Confusion, Polarization and Competition

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November 2018

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Introduction: Escaping the Bertrand Trap

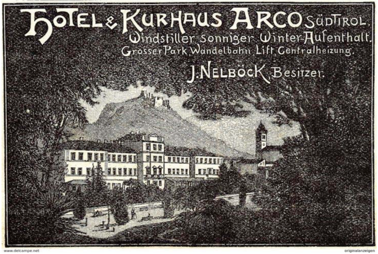
- **IO 101:** Oligopolistic producers of homogeneous products often suffer from the "Bertrand trap."
- Firms may want to escape the trap by confusing/deceiving consumers.
    - Fooling naive consumers by hiding add-on costs (Gabaix and Laibson, 2006; Heidhues et al., 2016).
    - Using complex price formats to impede comparison (Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013).

Introduction: Differentiated Goods

- **This paper:** Do firms producing differentiated goods also benefit from consumer confusion?
- The answer is not obvious, as firms are already earning positive profits in a transparent market.
- In reality, differentiated product markets do not seem to be equally transparent.

Market Transparency: Examples

Market Transparency: Examples



Preview: The Model

- We develop a general (and yet tractable) framework to study endogenous consumer confusion with differented products.
- In our model, firms can engage in some costless marketing activities before competing in prices.
- Consumers may incorrectly perceive the relative values of the products due to the confusion created by the mkt. activities.

Preview: Main Results

- Firms may *not* benefit from consumer confusion if the market features polarization.
  - Polarization: consumers are more likely to have strong opinions than being indifferent.
  - In this case, firms often prefer a transparent market.
- In contrast, if the market features indecisiveness, firms typically would want the consumers to be confused.

Literature

- **Competition with boundedly rational consumers:**
  - Gabaix and Laibson (2006); Heidhues et al. (2016); Spiegler (2006); Piccione and Spiegler (2012); Chioveanu and Zhou (2013), etc.: homogeneous products.
  - Our paper: differentiated products..
- **Manipulating preference distributions with marketing & product design:**
  - Johnson and Myatt (2006, AER): monopoly.
  - Our paper: duopoly.

The Model: Preferences

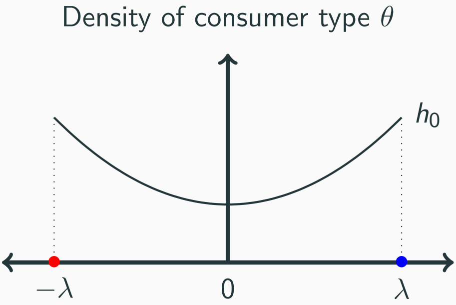
- Two firms $i, j = 1, 2$ compete for a continuum of consumers.
- Each firm i produces good i at zero marginal cost.
- Each consumer has a true valuation v_i for good i.
    - $(v_1, v_2) \sim F_0$ with density f_0 on $\mathbb{R}^2$.
- The true valuation difference $\Delta \equiv v_2 - v_1$ is distributed as

$$G_0(x) = \int_{-\infty}^{+\infty} \int_{-\infty}^{x} f_0\left(v, v + \tau\right) d\tau dv,$$

with the density function $g_0(x) = \int_{-\infty}^{+\infty} f_0(v, v + x) dv$.

Example: The Hotelling Line

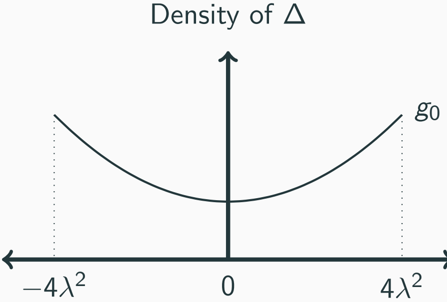
- Firm 1 locates at $x_1 = -\lambda$, and firm 2 is at $x_2 = \lambda$.



- True valuations: $v_i = \mu - (x_i - \theta)^2$, where $\mu > 0$.

Example: The Hotelling Line

- True valuation difference: $\Delta \equiv v_2 - v_1 = 4\lambda\theta$.



- $g_0(x) = h_0\left(\frac{x}{4\lambda}\right)$, $\forall x \in \mathbb{R}$.

