Haircut Cycles

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Abstract

This paper contributes to the literature on the effect of financial frictions on business cycle activity. We follow the "leverage cycles" approach in the spirit of Geanakoplos (2010) which argues that equilibrium fluctuations in collateral rates (equivalently haircuts, margins, or leverage), rather than just in interest rates, are a key driver of persistent fluctuations in economic activity. In particular, we focus on how adverse economic shocks can be amplified and prolonged by endogenous variations in haircuts in the standard macrofinance framework à la Kiyotaki and Moore (1997). In our model, collateral constraints are motivated by no-recourse loans, and the interest rate and the haircut are jointly determined as general equilibrium objects. We highlight the difference between the risk and the illiquidity of the collateral in determining the credit market equilibrium: an increase in risk increases both the interest rate and the haircut, while an increase in illiquidity increases the haircut but decreases the interest rate. Compared with the previous literature, our model allows us to decompose the transmission of adverse shocks through the credit market into the interest rate channel and the haircut channel, and evaluate their relative importance. The numerical exercises illustrate that risk shocks can generate sizable business cycle fluctuations through the credit market, and the haircut channel is dominant in times of low market liquidity. (JEL D53, E13, E32, E44, G01)

1 Introduction

This paper studies how credit market activity interacts with aggregate economic activity over the business cycle. In particular, we consider how adverse economic shocks can be amplified and prolonged by equilibrium fluctuations in haircuts and interest rates. We are motivated by the following three stylized facts: (1) the credit spread is counter-cyclical, and the leverage is pro-cyclical; (2) the credit market risk is counter-cyclical, and the magnitude of its fluctuation roughly agrees with that of the credit spread and the leverage; (3) in some recessions the credit spread is tighter, while in others the leverage is tighter. These facts are highlighted by Figure 1, which plots the indices for the credit spread and the leverage[1] composed by the Chicago Fed

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[1]There is no available data on the economy-wide haircut (the reciprocal of supply-side leverage), the systematic bookkeeping of which is a policy recommendation in this paper. For the motivational purpose here, a measure of aggregate leverage suffices.

as part of the National Financial Conditions Index (NFCI).

The first observation suggests that credit markets tighten in response to macroeconomic conditions, depriving firms of credit exactly when this is most needed. The idea that this mechanism may be an important feature of economic crises stretches back to at least Irving Fisher (1933), who conjectures that in response to adverse economic shocks, "banks curtail loans for self-protection" (haircuts rise), "money interest on safe loans falls, but money interest on unsafe loans rises" (credit spreads widen), and they in turn feed back to the recession. Fisher's insight has been repeatedly confirmed by the later crisis episodes, especially in the recent Great Recession (Brunnermeier, 2009; Gorton and Metrick, 2012).

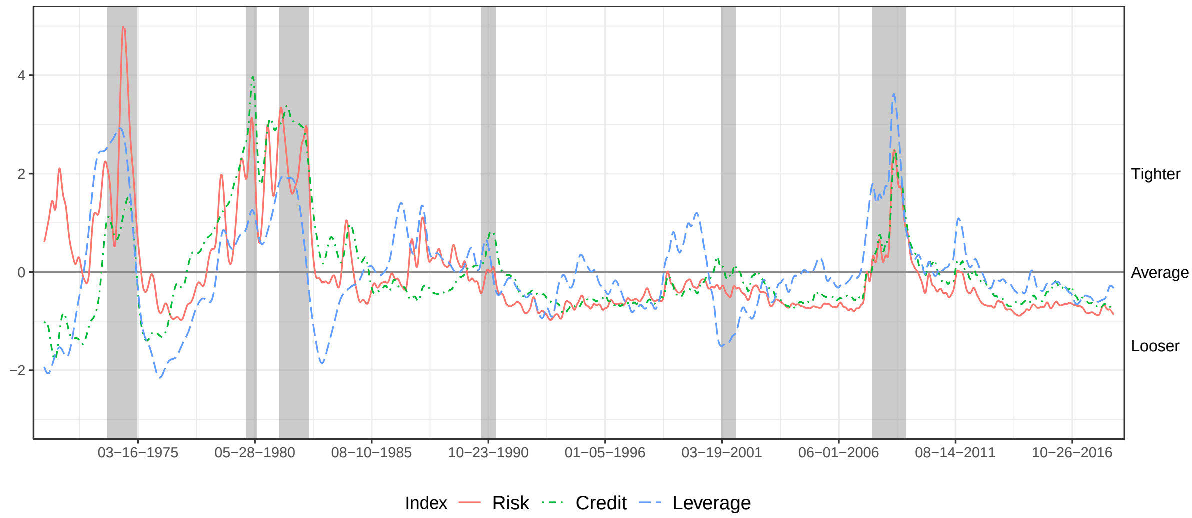


Figure 1: Credit Market Indices (Source: Chicago Fed NCFI)
The indices are based on 105 indicators of financial activity in the United States. Each is constructed to have an average value of zero and a standard deviation of one over the sample period. Positive values have been historically associated with tighter-than-average financial conditions, while negative values have been historically associated with looser-than-average financial conditions. More specifically, a positive value indicates higher-than-average credit spread and lower-than-average leverage. The shaded areas represent NBER recessions.

Despite its compelling nature, Fisher's idea has proven hard to formalize in the framework of standard business cycle theory. The difficulty comes from a peculiar feature of the credit market, as pointed out by Geanakoplos (2010): there is one good, the credit, but two prices, the haircut and the interest rate. Based on a simple price-taking equilibrium definition, equating the supply and demand for one good yields only one equation. To pin down a unique equilibrium, the state of the art approach is to treat haircut as being determined "outside" the credit market, either by exogenous shocks or by corporate finance techniques. Consequently, we have no satisfactory answers to the two natural questions concerning the understanding of fact (2) and (3): what adverse shocks drive the credit market prices? Whether the adverse shocks will manifest themselves as widening credit spreads or as surging haircuts? These questions are not only interesting for theoretical reasons, but also of crucial importance for policymakers. Recall that the Fed conducts monetary policy to support three specific goals: maximum sustainable employment, stable prices, and moderate long-term interest rates. If the movement of haircuts is at least as important as that of interest rates, should moderate long-term haircuts be the fourth goal of monetary policy?

This paper extends an otherwise standard business cycle model with a general equilibrium treatment of the credit market. This model can endogenously generate the comovements between the interest rate and the haircut as responses to risk shocks, and shows that the relative magnitude of their responses depends on the degree of market liquidity. As a presage, I first construct a two-period financial market model with one asset and two types of agents: the less productive households, and the productive entrepreneurs who have to borrow from the households. The credit market modeling follows the collateral general equilibrium tradition proposed by Geanakoplos (1997): there are a continuum of debt contracts, indexed by their haircuts, available at competitive prices (interest rates) on the credit market, and the agents choose their debt holdings as measures over the contract space. The contracts that are traded in nonzero quantity and their prices are determined in equilibrium. This paper differs slightly from Geanakoplos (1997) in that upon default, the lender suffers a proportional loss to the face value of the debt. This default cost can be interpreted as a measure of market illiquidity.

We show that the collateral equilibrium always exists, and the equilibrium is constrained efficient. The credit market prices can be obtained as the solution to a simple optimal contract problem, where the entrepreneurs maximize their return on equity subject to the households taking the equilibrium reservation return. The optimal contract problem yields two credit market equilibrium conditions, allowing us to determine both the interest rate and the haircut: one equation is the conventional interest rate Euler equation, which determines the households' expected return from the equilibrium debt contract; the other equation is a consensus valuation condition, which requires the marginal rates of substitution between the interest rate and the haircut to be equal between the households and the entrepreneurs. The latter is missing in a simple price taking equilibrium definition, which is not Walrasian when there are two prices and one good. In essence, the collateral equilibrium points out that in a market where the agents can earn leveraged return, the true object of interest is the return on equity, rather than the return on asset.

The most important parameter in the model is the risk of the asset. The reason is that collateralized debt contracts can be viewed as short put options on the borrower's asset (Merton, 1974), and option values depend crucially on the risk of the underlying due to their truncated payoff function. The comparative statics show that in the case where the entrepreneurs do not manage all the capital in the economy, when the risk increases, the equilibrium haircut and interest rate both increase to compensate the higher default risk. This prediction matches the empirical facts (1) and (2). Another important parameter for the model is the default cost: an increase in the default cost increases the haircut but decreases the interest rate. The reason is that higher haircut reduces the default probability, and therefore can act as an effective hedge against increases in default cost, while the case of the interest rate is the opposite. Therefore, the differential responses of haircuts and interest rates to risk shocks in different crisis episodes can be attributed to the variation in the degree of market illiquidity. This provides an explanation for fact (3).

Next, I extend the two-period model to infinite horizon to explore the impulse responses and propagation mechanisms. The dynamic model embeds the collateral equilibrium into the macrofinance framework of Kiyotaki and Moore (1997). Similar to the two-period model, there are two types of agents, productive entrepreneurs and less productive households. The aggregate capital is in fixed supply to abstract from aggregate capital accumulation. We also impose stochastic exit of entrepreneurs to avoid self-finance (Bernanke, Gertler and Gilchrist, 1999). We focus on the case of linear utility function to highlight the first-order importance of the option property of debt contracts in this model.

We study the impulse responses of the model to productivity shocks and risk shocks, and the role played by market illiquidity. Our model inherits the classic financial accelerator, where temporary adverse shocks have persistent effects on the economy by eroding the entrepreneurs' net worth, which takes time to rebuild. Its novelty lies in that it introduces a new channel, the haircut channel, in addition to the conventional asset price channel and credit spread channel, through which adverse shocks can trigger the finance accelerator. Since all the prices are endogenous, this model allows a clean decomposition of these three channels.

There are three important findings. First, the transmission of productivity shocks is mainly through the asset market in the same way as Kiyotaki and Moore (1997), while that of risk shocks mainly through the credit market. The intuition is that the asset price is a discounted sum of future dividends, so it is mostly affected by productivity shocks, and the debt contract is an option, so its price is mostly affected by risk shocks. Second, a positive risk shock increases both the equilibrium haircut and interest rate, reducing the entrepreneurs' borrowing capacity and increasing their borrowing cost. Moreover, the numerical examples show that the impulse responses to risk shocks are strong: a fifty percent increase in risk can cause a 2 percent output loss. Third, when the market is illiquid, the impact of risk shocks will be more loaded on the haircut. In our example, a fifty percent increase in default cost will offset most of the impact of risk shocks on the interest rate. Therefore, the haircut can be a much more precise indicator of financial market duress than the credit spread in some recessions. The work of Gorton and Metrick (2012) provides empirical evidence suggesting this was the case in the last recession. This paper highlights a mechanism that I believe is important for the understanding of the role of financial frictions in business cycle fluctuations. The above preliminary quantification supports my view, though a deeper quantitative exercise is deferred to future work.

The paper is structured as follows. Section 2 discusses related literature. Section 3 presents a two-period model highlighting the basic insights of the collateral equilibrium with productivity difference. Section 4 extends the two-period model to an infinite horizon macroeconomic model, and Section 5 illustrates the shock transmission mechanisms. Section 6 concludes.

2 Literature Review

The collateral general equilibrium was introduced by Geanakoplos (1997, 2003), Geanakoplos and Zame (2014). This equilibrium concept resolves the indeterminacy between the haircut and the interest rate in competitive credit markets. The modeling strategy typically requires that collateral acts a payment enforcement mechanism, and the agents are allowed to choose debt holdings as a measure over the contract space. The collateral equilibrium concept has been applied mostly in finite-periods belief-disagreement settings. Geanakoplos (2003, 2010), Fostel and Geanakoplos (2008, 2015) show that in a binomial economy, any collateral equilibrium is essentially equivalent to a no-default equilibrium. The binomial no-default theorem does not hold in alternative institutional settings. Geanakoplos (1997), Araujo, Kubler and Schommer (2012) provide examples where agents derive utility from the asset and there is equilibrium default. In Simsek (2013), where there are two types of agents, he shows that when their beliefs satisfy the hazard rate order, there exists an essentially unique equilibrium on a risky contract. Geerolf (2017) show that the complementarity of optimism between lenders and borrowers can generate Pareto distribution of firm leverages.

The pioneering works of Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997) laid the foundation for macrofinance. Take the latter as an illustration of the basic mechanism. There are two types of agents: productive entrepreneurs and less productive households. The entrepreneurs have to borrow from the households using their capital as collateral. A productivity shock depresses the asset price, erodes the net worth of entrepreneurs , and reduces the amount of capital managed by the entrepreneurs. This misallocation will be persistent, as net worth takes time to rebuild. Therefore, a recession is both amplified and prolonged by the financial friction.

