Topic 4 – Factor Proportions Theory

Prof. Ralph Ossa

Introduction

- Since the Ricardian model only allows for one factor of production it cannot shed light on (i) the effects of factor endowments on trade and (ii) the distributional effects of trade

- For example, do countries which are relatively well endowed with skilled labor export different goods than countries which are relatively well endowed with unskilled labor?

- Also, does everybody within a country reap some of the overall gains from trade or does trade perhaps generate winners and losers?

- In this lecture, we answer these questions by looking at the Heckscher-Ohlin model which analyzes the role of factor proportions in trade

Introduction (contd.)

- The Heckscher-Ohlin model delivers two main results: (i) countries export the goods which make intensive use of their abundant factors and (ii) the abundant factor gains from trade and the scarce factor loses from trade

- At the same time, trade is still driven by comparative advantage and brings about overall welfare gains so that the main lessons from the Ricardian model continue to apply

- In contrast to the Ricardian model, the Heckscher-Ohlin model is actually quite complicated if you want to understand it in every detail

- We will therefore only provide a selective treatment in an attempt to get across the main points without dedicating too much time

Overview of the lecture

- Autarky

- Basic properties

- Classic results

- Gains from trade

Autarky - Setup

- A single country ("Home") produces two goods ("cloth" and "food") from two factors of production ("labor" and "land") under perfect competition

- Production functions take the following Cobb-Douglas form, where $i = C$ refers to the cloth industry and $i = F$ refers to the food industry

$$Q_i = \left(\frac{L_i}{\beta_i}\right)^{\beta_i} \left(\frac{T_i}{1-\beta_i}\right)^{1-\beta_i} \; for \; i = \{C, F\}$$

- Q_i, L_i, and T_i denote the amount of output produced, the amount of labor employed, and the amount of land (T for terrain) used in these industries

Autarky – Setup (contd.)

- We assume that cloth production is **labor intensive** which means that the cloth industry employs relatively more labor compared to the food industry

$$\frac{L_C}{T_C} > \frac{L_F}{T_F}$$

- Notice that this concept is defined in relative terms and immediately implies that food production is **land intensive**

$$\frac{T_F}{L_F} > \frac{T_C}{L_C}$$

- We will see later on that this assumption is equivalent to the restriction $\beta_C > \beta_F$ in our Cobb-Douglas environment
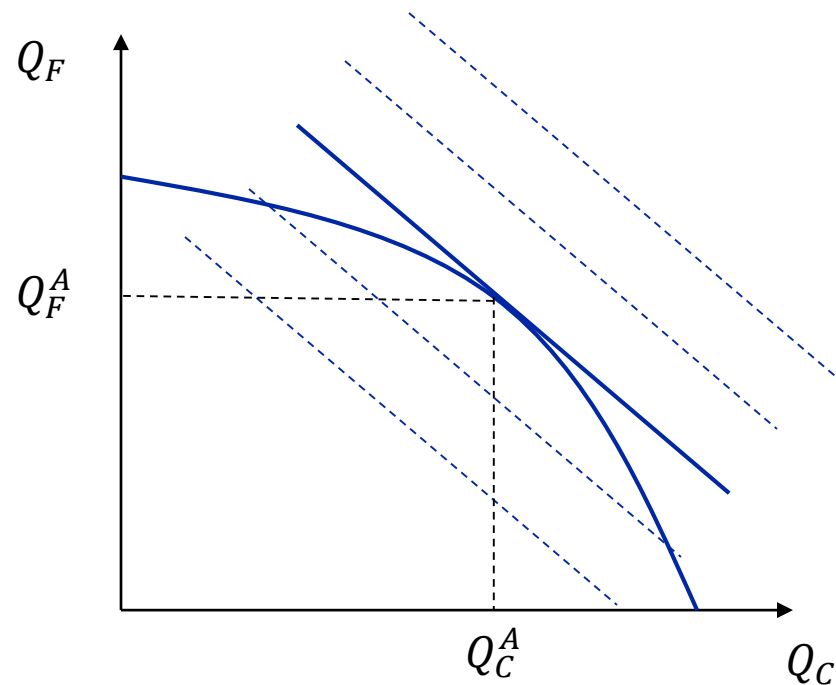
Autarky – PPF

- One can show that the opportunity cost of producing cloth in terms of food is increasing in this environment which implies that the PPF becomes concave
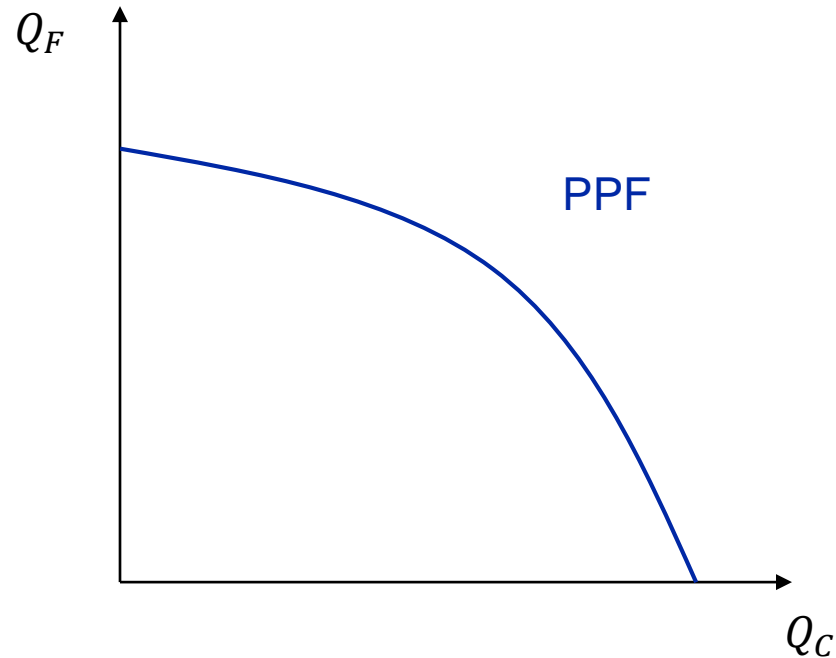
Autarky – Relative supply

- One can also show that the economy produces at the point which maximizes the value of production (you can verify that this is true in the Ricardian model too)


- The lines are **isovalue** lines which characterize combinations of cloth and food which have the same value