General Preferences

(A1) G_0 is symmetric at zero.

(A2) G_0 is log-concave on $supp(g_0)$.

(A3) g_0 is continuous at zero and $g_0(0) > 0$.

Two-Stage Competition

- **Stage 1:** Firms use marketing activities to influence consumer perception: $(v_1, v_2) \longrightarrow (\tilde{v}_1, \tilde{v}_2)$.
    - $\tilde{\Delta} = \tilde{v}_2 - \tilde{v}_1$: perceived valuation difference.
    - A consumer is confused if $\tilde{\Delta} \neq \Delta$.
- **Stage 2:** Firms compete in prices for perceived tastes.
    - A consumer will buy from firm i if $\tilde{v}_i - p_i > \tilde{v}_j - p_j$.
- **Solution concept:** Subgame Perfect Equilibrium (SPE).

Marketing Activities I

- The set of feasible marketing activities: A, with $\mathcal{A} \equiv A^2$.

- The firms' marketing choices jointly determine the dist. of $\tilde{\Delta}$:

$$\tilde{\Delta} = \Delta + \varepsilon_{\mathbf{a}}, \ \ \varepsilon_{\mathbf{a}} \sim \Gamma_{\mathbf{a}}, \ \ \forall \mathbf{a} = (a_1, a_2) \in \mathcal{A}.$$

- This is equivalent to the standard discrete choice specification

$$\tilde{v}_1 = v_1 + \varepsilon'_{\mathbf{a}}, \ \ \tilde{v}_2 = v_2 + \varepsilon''_{\mathbf{a}},$$

  where $\varepsilon'_{\mathbf{a}}$ and $\varepsilon''_{\mathbf{a}}$ may be correlated.

Marketing Activities II

(A4) $\forall \mathbf{a} \in \mathcal{A}$, $\Gamma_{\mathbf{a}}$ is symmetric at zero.

(A5) $\forall \mathbf{a} \in \mathcal{A}$, $\Gamma_{\mathbf{a}}$ is either degenerate or it has a density $\gamma_{\mathbf{a}}$ that is log-concave on $supp(\gamma_{\mathbf{a}})$.

Remarks on (A4) & (A5)

- (A4) implies that no firm has a systematic advantage over the other, no matter whether the consumers are confused or not.
  - In this sense, obfuscation is unbiased.
  - However, $\mathbb{E}[\tilde{v}_i] \neq v_i$ is allowed.

- **Examples:** $A \subset \mathbb{N}$ and $\Gamma_{\mathbf{a}}$ is

  (i) the uniform distribution on $[-(a_1 + a_2),\, a_1 + a_2]$, or

  (ii) the distribution of $\varepsilon_{\mathbf{a}} = \underbrace{\xi_1 + \ldots + \xi_n}_{a_1+a_2}$, where $\xi_k \sim U[-1, 1]$.

Pricing Equilibrium: Existence and Uniqueness

Proposition 1

Suppose that (A1)-(A5) hold. Then, in every pricing subgame with $\mathbf{a} \in \mathcal{A}$ *there exists a unique symmetric pure-strategy equilibrium. In eq., each firm chooses* $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$ *, where*

$$g_{\mathbf{a}}(0) = \int g_0(-\varepsilon)d\Gamma_{\mathbf{a}}(\varepsilon).$$

Pricing Equilibrium and Endogenous Confusion

- $g_{\mathbf{a}}(0)$: the mass of consumers who are indifferent according to the perceived tastes.

- Thus, firms benefit from confusing consumers if

$$g_{\mathbf{a}}(0) < g_0(0) \equiv \int f_0(v, v) dv.$$

- Roughly speaking, obfuscation softens competition if it creates spurious consumer loyalty.