The tightness of collateral constraints, or the haircut, is crucial for the performance of macrofinance models. There are several ways to pin down haircuts in the literature. The first is to treat the haircut as an exogenous constant or process. Among this literature, Kiyotaki and Moore (1997), Moll (2014), Midrigan and Xu (2014) take the haircut as an exogenous constant; Jermann and Quadrini (2012), Khan and Thomas (2013), Huo and Rios-Rull (2015) take it as an exogenous shock. The second is by a moral hazard constraint, including Bernanke, Gertler and Gilchrist (1999), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler and Kiyotaki (2015), Arellano, Bai and Kehoe (2016). Moral hazard constraints normally link the size of the debt to the firms' franchise value. During recessions, the firms' current asset value is lower compared with the franchise value, making adverse choices less profitable. This creates looser credit constraint during recessions and thereby the unappealing feature of countercyclical leverage (procyclical haircut), which has to be amended by other frictions if leverage is a target.

The third is to use other corporate finance techniques. For example, Bianchi, Ilut and Schneider (2018) assumes that the cost of debt is increasing in the face value, which has a similar flavor to the classical tradeoff between bankruptcy costs and tax benefits. As this approach creates constant leverage, equity adjustment cost is imposed to make sure the leverage is time-varying. The fourth is to generate risk-averse behavior of the entrepreneurs, and utilize the link between leverage and risk, as in Brunnermeier and Sannikov (2014). These two approaches determine the demand-side leverage, but still has to face the indeterminacy between the haircut and the interest rate in the credit market, or a flat credit surface. The collateral equilibrium approach used in this paper is supply-side, and therefore complements the existing literature. A simple question helps us better appreciate the difference between demand-side and supply-side leverage: during a recession, would the entrepreneurs be better-off if they were allowed to keep the pre-crisis leverage? The answer is yes for supply-side models but no for demand-side models. In other words, supply-side leverage variation acts as a shock amplifier, while demand-side leverage adjustment acts as a shock absorber.

There is a burgeoning literature studying the amplification of risk shocks by financial frictions. Among them, this paper is most similar to Christiano, Motto and Rostagno (2014), which also relies on the option property of collateralized debt contract. They create a short put option by allowing the firms to default when the realization of firm productivity is too low in a BGG framework. They find that risk shock is the most important shock in a macro model with financial frictions. Other papers also support the synergy between financial frictions and uncertainty in general, including Alfaro, Bloom and Lin (2016); Arellano, Bai and Kehoe (2016); Bianchi, Ilut and Schneider (2018).

Variation in firms' borrowing capacity has been shown to contribute significantly to the observed dynamics of real and financial variables. In Jermann and Quadrini (2012), firms' borrowing capacity is limited by an enforcement constraint which is subject to exogenous disturbances, which are called "financial shocks". They constructed the shock series using a residual approach, and found in a DSGE setting that financial shocks account for almost half of the volatility of output and 30 percent of the volatility of working hours. Khan and Thomas (2013) show that the economy's response to financial shocks is both quantitatively and qualitatively different from the response to productivity shocks, and their model can capture several aspects of the recent recession.

This paper also relates to the literature that uses wedges to study business cycle and financial frictions. The key idea is that financial friction manifests itself in the Euler equation of interest rate as an investment wedge, and this wedge becomes a sufficient statistic of the aggregate implication of financial frictions. The results are mixed: Chari, Kehoe and McGrattan (2007) reject the quantitative importance of the investment wedge, while Justiniano, Primiceri and Tambalotti (2010, 2011) support it; Buera and Moll (2015) show a financial shock is qualitatively isomorphic to a TFP shock. This paper shows that the collateral equilibrium concept is able to find an additional condition besides the interest rate Euler equation, and the accounting exercise should also account for the distortion of haircuts.

At last, this paper makes the first step to fill the gap between the macro literature and empirical studies of the last recession. In particular, Gorton and Metrick (2012) show that bilateral repo haircuts spiked between 2007 and 2009, leading to a severe credit crunch. They conjecture this "repo run" mechanism was at the nexus of the recession. Krishnamurthy, Nagel and Orlov (2014) and Copeland, Martin and Walker (2014) present differing evidence from the tri-party repo market, yet the increase in haircut in the overall repo market is largely uncontroversial. Note that the repo haircut is unequivocally a supply-side variable, with which cannot be dealt by the demand-side leverage models.

3 Two-period Model

In this section I present a simple two-period general equilibrium financial markets model where the interest rate and the haircut are endogenously determined, and establish the existence and efficiency of the general equilibrium.

3.1 Basic Environment

Our economy lasts for two time periods, 0 and 1. There are two types of agents , entrepreneurs and households, indexed by e and h, respectively. Each type of agents has a continuum of measure one so we do not need to distinguish between aggregate and individual variables. The entrepreneurs and households are all risk neutral and have a discount factor 1.

There is one productive asset in the economy, which we call capital. An entrepreneur is endowed with $k_0^e > 0$ units of capital, and a household is endowed with k_0^h units of capital. Let $k_0^e + k_0^h = 1$ so k's are also the capital shares held by each type. When held by an entrepreneur, a unit of asset generates Q_1 units of consumption at $t = 1$. Q_1 is stochastic and has c.d.f. F and p.d.f. f, with support $\mathbb{R}_+$. Moreover, I assume that f is strongly unimodal and continuously differentiable. When held by a household, a unit of asset generates $Q_1 - \kappa$ units of consumption, where $\kappa > 0$ represents the household's inefficiency in managing capital. Productivity difference makes sure that there is nonzero credit supply in equilibrium. At the end of $t = 1$ the agents consume everything.

3.2 Credit Constraint in General Equilibrium

There are two financial markets at time 0. The first is a spot asset trading market with asset price Q. The second is a credit market where borrowing must be collateralized by physical capital. The modeling of the credit market follows the general equilibrium tradition of Geanakoplos (1997). In this approach, all the debt contracts are traded as commodities in anonymous trading markets, where the prices of contracts are determined competitively. Since any debt contract can be uniquely characterized by its haircut $h \in [0, 1]$ subject to the normalization by the amount of collateral, the space of debt contracts is isomorphic to the unit interval $[0, 1]$. In this paper, we only consider non-contingent debt and rule out short selling[2]. Therefore, the price, or the interest rate of a debt contract, can be designated as $R(h)$.

By signing a collateralized debt contract, the creditor is de facto short a put option on the collateral. Since the debt is no-recourse, the gross return of debt contract h of unit face value at $t = 1$ is given by[3]

$$\min\left\{R(h), \frac{Q_1}{(1-h)Q}\right\}.$$

In Figure (2a), we plot as the solid line the gross return of a debt contract against the underlying price at $t = 1$. The strike price is the contractual payment $R(1 - h)Q$, above which the return is capped at R, and below which the return is $Q_1/(1 - h)Q$. Higher interest rate and haircut are both desirable to the lender, albeit in different manners: higher interest rate increases his exposure to up risk, while higher haircut reduces his exposure to down risk, as shown in Figure (2b). The option property of a collateralized debt contract foreshadows the importance of risk shocks in the dynamic model.

[2]For general treatments on richer contracts, the readers are referred to Geanakoplos and Zame (2014) and Simsek (2013).

[3]This framework captures many real-world debt contracts. For instance, in the repo market, $R - 1$ can be thought of as the repo rate and h the haircut; in the mortgage market, $R - 1$ can be thought of as the mortgage rate and h the downpayment ratio; in the corporate debt market, $R - 1$ can be thought of as the coupon rate and h the equity ratio.

In the rest of the paper, we further impose that default is costly to the lender. Upon default, the lender suffers a default cost which is proportional to the loan size, with the proportionality constant ξ. The default cost can be interpreted as market illiquidity, which can come from, for instance, transaction costs of houses, as in the mortgage market; bankruptcy costs of firms, as in the corporate debt market; transportation costs of physical collateral delivery, as in the repo market. The importance of liquidity in determining the haircut is well known in industry practices. For example, the article "What are haircuts?"on the ECB website explains that "the size of a haircut ... will depend on ... how risky that type of asset is, i.e., how volatile its price is, and how 'liquid' it is, i.e., how easy it is to sell it quickly without a loss of value"[4]. We can even find evidence dating back to the 18th century, in the Guidebook for Employees of a Tsing Dynasty Chinese pawnshop[5].

As will be articulated later, default cost happens to help generate inner solution equilibria, which are desirable in most real-world applications. The return to a debt contract when the lender takes into account the default cost is shown in Figure 2c and 2d. Default cost modifies the tradeoff between the interest rate and the haircut, rendering higher interest rate less desirable than under costless default.

Next we define the agents' debt holdings. Let $\mathcal{D}([0,1])$ denote the space of positive finite Borel measures over $[0,1]$. Agents j's debt holdings δ_+^j and δ_-^j are elements of $\mathcal{D}([0,1])$, where $j \in \{h, b\}$. The measure δ_+^j represents agent j's positive positions on contracts, or the contracts through which j lends. The measure δ_-^j represents agent j's negative positions on contracts, or the contracts through which j borrows. The price function $R : [0,1] \to \mathbb{R}_+$ is restricted to be Borel measurable and bounded. With debt holdings δ_+^j, δ_-^j, agent j's next period asset holding is given by

$$Qk_1^j = Qk_0^j - \int_0^1 d\delta_+^j + \int_0^1 d\delta_-^j. \tag{1}$$

The borrowing is subject to the collateral constraint

$$\int_0^1 \frac{1}{1-h} d\delta_-^j \leq Qk_1^j, \tag{2}$$

which says that all the asset of agent j can be pledged as collateral. This constraint can be viewed as a generalization of Kiyotaki and Moore (1997), where h is exogenous and constant for all contracts.

To keep notations simple, denote the households' gross return from direct capital holding by

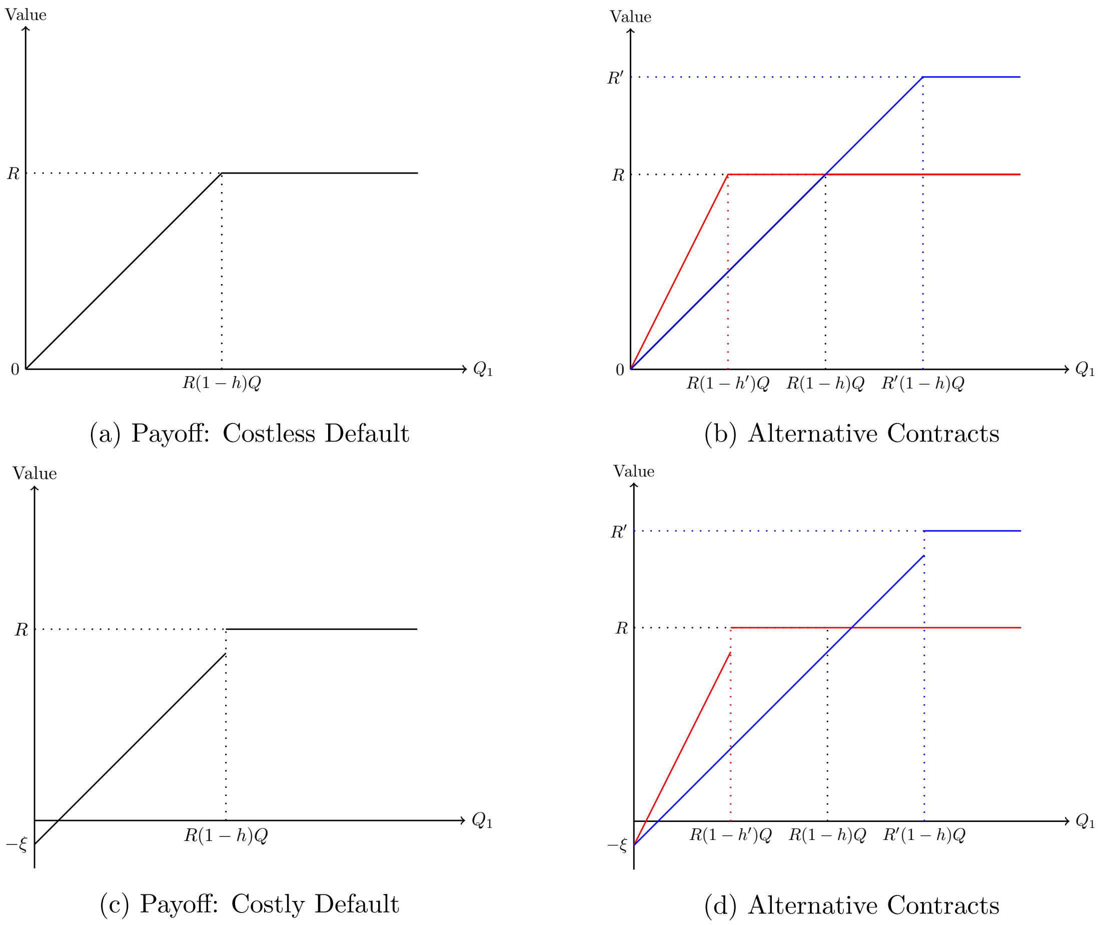
$$R^h \equiv \frac{Q_1 - \kappa}{Q},$$

the entrepreneurs' gross return from direct capital holding by

$$R^e \equiv \frac{Q_1}{Q},$$

[4] https://www.ecb.europa.eu/explainers/tell-me-more/html/haircuts.en.html.

