Journal of Economic Growth, 5: 185–206 (June 2000)

Schumpeterian Entrepreneurs Meet Engel's Law: The Impact of Inequality on Innovation-Driven Growth

JOSEF ZWEIMÜLLER

University of Zurich, CEPR-London and IZA-Bonn, Blümlisalpstrasse 10, CH-8006 Zurich

This article analyzes the impact of inequality on growth when consumers have hierarchic preferences and technical progress is driven by innovations. With hierarchic preferences, the poor consume predominantly basic goods, whereas the rich consume also luxury goods. Inequality has an impact on growth because it affects the level and the dynamics of an innovator's demand. It is shown that redistribution from very rich to very poor consumers can be beneficial for growth. In general, the growth effect depends on the nature of redistribution. Due to a demand externality from R&D activities, multiple equilibria are possible.

JEL classification: 014, 015, 031, H23

Keywords: innovation, growth, inequality, hierarchic demand, multiple equilibria

1. Introduction

This article investigates the impact of income inequality on economic growth when technical progress is driven by innovations and consumers have hierarchic preferences. When consumers have hierarchic preferences, the structure of demand is affected by the distribution of income. Poor people concentrate most of their expenditures on basic needs, whereas rich people direct their expenditures to luxurious goods. The empirical relevance of a hierarchic structure of demand is well documented: it is featured by Engel's law, according to which the expenditure share for food decreases with income.

When demand is affected by the income distribution, inequality may be an important determinant of innovation and growth. The empirical importance of the inequality-growth relationship is a matter of discussion in the empirical literature. A number of earlier studies have found a robust negative correlation between growth rates and income inequality in cross-country regressions (Persson and Tabellini, 1994; Alesina and Rodrik, 1994; Clarke, 1995; and in particular Perotti, 1996). While more recent work by Deininger and Squire (1998) casts doubt on the robustness of the relationship between growth and the distribution of *income*,[1] they find a strong negative relationship between long-run growth rates and initial inequality in the distribution of *assets* (as proxied by land distribution).

While recent research has extensively dealt with the question how income inequality affects the long-run growth performance of economies, little attention has been paid to the role of the income distribution for product demand and the resulting impact on innovations. Instead, much of the recent literature has either focused on the role of capital-market imperfections (see Galor and Zeira, 1993; Banerjee and Newman, 1993; Aghion and Bolton, 1997; and others) or on political mechanisms (Bertola, 1993; Persson and Tabellini, 1994; Alesina and Rodrik, 1994; and others).[2] In contrast, the present article focuses on the role played by inequality on the dynamics of an innovator's demand and relies on neither imperfect capital markets nor politico-economic arguments.

Demand effects on the incentives to innovate have not attracted much attention in the inequality-growth literature. This is surprising given the generally accepted view that innovations are an important source of economic development and technical progress. In the standard Schumpeterian growth models, consumers have homothetic preferences. By this assumption, the level of demand for the various goods—including the innovator's product—does not depend on income distribution. While the assumption of homothetic preferences has turned out convenient in incorporating monopolistic competition into a general equilibrium framework, it is highly questionable from an empirical point of view. The vast majority of studies of consumer behavior reject the hypothesis of homothetic preferences (see Deaton and Muelbauer, 1980).[3]

A hierarchy of wants implies that goods can be ranked according to their priority in consumption. In this article, hierarchic preferences are introduced in a stylized way. To satisfy a certain want, consumers buy one unit of an indivisible good. The utility of a consumer depends then on how many wants can be satisfied—that is, on the number of consumed goods. The level of market demand for a specific good is affected by the distribution of income because this determines how many consumers can afford it. In a dynamic context, hierarchic preferences imply that inequality determines how the level of demand for a particular good evolves. Today, the good of an innovator may be purchased only be a small group of rich people. But as incomes grow, the size of the market grows as less wealthy people also become willing to buy. The novel aspect of this article is to study how income distribution affects the *time path of demand* for the innovator's good and therefore the reward for an innovation.

As far as the supply side is concerned, the model captures the main features of the standard innovation-driven growth models. In particular, it is assumed that each innovation project increases the stock of public knowledge and thus the productivity level in the whole economy. However, there are differences between the present model and the standard ones. In the present model innovations are *new methods* to satisfy *wants*. These methods can be process innovations leading to more efficient production of a particular consumption good, or they can be product innovations replacing existing goods that are produced rather resource-intensive. This differs from the models of Romer (1987, 1990) and Grossman and Helpman (1991), where innovators introduce new intermediate inputs that are then used by all final output producers or where innovators introduce additional products. In the present model, innovators drive less efficient producers (a competitive fringe) out of the market but are never displaced themselves. This is different from Aghion and Howitt's (1992) framework, where successive innovations take place within the same market and the current innovator disappears with the next innovation.

As mentioned above, inequality affects growth because it affects the time path of demand faced by an innovator. The underlying mechanisms can be illustrated by studying a population with two groups of consumers who differ in income. We focus on the interesting case where the rich but not the poor are willing to buy the good of the most recent innovator. Within this set-up it will be shown that inequality is bad for growth. The intuition for this result is this: when income is concentrated among a few, the initial market for a new product is small; and when the remaining consumers are very poor, it takes a long time until the size of the market becomes larger. A redistribution from the rich to the poor does not change the initial market size of the most recent innovator if the rich reman rich enough to purchase this product. At the same time this redistribution leads to a more rapid increase in the size of the market of the most recent innovator because the improved income situation of the poor allows them to purchase this product in the nearer future. This increases the profitability of an innovation.

An interesting aspect of the model is the possibility of *multiple equilibria*. Two identical economies can end up in either a high- or low-growth regime. Multiplicity is the result of a complementarity between present and future R&D activities. If the expected future innovation rate and therefore the growth rate is high, current innovators can expect that their own markets will grow more rapidly. This creates an incentive to conduct more R&D today. This complementarity between present and future innovations comes into play for two reasons. On the one hand, higher growth leads to a more favorable time path of demand for an innovator and thus increases the incentives to innovate. On the other hand, due to technological spillovers, more innovative activities lead to a higher growth rate. In other words, the expected innovation rate creates a demand externality that influences the attractiveness of present innovations. If all agents are optimistic, innovation is profitable and growth will be high, with pessimistic expectations the opposite is the case.

The role of inequality and hierarchic preferences in the context of economic development has been studied in a number of other papers. The present article is related to that of Murphy, Shleifer, and Vishny (1989a). They show that the adoption of efficient methods of production requires large markets and that excessive concentration of wealth may be an obstacle to economic development. However, Murphy, Shleifer, and Vishny (1989a) focus on a static framework. As a consequence, changes in income distribution matter only if the demand of the marginal firm is affected. This is different from the present model, where not only the level but also the time path of demand affects growth. Moreover, the equilibrium in their model is always unique, whereas my model generates multiple equilibria.[4] Finally, their paper emphasizes on the importance of the agricultural sector in generating the necessary demand to promote industrialization. In contrast, the present article elaborates the idea that growth is driven by industrial R&D.

The importance of a hierarchic structure of demand is also emphasized by Eswaran and Kotwal (1993). If initially workers are too poor to buy manufacturing goods, productivity progress in the manufacturing sector will not trickle down and real wages cannot grow. A more even distribution of wealth as well as openness to international trade are mechanisms to escape underdevelopment. Baland and Ray (1991) consider a situation where a highly unequal distribution of assets generates a high demand for luxury goods. Since basic goods and luxuries are produced by the same resources, unemployment limits the demand for basic goods and allows to cover a high demand for luxuries. Like that of Murphy, Shleifer, Vishny (1989a) these papers stick to a static framework. Income distribution has an impact on the level of income but no effect on the rate of growth.[5]

Only recently has the literature begun to analyze the impact of income inequality and hierarchic demand on growth. Chou and Talmain (1996) highlight mechanisms also present in the current model. Their consumers have preferences over a standard commodity (leisure) and goods produced in a Grossman and Helpman–type innovation sector. If the demand for leisure is not linear in income, inequality affects growth. In the model of Chou and Talmain (1996) new goods are always consumed by all, rich *and* poor consumers. In contrast, I study the potentially important case in which not all consumers can afford an innovator's good. As a consequence both the level and time path of demand are affected by the distribution of income. Furthermore, in Chou and Talmain (1996) the equilibrium is always unique, whereas my model generates multiple equilibria. Also Falkinger (1994) studies the impact of income distribution on product development when consumers have a hierarchic structure of demand. He finds that the impact of income inequality on growth depends on the type of technological spillovers (innovations versus learning-by-doing). In this model firms live only for one period, whereas in my model innovators live forever. This allows me to study the behavior of innovators who have to consider how demand develops over time.