- They are implicitly defined by the equation $V = P_C Q_C + P_F Q_F$ which can be solved for

$$Q_F = \frac{V}{P_F} - \frac{P_C}{P_F} Q_C$$

- Production takes place at the tangency point shown in the diagram

Autarky – Relative supply

- The isovalue lines have slope $-\frac{P_C}{P_F}$ so that an increase in the relative cloth price leads to an increase in the relative cloth supply
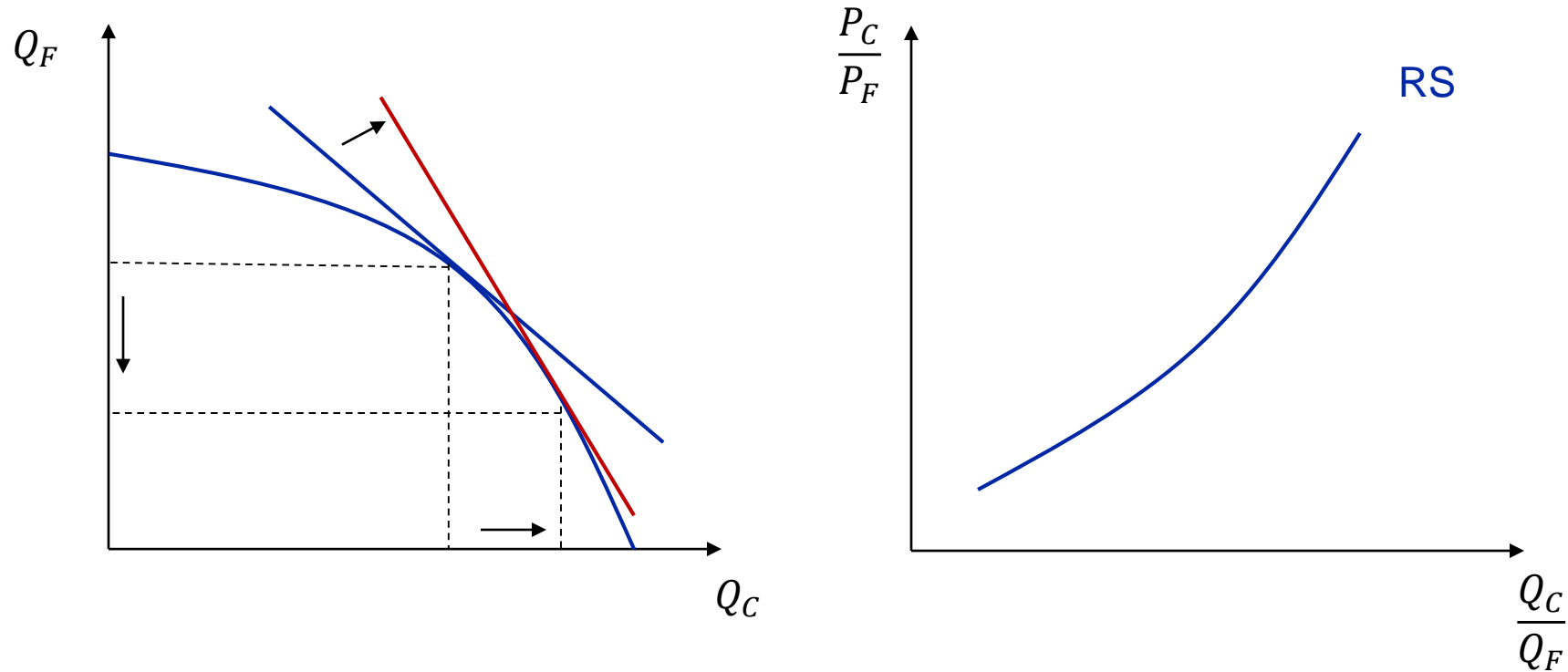
Autarky – Equilibrium

- To close the model, we again assume a downward sloping relative demand curve without specifying a particular functional form
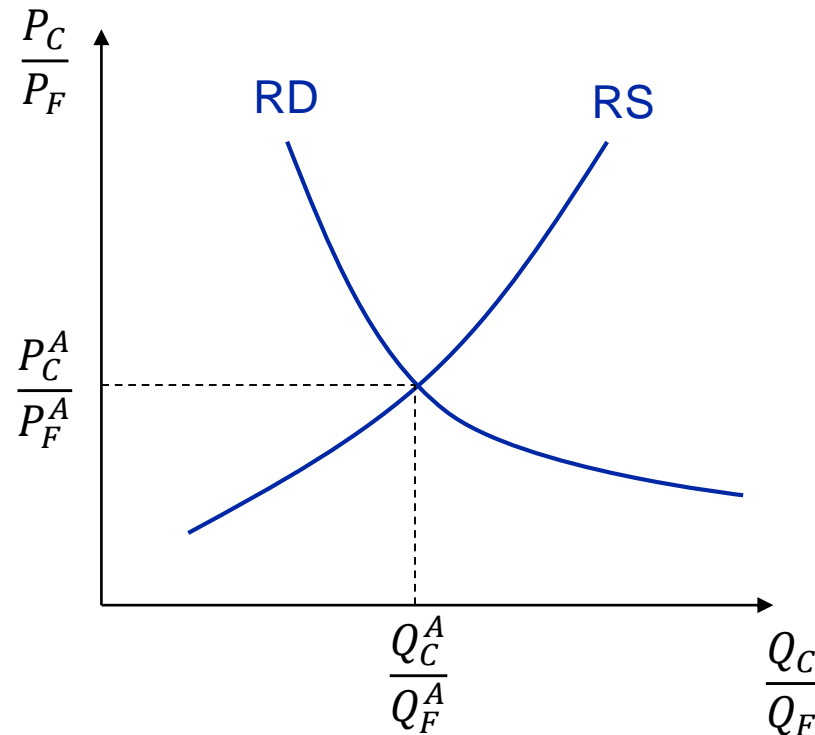
Basic properties – Overview

- In order to prepare for our derivation of the main results of the Heckscher-Ohlin model, it is useful to first go through a number of its basic properties

- These properties follow from the conditions that firms minimize costs, firms break even, and factor markets clear and thus apply under autarky and trade

- Keep in mind that they are building blocks for our later analysis so the point of deriving them will not always be immediately clear

Basic properties – Cost minimization

- Given our assumption of Cobb-Douglas production functions, we know that β_i is the share of labor costs in total costs and $1 - \beta_i$ is the share of land costs in total costs so that

$$\beta_i = \frac{wL_i}{wL_i + rT_i} \quad and \quad 1 - \beta_i = \frac{rT_i}{wL_i + rT_i}$$

- w is the wage rate and r is the land rental rate. You can think of r either literally as the rental price of land or as the opportunity cost of land if it is owned

- Taking ratios yields the following relationships between factor prices and factor intensities which we plot on the next slide

$$\frac{w}{r} = \frac{\beta_i}{1 - \beta_i} \frac{T_i}{L_i}$$
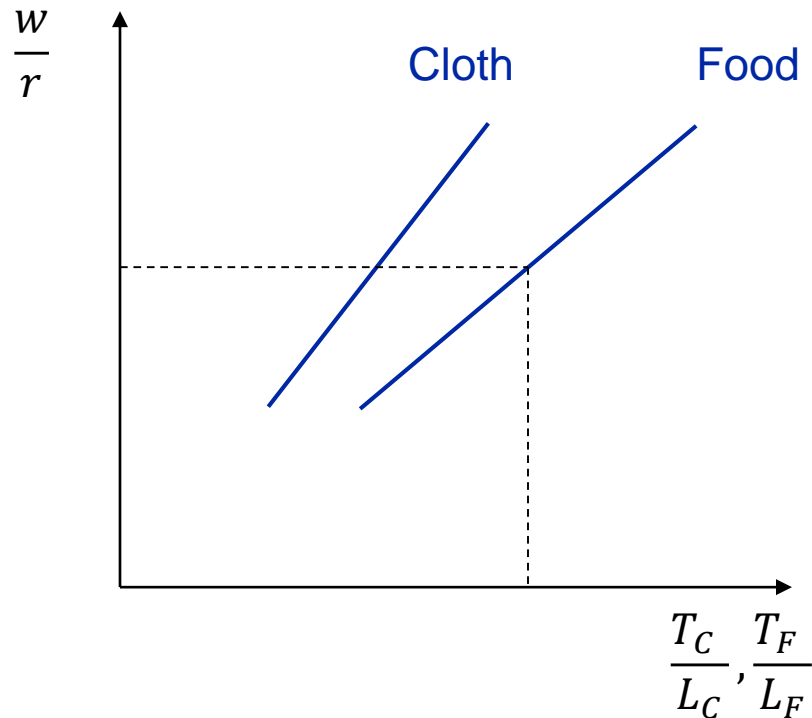
- A higher wage rental ratio makes firms substitute away from labor towards land

- By assumption, food production has a higher land intensity than cloth production

- Notice that relative factor prices directly pin down factor intensities

Clicker question:

How would this diagram change if cloth production was land intensive instead of labor intensive?

