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Selecting Valuation Distributions: Non-Price Decisions of Multi-Product Firms

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Selecting Valuation Distributions: Non-Price Decisions of Multi-Product Firms

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Abstract

This paper analyzes decisions of multi-product firms regarding product selection, innovation and advertising as choices of consumer valuation distributions. We show that a profit-maximizing monopolist chooses these distributions so as to maximize the dispersion of the valuation differences between goods across consumers. By contrast, she chooses the willingness-to-pay to be maximally or minimally dispersed, depending on the set of available distributions. In our benchmark model with uniform valuation differences, prices are increasing in valuation difference heterogeneity, but in more general settings this is not necessarily true. Moreover, the relation between willingness-to-pay heterogeneity and prices may well be non-monotone. Over wide parameter ranges, the firm's choice of valuation distribution does not maximize net consumer surplus. This problem is exacerbated when the firm has access to strategies that distort valuation heterogeneity or willingness-to-pay heterogeneity.

Keywords: product choice, multiproduct firms, product heterogeneity, valuation distributions, consumer confusion

JEL Classification: D43, L13, M30.

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1 Introduction

Multi-product firms are omnipresent. It is therefore hardly surprising that, from the early contributions of Ramsey (1929) to the recent work of Amir et al. (2016), Armstrong and Vickers (2018) and Nocke and Schutz (2018), a large literature has dealt with the pricing decisions of such firms. By contrast, their non-price decisions have received less attention. The analysis of topics such as product selection, innovation and advertising has mostly been carried out in models of single-product firms. This paper therefore proposes a theoretical approach to the non-price decisions of multi-product monopolists. It starts from the observation that such decisions can be usefully regarded as choices of valuation distributions subject to constraints. First, consider product selection. We think of a firm as having distributional knowledge of preferences for products that might be brought to the market. By selecting products from an existing set of alternatives, by improving existing products or creating novel products, the firm influences the distribution of product valuations that it offers to consumers. Second, on a related note, though advertising typically does not influence the consumers' *true* valuations for a firm's products, it may have an impact on *perceived* valuations. Thus, we can think of advertisers as selecting distributions of perceived valuations, with constraints given by the nature of the true valuation distribution.

We will deal with two related, but different types of questions. Taking the perspective of a profit-maximizing firm, we ask how it should optimally choose valuation distributions when it faces constraints that reflect technological conditions (in the case of product selection and innovation) or credibility restrictions (in the case of advertising). Taking the consumer perspective, we ask how the firm's choice of valuation distributions affects surplus, considering that it potentially also influences pricing decisions. We address these questions in a setting where the firm sells two imperfect substitutes. Its consumers buy one of the two products or none of them. The firm selects the valuation distribution from some fixed set of alternatives as well as the price for the two goods.[1] The firm has to take the average valuations across all consumers and goods as given; it can only influence the shape of the valuation distribution.

In our benchmark model, the average valuations for the two goods and the valuation differences are distributed uniformly and independently of each other, and the firm can choose the dispersion in each of these dimensions. In this setting, we first analyze the effects of greater heterogeneity in the differences between the valuations that consumers attach to its products. In the context of product selection, this amounts to asking how the firm should differentiate its products from each other. Should it focus on similar products, or should it sell very heterogeneous products? On the one hand, some differentiation should be profitable to improve match quality for consumers. On the other hand, the profitable extent of differentiation is limited by the heterogeneity of consumer preferences: It is pointless to sell products that are highly differentiated when consumer tastes are not. Against this background, our paper identifies a multi-product monopolist's profit-maximizing strategy for differentiating its products from each other. We find that the firm optimally chooses its products

[1] To focus on demand-side incentives, we abstract from different costs of the alternatives.

so as to maximize the dispersion of the distribution of valuation differences. Intuitively, while such increasing *valuation difference heterogeneity* will have ambiguous effects on the consumers' valuations of individual goods, it will increase the highest valuation. This results in a positive demand effect and thus in higher profits. By contrast, the effects on consumer surplus are mixed. Even though consumers benefit from the resulting increase in their highest valuation, increasing valuation difference heterogeneity induces higher prices. As a result, some consumers will lose. For some parameter regions, even the average consumer surplus (net of prices) will fall.

Second, we analyze the effects of increasing heterogeneity of the average valuations that consumers attach to a firm's products, which we refer to as *willingness-to-pay heterogeneity* or *WTP heterogeneity*. In the context of product selection, we ask: Should a firm choose a product range that appeals strongly to a small group of consumers, but much less to others, or should it act as a generalist by providing a reasonable fit for most potential consumers? For instance, consider a fashion firm that caters particularly to teenagers, but somewhat less to consumers in their twenties. Should the firm attempt to focus even more on the teenagers' tastes, even if it risks alienating the less enthusiastic older consumers? Intuitively, when consumers have similar (and sufficiently high) WTP, the monopolist will want to serve most or all of them. Consumers who are indifferent between buying and not buying therefore have below-average valuations; therefore, a marginal increase in heterogeneity reduces their valuations further, making them less likely to buy. This will reduce demand and profits. As heterogeneity increases further, the consumers who are indifferent between buying and not buying will ultimately have above-average WTP. Marginally increasing heterogeneity will therefore increase their WTP and thus demand and profits. Related to these observations, there are potential conflicts of interests between the firm and its consumers. While profit maximization requires very high or very low heterogeneity, this is not necessarily true for consumer surplus maximization.

While the profit effects of WTP-heterogeneity are similar to the single-product case (Johnson and Myatt (2006)), the price effects are more involved, and they depend on the extent of valuation difference heterogeneity. As long as valuation differences between the two goods are small (and, in particular, in the degenerate case of a single product), prices are U-shaped in WTP heterogeneity.[2] For higher valuation differences, prices can be N-shaped or increasing functions of WTP-heterogeneity. Thus, the pricing behavior of multi-product firms differs significantly from the behavior of single-product firms when valuation difference heterogeneity is large enough.

As mentioned above, our analysis not only applies to the choice of *true* valuation distributions (for instance, through product selection), but also to the choice of *perceived* valuation distributions (by advertising and, more generally by communication activities, e.g. by the way the firm presents products to consumers). This links our paper to a literature in marketing and behavioral industrial organization that has asked (usually in a competitive context) whether firms want to engage in activities that confuse consumers about the true valuations for their products. For a large part of the analysis, the distinction does not matter: Regardless of whether we think of the monopolist as

[2] When valuation differences are zero for all products, the analysis becomes equivalent to the single-product case.

choosing perceived or actual valuation distributions, the profit-maximizing choice is the same, as long as consumer behavior can be captured by the distribution. Our results on the optimal choice of valuation distributions therefore apply immediately to this alternative interpretation. We can use them to show that a monopolist will always want to exaggerate valuation differences between the products she sells, whereas the incentives to exaggerate WTP-heterogeneity are less clear.

The welfare analysis for the choice of perceived valuation distributions, however, is entirely different from the choice of true distributions. The potential for misalignment between consumers and firms is substantial: Consumer confusion may lead to various inefficiencies, as consumers may buy when they should rather choose the outside option (or vice versa), or they may end up buying the wrong product. We will show how the welfare effects will depend on the monopolist's available communication strategies and, in particular, on whether she can merely exaggerate the consumers' valuation differences between goods or the heterogeneity in willingness-to-pay across consumers or whether she can even influence the ordinal ranking between goods.

We extend our analysis to a more general setting where the willingness-to-pay and the valuation differences between the two products are not necessarily uniformly and independently distributed. In this setting, the positive effects of increasing valuation difference heterogeneity on profits and gross consumer surplus still hold, and we can find conditions under which the U-shaped relation between WTP heterogeneity and profits survives. By contrast, without the uniformity assumption, it is no longer clear that greater valuation difference heterogeneity increases prices. Intuitively, though demand increases, so does the sensitivity of demand to prices. We show that, for suitable valuation distributions, the latter effect can dominate, so that increasing valuation difference heterogeneity can reduce prices. Finally, we show how a simultaneous increase in the dispersion of valuation differences and willingness-to-pay has a non-monotone (U-shaped) effect on prices and profits, but a positive effect on consumer surplus.

The role of heterogeneity for a multi-product firm differs starkly from the case where each good is sold by a single-product duopolist. We illustrate this issue for the standard case that both types of heterogeneity are sufficiently small that the duopolists would cover the entire market. In this case, valuation difference heterogeneity fully determines the duopoly outcome: As heterogeneity decreases (the two products become more similar from the consumer perspective), the mass of indifferent consumers increases and firms respond with more aggressive pricing, which reduces their profits. By contrast, WTP heterogeneity has no effect as long as competition ensures full coverage. This obviously differs from our analysis of the multi-product monopoly where, starting from low initial heterogeneity, higher WTP-heterogeneity reduces profits.

Related Literature Johnson and Myatt (2006) analyze a single-product monopolist's incentive to engage in measures that increase the heterogeneity of the valuation distribution ("demand rotations"). They find that the monopolist will go for extremes. She will either aim at making consumer valuations very homogeneous, thus following a *mass-market strategy* and serving most or even all of them, or she will try to make them very heterogeneous by following a *niche strategy* where it focuses on the consumers with very large willingness-to-pay. Our paper confirms this logic for the multiproduct case, but it shows that the price effects are more complex. More importantly, it deals with valuation difference heterogeneity, which has no counterpart in the single-product case.

Anderson and Bedre-Defolie (2019) consider a multiproduct monopolist's incentive to provide different product varieties, and they compare the outcome to the social optimum. Contrary to our paper, they focus on the number of products rather than the type.[3] Another paper dealing with the choice of different valuation distributions is Roesler and Szentes (2017).[4] The authors ask how consumers want to be perceived by profit-maximizing single-product monopolists. Supposing that a consumer with a true valuation distribution receives an unbiased signal of his true valuation, they determine the signal distribution that maximizes consumer surplus if the monopolist chooses her optimal price. The structure of the problem constrains the choice to distributions that dominate the true distribution in a second-order stochastic dominance sense. Apart from the focus on single-product firms, the analysis differs from ours as the restrictions on the available distributions come from the interpretations as signal distributions of the underlying true distribution.

Our analysis of consumer confusion is related to Hefti et al. (2020) who consider duopolists that can confuse consumers by adding noise to their relative valuations. They identify properties of the underlying distribution of true preferences determining whether firms engage in confusion to soften competition. Surprisingly, we find that similar conditions for confusion to increase profits apply with multi-product monopolists, even though competition-softening is obviously not an issue in our context. More broadly, the issue of consumer confusion is related to a class of papers that discuss how firms can exploit consumer naïveté.[5] Most of this literature emphasizes the competition-softening role of consumer confusion in duopoly. Exceptions are Petrikaitė (2018) and Gamp (2019) who deal with the use of obfuscation techniques by a multiproduct monopolist. Both papers address very different questions than we do, focusing on how a monopolist can manipulate search costs to increase profits.

The paper is organized as follows. In Section 2, we introduce our benchmark model with uniformly and independently distributed willingness-to pay and valuation differences. Section 3 provides the results in this setting. Section 4 deals with a more general model, analyzing to which extent our insights are robust. Section 5 discusses the welfare effects of confusion. Section 6 concludes.

2 Benchmark Model

We now introduce our benchmark model. In Section 2.1, we state the assumptions that we will maintain throughout the paper, even when generalizing the benchmark model. Section 2.2 then

[3]More broadly related, several recent papers have revisited multiproduct pricing by monopolists (Amir et al. (2016) and Armstrong and Vickers (2018)) and oligopolists (Nocke and Schutz (2018)), respectively. None of these papers deals with the issue of choosing the product set.

[4]Their analysis is motivated by the emergence of consumer information apps such as Yelp.

[5]For instance, firms can use hidden fees (Gabaix and Laibson (2006); Heidhues et al. (2016)), spurious differentiation resulting from the credulity of consumers (Spiegler, 2006), complex price formats (Carlin, 2009; Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013), intransparent webpages (Ellison and Ellison, 2009), or higher consumer search costs (Ellison and Wolitzky, 2012).

specifies distributional assumptions that only apply in the benchmark model. Section 2.3 discusses the framework.

2.1 General Framework

A monopolist produces two goods $i = 1, 2$ at zero marginal cost; we abstract from fixed costs. There is a unit mass of consumers indexed by k. We assume that the monopolist chooses a distribution F of consumer valuations $(v_1, v_2) \in \mathbb{R}^2$, where his choice is restricted to some set $\mathcal{F}$. We maintain the following assumption unless otherwise mentioned.

Assumption 1. *The set $\mathcal{F}$ consists of distributions F with supports $S^F \subseteq \mathbb{R}^2$. These distributions (i) are symmetric at the diagonal given by $v_1 = v_2$;*
(ii) have the same average valuation $\overline{V}$ across consumers and products;
(iii) have a density f or finite support.