In the above papers income distribution affects the level of demand but has no impact on the prices of new goods. The papers by Glass (1996), Li (1996), and Zweimüller and Brunner (1996, 1998) are complementary to the present article because of their focus on prices. Within a quality ladder framework inequality has an impact on the incentives to innovate by affecting the willingness to pay for quality. The equilibrium price structure among goods of different qualities is affected by the income distribution.

This article is organized as follows. Section 2 presents the model, and Section 3 studies the innovation decision in detail. Section 4 studies the general equilibrium and analyzes the relationship between inequality and growth. In Sections 2 to 4, the focus is on a situation with only two groups of consumers, rich and poor. Section 5 extends the model to a general distribution. Section 6 concludes.

2. The Model

2.1. Technology

Consider an economy in which many different consumer goods are produced to satisfy the consumers' wants, indexed by j. A certain want j can either be satisfied by goods produced by a "modern" firm of by a "traditional" producer. Modern firms produce with an efficient increasing-returns technology, whereas the traditional firms produce with a less efficient constant-returns technology. While access to the traditional technology is free, access to the modern technology requires an "innovation."

Before describing the two technologies in more detail, it is important to clarify what is meant by *innovation* in this model. An innovation introduces a *new method* to satisfy a given *want*. Such a new method can either be a new method of production for an already existing good, or it can be a new (physically different) good that replaces an existing traditionally produced product. When a modern good and a traditionally produced good that satisfy the *same* want are physically different, they are assumed to be perfect substitutes in consumption. To give an example, consider the market for household services. In early stages of development, only the very rich had a market demand for household services, and the respective wants were satisfied with a relatively inefficient technology: through work done by domestic servants. As incomes grew over time, it became attractive for innovators to introduce new methods to satisfy the same wants like the washing machine, the vacuum cleaner, the dishwasher, and so on.[6]

Labor is the only production factor. To make an innovation at time t, a firm has to incur an R&D input of $a_r(t)$ labor units before output can be produced with a unit labor input $a_m(t)$. The traditional technology requires no R&D costs, and output can be produced with a unit labor input $a_c(t)$. Once invented, the modern technology is more efficient than the traditional technology—that is, $a_c(t) > a_m(t)$ for all t. It is also assumed that the input coefficients $a_c(t)$, $a_m(t)$ and $a_r(t)$ are the same for all wants—that is, independent of j.

Over time the economy becomes more productive as a result of innovations. As in most innovation-driven growth models, it is assumed that researchers of future generations build on experience of previous innovations. This means that the necessary R&D input $a_r(t)$ is decreasing in the amount of past innovations. It is also assumed that the labor requirements in final output production, $a_c(t)$ and $a_m(t)$, are decreasing in the amount of past innovations.[7] Denote by $n(t)$ the number of innovations that took place up to time t, then the labor input coefficients in the three sectors of the economy are

$$a_l(t) = a_l/n(t) \qquad \text{for} \qquad l = r, c, m. \tag{1}$$

2.2. *The Time Path of Prices and Wages*

The focus of this article is on a steady-state situation in which $n(t)$ grows at a constant but endogenously determined rate g. This means that the labor input coefficients $a_c(t)$, $a_m(t)$ and $a_r(t)$ are *decreasing* at rate g. If production of some good takes place with the constant-returns technology, the market for this good is competitive, and the price for such a product equals $p_c(t) = w(t)a_c(t)$. For the remainder of the article, I choose the competitively produced goods as the numeraire, so for all t, $p_c(t) = 1$. Using equation (1) the wage rate is then given by

$$w(t) = 1/a_c(t) = \frac{n(t)}{a_c}. \tag{2}$$

Equation (2) says that wages increase with productivity (the inverse of the unit labor requirement) and since productivity grows at the same rate as $n(t)$—the rate g—also the wage grows at rate g. Moreover, because $a_m(t)$ and $a_r(t)$ are decreasing at rate g, $w(t)a_m(t)$ and $w(t)a_r(t)$ are constant over time. It is assumed that the labor market is competitive, so also the R&D sector and the modern sector pay the wage $w(t)$.

If production takes place with the increasing-returns technology, the market is served by a single monopolist. If this monopolist charges a price higher than unity, he triggers entry from the competitive fringe and loses all customers. Consequently, no monopolistic firm has an incentive to set a price higher than unity. If the monopolist charges a price equal to unity, it is assumed that the monopolist gets the entire market and the traditional producers get nothing. It will be assumed throughout the article that no monopolist ever charges a price lower than unity. This assumption is made to keep the analysis tractable.[8]

When the price is unity, a monopolistic firm earns a unit profit $1 - w(t)a_m(t)$. Using equations (1) and (2) we can write $1 - w(t)a_m(t) = 1 - a_m/a_c = \pi > 0$. Notice that π is independent of t since the marginal cost $w(t)a_m(t)$ is constant over time.

2.3. Consumers

Consumers have preferences over a hierarchy of wants. A want in this hierarchy is captured by an index j, where $j \in [0, \infty)$. A low value of j indicates a basic need, whereas higher j's indicate more luxurious wants. To satisfy want j, a consumer has to buy one unit of a certain good, which is called good j. It is assumed that goods are indivisible and consumption is a zero-one choice. If want j is satisfied, the consumer derives additional utility $1/j$ when consuming a good $j > 1$ and additional utility 1 when consuming a $j \in [0, 1]$.[9]

Let us now consider the decision whether to consume good j. Denote by $\lambda_i(t)$ the marginal utility of wealth of consumer i at date t and recall that, for all j and t, the price is unity. Let us consider a good satisfying a want $j > 1$. The consumer will buy good j if the price in utility terms, which is $\lambda_i(t) \cdot 1 = \lambda_i(t)$, must be smaller than or equal to the additional utility a consumer derives when consuming good j, which is $1/j$. This means we must have $\lambda_i(t) \leq 1/j$. As far as the wants $j \in [0, 1]$ are concerned I assume that, at all times, all consumers can afford to satisfy these wants. Since the additional utility derived from satisfying wants $j \in [0, 1]$ is 1, this is the case if $\lambda_i(t) < 1$ for all i at all t.[10]

Notice that the price in utility terms $\lambda_i(t)$ is equal for all j but that the additional utility a consumer derives from consuming good j, $1/j$, is decreasing in j. Denote by $c_i(t)$ the value of j that satisfies $\lambda_i(t) = 1/c_i(t)$. Then we have $\lambda_i(t) \leq 1/j$ for all $j \leq c_i(t)$ and $\lambda_i(t) > 1/j$ for $j > c_i(t)$. This means consumer i satisfies the first $c_i(t)$ wants in the hierarchy, whereas no good that satisfies a want higher than $c_i(t)$ is purchased. This means that $c_i(t)$ is the most luxurious want in the hierarchy, that consumer i can afford to satisfy. Note also that $c_i(t)$ is a measure for the level of consumption since all goods in the interval $[0, c_i(t)]$ are consumed; and—since all prices are unity—it also measures consumption expenditures.

