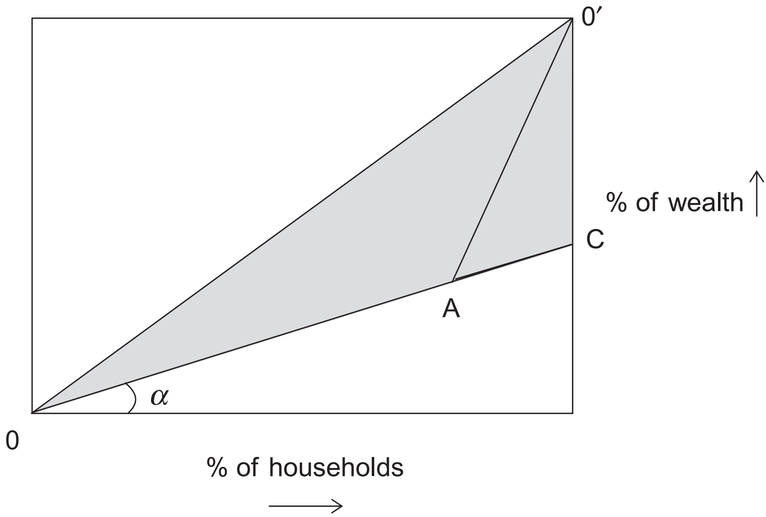

Figure 4. Inequality and growth: monopoly.

3.3.2 The rich buy the top quality, the poor buy the second-best quality

Proposition 3: In a regime where the rich buy the top quality and the poor the second-best quality:

(1) An increase in the population share of the poor β reduces the rate of innovation.
(2) Redistribution from the rich to the poor (an increase in the poor's wealth position d, holding population share constant), increases the rate of innovation, if and only if $\phi^* > \theta(2\beta - 1)(1 - \beta)$.

Proof: See appendix.

Remark: According to Proposition 3 (2) the impact of an increase in d is conditional upon ϕ^*, which is endogenous. However, it is easy to see that an increase in d unambiguously increases the rate of innovation if $\beta \leq \frac{1}{2}$, because then the right-hand side is non-positive and the condition is obviously fulfilled.

To understand the intuition behind Proposition 3 it is instructive to consider the impact of the poor's population share β and their wealth position d on the R-and the N-curve, respectively. We focus on horizontal shifts, that is on the implied changes in the research intensity ϕ^* given aggregate wealth v^*.

First we turn to the effect of an increase in the population share of the poor β on the profits that an innovator earns in his first and second period, Π_0 and Π_1. If β increases (and distribution parameter d remains constant), the relative wealth of the rich d_R increases (recall that d_R equals $(1 - \beta d)/(1 - \beta)$ which is increasing in β, see also figure 2). When the rich become wealthier, they are willing to pay a higher price for the quality good. Thus, *a priori*, the effect of a higher population share of the poor β on an innovators profit in his first period Π_0 is ambiguous: on the one hand, there is a smaller market for the quality leader. On the other hand he can charge a higher price for his product. It can be shown (see (A1) in the appendix) that the former effect always dominates the latter: a higher β unambiguously reduces profits for the top-quality producer, Π_0. The impact of β on profits for the second-best producer, Π_1, is always positive (see (A2)): the willingness to pay for quality by the poor is unaffected, but their group size increases.

How does this affect the incentive to innovate? It is clear that lower profits for the quality leader Π_0 decrease the incentive to innovate, whereas higher profits for the producer of the second-best quality Π_1 raise the incentive to innovate. From equations (3.7) and (3.8) we know how those profits are determined in terms of aggregate wealth v and the exogenous parameters of the model. It is straightforward to verify that the total sum of profits, $\Pi_0 + \Pi_1$, is reduced as the poor's population share β increases, in other words we have $-\partial\Pi_0/\partial\beta > \partial\Pi_1/\partial\beta$, (see (A1) and (A2) in the appendix). Moreover, as a matter of discounting, profits in period 0 have a higher weight than profits in period 1 in the calculation of the value of an innovation, see the right-hand-side of innovation equilibrium condition (2.7). In sum, an increase in the population share of the poor β reduces the present value of aggregate profits and therefore induces a lower research intensity ϕ, so the N-curve in figure 3 shifts to the left.

As β increases, also the R-curve shifts to the left, since the decrease of total profits means less expenditures on the quality good, and—for a given v—more on the standardized good. This requires additional resources in the standardized sector and leaves less resources for R&D, which implies a lower aggregate research intensity ϕ.

