Confusion, Polarization and Competition

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November 2018

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Introduction: Escaping the Bertrand Trap

- **IO 101**: Oligopolistic producers of *homogeneous* products often suffer from the “Bertrand trap.”

- Firms may want to escape the trap by confusing/deceiving consumers.
  - Fooling naive consumers by hiding add-on costs (Gabaix and Laibson, 2006; Heidhues et al., 2016).
  - Using complex price formats to impede comparison (Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013).
This paper: Do firms producing differentiated goods also benefit from consumer confusion?

The answer is not obvious, as firms are already earning positive profits in a transparent market.

In reality, differentiated product markets do not seem to be equally transparent.
Market Transparency: Examples

FREE from £26.00 per month...

- **Samsung Galaxy S3 Mini**
  - Compact design
  - S voice
  - Android Jelly Bean
  - 5 Megapixel camera
  - 4" touchscreen

- **HTC One SV**
  - 4G ready
  - 5 Megapixel camera
  - HD video record
  - Beats Audio
  - HTC Sync Manager

- **Nokia Lumia 820**
  - 4G ready
  - Windows phone
  - Wireless charging
  - 8 Megapixel camera
  - 4.3" OLED screen

FREE from £41.00 per month...

- **Samsung Galaxy S3 LTE**
  - 4G ready
  - Smart Stay
  - Voice commands
  - 8 Megapixel camera
  - HD video record

- **HTC Windows 8X**
  - Instant social updates
  - Windows phone
  - 1.5GHz processor
  - 4.3" touchscreen
  - Ultra-wide photos

- **Sony Xperia S**
  - 4.3" touchscreen
  - Scratch-resistant screen
  - Google Android 4
  - 1.5GHz processor
  - 12.1 Megapixel camera
Market Transparency: Examples
• We develop a general (and yet tractable) framework to study endogenous consumer confusion with differentiated products.

• In our model, firms can engage in some costless marketing activities before competing in prices.

• Consumers may incorrectly perceive the relative values of the products due to the confusion created by the mkt. activities.
Firms may not benefit from consumer confusion if the market features polarization.

- Polarization: consumers are more likely to have strong opinions than being indifferent.
- In this case, firms often prefer a transparent market.

In contrast, if the market features indecisiveness, firms typically would want the consumers to be confused.
• **Competition with boundedly rational consumers:**
  - Gabaix and Laibson (2006); Heidhues et al. (2016); Spiegler (2006); Piccione and Spiegler (2012); Chioveanu and Zhou (2013), etc.: homogeneous products.
  - **Our paper:** differentiated products.

• **Manipulating preference distributions with marketing & product design:**
  - **Our paper:** duopoly.
The Model: Preferences

- Two firms $i, j = 1, 2$ compete for a continuum of consumers.
- Each firm $i$ produces good $i$ at zero marginal cost.
- Each consumer has a true valuation $v_i$ for good $i$.
  - $(v_1, v_2) \sim F_0$ with density $f_0$ on $\mathbb{R}^2$.
- The true valuation difference $\Delta \equiv v_2 - v_1$ is distributed as
  \[
  G_0(x) = \int_{-\infty}^{+\infty} \int_{-\infty}^{x} f_0(v, v + \tau) d\tau dv,
  \]
  with the density function $g_0(x) = \int_{-\infty}^{+\infty} f_0(v, v + x) dv$. 
Example: The Hotelling Line

- Firm 1 locates at $x_1 = -\lambda$, and firm 2 is at $x_2 = \lambda$.

Density of consumer type $\theta$

- True valuations: $v_i = \mu - (x_i - \theta)^2$, where $\mu > 0$. 
Example: The Hotelling Line

- True valuation difference: \( \Delta \equiv v_2 - v_1 = 4\lambda \theta \).

- \( g_0(x) = h_0 \left( \frac{x}{4\lambda} \right), \ \forall x \in \mathbb{R} \).
General Preferences

(A1) $G_0$ is symmetric at zero.

(A2) $G_0$ is log-concave on $\text{supp}(g_0)$.