Let us now derive consumer i's instantaneous indirect utility. Recall that all goods in the range $[0, 1]$ yield utility 1, whereas higher-j goods yield utility $1/j$. When all goods up to some $c_i(t) > 1$ are consumed, the instantaneous indirect utility $u_i(t)$ is given by

$$u_i(t) = \int_0^1 1dj + \int_1^{c_i(t)} \frac{1}{j} dj = 1 + \ln(c_i(t)). \tag{3}$$

Assume that consumers have an infinite horizon and their lifetime utility is

$$U_i = \int_0^\infty u_i(t)e^{-\theta t} dt = \int_0^\infty [1 + \ln(c_i(t))] \, e^{-\theta t} dt,$$

where θ denotes the rate of time preference. While all consumers are assumed to earn the same wage $w(t)$, they are different with respect to asset ownership $A_i(t)$. (In Section 2.4 I discuss the wealth distribution in more detail.) There is a perfect capital market with interest rate r, which is constant over time since only steady states will be considered. No pure profits accrue in equilibrium (see Section 3). Consumers have perfect foresight and take as given that wages grow at rate g. The lifetime budget constraint can then be written as

$$\int_0^\infty c_i(t)e^{-rt}dt \leq A_{i0} + \int_0^\infty w_0 e^{-(r-g)t}dt,$$

where w_0 and A_{i0} are initial wages and assets, respectively. In the steady-state equilibrium also assets and consumption grow at rate g. It is straightforward to show that the consumption path is given by the following relations

$$g = r - \theta, \tag{4}$$

and

$$c_i(0) = w_0 + \theta A_{i0}. \tag{5}$$

Equation (4) is the Euler-equation that results from logarithmic preferences. Equation (5) states that the level of consumption is the sum of labor income and a fraction of assets equal to the rate of time preference.[11]

2.4. The Distribution of Wealth

To keep things simple, I first focus on a two-class society with rich (R) and poor (P) consumers. Section 5 extends the analysis to a more general distribution. Consumers of group R and P have equal preferences and earn the same wage but own different wealth levels. Denoting by L the size of the population and by β the group share of the poor, we have βL poor and $(1-\beta)L$ rich consumers. Furthermore, let d_i be the ratio of the value of assets owned by household i relative to the average. Poor consumers own less, so $0 \leq d_p < 1$. Rich consumers own more, so $d_R > 1$. The corresponding fractions in aggregate wealth are βd_p for the poor and $(1-\beta)d_R$ for the rich. These two terms must sum up to 1, and one can solve for $d_R = (1-\beta d_p)/(1-\beta) > 1$. Consequently, inequality decreases in d_p and increases in β (holding d_p constant).

Figure 1 shows the resulting Lorenz-curve. Given our assumptions, the Lorenz-curve is piecewise linear, with slopes d_p and $(1-\beta d_p)/(1-\beta)$ along $0A$ and AB, respectively. From inspection of the Lorenz-curve it is evident that increasing d_p, holding β constant, shifts the Lorenz-curve closer toward the diagonal. An increase in β, holding d_p constant, shifts A to A'—that is, a larger population share of group P shifts the Lorenz-curve out since, for given d_p, relative wealth of a rich household $d_R = (1-\beta d_p)/(1-\beta)$ becomes larger. In sum, an increase in d_p and a decrease in β (holding d_p constant) is associated with less inequality.

Aggregate wealth consists of firm shares. These firms earn a flow profit and the value of a firm k, $v_k(t)$, equals the present value of this flow profit. The value of wealth in the economy $V(t)$ is then the aggregate value of firms, that is the integral of $v_k(t)$ over the interval $[0, n(t)]$ where $n(t)$ is the measure of existing firms. The value of assets of consumer i is then

$$A_i(t) = d_i V(t)/L. \tag{6}$$

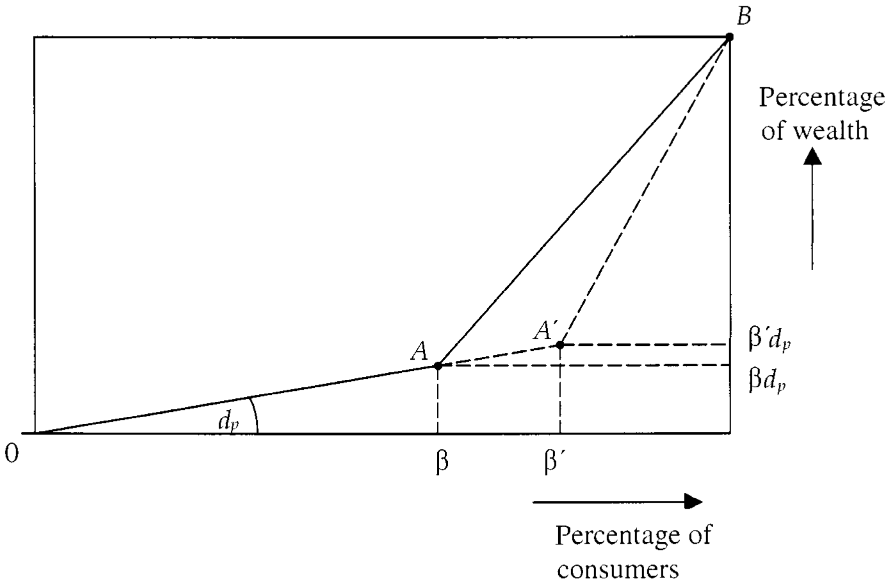

Figure 1. The distribution of assets.

2.5. The Resource Constraint

Total labor supply in the economy is L. The demand for labor comes from three different sectors: the R&D, the traditional sector, and the modern sector. Denote by $\dot{n}(t)$ the level of innovative activities at date t, then the number of R&D employees is $\dot{n}(t)a_r(t)$. Let $Y_m(t)$ denote total production in the monopolistic sector, then $Y_m(t)a_m(t)$ is employment in this sector. Finally, let total output by the competitive fringe be $Y_c(t)$, then $Y_c(t)a_c(t)$ is employment in the competitive sector. The labor-market equilibrium condition can be written as $L = \dot{n}(t)a_r(t) + Y_m(t)a_m(t) + Y_c(t)a_c(t)$.

The open question is: what determines the output levels in the competitive and the monopolistic sector? In other words: which wants are satisfied by monopolistic firms, and which ones are satisfied by the competitive fringe? It will become clear below that, to make it attractive for a monopolistic firm to incur the necessary R&D cost, it is crucial to have a large enough market. Clearly, the market is at its largest possible level for those goods that are consumed by all households. That is the case for all goods that satisfy wants $j \in [0, c_p(t)]$ where $c_p(t)$ is the most advanced want satisfied by the poor. These markets are the most attractive for modern producers. On the other end of the spectrum there are the most luxurious goods, which are bought only by the rich. These sectors are less attractive for modern producers, since demand is comparatively low.

So far, this is a static argument. In a dynamic context with ongoing innovations and growing incomes, the market size for goods that satisfy a certain want grows over time. Suppose that up to time $t, n(t)$ innovations have taken place. In a steady-state situation the chronological sequence of innovations follows the hierarchy of wants, meaning that the first innovations developed new methods for very low-j wants, whereas new methods to satisfy higher-j wants are introduced not until later stages of development. Suppose further that, once an innovator has conquered a market, he becomes the incumbent on this market and stays there forever. Then, at a given date t, the lowest-j goods are produced by monopolistic firms, whereas the highest-j goods are produced by the competitive fringe.

