Online Appendix

Hunting Unicorns? Experimental Evidence on Predatory Pricing Policies

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A Proofs

We now provide careful statements of our theoretical results and proofs. We start by formulating two tie-breaking rules.[1]

(T1) In any situation where both firms charge the same price, but a profitable downward deviation for exactly one firm is feasible, this firm wins the entire market (and becomes dominant).

The condition addresses two types of situations. First, it applies to cases where one firm is at its marginal cost (and thus cannot profitably reduce its price).[2] Second, it is relevant if one firm is at the Edlin constraint (and thus cannot reduce its price at all). The tie-breaking rule reflects the idea of a suitable discrete approximation.

Moreover, we have a tie-breaking rule regarding participation:

(T2) If firm H is indifferent between entering and not entering given the subsequent subgame strategies, it will not enter.

We maintain these tie-breaking assumptions throughout the paper.

[1]Our instructions assume that, for ties, firms share the profits 50-50. For the equilibrium analysis, we nevertheless need tie-breaking rules.

[2]In this case, the tie-breaking rule generates the standard textbook outcome that both firms charge at c_H, but firm L receives the entire demand. The relevant implication of this outcome that firm L serves the entire market at price c_H can be generated more precisely in an equilibrium where firm H mixes on an interval of prices c_H and higher (see Blume 2003, and Kartik 2011).

A.1 Laissez-Faire and Brooke Group Games

A.1.1 Proof of Proposition 1

The proof is identical for both games. There is a unique equilibrium strategy profile such that

(i) both firms play their respective monopoly price in subgames when they are alone in the market;

(ii) both firms choose the Bertrand duopoly price (c_H) in subgames when both firms are present;

(iii) firm L participates for every history; firm H does not participate for any history.

Clearly, in period 4, the described monopoly strategies are optimal; the strategy profile in the duopoly case is the unique pure-strategy equilibrium of the asymmetric Bertrand game. Anticipating this, firm H only participates in period 4 if firm L has previously exited. In period 3, suppose L is the monopolist. It anticipates being the monopolist in period 4 independent of pricing, so it sets $p^M(c_L)$. The argument is analogous if H is the monopolist (meaning that L has exited). If both firms are present, they anticipate that firm L will be the monopolist in period 4, independent of pricing in period 3. Thus, they set prices without taking period 4 into account, and the short-term Bertrand equilibrium emerges in period 3. Anticipating this pricing behavior, it is clear that only firm L is expected to earn positive net profits in the market, unless firm H is the monopolist in period 3. This gives the entry decision. The argument in the preceding periods is analogous.

A.2 Edlin

We first formulate a version of Proposition 2 that describes the equilibrium strategies in more detail. This result clearly implies the statement in the main text.

A.2.1 Re-Statement of Proposition

Proposition 1. *The Edlin Game has an SPE described in (i)-(ix) below. This equilibrium is not unique, but any SPE in pure strategies generates the same outcome.*

(i) Firm L participates in periods $t \leq 3$ *for arbitrary histories after which it has not previously exited. In period 4, L exits if and only if* $p_3^L > p^*$ *and H has not previously entered (or L was dominant in period* 3*).*

(ii) H participates in period t only if it has not previously exited and (a) L has exited, or (b) L was dominant in period $t-1$ *with* $p_{t-1}^L > p^*$*.*

(iii) In periods in which L is a monopolist, it sets prices as follows: If H has not yet entered in any period $t \leq 3$*, L sets* $p_t^L = p^*$ *(entry deterrence). Otherwise, L sets* $p_4^L = p^M(c_L) = 50$*.*

(iv) If L has previously exited, Player H sets $p_t^H = p^M(c_H) = 55$*.*

(v) If both firms participate in Period 4 and neither firm was dominant in period 3*,* $p_4^L = p_4^H = c_H$ *(and firm L takes the market given the tie-breaking rule).*

(vi) Suppose both firms participate in Period 4 and L was dominant in period 3*:*

(a) If $0.8p_3^L \in [0, c_H]$*,* $p_4^L = p_4^H = c_H$ *and firm L takes the market;*

(b) If $0.8p_3^L \in (c_H, 80]$*,* $p_4^L = p_4^H = 0.8p_3^L$ *and firm H takes the market.*[3]

(vii) Suppose both firms participate in Period 4 and H was dominant in period 3*:*

(a) If $0.8p_3^H \in [0, c_H]$*,* $p_4^L = p_4^H = c_H$

(b) If $0.8p_3^H \in (c_H, 80]$*,* $p_4^L = p_4^H = 0.8p_3^H$*;*

Firm L takes the market in both cases.

(viii) If both firms participate in periods $t = 2, 3$*, and L was dominant in period* $t-1$*, price-setting is as follows:*

(a) If $0.8p_{t-1}^L \in [0, c_H]$*, then* $p_t^L = p_t^H = 0.8p_{t-1}^L$ *and firm L takes the market;*

(b)If $0.8p_{t-1}^L \in \left(c_H, p^M(c_H)\right]$*, then* $p_t^L = p_t^H = 0.8p_{t-1}^L$ *and firm H takes the market;*

(c) If $0.8p_{t-1}^L \in \left(p^M(c_H), 80\right]$*, then* $p_t^L \geq 0.8p_{t-1}^L$*;* $p_t^H = p^M(c_H)$*, and firm H takes the market.*

(ix) If both firms participate in Period 3 *and firm H was dominant in period* 2*,*

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[3] In the special case that $0.8p_3^L = c_H$, the firms share the market, so that neither firm is dominant.

price-setting is as follows:
(a) If $0.8p_2^H \in [0, c_H]$*, then* $p_3^L = p_3^H = c_H$*;*
(b) If $0.8p_2^H \in (c_H, p^*]$*, then* $p_3^L = p_3^H = 0.8p_2^L$*;*
(c) If $0.8p_2^H \in (p^*, 80]$*, then* $p_3^L = p^*$*;* $p_3^H \geq 0.8p_2^H$*.*
Firm L takes the market in all three cases.
(x) If both firms participate in Period 3 and neither firm was dominant in period 2, both firms set c_H *and firm L takes the market.*

A.2.2 Proof of Proposition 3

We define the *quasi-cost* of a firm with cost type $\theta \in \{L, H\}$ that was dominant in period $t-1$ in period t as $\widehat{c}_\theta = \max\left(0.8p_{t-1}^\theta, c_\theta\right)$.[4]

Existence Period 4: For period 4, consider pricing: Obviously, setting the monopoly price is optimal for a monopolist as the competitor cannot be present in the future (see (iii) and (iv)). In any case where both firms are present in the last period, the game corresponds essentially to a static asymmetric Bertrand game with marginal costs of the dominant firm replaced with the quasi-marginal cost $\widehat{c}_\theta$. By arguments similar to those from the Bertrand game (together with the tie-breaking rule) a last-period price profile is a subgame equilibrium if and only if both firms price at $max\left(\widehat{c}_H, \widehat{c}_L\right)$. The winner is the unconstrained firm by the tie-breaking rule. Together, these arguments show (v)-(vii). They also show (iii) and (iv) for $t = 4$.

For period 4, consider participation. Anticipating the pricing behavior of L, H chooses to participate only if it is possible to break even in the last period, which requires one of the two conditions (a) and (b) in (ii): For (a), H earns the monopoly profit in period 4; for (b) she earns a non-negative net profit by setting $p_4^H = 0.8p_3^L$. If neither of these conditions hold, undercutting of firm L does not lead to positive net profits, so that entry is not profitable.
Anticipating the pricing behavior of H, Firm L only exits whenever she cannot set an entry-deterring price; that is, whenever L was dominant in period 3 with $p_3^L > p^*$.