Weak Polarization and Indecisiveness

Definition 1

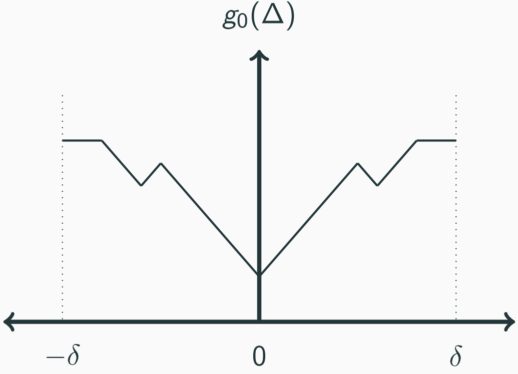
Suppose $\delta > 0$ *is such that* $[-\delta, \delta] \subseteq supp(g_0)$.

(i) *True prefs. are weakly* δ*-polarized if* $g_0(0) < g_0(x) \, \forall x \in (0, \delta]$.

(ii) *True prefs. are weakly* δ*-indecisive if* $g_0(0) > g_0(x) \, \forall x \in (0, \delta]$.

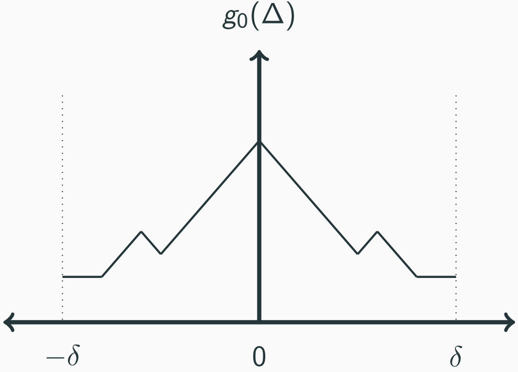
Remark. If $[-\delta, \delta] = supp(g_0)$, then δ can be interpreted as a measure of product differentiation.

Weakly Polarized Preferences: An Example



A weakly polarized preference with $v_2 = -v_1$.

Weakly Indecisive Preferences: An Example



A weakly indecisive preference with $v_2 = -v_1$.

A Richness Condition

(A6) The set A is "rich enough":

(i) $\exists\, \mathbf{a}_0 \in A$ s.t. $\varepsilon_{\mathbf{a}_0}$ is degenerate at zero (i.e., $\tilde{\Delta} = \Delta$ always).

(ii) $\forall a_j \in A$, $\exists\, a_i \in A$ such that $\varepsilon_{(a_i,a_j)} \neq \varepsilon_{\mathbf{a}_0}$.

Main Results: Equilibrium Confusion

Theorem 1

Suppose that (A1)-(A5) and (A6) hold. Further, suppose that $\exists\, \delta > 0$ s.t. $supp(\gamma_{\mathbf{a}}) \subseteq [-\delta, \delta]\ \forall \mathbf{a} \in \mathcal{A}$.

(i) *If preferences are weakly δ-polarized, then there exists an SPE without consumer confusion.*

(ii) *If preferences are weakly δ-indecisive, then there does not exist an SPE without consumer confusion.*

Intuition

- With noises ($\varepsilon_{\mathbf{a}} \neq \varepsilon_{\mathbf{a}_0}$), some truly indifferent consumers ($\Delta = 0$) become having a strict opinion ($\tilde{\Delta} \neq 0$).
- Similarly, some consumers with strong opinions will be rendered indifferent.
- In a polarized market, the second effect dominates, and thus $g_{\mathbf{a}}(0)$ will be minimized when $\varepsilon_{\mathbf{a}} = \varepsilon_{\mathbf{a}_0}$.
- In an indecisive market, the opposite logic applies.

Polarization and Indecisiveness

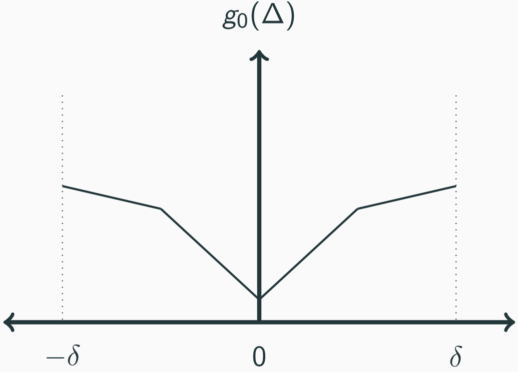
Definition 2

Suppose $\delta > 0$ *is such that* $[-\delta, \delta] \subseteq supp(g_0)$.