Now consider a situation where the most recent innovator—the firm that produces good $n(t)$—sells to the rich but not to the poor. This situation emerges when the rich can afford to satisfy more than the first $n(t)$ wants, whereas the poor cannot afford the first $n(t)$. For the first $n(t)$ wants, innovations have taken place and the corresponding goods are supplied by monopolistic firms. Thus the poor buy the subset $c_p(t) < n(t)$, all of which are produced by monopolistic suppliers, whereas the rich buy all $n(t)$ goods produced by monopolists plus additional goods to satisfy wants higher than $n(t)$. These goods are supplied by traditional producers on competitive output markets. Output in the monopolistic sector is then $Y_m(t) = c_p(t)\beta L + n(t)(1-\beta)L$, and output in the competitive sector is given by $Y_c(t) = [c_R(t) - n(t)]\,(1-\beta)L$. In a steady state, $n(t)$, $c_p(t)$, and $c_R(t)$ grow at the same rate g. Using equation (1) and the definitions $x_i = c_i(t)/n(t)$, $i = R, P$ and $g = \dot{n}(t)/n(t)$, the resource constraint can be written as

$$L = a_r g + a_m\left[(1-\beta) + \beta x_p\right]L + a_c(1-\beta)(x_R - 1)L. \tag{7}$$

The outcome that the rich buy from both the monopolistic and the competitive producers, whereas the poor buy only from monopolistic producers deserves some discussion. The result that the rich buy from competitive producers follows from the assumption that, in principle, *all* wants can be satisfy by products supplied by the inefficient traditional producers. Access to this technology is free, and whenever there is demand, traditional producers can enter without cost and serve the corresponding market. In contrast, modern technology is available only after an innovation. In the next section, we see that it pays to incur the necessary innovation cost only if there is sufficient demand for the corresponding good. If demand is too low—which is the case for more luxurious wants that only the rich are willing to satisfy—the R&D cost will not pay off. This means that the most advanced wants—"the fancies of the rich"—are satisfied by the competitive producers.[12]

The outcome that the poor buy only from monopolistic firms follows from the assumption that an innovator stays on the market forever, which could be the result of infinitely lived patents. The outcome is different when patents expire, so that after a while the monopoly position disappears and the market becomes competitive. Given the chronological sequence of innovations, the patents that expire first are those for the lowest-j markets. Thus, with *finite* patent duration, the poorest consumers will buy predominantly from competitive firms. Introducing finite patent length would complicate the model but would not add new insights into the inequality-growth tradeoff, as long as the poorest households will buy an innovator's good before patents expire. To keep things simple, it will be assumed throughout the article that innovators keep their monopoly position forever.

3. Innovations

3.1. Demand and the Value of a Monopolistic Firm

The level of demand of a monopolistic firm is equal to the number of consumers that can afford his good. With growing incomes, the level of demand for this good will change over time. The focus here is on a situation where initially only rich consumers buy; the monopolist can serve the entire market not until the poor have become rich enough. Denote by Δ the time it takes until the poor are willing to buy a good $j > c_p(t)$ that is currently purchased only by the rich. I can write $c_p(t+\Delta) = c_p(t)e^{g\Delta} = j$. Solving this for Δ yields $\Delta = -(1/g)\ln(c_p(t)/j)$. (Notice that Δ is positive since $\ln(c_p(t)/j) < 0$.) Obviously, Δ is decreasing both in g and in $c_p(t)$.

The value of a firm supplying a good $j \in \left[c_p(t), n(t)\right]$ can now be written as[13]

$$\begin{aligned} v_j(t) &= \pi \int_t^{\infty} D_j(\tau)e^{-r(\tau-t)}d\tau \\ &= \frac{L\pi}{r}\left[(1-\beta)+\beta e^{-r\Delta}\right] = \frac{L\pi}{r}\left[(1-\beta)+\beta\left(\frac{c_p(t)}{j}\right)^{r/g}\right]. \end{aligned}$$

While the level of demand makes a discrete jump at date $t+\Delta$, the value of a firm increases smoothly over time. The reason is that the cash flow to be earned from the poor is discounted and the corresponding discounted value increases as $t+\Delta$ approaches. From the above equation, the discount factor equals $[c_p(t)/j]^{r/g} < 1$. Clearly, discounting depends on the distance between the most luxurious good currently purchased by the poor, $c_p(t)$, and the good under consideration, j. As this gap decreases over time, the discount factor and the firm value increase smoothly. When the poor can afford good j, the firm value reaches $L\pi/r$ and stays there forever.

3.2. The Entry Decision

Entering a market is profitable as long as the necessary R&D costs are not larger than the reward to an innovation. Using equations (1) and (2) the R&D cost can be expressed as $a_r(t)w(t) = a_r/a_c$, constant over time.

The reward to an innovation equals the present value of the subsequent profit flow. Denoting by $n(t)$ the good supplied by the most recent innovator, replacing j by $n(t)$ in the above expression for $v_j(t)$ and using the definition $x_p = c_p(t)/n(t)$, the value of the most recent innovator is equal to $v_n(t) = \frac{L\pi}{r}\left[(1-\beta)+\beta x_p^{r/g}\right]$.

With free access to the R&D technology the equilibrium is characterized by zero profits, which means in equilibrium the value of an innovation may not be larger than the R&D costs. The equilibrium condition is therefore $a_r/a_c \geq v_n(t)$. This condition holds with equality when innovations take place in equilibrium—that is, when $g > 0$. Using the above

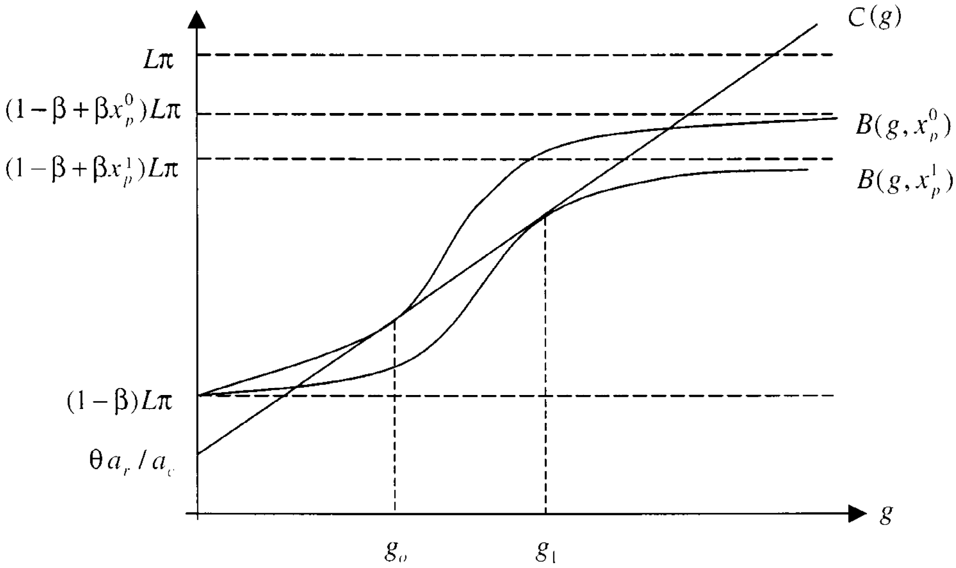

Figure 2. No-profit equilibria.

expression for $v_n(t)$ and the fact that from equation (4) the interest rate equals $r = g + \theta$, the zero-profit equilibrium can be expressed as

$$(g+\theta)(a_r/a_c) = L\pi\left[(1-\beta) + \beta x_p^{(g+\theta)/g}\right], \quad \text{or} \quad C(g) = B(g, x_p). \tag{8}$$

The left-hand side of (8) are the *current* costs of an innovation $C(g)$: the interest cost of investing in a new firm. In equilibrium these costs must not be smaller than the *current* returns from an innovation $B(g, x_p)$. These returns consist of a flow profit resulting from sales to the rich $(1-\beta)L\pi$ plus the increase in the firm value, $\beta L\pi(x_p)^{(g+\theta)/g}$.

Figure 2 draws both sides of equation (8) against g. $C(g)$ is a straight line with slope a_r/a_c and intercept $\theta(a_r/a_c)$. The innovation costs are increasing in g because in equilibrium a higher growth rate goes hand in hand with a higher interest rate (see equation (4)). $B(g, x_p)$ has the shape of a logistic curve. It equals $(1-\beta)L\pi$ for $g = 0$, is convex over the range $g \in [0, -\ln(x_p)\theta/2]$, and concave for $g \geq -\ln(x_p)\theta/2$. For $g \to \infty$, $B(g, x_p)$ approaches $(1-\beta+\beta x_p)L\pi$.