Taken together, these shifts of the N-curve and the R-curve imply that more inequality resulting from a higher population share of the poor β unambiguously reduces the equilibrium innovation rate ϕ^*. The impact of β on aggregate wealth v^* can be shown to be ambiguous.

The second part of Proposition 3 states that, for given population shares, a redistribution of wealth from the rich to the poor by increasing the poor's wealth position d may or may not increase the innovation rate ϕ^*. To appreciate this note first, from (A3) and (A4) in the appendix, that the sum of total profits, $\Pi_0 + \Pi_1$, is increasing in d. With a larger sum of these profits, there are less expenditures for the standard good at a given aggregate wealth level v. Hence there are free resources which can be employed in R&D. In other words, the R-curve shifts to the right.

It is, however, ambiguous how the N-curve is affected. The level of profits earned by the producer of the second-best quality Π_1 is increasing in the poor's wealth position d, because the poor are now willing to pay more for the second-best quality. This in turn affects the price of the top-quality producer positively, as the price that keeps the rich from buying the second-best quality can now be larger. This affects the profits of the quality leader Π_0 positively. On the other hand, a higher d also means that the rich have less assets which reduces their willingness to pay for the top quality. This implies that an increase in d has an ambiguous impact on the N-curve.

In sum, less inequality due to a higher wealth position of the poor d in general has an ambiguous effect on the equilibrium innovation rate. Proposition 3 (2), however, states that, if the group share of the rich is large enough ($\beta < \frac{1}{2}$), an increase in d increases the equilibrium research intensity ϕ^*. The reason is that, with a small β an increase in d leads to a small reduction in the wealth position of the rich d_R. This in turn implies that the willingness to pay for the top quality is not strongly reduced and overall the incentive to innovate becomes larger.

Finally, note that the effect of d on aggregate wealth v^* is ambiguous. A sufficient condition for an increase in v^* is that the N-curve does not shift to the right.

Again, one can illustrate the results of proposition 3 in a Lorenz-curve diagram. Figure 5a refers to the case $\phi^* > \theta(2\beta - 1)/(1 - \beta)$; a sufficient condition for an increase of ϕ^* is that A moves into the shaded area (increase of d and/or decrease of β). On the other hand, for the case $\phi^* < \theta(2\beta - 1)/(1 - \beta)$, in figure 5b the shaded area indicates movements of point A leading to a decrease of ϕ^*.

3.4 Selection of the equilibrium

Up to now we have discussed the determinants of the rate of innovation under two different scenarios: (1) when the quality leader serves the entire market, and (2) when the producer of the second-best quality also has a positive market share. So far, we have not explained under which circumstances

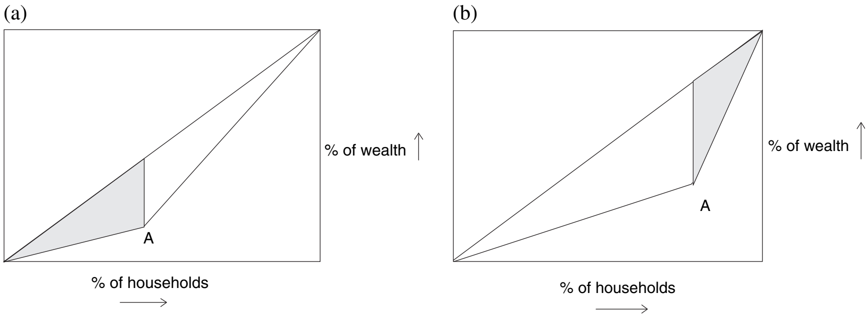

Figure 5. Inequality and growth: duopoly.

each of the regimes will occur and how the respective innovation rates can be compared.

Recall that we focus on steady-state equilibria, without specifying the dynamics out of equilibrium. Moreover, in order to determine the price equilibrium in the two regimes (see section 2.2) the firms have to anticipate correctly the endowments A_R and A_P of the consumers, which in turn depend on the equilibrium value v of aggregate wealth and are different in the two regimes.

Intuitively, it is obvious that, for a situation in which the quality leader sells to the rich and the producer of the second-best quality sells to the poor, the two groups must be sufficiently different. This means that the rich must be sufficiently wealthy (d sufficiently small) and the size of the market for the rich sufficiently large (β sufficiently small) in order to make it profitable to sell in the first period only to the rich and to the poor not until the second period. Clearly this depends on how strongly future profits are discounted, that is on the rate of time preference θ (note from the innovation equilibrium condition (2.7) that also the aggregate research intensity ϕ affects the discounting factor, in addition to θ).