[4] The motivation for calling this a quasi-cost is that the constraint to price above $0.8p_{t-1}^\theta$ has a similar effect on behavior as the constraint not to price below marginal cost.

Period 3 Pricing: In any subgame starting after the period 3 participation decisions, players anticipate that the competitor chooses the equilibrium strategy in period 4 after any period 3 pricing decision. In particular, firm H anticipates that, if firm L is in the market in period 3, it will stay and set prices in period 4 such that firm H cannot obtain a positive net profit unless firm L is dominant in period 3 with $p_3^L > p^*$; similarly, firm L anticipates that firm H will participate in period 4 only if $p_3^L > p^*$ and firm H was dominant in period 3. With this in mind, consider the pricing decisions in period 3.

Suppose first both firms are in and H was dominant in period 2 (see ix). First, let $0.8p_2^H \leq c_H$: With the proposed prices (and the tie-breaking rule), firm L earns a profit of $(c_H - c_L)\, D\,(c_H)$ in period 3. It induces exit of firm H and earns monopoly profits in period 4. Lower prices of Firm L in period 3 would reduce the period 3 profits of firm L without resulting in more profit in period 4. Any prices above c_H would lead to zero profits in period 3 and, at most the monopoly profit in period 4. Thus, firm L is best-responding. Firm H earns no profits, but cannot avoid this given that firm L undercuts in period 3 (and thereby also sets an entry-deterring price for period 4).

Second, let $0.8p_2^H \in (c_H, p^*]$: With the proposed prices (and the tie-breaking rule), firm L earns a profit of $\left(0.8p_2^H - c_L\right) D \left(0.8p_2^H\right)$ in period 3. It induces exit of firm H in period 4 and earns monopoly profits in period 4. Lower prices in period 3 would reduce period 3 profits without resulting in more profit in period 4. Higher prices in period 3 would lead to zero profits in period 3 and, at most the monopoly profit in period 4. Thus, firm L is best-responding. Firm H earns no profits, but cannot avoid this.

Finally, let $0.8p_2^H > p^*$. Firm H is constrained by the requirement that $p_3^H \geq 0.8p_2^H > p^*$. For any such price of firm H, by following the proposed strategy of setting $p_3^L = p^*$, firm L obtains a gross profit of $(p^* - c_L)\, D\,(p^*)$ in period 3; moreover L induces exit of firm H and earns the monopoly profit in period 4. This is a best response for L: For any higher price, firm H would participate in period 4, so that firm L would earn at most one monopoly profit (in period 3). For any lower price, firm L would still prevent entry and earn the monopoly profit in period 4, but period 3 profits would be lower than $(p^* - c_L)\, D\,(p^*)$. Firm H earns zero profits, but it cannot avoid this.

Next, suppose both firms are in and L was dominant in period 2 (viii): First consider $0.8p_2^L \leq c_H$: In the proposed equilibrium both firms set c_H. If $0.8p_2^L < c_H$, by the tie-breaking rule, firm L wins in this period (with the maximum possible period 3 profit given $p_3^H = c_H$); as $c_H < p^*$, she obtains the monopoly profit in period 4. Thus, L is best-responding. Firm H earns zero profits, but cannot avoid this; thus H is also best-responding. If $0.8p_2^L = c_H$, the argument is essentially the same, except that the two firms share the market in period 3 (as both firms are constrained below, the tie-breaking rule does not apply).
Second, suppose $0.8p_2^L \in (c_H, p^*]$: In the proposed equilibrium, firm H undercuts firm L and is thus dominant. Hence, in the next period firm H does not enter according to her strategy, and firm L earns the monopoly profit. Firm H is best-responding, because she cannot avoid having zero profits in period 4 (as $p_3^L = 0.8p_2^L \leq p^*$) and her profit in period 3 is maximal given the behavior of firm L. Firm L is also best-responding: She earns the monopoly profit in period 4; moreover, given her constraint and the behavior of firm H she cannot prevent losing in period 3.
Third, let $0.8p_2^L \in \left(p^*, p^M(c_H)\right]$: In the proposed equilibrium, firm H undercuts L in period 3. Firm L earns the monopoly profit in period 4. It cannot undercut firm H in period 3; so it is best-responding. Firm H obtains the maximal possible profit in this period, $\left(0.8p_2^L - c_H\right) D\left(0.8p_2^L\right) - F$, but no profit in period 4. The only potentially profitable alternative would be to set a higher price and thereby avoid undercutting L. Then L would exit in period 4 (as $0.8p_2^H > p^*$) given its strategy, and firm H would earn $\left(p^M(c_H) - c_H\right) D\left(p^M(c_H)\right) - F$ in period 4, but would also have to pay the additional fixed cost in period 3. Thus, H is best-responding if $\left(0.8p_2^L - c_H\right) D\left(0.8p_2^L\right) - F \geq \left(p^M(c_H) - c_H\right) D\left(p^M(c_H)\right) - 2F$. The right-hand side is 125. The condition holds for $0.8p_2^L \in [37.679, 70.811]$, and, in particular, in the interval $\left(p^*, p^M(c_H)\right] = (44.544, 50]$ under consideration.

Fourth, suppose $0.8p_2^L \in \left(p^M(c_H), 80\right]$. Firm H earns the monopoly profit in period 3, and firm L earns the monopoly profit in period 4. Firm L cannot undercut firm H in period 3; thus it is best responding. Firm H cannot prevent that firm L takes the market in period 4, unless it gives up on winning in period 3 and thus becoming dominant. Winning immediately, and thus not incurring the fixed cost in period 4 is preferable.

Now suppose both firms are in and neither was dominant in the previous period (x):

In this case, both firms are free to set arbitrary prices. The argument is as in an asymmetric Bertrand equilibrium, taking into account that firm H will exit in the next period and firm L will win the market.

Finally, consider situations with only one firm is in the market (iii) and (iv) for t=3. If firm H is monopolist, then this is because firm L has exited. There is no re-entry threat; firm H thus obtains the monopoly profit in two periods by applying her strategy, which clearly is optimal. If firm L is monopolist, this could be because firm H has exited or because it has never entered. In the former case, there is no re-entry threat; firm L thus obtains the monopoly profit in two periods by applying her strategy, which clearly is optimal. In the latter case, firm L has to take into account that firm H will enter in period 4 if and only if $p_3^L > p^*$. In the proposed equilibrium, L deters entry by setting $p_3^L = p^*$. She thus earns a positive profit $(p^* - c_L)\, D(p^*) - F$ if she adheres to the equilibrium strategy. This is clearly the best possible response among those that deter entry. The best way not to deter entry is to set $p_3^L = p^M\,(c_L)$. This way, however, firm L only obtains the monopoly profit once, which is less than with the proposed equilibrium behavior.

Next, consider participation decisions in period 3.
First, consider player L. According to the proposed equilibrium strategies, she stays in the market. If she was not dominant in period 2, she expects a positive profit in both periods, so staying is optimal. If she was dominant in period 2 with $p_2^L < p^*$, this is also true, because she expects firm H to exit. If she was dominant with $p_2^L > p^*$, L expects that H will enter and undercut in period 3, so that L will earn no profit in period 3, and she will pay the fixed cost of 300. However, in period 4, she will be the monopolist and earn the net profit 600. Thus, staying is a best response.

Second, consider player H. Clearly, if L has exited, participation (as proposed by the strategy) is optimal for H. If L is in period 3, H participates only if L was dominant in period 2 with $p_2^L > p^*$. By staying if $p_2^L > p^*$, H earns positive net profits in period 3. She will exit thereafter, but participating in period 3 is nevertheless profitable. By not participating if $p_2^L \leq p^*$, she earns zero profits in periods 3 and 4. Given the strategy of player L, she cannot avoid this.