(A3) $g_0$ is continuous at zero and $g_0(0) > 0$. 
Two-Stage Competition

- **Stage 1:** Firms use marketing activities to influence consumer perception: $(v_1, v_2) \rightarrow (\tilde{v}_1, \tilde{v}_2)$.
  - $\Delta = \tilde{v}_2 - \tilde{v}_1$: perceived valuation difference.
  - A consumer is confused if $\Delta \neq \tilde{\Delta}$.

- **Stage 2:** Firms compete in prices for perceived tastes.
  - A consumer will buy from firm $i$ if $\tilde{v}_i - p_i > \tilde{v}_j - p_j$.

- **Solution concept:** Subgame Perfect Equilibrium (SPE).
• The set of feasible marketing activities: \( A \), with \( A \equiv A^2 \).

• The firms’ marketing choices jointly determine the dist. of \( \tilde{\Delta} \):

\[
\tilde{\Delta} = \Delta + \varepsilon_a, \quad \varepsilon_a \sim \Gamma_a, \quad \forall a = (a_1, a_2) \in A.
\]

• This is equivalent to the standard discrete choice specification

\[
\tilde{v}_1 = v_1 + \varepsilon'_a, \quad \tilde{v}_2 = v_2 + \varepsilon''_a, \\
\]

where \( \varepsilon'_a \) and \( \varepsilon''_a \) may be correlated.
(A4) \( \forall a \in A, \Gamma_a \) is symmetric at zero.

(A5) \( \forall a \in A, \Gamma_a \) is either degenerate or it has a density \( \gamma_a \) that is log-concave on \( supp(\gamma_a) \).
Remarks on (A4) & (A5)

- (A4) implies that no firm has a **systematic advantage** over the other, no matter whether the consumers are confused or not.
  - In this sense, obfuscation is **unbiased**.
  - However, $\mathbb{E}[\tilde{v}_i] \neq v_i$ is allowed.

- **Examples:** $A \subset \mathbb{N}$ and $\Gamma_a$ is

  (i) the uniform distribution on $[-(a_1 + a_2), a_1 + a_2]$, or

  (ii) the distribution of $\varepsilon_a = \underbrace{\xi_1 + \ldots + \xi_n}_{a_1 + a_2}$, where $\xi_k \sim U[-1, 1]$. 
Proposition 1

Suppose that (A1)-(A5) hold. Then, in every pricing subgame with \( a \in A \) there exists a unique symmetric pure-strategy equilibrium. In eq., each firm chooses \( p^*_a = \frac{1}{2g_a(0)} \), where

\[
g_a(0) = \int g_0(-\varepsilon) d\Gamma_a(\varepsilon).
\]
• $g_a(0)$: the mass of consumers who are \textit{indifferent} according to the \textit{perceived} tastes.

• Thus, firms benefit from confusing consumers if

$$g_a(0) < g_0(0) \equiv \int f_0(v, v) dv.$$ 

• Roughly speaking, obfuscation \textit{softens competition} if it creates \textit{spurious consumer loyalty}.
### Definition 1

Suppose $\delta > 0$ is such that $[-\delta, \delta] \subseteq \text{supp}(g_0)$.

(i) True prefs. are weakly $\delta$-polarized if $g_0(0) < g_0(x) \forall x \in (0, \delta]$.

(ii) True prefs. are weakly $\delta$-indecisive if $g_0(0) > g_0(x) \forall x \in (0, \delta]$.

**Remark.** If $[-\delta, \delta] = \text{supp}(g_0)$, then $\delta$ can be interpreted as a measure of product differentiation.
A weakly polarized preference with $\nu_2 = -\nu_1$. 
Weakly Indecisive Preferences: An Example

A weakly indecisive preference with $v_2 = -v_1$. 
(A6) The set $A$ is “rich enough”:

(i) $\exists a_0 \in A$ s.t. $\varepsilon_{a_0}$ is degenerate at zero (i.e., $\tilde{\Delta} = \Delta$ always).

(ii) $\forall a_j \in A$, $\exists a_i \in A$ such that $\varepsilon_{(a_i, a_j)} \neq \varepsilon_{a_0}$. 
Main Results: Equilibrium Confusion

**Theorem 1**

Suppose that (A1)-(A5) and (A6) hold. Further, suppose that \( \exists \delta > 0 \) s.t. \( \text{supp}(\gamma_a) \subseteq [-\delta, \delta] \) \( \forall a \in A \).