[5] "If (the clothes being pledged) is common, easy to sell, it is fine to lend even if the realized return from liquidation will be low; if it is damaged silk clothes, and hard to modify, lend less even when the realized return from liquidation will be high." The Pawnshops of Eighteenth-Century Huizhou Merchants: The Pawnshops of the Wu Family from Mingzhou, Xiuning, Feng Yuejian.



(a) Payoff: Costless Default (b) Alternative Contracts

(c) Payoff: Costly Default (d) Alternative Contracts

Figure 2: Option Property of a Collateralized Debt Contract

the gross return from debt by

$$R^d \equiv \min\left\{R, \frac{Q_1}{(1-h)Q}\right\},$$

and the threshold of default by

$$\tilde{Q} = R(1-h)Q.$$

Agent j's objective is to maximize his expected net worth at the end of time 1

$$\max_{k_1^j, \delta_+^j, \delta_-^j} \quad \mathbb{E}[R^j Q k_1^j] + \mathbb{E}\left[\int_0^1 (R^d(h) - \mathbb{I}_{\tilde{Q}>Q_1}\xi) d\delta_+^j\right] - \mathbb{E}\left[\int_0^1 R^d(h) d\delta_-^j\right], \tag{3}$$

subject to (2) and (1),

where $\mathbb{I}$ is the indicator function.

Definition 1 (General Equilibrium)**.** A *general equilibrium* consists of asset and debt contract holdings $(k_1^j, \delta_+^j, \delta_-^j)_{j\in\{h,b\}}$, the asset price $Q \in \mathbb{R}_+$, the credit market price $R : [0,1] \to \mathbb{R}_+$, such that the asset and debt holdings solve problem (3) subject to (2) and (1), the asset market clears, $\sum_{j\in\{h,b\}} k_1^j = 1$, and the debt market clears, $\sum_{j\in\{h,b\}} \delta_+^j = \sum_{j\in\{h,b\}} \delta_-^j$.

3.3 Solution to the General Equilibrium

In this subsection I first prove that the general equilibrium always exists and is constrained efficient. Then I proceed to show how the equilibrium can be found by solving a simple optimal contract problem. At the end, some comparative statics are presented to illustrate how the equilibrium outcomes respond to the volatility in asset price and the magnitude of default cost.

Theorem 1 (Existence & Constrained Efficiency of GE)**.** Under the specified assumptions on utility, technology, and institution, there always exists a general equilibrium, and the equilibrium is Pareto efficient. There exists $v^h \in [1, \mathbb{E}R^e]$, such that the equilibrium credit market prices are the solution to the following optimal contract problem

$$\begin{aligned} v^e = \max_{R\in[1,\infty),h\in[0,1]} \quad & \frac{\mathbb{E}R^e - \mathbb{E}R^d(1-h)}{h} \\ \text{s.t.} \quad & \mathbb{E}R^d - F(\tilde{Q})\xi = v^h, \end{aligned} \tag{4}$$

where $Q = \mathbb{E}[Q_1 - \kappa]$.

As shown in the formal proof in Appendix B, the objective function of problem (4) is the entrepreneur's return on equity (ROE) when their collateral constraint is binding, and the budget constraint requires the households' expected return from debt minus default cost to be equal to v^h. v^h measures the market power of the households: when $v^h = 1$, the households have zero market power as they make zero profit in the credit market compared with direct capital holding; when $v^h = \mathbb{E}R^e$, the households have all the market power as the entrepreneurs earn zero profit in the credit market compared with direct capital holding. In the macro-finance literature, the credit market equilibrium condition is typically the constraint of problem (4) alone, in the form of some Euler equations. Essentially, the collateral equilibrium formulation allows the agents to make decisions based on the return on equity, rather than the return on asset, in a market where the agents can earn leveraged return.

We illustrate problem (4) graphically on the R-h plane in Figure 3. The budget constraint is represented by the red line, and it moves towards the northeast as v^h increases. The indifference curve for the objective function is shown as the blue line, and the improving direction is southwest. The solution to the problem is the point where the entrepreneurs' indifference curve is tangent to the households' participation constraint. The reason why the tangent point can be on the upper quadrant is the default cost, the discussion of which we postpone until the Remark in section 3.3.2. It is worth commenting that the existence and efficiency results are shown to hold under more general settings in Geanakoplos (1997).

It can be shown that the objective function of problem (4) is not concave, and the budget set including the improving region is not convex for most parameters of interest. In general, depending on the shape of F, the problem can have multiple solutions. To simplify the analysis, I will first restrict attention to the region of the parameter space in which the solution is unique. The general case is analyzed in Appendix E.

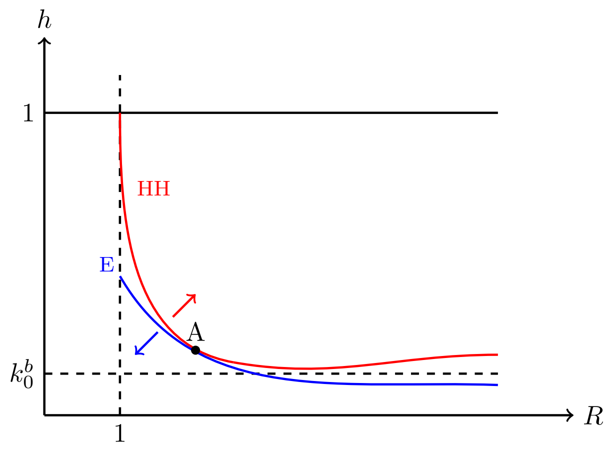


Figure 3: Illustration of problem (4)

Theorem 2. Assume that problem (4) permits a unique[6] solution $(R^{**}(v^h), h^{**}(v^h))$. There exists an essentially unique general equilibrium where

1. Q^* is uniquely determined by $\mathbb{E}R^h = 1$;
2. only one contract h^* is traded in non-zero quantity in equilibrium and its price R^* is uniquely determined;
3. The equilibrium credit market prices (R^*, h^*) are found by solving program (4):
   (a) if $h^{**}(1) > k_0^e$, $(R^*, h^*) = (R^{**}(1), h^{**}(1))$;
   (b) otherwise, the equilibrium haircut is $h^* = k_0^e$. Let v^{h*} be such that $h^{**}(v^{h*}) = k_0^e$. The equilibrium interest rate R^* can be solved from the constraint of program (4) by setting $h = h^*$ and $v^h = v^{h*}$.

There are two possible cases for the credit market equilibrium. In case (a), the equilibrium haircut is higher than the entrepreneurs' initial capital share, and the households hold some capital in equilibrium. The equilibrium features *loose credit supply*, and the households make zero profit in the credit market. Here the general equilibrium is equivalent to a principal-agent equilibrium where the entrepreneurs have all the market power. Note that in this case the equilibrium allocation of market power is the same as Kiyotaki and Moore (1997), albeit there the haircut is not a choice variable.

In case (b), the equilibrium haircut is equal to the entrepreneurs' initial capital share, and the households do not hold any capital in equilibrium: the equilibrium features *tight credit supply*. In this case, the entrepreneurs' aggregate endowment share is large, and competition among them drives up the cost of credit until there is no more excess credit demand. The equilibrium is still constrained efficient, but the households possess some market power and earn positive profits. The uniqueness of equilibrium in both cases is guaranteed by the assumption.

[6]Ideally, since the property of the solutions of problem (4) depends entirely on F, we would like to have assumptions on F to guarantee that the assumption in Theorem 3 holds. Unfortunately, the fact that objective function (4) is non-concave and the default cost setting render this task difficult.

The asset price in this model is always equal to the households' marginal productivity. The reason is that collateral constraint (2) accidentally guarantees that the asset supply is always abundant in a one-asset economy: the entrepreneurs can only purchase as much asset as they can borrow from the households. Alternative forms of collateral constraints can introduce interesting asset pricing behavior but will not be the focus of this paper[7].

3.3.1 Comparative Statics

Conventional wisdom suggests that higher haircuts and higher interest rates are in favor of the lender, and they should both occur to compensate the increase in expected loss when the quality of the collateral worsens, i.e., when the risk or the illiquidity of the collateral increases. However, our model shows that risk and illiquidity have different roles in determining the credit market equilibrium: while in most times an increase in risk increases both the interest rate and the haircut, an increase in illiquidity increases the haircut but decreases the interest rate.

In Figure 4 we show some comparative statics of the collateral equilibrium by a numerical example, and the parameters are listed in Table 1. We choose Q_1 to be lognormally distributed with parameters μ and σ. The production inefficiency is around 2.5% of the expected asset value to the entrepreur. The parameters of interests, σ and ξ, varies from 0.05 to 0.12 and from 0.03 to 0.07, respectively. We numerically verified that the uniqueness assumption is satisfied under the chosen parameters.

μ	3.6
κ	1
ξ	0.03-0.07
σ	0.05-0.12
k_0^e	0.05 & 0.1

Table 1: Parameters for Comparative Statics

The solid line in Figure 4 shows how the credit variables change against σ_Q when $k_0^e = 0.05$ and $\xi = 0.05$. For all $\sigma \in [0.05, 0.12]$, $h^{**}(1)$ is larger than k_0^e, so we are in the loose credit supply case (a). As predicted, due to the option property of the debt contracts, the volatility of the asset price is important for the credit market equilibrium. As risk decreases, R and h both decrease due to the lower default risk. The entrepreneurs' value per net worth increases along the process, while the households' value per net worth remains constant.

When $k_0^e = 0.1$, the comparative statics, represented by the dashed line in Figure 4, is more delicate. As risk decreases, the model behaves the same as when $k_0^e = 0.05$ until $h^{**}(1)$ drops to k_0^e, upon which we enter the tight credit supply case (b). After this point, as risk further decreases, the borrowing cost has to rise to offset the entrepreneurs' excess credit demand. A decrease in σ will exert two forces of opposite direction on the interest rate: a downward pressure as the default risk is lower, and an upward pressure as it increases the excess credit demand and thus the competition between entrepreneurs. The competition effect reduces the

[7] For example, a more literal generalization of the Kiyotaki and Moore (1997) collateral constraint takes the form $\int_0^1 \frac{R(h)}{1-h} d\delta_-^j \leq \mathbb{E}Q_1 k_1^j$. It can be easily verified that in case (b), $Q^* = \mathbb{E}[Q_1 - \kappa]$ leads to excess asset demand, and cannot be the equilibrium asset price. To solve for the equilibrium, we need to equate the asset demand derived from the budget constraint (1) to the asset supply of the household, and jointly solve Q^* and (R^*, h^*).

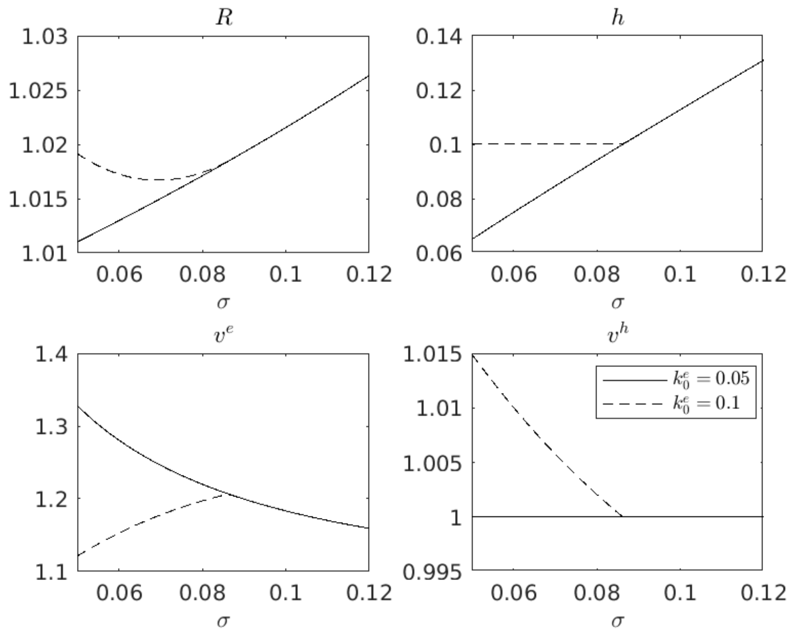


Figure 4: Comparative Statics on Risk

entrepreneurs' value per net worth, but increases the households' value per net worth. When σ is sufficiently small, the interest rate actually increases as σ decreases since the competition effect starts to dominate the risk-hedging effect.

