Lecture 9: General Equilibrium with Incomplete Markets

Florian Scheuer

1 Plan

• Think about general equilibrium in a world of incomplete markets. Both
  – households’ risky income stream and
  – the interest households’ earn on their savings

come from the aggregate production function

2 Model

2.1 Households

• Represent labor-income risk as a stochastic process for the effective supply of labour:
  \[ y^i = wl^i \]

where \( l^i \) is household-specific

• Shocks can represent
  – Unemployment
  – Shocks to individual ability
  – Shocks to the quality of the employer-employee match

• Households solve income-fluctuations problem taking \( w \) and \( R \) as given and constant

• State variables for individual household problem can be
  – \( A \) and \( s \)
or

\[ x \equiv RA + y(s) \text{ and } s \text{ as we did before} \]

- As long as \( \beta R < 1 \) and \( \lim_{c \to \infty} -\frac{u''(c)}{u'(c)} = 0 \), there will be an invariant joint distribution of \( A \) and \( s \) for given factor prices. We didn’t quite prove this, it’ll be part of the problem set. Call this

\[ F(A, s; R, w) \]

and denote the marginal distribution of \( A \) by

\[ F(A; R, w) \]

- Total assets held by all the individuals of the economy in steady state are:

\[ A(R, w) = \int A dF(A; R, w) \]

- Properties of \( A(R, w) \):

  - \( \lim_{R \to \frac{1}{\beta}} A(R, w) = \infty \)
  - \( \lim_{R \to 0^+} A(R, w) = -b \)
  - Not necessarily monotonic in \( R \)
2.2 Firms

- Competitive firms, so factor prices are

\[ w = F_L(K, L) \]

\[ R = F_K(K, L) + 1 - \delta \]

2.3 Market clearing

- Analyze steady state (transitional dynamics and aggregate shocks not easy to compute...)

- Aggregate labor supply is fixed (no aggregate risk)

\[ L = \sum_i l_i = 1 \]

- Capital market clearing:

\[ A(R, w) = K \]  \hspace{1cm} (1)

Equation (1) is the whole point of Aiyagari [1994]

- Express (1) in terms of capital supply and demand, i.e. call the LHS capital supply and the RHS capital demand

  – Demand:

\[ K^D(R) = F_K^{-1}(R - 1 + \delta) \]

  – Supply:

\[ K^S(R) = A(R, w(R)) \]

where

\[ w(R) = F_L\left(K^D(R), 1\right) \]

i.e. we also implicitly clear the labor market

- \( w(R) \) is decreasing \( \Rightarrow \) additional source of possible non-monotonicity in \( K^S(R) \)!

- (but monotone in most examples)

- Note that we immediately know that in equilibrium \( R < \frac{1}{\beta} \). Why?
2.4 Computation

An algorithm:

1. Guess $R$

2. Compute $K^D(R)$ and $w(R)$

3. Solve consumer’s dynamic programing problem ⇒ obtain policy function $c(x,s)$ or $c(A,s)$

4. Simulate the life of one consumer for many periods ⇒ obtain invariant distribution of assets

   - or, alternatively, compute the invariant distribution directly from the transition matrix of the consumer’s state

5. Use invariant distribution of assets to compute $K^S(R) = A(R, w(R))$

6. Adjust the guess of $R$ and repeat 1-5 until $K^S(R) = K^D(R)$

   - One way to update the guess of $R$ is $R^{NEW} = \mu R + (1 - \mu) [F_K(A(R) + 1 - \delta, 1)]$
2.5 Aiyagari’s calibration

- $\beta = 0.96$
- $F = K^\alpha$ with $\alpha = 0.36$
- $\delta = 0.08$
- $u = \frac{\sigma^{-1} - \sigma}{1 - \sigma}$, $\sigma = 1, 3, 5$
- Labour income process:

$$\log l_t = \rho \log l_{t-1} + \sqrt{(1 - \rho^2)}\epsilon_t$$

with $\rho = \{0, 0.3, 0.6, 0.9\}$ and $Var(\epsilon_t) = \{0.2, 0.4\}$, approximated with a discrete Markov chain