The argument in period 2 is analogous to the argument for period 3. The only slightly larger difference concerns the case that L is in and $0.8p_1^L \in \left(p^*, p^M(c_H)\right]$: In the proposed equilibrium, firm H undercuts L in period 2 and Firm L earns the monopoly profit in periods 3 and 4. L cannot undercut firm H in period 2; so it is best-responding. Firm H obtains the maximal possible profit in this period given the price of firm L, $\left(0.8p_1^L - c_H\right) D\left(0.8p_1^L\right) - F$, but no profit in periods 3 and 4. The alternative would be to avoid undercutting L. However, then L would still remain in the market in period 3 given its strategy, and firm H would earn at most $\left(p^M(c_H) - c_H\right) D\left(p^M(c_H)\right) - F$ in period 3 (and no profit in period 4). The deviation is thus even less attractive than in the corresponding situation in period 3 for $0.8p_2^L \in \left(p^*, p^M(c_H)\right]$.

In Period 1, the only pricing deviation for firm 1 worth considering is that it chooses the monopoly price. Doing this yields a short term gain of $650 - 620.23 = 29.77$, but a loss of the monopoly profit (650) in the next period. Thus, the proposed strategy profile is a subgame perfect equilibrium.

Uniqueness We already saw that we cannot hope for more than outcome uniqueness. Clearly, any SPE must involve the above-described behavior in Period 4.

Consider pricing in Period 3.

Suppose both firms are in and H was dominant in period 2.
First suppose $0.8p_2^H \in [0, p^*]$. Consider an arbitrary equilibrium price candidate $p_3^H \in (c_L, p^*]$. Then it is always the unique best response of firm L to undercut marginally: Given period 4 behavior, this yields the monopoly profit in period 4 and, given p_3^H, the maximum possible profit in period 3. Thus, any equilibrium with $p_3^H \in (c_L, p^*]$ must have $p_3^L = p_3^H$ and firm L taking the market. Moreover, any such equilibrium must have $p_3^H = \widehat{c}_H$, for otherwise firm H could profitably deviate downwards. Clearly, there can be no equilibrium with $p_3^H > p^*$: In this case, firm H could profitably undercut whenever $p_3^L > 0.8p_2^H$. If $p_3^L \leq 0.8p_2^H$, firm L could profitably deviate by increasing the price to p^*. Finally, equilibria with $p_3^H \leq c_L$ can clearly not exist. Thus, all equilibria must contain prices in period 3 as described in (ix)
Second suppose $0.8p_2^H > p^*$. Then it is clearly the unique best response for firm L

to set $p_3^L = p^*$. As (ix) allows for arbitrary behavior of H that is consistent with the Edlin constraint, there can be no other equilibrium than those mentioned in (ix).

Suppose both firms are in and L was dominant in period 2.

Suppose $0.8p_2^L \in [0, c_H]$. There can be no equilibrium with $p_3^L > p^*$, as firm L would thereby giving up period 4 monopoly profits, which it could avoid by setting p^* (this deviation could potentially lower period 3 profits, but never by an amount that would not make it worthwhile).

There can be no equilibrium with $p_3^H > c_H$ If firm H wins in such an equilibrium and $p_3^H < p^*$, firm L could undercut and earn a positive profit in period 3 (and still the monopoly profit in period 4). Now suppose firm L wins in such an equilibrium. We already saw there can be no equilibrium such that $p_3^L > p^*$. Thus suppose $p_3^L \leq p^*$. Then H can increase profits by marginally undercutting. Clearly there can be no equilibrium with $p_3^H = p_3^L = p^*$.

There cannot be an equilibrium with $p_3^\theta \in [0, c_L)$ for at least one $\theta \in \{L, H\}$ either. As firm H would earn (avoidable) negative gross profits, there can be no equilibrium with $p_3^H \leq p_3^L$ and $p_3^H \in [c_L, c_H)$. There can be no equilibrium with $p_3^L < p_3^H$ and $p_3^H \leq p^*$, as firm L could increase profits by increasing price (while still inducing exit). Finally, consider $p_3^H \in [c_L, c_H)$ and $p_3^H > p_3^L$. Firm L could profitably increase prices.
If $p_3^H \leq p_3^L$, firm L could profitably undercut firm H. Thus, the only remaining possibility is $p_3^H = p_3^L = c_H$.

Suppose $0.8p_2^L \in (c_H, p^*]$. For any allowable $p_3^L \in \left[0.8p_2^L, p^*\right)$, firm H knows that it will earn zero profits in period 4 no matter what it does in period 3. Thus, it has the unique best response to undercut marginally, so that $p_3^H = p_3^L$. Clearly, there can be no such equilibrium with $p_3^L > 0.8p_2^L$, as both firms could profitably undercut. Thus, we must indeed have $p_3^L = p_3^H = 0.8p_2^L$. Next consider $p_3^L > p^*$. If $p_3^L < p_3^H$, firm L would become dominant in period 3 and thus give up the monopoly profit in period 4. It could thus profitably deviate to p^*. If $p_3^H < p_3^L$, firm H would benefit from increasing its price slightly. If $p_3^H = p_3^L$, H would benefit from undercutting marginally. Finally, suppose $p_3^L = p^*$ and $0.8p_2^L < p^*$. If $p_3^H < p_3^L$, firm H would benefit from increasing its price slightly. If $p_3^H \geq p_3^L$, H would benefit from undercutting.

Suppose $0.8p_2^L \in \left(p^*, p^M\left(c_H\right)\right]$. Given the restrictions of L, firm H can always undercut marginally and thereby secure a profit of $\left(p_3^L - c_H\right) D\left(p_3^L\right) - F$. This is always a best response: Not undercutting would mean that she would induce exit of firm L, so that she would earn the monopoly profit in period 4, but she would have to pay the fixed cost twice. By previous arguments, not undercutting is not a profitable deviation. Pricing lower than p_3^L would mean profit losses in period 3, without any compensating gains. Finally, an equilibrium with prices above $0.8p_2^L$ cannot exist: Firm H could always increase profits by reducing its price, without any compensating losses.

Suppose $0.8p_2^L \in \left(p^M\left(c_H\right), 80\right]$. As (viii) does not restrict the strategy of L beyond the Edlin restriction, we only check whether there can be another price of H than $p^M\left(c_H\right)$ in any equilibrium. For any price of L, this would have to lead at least to one monopoly profit of firm H (as H can always secure this by setting $p^M\left(c_H\right)$). Given the strategy of L in period 4, getting a gross monopoly profit in period 4 is only possible for H if it does not undercut L in period 3 who then cannot avoid that firm H enters in period 4. However, any such strategy of firm 3 means that it wins only one (gross) monopoly profit, but has to pay the fixed costs twice. As the alternative of winning the monopoly profit immediately and paying the fixed cost only once is always available (independent of the behavior of L), pricing above L in period 3 cannot be a best response.

Remaining Arguments

The above arguments on pricing in Period 3 imply the statements on participation (Part (i) and (ii) of Proposition 4). The argument for the uniqueness of pricing behavior in Period 2 duopolies is similar as for Period 3 except that it is simpler because the incumbent is always dominant. Thus behavior from period 2 on in any equilibrium is as described in Proposition 4. As the first-period behavior of firm L described in Proposition 4 is strictly optimal given the reactions in periods 2 and following, there can be no other equilibrium.

A.3 Baumol

We start by presenting a version of Proposition 3 that specifies the strategies in more detail. We then prove the result.

A.3.1 Re-Statement of Proposition 3

In the Baumol case, we will occasionally invoke a tie-breaking rule which applies in specific constellations when neither firm prices at its quasi-cost. It deals with situations where no firm wants to let the other firm win (so as to avoid being constrained in the next period), whereas the other firm is happy to win (and will exit in the next period).

