International arbitrage and the extensive margin of trade between rich and poor countries

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Abstract

Incorporating consumption indivisibilities into the Krugman-model, we show that an importer’s per-capita income becomes a primary determinant of “export zeros”. Households in the rich North (in the poor South) are willing to pay high (low) prices for consumer goods, hence unconstrained monopoly pricing generates arbitrage opportunities for internationally traded products. Export zeros arise because some northern firms abstain from exporting to the South, to avoid international arbitrage. We show that rich countries benefit more from a trade liberalization than poor countries, and that the latter may even lose. These results hold also under more general preferences (that feature an intensive and extensive consumption margin). U.S. firm-level data as well as disaggregate trade data show a robust negative association between export zeros and (potential) importers’ per-capita income. This evidence is consistent with the predictions of our model.

JEL classification: F10, F12, F19

Keywords: Non-homothetic preferences, parallel imports, arbitrage, extensive margin, export-zeros

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1 Introduction

We study a model of international trade in which an importer’s per-capita income is a primary determinant of the extensive margin of international trade. Two facts motivate our analysis. First, there are huge differences in per-capita incomes across the globe, and these differences may have important consequences for patterns of international trade. Ranking all potential trade flows by the trading partners’ per-capita income ratio reveals that the median trade relation features an income ratio of 4; the 25th percentile features a ratio of 2, and even at the 10th percentile the income ratio is as high as 1.5. In other words, the typical (potential) trade relation is one between a rich and a significantly poorer country. Second, per-capita incomes indeed correlate with the extensive margin of trade: In disaggregate trade data, export probabilities are strongly increasing in (potential) importers’ per-capita income. In 2007, for example, the probability that the U.S. exports a given HS 6-digit product to other high-income countries was 63.4 percent, while the export probabilities to upper-middle, lower-middle, and low-income destinations were only 48.8 percent, 36.6 percent, and 13.6 percent, respectively. Furthermore, also U.S. firm-level data show a positive correlation between export probabilities and destinations’ per-capita incomes (Bernard, Jensen, and Schott 2009).

Recent research has emphasized the presence of “zeros” in bilateral trade data, see e.g. Helpman, Melitz, and Rubinstein (2007) at the country-pair level; Hummels and Klenow (2005) at the product level; and Bernard, Jensen, Redding, and Schott (2007) at the firm level. However, the literature did not systematically explore the role of per-capita incomes. The standard explanation for export zeros relies on heterogeneous firms and fixed export-market entry costs (Melitz 2003, Chaney 2008, Arkolakis, Costinot, and Rodriguez-Clare 2012). Export zeros arise when firms in the source country have too high (trade and production) costs, or when market size (as measured by aggregate GDP) in the destination country is too low. There is no separate role for per-capita incomes due to the assumption of homothetic preferences: It is irrelevant whether a given aggregate GDP arises from a large population and a low per-capita income, or vice versa.

This is different in our framework where per-capita income differences are a crucial determinant of export zeros and where zeros arise even in the absence of firm heterogeneity. We assume that consumer goods are indivisible and households purchase either one unit of a particular product or do not purchase it at all. This lets households respond only along the extensive margin of consumption and contrasts to CES-preferences where consumers respond only along the intensive margin. Incorporating such “0-1” preferences into an otherwise standard Krugman (1980) framework has important implications for general equilibrium outcomes. By generating export zeros solely from per-capita income gaps, our analysis emphasizes the demand side and is complementary to standard approaches which emphasize the supply side.

The key contribution of our paper is the recognition that firms from rich countries might not export to a poor country due to a threat of international arbitrage. Consider a U.S. firm

\[^{1}\text{This heterogeneous-firm framework has proven to be useful in explaining firm-level evidence on export behavior. For a recent survey, see Bernard, Jensen, Redding, and Schott, 2012.}\]
that sells its product both in the U.S. and in China. Suppose this firm charges a price in
China equal to the Chinese households’ (low) willingness to pay and a price in the U.S. equal
to the U.S. households’ (high) willingness to pay. When price differences are large, arbitrage
opportunities emerge: arbitrageurs can purchase the good cheaply on the Chinese market, ship
it back to the U.S., and underbid local U.S. producers. In equilibrium, U.S. firms will adjust
their pricing and export behavior, anticipating this threat of arbitrage. To avoid arbitrage,
a U.S. exporter has basically two options: (i) charge a price in the U.S. sufficiently low to
eliminate arbitrage incentives; or (ii) abstain from selling the product in China (and other
equally poor countries) thus eliminating arbitrage opportunities. These two options involve a
trade-off between market size and prices: firms that export globally have a large market but
need to charge a low price; firms that sell exclusively on the U.S. market (and other equally
rich countries) can charge a high price but have a small market. In an equilibrium with ex-ante
identical firms, the two options yield the same profit.