Basic properties – Cost minimization (contd.)

- The relationships $\frac{w}{r} = \frac{\beta_i}{1-\beta_i} \frac{T_i}{L_i}$ can be combined with the production functions $Q_i = \left(\frac{L_i}{\beta_i}\right)^{\beta_i} \left(\frac{T_i}{1-\beta_i}\right)^{1-\beta_i}$ to solve for the factor demands

$$L_i = \beta_i \left(\frac{r}{w}\right)^{1-\beta_i} Q_i \quad and \quad T_i = (1-\beta_i) \left(\frac{w}{r}\right)^{\beta_i} Q_i$$

- Notice that they confirm that cloth is labor intensive if and only if it has the higher labor cost share (which is already implicit in the graph on the previous slide)

$$\frac{L_C}{T_C} > \frac{L_F}{T_F} \Leftrightarrow \beta_C > \beta_F$$

Basic properties – Break even conditions

- Substituting the above factor demands into $c_i = \frac{wL_i + rT_i}{Q_i}$ yields the average cost functions
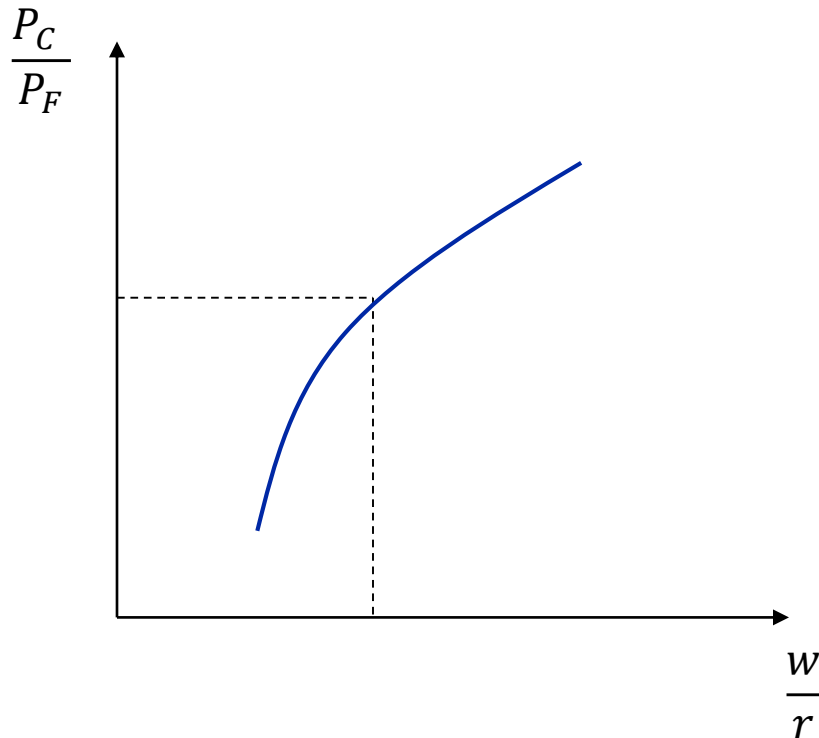
$$c_i = w^{\beta_i} r^{1-\beta_i}$$

- Prices have to equal average costs to ensure that firms break even so that

$$P_i = w^{\beta_i} r^{1-\beta_i}$$

- This implies the following relationship between relative factor prices and relative goods prices which we plot on the next slide

$$\frac{P_C}{P_F} = \left(\frac{w}{r}\right)^{\beta_C - \beta_F}$$

Basic properties – Break even conditions (contd.)

- Higher wages increase the costs of cloth production by more than the costs of food production because cloth is labor intensive

- Notice that relative goods prices directly pin down relative factor prices (and thus also factor intensities)

Clicker question:

How would this diagram change if cloth production was land intensive instead of labor intensive?

Basic properties – Factor market clearing

- We again assume full employment implying that the wage and the land rental rate adjust to make sure that the labor and land markets clear

$$a_{LC}(w,r)Q_C + a_{LF}(w,r)Q_F = L \quad and \quad a_{TC}(w,r)Q_C + a_{TF}(w,r)Q_F = T$$

- In the above conditions, I have defined the **unit labor uses** $a_{Li}(w,r)$ and **unit land uses** $a_{Ti}(w,r)$ emphasizing that they depend on factor prices (w,r)

$$a_{Li}(w,r) = \beta_i \left(\frac{r}{w}\right)^{1-\beta_i} \quad and \quad a_{Ti}(w,r) = (1-\beta_i)\left(\frac{w}{r}\right)^{\beta_i}$$

- These equations for the unit factor uses follow directly from the above equations describing the factor demands (they are really just summarizing some of the terms)

Basic properties – Factor market clearing (contd.)