(T3) Suppose in $t = 2, 3$ both firms charge the same price, but neither firm prices at its quasi-cost. Fix continuation strategies in $t+1$ for both firms. Suppose firm i prefers leaving the market to j in period t rather than taking the market (and conversely for j). Then j wins the market.

Proposition 2. *The Baumol Game has a continuum of SPE described below, and all equilibria have these properties:*[5]
Periods $t = 3, 4$*: If a firm* θ *is alone in the market after the other firm has previously exited, and* θ *was dominant in the period* s *in which the other firm exited,* θ *sets* $p_\theta^\beta = min(p^M(c_\theta), p_s^\theta)$*. In any other situation such that* θ *is alone in the market, it sets* $p^M(c_\theta)$*. If both firms are in the market, they set* $p_t^H = p_t^L = c_H$*. Only L participates unless (i) L has exited in period s<t and or (ii) L was dominant in period t-1 and* $p_{t-1}^L < p_L^B$*. In case (i), only H participates if (a) firm L exited before there was a duopoly or (b) L exited after a duopoly period in which* $p_{s-1}^H \geq p_B^H$*; if* $p_{s-1}^H < p_H^B$ *nobody participates. In case (ii), both firms mix over participation decisions and earn zero expected profits.*
Period 2: Everything is as in periods 3 and 4, except in the duopoly situation. Then both firms set arbitrary but identical prices in $[c_H, \widetilde{p}_2]$ *such that*

$$3\left(\widetilde{p}_2 - c_L\right)\left(80 - \widetilde{p}_2\right) = 2\left(p^M\left(c_L\right) - c_L\right) D(p^M\left(c_L\right))$$

and firm H takes the market. $\widetilde{p}_2$ *is approximately* 32.679*.*
L participates and H does not.
Period 1: L participates and sets the monopoly price.

[5] For simplicity, we continue to treat the pricing games in the heuristic textbook manner rather than identifying mixed-strategy equilibria as Blume (2003) and Kartik (2011); see footnote 2.

A.3.2 Proof of Proposition 5

Period 4: The only non-trivial part of the argument concerns the participation subgame in the case that there was a duopoly in period 3 where firm L was dominant with $p_3^L < p_L^B$. In this case, it is straightforward to show that there can be no pure-strategy participation equilibrium: If neither firm participates, H can profitably enter. If both firms participate, H earns negative profits. If only L participates, it earns negative profits (as it is forced to price below break even). If only H stays, L would benefit from staying (and undercutting).

However, there is a mixed strategy equilibrium (with zero expected profits) where firm H stays with probability $r = \left\{ \frac{\left(p_3^L - 20\right)\left(p_3^L - 80\right) + 300}{\left(p_3^L - 20\right)\left(p_3^L - 80\right) + 500} \right\}$ and firm L stays with probability $q = \frac{11}{25}$. As usual, this comes from the indifference conditions (between staying and exiting). For firm L this is

$$r\left(\left(c_H - c_L\right)\left(80 - c_H\right)\right) + \left(1 - r\right)\left(p_3^L - c_L\right)\left(80 - p_3^L\right) - F = 0.$$

As Firm H earns the monopoly profit if it participates and L is not participating, indifference between staying and exiting requires

$$\left(1 - q\right)\left(p^M\left(c_H\right) - c_H\right)\left(80 - p^M\left(c_H\right)\right) - F = 0.$$

Together, these conditions give the above (uniquely defined) mixing probabilities. The function below plots r as a function of p_3^L: The lower the previous price was, the higher the required participation probability of firm H that makes L want to participate:

Period 3:

If a firm θ is alone in the market, because the other firm has previously exited and θ had a market share of one in the period that the other firm exited, θ sets p_θ^β.
It cannot set a higher price. A lower price will increase losses.
In any other situation such that θ is alone in the market, it sets $p^M(c_\theta)$.
This is obvious.
If both firms are in the market, they set $p_3^H = p_3^L = c_H$.
Firm H earns nothing, but cannot avoid this.

Firm L earns $2\left(c_H - c_L\right)\left(80 - c_H\right) - 2F = 400$ this way (because she takes

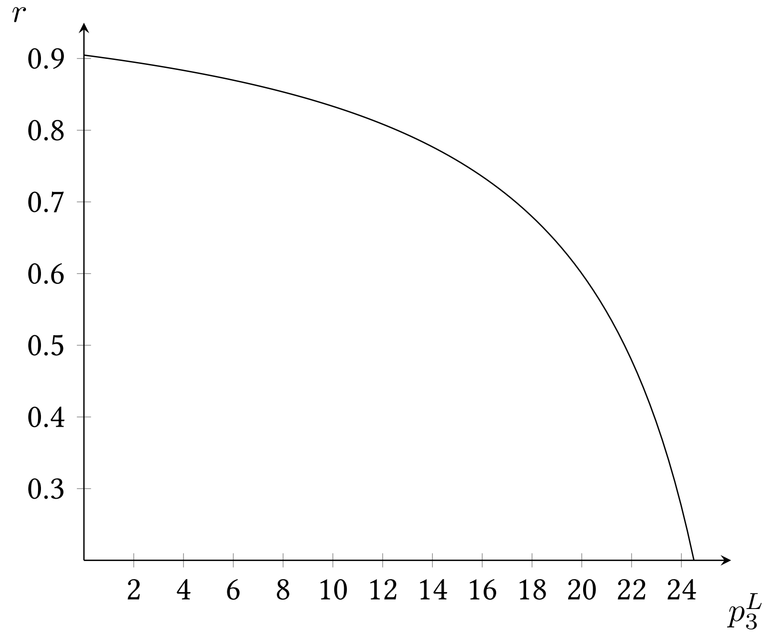

Figure A1: Baumol Game: Mixed strategy of the entrant in Period 4

the market today, she is Baumol constrained tomorrow). The only conceivable profitable deviation would be to let H win today. Then L could set the monopoly profit in period 4 and earn a total profit $\left(p^M(c_L) - c_L\right) D(p^M(c_L)) - 2F = 300$. This deviation is not profitable. The uniqueness argument is similar to standard arguments in the static Bertrand game; in addition, one has to take into account that, for higher prices, L would be prepared to undercut, because the short-term profits would be sufficiently attractive to make up for the (relatively small) long-term losses from the Baumol restriction.

In the participation subgame, there is a unique subgame equilibrium such that only L participates unless (i) L has previously exited in period s and or (ii) firm L was dominant in period 2 with $p_2^L < p_L^B$. In case (i), there is a unique subgame equilibrium such that only H participates if $p_s^H \geq p_H^B$; otherwise nobody participates. In case (ii), there is no pure-strategy SPE of the period 3 subgame. However, there is an equilibrium where both firms mix over participation decisions. In this equilibrium, both firms earn zero profits.

This is all straightforward except for the mixing equilibrium in participation decisions. Thus consider the case that L was dominant with $p_2^L < p_L^B$.

The argument for why there is no subgame equilibrium is essentially as in period 4: If neither firm participates, H can profitably enter. If both firms participate, H earns negative profits in period 3 (asymmetric Bertrand equilibrium); in the ensuing subgame in period 4 it earns zero on expectation. If only L participates, it earns negative profits (as it is forced to price below break even). If only H stays, L would benefit from staying (and undercutting); thereby earning positive profits in periods 3 and 4.