The current returns to an innovation $B(g, x_p)$ depend on the growth rate because the discount factor $x_p^{(g+\theta)/g}$ depends on the growth rate. There are two different effects of g on $B(g, x_p)$. On the one hand, a rise in g means that the profits from the poor accrue earlier since their incomes grow faster. This increases the incentive to innovate. On the other hand, a larger g increases the interest rate: future profits have to be discounted at a higher rate, which reduces the reward to an innovation. The former effect always dominates the latter meaning that $B(g, x_p)$ is increasing in g. But for $g \to 0$ and $g \to \infty$ the net effect goes to zero.

The current reward to an innovation is not only affected by the growth rate but also by the level of consumption of poor people x_p. When the poor have already a high consumption level, it takes not much time until they can afford the innovator's product. This raises the payoff of an innovation. In Figure 2, a higher x_p means that $B(g, x_p)$ shifts upward.

It is straightforward to verify from Figure 2 under which conditions an equilibrium exists. $C(g)$ is linear in g and $B(g, x_p)$ tends to $(1 - \beta + \beta x_p)L\pi$ as g becomes large. Thus the two lines in Figure 2 have at least one intersection, as long as $B(0, x_p) > C(0)$—that is, the benefit-curve starts above the cost-curve in Figure 2, which is the case if $\theta(a_r/a_c) < (1 - \beta)L\pi$. This intersection is unique if the rate of time preference θ is sufficiently small.[14]

For larger θ multiple equilibria are possible. θ has an impact on the curvature of the $B(g, x_p)$-function, and a situation like the one depicted in Figure 2 can arise. In this case there may either be a unique equilibrium ($x_p > x_p^0$ or $x_p < x_p^1$) or multiple equilibria ($x_p^0 \geq x_p \geq x_p^1$).[15]

4. Growth and Income Distribution

The previous section was concerned with the equilibrium innovation rate g taking the consumption level x_p as given (equation (8)). This is only a partial equilibrium. The general equilibrium has also to consider which combinations of growth and consumption are feasible given the economy's resource constraint (equation (7)). And it has to take into account that consumption choices have to be optimal. From equation (5), using (1), (2), and (6), the optimal consumption levels of household i, $x_i = c_i(t)/n(t)$ can be expressed as

$$x_i = 1/a_c + \theta d_i \overline{v}/L, \ i = R, P, \tag{9}$$

where $\overline{v} = V(t)/n(t)$ denotes the average value of all monopolistic firms.[16]

Equations (7) to (9) form a system of four equations with four unknowns: x_p and x_R, g, and $\overline{v}$. I discuss the equilibrium by reducing this system to two equations in g and x_p. The first one is equation (8), where only g and x_p appear. The second equation is obtained from (7) and (9), both of which are linear and can be conveniently reduced to one equation in g and x_p.[17] This yields

$$x_p = \frac{1}{a_c} + d_p \frac{L\pi(1 - \beta + \beta/a_c) - g a_r/a_c}{L(1 - \beta d_p \pi)}. \tag{10}$$

From equations (8) and (10) the equilibrium growth rate g and the equilibrium level of consumption of the poor x_p can be determined. I proceed as follows. In Section 4.1 I establish under which conditions a unique general equilibrium exists. Section 4.2 discusses the main topic of this article—the relationship between inequality and growth. Section 4.3 considers the possibility of multiple equilibria. The latter possibility arises because the zero-profit equilibrium may not be unique (see Section 3).

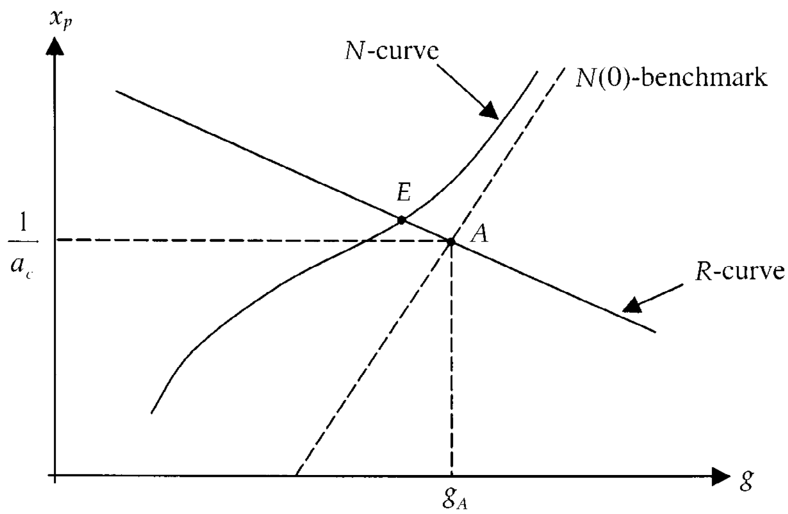

Figure 3. A unique general equilibrium.

4.1. A Unique General Equilibrium

To study the conditions under which an equilibrium exists and when it is unique, it is convenient to draw equations (8) and (10) in (g, x_p)-space (Figure 3). The N-curve in Figure 3 represents the no-profit condition (8). The R-curve represents equation (10), which captures the resource constraint plus optimal consumption choices.

The shape of the N-curve follows from the discussion in Section 3. There we have seen that the no-profit equilibrium is unique for all consumption levels of the poor x_p, if the rate of time preference θ is sufficiently low. It can be easily verified from Figure 2 in Section 3 that an increase in x_p shifts the B-curve up and leads to an unambiguous increase in the growth rate g if the no-profit equilibrium is unique. In terms of Figure 3, the N-curve is upward sloping. This has a clear intuition: more consumption by the poor makes innovation more profitable and thus lead to higher growth.

The R-curve is linear and has a negative slope, see equation (10). This has also a clear intuition: if the poor choose a high consumption level, a high fraction of available resources is needed for the production of consumer goods. Consequently, only a small amount of resources are left for R&D, and only a low growth rate is feasible.

It remains to show under which conditions the N-curve and the R-curve in Figure 3 intersect at a positive g. Recall that this intersection must be over the range $[1/a_c, 1)$ because consumption never falls below the wage rate ($x_p \geq 1/a_c$, see equation (9)) and because the poor do not buy the good of the most recent innovator ($x_p < 1$).[18] To show that such an intersection exists, it turns out convenient to consider the hypothetical N-curve when the rate of time preference θ becomes 0 (the $N(0)$-curve in Figure 3). For $\theta > 0$ the N-curve lies to the left of this benchmark and approaches it smoothly as θ tends to zero.[19]

When $\theta = 0$, the no-profit condition (8) simplifies to $g(a_r/a_c) = L\pi[(1-\beta)+\beta x_p]$; thus the $N(0)$-curve is a straight line. It is straightforward to verify that the $N(0)$-curve and the R-curve (equation (10)) intersect at $x_p = 1/a_c$ and $g_A = L\pi(1 - \beta + \beta/a_c)/(a_r/a_c) > 0$ (point A in Figure 3). Since the N-curve approaches the $N(0)$-benchmark smoothly as θ tends to zero, there is always a sufficiently small θ that guarantees an intersection with positive growth (point E in Figure 3). Moreover, the equilibrium growth rate will be strictly smaller than g_A, and the consumption level of the poor will not fall below $1/a_c$. (A sufficiently small wealth position d_p guarantees furthermore that the consumption level of the poor will not rise above unity and remains in the required range $x_p \in [1/a_c, 1)$.)