$$Slope = \frac{T_F}{L_F} = \frac{a_{TF}(w,r)}{a_{LF}(w,r)}$$

$$Slope = \frac{T_C}{L_C} = \frac{a_{TC}(w,r)}{a_{LC}(w,r)}$$
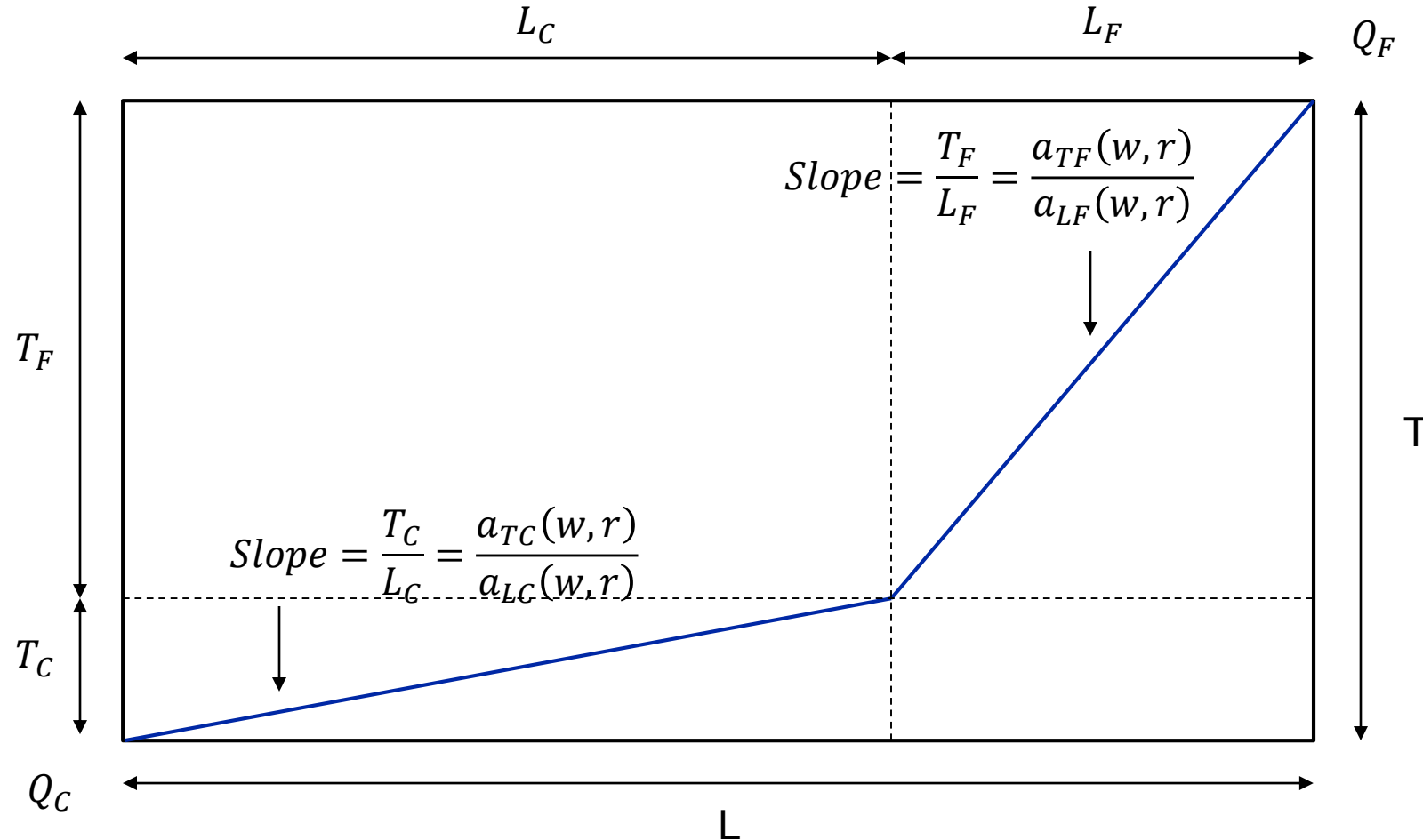
Classic results - Overview

- Armed with these basic properties, we are now ready to go through four classic results of the Heckscher-Ohlin model:

1. Rybczynski theorem

2. Heckscher-Ohlin theorem

3. Stolper-Samuelson theorem

4. Factor price equalization theorem

Classic results – Rybczynski theorem

Theorem: *For given relative goods prices, an increase in the endowment of a factor will increase the output of the industry using this factor intensively and decrease the output of the other industry*

- Recall from the above discussion that relative goods prices pin down relative factor prices and relative factor prices pin down factor intensities

- Hence, saying "for given relative goods prices" is equivalent to saying "for given relative factor prices" which is, in turn, equivalent to saying "for given factor intensities"

- The theorem then follows from the following diagram which illustrates that, for given factor intensities, an increase in the endowment of land will increase the output of food and decrease the output of cloth

- By taking relative goods prices as given, we are implicitly considering the case of a small open economy whose goods prices are determined in world markets

Classic results – Rybczynski theorem (contd.)
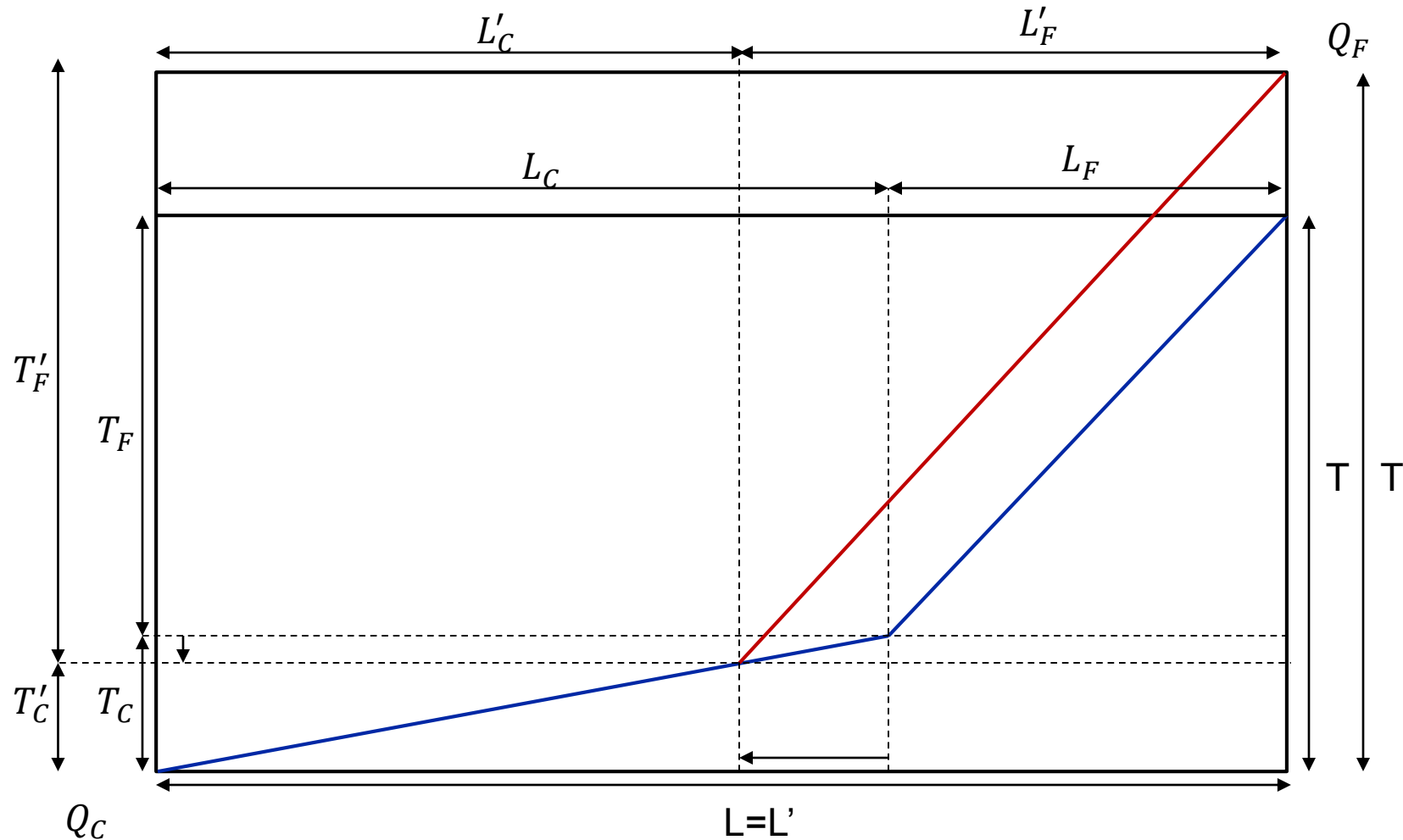
$\downarrow L_C, \downarrow T_C, \uparrow L_F, \uparrow T_F$

$\Rightarrow \downarrow Q_C, \uparrow Q_F$

Clicker question:

Why do we keep the slopes of the blue and red lines constant in this thought experiment?