Similarly, the solid line in Figure 5 shows how the credit variables change against ξ when $k_0^e = 0.05$ and $\sigma = 0.085$. For all $\xi \in [0.03, 0.07]$, $h^{**}(1)$ is larger than k_0^e, so we are in the loose credit supply case (a). As default cost decreases, h decreases as the expected loss upon default becomes smaller. The entrepreneurs' value per net worth increases along the process, while the households' value per net worth remains constant. The less intuitive phenomenon is that the interest rate increases as default cost decreases, against the conventional wisdom that credit spreads should widen when illiquidity is high. The reason is that in this model, the interest rate is such an ineffective hedge against default cost that the entrepreneurs would rather sacrifice their valuable leverage in times of low market liquidity. When $k_0^e = 0.1$, we reach case (b) when ξ is small enough, and the competition effect kicks in. The comparative statics is similar to that of risk and will not be repeated here.

The findings in Figure 4 and Figure 5 imply that in certain crisis episodes when a risk shock is accompanied by an illiquid financial market, the haircut can increase much while the interest rate moves little. We can find at least two empirical documentations of such crises in the existing literature.

The first example is the recent crisis. Gorton and Metrick (2012) document that in the bilateral repo market, there was a six-fold increase in their index for repo rate from 2007 to 2008, but a fourteen-fold increase in the index for repo haircut. Moreover, their regression results show the proxy for the volatility of collateral value only has explanatory power on repo haircuts but not on repo spreads. They lament for the lack of theoretical models to explain this phenomenon: "It could seem natural that repo spreads and repo haircuts should be jointly

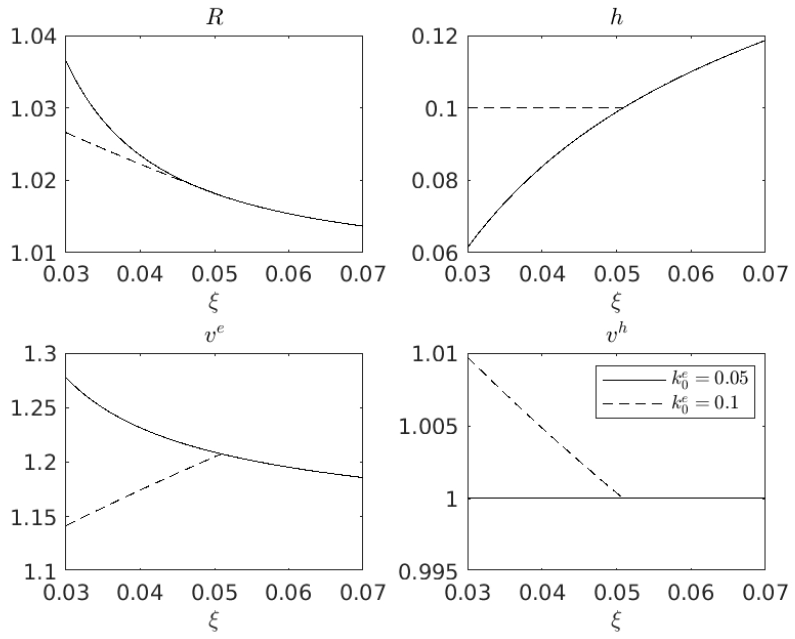


Figure 5: Comparative Statics on Default Cost

determined. Unfortunately, the theory is not sufficiently developed to provide much guidance here". The underlying mechanism conjectured by them, however, is exactly what we have in this model: "In reality, collateral pricing can be uncertain, and illiquidity and volatility in the secondary markets for this collateral can induce large transactions costs following a default. ... Higher haircuts could occur to adjust for the uncertain value of the collateral, because each dollar of collateral could be worth much less by the time it can be sold." Mian and Sufi (2014) also provide evidence that the increase in the credit spread could not explain the scale of the credit crunch (p.130) in the Great Recession.

The second example took place in the margin loans market on the Amsterdam stock exchange in 1772, following the bankruptcy of an investor syndicate speculation. Koudijs and Voth (2016) show that the major lenders to the stricken syndicate tightened their collateral requirements, which verifies the importance of perceived risk in determining the credit market prices. They rule out various alternative explanations, such as changes in regulatory constraints, asset price decline, and losses among intermediaries. At the same time, the required interest rates of these lenders remained the same, which, by our model, should be caused by market illiquidity. Indeed, in the same paper, they also document that from December 28 onward when a string of margin calls were issued and not met by the bankrupted speculators, the lenders had the right to sell the collateral immediately, but most of the liquidation transactions were delayed until the end of January 1773. The contract design in that time means there was no upside in this delay for the lenders. They conclude it is very likely that liquidity on the Amsterdam exchange dried up. Note that although no-default models can generate similar predictions, they are not suitable for this incident: in the Amsterdam stock market between January 1770 to January 1773, the collateral value fell below the loan value in 7 out of 100 loan contracts.

3.3.2 Equivalence with the Principal-Agent Equilibrium in Case (a)

Since in the real world unproductive agents do seem to hold some capital, we will be working exclusively with the loose credit supply case (a) in the dynamic model of Section 4. In the proof of Theorem 3, we note that the general equilibrium in case (a) is equivalent to a principal-agent equilibrium where the entrepreneurs have all the market power. We now formally define this principal-agent equilibrium and state the equivalence result as a corollary, which will allow us to construct the dynamic model in the conventional form without resorting to measure theory.

Definition 2 (Principal-Agent Equilibrium)**.** A *principal-agent equilibrium* consists of asset holdings k_1^j, debt holdings d^j, the asset price Q, the credit market prices R, h, such that the entrepreneurs' problem

$$\begin{aligned}
\max_{(k_1^e,d,R,h)\in[0,1]\times\mathbb{R}_+^2\times[0,1]} \quad & \mathbb{E}[R^e Q k_1^e - R^d d] \\
\text{s.t.} \quad & d \leq (1-h)Qk_1^e, \\
& Qk_1^e = Qk_0^e + d \\
& \mathbb{E}R^d = \mathbb{E}R^h + F(\tilde{Q})\xi,
\end{aligned}$$

is satisfied and the asset market clears.

From the comparative statics we know roughly that case (a) happens when k_0^e is small enough and ξ is large enough, given the distribution of Q_1. In the following corollary we show such bounds of k_0^e and ξ indeed exist.

Corollary 2.1 (Equivalence Between GE & PAE)**.** Assume that problem (4) permits a unique solution for $v^h = 1$. Given the distribution of the asset price, there exists $\underline{\xi}$, $\bar{k}_0^e(\underline{\xi})$ such that when $\xi > \underline{\xi}$, $k_0^e < \bar{k}_0^e(\underline{\xi})$,

1. there exist a unique principal-agent equilibrium;
2. $k_1^{h*} > 0$ in equilibrium;
3. the equilibrium Q^* is given by
$$Q^* = \mathbb{E}[Q_1 - \kappa]. \tag{5}$$
4. the equilibrium (R^*, h^*) are the solution of the system of equations
$$\mathbb{E}R^d = 1 + F(\tilde{Q})\xi \tag{6}$$
$$\frac{[1-F(\tilde{Q})](1-h) - \xi f(\tilde{Q})(1-h)^2 Q}{[1-F(\tilde{Q})]R - \xi f(\tilde{Q})\tilde{Q} - (1+\xi F(\tilde{Q}))} = \frac{[1-F(\tilde{Q})](1-h)}{[1-F(\tilde{Q})]R - \frac{\mathbb{E}R^e - \mathbb{E}R^d(1-h)}{h}}. \tag{7}$$
5. the general equilibrium is essentially equivalent to the principal-agent equilibrium in the sense that only one contract h^* will be traded and its price is given by R^*, and other equilibrium variables coincide.

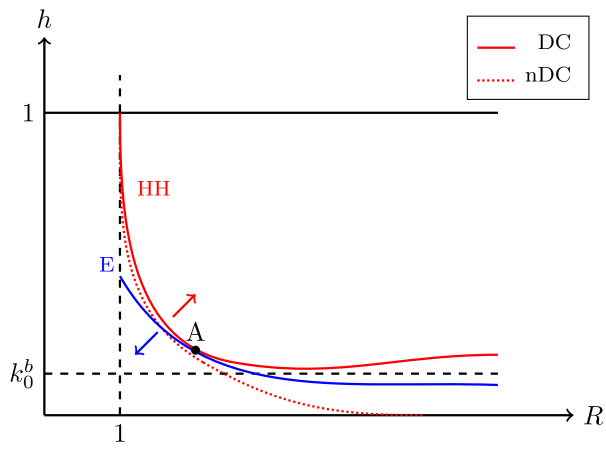


Figure 6: Credit Market Equilibrium with Costly Default

Remark 1 (The Role of Default Cost)**.** If $\xi = 0$, problem (4) is reduced to

$$\max_{h \in [0,1]} \quad \frac{\mathbb{E}R^e - v^h}{h} + v^h.$$

If $\mathbb{E}R^e - v^h > 0$, the entrepreneurs will always choose $h = 0$. Therefore, the only way to clear the credit market is the corner solution $v^h = \mathbb{E}R^e$ and $h^* = k_0^e$. Intuitively, in a collateralized debt market where the entrepreneurs earn leveraged return, absent other frictions, they value haircut more than the households, and are always willing to give up the interest rate for a lower haircut. Mathematically, the slope of the entrepreneurs' indifference curve is always flatter than that of the households, rendering interior tangent points impossible (see Appendix A for the algebra). In the presence of costly default, using R to satisfy the households becomes increasingly costly as the default probability increases. Costly default establishes a non-trivial trade-off between R and h, and thereby generates inner solution equilibria.

Corner solution also shows up in the original belief disagreement settings (Fostel and Geanakoplos, 2015), albeit in the form that the equilibrium traded contracts are only riskless contracts. To generate an inner solution, Simsek (2013) assumes that the belief disagreement satisfies the hazard rate order condition, which requires the entrepreneurs to be increasingly more optimistic than the households along the support of Q_1. Similar to our method in essence, using R to satisfy the households becomes increasingly expensive due to the widening belief gap, creating a non-trivial trade-off between R and h. The analog is so close that we can set the default cost ξ to 0 and plug belief disagreements into Equation (7) to get the identical consensus valuation equation as Simsek (2013):

$$\frac{1 - F^e(\tilde{Q})}{1 - F^h(\tilde{Q})} = \frac{\mathbb{E}R^e - \mathbb{E}R^d(1-h)}{h}, \tag{8}$$

where F^e and F^h are the agents' beliefs on the distribution of Q_1.

It is unfortunate that Equation (7) from the default cost approach does not have a similarly simple structure, and there is no easy characterization of F such that the system of equations (6) and (7) has a unique solution. The loss of theoretical beauty is partially compensated by the closer connection to the macro and empirical finance literature.

4 Dynamic Model

In this section, I extend the static model discussed in the previous section to an infinite horizon macroeconomic model to study the dynamic transmission mechanism. We focus on the range of parameters where case (a) holds, and simplify the formulation of our general equilibrium model using Corollary 2.1.

4.1 Production Technology and Institutional Setting

There are two types of agents in this economy: productive entrepreneurs and less productive households. Each type of agents has a unit measure so we do not need to distinguish between aggregate and individual variables. The aggregate productivity follows a standard AR(1) process

$$\log Z_{t+1} = \rho_z \log Z_t + \Sigma_t \epsilon_{t+1},$$

except that Σ_t is time-varying and also follows an AR(1) process

$$\log \Sigma_{t+1} = c_\sigma + \rho_\sigma \log \Sigma_t + \eta \varepsilon_{t+1}.$$

Here ϵ_t and ε_t are two independent i.i.d. Gaussian white noise processes with unit variance, and η is the standard deviation of the innovation to $\log \Sigma_t$.

The production function of the entrepreneurs is constant return to scale:

$$Y_t^e = Z_t K_{t-1}^e, \tag{9}$$

where K_{t-1}^e is the capital held by the entrepreneurs. The production function of the households is given by

$$Y_t = (Z_t - \kappa) K_{t-1}^h, \tag{10}$$

where $\kappa > 0$ represents the households' inefficiency in managing capital. The households can either use their capital to produce, or lend to the entrepreneurs. The aggregate capital is in fixed supply and does not depreciate:

$$K_t^e + K_t^h = 1.$$

The only available contracts in the credit market are non-contingent debt contracts, enforced by physical collateral. Denote the haircut in a debt contract by h_t and the interest rate by R_{t+1}. The default cost is ξ fraction of the face value of the debt. The dynamic assumption on the entrepreneurs' endowment is stated in the next subsection. This simplified formulation of the credit market is justified by Corollary 2.1. We again introduce some simplifying notations:

- the households' gross return from direct capital holding

$$R_{t+1}^h \equiv \frac{Q_{t+1} + Z_{t+1} - \kappa}{Q_t};$$

- the entrepreneurs' gross return from direct capital holding

$$R^e_{t+1} \equiv \frac{Q_{t+1} + Z_{t+1}}{Q_t};$$

- the gross return from debt

$$R^d_{t+1} \equiv \min\left\{R_{t+1}, \frac{Q_{t+1}}{(1-h_t)Q_t}\right\};$$

- the threshold of default

$$\tilde{Q}_{t+1} = R_{t+1}(1-h_t)Q_t.$$

4.2 Agents' Problems

A household faces flow the budget constraint

$$Q_t K^h_t = R^h_t Q_t K^h_{t-1} + (R^d_t - \mathbb{I}_{\tilde{Q}_t > Q_t}\xi)D_{t-1} - C^h_t - D_t, \tag{11}$$

where D_t is the new loans issued, C^h_t is the period t consumption, Q_t is the price of capital, and $\mathbb{I}$ is the indicator function. The household's utility function is a discounted sum of future consumption

$$\sum_{t=0}^{\infty} \beta^t C^h_t.$$

To prevent the entrepreneurs from saving themselves out of the collateral constraint, we follow Bernanke, Gertler and Gilchrist (1999) by assuming that entrepreneurs exit with a constant probability $1-\gamma$, and they are forced to consume all their net worth when exit. To fill the gap left by the exiting entrepreneurs, every period new entrepreneurs enter with endowment w^e. The number of entering entrepreneurs equals the number of exiting entrepreneurs.