Recall from the end of section 2.2. that for the choice of the particular regime, the quality leader plays the decisive role. This means that, for a given value of aggregate wealth v, a specific regime can represent an equilibrium only if the top-quality producer does not prefer the alternative regime. In particular, a necessary condition for a duopoly to occur is (superscripts M and D refer, respectively, to the monopoly situation, where the quality leader serves the entire market, and the duopoly case, where the quality leader sells to the rich and the producer of the second-best quality to the poor; in addition, the dependency of profits on v and θ is written explicitly):

$$\frac{\Pi_0^{\mathrm{D}}(v^{\mathrm{D}},\theta)}{\phi^{\mathrm{D}}+\theta}+\frac{\Pi_1^{\mathrm{D}}(v^{\mathrm{D}},\theta)\phi^{\mathrm{D}}}{(\phi^{\mathrm{D}}+\theta)^2}\geq\frac{\Pi_0^{\mathrm{M}}(v^{\mathrm{D}},\theta)}{\phi^{\mathrm{D}}+\theta} \tag{3.11}$$

that is, expected profits in case of a duopoly must be higher than with a monopoly, given equilibrium values of wealth and of the rate of innovation in the duopoly regime. Next note that $v^{\mathrm{D}} > v^{\mathrm{M}}$ (for fixed parameters w, F, a, k, θ), as was mentioned at the end of section 3.2. Profits increase in v, hence we get with (3.11)

$$\Pi_0^{\mathrm{D}}(v^{\mathrm{D}},\theta)+\frac{\Pi_1^{\mathrm{D}}(v^{\mathrm{D}},\theta)\phi^{\mathrm{D}}}{\phi^{\mathrm{D}}+\theta}>\Pi_0^{\mathrm{M}}(v^{\mathrm{M}},\theta) \tag{3.12}$$

As $\Pi_1^{\mathrm{M}} = 0$, the equilibrium condition (2.7) can be written as:

in the regime where the quality leader sells to rich and to poor consumers

$$\phi^{\mathrm{M}}wF+\theta wF=\Pi_0^{\mathrm{M}}(v^{\mathrm{M}},\theta) \tag{3.13}$$

in the regime where the quality leader sells to the rich and the producer of the second-best quality sells to the poor

$$\phi^{\mathrm{D}}wF+\theta wF=\Pi_0^{\mathrm{D}}(v^{\mathrm{D}},\theta)+\frac{\Pi_1^{\mathrm{D}}(v^{\mathrm{D}},\theta)}{\phi^{\mathrm{D}}+\theta} \tag{3.14}$$

Equation (3.12) says that the right-hand side of (3.14) is larger than that of (3.13). The same must hold for the left-hand sides, which proves the following

Proposition 4: If, for given inequality parameters, a regime where both the quality leader and the producer of the second-best quality have a positive market share occurs, then the rate of innovation is higher than it would be with a monopoly.

Thus, if the top-quality producer regards a duopoly as more profitable than serving the whole market, this is also preferable as far as growth is concerned. How do these considerations affect the relationship between inequality and growth? Consider an increase in inequality starting from a monopoly regime. As long as the top-quality producer regards it as still profitable to serve the whole market, an increase in inequality decreases the rate of innovation. However, the top-quality producer might also consider selling only to rich consumers thus inducing a switch of the regime. Proposition 4 implies that such a regime switch from a monopoly (which is more likely when inequality is low) to a duopoly (which tends to occur when there is high inequality) leads to a higher innovation rate. In sum, while inequality and the innovation rate are negatively correlated *within* a regime, an increase in inequality may raise the innovation rate when higher inequality leads to a switch from a monopoly to a duopoly regime.

4. CONCLUSION

In this work we have studied the impact of the distribution of income on the rate of innovation. Inequality plays a role because consumers purchase the quality good in a fixed quantity, implying that richer households have a higher willingness to pay for quality. As a result, inequality affects the prices for the quality good and therefore the profitability of conducting R&D. We have concentrated on a set-up where consumers differ in the ownership of wealth but earn the same wage income. However, it is straightforward to show that exactly the same effects of inequality occur when inequality results from differences in the wage rates rather than in wealth. The reason is that differences in income flows between rich and poor consumers drive price differences between quality goods. The source of the income flows does not play any separate role.[12]

In equilibrium, not only the quality leader but also producers of worse qualities may have a positive market share. The analysis was confined to two groups of consumers, which lead to two possible regimes; one where only the best quality is produced; and a second regime where both the best and the second-best quality have a positive market share. The latter case is different from previous endogenous growth models where only the top-quality producer is in the market. We have shown that the top quality is purchased by all households only if the degree of heterogeneity between consumers is sufficiently small. When the distribution is sufficiently unequal there is scope also for the producer of the second-best quality.