Let t be the staying probability of H in period 3. Indifference of firm L requires that $t = \frac{\left(p_2^L-20\right)\left(p_2^L-80\right)+300}{\left(p_2^L-20\right)\left(p_2^L-80\right)+700}$ (see the plot below)

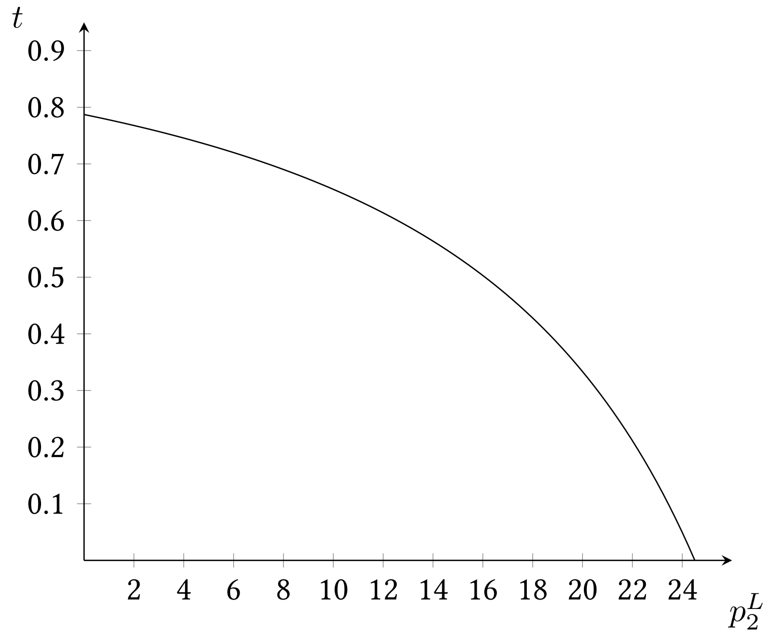

Figure A2: Baumol Game: Mixed strategy of the entrant in Period 3

To see this, suppose L stays. If H does not stay in period 3, firm L will make losses because of the Baumol constraint, and it will not be present in period 4 to avoid further losses. This gives a profit contribution $(1-t)\left(\left(p_2^L-20\right)\left(80-p_2^L\right)-F\right) < 0$. If H stays in period 3, firm L will earn $(c_H-c_L)(80-c_H)-F$ in the asymmetric Bertrand equilibrium in period 3. Because $p_3^L = c_H > p_L^B$, firm H will exit and L will stay, but is constrained to set $p_4^L = c_H$. The expected profits from staying are thus

$$(1-t)\left(\left(p_2^L-20\right)\left(80-p_2^L\right)-F\right)+t\left(2\left(c_H-c_L\right)\left(80-c_H\right)-2F\right).$$

Inserting parameters and setting this equal to zero gives the mixing probability.

If firm H stays, it earns positive profits only if firm L exits in Period 3. (If firm L stays, H will clearly not earn profits in period 3 and it will earn the zero expected MSE profits in period 4).

Let s be staying probability of L. If L does not stay, H wins the net monopoly profit in both periods. If L stays in period 3, H earns nothing in the resulting asymmetric Bertrand game. As L has then reverted to setting $p_3^L = c_H$ and hence above the Baumol price, H exits thereafter, thus earning a total net profit of -300 in periods 3 and 4. Thus, the indifference condition is $(1-s)(650) - s(300) = 0$, yielding $s = \frac{13}{19}$.

Next consider period 2.

In this period, the Baumol constraint has no bite, because there has been no previous duopoly interaction. Thus, any firm that is in the market is free to set its price without any constraints. However, in the duopoly case, the firms will be concerned about constraints that there pricing has on future prices in case of exit of the competitor.

In period 2, consider the suggested prices in $[c_H, \widetilde{p}_2]$: Firm L leaves the market to firm H in period 2 (but pays the fixed cost) and then earns twice the monopoly profit in the remaining periods; thus total profits are $2\left(p^M(c_L)-c_L\right)(80 - p^M(c_L)) - 3F$. The only conceivable profitable deviation would be to underbid the competitor; which would then also mean L has to set the same price in the future. This would give profits of at most $3(\widetilde{p}_2 - c_L)(80-\widetilde{p}_2) - 3F$. By construction of $\widetilde{p}_2$, firm L is indifferent between these two profits if $p = \widetilde{p}_2$ because

$$3(\widetilde{p}_2 - c_L)(80-\widetilde{p}_2) = 2\left(p^M(c_L)-20\right)(80-p^M(c_L)) = 1800$$

For any price equilibrium candidate p_2 in $[c_H, \widetilde{p}_2)$, $3(p_2-c_L)(80-p_2) < 3(\widetilde{p}_2 - c_L)(80-\widetilde{p}_2)$; firm L thus strictly prefers firm H to win.

Given that both firms set the same price and firm H wins the market with a positive gross profit, she cannot earn higher short-term profits. By avoiding to undercut, H would earn zero gross profits today. But this would not increase profits in the next period, as it would exit anyway (as any price in $[c_H, \widetilde{p}_2]$ is above the break-even price of L, who will stay).

Period 1: By setting the monopoly price, L can guarantee itself four times the monopoly profit. Clearly this is optimal.

References

Blume, A. (2003). Bertrand without fudge. *Economics Letters 78*(2), 167–168.

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B Instructions

[Instructions for L'FAIRE, translated from German. The parts that are different in the instructions for BROOKE, EDLIN, and BAUMOL are reported in boxes]

General Instructions: We are pleased to welcome you to this economic study. Please read the following instructions carefully. During this study, you have the opportunity to earn a fair amount of money in addition to the 5 Euros that you receive as an initial endowment for participating. The exact amount depends on your decisions and the decisions of the other participants. You remain anonymous during the entire study.
During the study, we do not speak of Euros but of points. Your entire income will first be calculated in points. The total amount of points you earn will be converted to Euros at the end of the study. The following conversion rate applies:

600 points = 1 Euro

To start with, you receive 1500 points to cover potential losses. At the end of today's session, you will receive your earnings from the study plus the initial endowment of 5 Euros in cash.
We will explain the exact procedure of the study in the next pages. These instructions are solely for private use, please do not communicate with the other participants during the study. If you have any questions, please contact the supervisors.

The Study: This study is divided into 7 separate rounds. In each round, you are paired with another participant selected at random from those present in the room. In each round, you are assigned one of two roles. Either you are firm A or firm B. If you are assigned the role of firm A, the other participant in your group is assigned the role of firm B and vice-versa. These roles are randomly allotted at the beginning of each round and remain unchanged throughout this round. Each round consists of four periods.
Firms A and B produce a homogenous good and sell this in the same market. In each period that your firm participates in the market, you have to set the price at

which you want to sell the good. At the beginning of each period, if you currently participate in the market, you can decide to exit the market and, if you currently do not participate in the market you can decide to enter it for this period. If you decide to exit the market, you will not be able to enter it anymore for the entire round.
Firm A and firm B differ in two respects:

1. Firm A produces the good at a lower cost than firm B.
2. Firm A starts off in the market at the beginning of period 1 whereas firm B can enter the market only at the beginning of period 2.

The procedure in a particular round is as follows (see figure A3).

Period 1:
(1) Firm A is alone in the market and sets a price (see (iii)).
(2) The profit of firm A is realized. Both firms learn the price, the quantity sold and the profit of firm A (see (iv)).

Period 2 and all subsequent periods until the round ends:
(1) Each firm that participates in the market decides whether to exit the market. Once a firm has exited the market, it cannot enter it anymore in this round. In each period in which firm B has not entered the market previously in this particular round, it decides whether to enter the market (see (i)).
(2) Each firm learns the exit, respectively, entry decision of the other firm (see (ii)).
(3) Each firm that participates in the market sets a price (see (iii)).
(4) Profits are realized. Both firms learn prices, quantities sold and profits (see (iv)).