Classic results – Rybczynski theorem (contd.)

- Zimring (2017) tests the Rybczynski theorem using the near elimination of commuting from the West Bank to Israel in 2000 as a natural experiment

- Until then, 20% of the West Bank population commuted to work in Israel on a daily basis. Following the outbreak of the Second Intifada, this was severely restricted by the Israeli government

- Zimring (2017) interprets this as an increase in the West Bank's labor endowment and studies the effects it has on the West Bank's production pattern

- Trade remained free and the West Bank plausibly takes goods prices as given so that the Rybczynski assumptions are satisfied

Classic results – Rybczynski theorem (contd.)

Figure 1: Commuters to Israel and Employment Rate in the West Bank
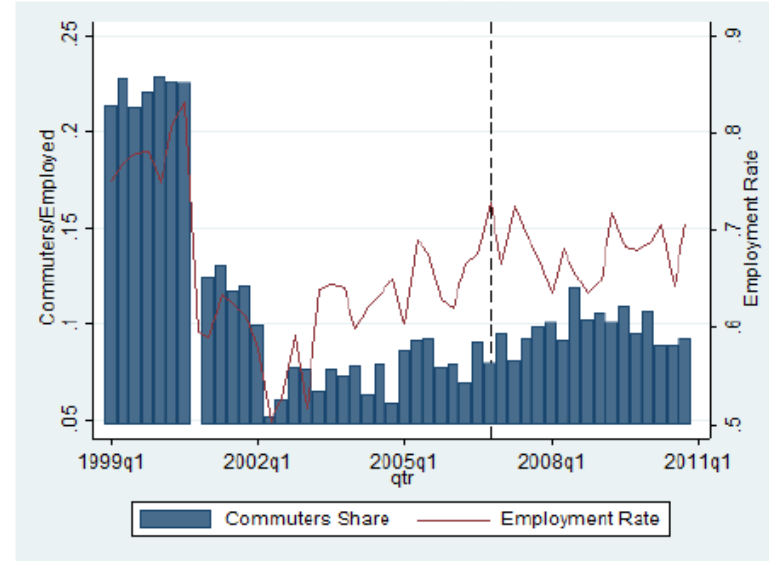
Notes: Data are at a quarterly frequency and are taken from the Palestinian Central Bureau of Statistics Labor Force Survey. Commute to Israel is as reported by workers. Share of commuters (left axis) is the number of persons who live in the West Bank and report commuting to work in Israel out of total employed persons who live in the West Bank. Employment rate (right axis) is total employed persons divided by all persons of prime working age (25-55).

Source: Zimring (2017)

Classic results – Rybczynski theorem (contd.)

Figure 3: Employment Growth 1999-2007, and Share of Commuters in 1999
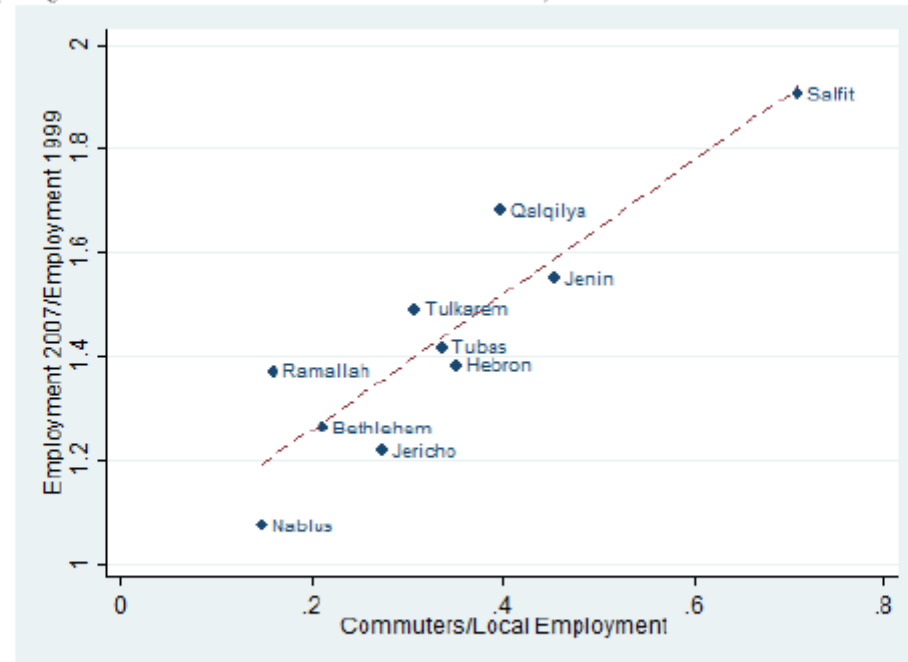
Notes: Data are from the Palestinian Central Bureau of Statistics Labor Force Survey. Commute to Israel is as reported by workers.

Source: Zimring (2017)

Classic results – Rybczynski theorem (contd.)

- Zimring (2017) exploits that there is large variation within the West Bank in terms of these labor supply shocks

- This variation is driven by variation in the proximity to the Israeli border since the share of commuters was lower in more remote districts

- Consistent with the Rybczynski theorem, Zimring (2017) finds that the composition of production shifted more towards labor intensive goods in districts which experienced larger labor supply shocks

Classic results – Heckscher-Ohlin theorem

Theorem: *Version 1: A country has a comparative advantage in the good which uses its abundant factor intensively. Version 2: A country exports the good which uses its abundant factor intensively.*

- We now suppose that there is a second country ("Foreign") which produces the same goods as Home using identical preferences and technologies (there are no Ricardian productivity differences)

- We also assume that Home is **labor abundant** in the sense that it is endowed with relatively more labor than Foreign (which immediately implies that Foreign is **land abundant**)

$$\frac{L}{T} > \frac{L^*}{T^*}$$

- Under these assumptions, the Heckscher-Ohlin theorem applies which summarizes the first main result of the Heckscher-Ohlin model

Classic results – Heckscher-Ohlin theorem (contd.)