Parts (i) is a particularly stark way of expressing the horizontal relation between the different goods – neither one of them has a systematic advantage over the other. (ii) implies that the monopolist cannot influence the overall valuation (ii) by his activities. Such unbiasedness assumptions are commonly used in the literature as a disciplining device.[6] In a monopolistic setting, (ii) is particularly useful because it allows us to focus on non-trivial choices of the monopolist.[7] By contrast, Part (iii) of Assumption 1 is made for convenience only. For ease of exposition, we will focus on the case with a density in the general analysis; the case with finite support will be used only in an illustrative example in Section 4.3.
Moreover, the monopolist chooses a price vector $\mathbf{p} = (p_1, p_2) \in \mathbb{R}^2_+$. Each of the consumers buys at most one of the two goods. A consumer k will acquire the alternative with the highest net utility $v_i(k) - p_i$, $i \in \{1, 2\}$ or choose an outside option with value normalized to 0. Thus, we obtain the demand function for product i as

$$D_i^F(p_i, p_j) = Pr^F\Big(v_i(k) - p_i \geq max\big\{v_j(k) - p_j, 0\big\}\Big),^8 \tag{1}$$

where $j \neq i$ and $Pr^F(.)$ stands for the probability of the event given the valuation distribution F. We first consider the monopolist's choice of prices for fixed valuation distribution F. The firm solves the following optimization problem:

$$\max_{p_1, p_2} \Pi^F(p_1, p_2) = p_1 D_1^F(p_1, p_2) + p_2 D_2^F(p_2, p_1) \tag{2}$$

Once we have dealt with the pricing problem, we will ask how the monopolist optimally chooses the valuation distribution from the set $\mathcal{F}$. For the welfare analysis, we note that total surplus if F

[6] See the discussion in Hefti et al. (2020) for details.

[7] If the monopolist had the opportunity to increase average consumer valuations at zero cost, she would always benefit from doing so.

[8] When F is finite, this expression has to be modified by introducing an assumption that the firms share the demand of consumers for which $v_i(k) - p_i = v_j(k) - p_j$, equally.

has a density is

$$TS^F(p_1,p_2) = prob\Big(v_1 - p_1 \geq max\{v_2 - p_2, 0\}\Big) E\Big[v_1 \big| v_1 - p_1 \geq max\{v_2 - p_2, 0\}\Big] + \\ prob\Big(v_2 - p_2 \geq max\{v_1 - p_1, 0\}\Big) E\Big[v_2 \big| v_2 - p_2 \geq max\{v_1 - p_1, 0\}\Big],$$

where $E[...]$ is the expectation of the respective valuation conditional on that good being purchased. Therefore,

$$TS^F(p_1,p_2) = \int_{p_1}^{\infty} \int_0^{v_1+p_2-p_1} v_1 f(v_1,v_2) dv_2 dv_1 + \int_0^{v_2+p_1-p_2} \int_{p_2}^{\infty} v_2 f(v_1,v_2) dv_2 dv_1. \tag{3}$$

(Net) Consumer surplus is

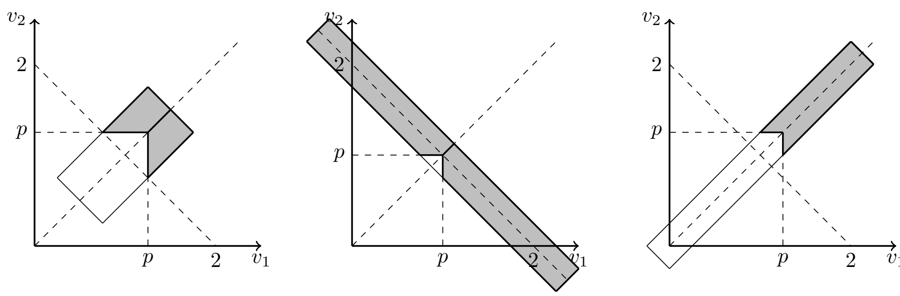
$$CS^F(p_1,p_2) = TS^F(p_1,p_2) - \Pi^F(p_1,p_2). \tag{4}$$

2.2 Distributional Assumptions of the Benchmark Model

We illustrate our ideas in a simple benchmark model. We start with the observation that the preferences of a consumer with valuation vector $(v_1(k), v_2(k))$ can equivalently be characterized by his average willingness-to-pay $\overline{v}_k \equiv \frac{v_1(k)+v_2(k)}{2}$ and his valuation difference $\Delta(k) \equiv v_2(k) - v_1(k)$. Next, we assume that $\overline{v}_k$ is uniformly distributed on an interval $[1-W, 1+W]$, where W captures WTP-heterogeneity; in particular, the average WTP in the population, $\overline{V}$, is 1. The valuation difference Δ is uniformly distributed on an interval $[-2E, 2E]$ (independently of the average WTP); where E captures the extent of valuation difference heterogeneity in the population. Thus the valuation distribution has density $f(E,W) \equiv \frac{1}{8EW}$ on its support $[-2E, 2E] \times [1-W, 1+W]$, and it is symmetric in two ways. First, products are symmetric in the sense that average valuations are the same for both goods. Second, the population is symmetric with respect to the average WTP, so that upward deviations from the average WTP are equally likely as downward deviations of the same size.

Figure 1 illustrates the monopolist's demand at symmetric prices p, with total demand consisting of the mass of all valuation vectors for which at least one good has a value of at least p.[9] The figure distinguishes between three different cases. In all three cases, the rectangle with corners $(1-W-E, 1-W+E)$, $(1-W+E, 1-W-E)$, $(1+W-E, 1+W+E)$ and $(1+W+E, 1+W-E)$ describes the support of the joint valuation distribution. Case (a) assumes that $W < 1-E$, so that all consumers value both goods positively. The zero-cost assumption implies that, in this case, serving all consumers would be efficient (maximize total surplus for the given valuation distribution). We do not rule out that some consumers have negative valuations for one or

[9]The solution of the firm's optimization problem is symmetric.



(a) $W < 1 - E$ **(b)** $1 - E < W < 1$ **(c)** $W > 1$

Note: In each case, demand is determined by the size of the (shaded) region of valuation vectors (v_1, v_2) with $\max(v_1, v_2) \geq p$.

Figure 1: The monopolist's demand at symmetric prices

both goods: This will happen for sufficiently large WTP heterogeneity and/or valuation difference heterogeneity ($W > 1 - E$): If $1 - E < W < 1$ (Case (b)), then consumers with pronounced preferences for one of the two goods may have negative valuations for the other good; if $W > 1$ (Case (c)), then some consumers even have negative valuations for both goods. We will restrict the monopolist's choices of valuation distributions as follows.

Assumption 2. *The set $\mathcal{F}$ consists of all distributions corresponding to (E, W) such that $E \in [\underline{E}, \overline{E}]$ for some $\overline{E} \geq \underline{E} \geq 0$ and $W \in [\underline{W}, \overline{W}]$ for some $\overline{W} \geq \underline{W} \geq 0$. Unless otherwise mentioned, we set $\underline{E} = \underline{W} = 0$.*

The monopolist therefore faces substantive restrictions when choosing valuation distributions. She cannot induce arbitrary valuation distributions. Because $E = 0$ corresponds to the case that both goods are valued equally by each consumer, the assumption that $\underline{E} = 0$ effectively implies that the monopolist has the option to choose only one product. The assumption that $\underline{W} = 0$ corresponds to the case that the monopolist has the option to offer two goods that every consumer values the same on average.

2.3 Discussion

We now discuss our framework. We first discuss two alternative interpretations to which it applies. Thereafter, we relate our concept of valuation difference heterogeneity to standard notions of product differentiation.

2.3.1 Interpretation of the Choice Problem

We have two main interpretations of the monopolist's choice of valuation distributions. First, one may think of the monopolist as having access to an exogeneously given set $\mathcal{I}$ of goods i. For each of these goods, each consumer k has some valuation $v_i(k)$, which we take as exogenously given for the monopolist. However, we suppose that the monopolist has to select two of the goods ($i = 1, 2$) from

$\mathcal{I}$, for instance, because of capacity constraints and diseconomies of scope. This selection results in a valuation vector $\big(v_1(k), v_2(k)\big)$ for each consumer k, so that the monopolist effectively chooses a distribution F of consumer valuations $(v_1, v_2) \in \mathbb{R}^2$. The restriction to the exogenously given set $\mathcal{F}$ of distributions reflects the set of available products $\mathcal{I}$ and consumer preferences.

In the second interpretation, which we will return to in Section 5, the set of available products is fixed. However, the monopolist can influence the consumers' perceptions of products by choosing her advertising and communication policies. We can then distinguish between true valuations, which correspond to those actually experienced in consumption, and perceived valuations, which correspond to those believed to be correct after exposure to firm communication. We ask whether the monopolist wants to confuse consumers by distorting their perceptions or whether she aims at limiting confusion. In this interpretation, the set $\mathcal{F}$ will correspond to the set of perceived distributions that the monopolist can induce. The restrictions on the set of feasible distributions will reflect the existence of an underlying distribution of true valuations which the monopolist cannot credibly distort beyond limits.

In most of this paper, we will use the first interpretation of the monopolist's choice of valuation distribution. As far as monopoly profits and prices are concerned, this will make no difference. When dealing with consumer surplus, more care will be necessary, as we will see in Section 5, where we focus specifically on the welfare effects of confusion strategies on consumer surplus.

2.3.2 Valuation Difference Heterogeneity vs. Product Differentiation

Before returning to the main line of argument in Section 3, we clarify the relation between our concept of valuation difference heterogeneity and the standard notion of product differentiation in a locational model of the Hotelling type. We only sketch the argument here, the details are in Appendix A.3.1. The model of Section 2.2 with $W = 0$ translates directly into a model where the consumers are uniformly distributed on a Hotelling line, the products of the monopolist are at the opposing ends of the line and the consumers face linear transportation costs that have to be subtracted from the gross valuation for each product. In such a Hotelling model, a standard way to capture increasing product differentiation would be an increase in transportation costs. While such an increase does lead to higher valuation differences, it typically also reduces the consumers' overall valuation for the products – which contrasts with our definition of an increase in valuation difference heterogeneity.[10] Instead, an appropriate way to capture valuation difference heterogeneity in the Hotelling setting is as the combined effect of an increase in transportation costs and an (appropriately sized) simultaneous increase in gross valuation.

[10] An alternative way to capture product differentiation in a Hotelling setting is by an outward shift of the product locations (for fixed consumer location). Again, such a change would potentially have the feature of reducing overall valuations and therefore does not correspond to an increase in valuation difference heterogeneity.

3 Results for the Benchmark Model

We now suppose the monopolist who faces a population of consumers can influence the valuation distribution by choosing heterogeneity as captured in the parameters E and W, respectively. To simplify the discussion of the intuition, we consider *monotone dispersions* of willingness-to-pay and valuation differences, where the dispersion increases the deviation of the respective variable from the average *for each consumer proportionally.* We ask how these changes affect prices, profits and consumer surplus. The results will highlight conflicts of interest between consumers and the firm. We write the total surplus and consumer surplus for symmetric prices $(p_1, p_2) = (p, p)$ and parameter values E and W as $TS(p; E, W)$ and $CS(p; E, W)$, respectively. Evaluated at profit-maximizing prices $p^*(E, W)$, the surplus levels become $TS^*(E, W) := TS(p^*(E, W); E, W)$ and $CS^*(E, W) := CS(p^*(E, W); E, W)$. The *total effect* of a marginal change in the parameter $\theta \in (E, W)$ can be decomposed as follows:

$$\frac{dTS}{d\theta} = \frac{\partial TS}{\partial \theta} + \frac{\partial TS}{\partial p}\frac{\partial p^*}{\partial \theta}, \tag{5}$$

with all derivatives evaluated at $p = p^*(E, W)$.[11] Here, we refer to $\frac{\partial TS}{\partial \theta}$ as the *direct effect* of the parameter change and to $\frac{\partial TS}{\partial p}\frac{\partial p^*}{\partial \theta}$ as the *indirect effect.* We use an analogous terminology for consumer surplus CS.[12] We start with the effect of higher valuation difference heterogeneity E.

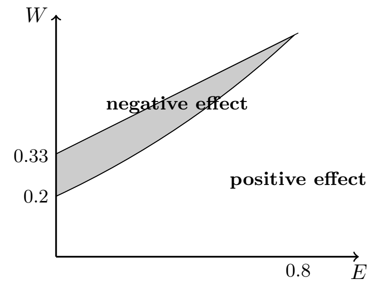
Proposition 1. *Consider the benchmark model of Section 2.2.*
(i) An increase in valuation difference heterogeneity E has a positive direct effect on consumer surplus; moreover, it increases profits and prices.
(ii) There exists a parameter region in which E has a negative total effect on consumer surplus.

The proposition is an immediate implication of Lemma A.1 in the appendix, which characterizes the profit-maximizing price vector, and Lemma A.3, which characterizes consumer surplus. To understand the intuition for (i), note that a monotone valuation dispersion increases the higher of the two valuations for each consumer, which is the one determining the maximal price she is prepared to pay. Accordingly, an increase in E has a positive direct effect on consumer surplus, and it increases demand at fixed prices. As a result, it leads to higher profits. Moreover, the dispersion increases prices because it not only increases demand, but simultaneously reduces the sensitivity of demand to price changes: The uniform spread in the distribution means that there are less critical consumers at any given price level, so that a marginal price increase reduces demand less than before the dispersion.

Proposition 1(ii) reflects the intuition that this price increase counteracts the direct positive effect of prices on consumer surplus: For those consumers who hardly benefit from the increasing heterogeneity, the adverse price effect dominates and they are worse off. As a result, it is possible that net consumer surplus falls. Thus, in spite of the direct positive effect on consumer surplus, the

[11] The relevant derivatives generically exist; see Appendix A.1.1.