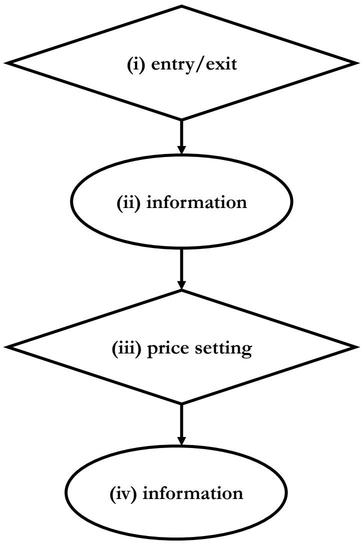


Figure A3: Sequence of Events

The profits are summed up across all periods in each round. After four periods a round ends, and a new round begins. For the new round, participants are newly paired, and the roles of firm A and B are randomly reassigned within these groups. In every round, you are informed only about the decisions of the other firm in your group. When a new round starts and you are matched with a new firm, neither of you will know anything about the decisions of the other firm in prior rounds.

Per-Period Profit: Your profit in each period depends on whether you participate in the market or not:

(i) Each period in which you do not participate in the market, you earn a fixed amount of 50 points with certainty.

(ii) Each period in which you participate in the market, the factors market demand, costs and price setting behavior determines your profit. How these factors determine your profit is explained below.

Market Demand: In each period, each firm that participates in the market has to decide which price to set. Both firms set their price at the same time. However, only the firm with the lower price can sell the good. If the lower price is P the firm who sets this price sells Q units of the good. The quantity Q is determined as follows:

$$Q = 80 - P$$

Example: Suppose that you set a price of 70, and you are the firm with the lower price. In this case, you sell 10 units. However, if you are the firm that sets the higher price you do not sell anything. If both of you chose the same price then you both sell half of the quantity Q. For example, both set a price of 10 and share the resulting quantity (70) and sell 35 units each.

Costs: Each firm has two kinds of costs:

(i) Each selling firm pays a unit cost of production. This cost is 20 per unit for firm A and 30 per unit for firm B.
Example: Suppose that you are firm A and sell 10 units. Your production cost is then $10 \times 20 = 200$. If you are firm B and you sell 10 units, your production cost is $10 \times 30 = 300$.

(ii) For each period in which a firm participates in the market, it has to pay a fixed cost of 250. This cost is the same for firms A and B, and it is independent of production.
Example: Suppose that you participate in the market but do not sell anything. Hence, you make a revenue of zero and pay no unit cost of production. However, you have to pay the fixed cost, and, thus, you make a loss of 250.

Price Setting: Prices are integers in between 0 and 80. You can choose any price in this range.

Price Setting BROOKE:

In certain situations, however, not the entire price range between 0 and 80 is available. The graph below illustrates these situations. The upper part of the graph corresponds to the market situation in an arbitrary period t. Three situations are to be distinguished:

- In Situation 1, only you sell the good (black box) in period t while the other firm does not sell anything (white box). The reason why the other firm does not sell anything in period t may be twofold (dashed line):

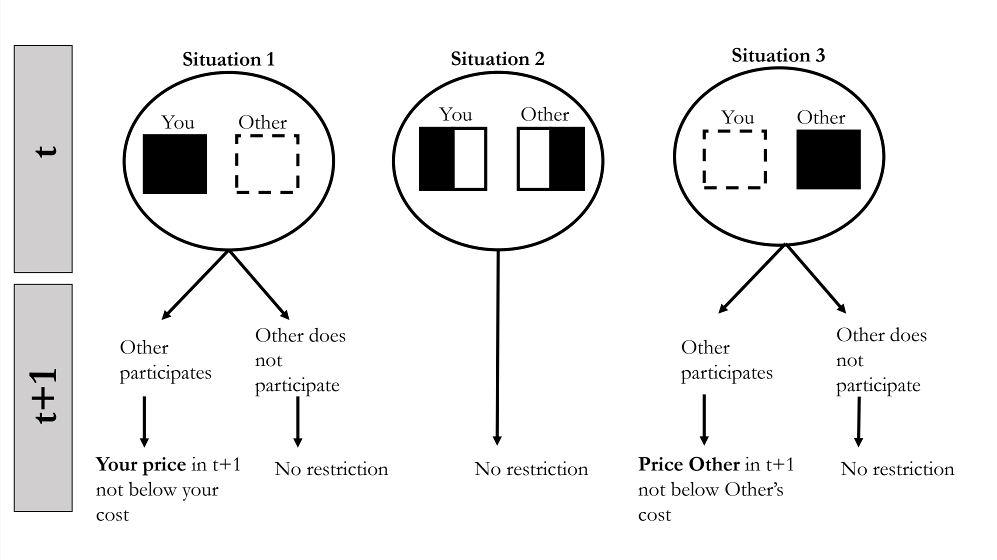
    1. The other firm does not participate in the market in period t or
    2. The other firm set a higher price than you did in period t.

    If now the other firm participates in the market in the next period $t+1$ you are not allowed to set a price in period $t+1$ that is below your own unit cost of production. No restriction applies to the other firm. If the other firm does not participate in the market in period $t+1$ no restriction applies to you.
    Example: Suppose that your unit cost of production is 20 and that, currently, only you sell 10 units at a price of 70. If the other firm participates in the market in the next period, you are not allowed to set a price in the next period that is below 20. The other firm can set any price between 0 and 80.

- In Situation 2, both you and the other firm set the same price in period t and thus, both sell half of the total quantity each. In the next period $t+1$, neither you nor the other firm are restricted in their price setting.

- In Situation 3, only the other firm sells the good in period t. You do not sell anything because, either you do not participate in the market in period t or you set a higher price than the other firm did. If now the other firm participates in the market in the next period $t+1$, it is not allowed to set a price in period $t+1$ that is below its own unit cost of production. If it does not participate in the market, it cannot set any price. In period $t+1$, no restriction applies to you.

Price Setting BROOKE:



In a nutshell, the following rule applies: the firm that sells the good in t, must not set a price in $t+1$ that is below its own unit cost of production if the other firm participates in the market in $t+1$.

Price Setting EDLIN:

In certain situations, however, not the entire price range between 0 and 80 is available. The graph below illustrates these situations. The upper part of the graph corresponds to the market situation in an arbitrary period t. Three situations are to be distinguished:

- In Situation 1, only you sell the good (black box) in period t while the other firm does not sell anything (white box). The reason why the other firm does not sell anything in period t may be twofold (dashed line):

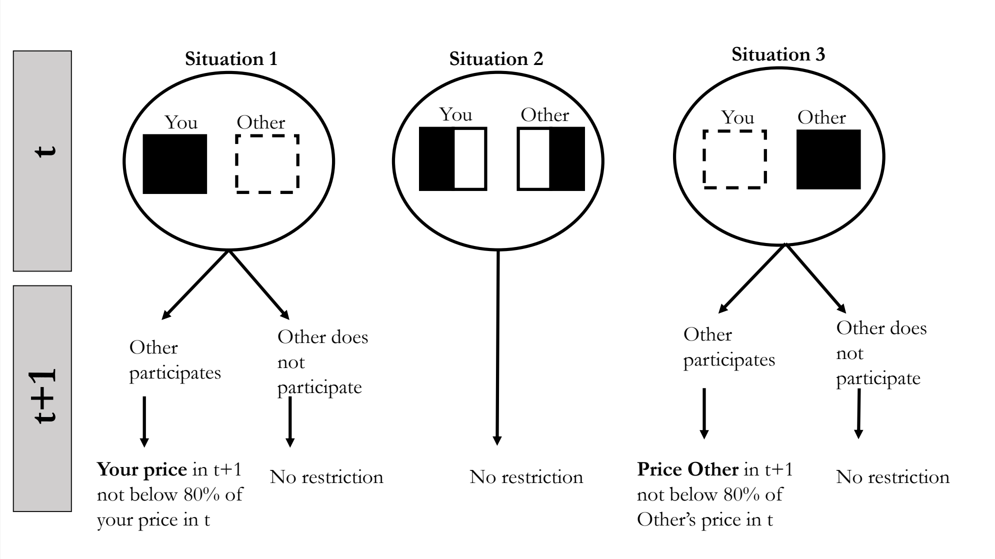
    1. The other firm does not participate in the market in period t or
    2. The other firm set a higher price than you did in period t.