(i) *True preferences are* *δ-polarized* *if* g_0 *is strictly* $\uparrow$ *on* $[0, \delta]$.

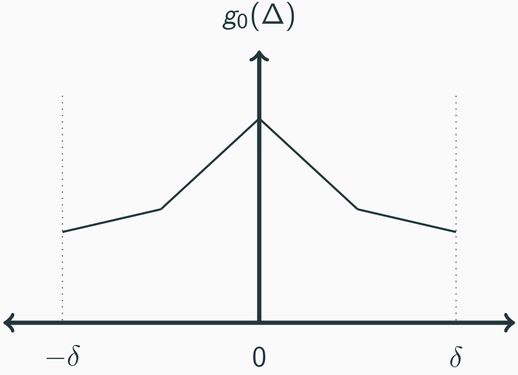
(ii) *True preferences are* *δ-indecisive* *if* g_0 *is strictly* $\downarrow$ *on* $[0, \delta]$.

Polarized Preferences: An Example



A polarized preference with $v_2 = -v_1$.

Indecisive Preferences: An Example

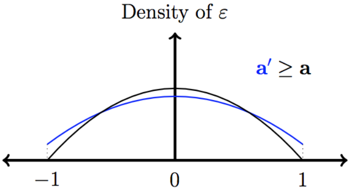


An indecisive preference with $v_2 = -v_1$.

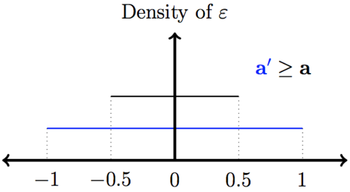
Sidewise Single-Crossing Property

(A6') $A \subset \mathbb{R}_+$ is compact, and $\forall \mathbf{a} \neq \mathbf{a}'$ with $\mathbf{a} \leq \mathbf{a}'$, we have $supp(\gamma_{\mathbf{a}}) \subseteq supp(\gamma_{\mathbf{a}'})$, & $\forall e, e' \in supp(\gamma_{\mathbf{a}'})$ with $e' > e \geq 0$,

$$\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) \geq 0 \implies \gamma_{\mathbf{a}'}(e') - \gamma_{\mathbf{a}}(e') > 0.$$



(a) $\gamma_{\mathbf{a}'} \succ_{S.S.C.} \gamma_{\mathbf{a}}$, fixed supports.



(b) $\gamma_{\mathbf{a}'} \succ_{S.S.C.} \gamma_{\mathbf{a}}$, variable supports.

Main Results: Uniqueness and Monotonicity I

Theorem 2

Suppose that (A1) - (A5) and (A6') hold. Further, suppose that $\exists\, \delta > 0$ *s.t.* $supp(\gamma_{\mathbf{a}}) \subseteq [-\delta, \delta]\ \forall \mathbf{a} \in \mathcal{A}$.

(i) *A unique SPE exists if prefs. are* δ*-polarized or* δ*-indecisive.*

(ii) *In the case of polarization, consumer confusion is minimal, i.e.,* $a_1^* = a_2^* = \min A$.

(iii) *In the case of indecisiveness, consumer confusion is maximal, i.e.,* $a_1^* = a_2^* = \max A$.

Weaker Orders on Distributions

- The sidewise single-crossing property is a rather incomplete order on distributions.
- Some applications may require considering weaker orders such as mean-preserving spread ($\Longleftrightarrow$ SOSD).
- The uniqueness & monotonicity result can be extended to the order of MPS, at the cost of more restrictive preferences.

Strong Polarization and Indecisiveness

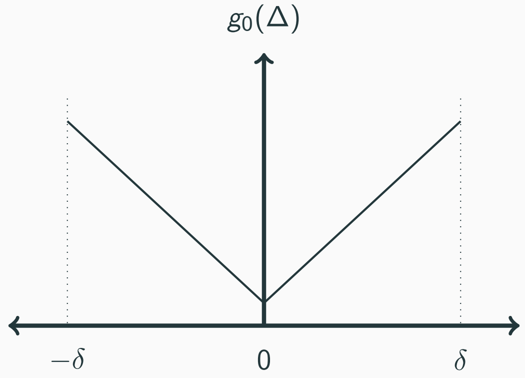
Definition 3

Suppose $\delta > 0$ *is such that* $[-\delta, \delta] \subseteq$ *supp*(g_0).