[12] For profits, the envelope theorem implies that there is no indirect effect, so that the total and direct effects of the parameter changes coincide.



Note: The shaded area consists of all (E,W) for which a marginal increase in E decreases consumer surplus (net of prices).

Figure 2: Consumer Surplus and Valuation Difference Heterogeneity "E"

firm's optimal (high) choice of the dispersion parameter E does generally not maximize consumer surplus. Figure 2 illustrates this point. The figure highlights the parameter region in which (net) consumer surplus is decreasing in E. The right border of the region therefore characterizes the values of E that minimize net consumer surplus locally for fixed W. To the right of this line, consumer surplus increases in E again. Thus, for fixed W, the profit-maximizing firm's choice of E (which, by Proposition 1(i), is $\overline{E}$, the maximal available E) does not necessarily maximize consumer surplus.[13]

The following proposition (illustrated in Figure 3) describes the more complex effect of increasing WTP heterogeneity W.

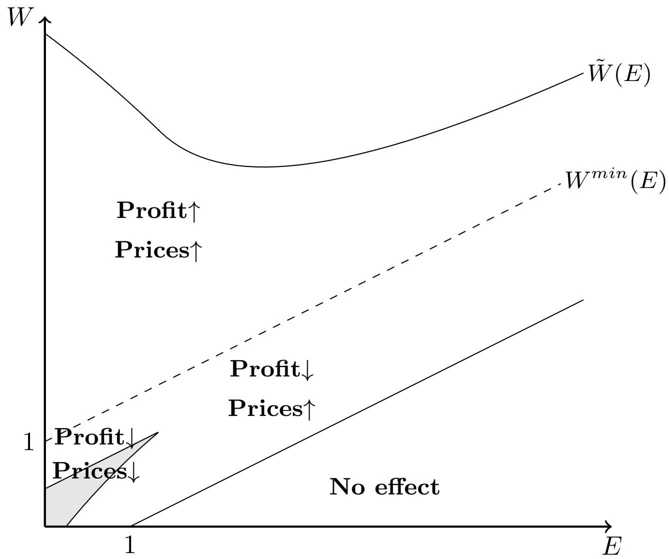
Proposition 2. *Consider the benchmark model of Section 2.2.*
(i) The direct effect of W on consumer surplus is positive.
(ii) For any $E \geq 0$, profits are first weakly decreasing in W; beyond a threshold they become weakly increasing; for any given E, the profit-maximizing price of the firm is $W = \underline{W} = 0$ if $\overline{W}$ is below a threshold $\tilde{W}(E)$ and $W = \overline{W}$ if $\overline{W} > \tilde{W}(E)$.
(iii) For sufficiently small $E \geq 0$, prices are U-shaped in W. For intermediate values of E, prices are N-shaped in W (first increasing, then decreasing, then increasing); for high values of E, prices are monotone increasing in W.

The clear monotonicity result (i) may seem surprising as the effect of W on an individual consumer's average WTP is positive if the WTP is above average and negative if it is below average. However, as the consumers who buy at any given price are those with relatively high valuations, the former effect is more important than the latter, resulting in the positive overall effect.[14] Figure 3 illustrates parts (ii) and (iii) of Proposition 2. Below the dashed line $W^{min}(E)$, higher WTP heterogeneity

[13] This is most obvious for W between 0.2 and 0.33, where consumer surplus is monotone decreasing as long as E is sufficiently small. For these W, assuming that $\overline{E}$ is sufficiently small, consumer surplus is maximal for $E = 0$, whereas the firm wants to choose E as high as possible.

[14] This argument needs to be modified with more general distributions, where the average willingness to pay is not symmetrically distributed; for instance, when high WTP-consumers are rare, then the adverse effect on low WTP-consumers will dominate.

weakly reduces profit.[15] Above the dashed line, the firm's profit is increasing in WTP heterogeneity. It reaches the level corresponding to $W = 0$ again at $\tilde{W}(E)$, then increases further. Thus a firm will either choose minimal heterogeneity or, if it has access to a sufficiently high level of heterogeneity, it will choose the maximal available W. The result reflects the relation between W and demand. Intuitively, for sufficiently low initial heterogeneity, the firm wants to keep a large share of consumers on board, including some with below-average WTP. When W increases slightly, the WTP of these marginal consumers (who have below-average WTP) falls, resulting in a lower total demand at the given price and thus in lower profits. As W increases further, it will eventually cross a threshold beyond which the price is set so high that only consumers with above-average valuation demand the good. As their valuation is positively affected by a further increase in W, such an increase has a positive effect on total demand and profit.



Note: In each region, an upward-pointing (downward-pointing) arrow stands for a positive (negative) effect of a small increase of W on the respective variable

Figure 3: Profits, Prices and WTP Heterogeneity

In the small shaded region near the origin in Figure 3, prices are decreasing in W; outside that region they are increasing. Thus, the relation between WTP heterogeneity W and prices is non-monotone only for small values of E. To understand this relation, note that, as usual, an increase in prices has a positive effect on revenue per consumer, but reduces demand. An increase in W mitigates the latter effect, because it reduces the mass of marginal consumers for any given interior price. Therefore, as shown in Figure 3, prices can be increasing in W even in a part of the region where demand and profits are decreasing in W.

Proposition 2 illustrates the conflict between firm and consumers concerning the choice of WTP-heterogeneity. Starting from low levels of W, profits fall. By contrast, the effect on consumer

[15] Below the line $W = \frac{E}{2} - \frac{1}{2}$, demand and thus profits are independent of W.

surplus may well be positive. As the *direct* effect of W on consumer surplus is everywhere positive by part (i), the *total* effect must at least be positive in the area where prices are decreasing in W (the shaded area in Figure 3; see part (iii)).[16] Thus, consumers may want some heterogeneity whereas the firm does not. For higher initial values of W, a different type of conflict may emerge, because both profits and prices are increasing in W. The adverse price effect on consumer surplus in this region may dominate the positive direct effect, so that the interests of the firm and consumers are again not aligned – now it is the firm who prefers greater heterogeneity while consumers do not. Figure A.5 in Appendix A.3.2 illustrates the areas where conflicts between profit maximization and consumer surplus maximization emerge. In line with the above arguments, there is a region where consumer surplus is increasing in WTP-heterogeneity, but profits are decreasing. Conversely, there is a region where consumer surplus is decreasing in WTP-heterogeneity, but profits are increasing. Whether the conflicts between firm and consumers result in a suboptimal choice of W from the consumer perspective depends on the constraint set.[17] If $\overline{W}$ is sufficiently large (above $\tilde{W}(E)$ for the firm and above a line $\tilde{W}^{CS}(E)$ defined analogously for consumers), then profit maximization and maximization of consumer surplus both demand choosing $W = \overline{W}$, that is, as high as possible. For smaller values of $\overline{W}$, the optimal choice of the firm need not maximize total consumer surplus.

Comparison with Duopoly The effects of increasing heterogeneity for a multiproduct monopolist differ from those arising when each product is offered by a duopolist. We will illustrate this for the standard case that competition is sufficiently intense that the duopoly market is fully covered. As $v_i - v_j$ is distributed uniformly on $[-2E, 2E]$, in the full-coverage case, the duopoly version of our model is equivalent to a Hotelling duopoly with firms located at $L = -0.5$ and $L = 0.5$ and linear transportation cost $t = 2E$. Using standard arguments, the candidate equilibrium in this case is $p^d = 2E$.[18] Full coverage requires $1 - W \geq p^d$, so that the consumer with the minimal valuation still buys one of the products. Hence, a necessary condition for existence of this equilibrium is $W \leq 1 - 2E$. In Appendix A.3.4, we show that this condition is also sufficient for this full coverage equilibrium to exist. Equilibrium profits per firm are $\Pi = t/2 = E$. Thus, as long as $W \leq 1 - 2E$, prices and profits are monotone increasing in valuation difference heterogeneity as in the monopoly case. Intuitively, increasing valuation difference heterogeneity softens competition, so that duopolists can safely increase prices. Contrary to the multi-product case, however, sufficiently small changes of WTP heterogeneity have no effect: As W does not influence the mass of indifferent consumers, prices and profits are independent of WTP heterogeneity. As soon as W becomes high enough that the regime boundary is reached, it clearly does have an effect, as the full-coverage equilibrium breaks down.

[16] Appendix A.3.2 provides a calculation of consumer surplus, as well as a graphical description of the areas where W has negative or positive effects on consumer surplus, respectively.

[17] See Appendix A.3.2 for details.

[18] Note that this is not only true for $W = 0$, but for all W for which full coverage applies.

4 General Valuation Distributions

In the following, we extend the analysis beyond our benchmark model by relaxing the independence and uniformity assumptions on the valuation distribution. While several of the main insights of the previous section generalize, others need to be qualified.

4.1 Assumptions and Terminology

We maintain the assumptions of Section 2.1. In addition, we introduce some further terminology and assumptions. We continue to denote consumer k's *valuation difference* as $\Delta(k) = v_2(k) - v_1(k)$ and his average valuation or average *willingness-to-pay (WTP)* as $\overline{v}(k) \equiv \frac{v_1(k)+v_2(k)}{2}$. Moreover, we denote the average valuation across all consumers and goods as $\overline{V}$. While this overall average valuation is fixed across all distributions, the average valuation of an individual consumer for the two goods may differ across distributions.

Definition 1. *Fix some valuation distribution $F \in \mathcal{F}$.*
*(i) An average valuation $\overline{v}$ is **conceivable** if there exists (v_1, v_2) in S^F such that $\frac{v_1+v_2}{2} = \overline{v}$.*
*(ii) A valuation difference Δ is **conceivable** if there exists (v_1, v_2) in S^F such that $v_2 - v_1 = \Delta$.*

Thus, in the benchmark model, the overall average valuation is 1, but the conceivable average valuations of individual players are those in $[1-W, 1+W]$. The conceivable valuation differences are those which are in $[-2E, 2E]$. Contrary to the benchmark model, we no longer assume that valuation differences and average valuations are distributed independently of each other. Accordingly, we now introduce the notation $G_{\overline{v}}$ and $g_{\overline{v}}$ for the conditional distribution of valuation differences and its density, respectively, induced by restricting average valuations to $\frac{v_1+v_2}{2} = \overline{v}$ for some conceivable $\overline{v}$. The symmetry assumption for F from Assumption 1 directly translates into a symmetry condition for $g_{\overline{v}}$:

$$g_{\overline{v}} \text{ is symmetric at zero for all } \overline{v} \geq 0. \tag{6}$$

Hence, for any given average valuation $\overline{v}$ and valuation advantage Δ of good 2 over good 1, it is equally likely that good 1 has the same valuation advantage over good 2. We further denote the distribution of average valuations induced by F as $\overline{F}$. Finally, for every conceivable valuation difference Δ, we denote the distribution of average valuations conditional on Δ as $\overline{F}_\Delta$, its density as $\overline{f}_\Delta$, and its expectation as $E(\overline{F}_\Delta)$.

Definition 2. *(i) Two valuation distributions F^1 and F^2 are **WTP-equivalent** if the induced distributions $\overline{F}^1$ and $\overline{F}^2$ of average valuations are identical.*
*(ii) Two valuation distributions F^1 and F^2 are **valuation-difference-equivalent** if the induced distributions G^1 and G^2 of valuation differences $v_2(k) - v_1(k)$ are identical.*

WTP-equivalent distributions can differ only in the distribution of consumer valuation differences, not in the distribution of their average WTP.[19] Thus, in our benchmark model two WTP-equivalent

[19] For instance, if consumers' positions are shifted from the interior of a Hotelling line towards the end, this induces a WTP-equivalent dispersion of the valuation distribution.

distributions have the same W, but not necessarily the same E. Conversely, valuation-difference-equivalent distributions can differ only with respect to the WTP distribution, not with respect to the distribution of valuation differences. In the benchmark model, E must therefore be the same in both cases, but not W. We now introduce different notions of dispersion of valuations.

Definition 3. *(i) Consider two WTP-equivalent valuation distributions F^1 and F^2 with corresponding difference distributions G^1 and G^2. F^2 induces a **valuation difference dispersion** of F^1 if, for every conceivable $\overline{v}$ and $v_2 > v_1$, $G^2_{\overline{v}}(v_2 - v_1) \leq G^1_{\overline{v}}(v_2 - v_1)$, with strict inequality for (v_1, v_2) on a subset of S^{F^1} with positive measure.*
*(ii) Consider two valuation-difference-equivalent valuation distributions F^1 and F^2 with corresponding WTP-distributions $\overline{F}^1$ and $\overline{F}^2$. F^2 induces a **WTP dispersion** of F^1 if the following properties hold. First, for every conceivable Δ, $E(\overline{F}^1_\Delta) = E(\overline{F}^2_\Delta)$. Second, for every conceivable Δ, $\overline{F}^1_\Delta(\overline{v}) \geq \overline{F}^2_\Delta(\overline{v})$ if $\overline{v} > E(\overline{F}^1_\Delta)$, $\overline{F}^1_\Delta(\overline{v}) \leq \overline{F}^2_\Delta(\overline{v})$ if $\overline{v} < E(\overline{F}^1_\Delta)$, with strict inequalities for (v_1, v_2) on a subset of S^{F^1} with positive measure.*

As valuation differences are symmetrically distributed around zero, a valuation difference dispersion makes the valuation differences more heterogeneous (while the WTP remains unchanged); generalizing the increase in E in the benchmark model, it shifts mass from valuation profiles close to the diagonal $v_1 = v_2$ to locations that are further away. Conversely, generalizing the increase in W in the benchmark model, a WTP dispersion increases the heterogeneity of average valuations, but does not affect valuation differences. Thus, it stretches valuation profiles along (parallel to) the diagonal.