    If now the other firm participates in the market in the next period $t+1$ you are not allowed to set a price in period $t+1$ that is below 80% of your price in period t (rounded to integers). No restriction applies to the other firm.
    Example: Suppose that, currently, only you sell 10 units at a price of 70. If the other firm participates in the market in the next period, you are not allowed to set a price in the next period that is below 56 ($= 0.8 \times 70$) (see supplementary sheet). The other firm can set any price between 0 and 80.

- In Situation 2, both you and the other firm set the same price in period t and thus, both sell half of the total quantity each. In the next period $t+1$, neither you nor the other firm are restricted in their price setting.

- In Situation 3, only the other firm sells the good in period t. You do not sell anything because, either you do not participate in the market in period t or you set a higher price than the other firm did. If now the other firm participates in the market in the next period $t+1$, it is not allowed to set a price in period $t+1$ that is below 80% of its price in period t. In period $t+1$, no restriction applies to you.

Price Setting Edlin:



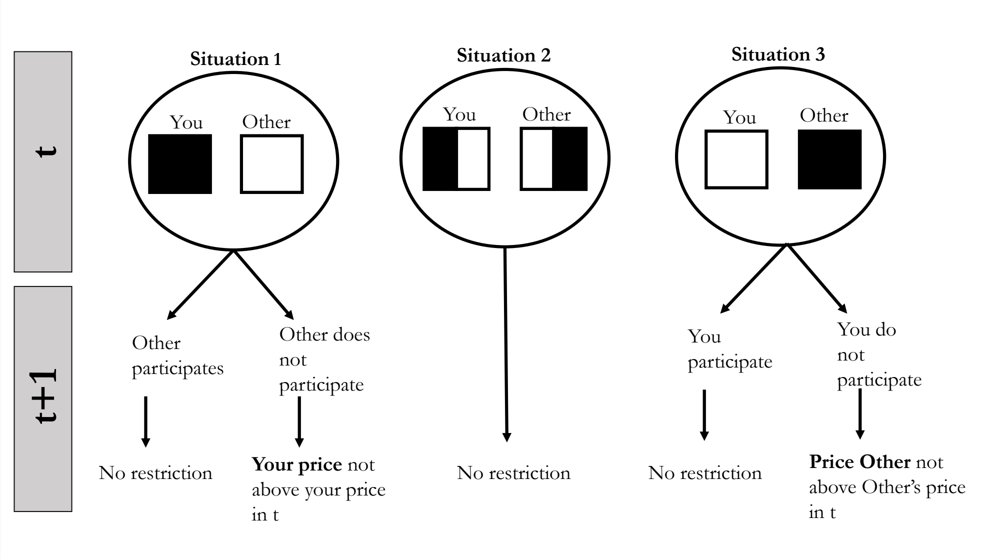
In a nutshell, the following rule applies: the firm that sells the good in t, must not set a price in $t + 1$ that is below 80% of its price in t (rounded to integers) if the other firm participates in the market in $t + 1$.

Price Setting BAUMOL:

In certain situations, however, not the entire price range between 0 and 80 is available. The graph below illustrates these situations. The upper part of the graph corresponds to the market situation in an arbitrary period t. Three situations are to be distinguished:

- In Situation 1, both you and the other firm participate in the market but only you sell the good in period t; the other firm sells nothing. If now the other firm exits the market and thus does not participate in the market in the next period $t+1$, you are not allowed to increase your price in $t+1$ and in all subsequent periods in this round. If the other firm participates in the market in the next period $t+1$ no restriction applies to you or the other firm.
  Example: Suppose that, currently, both participate in the market, but that you are the one who set the lower price, say, 70. In the current period, only you sell the good, namely, 10 units. If the other firm exits the market in the next period you are not allowed to increase your price above 70 in the next and all subsequent periods in this round.
- In Situation 2, both you and the other firm set the same price in period t and thus, both sell half of the total quantity each. In the next period $t+1$, neither you nor the other firm are restricted in their price setting.
- In Situation 3, both you and the other firm participate in the market but only the other firm sells the good in period t; you sell nothing. If now you exit the market and thus do not participate in the market in the next period $t+1$, the other firm is not allowed to increase its price in $t+1$ and in all subsequent periods in this round. If you participate in the market in the next period $t+1$ no restriction applies to you or the other firm.

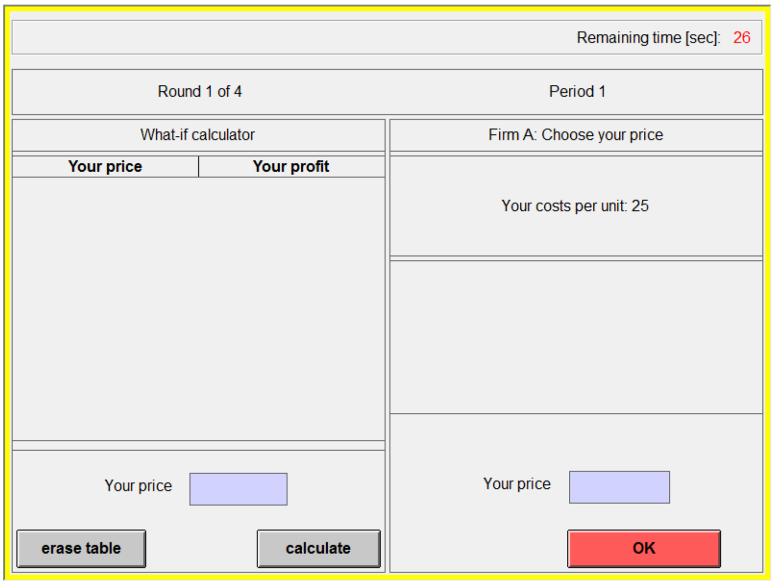
Price Setting BAUMOL



In a nutshell, the following rule applies: the firm that sells the good in t, must not increase its price in $t+1$ and until the end of this round if the other firm exits the market in $t+1$.

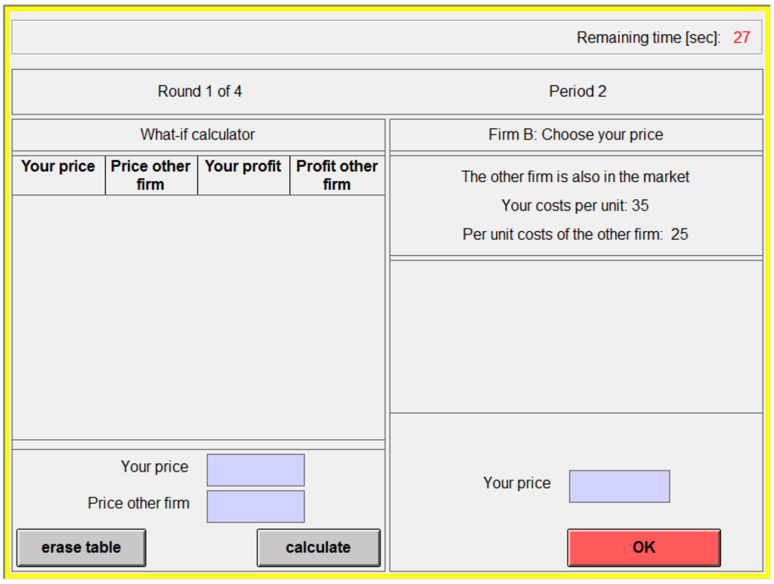
Calculation of Per-Period Profit: The per-period profit is calculated as follows: each firm makes a revenue which equals price times units sold ($P \times Q$). Subtracting the total cost incurred, that is, the sum of the production cost and the fixed cost, gives the per-period profit.
Example: Suppose that you currently sell 10 units at a price of 70. With the cost of firm A, the profit yield is $70 \times 10 - 10 \times 20 - 250$, that is, 250. With the cost of firm B, the profit yield is $70 \times 10 - 10 \times 30 - 250$, that is, 150.