4.2. Inequality and Growth

The main goal of this article is to analyze the impact of inequality on growth. The parameters of particular interest are therefore the distribution parameters d_p and β. In the following I consider the impact of changes in these two parameters on the equilibrium growth rate under the assumption that there is a unique general equilibrium. (Multiple equilibria will be discussed in Section 4.3). It turns out convenient to do this comparative steady-state analysis graphically. We therefore have to look at how changes in the parameters d_p and β shift the N- and the R-curves.

4.2.1. An Increase in the Wealth Position of the Poor $\boldsymbol{d_p}$ An increase in d_p represents a decrease in inequality. How does such a change affect the growth rate? From equation (8) it is evident that the N-curve is independent of the distribution parameter d_p. The R-curve, however, is affected by a change in d_p. For $d_p = 0$, the R-curve equals the horizontal line $x_p = 1/a_c$ and rotates clockwise around the point A as d_p increases. Point A in Figure 4 corresponds to $x_p = 1/a_c$ and $g_A = L\pi(1 - \beta + \beta/a_c)/(a_r/a_c)$. Recall from Figure 3 above that a possible intersection between the R- and and N-curves occurs at $g < g_A$. This means that an increase in d_p shifts the R-curve up, over the relevant range of g. The impact of an increase in d_p is an unambiguous increase in g (from point E to point E_1 in Figure 4).

The reason that an improved wealth position of the poor increases growth is the change in the composition of aggregate consumer demand and its impact on the efficiency of production. The most luxurious goods consumed by the rich are produced with the inefficient technology, whereas the most luxurious goods consumed by the poor are produced with the more efficient modern technology. As a result of the redistribution the rich reduce and the poor increase their consumption. In the new equilibrium a higher proportion of final output is produced in the modern sector, and production becomes more efficient. This gain in productivity releases resources that can be employed in the R&D sector, resulting in higher growth.

In the new equilibrium the consumption level of the rich x_R is lower, meaning the increase in growth and in consumption by the poor go at the expense of consumption by the rich. The fact that growth increases means that absolute consumption by the poor $c_p(t)$ initially jumps to a higher level and has then a steeper path. Consumption by rich people $c_R(t)$ jumps to a lower level but, due to higher growth, will be above the original path in finite time.

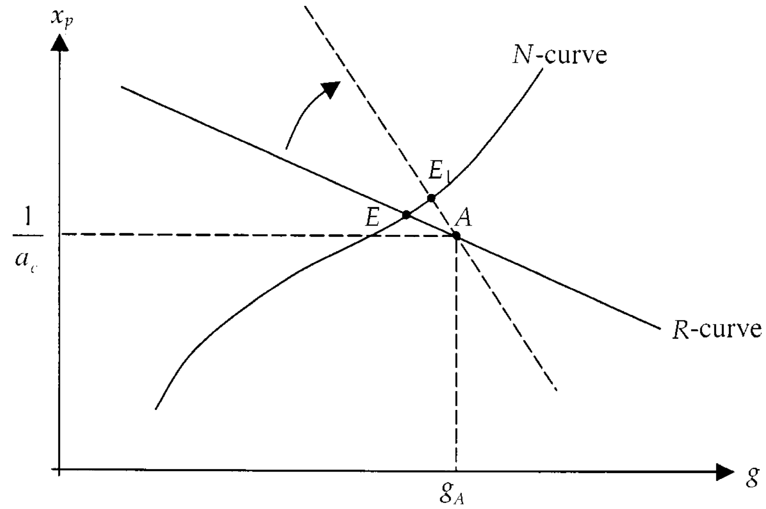

Figure 4. An increase in d_p.

4.2.2. An Increase in the Population Share of the Poor β *(Holding* $\boldsymbol{d}_p$ *Constant)* If β increases and d_p is held constant, this leads to an outward shift of the Lorenz curve (see Figure 1) and therefore tends to increase inequality. A change in β affects both the N-curve and the R-curve. If β increases, it is straightforward to verify from equation (8) that the N-curve shifts up.[20] To see how the R-curve is affected, note that an increase in β leads to a clockwise rotation of the R-curve around the point B in Figure 5—that is, at $x_p = 1$ and $g_B = [\pi - (1 - 1/a_c)/d_p]L/(a_r/a_c)$. Since we consider only values of $x_p < 1$, this means the R-curve shifts down over the relevant range. In sum, an increase in β leads to a reduction in g (from E to E_2 in Figure 5). Because, for a given d_p, a higher β is associated with higher inequality, this also establishes a negative impact of inequality on growth.

The reason why a higher population share for the poor leads to a reduction in the growth rate is twofold. On the one hand, a higher population share of the poor means there are fewer rich people, and this makes innovations less profitable. This is because the size of the market in the early stage of production becomes smaller. This is the reason for the shift in the N-curve. On the other hand, a higher population share for the poor β (holding d_p constant) increases the wealth position of the rich (recall $d_R = (1 - \beta d_p)/(1 - \beta)$). The resulting increase in consumption is directed toward the competitive fringe and aggregate production becomes less efficient. This leaves less resources for R&D and leads to a lower growth rate. For this reason, the R-curve shifts upward. In the new equilibrium the growth rate g is lower, the impact on the level of consumption of the poor x_p is ambiguous, and the consumption level of the rich x_R is higher.

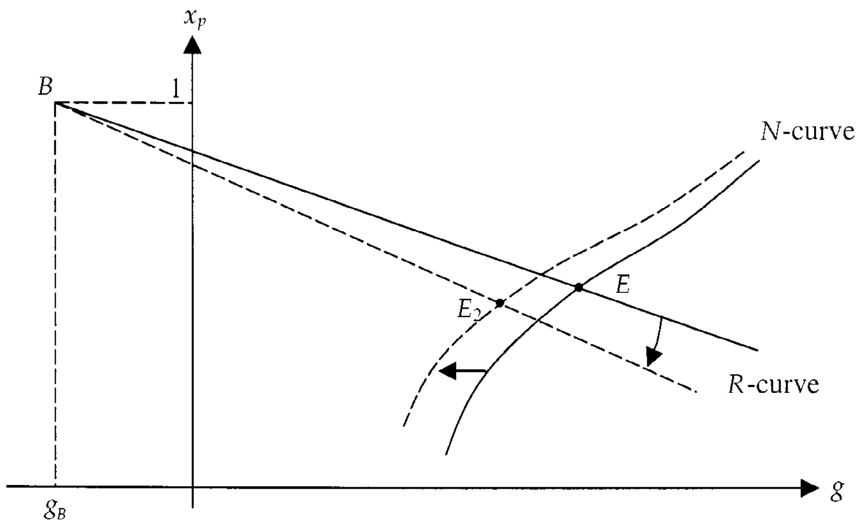

Figure 5. An increase in β.

4.3. Multiple Equilibria

The possibility of multiple general equilibria comes from the fact that, depending on the rate of time preference θ, there may be multiple no-profit equilibria (see Section 3). It is therefore instructive to look more closely at the shape of the N-curve for different values of θ. If the rate of time preference is high—that is, if $\theta \geq L\pi/(a_r/a_c)$—innovation is not profitable and the N-curve coincides with the vertical axis.[21] On the other hand, if $\theta \to 0$, the N-curve tends toward a straight line with a positive slope (the $N(0)$-curve of Figure 3). Figure 6 depicts an intermediate value of θ that is sufficiently large to generate a nonmonotonic N-curve. There are three points of intersection between the N- and the R-curves. The steady-state equilibrium could either be characterized by high growth and low consumption (like E_3 in Figure 6), low-growth and high consumption (E_5 in Figure 6) or something in between (E_4).[22]

Multiple equilibria arise in this model because innovation activities of present and future innovators are complementary. If current innovators expect high-innovation activities of future generations, they have an incentive to conduct more R&D. This complementarity is due to technological spillovers: a higher level of R&D activities generates a higher growth rate, and a higher growth rates leads to a more rapid development of an innovator's market. In other words, a higher expected innovation rate creates a positive demand externality that makes present innovations more profitable. Multiple equilibria are the result of a coordination problem in which the expectations about the future innovation rate determine whether the economy experiences high or low growth. The economy will be trapped in underdevelopment E_5 if agents are pessimistic and expect low growth. The prosperity path E_3 would be feasible, but there is no possibility to syn-