To avoid technicalities, we state the following assumption directly rather than in terms of distributional conditions. Again, the assumption holds in the benchmark model.

Assumption 3. *Every valuation distribution $F \in \mathcal{F}$ is such that the firm's maximization problem has a unique solution, which involves symmetric prices $(p^*(F), p^*(F))$.*

Note that we do not explicitly require concavity of profit functions here, but we will impose it where necessary.

4.2 Analysis

We first focus on the optimal choice of prices for a fixed valuation distribution. By Assumption 3, the firm's profit function simplifies to:

$$\Pi^F(p) = pPr^F(\max\{v_1, v_2\} \geq p). \tag{7}$$

We denote the profit resulting from this problem as $\Pi^*(F)$. In the case that F has a density, the profit function becomes

$$\Pi^F(p_1, p_2) = p_1 \int_{p_1}^{\infty} \int_0^{v_1+p_2-p_1} f(v_1, v_2) dv_2 dv_1 + p_2 \int_0^{v_2+p_1-p_2} \int_{p_2}^{\infty} f(v_1, v_2) dv_2 dv_1 \tag{8}$$

The first-order condition for the profit-maximizing choice of p is thus

$$(\Pi^F)'(p) = \left(1 - \int_0^p \int_0^p f(v_1, v_2) dv_2 dv_1\right) - \left(2p \int_0^p f(p, v_2) dv_2\right) = 0 \tag{9}$$

The first bracketed term is the benefit from serving inframarginal consumers at a higher price. The second bracketed term captures the demand losses resulting from higher prices. The following result is useful to understand the comparative statics of profits and prices, respectively.

Lemma 1. *Consider the general model introduced in Section 4.1. Suppose $F^2 \neq F^1$.*
(i) If $D^{F^2}(p^*(F^1), p^*(F^1)) > D^{F^1}(p^*(F^1), p^*(F^1))$, where $p^*(F^1)$ is the solution of the maximization problem for F^1, then $\Pi^*(F^2) > \Pi^*(F^1)$.
(ii) Suppose F^2 has a density and $\Pi^F(p)$ is strictly concave. Then $p^*(F^2) > p^*(F^1)$ if and only if

$$\begin{aligned}(\Pi^{F^2})'(p^*(F^1)) =& D^{F^2}(p^*(F^1), p^*(F^1)) + p^*(F^1)\left(\frac{\partial D^{F^2}}{\partial p_1}(p^*(F^1), p^*(F^1)) + \frac{\partial D^{F^2}}{\partial p_2}(p^*(F^1), p^*(F^1))\right) = \\ & 1 - \int_0^{p^*(F^1)} \int_0^{p^*(F^1)} f^2(v_1, v_2) dv_2 dv_1 - 2p^*(F^1) \int_0^{p^*(F^1)} f^2(p^*(F^1), v_2) dv_2 > 0\end{aligned}$$

Part (i) of Lemma 1 generalizes the logic underlying the profit effects in the benchmark model where positive demand effects of increasing heterogeneity translated into increasing profits. This logic holds *for any demand increase* because a monopolist can always react to increasing demand by leaving prices unchanged – thus an activity that increases demand is always profitable. By (ii), whether changes in valuation distributions increase prices depends on the effects on the mass of inframarginal and marginal consumers, respectively. Intuitively, moving from F^1 to F^2 will increase prices if, at the initial equilibrium with prices $p^*(F^1)$, the marginal profit effect of increasing both prices at the same time is positive. The first line of the expression in (ii) captures this marginal profit effect; the second line translates it into a distributional condition.

4.3 Increasing Heterogeneity

We now suppose that $\mathcal{F}$ contains distributions that can be compared with respect to valuation difference dispersion or WTP dispersion, and we ask how these dispersions affect prices, profits and surplus.

Increasing valuation difference heterogeneity We immediately obtain the following result, which generalizes the profit effect discussed in Proposition 1(i).

Proposition 3. *Consider the general model introduced in Section 4.1. A valuation difference dispersion has a positive direct effect on consumer surplus, and it strictly increases profits. In particular, a profit-maximizing choice of $F \in \mathcal{F}$ must maximize valuation difference dispersion.*

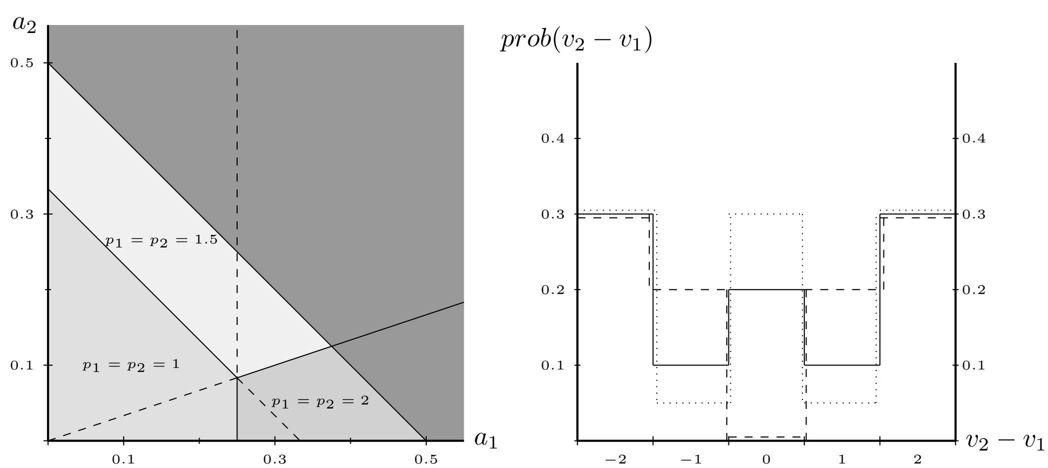
We only sketch the proof, which is straightforward. Intuitively (again arguing for a monotone valuation difference dispersion as in the benchmark model), though the average valuation remains unchanged, a higher *maximal* willingness to pay, $\max_{i\in 1,2} v_i(k)$, becomes more likely. Thus, for a fixed symmetric price vector, demand and consumer surplus increase as well. For a multiproduct monopolist who can sell at most one good to each consumer, the change is profitable.

Proposition 3 does not contain a statement about the relation between valuation difference heterogeneity and prices. Contrary to the benchmark model, higher valuation difference heterogeneity does not necessarily induce higher prices. We provide an instructive discrete counterexample that has the spirit of a Hotelling model.

Example 1 Suppose given two non-negative real numbers a_1 and a_2 such that $a_1 + a_2 \leq 1$. Consider the discrete valuation distribution F with $prob(2,0) = prob(0,2) = a_1$, $prob(1.5, 0.5) = prob(0.5, 1.5) = a_2$ and $prob(1,1) = 1 - 2a_1 - 2a_2 = a_3$. Table 1 shows demand and profits for the three possible candidates for an optimal price.

Price	Demand	Profit
2	$2a_1$	$4a_1$
1.5	$2a_1 + 2a_2$	$3a_1 + 3a_2$
1	1	1

Table 1: Demand and Profit for Optimal Price



(a) Optimal Prices (b) Valuation Difference Distributions

Note: (a) depicts the parameter regions in which the respective optimal price emerges. (b) displays distributions of valuation differences. In all cases $a_1 = 0.3$, whereas $a_2 = 0.1$ (solid); $a_2 = 0.2$ (dashed) and $a_2 = 0.05$ (dotted).

Figure 4: Price Effects of Valuation Difference Heterogeneity: Counterexample

Using the right column of Table 1, it is straightforward to show how the optimal price depends on the probabilities a_1 and a_2; Figure 4(a) depicts the regions in $(a_1; a_2)$-space where each of the three prices is optimal. There are several ways to capture increasing valuation difference heterogeneity in this parameterized example. We focus on the effect of increasing a_2 at the expense of a_3, while keeping a_1 fixed (that is, moving mass from zero valuation differences to small valuation differences), because it is most subtle.[20] When a_1 is small (below 0.25), the increase in heterogeneity (higher a_2) weakly increases prices, as the firm eventually gives up serving indifferent consumers. When a_1 is higher, the increase in heterogeneity weakly reduces prices, as the firm eventually gives up the exclusive focus on consumers with very high valuation differences. Figure 4(b) illustrates the last observation. For all three distributions depicted there, $a_1 = 0.3$. The distribution captured by the solid line corresponds to $a_2 = 0.1$ (and thus $a_3 = 0.2$). In Figure 4(a), it is represented by a point on the boundary between the regions with equilibrium prices 2 and 1.5. For the dashed distribution, which results from a valuation difference dispersion of the previous distribution ($a_2 = 0.2$), the profit-maximizing prices are $p_1 = p_2 = 1.5$, whereas for the dotted distribution, for which the valuation difference dispersion is lower than for the original one ($a_2 = 0.05$), the profit-maximizing prices are $p_1 = p_2 = 2$. Contrary to the benchmark model with uniformly distributed valuation differences, the effects of increasing heterogeneity depend on whether high valuation differences become more common relative to low valuation differences or low valuation differences become more common relative to no valuation differences. Prices tend to increase in the former case, but not in the latter.

Increasing WTP heterogeneity We have seen in the benchmark model that profits are U-shaped in WTP heterogeneity. We now want to use the more general setting to identify the driving forces behind this result. To this end, we require an assumption on the set $\mathcal{F}$ of available distributions. This assumption is satisfied in the benchmark model if we consider the set of all (E, W) for which E is fixed at some level, whereas $W \in [0, \overline{W}]$ for sufficiently large $\overline{W}$.

Assumption 4. *The set $\mathcal{F}$ of valuation distributions has the following properties:*
(i) $\mathcal{F}$ is parameterized as $\mathcal{F}_\tau$ by $\tau \in [0, T)$ for some $T > 0$, so that an increase in τ corresponds to a WTP-dispersion.
(ii) The optimal price for F_τ, p_τ, is a continuous function of τ.
(iii) There exists an open left neighbourhood of $\overline{V} \in \mathbb{R}^+$ in which $\Pi^{F_0}(p) > \Pi^{F_0}(\overline{V})$.
(iv) Define $\overline{V}^{max}$ as the maximum value of v_1 among all (v_1, v_2) in the support of F_τ for some $\tau \in [0, T)$ for which the average valuation is $\overline{V}$. Then there exists a $\tau^* > 0$ such that $\Pi^{F_{\tau^*}}(p) > \Pi^{F_{\tau^*}}(\overline{V}^{max})$ for p in an open right neighbourhood of $\overline{V}^{max}$.

This assumption captures those features of the benchmark model which are decisive for the non-monotone effect of WTP-heterogeneity: We show that the profit effect of WTP heterogeneity is

[20] Figure 4(a) immediately shows that two other increases in valuation difference heterogeneity weakly increase prices. (i) An increase in a_1 with a simultaneous reduction in a_2 by the same amount, so that a_3 remains fixed, which corresponds to putting more mass on large valuation differences rather than on small valuation differences; (ii) a proportional increase in a_1 and a_2 (a multiplication by the same factor above 1); graphically, this shifts out (a_1, a_2) along a ray from the origin, moving mass away from identical valuations to small as well as large valuation differences.

negative when dispersion is sufficiently small and positive when dispersion is large.

Proposition 4. *Suppose that $\mathcal{F}$ satisfies Assumption 4 and that $\Pi^F(p)$ is strictly concave. Then, starting from F_0, the WTP dispersion resulting from a marginal increase of τ reduces profits. For sufficiently high initial levels of WTP-heterogeneity ($\tau > \tau^*$), a further dispersion increases profits.*

Intuitively, Assumption 4(iii) requires that average valuations have to be sufficiently high. It guarantees that, with minimal WTP heterogeneity ($\tau = 0$), the firm sets its price in such a way that all consumers whose WTP is at least average buy (as well as some consumers with below-average WTP), so that the dispersion reduces the valuations of every consumer who does not buy or is indifferent. Moreover, there is a non-degenerate set of buyers whose valuation falls after a WTP-dispersion. Hence, the dispersion reduces demand. Condition (iv) guarantees that, for sufficiently high WTP heterogeneity, the price will be so high that the marginal buyer has an average valuation above $\overline{V}$. Increasing WTP therefore increases the gross consumer surplus of the marginal buyer and thus increases demand and profits.[21]

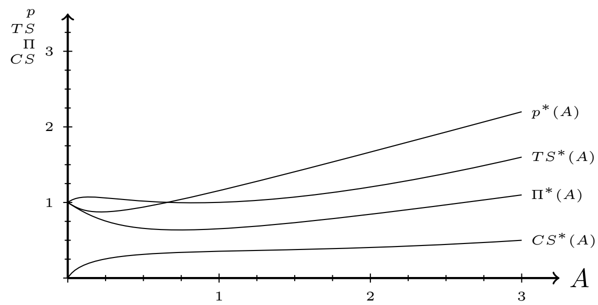
4.4 Example: Independent Product Valuations

In the benchmark model, average valuations and valuation differences are distributed independently of each other, but individual valuations are not. As another example of the general setting of Section 4.1, we now deal with the converse case that product valuations are distributed independently of each other, but average valuations and valuation differences are not. We assume that the set $\mathcal{F}$ contains only distributions such that the individual valuation distributions for each good are i.i.d. and uniform. The set of these single-good distributions is parameterized by an $A \geq 0$; with A corresponding to the uniform distribution on $[1 - A, 1 + A]$. Thus, in contrast with the benchmark model, there is only one parameter capturing heterogeneity: As A increases, consumers simultaneously become more heterogeneous with respect to willingness-to-pay and valuation differences. Moreover, WTP and valuation differences are clearly not distributed independently of each other.[22]
In Appendix A.3.3, we sketch the solution to this simple example. Figure 5 summarizes the main results. Prices and profits are both non-monotone (U-shaped) in A. Comparison with the benchmark model is instructive. There, prices and profits are both increasing in valuation difference heterogeneity, but usually not for WTP-heterogeneity.[23] Intuitively, the U-shaped relation between A and prices as well as profits reflects the influence of WTP-heterogeneity, whereas the influence of valuation difference heterogeneity is obscured. By comparison, our benchmark model thus has the advantage that it makes the differences between both types of heterogeneity transparent.
Figure 5 also shows that the effect of A on consumer surplus is strictly positive. Again, the result reflects the interplay of the effects of WTP-heterogeneity and valuation difference heterogeneity. As

[21] The result is closely related to the analysis of Johnson and Myatt (2006) for the single-product monopolist.