We find that less income inequality tends to improve the profitability of innovations. This is because a more equal distribution allows quality producers to charge higher prices. If only the quality leader is on the market,

[12] In a formal model, this can be shown by assuming that w is the wage per efficiency unit of labour and the two groups of individuals differ in labour productivity. Let l be the productivity of the low-income group, then normalizing total efficient labour supply to L gives us for the productivity of the high-income group: $l_R = (1 - \beta l)/(1 - \beta)$. With l instead of d all statements remain valid.

the price of his product can be larger if, due to more income, the poor are willing to pay more for quality. If also the second-best quality is on the market, our results are ambiguous. The ambiguity comes from the fact that with more equality the price for the worse quality can be higher, but the impact on the price of the top quality is indeterminate. We have established sufficient conditions under which more equality leads to a higher incentive to innovate. A further ambiguity comes from the fact that an increase in inequality may lead to a switch from a monopoly to a duopoly regime. When such a switch occurs the most likely outcome is an increase in the innovation rate.

APPENDIX

Proof of Lemma 3:

(1) φ^R is implicitly defined by writing (3.4) as $R(\phi, v, \beta, d) = 0$ with $R(\cdot) = -wF + (\Pi_0 + \Pi_1 - \theta v)/\phi$. As noted in the text, Π_0 and Π_1 are linear in v and independent of ϕ, hence φ^R is a linear relation. Its slope is determined by implicit differentiation: $\partial\varphi^R/\partial\phi = -R_\phi/R_v$, with the partial derivatives $R_\phi = -(\Pi_0 + \Pi_1 - \theta v)/\phi^2$ and $R_v = (\partial\Pi_0/\partial v + \partial\Pi_1/\partial v - \theta)/\phi$. $R_\phi < 0$ follows from $\Pi_0 + \Pi_1 - \theta v = \phi wF > 0$ for $\phi > 0$ (see (3.4)), while $R_v < 0$ (= 0, if $\theta = 0$) is seen from differentiation of either (3.5), (3.6), which gives $\partial\Pi_0/\partial v + \partial\Pi_1/\partial v = d\theta(k-1)/k < \theta$, or (3.7), (3.8), which gives

$$\partial\Pi_0/\partial v + \partial\Pi_1/\partial v = \theta[(k-1)(1-\beta d) + (k-1)(1-\beta)d/k + (k-1)\beta d]k$$
$$< \theta[k - 1 + (k-1)(1-\beta)/k]/k = \theta[k^2 - 1 - (k-1)\beta]/k^2 < \theta \text{ (note } d < 1)$$

Thus $\partial\varphi^R/\partial\phi < 0$.

(2), (3) φ^N is implicitly defined by writing (2.7) as $N(\phi, v, \beta, d) = 0$ with $N(\cdot) = -wF + \Pi_0/(\phi + \theta) + \phi\Pi_1/(\phi + \theta)^2$, where Π_0 and Π_1 are determined by (3.5), (3.6) or (3.7), (3.8), depending on the regime. We have $N_\phi = -\Pi_0/(\phi + \theta)^2 - \Pi_1(\phi - \theta)/(\phi + \theta)^3$. $\phi > 0$ and $\theta \geq 0$, hence N_ϕ has the same sign as $-\Pi_0 + \Pi_1(\theta - \phi)/(\theta + \phi)$, which is negative in case of a monopoly, because then $\Pi_1 = 0$. In case of a duopoly, one finds $\bar{\phi} = \theta(\Pi_1 - \Pi_0)/(\Pi_1 + \Pi_0)$ such that $N_\phi < 0$ for $\phi > \bar{\phi}$. We set $\bar{\phi} = 0$ if $\Pi_1 \leq \Pi_0$. Otherwise, $\bar{\phi} > 0$ and $\bar{\phi} \to 0$ for $\theta \to 0$.

$N_v = (\partial\Pi_0/\partial v)/(\phi + \theta) + (\partial\Pi_1/\partial v)\phi/(\phi + \theta)^2 > 0$ follows from differentiation of Π_0 and Π_1 in the two regimes. The assertions follow from implicit differentiation, $\partial\varphi^N/\partial\phi = -N_\phi/N_v$.