Without loss of generality, we assume that entrepreneurs can only consume in the period they exit. An entrepreneur has the preference

$$\mathbb{E}_t \sum_{i=1}^{\infty} (1-\gamma)\gamma^i \beta^i c^e_{t+i},$$

where $(1-\gamma)\gamma^t$ is the probability of exiting at date t, and c^e_t is the terminal consumption if the entrepreneur exits at t. The net worth of a surviving entrepreneur is

$$n_t = R^e_t Q_t k^e_{t-1} - R^d_t d_{t-1}. \tag{12}$$

The net worth of an entering entrepreneur is simply his endowment

$$n_t = w^e. \tag{13}$$

At each period t, an entrepreneur finances his asset holdings $Q_t k^e_t$ with new debt and his net worth

$$Q_t k_t^e = n_t + d_t,$$

where the size of the debt is subject to the collateral constraint

$$d_t \leq (1 - h_t) Q_t k_t^e.$$

4.3 Equilibrium and Aggregation

As long as we are in case (a), the market price of capital is equal to the marginal product of capital for the households

$$1 = \beta \mathbb{E}_t R_{t+1}^h, \tag{Q}$$

and the expected return from the debt is equal to the households' return from direct capital holding

$$1 = \beta [\mathbb{E}_t R_{t+1}^d - \xi F(\tilde{Q}_{t+1})]. \tag{RH1}$$

Note that allocation is absent from the asset pricing equations. This simplification comes from that the linear utility function takes away stochastic discount factors, and the linear production function takes away the effect of misallocation. The entrepreneurs' ROE is

$$\frac{\mathbb{E}_t R_{t+1}^e - \mathbb{E}_t R_{t+1}^d (1 - h_t)}{h_t}. \tag{14}$$

The equilibrium credit market prices R_{t+1} and h_t are solved by maximizing (14) over R_{t+1} and h_t, subject to (RH1). The solution can be found by solving a system of two equations consisting of Equation (RH1) and

$$\frac{[1 - F(\tilde{Q}_{t+1})](1 - h_t) - \xi f(\tilde{Q}_{t+1})(1 - h_t)^2 Q_t}{[1 - F(\tilde{Q}_{t+1})] R_{t+1} - \xi f(\tilde{Q}_{t+1}) \tilde{Q}_{t+1} - (1 + \xi F(\tilde{Q}_{t+1}))} = \frac{[1 - F(\tilde{Q}_{t+1})](1 - h_t)}{[1 - F(\tilde{Q}_{t+1})] R_{t+1} - \frac{\mathbb{E} R_{t+1}^e - \mathbb{E} R_{t+1}^d (1 - h_t)}{h_t}}. \tag{RH2}$$

As in the two-period model, this equation is the consensus valuation condition, which requires the marginal rate of substitution between the interest rate and the haircut to be equal between the entrepreneurs and the households.

Since h_t is independent of the entrepreneurs' net worth, we can aggregate across entrepreneurs to get the evolution of aggregate entrepreneurs' capital

$$Q_t K_t^e = \frac{N_t}{h_t}, \tag{DY}$$

where the entrepreneurs' net worth is the sum of the net worth of surviving entrepreneurs and entering entrepreneurs

$$N_t = \gamma [(Z_t + Q_t) K_{t-1}^e - R_t^d D_{t-1}] + (1 - \gamma) w^e,$$

where the aggregate credit supply is subject to the collateral constraint

$$D_{t-1} = (1 - h_{t-1})Q_{t-1}K^e_{t-1} \tag{15}$$

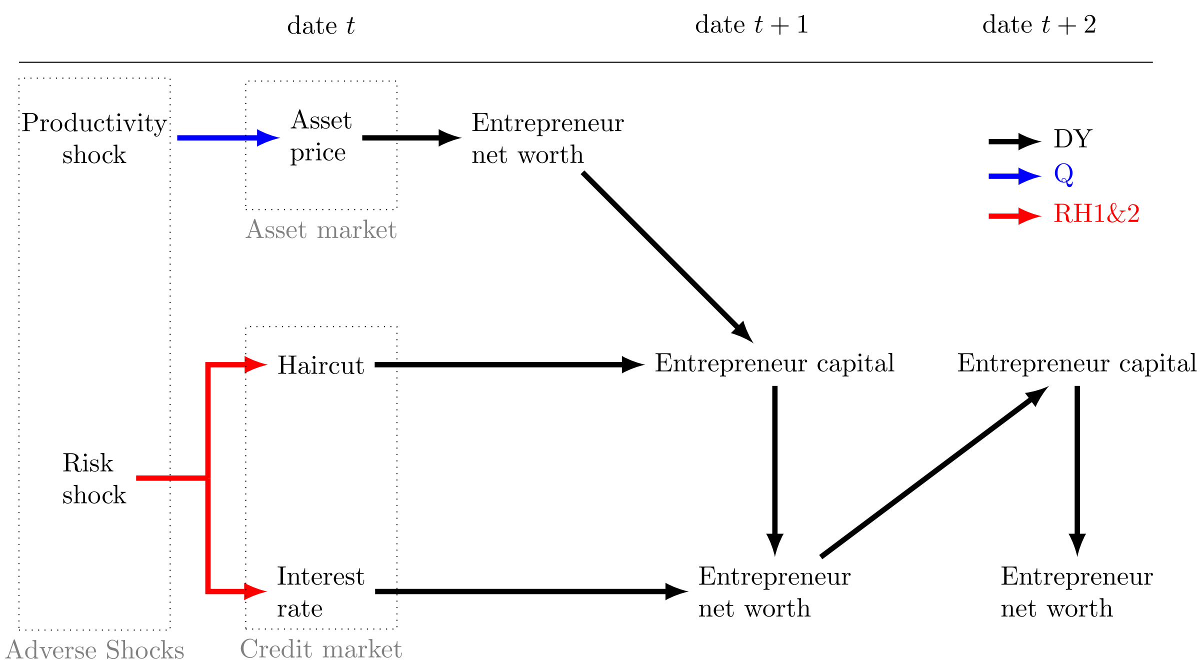


Figure 7: An Anatomy of Macrofinance Models

We illustrate the basic mechanism of the dynamic model using the flow chart Figure 7. The black arrows represent Equation (DY), which governs how the financial markets affect the real economy, and how recessions can be amplified and prolonged by financial frictions. (DY) is a variant of the famous Equation 7 in Kiyotaki and Moore (1997), and can be found in many macro models with financial frictions. It uncovers how the three financial market prices, Q, R and h, affect the entrepreneurs' asset under management: a decrease in Q and an increase in R reduces the entrepreneurs' net worth, and thereby reduces the entrepreneurs' capital indirectly; an increase in h directly reduces the entrepreneurs' capital. Therefore, there are three distinct channels through which the impact of macroeconomic shocks can be amplified and prolonged by financial frictions: the asset price channel, the haircut channel and the interest rate channel, depending on whether the shocks affect Q, h or R.

The asset price channel in this model is described by the asset pricing equation (Q) and represented as the blue arrow in Figure 7. The haircut channel and the interest rate channel are jointly described by Equations (RH1) and (RH2), and are represented as the red arrows. The previous macrofinance literature only contains variants of the interest rate Euler equation (RH1), but not the consensus valuation equation (RH2), which is required by the collateral general equilibrium.

Continuing describing the model, the exiting entrepreneurs consume all their net worth

$$C^e_t = (1 - \gamma)[(Z_t + Q_t)K^e_{t-1} - M_t]. \tag{16}$$

The total output is the sum of the output from both the households and the entrepreneurs, minus the management cost

$$Y_t = Z_t - \kappa K^h_{t-1} + (1-\gamma)w^e. \tag{17}$$

The output is either used for default cost, or consumed by households and entrepreneurs

$$Y_t = C^e_t + \mathbb{I}_{\tilde{Q}_t > Q_t} \xi D_{t-1}/Q_t + C^h_t. \tag{18}$$

This completes the description of the dynamic model.

5 Impulse Responses and Propagation Mechanisms

The simplification of the linear model mainly comes from the insulation of the asset pricing equation from allocation. Using the first order perturbation method on Equation (Q), we find that the asset price is equal to

$$\log Q_t = q_0 + A_{qz} z_t, \text{ where } A_{qz} = \frac{\beta\rho_z}{Q_{ss}(1-\beta\rho_z)}.$$

Therefore, conditional on the information at time t, $\log Q_{t+1}$ is normally distributed with mean $q_0 + A_{qz}\rho_z \log Q_t$ and standard deviation $A_{qz}\sigma_t$. We can then use Equations (RH1) and (RH2) to solve for the credit market prices. Our goal is to provide a suggestive numerical example to illustrate the basic mechanism of this model. Given its simplicity, these numerical exercises are not precise estimates.

5.1 Parameters and Steady States

Table 2 lists the parameter values for our model. There are in total eight parameters, β, ρ_z, ρ_σ, γ, w^e, κ, ξ, c_σ. β and ρ_z are conventional and directly taken from the RBC literature. γ, w^e and κ are only seen in the macrofinance literature, and are rescaled to resemble Gertler and Kiyotaki (2015). Two parameters are the most important to this model: the steady-state risk of the entrepreneurs' productivity, c_σ, and the default cost, ξ. We choose them to match the steady state entrepreneur leverage of 8 and credit spread of 0.5%. The choice of $\xi = 0.05$ is within the range of the empirical findings in the bankruptcy cost studies, ranging from less than 1% to more than 15%[8]. The choice of c_σ is such that the steady state Σ is 0.23, which is within the range of steady state firm-level risk used in the previous literature[9]. We did not pick a value for η as it does not enter into our linear model when the asset price is solved using a first-order approach. These parameter choices are meant to be suggestive, and the results to be shown are robust to a wide range of parameter specifications.

As in other models with uncertainty, the meaningful steady state here is a stochastic one. In particular, I first calculate the steady state prices under the steady state productivity 1 and risk $\bar{\Sigma}$. The steady state allocations are achieved by simulating the economy for a long enough

[8]See Altman (1984); Ang, Chua and McConnell (1982); Warner (1977).

[9]See Bloom et al. (2016); Arellano, Bai and Kehoe (2016); Christiano, Motto and Rostagno (2014).

ρ_z	0.95	Productivity persistence
ρ_σ	0.8	Risk persistence
$\bar{\Sigma}$	0.23	Steady state risk
β	0.99	Discount factor
γ	0.93	Entrepreneur survival probability
w^e	0.1	Entrepreneur endowment
κ	0.5	Household management cost
ξ	0.05	Default cost

Table 2: Parameters

period. The steady state values are listed in Table 3. To analyze the propagation mechanisms, three types of shocks are considered[10]: a productivity shock with initial magnitude -1% and persistence ρ_z, a risk shock with initial magnitude 50% and persistence ρ_σ, and a default cost increase with initial magnitude 50% and persistence ρ_σ. An increase in default cost can be interpreted as deterioration of market liquidity.

Q	49.500
h	0.125
R	1.014
Leverage	7.976
D	3.909
K^e	0.090
Y	0.552
N^e	0.560

Table 3: Steady State Values

5.2 Response to a Risk Shock

Figure 8 shows the impulse response of the economy to a risk shock. Since the creditor bears only down risk, a mean-preserving spread requires the borrower to compensate him with higher haircut and interest rate. The haircut increases from 0.125 to 0.154, which reduces the entrepreneurs' leverage from 8 to 6.5, a direct 19% reduction in entrepreneur asset. At the same time, the credit spread widens by 27 basis points, further slowing down the recovery of the entrepreneurs' asset under management.

Note that the asset price is perfectly insulated from the risk shock. Recall that asset can be used for two purposes: collateral and productive capital. When the agents are risk-neutral, risk only affects the valuation of the asset as collateral but not as productive capital. Since the marginal investors in our model are always the households, risk does not enter into the equilibrium asset price. Similarly, liquidity does not enter into the asset price, as will be shown in the next numerical exercise. This feature is specific to our framework and not generic to the collateral equilibrium models.