[22] For very high and very low WTP (near $1 + A$ and $1 - A$, respectively), the valuation differences are concentrated on a small interval compared with the case that the WTP is intermediate (closer to 1).

[23] Recall that profits were decreasing in W for low values of W and arbitrary E, whereas prices were only decreasing for low values of W and E.



Note: The figure describes equilibrium prices, profits, total surplus and consumer surplus as functions of A

Figure 5: Example: Independent Product Valuations

we saw in the benchmark model, each type of heterogeneity can have negative effects on consumer surplus. By aggregating the two sources of heterogeneity in one parameter, these subtleties are obliterated. Essentially, the positive effects of heterogeneity in one dimension dominate the negative effects in the other dimension.

5 The Welfare Effects of Confusing Consumers: A Taxonomy

We have argued that our framework can represent a firm's product choice or its communication strategy towards consumers. So far, however, we have focused on the former interpretation. We now discuss how the framework can shed light on communication strategies. To this end, we assume that the firm can choose these strategies in such a way that they potentially affect each consumer's perception of the value of each good, without having any effect on the true value of the good. The latter restriction is a natural disciplining device to guarantee that manipulating perceptions comes at a cost, which has many well-known analogues in the literature (see the discussion in Hefti et al. (2020)). More precisely, the true valuation distribution is now exogenously fixed at F_0, but the firm can select a valuation distribution from some set $\mathcal{F}$. Each choice thus corresponds to a different distribution of perceived valuations. We assume that each consumer makes his consumption decision based on these perceived valuations. Thus, he is taking the perceived valuation for granted, without questioning whether it might have been manipulated by the firm. Therefore, from the firm's perspective, the effect of choosing a certain distribution of perceived valuations from $\mathcal{F}$ is entirely analogous to the effect of choosing a distribution of true valuations. Hence, all our previous results on profits and prices directly carry over to the new interpretation. The differences exclusively concern the results on consumer surplus. For definiteness, we suppose that full revelation is feasible:

Assumption 5. *The set $\mathcal{F}$ contains the distribution F_0 of true valuations.*

Together with Assumption 1, Assumption 5 means that all communication strategies induce the true valuation on average. However, in addition to the true distribution, the firm can induce distributions of perceived valuations where some or all consumers misperceive the valuations of the goods at the moment of purchase. Then, for any consumer who would have bought the good under the truthful strategy as well as under the distorted strategy at the given price, confusion has no effect on the utility from consumption. This contrasts with the case of product choice analyzed before where we assumed that the firm's choices actually affect true valuations.

The firm's communication decisions can nevertheless potentially affect consumer surplus through three different channels. First, following the logic of Sections 3 and 4, the choice of the distribution of (perceived) valuations affects prices, so that it even influences net consumer surplus for those consumers whose consumption decision it does not affect. Second, even without any price effect, the communication strategy can affect whether a consumer purchases the good: A consumer may be misled into buying if the perceived valuation is higher than the true valuation; conversely, he may be misled into abstaining from consumption if the perceived valuation is lower than the true valuation. Third, the communication strategy can affect which good the consumer buys: It can bias some consumers in favor of one good and others in favor of the other. In the following, we will discuss the role of these channels in more detail for three different classes of obfuscating communication policies. We will carry out the entire discussion below in the context of the benchmark example (with the obvious reinterpretation that the valuation distributions reflect the firm's communication policy).

5.1 Exaggerating Valuation Differences

First, we suppose that the firm can (only) affect the consumers' perceptions of the difference between the valuations for the two products, thereby creating a valuation difference dispersion. We assume that the set $\mathcal{F}$ of available distributions of perceived valuations can be totally ordered by valuation difference heterogeneity, with a minimum and a maximum, where the minimum corresponds to the truthful policy. Because the firm benefits from greater dispersion of perceived valuation differences, it will choose the maximum instead. There are several channels through which this strategy may adversely affect consumers. First, at least in the benchmark model, the firm will choose higher prices as perceived valuations become more dispersed (keeping average valuations fixed); see Proposition 1. Second, the confusion strategy will induce some consumers to buy at some given price when they should have abstained from consumption, as they overestimate the value of their preferred good. For monotone valuation dispersions, there are no other effects on consumer welfare. Clearly, when valuation differences are exaggerated, no consumer will perceive the valuation of his preferred product as lower than it is, so that confusion does not induce consumers to abstain from consumption when they should buy. Similarly, because the preferred product is not affected by the confusion strategy, a pure exaggeration of valuation differences does not induce consumers to buy the wrong good.

5.2 Exaggerating WTP differences

Next, we suppose that, through its communication and advertising strategies, the firm can increase its appeal to some consumers, at the cost of making it appear less attractive to others. Specifically, a less truthful communication strategy increases the perceived values of both goods for consumers with above-average valuations and decreases them for consumers with below-average valuation, thus exaggerating WTP differences.[24] Accordingly, we assume that the set $\mathcal{F}$ of available distributions of perceived valuations can be totally ordered by WTP heterogeneity. For definiteness, we again assume that F has a minimum and a maximum with respect to WTP heterogeneity, with the minimum corresponding to truthful communication.

Clearly, as such communication strategies do not affect perceived valuation differences, they cannot induce a consumer to buy the wrong good. However, it might appear that, by exaggerating WTP differences, that is, by increasing the dispersion of the distribution of perceived WTP, the firm will induce some low-valuation consumers to exit the market, even though, according to their true valuations they should purchase one of the goods. However, this argument fails to take the firm's incentives into account. We know from Sections 3 and 4 that, contrary to valuation difference heterogeneity, the firm does not necessarily want WTP heterogeneity. More precisely, according to Lemma 1, the firm wants WTP heterogeneity only if this increases demand, keeping prices fixed. This corresponds to situations where the firm focuses on high-valuation consumers whose perceived valuations increase as a result of greater dispersion. It is easy to see that, in the benchmark example, the parameter region for which higher W increases profits is $W > 1 + E/2$. In this region, the optimal price is $p = \frac{2W+E+2}{4} > 1$. Thus, the firm only sells to consumers with above-average valuations. These consumers will have higher perceived valuations than under the true valuation distribution, so that, at the given price, they will never abstain from buying as a result of a WTP heterogeneity exaggeration strategy voluntarily adopted by a profit-maximizing firm. Thus, at least in this example, apart from price increases, if a firm optimally chooses to exaggerate WTP differences, this can only affect consumers by misleading some to buy a good when they should rather not buy at all. We therefore cannot take it for granted that a firm will confuse some consumers in such a way that they erroneously abstain from buying.

5.3 Noisy Information

So far, we assumed that confusion strategies work by simultaneously exaggerating positive and negative opinions of consumers. An alternative approach has been suggested by Hefti et al. (2020) who assume that (duopolistic) firms can add noise to the perception of valuation differences, which is independent of a consumer's true preferences. Thus, contrary to the assumptions of Section 5.1, consumers with true preferences for one good will not necessarily have stronger perceived preferences for this good as a result of confusion; the valuation difference may be perceived as smaller than it is, or it may even shift sign, so that a consumer is fooled into believing he prefers

[24] By reversing the arguments, the analysis applies to the opposite case that the true WTP difference distribution is dispersed and the firm can use communication to make these differences less visible.

the "wrong" good. As argued in much detail in Hefti et al. (2020) for the duopoly case, various common marketing activities have such effects. Firms may generate information overload for their consumers, for instance, by making products unnecessarily complex or relying on intransparent product labels. They may hide important information to impede product comparison or they may describe their products in a vague way. This can lead to noisy perceptions of relative product valuations. To formalize the relevant dispersion concepts, we assume as in the benchmark case that the willingness-to-pay and the valuation differences are independently distributed of each other. However, we do not impose uniformity.

Definition 4. *An* ***unbiased valuation difference confusion activity*** *is described by a random variable $\varepsilon = (\varepsilon_1, \varepsilon_2)$ drawn from a symmetric distribution Φ on $\mathcal{R}$ with density ϕ and mean 0. As a result of the confusion activity, perceived valuations differences are distributed as $\tilde{F} = F + \Phi$.*

Therefore, an unbiased valuation difference confusion activity adds noise to the true valuations, without affecting average valuations. It does not affect WTP, and it does not systematically increase or decrease valuation differences. For some consumers, it makes one good more attractive, for others the other one.

5.3.1 Results

Incentives for confusion will turn out to depend on properties of the true valuation distribution. The following definition specifies the relevant properties.[25] In this definition, we suppose the true valuation difference distribution, conditional on the average valuation being $\overline{v}$, is given as $G^0_{\overline{v}}$, with density $g^0_{\overline{v}}$ and support $[-E_{\overline{v}}, E_{\overline{v}}]$ for some $E_{\overline{v}} > 0$.

Definition 5. *(i)* ***(Indecisiveness)*** *True preferences are* ***indecisive*** *if $g^0_{\overline{v}}$ is strictly increasing on $[-E_{\overline{v}}, 0]$ and strictly decreasing on $[0, E_{\overline{v}}]$ for every conceivable $\overline{v}$.*
(ii) ***(Polarization)*** *True preferences are* ***polarized*** *if $g^0_{\overline{v}}$ is strictly decreasing on $[-E_{\overline{v}}, 0]$ and strictly increasing on $[0, E_{\overline{v}}]$ for every conceivable $\overline{v}$.*

Thus, with indecisive preferences, small valuation differences are more likely than larger valuation differences, and conversely with polarized preferences. Proposition 3 has immediate implications for the firm's incentive to engage in confusion. The conclusion depends on the shape of the distribution of true valuation differences.

Proposition 5. *(i) With indecisive preferences, unbiased valuation difference confusion strictly increases profits.*
(ii) With polarized preferences, unbiased valuation difference confusion strictly reduces profits.

The result is intuitive. With indecisive preferences, for any fixed average valuation, valuation profiles arise more frequently the closer to indifference they are. For any given price threshold, adding noise to the true valuation distribution thus tends to make more consumers believe their

[25]The definition is a slight modification of the corresponding definition in Hefti et al. (2020).

maximal valuation is beyond the threshold when it actually is not than vice versa. As a result, demand and profits increase. Conversely, when preferences are polarized, adding noise reduces demand and thus profits.[26] When the firm selects a confusion strategy, this will have qualitatively different welfare effects than when valuation differences of WTP-differences are exaggerated. It can now happen that consumers abstain from buying even though it would be optimal for them to buy, and it is also possible that consumers are confused into buying the wrong product.

6 Conclusions

This paper deals with a multiproduct monopolist's incentives to engage in marketing activities that affect consumers' valuation distributions, and we identify the effects of these choices on consumer surplus. The firm unambiguously benefits from greater consumer heterogeneity with respect to valuation differences between the two goods. While consumers benefit from the resulting increase in the value of the preferred good, they may suffer from higher prices set by the firm. We also find that a multiproduct monopolist aims at extreme distributions of *willingness-to-pay*, favoring very homogeneous and very heterogeneous populations relative to intermediate cases, while the price effects are more complex. Again, we identify conflicts of interests between the firm and its consumers with respect to the desirable amount of heterogeneity.

Applied to communication strategies, our results show that a multiproduct firm usually wants to engage in activities that exaggerate valuation difference heterogeneity, whereas the case for exaggerating willingness-to-pay heterogeneity is less clear. Either way, the firm might then induce consumers to buy a product where they should not. Moreover, the firm only wants to engage in confusion activities that introduce noise into valuation differences if the true valuations in the population display indecisiveness, meaning that they are concentrated around indifference. If preferences are polarized, meaning that indifference is relatively rare, a firm wants to educate consumers about relative valuations. Surprisingly, this conclusion is similar to Hefti et al. (2020) who show that duopolists only confuse consumers if preferences are indecisive (rather than polarized), because this softens competition. Here we find similar results without competition, reflecting the demand effects of confusion.