Proof of Proposition 1: We compute the root ϕ_0^R of φ^R (where $\varphi^R(\phi_0^R) = v = 0$) as $\phi_0^R = (\Pi_0(0) + \Pi_1(0))/wF$, where $\Pi_t(0)$, $t = 0, 1$, denotes profits at $v = 0$. ϕ_0^R is independent of θ, it coincides with the root $\phi_0^{N,\theta=0}$ of φ^N, if $\theta = 0$. $\phi^N(0)$ is negative for sufficiently small θ, as is seen from solving $N(0, v, \beta, d) = 0$, (see proof of Lemma 3), i.e. $-wF + \Pi_0(v)/\theta = 0$ for v and noting that Π_0 increases linearly with θv (say $\Pi_0 = D + E\theta v$, where $D, E > 0$, which gives $v = wF/E - D/(E\theta)$, which is negative for sufficiently small θ).

We know from Lemma 3 that the slope $\partial\varphi^N/\partial\phi$ may be negative for small $\phi > 0$, but is positive for $\phi > \bar{\phi}$. Hence $\varphi^N(0) < 0$ implies that $\partial\varphi^N/\partial\phi$ is positive at the root $\phi_0^{N,\theta}$, for small enough $\theta > 0$. Then Proposition 1 follows from continuity of φ^N with respect to θ and from the fact that for small enough θ the root $\phi_0^{N,\theta}$ decreases with increasing θ: $\partial\phi_0^{N,\theta}/\partial\theta = -N_\theta/N_\phi < 0$[13] (we know $N_\phi < 0$ from the proof of Lemma 3(1); $N_\theta = -\Pi_0/(\phi + \theta)^2 - \Pi_1/(\phi + \theta)^3 < 0$, as $\partial\Pi_t/\partial\theta = 0$ for $v = 0$).

Proof of Proposition 3: We consider the shifts of the φ^R- and φ^N-curves caused by an increase in β and d, respectively. These shifts are determined by implicit differentiation of ϕ with respect to β and d, respectively, using the equation $R(\cdot) = 0$ and $N(\cdot) = 0$, as defined in the proof of Lemma 3.

As a preparation we compute from (3.7), (3.8):

$$\frac{\partial\Pi_0}{\partial\beta} = -\frac{k^2-1}{k^2}Lw(1-a) - \frac{k-1}{k}d\theta v - \frac{k-1}{k^2}d\theta v \tag{A1}$$

$$\frac{\partial\Pi_1}{\partial\beta} = \frac{k-1}{k}Lw(1-a) + \frac{k-1}{k}d\theta v \tag{A2}$$

$$\frac{\partial\Pi_0}{\partial d} = -\frac{k-1}{k}\beta\theta v + \frac{k-1}{k^2}(1-\beta)\theta v \tag{A3}$$

$$\frac{\partial\Pi_1}{\partial d} = \frac{k-1}{k}\beta\theta v \tag{A4}$$

To find the effect of an increase of β we use $\partial\phi^N/\partial\beta = -N_\beta/N_\phi$ and $\partial\varphi^R/\partial\beta = -R_\beta/R_\phi$. $N_\phi < 0$ and $R_\phi < 0$ is known from the proof of Lemma 3. $N_\beta = (\partial\Pi_0/\partial\beta)/(\phi + \theta) + (\partial\Pi_1/\partial\beta)\phi/(\phi + \theta)^2 < 0$ follows from (A1) and (A2) and the fact that $\phi/(\phi + \theta)^2 < 1/(\phi + \theta)$ and $(k - 1)/k < (k^2 - 1)/k^2$, due to $k > 1$.

[13] N_θ and N_ϕ are evaluated at $v = 0$.

Analogously, we have $R_\beta = (\partial\Pi_0/\partial\beta + \partial\Pi_1/\partial\beta)/\phi < 0$. Altogether, both φ^R and φ^N are shifted to the left by an increase of β, which proves part (1).

To determine the effect of d we implicitly differentiate the solution ϕ^*, v^* of the system of two equations $N(\cdot) = 0$, $R(\cdot) = 0$ with respect to d, which gives $\partial\phi^*/\partial d = -(R_v N_d - N_v R_d)/(R_v N_\phi - N_v R_\phi)$. It is already known that the denominator is positive, hence the sign of $\partial\phi^*/\partial d$ depends on the numerator, which can be computed as $(k-1)\theta^2[\phi^* - \beta\phi^* + \theta - 2\beta\theta]v/(\phi^* k^2(\phi^* + \theta)^2)$. The expression in square brackets is positive if and only if $\phi^* > \theta(2\beta - 1)/(1 - \beta)$.

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