- To see this, notice that Home's and Foreign's relative supply curves would be the same if Home and Foreign had identical factor endowments

- But Home is labor abundant and cloth is labor intensive so that Home's relative supply curve is actually to the right of Foreign's

- In particular, we know from the Rybczynski theorem that an increase in the labor endowment increases cloth output and decreases food output for a given relative price thereby shifting the relative supply curve out

- Also, we know from the production functions that any proportional change in factor endowments leaves the relative supply curve unchanged (constant returns to scale)

Classic results – Heckscher-Ohlin theorem (contd.)
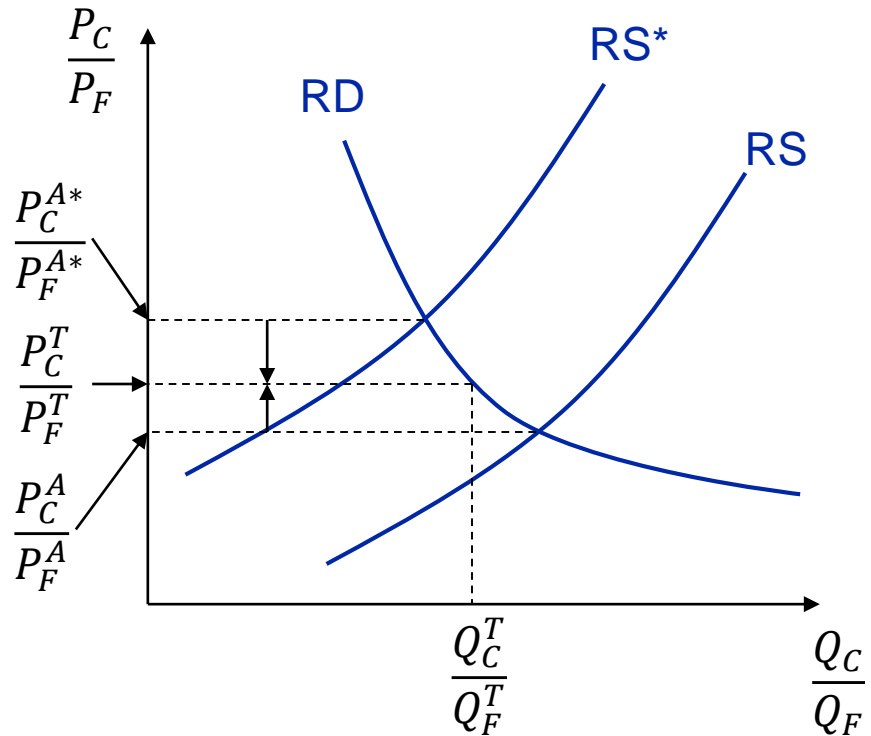
Clicker question (return after next slide):

How do we know that the trade relative price must lie between the two countries' autarky relative prices?

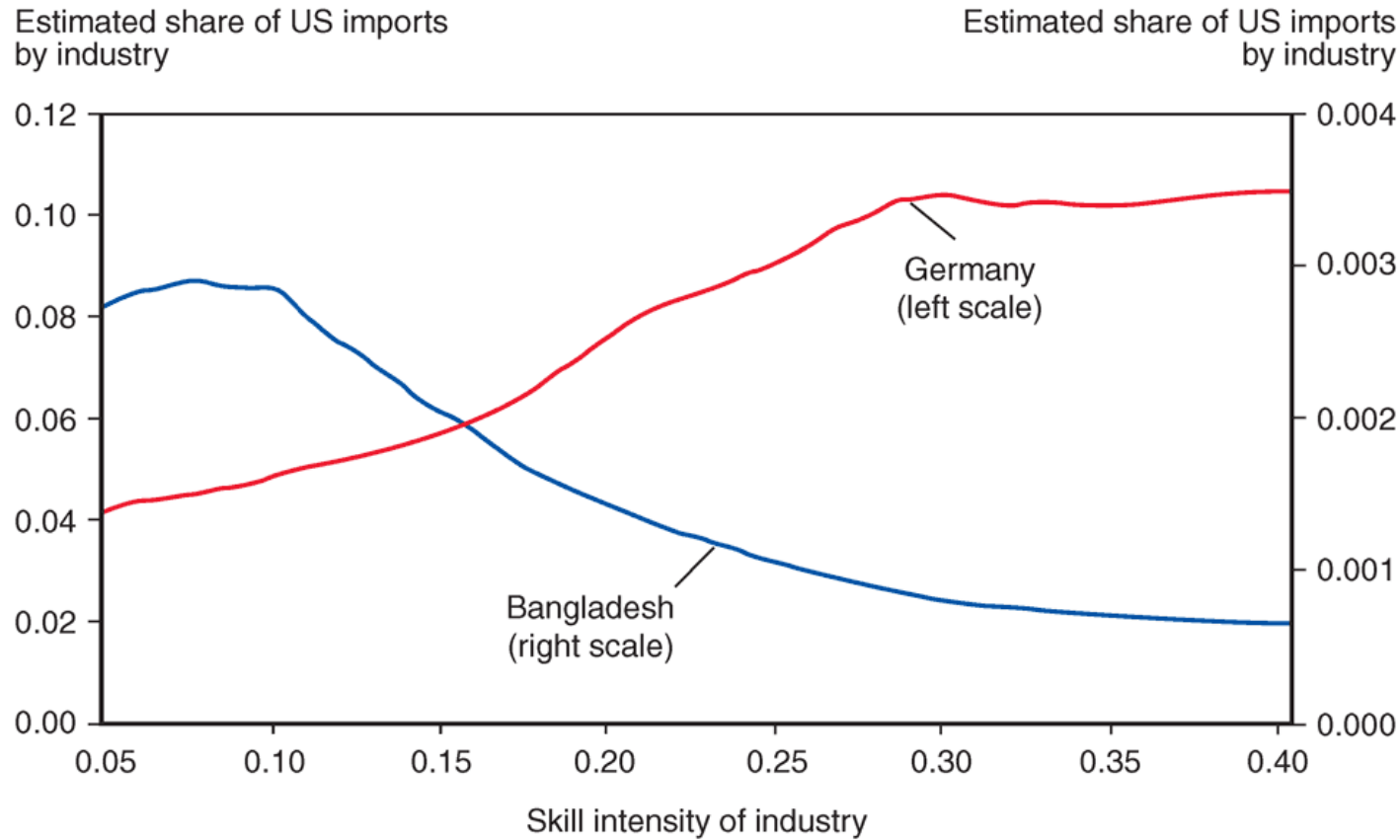# Classic results – Heckscher-Ohlin theorem (contd.)

- Hence, Home's autarky relative cloth price is lower than Foreign's autarky relative cloth price which means that Home produces on a flatter part of its PPF

- This implies that Home has the lower opportunity costs of cloth production and therefore a comparative advantage in cloth (version 1 of the theorem)

- Moreover, the trade relative cloth price has to be somewhere between Home's and Foreign's autarky relative cloth prices so that it increases in Home and decreases in Foreign upon trade liberalization

- This implies that Home becomes an exporter of cloth and an importer of food, and Foreign becomes an importer of cloth and an exporter for food (version 2 of the theorem)

Classic results – Heckscher-Ohlin theorem (contd.)