[26]The logic of Proposition 5 can be adjusted to apply to the case that confusion affects only WTP, while leaving valuation differences untouched. In this case, the shape of the density $\overline{f}_{\triangle}$ determines whether confusion increases or decreases profits.

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A Appendix

This appendix contains three parts. Section A.1 proves the propositions of the benchmark model stated in the main text. Section A.2 gives proofs for the results of the general model. Finally, in Section A.3, we provide formal details of those parts of the analysis that were only sketched in the main text.

A.1 The Benchmark Model

In the following, we prove the results for the benchmark model. We start with some auxiliary results, before we prove Propositions 1 and 2.

A.1.1 Auxiliary Results: Prices and Surplus

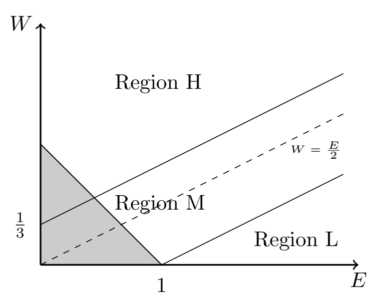
For later reference, we first provide the characterization of the profit-maximizing (symmetric) price vector on which the results in Section 3 rely.

Lemma A.1. *(a) If $W \geq \frac{1}{2}E + \frac{1}{3}$ (Region H), the profit-maximizing price is $p^* = \frac{2W+E+2}{4}$.*
(b) If $W \in (\frac{1}{2}E - \frac{1}{2}, \frac{1}{2}E + \frac{1}{3})$ (Region M), then the profit-maximizing price is

$$p^* = \frac{1}{3}\sqrt{12EW + (1-W)^2} + \frac{2}{3}(1-W).$$

(c) If $W \leq \frac{1}{2}E - \frac{1}{2}$ (Region L), the profit-maximizing price is $p^* = \frac{1+E}{2}$.

In Figure A.1, we illustrate the parameter regions referred to in Lemma A.1. The shaded area depicts the parameter region with non-negative valuations for all consumers, s.t. $E + W \leq 1$. The dotted line separates the parameter region according to which side of the rectangle forming the support is longer; the proof will be slightly different in the two cases.



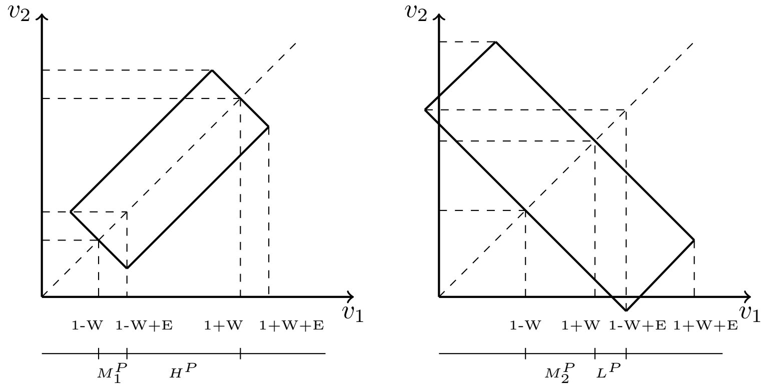
Note: The figure describes the parameter regions corresponding to parts (a), (b) and (c) of Lemma A1, respectively

Figure A.1: Parameter regions from Lemma A.1

Proof. We start with the details for the case that

$$W \geq E/2, \tag{10}$$

corresponding to Figure A.2(a).[27]



(a) $W \geq \frac{E}{2}$ **(b)** $W \leq \frac{E}{2}$

Note: The figure describes the equilibrium price regions used in the derivation/proof of the results of Lemma A1 and in Table 2.

Figure A.2: Optimal Price Regions

Step 1: *For $W \geq E/2$, the demand function is given as*

$$D(p,p) = \begin{cases} 1 & \text{if } p \leq 1-W \\ 1 - \frac{(p-1+W)^2}{4EW} & \text{if } 1-W \leq p \leq 1-W+E \\ \frac{1+W-p}{2W} + \frac{E}{4W} & \text{if } 1-W+E \leq p \leq 1+W \\ \frac{(1+W+E-p)^2}{4EW} & \text{if } 1+W \leq p \leq 1+W+E \\ 0 & \text{if } 1+W+E \leq p \end{cases}. \quad (11)$$

To see this, note that demand at price p is the probability that $\max\{v_1, v_2\} \geq p$, which is the ratio between the area of the set of valuations in the support satisfying this condition and the total area of the rectangle, which is $8EW$. Obviously, for $p \leq 1 - W = \underline{W}$) demand is 1 and for $p \geq 1 + W + E = \overline{W}$) it is 0. For the remaining regions, which correspond to different positions of p relative to the corner points in Figure A.2(a), demand is easily calculated as in (11).[28]

The following Steps 2-5 investigate potential candidate optima in each of the four intervals for which demand is positive.

Step 2: *For $W \geq E/2$, there is no maximum with $p \leq 1 - W$.*
This statement is trivial unless $W < 1$, in which case the only candidate is $p = 1 - W$. Using the second subcase of (11), a slight increase of p would give a profit of $p\left(1 - \frac{(p-1+W)^2}{4EW}\right)$ as $1-W < p < 1-W+E$. The derivative of this expression with respect to p is positive for $p = 1-W$.

Step 3: *For $W \geq E/2$, a maximum with $1 - W < p \leq 1 - W + E$ exists if and only if*

[27] In Figure 1, this case applies in Region H and in the upper part of region M.

[28] For $W > E/2$, the price intervals given in Condition (11) are all non-degenerate. For $W = E/2$ we have $1 - W + E = 1 + W$, so that the third of the five cases in (11) is superfluous.

$W \leq E/2 + 1/3$. *It is given by*

$$p = \frac{1}{3}\sqrt{12EW + (1-W)^2} + \frac{2}{3}(1-W).$$

Obviously, a maximum in this region can only exist if $W < 1 + E$. Using the second subcase of (11), the profit in the region is $p\left(1 - \frac{(p-1+W)^2}{4EW}\right)$. Hence, the first-order condition is $4pW - 2W - 4WE - 4p + 3p^2 + W^2 + 1 = 0$. Straightforward calculations show that the unique candidate solution satisfying the second-order condition is $p = \left(\frac{1}{3}\sqrt{12WE - 2W + W^2 + 1} - \frac{2}{3}W + \frac{2}{3}\right)$. This price always satisfies $p > 1 - W$ and it satisfies $1 - W + E \geq p$ if and only if $W \leq \frac{E}{2} + \frac{1}{3}$. The resulting equilibrium candidate can easily be shown to be global optimum under the given parameter conditions.[29]

Step 4: *For* $W \geq E/2$, *a maximum with* $1-W+E < p < 1+W$ *exists if and only if* $W > E/2+1/3$. *It is given by*

$$p = \frac{2W + E + 2}{4}$$

Using (11), profit in this region is $p\frac{2+2W-2p+E}{4W}$. The first-order condition immediately gives a candidate solution $p = \frac{2W+E+2}{4}$, which satisfies the second-order condition. For this candidate, $1 + W - p > 0$ always holds, whereas $p - (1 - W + E) > 0$ only holds if $W > E/2 + 1/3$, which holds by assumption. The resulting equilibrium candidate can easily be shown to be global optimum under the given parameter conditions.[30]

Step 5: *For* $W \geq E/2$, *there is no maximum with* $p \geq 1 + W$.

Using (11), the profit in $[1+W, 1+W+E]$ is $p\frac{(1+W+E-p)^2}{4EW}$. The first-order condition has $p = \frac{1+W+E}{3}$ as a unique solution. Since $W \geq E/2$, it follows that the candidate solution satisfies $p < 1 + W$, so that there can be no interior solution. As $p = 1+W+E$ yields zero demand, the only candidate for a boundary solution is $p = 1+W$. A slight decrease of p would give a profit of $\Pi(p) = p\left(\frac{1+W-p}{2W} + \frac{E}{4W}\right)$ as $1 - W + E \leq p < 1 + W$. Inserting $p = 1 + W$ gives $\frac{\partial \Pi}{\partial p} = \frac{E - 2(1+W)}{4W} < 0$, so that this deviation is profitable.

These arguments prove Lemma A.1(a); they also show that (b) would hold as long as $W \geq E/2$. We now turn to the case that

$$W < E/2. \tag{12}$$

The steps and arguments for this case are similar as in the previous analysis; the main difference is that under condition (12) $1 - W + E \geq 1 + W$, which corresponds to Figure A.2(b). The demand function (Step 1) differs slightly from (11), Step 2 remains identical and in Step 3-5 the resulting parameter conditions are adjusted accordingly (see Figure A.2(b):

Step 1': *For* $W < E/2$, *the demand function is given as*

[29] The parameter region corresponds to the part of region M in Figure A.1, which lies above the dotted line.

[30] The parameter region corresponds to region H in Figure A.1.

$$D'(p,p) = \begin{cases} 1 & \text{if } p \leq 1-W \\ 1 - \frac{(p-1+W)^2}{4EW} & \text{if } 1-W \leq p \leq 1+W \\ \frac{1+E-p}{E} & \text{if } 1+W \leq p \leq 1-W+E \\ \frac{(1+W+E-p)^2}{4EW} & \text{if } 1-W+E \leq p \leq 1+W+E \\ 0 & \text{if } 1+W+E \leq p \end{cases} \tag{13}$$

Step 3': *For $W < E/2$, a maximum with $1-W \leq p < 1+W$ exists if and only if $W > E/2-1/2$. It is given by*

$$p = \frac{1}{3}\sqrt{12EW + (1-W)^2} + \frac{2}{3}(1-W)$$

The arguments are similar to those in Step 3 above.[31]

Step 4': *For $W < E/2$, a maximum with $1+W \leq p \leq 1-W+E$ exists if and only if $W \leq E/2-1/2$. It is given by*

$$p = \frac{1+E}{2}$$

The arguments are similar as in Step 4 above.[32]

Step 5': *For $W < E/2$, there is no maximum with $p \geq 1-W+E$.*
The arguments follow those for Step 5 above.

The remaining results of Lemma A.1 now follow. □

Next, we turn towards total surplus. We summarize the relation between parameter regions and price regions (that is, the position of the optimal price relative to the corners of the support as in Figures A.2) in Table 2. In this table, we divide region M into regions M_1 and M_2 according to whether $W \geq E/2$ or $W \leq E/2$.[33]

Parameter Regions	$W \geq \frac{E}{2} + \frac{1}{3}$ Region H Lemma $A1$ Step 4	$\frac{E}{2} + \frac{1}{3} > W \geq E/2$ Region M_1 Lemma $A1$ Step 3	$E/2 > W > \frac{E}{2} - \frac{1}{2}$ Region M_2 Lemma $A1$ Step 3'	$\frac{E}{2} - \frac{1}{2} \geq W$ Region L Lemma $A1$ Step 4'
Price Regions	$[1-W+E, 1+W]$ Region H^P	$[1-W, 1-W+E]$ Region M_1^P	$[1-W, 1+W]$ Region M_2^P	$[1+W, 1-W+E]$ Region L^P

Table 2: Relation between parameter regions and price regions (derived from Lemma A.1)

Since we are primarily interested in local behavior around the firm's optimum, we state the next lemma accordingly: For each of the parameter Regions $R \in \{L, M, H\}$, we provide concrete ex-

[31] The parameter region where this price emerges is the part of the area M in Figure A.1 for which $W < E/2$.

[32] The resulting parameter region is Region L, in Figure A.1. Observe that, since $W \geq 0$ by assumption, this region only occurs for $E \geq 1$. This price can therefore only ever be optimal, when we allow for negative valuations, i.e. $W + E > 1$.

[33] The result follows from Steps 3, 4, 3' and 4' in Lemma A.1.

pressions for the total surplus $TS^R = TS^R(p;W,E)$ that apply for prices p that are interior in the corresponding price region R^P (depicted in Table 2), i.e. for p near the optimum $p^*(W,E)$ derived in Lemma A.1.

Lemma A.2. *(a) Suppose $W \geq \frac{1}{2}E + \frac{1}{3}$ (Region H). Total surplus for $p \in H^P$ is*

$$TS^H = \frac{1}{12W}\Big(E^2 + 3(1+W)(1+W+E) - 3p^2\Big).$$

(b) Suppose $W \in (\frac{1}{2}E - \frac{1}{2}, \frac{1}{2}E + \frac{1}{3})$ (Region M). Total surplus for $p \in M^P$ is

$$TS^M = \frac{1}{12EW}\Big(W^3 - 3W^2 + 3W\big(1+4E+2E^2\big) + 3p^2(1-W) - 2p^3 - 1\Big).$$

(c) Suppose $W \leq \frac{1}{2}E - \frac{1}{2}$ (Region L). Total surplus for $p \in L^P$ is

$$TS^L = \frac{1}{6E}\big(W^2 + 3E(2+E) - 3p^2 + 3\big).$$

Proof. To obtain the general expressions for total surplus at given prices, we note that it results from integrating the maximal valuations of those consumers who actually buy at those prices. Thus, the total surplus from consumers with $v_1 \geq v_2$, which, by symmetry is half of total surplus $TS(p;W,E)$, has the form

$$TS_{v_1}(p;W,E) = \int_p^{1+W+E}\int_{b_1(v_1)}^{b_2(v_1)} \frac{v_1}{8EW} dv_2 dv_1$$

Here $b_1(v_1)$ and $b_2(v_1)$ are the lower and upper bounds of the area in (v_1,v_2)-space for which (v_1,v_2) is in the support of F. For these functions we obtain:

$$b_1(v_1) = \begin{cases} -v_1 + 2(1-W) & \text{if } v_1 \in [1-W, 1-W+E] \\ v_1 - 2E & \text{if } v_1 \in [1-W+E, 1+W+E] \end{cases} \tag{14}$$

$$b_2(v_1) = \begin{cases} v_1 & \text{if } v_1 \in [1-W, 1+W] \\ -v_1 + 2(1+W) & \text{if } v_1 \in [1+W, 1+W+E] \end{cases} \tag{15}$$

Using this logic, it is straightforward to calculate the expressions for total surplus.