[10]Since in our model entrepreneurs' productivity is homogeneous, if default happens, all entrepreneurs will default, creating a large discontinuity in output. To avoid this uninteresting phenomenon, the shocks are chosen such that default does not actually happen. To generate realistic proportions of firm-level default, the model should be extended with heterogeneous productivity.

Now we turn to the real economy. We decompose the credit market response to a risk shock into the haircut channel, represented by the grey area, and the interest rate channel, represented by the white area. The increase in haircut causes an immediate credit crunch and reduction of entrepreneur asset, which translates into more severe misallocation and a recession. The increase in credit spread works differently: it does not reduce the entrepreneurs' asset directly but instead indirectly by eroding the net worth of the entrepreneur, and thus its effect is slower but more persistent.

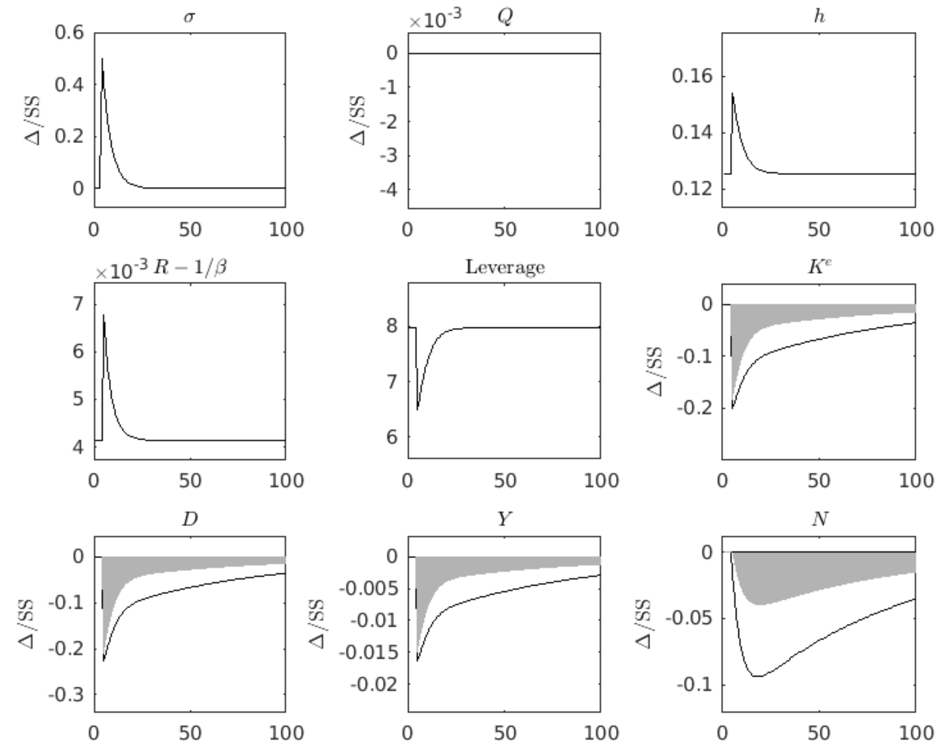


Figure 8: Impulse Response to a Risk Shock

Note: A Δ/SS on the y-axis label means that this subplot describes the deviation of a variable from its steady state over its steady state value. For example, subplot D plots the evolution of $(D_t - D_{SS})/D_{SS}$.

5.3 Illiquidity and (Credit) Spread-less Recessions

Figure 8 clearly shows that the credit spread is not a sufficient statistics measure of financial frictions in the credit market, and the haircut can account for a large proportion of the propagation of risk shocks. Since we are also interested in what determines the distribution of the impact of risk shocks on the interest rate and the haircut, we perform another experiment in Figure 9, where the economy is hit by both a risk shock and an increase in the default cost. The rise in the default cost changes the trade-off between the haircut and the interest rate, and now the lenders prefer higher haircut over interest rate for fear of default. The propagation of risk shock is now nearly perfectly loaded on the haircut, and the credit spread barely moved at all.

Therefore, this model supports the policy suggestions of better monitoring and regulation of supply-side leverage (Geanakoplos, 2010; Gorton and Metrick, 2012), as it can be much more important than the credit spread as an indicator of banking duress during crises times, which are more than often marked by both higher economic volatility and lower market liquidity.

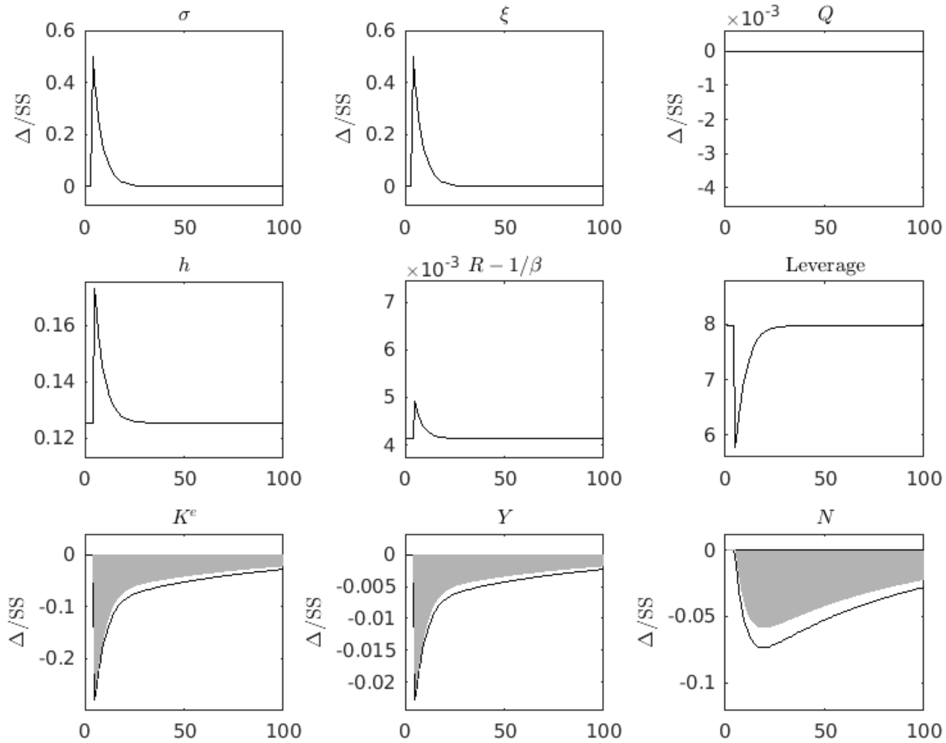


Figure 9: Risk Shock When Market Are Illiquid

5.4 Risk Shock versus Productivity Shock

It is useful to compare the propagation of risk shocks to that of productivity shocks. In Figure 10, the dashed line represents the impulse response to a productivity shock, and the solid line represents that to a risk shock. We immediately notice the "orthogonality" of the productivity shock and the risk shock in terms of transmission mechanisms. In particular, the productivity shock goes through the asset market, and the risk shock goes through the credit market. By decreasing the asset price, a productivity shock erodes the net worth of the entrepreneurs and thereby initiates the financial accelerator, whereas the risk shock does so by increasing the haircut and the interest rate.

Since the productivity shock mechanically lowers output, we represent this "external propagation" by the shaded area in the last subplot, and the "internal propagation", or the financial accelerator effect by the region between the shade and the dashed line. The lack of internal propagation through the asset price channel is well known as the Kocherlakota (2000) critique. The intuition comes from the asset pricing equation (Q), which says Q_t is a discounted sum of future dividends. Therefore, a 1% productivity shock will translate into around 1% asset price decline. Equation (15) then implies the aggregate credit supply will drop by around 1%, the misallocation caused by which is insignificant to induce large output losses. In contrast, the loss of entrepreneurs' capital by a risk shock through the credit market is about 20 times larger, which creates huge output losses without resorting to any external propagation. Therefore, besides the richer credit market dynamics, the framework proposed here also helps resolve the quantitative insignificance of the classical financial accelerator.

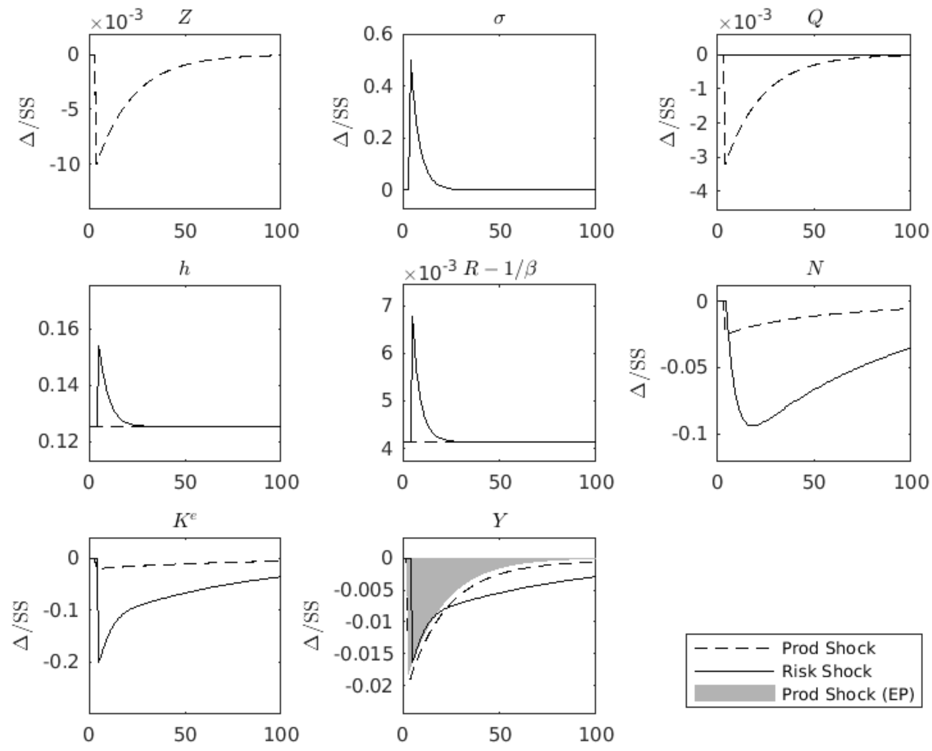


Figure 10: Risk Shock vs. Productivity Shock

5.5 Endogenous Haircut versus Exogenous Haircut

In this subsection we perform two additional numerical exercises to demonstrate how exogenous haircut models can create misleading predictions against risk shocks. In both exercises, we consider a new model where h_t is exogenously chosen to be always equal to the steady state value. Therefore, the credit market equilibrium condition is the interest rate Euler equation (RH1) alone. In the first exercise, we shock this economy with the same risk shock process as Section 5.2. The impulse response is plotted as the dashed line in Figure 14, and the impulse response of Section 5.2 is plotted as the solid line for comparison.

In Figure 14, the most noticeable change brought by exogenous haircut is the large spike in credit spread (105 basis points versus 27 basis points before). Since the interest rate is inefficient as a hedge against an increase in risk, loading all the shock impact to the interest rate exaggerates the effect of risk shocks on the economy (Christiano, Motto and Rostagno, 2014; Gilchrist, Sim and Zakrajšek, 2014), as shown in the subfigure of Y. It also overstates the decline of entrepreneurs' net worth. If we were to fix the haircut as exogenous in the illiquidity exercise in Section 5.3, the overestimation will be even larger.

On the flip side of the same coin, exogenous haircut models require much larger credit spread movements to generate the observed business cycle dynamics. In the second exercise, we use a risk shock of 42% on the exogenous haircut model to generate a comparable recession to the one shown in Section 5.2, and leave the figure to Appendix G. We find that the credit spread movement is 3 times that of the endogenous haircut model (84 basis points versus 27 basis points). The over-reliance of recent macrofinance models on credit spreads has drawn strong criticism from other fields (Mian and Sufi, 2014) and cast doubts over the credibility of the banking lending view of recessions. This is the compelling practical reason why endogenizing haircut is so important for macrofinance.