- Early test of the Heckscher-Ohlin theorem maintained the assumption that countries produce with identical technologies and did not deliver particularly promising results

- But this approach seems like a non-starter from today's perspective since we now know that there are large cross-country differences in technology

- More recent tests which drop this and sometimes also other unrealistic assumptions usually provide more favorable results
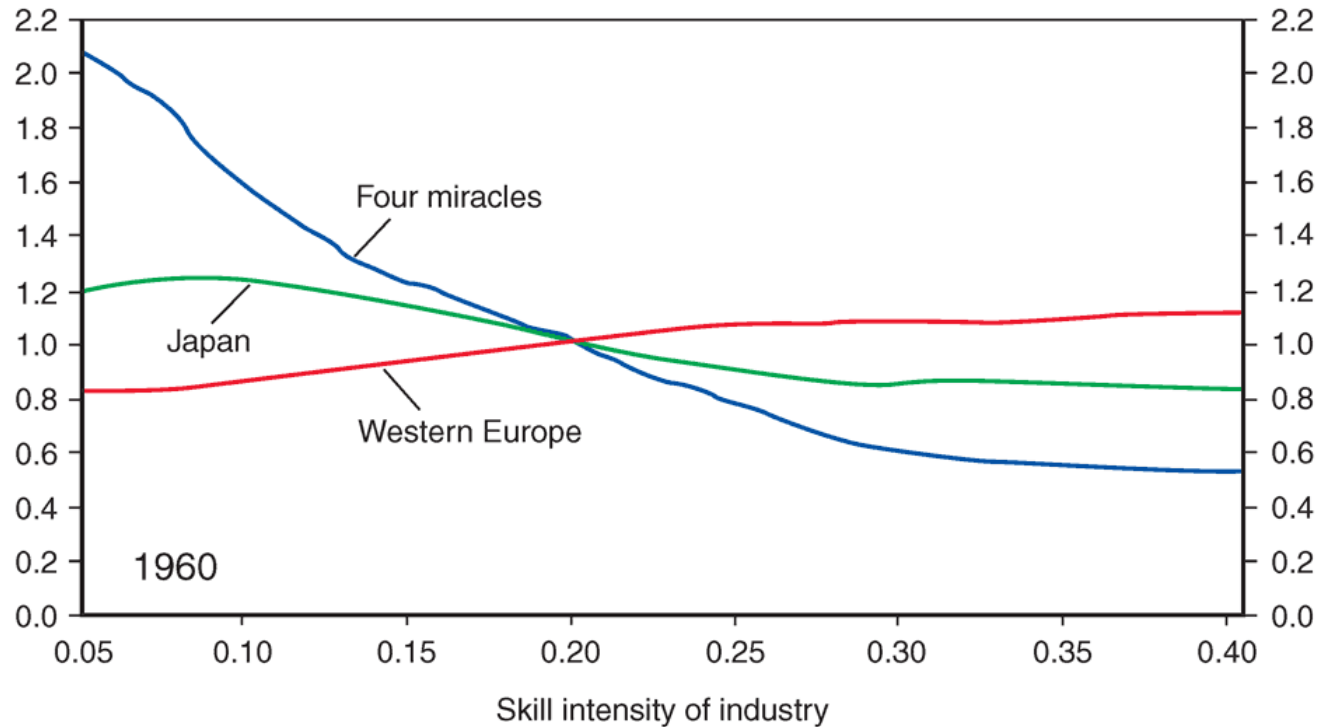
Classic results – Heckscher-Ohlin theorem (contd.)

Source: Romalis (2004)

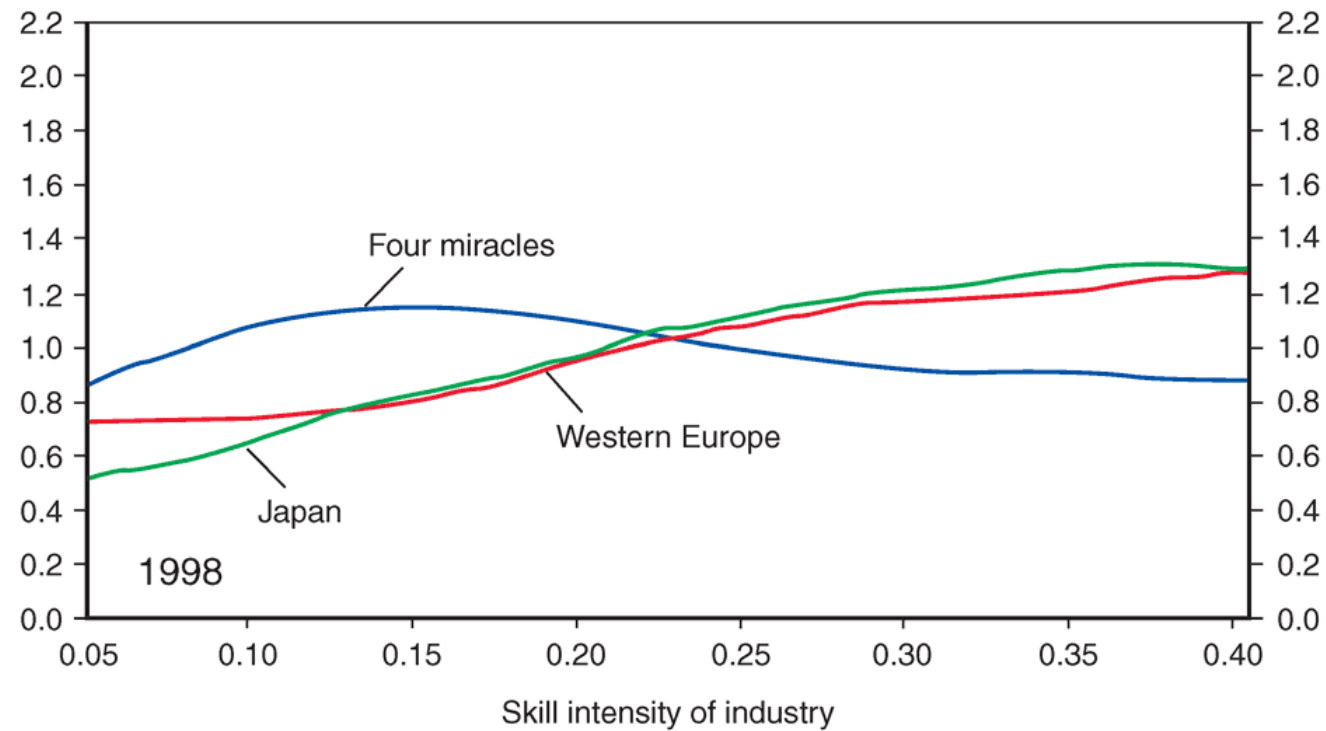# Classic results – Heckscher-Ohlin theorem (contd.)

Source: Romalis (2004)

Classic results – Heckscher-Ohlin theorem (contd.)

(a) 1998

Source: Romalis (2004)

Classic results – Stolper-Samuelson theorem

Theorem: *An increase in the relative price of a good will increase the real return to the factor used intensively in the production of that good, and reduce the real return to the other factor, in terms of both goods*

- Taking logs and then totally differentiating the break even conditions reveals that changes in goods prices are weighted averages of changes in factor prices (notation: $\hat{x} = \mathrm{dln}(x) = \frac{dx}{x}$)

$$\widehat{P_C} = \beta_C \hat{w} + (1 - \beta_C)\hat{r} \quad and \quad \widehat{P_F} = \beta_F \hat{w} + (1 - \beta_F)\hat{r}$$

- By the properties of weighted averages, this immediately implies that goods price changes bring about the following magnified factor price changes

$$\hat{w} > \widehat{P_C} > \widehat{P_F} > \hat{r} \quad or \quad \hat{r} > \widehat{P_F} > \widehat{P_C} > \hat{w}$$

Classic results – Stolper-Samuelson theorem (contd.)