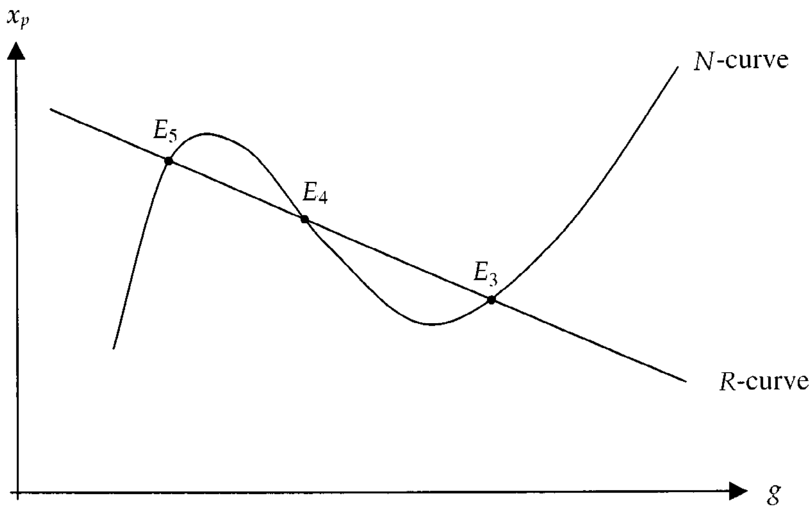

Figure 6. Multiple equilibria.

chronize expectations.[23] No agent has a reason to expect high growth when all others are pessimistic.

5. The Distribution of Wealth

The results derived in the previous section refer to a situation with only two groups of consumers. In this section I show that the results of this model carry over to the case of a general distribution. Suppose there are many different types $i = (1, \ldots, k, \ldots, K)$, ranked by wealth, so that a higher i indexes a type with more assets. Denote by k the number of types who are too poor to purchase the good of the most recent innovator. The remaining $K - k$ types can afford all goods supplied by monopolists plus luxuries from the competitive fringe.

Before we study the general case, assume first that $k = 1$ but $K - k \geq 2$. There continues to be only one group that cannot afford the most recent innovator's good, but there is more than one group that can afford this good. A redistribution between the latter relatively rich types has no impact on the demand of the most recent innovator because all these consumers buy this good anyway. Thus the most recent innovator's demand does not change, and the incentive to innovate remains the same. This means the N-curve remains unchanged. Also the R-curve is unaffected. The considered redistribution concerns only consumers who purchase luxuries produced by the competitive fringe. These consumers change their consumption of these luxuries (those who gain from the redistribution consume more, those who lose consume less), but the overall level of production in the competitive sector stays the same and thus the R-curve remains unchanged. As a result, redistribution among these relatively rich consumers has no impact on growth.

Now consider the general case when $k \geq 2$, so there are many types of consumers who cannot afford the product of the most recent innovator. Denote by β_i is the population share of type i and by $\beta = \sum_{i=1}^{k} \beta_i$ the population share of those who cannot afford the most recent innovator's product. Moreover denote by $\overline{x}_P = (1/\beta)\sum_{i=1}^{k} \beta_i x_i$ the average consumption level of those who *cannot* and by $\overline{x}_R = (1/(1-\beta))\sum_{i=k+1}^{K} \beta_i x_i$ the consumption level those who *can* afford all monopolistic goods. This leads to the same R-curve as in equation (7) (we only have to replace x_P and x_R by $\overline{x}_P$ and $\overline{x}_R$). However, the N-curve now changes to

$$\begin{aligned}(g+\theta)(a_r/a_c) &= L\pi\left[(1-\beta)+\sum_{i=1}^{k}\beta_i(x_i)^{(g+\theta)/g}\right] \\ &\geq L\pi\left[(1-\beta)+\overline{x}_p^{(g+\theta)/g}\right], \qquad (8')\end{aligned}$$

where the last relation is due to Jensen's inequality. What is the impact of a redistribution between households that cannot afford the most recent innovator's product? Equation (8) says that a more dispersed distribution among the relatively poor types is favorable for innovators. As a result of discounting it is favorable to shift a given profit flow closer to the present. (More precisely, it is better to have a profit flow of $\pi/2$ starting next year and another $\pi/2$ starting three years from now as opposed to a profit flow of π starting two years from now.) It follows that more inequality within relatively poor consumers enhances growth.

So far, we have focused on the impact of inequality *within* those who can and those who cannot afford the good of the most recent innovator. It remains to discuss the effect of redistribution *between* these two groups. However, this is what we have discussed in Section 4 above. For any redistribution between a type $i \leq k$ and a type $i > k$ there are exactly the same mechanisms at work as for a redistribution between type P and R in Section 4. From this discussion we know that more inequality *between* those who can and those who cannot afford the good of the most recent innovator is harmful for growth. A redistribution from the very rich to the very poor will be a redistribution from consumers who can afford the good of the most recent innovator to consumers who cannot. In this sense, redistribution from the very rich to the very poor increases the growth rate.

We can now easily characterize the distribution that maximizes the growth rate: the growth-maximizing distribution is the distribution that maximizes the level of demand of the most recent innovator. There are two possible scenarios. The first one allows a situation where even the poorest can afford the good of the most recent innovator. However, this is feasible only if the efficiency in production of the monopolistic sector is sufficiently high—that is, a_m must be sufficiently low.[24] The alternative scenario arises when the labor input in the monopolistic sector is high, so that a situation where all consumers buy all goods produced in the monopolistic sector is not feasible. In this case the wealth distribution that maximizes growth is such that the rich consumers have just enough wealth to consume all goods in the monopolistic sector but no goods from the competitive fringe. The poor consumers own no assets, earn only wage income, and consume only a subset of the monopolistic goods.

6. Conclusions

When consumers have hierarchic preferences, the structure and the dynamics of demand are affected by the distribution of income. Poor people consume more basic goods, whereas rich people direct their expenditure to more luxurious goods. The long-run growth rate depends on the distribution of income because it affects the time path of an innovator's demand.

How a change in income inequality affects the long-run growth rate depends on the optimal consumption levels of the consumers affected by the redistribution. First, a redistribution from consumers who can to those who cannot afford the good supplied by the most recent innovator leads to an increase in the growth rate. The reason is that after such a redistribution the market of an innovator grows faster, which increases the incentive to innovate. Second, redistribution among consumers who can afford the most recent innovator's product has no effect on the growth rate. This is because the level and the dynamics of demand for an innovator remain unaffected. Finally, a redistribution between households both of whom cannot afford the most recent innovator's product reduces the growth rate. This is because innovators are increasingly worse off when a given profit flow is shifted toward the future.

The model may generate multiple steady-state equilibria. With a sufficiently unequal distribution and a sufficiently high rate of time preference, two identical economies can end up in different growth regimes. In one regime the growth rate is high, and few resources are devoted to the inefficient competitive sector. In another regime, the competitive sector has high demand and R&D activities remain on a low level. Multiple equilibria are the result of a complementarity between present and future R&D activities. If current innovators expect a high future innovation rate, they have an incentive to conduct more R&D today. This complementarity is the result of the fact that innovations drive growth and that the economywide growth rate has a positive impact on the evolution of an innovator's market.

Acknowledgments

Part of this research was carried out during a stay at the University of California at Berkeley and at Stanford University. My special thanks to to Josef Falkinger. I also am grateful to Pranab Bardhan, Giuseppe Bertola, Francois Bourguignon, Martin Brown, Johann K. Brunner, Reto Föllmi, Simon Gächter, Markus Knell, Maurice Kugler, Gabriel Lee, Kiminori Matsuyama, Kalle Moene, Kevin Murphy, Arno Riedl, Paul Romer, Ragnar Torvik, and seminar participants at Linz, Oslo, Vienna, Humboldt Berlin, Zurich, CentER Tilburg, Bern, the meetings of the Royal Economic Society at Canterbury, the Verein für Socialpolitik at Jena, and the Macroeconomics Group of the Verein für Socialpolitik at Vienna for comments. Financial support was provided by the Austrian Fonds zur Föderung der wissenschaftlichen Forschung under the project JO548-SOZ.