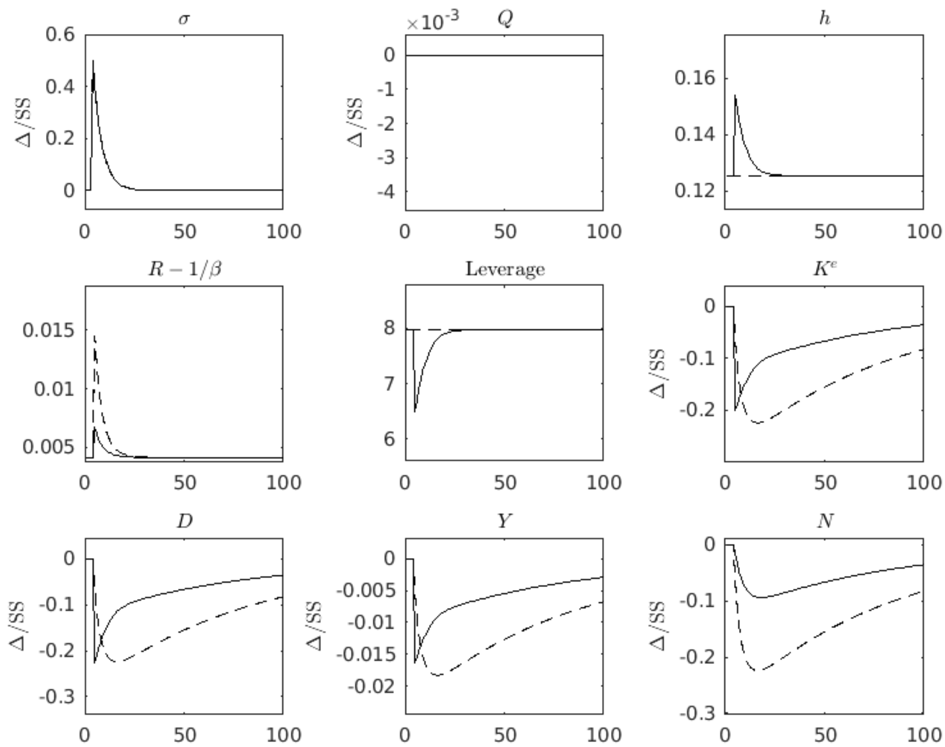


Figure 11: Endogenous Haircut vs. Exogenous Haircut

6 Conclusion

This paper presented an infinite-horizon macroeconomic model with endogenous collateral requirements, motivated by no-recourse loans. In essence, it is a combination of the "credit cycle" framework by Kiyotaki and Moore (1997) and the "leverage cycle" framework by Geanakoplos (1997). Compared with the previous macroeconomic models with financial frictions, this paper is capable of endogenously determining supply-side leverage as a general equilibrium object. It shows that when the entrepreneurs do not manage all the capital in the economy, the interest rate and the haircut both increase after a positive risk shock. Moreover, liquidity plays a crucial role in determining how risk shocks transmit through the haircut channel and the interest rate channel: when market liquidity is low, the haircut channel becomes more important, and the inverse holds.

The dynamic model in Section 4 is a highly stylized one. Although this approach is successful in communicating the fundamental mechanism, it leaves open the interesting question of how the credit market equilibrium interacts with other common macroeconomic frictions, such as price rigidity, wage rigidity, investment adjustment cost. The numerical challenge in solving a full-scale DSGE lies in that nonlinearity will invalidate the simple solution given in Corollary 2.1, and force us to jointly solve a nonlinear version of problem (4) with other market equilibrium conditions. This process is very costly numerically, and I have not succeeded in designing an algorithm with reasonable speed. It is thus a pressing next step to find an efficient numerical algorithm for the haircut cycles model to evaluate its quantitative performance. We also notice that endogenizing market liquidity can be a promising direction of future research.

The framework offered in this paper is a very basic and thus flexible one. It can be easily extended to incorporate richer types of investor heterogeneity, such as differences in risk attitudes, hedging techniques, and belief disagreements; richer financial market institutions, such as different collateral constraints, bank runs, and multi-period debt; other aggregate shocks. Less straightforward extensions which are promising include allowing for short-selling, idiosyncratic firm-level productivity shocks, and asymmetric information.

Among all the extensions I believe adverse selection to be the most promising, as shrewd readers might have already noticed the striking similarity between the contractual settings in this paper and Stiglitz and Weiss (1981). Recall from Figure 4 that the interest rate increases as risk decreases in case (b). The reason lies in that the households' market power, as reflected by v^h, endogenously increases as risk decreases, which has the counter-intuitive prediction that the entrepreneurs have all the market power in crisis times when risk is the highest. This leads us to reflect upon what deviations from a competitive benchmark can change this prediction. My preliminary answer is adverse selection: higher risk will aggravate the adverse selection problem (Stiglitz and Weiss, 1981) and shift the market power to the households in times of crises. Since the haircut is exogenous in their paper, it is exciting to see how the collateral equilibrium theory will affect the prediction of credit rationing when both the haircut and the interest rate serve as screening mechanisms.

A Why not Simple Price Taking?

Defining credit market general equilibrium in the form of Definition 1 is first proposed by Geanakoplos (1997). This is different from the simple price taking definition:

Definition 1A (Simple Price Taking Definition)**.** A general equilibrium consists of asset holdings k_1^j, debt holdings d^j, asset market price Q, credit market price R, h, such that asset and debt holdings solve the following problem

$$\begin{aligned} \max_{k_1^j, d^j} \quad & \mathbb{E}[R^j Q k_1^j - R^d d^j] \\ \text{s.t.} \quad & d^j \le (1-h) Q k_1^j, \quad \text{if } d_j > 0 \\ & Q k_1^j = Q k_0^j + d^j, \end{aligned}$$

the asset market clears, $\sum_{j \in \{h,b\}} k_1^j = 1$, and debt market clears, $\sum_{j \in \{h,b\}} d^j = 0$.

Definition 1A might be the first equilibrium definition that comes to a reader's mind when trying to define a competitive equilibrium with collateralized borrowing. This definition views (R, h) as prices, and let the agents be price takers in the credit market. Using this definition, any (R, h) pair such that $\mathbb{E}R^d = 1$ is an equilibrium credit market price.

However, Definition 1A is inconsistent with the essence of a Walrasian equilibrium in the sense that the agents have incentive to alter the prices that equate demand and supply. To see this, let us denote A as a credit market price pair (R, h), such that $h > k_0^e$ and $\mathbb{R}^d = 1$. A is a general equilibrium credit market price according to Definition 1A. In Figure 12, we draw the households' and entrepreneurs' indifference curve through A on the R-h plane as the red line and blue line, respectively. The arrows represent the improving directions. The entrepreneurs' indifference curve is always flatter as they earn leveraged return and are less willing to substitute h with R. Now we look at a new price B, southeast of A between the two indifference curves. We immediately see that both types will have incentives to alter the price to B, and A cannot be part of a competitive equilibrium. This logic can continue until h reaches k_0^e. The algebraic proof of this argument can be found after the next paragraph.

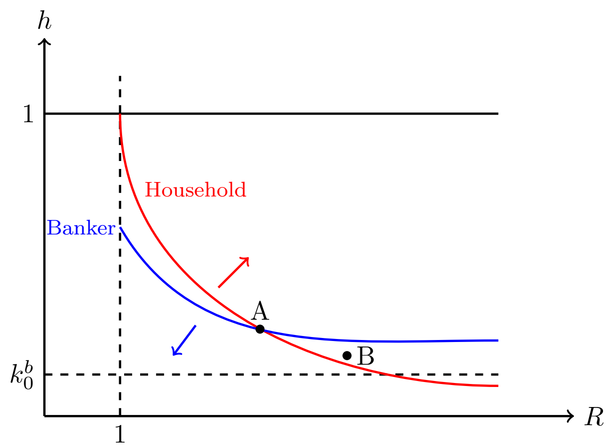


Figure 12: Credit Market Equilibrium

Theoretically, the simple price taking assumption implicitly restricts the set of possible price deviations to be uni-dimensional. When there are two prices for one good, as in the credit market, it will fail to account for the type of "double deviations" proposed in Figure 12. The argument above hints the possibility of alternative definitions, for example, letting the contract space be $\mathbb{R}_+$, and haircut be the price of contracts: $h : \mathbb{R}_+ \to [0, 1]$ will also work.

Proof. Assume that (R, h) is the equilibrium credit market price, and $h > k_0^e$. This implies that

$$\mathbb{E}R^h = \mathbb{E}R^d = 1.$$

We look at the indifference curves of the entrepreneurs' and households' return. The entrepreneurs' ROE can be shown to be

$$\frac{\mathbb{E}R^e - \mathbb{E}[\min\{\tilde{Q}, Q_1\}]/Q}{h}.$$

The partial derivative of the entrepreneurs' ROE w.r.t. to h is

$$\frac{[1 - F(\tilde{Q})]R}{h} - \frac{\mathbb{E}R^e - \mathbb{E}[\min\{\tilde{Q}, Q_1\}]/Q}{h^2}.$$

Increasing h has two effects on the entrepreneurs' ROE: the first term is the reduction in debt repayment if he does not default, and the the second term is the loss of leveraged return. The partial derivative w.r.t. R is

$$-\frac{[1 - F(\tilde{Q})](1 - h)}{h}.$$

Increasing R only increases the repayment when he does not default, so there is only a negative term.

Next we look at the households' return. It is easier to work with the return on a unit of collateral, instead of a unit of debt:

$$\mathbb{E}[\min\{\tilde{Q}, Q_1\}] - (1 - h)Q = 0.$$

The partial derivatives for the households' return w.r.t. h is

$$-[1 - F(\tilde{Q})]RQ + Q.$$

There are two effects of increasing h: the loss in return if the entrepreneur does repay from smaller loan size, and the cost saved from smaller loan size (less principal invested). The partial derivatives for the households' return w.r.t. R is

$$[1 - F(\tilde{Q})](1 - h)Q.$$

Increasing R increases the return if the entrepreneur does repay. Compared with the entrepreneur, the households' marginal return is not leveraged. This will create a discrepancy in the valuation of contracts between the entrepreneurs and the households.

Denote the slope of the isoquant curve (dh/dR) of the entrepreneurs' ROE on the R-h plane by SL^e, and that of household j's objective function by SL^h. Intuitively, we need to increase h by $|SL^j|dh$ for a decrease in R by dR to keep agent j's payoff constant. We have

$$SL^e = \frac{[1-F(\tilde{Q})](1-h)}{[1-F(\tilde{Q})]R - \mathrm{ROE}^e}$$
$$SL^h = \frac{[1-F(\tilde{Q})](1-h)}{[1-F(\tilde{Q})]R - 1}.$$

It is easily seen that ,

$$SL^h < SL^e < 0. \tag{19}$$

Now consider the following deviation $(R+\Delta R, h-\Delta h)$, where

$$\Delta R > 0, \text{ and } \Delta h = |SL^e|\Delta R.$$

Under the new price, the households earn positive expected return, and the entrepreneurs' ROE is larger than under (R, h) so they will accept. This double deviation works for any (R, h) on the households' zero profit line until $h = k_0^e$. □

B Proof of Theorem 1

Proof. Let $R_{j,\text{bid}}(h)$ and $R_{j,\text{ask}}(h)$ be agent j's bid price and ask price for contract h, respectively. We also denote by

$$R^d_{j,\text{bid}} = R^d(R_{j,\text{bid}}(h), h), \; R^d_{j,\text{ask}} = R^d(R_{j,\text{ask}}(h), h).$$

In view of Definition 1, market clearing for debt contract requires

$$\min_j R_{j,\text{ask}}(h) \geq R(h) \geq \max_j R_{j,\text{bid}}(h), \quad \text{for all } h.$$

In addition, a contract $\hat{h}$ is traded in positive quantities only if

$$R_{i,\text{ask}}(\hat{h}) = R(\hat{h}) = R_{j,\text{bid}}(\hat{h}), \quad \text{for some } \{i, j\} = \{h, b\}.$$

The proof goes by five steps. The first step provides some simplifying observations. The second step shows that in equilibrium household will not borrow. The third step solves for the asset price. The fourth step characterizes the equilibrium with an optimal contract problem and establishes the Pareto efficiency of any collateral equilibrium. The last step shows that an equilibrium always exists.

Step 1: some simplifying observations. The entrepreneurs' return from direct capital holdings is always larger than the households', which is hard-wired in the productivity difference model

$$\mathbb{E}R^h < \mathbb{E}R^e. \tag{20}$$

To pin down the agents' bid and ask price, we need to know their value per net worth. Since the problem (3) is linear, the agent's value per net worth, v^j, is a linear multiplier on their net worth: $v^j Q k_0^j$. This states that v^j is agent j's maximum return on equity (ROE).

The agents' ask prices equate the return of debt to their ROE

$$\mathbb{E}R^d_{j,\text{ask}} - \xi F(\tilde{Q}_{j,ask}) = v^j. \tag{21}$$

The agents' bid prices equate their leveraged return from a unit net worth to their ROE

$$\frac{\mathbb{E}R^j - \mathbb{E}R^d_{j,\text{bid}}(1-h)}{h} = v^j. \tag{22}$$

Step 2: households will not borrow, i.e., $R_{b,\text{ask}} > R_{h,\text{bid}}$ for any contract $h < 1$. Suppose this is not true, and for some $h < 1$ there is $R_{b,\text{ask}} \leq R_{h,\text{bid}}$. By the monotonicity of $\mathbb{E}R^d$, $\mathbb{E}R^d_{b,\text{ask}} \leq \mathbb{E}R^d_{h,\text{bid}}$. Plug it into (22) to get

$$\begin{aligned} v^h &\leq \frac{\mathbb{E}R^h - \mathbb{E}R^d_{b,\text{ask}}(1-h)}{h} \\ &\leq \frac{\mathbb{E}R^h - \mathbb{E}R^e(1-h)}{h} \\ &= \frac{\mathbb{E}R^h - \mathbb{E}R^e}{h} + \mathbb{E}R^e. \end{aligned}$$

The second inequality uses Equation (21) and the fact that the entrepreneurs can always hold capital directly to achieve $v^e \geq \mathbb{E}R^e$. Note that for any $h < 1$ the right hand side is strictly smaller than $\mathbb{E}R^h$, the return the households can achieve by direct capital holding and no borrowing. Therefore, $R_{b,\text{ask}} > R_{h,\text{bid}}$ in equilibrium and the households do not borrow, i.e., $\delta^h_- = \delta^e_+ = 0$.