Procedure: In each period, each firm that participates in the market sees the input screen. In period 1, this screen appears only for firm A. Firm A thus sees the following input screen:



The upper part of the screen shows the round in which you are in on the left; in this example, it is round 1 of 7. On the right, you see a time specification in seconds which indicates how much time you have left to enter your price. Please try to reach your decision in the given time. Below the time indication, you see in which period you are in. In this example, it is period 1. The remaining part of the screen is divided into two sections. On the left, you can see the "What-if-calculator". You can use this tool to determine your per-period profit using different prices. On the right, the screen reminds you of your unit cost of production; in this example, 20 is shown as a value. Below the cost information, you can enter the price you want to set in this period. In order to confirm a price, you must click on the "OK" button. You can revise your price until you click on this button. Once you have done this, you can no longer revise your decision for this period.

Notice that the layout of this screen as well as the "What-if-calculator" adapts to the situation, namely, whether only one or both firms are in the market. In particular, when both firms participate in the market, the input screen appears as follows:

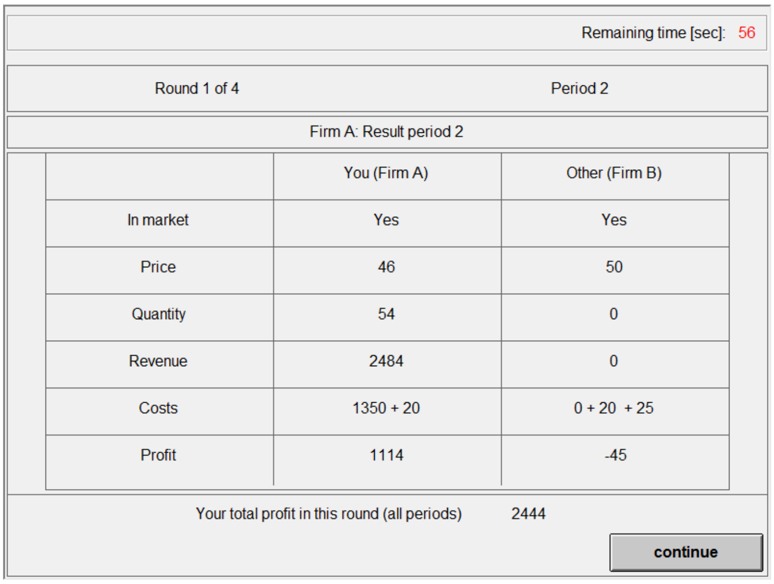


Here, in the example, you see the input screen of firm B in period 2 of round 1. The left part of the screen shows again the "What-if-calculator". Now you can calculate your profit as a combination of your price and the price of the other firm. On the right, you can see, in addition to your unit production cost, the unit production cost of the other firm as well as whether this firm participates in the market.

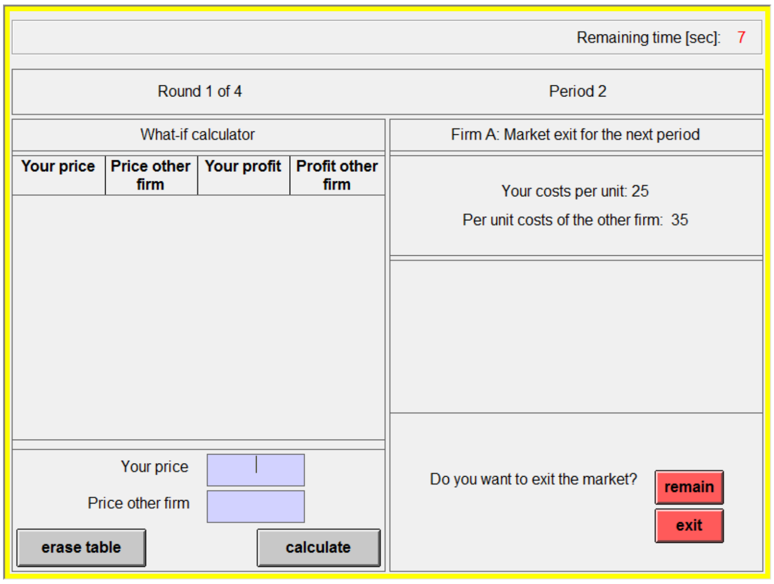
Brooke, Edlin, Baumol

If either your or the other firm's price range is restricted, it is always indicated on this input screen below the cost information. If none of the price ranges is restricted in any way, nothing is indicated. In this example, neither your not the other firm's price range is restricted: You can both set any price between 0 and 80.

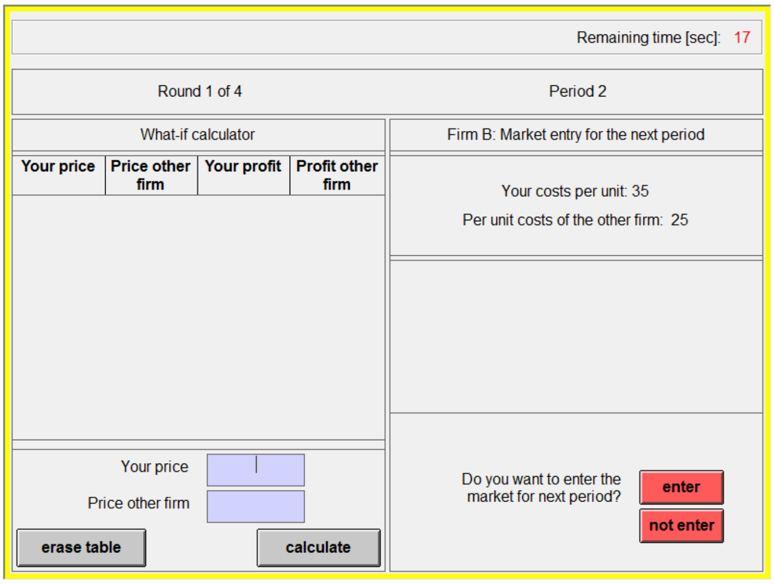
After both firms have entered their price, it will be determined who serves the market. Both firms will be informed on the prices, quantities and profits in this period. Note that you can also incur losses.



Once you have read this information, please click on the "Continue" button.
In each period (other than period 1), prior to the price setting decision, each firm that currently participates in the market has to decide whether to exit the market for this and all subsequent periods in this particular round. Once a firm has exited the market, it cannot enter it anymore in this round.



In each period (other than period 1) in which firm B has not entered the market previously in this particular round, firm B has to decide whether to enter the market for this period. Only if firm B enters the market, it can set a price. Below the cost information, the screen reminds firm B of the price that firm A set in the previous period.



Each firm learns the exit, respectively, entry decision of the other firm. Each firm that participates in the market in this period then goes on to price setting via the input screen as it is explained above. Each firm that does not participate in the market in this period can set no price and will be informed on the price, quantity sold and potential profits at the end of this period.

Do you have any further questions? If so, please raise your hand. The supervisors will come to you at your workplace. Otherwise, we kindly ask you to answer the control questions on your screen.