Notes

1. Barro (1999) finds that the inequality-growth correlation is negative for developing countries but positive for developed economies.

2. For recent surveys on income distribution and growth, see Benabou (1996), Aghion and Howitt (1998, ch. 9 and 10), and Bertola (forthcoming).
3. See Jackson (1984) for direct evidence in favor of hierarchical structure of demand. Falkinger and Zweimüller (1996) provide similar evidence using aggregate consumption data from the International Comparison Project of the United Nations. Bils and Klenow (2000) show evidence for Engel-curves for product quality.
4. It is interesting to note that the results in this article encompass the results of two different papers by Murphy, Shleifer, and Vishny (1989a, 1989b). In their "Big Push" paper, which elaborates an earlier idea by Rosenstein-Rodan (1943), multiple equilibria are essential, but income inequality plays no role. In the paper on market size and income distribution, multiple equilibria cannot arise. In contrast, the conditions for multiple equilibria in my article include a sufficient degree of inequality. Matsuyama (1993) studies multiple equilibria in a dynamic version of the "Big Push." Contrary to the present article, in that paper multiple equilibria are driven by history rather than by expectations, and inequality-effects are not studied.
5. See also Bourguignon (1990) for a static model of a dual economy where income distribution affects the equilibrium outcome because the composition of demand varies across income classes.
6. A similar example concerns the wants for entertainment. In early stages of development, the rich had their own orchestras, theater performances, and so on. Once there was growing demand for entertainment also by the poorer classes, it became attractive to introduce radio, television, CD-players, and so on. Many other examples can be given along these lines.
7. See Young (1993) for a justification of such an assumption.
8. This assumption means that the monopolistic firms do not necessarily choose the profit-maximizing price. Allowing prices lower than unity would lead to considerable complications, and it would no longer be possible to solve the model. The reason is that the monopoly price not only is different for the different goods but also depends on the wealth distribution. Since the wealth level is an endogenous variable that depends on the growth rate and all other endogenous variables of the model, each price depends on all endogenous variables. Notice, however, that appropriate restrictions on wealth distribution can generate a situation in which the profit-maximizing price is unity. To guarantee that no monopolist has an incentive to set a price lower than unity, the monopolist's demand curve must be sufficiently inelastic over the range $p(j,t) \in (1-\pi, 1)$, where $1-\pi$ are marginal cost. A sufficiently inelastic demand means that not many additional customers can be attracted by lowering the price. The demand conditions depend on the wealth distribution (see Section 2.4), and there are appropriate restrictions on this distribution such that, for all goods j at all times t, it is not optimal for any monopolist to set a price lower than unity. An appendix where sufficient conditions on the wealth distribution are derived is available from the author.
9. These preferences can be represented by the utility function $\int_0^1 1 \cdot z(j,t)dj + \int_1^\infty \frac{1}{j} \cdot z(j,t)dj$, where $z(j,t)$ is an indicator that takes the value 1 if good j is consumed and the value 0 if not.
10. The condition under which $\lambda_i(t) < 1$ is discussed in note 11.
11. Above I have assumed that, for all t, all consumers can afford all goods in the interval [0,1]. Since all consumers earn the same wage and since wages grow over time, the income of the poorest is at least w_0 for all t. According to equation (5) using (2), $c_i(t) > 1$, if $w_o = n_0/a_c > 1$, where n_0 is the initial number of goods that can be produced with the increasing return technology.
12. See Murphy, Shleifer, and Vishny (1989a) for a similar structure in static context.
13. If the firm produces a good $j > c_R(t)$, analogous arguments as above lead to a value

$$v_j(t) = (L\pi/r)\left[(1-\beta)\,(c_R(t)/j)^{r/g} + \beta(c_p(t)/j)^{r/g}\right]$$

14. Necessary and sufficient conditions that guarantee a unique no-profit equilibrium are conditions on the rate of time preference θ, which determines the curvature of the B-function. From Figure 1, a necessary condition for uniqueness is $\theta < [a_c/a_r]\,(1-\beta)L\pi$. To establish a sufficient condition first note that a unique zero-profit equilibrium for $x_p = 1/a_c$ implies uniqueness for $x_p > 1/a_c$. (Recall that the wage rate $1/a_c$ is lowest value that x_p can take.) The sufficient condition is then $(\tilde{g}+\theta)(a_r/a_c) < (1-\beta)L\pi$, where $\tilde{g}$ is the rate of growth and where the $B(1/a_c, g)$-curve has its steepest slope. It is straightforward to verify that $\tilde{g} = \ln(a_c)\theta/2$. The sufficient condition is then $\theta < \frac{a_c(1-\beta)L\pi}{a_r[1+\ln(a_c)/2]}$.

15. Figure 1 assumes that $\theta < [a_c/a_r](1-\beta)L\pi$—that is, the B-curve starts above the C-curve at $g = 0$. If instead $\theta \geq [a_c/a_r](1-\beta)L\pi$, $g = 0$ is always an equilibrium (benefits are not larger than costs).
16. Using the definition of $V(t)$ and the expression for the value of a firm $v_j(t)$ derived in Section 3.1, it is straightforward to show that $V(t)$ grows at the same rate as $n(t)$ so that $\overline{v}$ is constant over time.
17. Replacing x_p and x_R in equation (7) by the corresponding expressions from (9) yields an equation in g and $\overline{v}$. From this equation, using $\pi = (a_c - a_m)/a_c$, it is straightforward to calculate $\overline{v} = \frac{L\pi(1-\beta+\beta/a_c)-ga_r/a_c}{\theta(1-\beta d_p\pi)}$. Using this in the expression for x_p from (9) yields (11).
18. We also have to make sure that the rich do buy the goods of all innovators—that $x_R > 1$. This requires that $d_R = (1-\beta d_p)/(1-\beta)$ is large enough (which will be the case if d_p is close to zero and/or β is sufficiently close to unity).
19. Again, this can be verified by inspecting Figure 1 in Section 3. If θ increases, the discount factor $x_p^{(g+\theta)/g}$ decreases (since $x_p < 1$) and the $B(g, x_p)$-curve shifts downward. When the no-profit-equilibrium is unique, this leads always to a lower g for a given x_p.
20. Implicit differentiation of equation (8) shows that for $x_p < 1$ we get $\partial x_p/\partial\beta > 0$.
21. a_r/a_c is the capital that has to be invested to make an innovation, and $L\pi$ is the highest possible profit resulting from this investment, so $L\pi/(a_r/a_c)$ is the highest rate of return of an innovation. Since the interest rate is never smaller than θ (see equation (4)), innovation is not profitable when $\theta \geq L\pi/(a_r/a_c)$ and $g = 0$, irrespective of the x_p.
22. As an example for a situation like in Figure 6, assume that $\theta = 0.02$, $\beta = 0.9$, and $L\pi/(a_r/a_c) = 0.5$ and that $d_p = 0$ and $1/a_c = 0.28$ (that is, the R-curve is horizontal at $x_p = 0.28$). Then there are three intersections between the R- and N-curves: at $g_1 = 0.00576$, $g_2 = 0.01023$, and $g_3 = 0.03615$.
23. For a discussion of path-dependent versus expectation-determined equilibria, see Krugman (1991) and Matsuyama (1991).
24. When all consumers buy the good of the most recent innovator, the resulting labor demand is $a_m L$. From resource constraint (7) it is evident that a necessary condition that all households consume all monopolistic goods is feasible only if $a_m < 1$.

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