For Region H, the corresponding price region is $H^P = [1-W+E, 1+W]$ by Table 1, so that

$$\begin{aligned} TS^H &= \frac{1}{4EW}\left(\int_p^{1+W}\int_{v_1-2E}^{v_1} v_1 dv_2 dv_1 + \int_{1+W}^{1+W+E}\int_{v_1-2E}^{-v_1+2+2W} v_1 dv_2 dv_1\right) \\ &= \frac{1}{12W}\Big(E^2 + 3(1+W)(1+W+E) - 3p^2\Big). \end{aligned}$$

For Region M_1, the price region is $M_1^P = [1-W, 1-W+E]$. Thus

$$\begin{aligned} TS^{M_1} &= \frac{1}{4EW}\left(\int_p^{1-W+E}\int_{-v_1+2-2W}^{v_1} v_1 dv_2 dv_1 + \int_{1-W+E}^{1+W}\int_{v_1-2E}^{v_1} v_1 dv_2 dv_1 + \int_{1+W}^{1+W+E}\int_{v_1-2E}^{-v_1+2+2W} v_1 dv_2 dv_1\right) \\ &= \frac{1}{12EW}\Big(W^3 - 3W^2 + 3W\big(1+4E+2E^2\big) + 3p^2(1-W) - 2p^3 - 1\Big). \end{aligned}$$

The same expression applies for Region M_2 by taking into account that $M_2^P = [1-W, 1+W]$ according to Table 1. Finally, using $L^P = [1+W, 1-W+E]$ we obtain

$$\begin{aligned} TS^L &= \frac{1}{4EW}\left(\int_p^{1-W+E}\int_{-v_1+2-2W}^{-v_1+2+2W} v_1 dv_2 dv_1 + \int_{1-W+E}^{1+W+E}\int_{v_1-2E}^{-v_1+2+2W} v_1 dv_2 dv_1\right) \\ &= \frac{1}{6E}\left(W^2 + 3E(2+E) - 3p^2 + 3\right) \end{aligned}$$

□

A.1.2 Proof of Proposition 1

We start with the results on firm profits in part (i) of the proposition. By symmetry, the profit is $\Pi(p, W, E) = p \cdot D(p, W, E)$. By Lemma A.1, in the optimum, the firm chooses

$$p^*(W,E) = \begin{cases} \frac{2W+E+2}{4} & \text{for } W \geq \frac{E}{2} + \frac{1}{3} \\ \frac{1}{3}\sqrt{12WE + (1-W)^2} + \frac{2}{3}(1-W) & \text{for } W \in \left(\frac{1}{2}E - \frac{1}{2}, \frac{1}{2}E + \frac{1}{3}\right) \\ \frac{1+E}{2} & \text{for } W \leq \frac{E}{2} - \frac{1}{2} \end{cases}$$

To obtain demands at these profit-maximizing prices, we note that demand function (11) applies in Region H (where $D(p,p) = \frac{E+2(1+W-p)}{4W}$) and in the part of Region M for which $W \geq E/2$ (where $D(p,p) = 1 - \frac{(p-(1-W))^2}{4EW}$), whereas (13) applies in Region L (where $D(p,p) = \frac{1+E-p^*}{E}$) and in the part of Region M for which $W \leq E/2$. Thus, the profit at profit-maximizing prices is:

$$\Pi(p^*, W, E) = \begin{cases} p^*\left(\frac{E+2(1+W-p^*)}{4W}\right) & \text{for } W \geq \frac{E}{2} + \frac{1}{3} \\ p^*\left(1 - \frac{(p^*-(1-W))^2}{4EW}\right) & \text{for } W \in \left(\frac{1}{2}E - \frac{1}{2}, \frac{1}{2}E + \frac{1}{3}\right) \\ p^*\left(\frac{1+E-p^*}{E}\right) & \text{for } W \leq \frac{E}{2} - \frac{1}{2} \end{cases} \tag{16}$$

Inserting the respective values for $p^*(W,E)$ one quickly obtains

$$\frac{\partial \Pi^*(W,E)}{\partial E} > 0 \text{ and } \frac{\partial p^*(W,E)}{\partial E} > 0 \tag{17}$$

This proves the results for the firm in part (i) of the proposition.

Next, we consider the results on consumer surplus in part (i) and (ii) of the proposition. We are primarily interested in local behavior around the firm's optimum and state the lemma accordingly: Using Lemma A.2 and the profit expression (16), we obtain consumer surplus for parameters in Region $R \in \{L, M, H\}$, denoted as $CS^R = CS^R(p; W, E) = TS^R - \Pi^R$:

Lemma A.3. *(a) Suppose $W \geq \frac{1}{2}E + \frac{1}{3}$ (Region H). Consumer surplus for $p \in H^P$ is*

$$CS^H = \frac{1}{12W}\left(E^2 + 3E(1-p+W) + 3(1-p+W)^2\right)$$

(b) Suppose $W \in (\frac{1}{2}E - \frac{1}{2}, \frac{1}{2}E + \frac{1}{3})$ (Region M). Consumer surplus for $p \in M^P$ is

$$CS^M = \frac{1}{12EW}\Big(3W\left(2E^2 - 4E(p-1) + (p-1)^2\right) + 3(p-1)W^2 + (p-1)^3 + W^3\Big)$$

(c) Suppose $W \leq \frac{1}{2}E - \frac{1}{2}$ (Region L). Consumer surplus for $p \in L^P$ is

$$CS^L = \frac{1}{6E}\Big(3(1+E-p)^2 + W^2\Big)$$

Finally, we use Lemma A.3 to show how consumer surplus depends on parameters. Standard calculations show that $\frac{\partial CS(p;W,E)}{\partial E} > 0$ always holds, so that the direct effect of E on consumer surplus is positive (as claimed in Part (i) of the proposition. To capture the total effect of E on consumer surplus when taking into account the firms' price adjustments, recall that $CS^*(W,E) := CS(p^*(W,E);W,E)$. Further, define

$$s(E,W) := \sqrt{12EW + (1-W)^2}. \tag{18}$$

Straightforward calculations show that $\frac{\partial CS^*}{\partial E} > 0$ in Regions H and L. In Region M, $\frac{\partial CS^*}{\partial E} < 0$ if and only if

$$\Bigg[2(-1+s(E,W)) - W\Big(-8 + 12E(-1+W)^2 + 6s(E,W) + 9E^2(32W - 9s(E,W)) + 2W\big(6 - 3s(E,W) + W(-4+W+s(E,W))\big)\Big)\Bigg] > 0$$

It is straightforward to show the requirement of part (ii) of the proposition that there exist parameter values for which the last condition holds; these parameter values correspond to the shaded areas in Figure 2.

A.1.3 Proof of Proposition 2

(i) Using Lemma A.3, we get $\frac{\partial CS^R(p;W,E)}{\partial W}$ for each Region $R \in \{L, M, H\}$. Evaluating the derivatives at the respective optimal price $p^*(W;E)$ (see Lemma A.1) in the respective relevant region R, simple calculations show that $\frac{\partial CS^R(p;W,E)}{\partial W} > 0$ always holds and the functions $CS^R(p;W,E)$ coincide at the regime boundaries. Therefore the direct effect of W on consumer surplus is positive.
(ii) Simple calculations show that $\frac{\partial \Pi^*}{\partial W} < 0$ in region M. In region H, $\frac{\partial \Pi^*}{\partial W} \geq 0$ if and only if $W \geq 0.5E + 1$. In region L, the profit is independent of W. Putting these results together, we find that profits are (weakly) decreasing in W below the threshold $W = 0.5E + 1$ (depicted in Figure 3), where the minimum obtains, and increasing above the threshold.
Next, we show how the global maximum depends on the feasible $\overline{W}$. From the above result, the first candidate is the minimal choice of W, i.e. by Assumption 2, $W = \underline{W} = 0$. To prove the claim in the proposition, it now suffices to show that for every $E \geq 0$ there exists a $\tilde{W}(E) > 0.5E + 1$ so that the firm's profits at $(\tilde{W}(E), E)$ are the same as for the minimal choice of W. $\tilde{W}(E)$ is defined

implicitly by

$$\Pi\Big(p^*(\tilde{W}(E),E),p^*(\tilde{W}(E),E);\tilde{W}(E),E\Big)=\Pi\Big(p^*(0,E),p^*(0,E);0,E\Big)$$

By Lemma A.1, $\Pi(p^*(\tilde{W}(E),E),p^*(\tilde{W}(E),E;\tilde{W}(E),E)=\frac{(2+E+2\tilde{W})^2}{32\tilde{W}}$, given that $(\tilde{W}(E),E)$ is in region H $\forall E$. The expression for $\Pi(p^*(0,E),0,E)$ depends on the region: For $E<1$, $(0,E)$ is in region M so that $\Pi(p^*(0,E),0,E)=1$. For $E\geq 1$, $(0,E)$ is in region L so that $\Pi(p^*(0,E),0,E)=\frac{(1+E)^2}{4E}$.[34] Straightforward calculations yield the line $\tilde{W}(E)$ depicted in Figure 3:

$$\tilde{W}(E)=\begin{cases}\frac{1}{2}(6-E+4\sqrt{2-E}) & \text{for } E\in[0,1]\\ \frac{(2+E)^2}{2E} & \text{for } E\geq 1\end{cases}$$

(iii) To understand the price effect, note from Lemma A.1 that $\frac{\partial p^*}{\partial W}=0$ in Region L and $\frac{\partial p^*}{\partial W}>0$ in Region H. In Region M, $\frac{\partial p^*}{\partial W}<0$ if and only if $6E\leq 1-W+2\sqrt{(1-W)^2+12EW}$.

A.2 Proofs of General Results

A.2.1 Proof of Lemma 1

(i) As the firm can always choose $p^*(F^1)$ together with distribution F^2, we obtain
$\Pi^*(F^2)\geq p^*(F^1)D^{F^2}(p^*(F^1),p^*(F^1))>p^*(F^1)D^{F^1}(p^*(F^1),p^*(F^1))=\Pi^*(F^1)$.
(ii) By strict concavity, a change in the valuation distribution from F^1 to F^2 increases p^* if and only if $\frac{\partial\Pi^{F^2}}{\partial p}(p^*(F^1))>\frac{\partial\Pi^{F^1}}{\partial p}(p^*(F^1))$. The condition in (ii) makes sure this requirement holds.

A.2.2 Proof of Proposition 3

Denote the maximum and minimum of v_1 and v_2 as v^{max} and v^{min}, respectively. By definition, a valuation difference dispersion corresponds to a first-order stochastic dominance shift of the distribution of $v^{max}-v^{min}$ for fixed average valuation. Denote the distribution of the second-order statistic of F^i, that is, of v^{max}, with $F^i_{(2)}$. As $v^{max}=\frac{v^{max}+v^{min}}{2}+\frac{v^{max}-v^{min}}{2}$ and $\frac{v^{max}+v^{min}}{2}$ is the same for F^2 and F^1, a valuation difference dispersion from F^1 to F^2 therefore also induces a first-order shift of $F^i_{(2)}$, and thus increases average gross consumer surplus. As $1-F^2_{(2)}(p)>1-F^1_{(2)}(p)$ for every p, the valuation difference dispersion increases demand. Thus by Lemma 1(i), it increases profits.

A.2.3 Proof of Proposition 4

Together with strict concavity, Assumption 4(iii) implies that the optimal price $p(0)$ for $\tau=0$ satisfies $p(0)<\overline{V}$. Thus, the marginal buyers have average valuations below $\overline{V}$, and a WTP dispersion reduces these marginal buyer's WTP and thus demand. By Assumptions 4(i) and 4(ii),

[34] Recall that in region L the profit is independent of W. Therefore, for $E\geq 1$, any choice of $W\in[0,\frac{1}{2}E-\frac{1}{2}]$ yields the same profit and hence corresponds to the minimal choice of W.

after a sufficiently small increase in τ, the new marginal buyer will still have marginal valuations below $\overline{V}$, so that further marginal increases in WTP-heterogeneity reduce demand and profits.
Next, note that the highest price at which some buyer with average valuation will still buy is $p = \overline{V}^{max}$. Starting from this price, the marginal effect of a price increase on profits under F_{τ^*} is positive by Assumption 4(iv). Thus, for this F_{τ^*} it is profitable to increase prices to a level at which no consumer with average valuation buys. Then a further increase in WTP heterogeneity will increase the marginal buyer's maximal willingness to pay and therefore increase demand and profit.