(i) If preferences are weakly \( \delta \)-polarized, then there exists an SPE without consumer confusion.

(ii) If preferences are weakly \( \delta \)-indecisive, then there does not exist an SPE without consumer confusion.
- With noises ($\varepsilon_a \neq \varepsilon_{a_0}$), some truly indifferent consumers ($\Delta = 0$) become having a strict opinion ($\tilde{\Delta} \neq 0$).

- Similarly, some consumers with strong opinions will be rendered indifferent.

- In a polarized market, the second effect dominates, and thus $g_a(0)$ will be minimized when $\varepsilon_a = \varepsilon_{a_0}$.

- In an indecisive market, the opposite logic applies.
Definition 2

Suppose $\delta > 0$ is such that $[-\delta, \delta] \subseteq \text{supp}(g_0)$.

(i) True preferences are $\delta$-polarized if $g_0$ is strictly $\uparrow$ on $[0, \delta]$.

(ii) True preferences are $\delta$-indecisive if $g_0$ is strictly $\downarrow$ on $[0, \delta]$. 
A polarized preference with $v_2 = -v_1$. 
An indecisive preference with $v_2 = -v_1$. 
(A6’) $A \subset \mathbb{R}_+$ is compact, and $\forall a \neq a'$ with $a \leq a'$, we have $\text{supp}(\gamma_a) \subseteq \text{supp}(\gamma_{a'})$, & $\forall e, e' \in \text{supp}(\gamma_{a'})$ with $e' > e \geq 0$, 

$$\gamma_{a'}(e) - \gamma_a(e) \geq 0 \implies \gamma_{a'}(e') - \gamma_a(e') > 0.$$ 

(a) $\gamma_{a'} \succ S.S.C. \gamma_a$, fixed supports. 

(b) $\gamma_{a'} \succ S.S.C. \gamma_a$, variable supports.
Theorem 2

Suppose that (A1) - (A5) and (A6’) hold. Further, suppose that \( \exists \delta > 0 \text{ s.t. } \text{supp}(\gamma_a) \subseteq [-\delta, \delta] \forall a \in A. \)

(i) A unique SPE exists if prefs. are \( \delta \)-polarized or \( \delta \)-indecisive.

(ii) In the case of polarization, consumer confusion is minimal, i.e., \( a_1^* = a_2^* = \min A. \)

(iii) In the case of indecisiveness, consumer confusion is maximal, i.e., \( a_1^* = a_2^* = \max A. \)
Weaker Orders on Distributions

- The sidewise single-crossing property is a rather incomplete order on distributions.

- Some applications may require considering weaker orders such as mean-preserving spread ($\Leftrightarrow$ SOSD).

- The uniqueness & monotonicity result can be extended to the order of MPS, at the cost of more restrictive preferences.
### Definition 3

Suppose $\delta > 0$ is such that $[-\delta, \delta] \subseteq \text{supp}(g_0)$.

(i) True preferences are strongly $\delta$-polarized if $g_0$ is strictly convex on $[-\delta, \delta]$.

(ii) True preferences are strongly $\delta$-indecisive if $g_0$ is strictly concave on $[-\delta, \delta]$. 

A strongly polarized preference with $\nu_2 = -\nu_1$. 
A strongly indecisive preference with $v_2 = -v_1$. 
(A6")  $A \subset \mathbb{R}_+$ is compact, and $\forall a \neq a'$ with $a \leq a'$, we have

$$\varepsilon_{a'} \overset{d}{=} \varepsilon_a + \eta,$$

where $\eta$ is non-degenerate and $\mathbb{E}[\eta | \varepsilon_a] = 0$. 
Theorem 3

Suppose that (A1) - (A5) and (A6") hold. Further, suppose that
\[ \exists \delta > 0 \text{ s.t. } \text{supp}(\gamma_a) \subseteq [-\delta, \delta] \quad \forall a \in A. \]

(i) A unique SPE exists if preferences are strongly \( \delta \)-polarized or strongly \( \delta \)-indecisive.

(ii) In the case of strong polarization, consumer confusion is minimal, i.e., \( a_1^* = a_2^* = \min A \).

(iii) In the case of strong indecisiveness, consumer confusion is maximal, i.e., \( a_1^* = a_2^* = \max A \).
Massive Obfuscation

- Theorems 1 - 3 require that the scope of obfuscation ($\text{supp}(\gamma_a)$) is limited by true product differentiation.