Step 3: the households price the asset. The reason is that the collateral constraint (2) restricts the entrepreneurs to buy no more asset than what the households have. Essentially, there is always excess supply in the asset market, and the equilibrium asset price is equal to the households' expected return from direct capital holding. Therefore, we have

$$1 = \mathbb{E}R^h.$$

Step 4: determining v^e and v^h. Since the entrepreneurs can always choose to invest in contracts with a positive excess return, without loss of generality we only need to consider the case where the collateral constraint is binding. The entrepreneurs' value per net worth can be calculated by solving the following problem

$$\begin{aligned} v^e Q k_0^e = \max_{k_1^e, \delta_-^e} \quad & \mathbb{E}R^e Q k_1^e - \int_0^1 \mathbb{E}R^d_{h,\text{ask}} d\delta^e_-, \\ \text{s.t.} \quad & Q k_1^e = Q k_0^e + \int_0^1 d\delta^e_-, \\ & \int_0^1 \frac{1}{1-h} d\delta^e_- = Q k_1^e. \end{aligned}$$

By the envelope condition, v^e is the Lagrange multiplier of the budget constraint. The first order conditions are

$$\mathbb{E}R^e - v^e + \lambda^e = 0$$
$$-\mathbb{E}R^d_{h,\text{ask}} + v^e - \lambda^e \frac{1}{1-h} \leq 0.$$

Combining them we get the entrepreneurs' value per net worth

$$v^e \geq \frac{\mathbb{E}R^e - \mathbb{E}R^d_{h,\text{ask}}(1-h)}{h}, \text{ with equality only if } h \in \operatorname{supp}(\delta^e_-), \tag{23}$$

where $R(h)$ is implicitly defined by (21). Note that finding $\operatorname{supp}(\delta^e_-)$ is equivalent to solving problem (4). This proves that any general equilibrium must be constrained efficient.

Step 5: the solution to problem (4) always exists. Since the objective function is strictly decreasing in R, we can strengthen the budget constraint with $R \in [1, \bar{R}]$, where $\bar{R}$ is a very large number. There are then two cases. If the budget constraint does not cross the horizontal axis, we have a continuous function on a compact set, which must reach its maximum. If the budget constraint crosses the horizontal axis, we only have to consider the discontinuity at $h = 0$. Here the maximum of the objective function is reached at $h = 0$ and any R such that $\mathbb{E}R^e > \mathbb{E}R^d$. Therefore, a solution to problem (4) always exists, and so is a general equilibrium. □

C Proof of Theorem 3

Proof. By assumption, problem (4) has a unique solution. There are two possible cases.

Case (a): $h^{**}(1) > k^e_0$ in the solution. Since all the entrepreneurs hold h^{**}, the households end up holding some capital in equilibrium. This in turn means the households have to earn the same return in the credit market as in the asset market, so $v^h = 1$. Note that case (a) can only happen when $\xi > 0$, and therefore increasing v^h will increase $h^{**}(v^h)$, and there can be no other equilibria.

Case (b): $h^{**}(1) \leq k^e_0$ in the solution. The entrepreneurs demand more credit than the households' supply if the households were to earn zero profit. To eliminate the excess demand, we need to find the most efficient way to increase the households' profit, which is the procedure described in Step 3(b) in Theorem 3.

After we have solved for the equilibrium prices, other equilibrium variables can be found by the market clearing conditions and budget constraints. □

D Proof of Corollary 2.1

Proof. Note that the entrepreneurs will never choose contracts with $\mathbb{E}R^e < \mathbb{E}R^d$, as they make losses on these contracts. Therefore, without loss of generality, we consider the feasible set strengthened by the condition

$$\mathbb{E}R^e > 1 + \xi F(\tilde{Q}),$$

which implies that the collateral constraint is binding. This requirement is equivalent to

$$h > 1 - \frac{F^{-1}((\mathbb{E}R^e - 1)/\xi)}{QR}.$$

Let $\underline{\xi}$ be such that the right hand side is equal to 0. When $\xi > \underline{\xi}$, the equilibrium h^* must be positive. Therefore, there exists $\bar{k}_0^e$, dependent on ξ, such that $h^* > \bar{k}_0^e$. When $k_0^e < \bar{k}_0^e$ and $\xi > \underline{\xi}$, the asset price be equal to the households' return from direct capital holding, which yields the asset pricing equation (5). We can simplify the entrepreneurs' problem into

$$\max_{(R,h)\in\mathbb{R}_+\times[0,1]} \quad \frac{\mathbb{E}R^e - \mathbb{E}R^d(1-h)}{h}$$
$$\text{s.t.} \quad \mathbb{E}R^d = 1 + F(\tilde{Q})\xi,$$

which yields the first order conditions (6) and (7). □

E The Case of Multiple Equilibria

Theorem 3. Designate the solutions to (4) by two ordered sets $R^{**}(v^h)$ and $h_i^{**}(v^h))$. The general equilibria exist and

1. Q^* is uniquely determined by $\mathbb{E}R^h = 1$;
2. a set of contracts h^* are traded in non-zero quantity in equilibrium and their prices R^* are uniquely determined;
3. R^* and h^* are found by solving program (4):
   (a) if $\min h^{**}(1) > k_0^e$, $R^* = (R^{**}(1)$, $h^* = h^{**}(1)$, and the entrepreneurs' debt holding over h^* is indeterminate as long as market clears;
   (b) otherwise, h^* reduces to a singleton set $\{k_0^e\}$. R^* can be solved from the constraint of problem (4).

Proof. Step 1-4 remain the same as in the proof of Theorem 1.

Step 5: Case (a) is straightforward. As long as market clears, it matters not what the debt holdings of the entrepreneurs are. To prove the solution under case (b), we first observe that the multiple equilibria in this model comes from the potential multiple solution of problem (4). Therefore, there is no sunspot, and the entrepreneurs' ROE on each contract actively traded in equilibrium must be the same. Using this observation, we can establish that any $h < k_0^e$ will not be traded in equilibrium. Assume, by contradiction, that two contracts $h_0 < k_0^e$ and $h_1 \geq k_0^e$ are traded in equilibrium (we need not consider the case where all traded contracts are $h < k_0^e$ since market does not clear there). Consider the contract h_0. If the entrepreneurs put all their debt holdings to this contract, their collateral constraint is not binding. It is easy to find another contract $h_0 < h_1 \leq k_0^e$ with the same price R_0, which keeps the entrepreneurs' return constant while making the households strictly better offer. Therefore, any contracts with $h < k_0^e$ cannot be actively traded in equilibrium.

By this logic, we have to raise v^h until $\min h^{**}(v^{h*}) = k_0^e$. Recall that only the contract $h^* = k_0^e$ can sustain an equilibrium where $v^h > 1$. Therefore, only h^* is traded in equilibrium, and its price can be solved from the budget constraint of problem (4). □

F Convex Management Cost

Some macrofinance models assume convex management cost to improve quantitative performance and introduce richer asset price dynamics (Gertler and Kiyotaki, 2015; Kiyotaki and Moore, 1997). In this section we illustrate how convex management cost affects the results of our paper. The model is the same as the 2-period model in section 3, except that the households' management cost takes the form $\kappa/2(k_1^h)^2$, i.e., the households are increasingly ineffective when they hold more capital directly. This will change the asset pricing equation into

$$Q = \mathbb{E}Q_1 - \kappa k_1^h,$$

while leaving the rest of the equilibrium unchanged. This equation says that the asset price is lower the more asset is held by the households. We adjust κ to 4 to make sure the equilibrium haircut is still larger than the entrepreneurs' initial endowment, and plot the comparative statics in Figure 13.

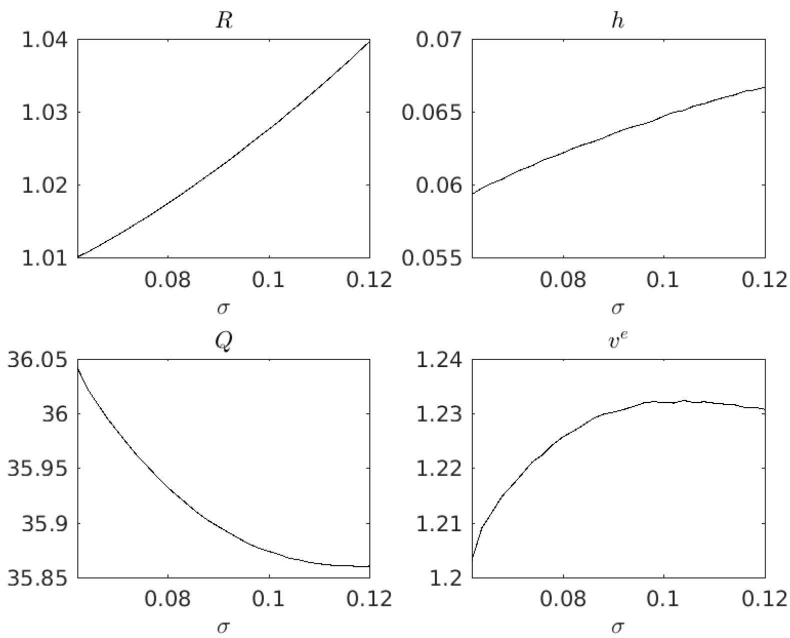


Figure 13: Comparative Statics with Convex Management Cost

When risk increases, the interest rate and the haircut both increase, and the asset price declines. The haircut response is weakened as the lower asset price increases the entrepreneurs' return from managing capital, and they are now willing to give up more interest rate to leverage up. However, there are several reasons why this result should not be interpreted as a challenge to the importance of the haircut channel. First, there is no empirical support for this form of management cost, and the sole purpose of it seems to be quantitative expediency in response to the Kocherlakota (2000) critique. Second, it creates richer asset price dynamics at the expense of other virtues of this model. In particular, with linear management cost, the entrepreneurs' value per net worth, v^e, decreases as σ increases, while with convex management cost, it increases for most values of σ. Put in other words, the entrepreneurs love crises, as their return on equity is mostly increasing in risk. This is generally considered a shortcoming that should be overcome in these models. There might be no quick solution to introduce asset price responses to risk shocks without fundamental modification to our current framework, and this is an important direction for future exploration.

G Excessive Credit Spread Fluctuations

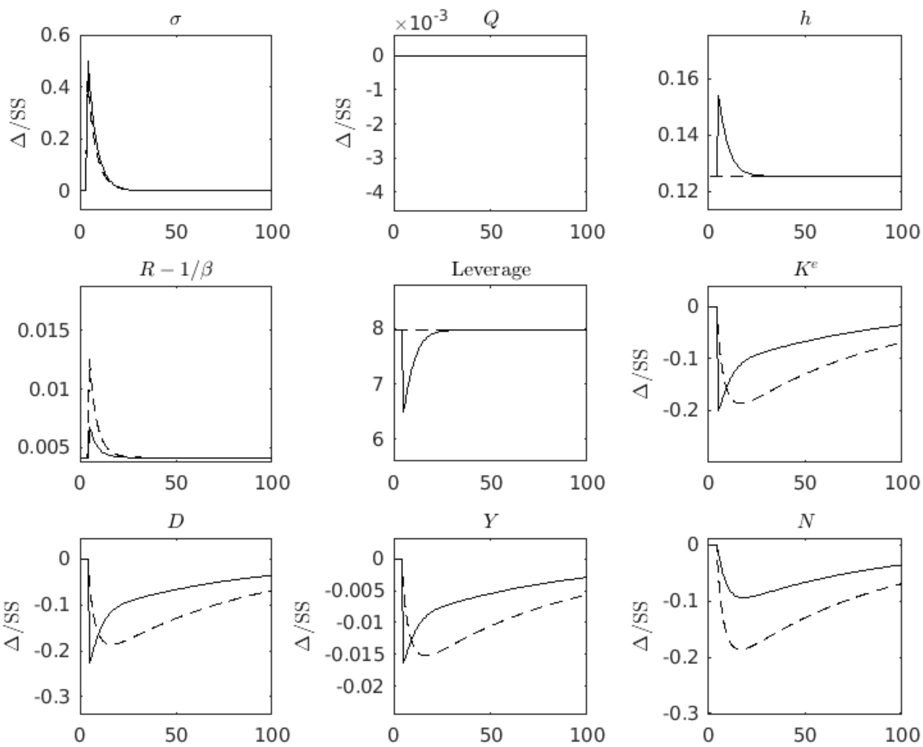


Figure 14: Endogenous Haircut vs. Exogenous Haircut

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