- This proves the theorem because the inequality on the left says that an increase in the relative price of cloth leads to an increase in the real wage and a decrease in the real land rental rate, in terms of both goods

$$\uparrow \frac{P_C}{P_F} \Rightarrow \uparrow \frac{w}{P_C}, \uparrow \frac{w}{P_F}, \downarrow \frac{r}{P_C}, \downarrow \frac{r}{P_F}$$

- Similarly, the inequality on the right says that an increase in the relative price of food leads to an increase in the real land rental rate and a decrease in the real wage, in terms of both goods

$$\uparrow \frac{P_F}{P_C} \Rightarrow \uparrow \frac{r}{P_C}, \uparrow \frac{r}{P_F}, \downarrow \frac{w}{P_C}, \downarrow \frac{w}{P_F}$$

Clicker question:

What happens to the welfare of workers if their real wages increase in terms of both goods?

Classic results – Stolper-Samuelson theorem (contd.)

- Together with the Heckscher-Ohlin theorem, the Stolper-Samuelson theorem implies that the owners of the abundant factor gain and the owners of the scarce factor lose from trade

- In particular, the Heckscher-Ohlin theorem predicts that the relative price of the good which makes intensive use of the abundant factor increases following trade liberalization

- The Stolper-Samuelson theorem then adds that this increases the real return to that factor and decreases the real return to the other factor in terms of both goods

- This is the second main result of the Heckscher-Ohlin model

Classic results – Stolper-Samuelson theorem (contd.)

- One implication of the Stolper-Samuelson theorem is that skilled-workers should gain and unskilled workers should lose from trade in skilled-labor abundant countries

- As we will discuss in more detail in the next lecture, this makes trade a potential explanation for the increase in wage inequality observed in the US and other rich countries

- However, we will see that rising wage inequality is most likely brought about by other forces such as technological progress and outsourcing

- This is not evidence against the Stolper-Samuelson theorem but just says that other mechanisms played a more important role

Classic results – Factor price equalization theorem

Theorem: *If two countries are engaged in free trade, have identical technologies, and produce both goods, then they must have the same factor prices*

- This follows immediately from the fact that both countries have identical break even conditions if they face identical goods prices due to free trade

- We explicitly assumed that both countries have identical technologies. Without this assumption, they would not have identical break even conditions

- We implicitly assumed that both countries produce both goods. Without this assumption, they would not have to break even in both industries

Classic results – Factor price equalization theorem (contd.)

- Hence, international trade is a perfect substitute for international factor mobility in this model in the sense that it also equalizes factor prices

- Intuitively, this is because the countries export the goods which make intensive use of their abundant factor which is as if they exported the factor directly

- A related result is the so-called factor price insensitivity result which says that, for given goods prices, changes in factor endowments have no effect on factor prices

- Mathematically, this follows from the fact that the break even conditions can be solved for factor prices in terms of goods prices with no factor endowments appearing anywhere

# Classic results – Factor price equalization theorem (contd.)

- Needless to say, the prediction of factor price equalization is flatly rejected by the data. For example, wages in Switzerland are not the same as in Bangladesh

- The main culprit is again our assumption that countries have identical technologies which makes it a nonstarter empirically

- Interestingly, however, Zimring (2017) finds support of the factor prices insensitivity prediction in the West Bank study we discussed earlier

- In particular, wages in districts which received larger labor supply shocks due to the commuting ban did not fall faster than wages elsewhere in the West Bank

Gains from trade

- It can be shown that there are still gains from trade in the Heckscher-Ohlin model in the sense that the winners win more than the losers lose

- This means that it is possible for the winners to compensate the losers sufficiently for everyone to be better off

- Just as in the Ricardian model, the reason is that consumption possibilities are larger under trade than under autarky because production and consumption choices then no longer have to coincide

- We are now referring to the consumption possibilities of a representative household which collects all factor incomes in the economy

Gains from trade (contd.)
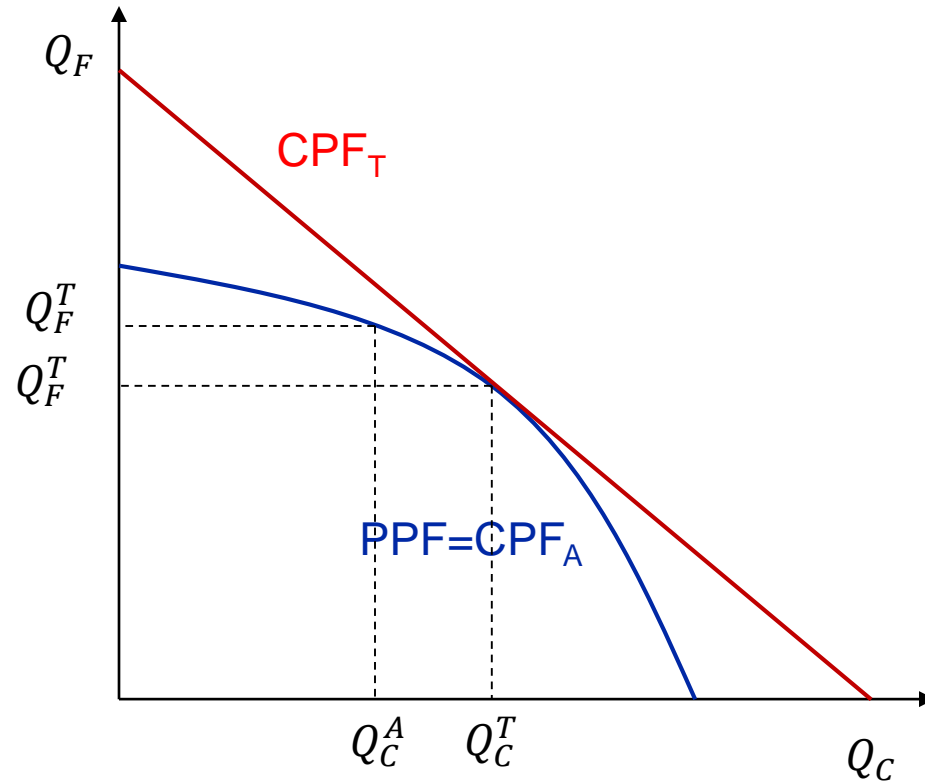
- Under autarky, consumption possibilities are limited by production possibilities so that CPF and PPF coincide

- Under trade, the consumption possibilities frontier is instead given by the isovalue line which is tangent to the PPF at the new production point

- This is simply because this line describes all the consumption bundles consumers can afford at trade prices

Conclusion

- We discussed the Heckscher-Ohlin model which analyzes the role of factor proportions in trade

- It delivers two main results: (i) countries export the goods which make intensive use of their abundant factors and (ii) the abundant factor gains from trade and the scarce factor loses from trade

References

- *J. Romalis. 2004. "Factor Proportions and the Structure of Commodity Trade". American Economic Review*

- *A. Zimring. 2017. "Testing the Heckscher-Ohlin-Vanek Theory with a Natural Experiment". Cornerstone Research Working Paper*