A.2.4 Proof of Proposition 5

(i) Fix a conceivable average true valuation $\overline{v}$. Suppose for given $\overline{v}$ the difference of the noise terms is distributed as $G_{\overline{v}}$. Suppose the support of $G_{\overline{v}}$ is given as $[-E_{\overline{v}}, E_{\overline{v}}]$ for some $E_{\overline{v}} > 0$. For any fixed price p, denote the valuation difference of the critical consumer who is indifferent between buying good 1 and not buying it as $\Delta^* \equiv \Delta^*(p, \overline{v})$. Suppose that $\Delta^* > 0$; the case that $\Delta^* < 0$ is analogous. The consumers whose perceived valuation difference increases from below Δ^* to above Δ^* due to obfuscation are those with $\Delta \in (\Delta^* - E_{\overline{v}}, \Delta^*)$ and a difference of noise terms $\varepsilon > \Delta^* - \Delta$; the corresponding mass is $\int_{\Delta^*-E_{\overline{v}}}^{\Delta^*} g_{\overline{v}}(\Delta)(1 - G_{\overline{v}}(\Delta^* - \Delta))d\Delta$. Similarly, the mass of consumers whose valuations fall from above Δ^* to below Δ^* due to obfuscation are those with $\Delta \in (\Delta^*, \Delta^* + E_{\overline{v}})$ and $-\varepsilon > \Delta - \Delta^*$; the corresponding mass is $\int_{\Delta^*}^{\Delta^*+E_{\overline{v}}} g_{\overline{v}}(\Delta)(1 - G_{\overline{v}}(\Delta^* - \Delta))d\Delta$. Indecisiveness implies that the former term is larger than the latter, so that the number of consumers who mistakenly buy the good as a result of obfuscation is larger than the number of consumers who mistakenly do not buy. Integrating over all conceivable $\overline{v}$ shows that confusion increases total demand and profits when preferences are indecisive.
(ii) is analogous.

A.3 Additional Details

We now substantiate several of the claims made in Sections 2 and 3 for the benchmark model.

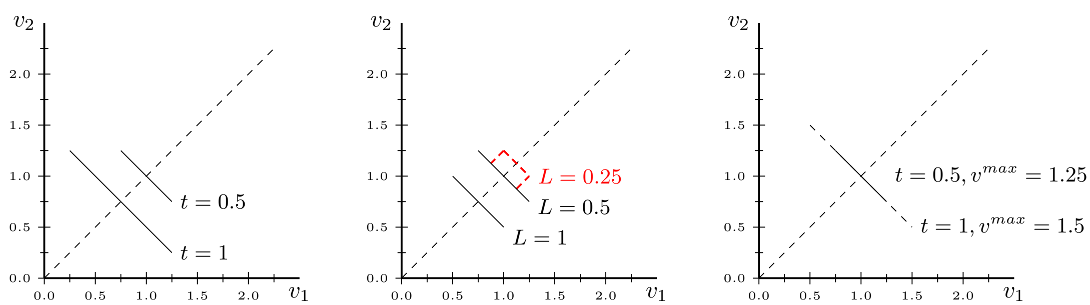
A.3.1 Valuation Difference Heterogeneity vs. Product Differentiation

We compare our framework with a Hotelling model, where consumers are uniformly distributed on the interval $[-0.5, 0.5]$, with consumer k located at $x(k)$. Product 1 is located at $-L$, product 2 at L for some $L > 0$. Consumer k has utility $v^{max} - t|x(k) + L|$ for some $t > 0$ when buying product 1, $v^{max} - t|x(k) - L|$ when buying product 2, where $v^{max} > 0$ is the gross valuation for the product (commonly shared by all consumers). Our model with $W = 0$ and $E > 0$ is equivalent with a Hotelling Model with $v^{max} = 1 + E$, $L = 0.5$ and $t = 2E$.
The short line in Figure A.3(a) shows the support for our benchmark model with $W = 0$ and $E = 0.25$ (or equivalently, the Hotelling model with $v^{max} = 1.25$, $L = 0.5$ and $t = 0.5$). A standard way to capture greater product differentiation in the Hotelling model is by an increase in t. The longer line in Figure A.3(a) shows how the support of the valuation distribution changes as t

increases from 0.5 to 1: The length of the support increases because consumers attach larger value differences to the products. Crucially, however, the support also moves closer to the origin: As t increases, match values fall. Thus, contrary to an increase in valuation difference heterogeneity in our model, an increase in transportation costs reduces average valuations.

Alternatively, one can capture changes in differentiation through changes in product location. The upper of the two solid straight lines in Figure A.3(b) corresponds to the benchmark case in which $t = 0.5$ and $L = 0.5$; the lower solid line corresponds to $L = 1$. Like an increase in t, the increase in L (an outward shift of the product locations for fixed consumer locations), shifts the support towards the origin, in this case, without the simultaneous increase in the length of the line. Thus, there is no increase in valuation difference heterogeneity. Intuitively, here products are differentiated in a way that is excessive, resulting in lower valuations for every consumer. Conversely, a reduction in product differentiation, where the products are moved into the interior of the interval $[-0.5, 0.5]$ leads to a support that is only piecewise linear (see the red line, which is drawn for $L = 0.25$).[35] Again, this change has an effect on average valuation differences.

These considerations show that, in the Hotelling Model, an increase in valuation difference heterogeneity cannot be captured directly as an increase in horizontal product differentiation, no matter whether this is interpreted as an increase in t or as an outward shift in product locations L. Instead, it corresponds to a simultaneous increase in t and in the highest consumer valuation v^{max} such that the average consumer valuation $\overline{V} = v^{max} - t/2$ is not affected. Figure A.3(c) depicts the effects of such a change, fixing $L = 0.5$: Instead of the original solid line, the support now corresponds to the dashed line.



(a) Effects of increasing t (for $v^{max} = 1.25, L = 0.5$)

(b) Effects of increasing L (for $v^{max} = 1.25, t = 0.5$)

(c) Effects of simultaneously increasing t and v^{max} (for $L = 0.5$)

Figure A.3: Valuation Difference Heterogeneity vs. Product Differentiation

A.3.2 The relation between WTP-heterogeneity and consumer surplus

We first provide the arguments concerning the relation between W and consumer surplus. The results all rely on the expression for consumer surplus given in Lemma A.3.

[35] The upward-sloping parts of the line correspond to consumer locations to the left of -0.25 and to the right of 0.25, respectively.

First, starting from Lemmas A.1 and A.3, tedious calculations identify two regions where the total effects on consumer surplus are negative. In region H, $\frac{\partial CS^*}{\partial W} < 0$ if and only if $\frac{12W^2-(12+12E+7E^2)}{192W^2} < 0$. In Region M, $\frac{\partial CS^*}{\partial W} < 0$ if and only if

$$-1 + s(E,W) + W(1 - 6E(-1+W)^2 - 144E^2W + 63EWs(E,W) + W(-3+2W)(-1+W+s(E,W))) < 0,$$

where $s(E,W)$ is defined as in (18).

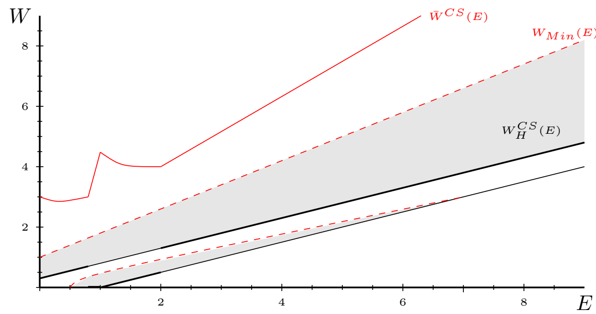


Figure A.4: The Relation between WTP Heterogeneity W and Consumer Surplus

These two regions are shaded in Figure A.4. Moreover, the dashed lines depict local minima of net consumer surplus as a function of W for each E. The solid lines depict local maxima, with the parts emphasized in bold corresponding to the higher of these two local maxima, which we denote as $W_H^{CS}(E)$. The comparison of the two local maxima shows that

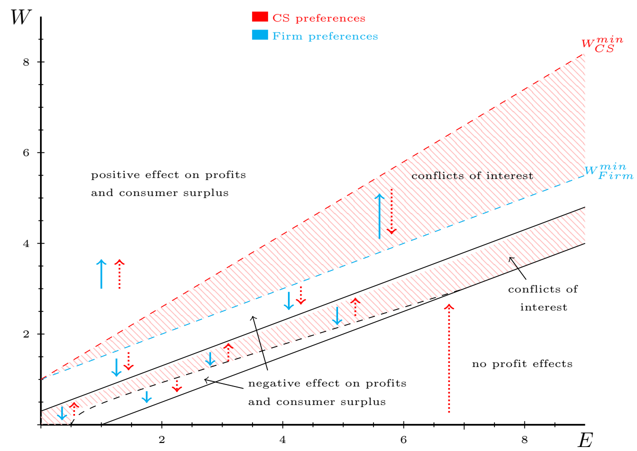
$$W_H^{CS}(E) = \begin{cases} \frac{1}{2}E + \frac{1}{3} & \text{for } E \in [0, E_1] \\ 0 & \text{for } E \in [E_1, 1] \\ \frac{1}{2}E - \frac{1}{2} & \text{for } E \in [1, E_2] \\ \frac{1}{2}E + \frac{1}{3} & \text{for } E \geq E_2 \end{cases},$$

where the regime boundaries are given approximately as $E_1 \approx 0.8165$ and $E_2 \approx 2$. The emphasized black discontinuous solid line in Figure A.4 depicts this line.

The figure also illustrates the claim that, for sufficiently high initial values of W, consumer surplus is increasing in W: The line $\tilde{W}^{CS}(E)$ corresponds to the values of W for which consumer surplus is as high as at $W_H^{CS}(E)$. It can be calculated as:

$$\tilde{W}^{CS}(E) = \begin{cases} \frac{12+12E+7E^2}{4+6E} & \text{for } E \in [0, E_1] \\ -1 + 3.5E + 0.57735\sqrt{E(-24+35E)} & \text{for } E \in [E_1, 1]. \\ \frac{1}{6E}\left(8 + 2E + 5E^2 + 2\sqrt{16 + 8E + 12E^2 - 4E^3 + E^4}\right) & \text{for } E \in [1, E_2] \\ \frac{12+12E+7E^2}{4+6E} & \text{for } E \geq E_2 \end{cases}$$

Finally, by combining Figures 3 and A.4, we arrive at Figure A.5, which separates the regions where there are conflicts of interest between consumers and the firm. In the shaded regions, the signs of the effect of W on profits and consumer surplus differ. In the other regions the sign of the effect of W on consumer surplus and profits is the same.



Note: Solid arrows correspond to firm profits, dashed arrows to consumer surplus. An upward-pointing arrow corresponds to positive effects of marginal WTP-increases on the respective quantity, a downward-pointing arrow to a negative effect.

Figure A.5: Conflicts of Interest

A.3.3 One-dimensional Heterogeneity

We now derive the formulas on which Figure 5 in the example of Section 4.4 is based on.

Demand for symmetric prices (p, p) is given as $D(p, p; A) = Prob\big(\max\{v_1, v_2\} \geq p\big) = 1 - \left(\frac{p-(1-A)}{2A}\right)^2$ and profits are $\Pi(p, p; A) = pD(p, p; A)$. Straightforward calculations then show that the optimal choice of prices is given as

$$p^*(A) = \frac{1}{3}\Big(2 - 2A + \sqrt{1 - 2A + 13A^2}\Big)$$

Inserting these prices into $\Pi(p, p; A) = pD(p, p; A)$ immediately gives profits as a function of A:

$$\Pi^*(A) = \frac{1}{3}\left(2 - 2A + \sqrt{1 - 2A + 13A^2}\right)\left(1 - \frac{\left(\sqrt{1 - 2A + 13A^2} - (1 - A)\right)^2}{36A^2}\right)$$

Finally, total surplus is given as

$$TS(p, p; T) = \int_p^{1+A}\int_{1-A}^{v_1} \frac{v_1}{2A^2} dv_2 dv_1 = \frac{(1 + A)^2(5A - 1) - 3(A - 1)p^2 - 2p^3}{12A^2}$$

and consumer surplus, defined by $CS(p,p;A) = TS(p,p;A) - \Pi(p,p;A)$, becomes:

$$CS(p,p;A) = \int_{p}^{1+A} \int_{1-A}^{v_1} \frac{v_1 - p}{2A^2} dv_2 dv_1 = \frac{(1+A-p)^2(5A+p-1)}{12A^2}.$$

A.3.4 Duopoly

We show that there can be no profitable upward deviation from the candidate equilibrium $p^* = 2E$. To this end, we calculate an upper bound to the deviation profit that ignores the fact that the deviating firm my lose some consumers to the outside option. We can calculate the demand of a firm i after an upward deviation to a price p under the assumption that it will maintain all consumers for whom $v_i - p_i \geq v_j - p_j$ no matter whether $v_i - p_i \geq 0$ or not as $\frac{(4E-p_i)}{4E}$. Using this upper bound for the demand after deviation, we obtain an upper bound for the deviation profit as $\frac{(4E-p)p}{4E}$. This expression is maximized for $p = 2E$, in which case it becomes E. As this is the profit in the proposed equilibrium, there cannot be a profitable deviation.