- If obfuscation can be massive (i.e., $\text{supp}(\gamma_a) \supset \text{supp}(g_0)$), then the true preferences may play no vital role.

- For example, suppose that $\varepsilon_a \sim U[-\omega, \omega]$, where $\omega \geq 0$.

- Quite generally, we have $\lim_{\omega \to +\infty} g_a(0) = 0$, and thus
  \[
  \lim_{\omega \to +\infty} p_a(0) = +\infty.
  \]

- Hence, regardless of the shape of true preferences, firms would want to obfuscate massively if possible.
Recall the Hotelling model with $x_1 = \lambda, x_2 = -\lambda$. Let $\lambda = 1$.

Consumer types $\theta \sim H_0$, with the following density function:

$$h_0(\theta) = \begin{cases} 
\alpha \theta^2 + \beta, & \text{if } \theta \in [-\lambda, \lambda], \\
0 & \text{otherwise},
\end{cases}$$

where $\beta = \frac{1}{2\lambda} - \frac{\alpha \lambda^2}{3}$ and $\alpha \in \left[ -\frac{3}{4\lambda^3}, \frac{3}{2\lambda^3} \right]$. 
Measure of Indecisiveness/Polarization

Density of $\theta$ (and $\frac{\Delta}{4\lambda}$)

![Graph showing density of $\theta$ and $\frac{\Delta}{4\lambda}$]

Polarized Distributions: $\alpha > 0$, $\lambda = 1$.

$\alpha = \frac{3}{4}$

$\alpha = \frac{1}{2}$
Measure of Indecisiveness/Polarization

Indecisive Distributions: \( \alpha < 0, \lambda = 1 \).
Proposition 2

Suppose that $\alpha \leq \hat{\alpha} \equiv (6 - 3\sqrt{3})/4\lambda^3$. Then, there exists a unique symmetric pure-strategy equilibrium in every pricing subgame, where each firm chooses the price $p^*_a = \frac{1}{2g_a(0)} \forall a \in A$.

Proof. $H_0$ is log-concave on if $\alpha \leq \hat{\alpha}$, and $G_0(\Delta) = H_0 \left( \frac{\Delta}{4\lambda} \right)$. □

- Hence, if $A$ and $\Gamma_a$ satisfy the previous stated assumptions, the general results apply.

- The locational structure also allows us to derive some comparative statics results about consumer welfare.
Proposition 3

Consider the Hotelling model. Suppose that $\varepsilon_{a^*} \sim U[-\omega_{a^*}, \omega_{a^*}]$, where $\omega_{a^*} > 0$. The expected welfare loss $L$ is

(i) strictly increasing in $\omega_{a^*}$,

(ii) strictly decreasing in $\alpha$ if $\omega_{a^*} < \hat{\omega} \equiv 64\lambda^2 / 15$, and

(iii) strictly increasing in $\alpha$ if $\omega_{a^*} > \hat{\omega}$. 
• When obfuscation is constrained to be small or moderate:

\[ L_{\text{Indecisive}} > L_{\text{polarized}} = 0. \]

• When obfuscation can be massive:

\[ L_{\text{polarized}} > L_{\text{Indecisive}} > 0. \]
Policy Intervention: Information Disclosure

• Suppose a regulating authority has some information that can help consumers perceive the product values correctly.

• With indecisive preferences, information disclosure always benefits consumers: better matches & lower prices.

• With polarized preferences, the effect is less clear:
  • If firms can educate consumers, they will do it already.
  • If they can’t, making the market transparent reduces mismatches but increases prices.
Policy Intervention: Outside Options

- Suppose the regulating authority can supply (or mandate the provision of) an “outside option” to the consumers.
  - E.g., basic v.s. additional insurances in Switzerland.

- In general, this can constrain the firms’ power of price setting.

- However, a potential downside is that some consumers may opt out even if it would have been efficient for them to buy.
Conclusion

- With differentiated goods, firms need not benefit from obfuscation, and it does not necessarily arise in equilibrium.

- With polarized consumers, obfuscation is often unattractive to firms.

- With indecisive consumers, obfuscation remains attractive and is bad from a welfare perspective.