- $b = 0$
- Results:

| TABLE II |
|-----------------|-----------------|-----------------|
|                | A. Net return to capital in %/aggregate saving rate in % $\ (Var(\epsilon) = 0.2)$ |                |
| $\rho$         | $\sigma = 1$    | $\sigma = 3$    | $\sigma = 5$    |
| 0              | 4.1666/23.67    | 4.1456/23.71    | 4.0858/23.83    |
| 0.3            | 4.1365/23.73    | 4.0432/23.91    | 3.9054/24.19    |
| 0.6            | 4.0912/23.82    | 3.8767/24.25    | 3.5857/24.86    |
| 0.9            | 3.9305/24.14    | 3.2903/25.51    | 2.5260/27.36    |
|                | B. Net return to capital in %/aggregate saving rate in % $\ (Var(\epsilon) = 0.4)$ |                |
| $\rho$         | $\sigma = 1$    | $\sigma = 3$    | $\sigma = 5$    |
| 0              | 4.0649/23.87    | 3.7816/24.44    | 3.4177/25.22    |
| 0.3            | 3.9554/24.09    | 3.4188/25.22    | 2.8032/26.66    |
| 0.6            | 3.7567/24.50    | 2.7835/26.71    | 1.8070/29.37    |
| 0.9            | 3.3054/25.47    | 1.2894/31.00    | -0.3456/37.63   |
• Compare to full insurance:

\[
R - 1 = \frac{1}{\beta} - 1 = 4.17\%
\]

\[
\frac{s}{Y} = 23.67
\]

• The savings rate comes from:

\[
\alpha K^{\alpha - 1} + (1 - \delta) = \frac{1}{\beta}
\]

\[
\Rightarrow K = \left( \frac{\frac{1}{\beta} - 1 + \delta}{\alpha} \right)^{\frac{1}{\alpha - 1}}
\]

\[
sK^\alpha = \delta K
\]

\[
\Rightarrow s = \delta K^{1 - \alpha}
\]

\[
= \delta \frac{\alpha}{\frac{1}{\beta} - 1 + \delta}
\]

\[
= 0.08 \frac{0.36}{1.0417 - 1 + 0.08}
\]

• Conclusion: for middle-of-the-range parameters, not a big deal!

• Self-insurance quite effective: welfare gain of 14 percent of consumption compared to autarky (for a single individual, no GE effects)

• In the cross section, \( \text{Var}(c) < \text{Var}(y) < \text{Var}(A) \), which coincides with data. Quantitatively, model under-predicts wealth inequality, e.g. compared to top wealth shares in Piketty (2014), Saez-Zucman (2016)

• Is this the wrong model for the top of the wealth distribution?


• Carroll [1997]: The right calibration of this type of model has:

  – Temporary and permanent income shocks
  – \( \beta \ll \frac{1}{R} \)

  \( \Rightarrow \) “Buffer stock” behavior

  * Hold a small amount of assets to smooth temporary shocks
  * But not large because \( \beta R < 1 \)
* Do not smooth permanent shocks
* An individual’s consumption closely tracks their income

3 Welfare implications of market incompleteness

- Market incompleteness: 1st welfare theorem does not hold
- Formally, it comes from no single budget constraint (because it must hold state by state)
- Average consumption higher than with complete markets (unless past golden rule)
- Steady state utility may be higher than with complete markets (higher average consumption but more risk)
  - (but take into account transition)

3.1 Constrained efficiency

- Allocation not Pareto efficient
- When 1st welfare theorem holds, it’s a very strong result:
  - We don’t really think a planner could solve the “planner problem”
  - But even if they could, they still wouldn’t improve on the market
- When 1st welfare theorem doesn’t hold, should we conclude that the market does not allocate resources well?
- Clearly a planner who could solve the planner problem would do better
- But what if we made the planner less powerful?
- What is a fair comparison between what the markets and a hypothetical planner would do?
- Suppose we impose “the same” constraints on the planner as on the market
- What do we mean by “the same constraints”?
  - The planner can tell people how much to trade in the existing markets
  - The planner cannot create new markets
- Justification:
– Maybe there is a reason why markets are not complete
– Comparing equilibrium to first-best allocation may be too much to ask of the markets
– If we could prove that the planner cannot improve upon the market, this would be a constrained form of the 1st welfare theorem
– If we were persuaded that we could not fix the market incompleteness, there would be a good case not to intervene in the market

• “Constrained efficiency”
  – Could a planner make everyone be made better off by using only the existing markets?
  – In general, yes! Hart [1975], Stiglitz [1982], Geanakoplos and Polemarchakis [1986]
  – i.e. incomplete market allocations are not constrained efficient in general

• Constrained efficiency is more of a case-by-case definition of what powers we imagine the planner has than a generally-well-defined concept that can be used in any context.
  – Need to be careful in spelling out what powers the planner is assumed to have and what powers the planner does not.
  – Policy implications may or may not follow. i.e. to go from “constrained inefficiency” to “this policy should be pursued” one needs to make the case that the planner’s powers correspond to some plausible policy instruments.

• In insurance example, because marginal rates of substitution are not equalized, prices affect the degree of insurance. A planner could take that into account, while the market does not

• Davila et al. [2005] apply this reasoning to the Aiyagari [1994] model.

References