(i) *True preferences are strongly* δ*-polarized if* g_0 *is strictly convex on* $[-\delta, \delta]$.

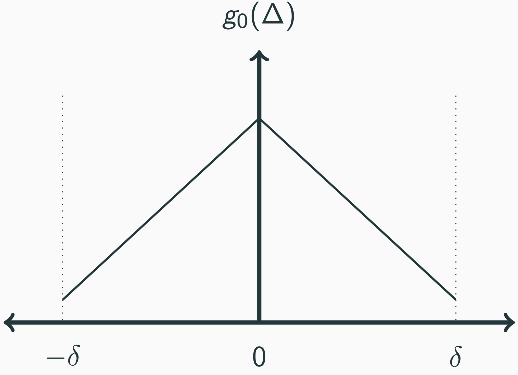
(ii) *True preferences are strongly* δ*-indecisive if* g_0 *is strictly concave on* $[-\delta, \delta]$.

Strongly Polarized Preferences: An Example



A strongly polarized preference with $v_2 = -v_1$.

Strongly Indecisive Preferences: An Example



A strongly indecisive preference with $v_2 = -v_1$.

Mean-Preserving Spread Property

(A6”) $A \subset \mathbb{R}_+$ is compact, and $\forall \mathbf{a} \neq \mathbf{a}'$ with $\mathbf{a} \leq \mathbf{a}'$, we have

$$\varepsilon_{\mathbf{a}'} \overset{d}{=} \varepsilon_{\mathbf{a}} + \eta,$$

where η is non-degenerate and $\mathbb{E}[\eta \,|\, \varepsilon_{\mathbf{a}}] = 0$.

Main Results: Uniqueness and Monotonicity II

Theorem 3

Suppose that (A1) - (A5) and (A6") hold. Further, suppose that $\exists\, \delta > 0$ *s.t.* $supp(\gamma_{\mathbf{a}}) \subseteq [-\delta, \delta]\ \forall \mathbf{a} \in \mathcal{A}$.

(i) *A unique SPE exists if preferences are strongly* δ*-polarized or strongly* δ*-indecisive.*

(ii) *In the case of strong polarization, consumer confusion is minimal, i.e.,* $a_1^* = a_2^* = \min A$.

(iii) *In the case of strong indecisiveness, consumer confusion is maximal, i.e.,* $a_1^* = a_2^* = \max A$.

Massive Obfuscation

- Theorems 1 - 3 require that the scope of obfuscation ($supp(\gamma_{\mathbf{a}})$) is limited by true product differentiation.

- If obfuscation can be massive (i.e., $supp(\gamma_{\mathbf{a}}) \supset supp(g_0)$), then the true preferences may play no vital role.

- For example, suppose that $\varepsilon_{\mathbf{a}} \sim U[-\omega, \omega]$, where $\omega \geq 0$.

- Quite generally, we have $\lim_{\omega \to +\infty} g_{\mathbf{a}}(0) = 0$, and thus

$$\lim_{\omega \to +\infty} p_{\mathbf{a}}(0) = +\infty.$$

- Hence, regardless of the shape of true preferences, firms would want to obfuscate massively if possible.

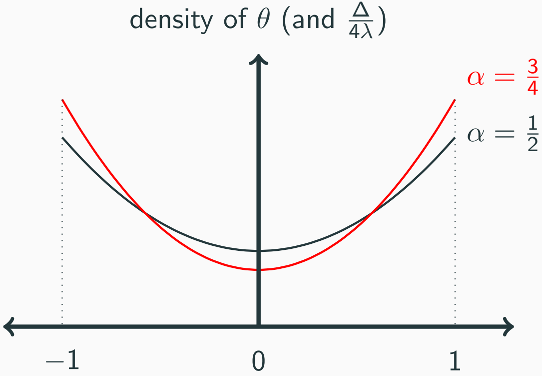
Application: Competition On the Line

- Recall the Hotelling model with $x_1 = \lambda, x_2 = -\lambda$. Let $\lambda = 1$.
- Consumer types $\theta \sim H_0$, with the following density function:

$$h_0(\theta) = \begin{cases} \alpha\theta^2 + \beta, & \text{if } \theta \in [-\lambda, \lambda], \\ 0 & \text{otherwise}, \end{cases}$$

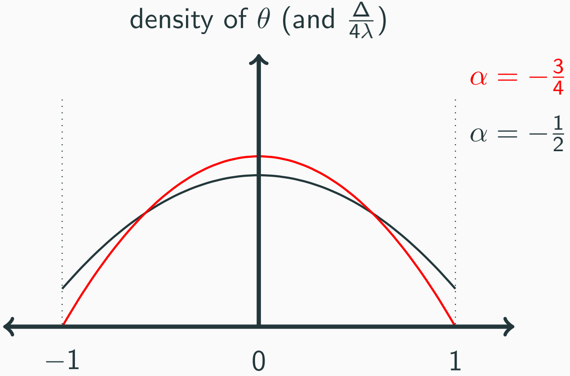
where $\beta = \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}$ and $\alpha \in \left[-\frac{3}{4\lambda^3}, \frac{3}{2\lambda^3}\right]$.

Measure of Indecisveness/Polarization



Polarized Distributions: $\alpha > 0$, $\lambda = 1$.

Measure of Indecisveness/Polarization



Indecisive Distributions: $\alpha < 0$, $\lambda = 1$.

Locational Model: Results

Proposition 2

Suppose that $\alpha \leq \hat{\alpha} \equiv (6-3\sqrt{3})/4\lambda^3$. *Then, there exists a unique symmetric pure-strategy equilibrium in every pricing subgame, where each firm chooses the price* $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}$ $\forall \mathbf{a} \in \mathcal{A}$.

PROOF. H_0 is log-concave on if $\alpha \leq \hat{\alpha}$, and $G_0(\Delta) = H_0\left(\frac{\Delta}{4\lambda}\right)$. □

- Hence, if $\mathcal{A}$ and $\Gamma_{\mathbf{a}}$ satisfy the previous stated assumptions, the general results apply.
- The locational structure also allows us to derive some comparative statics results about consumer welfare.

Welfare Analysis

Proposition 3

Consider the Hotelling model. Suppose that $\varepsilon_{\mathbf{a}^*} \sim U[-\omega_{\mathbf{a}^*}, \omega_{\mathbf{a}^*}]$, *where* $\omega_{\mathbf{a}^*} > 0$. *The expected welfare loss* L *is*

(i) *strictly increasing in* $\omega_{\mathbf{a}^*}$,

(ii) *strictly decreasing in* α *if* $\omega_{\mathbf{a}^*} < \hat{\omega} \equiv 64\lambda^2/15$, *and*

(iii) *strictly increasing in* α *if* $\omega_{\mathbf{a}^*} > \hat{\omega}$.

Welfare Analysis: Endogenous Confusion

- When obfuscation is constrained to be small or moderate:

$$L_{Indecisive} > L_{polarized} = 0.$$

- When obfuscation can be massive:

$$L_{polarized} > L_{Indecisive} > 0.$$

Policy Intervention: Information Disclosure

- Suppose a regulating authority has some information that can help consumers perceive the product values correctly.
- With indecisive preferences, information disclosure always benefits consumers: better matches & lower prices.
- With polarized preferences, the effect is less clear:
    - If firms can educate consumers, they will do it already.
    - If they can't, making the market transparent reduces mismatches but increases prices.

Policy Intervention: Outside Options

- Suppose the regulating authority can supply (or mandate the provision of) an "outside option" to the consumers.
  - E.g., basic v.s. additional insurances in Switzerland.
- In general, this can constrain the firms' power of price setting.
- However, a potential downside is that some consumers may opt out even it would have been efficient for them to buy.

Conclusion

- With differentiated goods, firms need not benefit from obfuscation, and it does not necessarily arise in equilibrium.
- With polarized consumers, obfuscation is often unattractive to firms.
- With indecisive consumers, obfuscation remains attractive and is bad from a welfare perspective.