A second main result relates to gains from trade and the welfare effects of trade liberal-
izations. When per-capita income gaps are small, so that firms in the richer country are not
constrained by arbitrage, lower trade costs increase welfare in both countries. However, when
per-capita income gaps are large, only the rich country gains, while the poor country loses from
a trade liberalization. The reason is that lower trade costs tighten the arbitrage constraint.
To avoid arbitrage, northern firms that trade their product globally need to reduce their prices
on the northern markets. This makes selling the product solely in the northern market more
attractive, decreasing the fraction of firms exporting to the South. The associated loss in
varieties supplied to households in the poor country is harmful for their welfare.

Our analysis highlights three further points. First, we make precise the differential con-
sequences of an increase in aggregate GDP due to a higher per-capita income compared to a
larger population. A higher per-capita income in the South raises poor households’ willingness
to pay, increasing northern firms’ incentive to sell their products internationally. In equilib-
rium, a larger fraction of northern firms export their product to the South. In contrast, a
larger population in the South leaves southern households’ demand for varieties unchanged but
allows for the production of more varieties. This increases the world’s per-capita consumption
due to a scale effect; increases the volume of trade and may or may not increase trade intensity.
Moreover, a larger population in the poor country may or may not increase the probability
that a northern firm exports to the South. In sum, our model predicts that per-capita income
has a stronger effect than population size on the probability that a northern firm exports to
the South.

A second point shows that the result of detrimental effects of trade liberalizations (on a
poor country’s welfare) needs to be qualified in a multi-country setting. When there are many
rich and many poor countries, a multilateral trade liberalization still reduces North-South
trade due to tighter arbitrage. However, it also stimulates South-South trade because the
arbitrage constraint is not binding when trading partners have a similar per-capita income.
Hence a multilateral trade liberalization benefits the welfare of poor households if the increase
in South-South trade overcompensates the fall in North-South trade. The multi-country setting
is also useful because it delivers empirical predictions. The prediction is that a northern firm will export to all other northern countries, while the probability that it sells to southern countries is strictly less than unity and decreases in the per-capita income gap between North and South.

A third point analyzes the conditions under which the basic logic of our “0-1” preferences carries over to general preferences that allow for an intensive margin of consumption. Assuming a general subutility function \( v(c) \), we show that the results of our simple model hold for a large class of preferences that satisfy certain regularity conditions (in particular, a finite reservation price). We provide numerical examples assuming that \( v(c) \) belongs to a subset of the HARA-class. Numerical examples show that arbitrage arises for a wide range of parameter values. We also show that disregarding arbitrage might lead to misleading conclusions regarding the welfare effects of trade liberalizations.

The present paper connects to various strands of the literature. First, it is related to the literature on pricing-to-market which focuses on the cross-country dispersion of prices of tradable goods. Atkeson and Burstein (2008) generate pricing-to-market in a model with Cournot competition and variable mark-ups. However, their focus is on the interaction of market structure and changes in marginal costs rather than on per-capita income effects. Hsieh and Klenow (2007), Manova and Zhang (2009), and Alessandria and Kaboski (2011), among others, document that prices of tradable consumer goods show a strong positive correlation with per-capita incomes in cross-country data. Simonovska (2011) provides a theoretical framework in which richer consumers are less-price sensitive, so mark-ups and prices are higher in richer countries. A similar mechanism is also at work in the papers by Markusen (2011), Sauré (2010), and Bekkers, Francois, and Manchin (2011). Variable mark-ups and pricing-to-market driven by per-capita income are also a crucial feature in our framework. Our paper extends this literature by showing that export zeros arise from the (threat of) international arbitrage, a feature not considered in previous papers.

Second, the paper is related to the literature on parallel trade (surveyed in Maskus, 2000 and Ganslandt and Maskus, 2007). The key difference lies in our emphasis on the role of general equilibrium effects. To see why this is important, consider for example the recent contribution by Roy and Saggi (2012). They show that in an international duopoly, parallel trade induces the Southern firm to charge an above monopoly price in the South in order to be able to charge a high price in the North. Softer competition in the North then induces the Northern firm to sell only in its home market at a high price, which harms northern consumers. We show that considering the general equilibrium uncovers an opposing force working through the economy wide resource constraint: It is still true that a subset of northern firms will find it optimal to sell only in their home market at a high price. But this means that less northern resources are used to produce goods for the South, which increases the numbers of available

varieties in the North and thus welfare. So the welfare effects of parallel trade rules goes in the opposite direction when considering the general equilibrium.

Third, the presence of a trade participation margin links the present paper to a third literature that builds on Melitz (2003) and explores demand- and/or market-size effects in the context of heterogeneous firm models. Arkolakis (2010) incorporates marketing costs into that framework, generating a role of population size on export markets in addition to aggregate income. Eaton, Kortum, and Kramarz (2011) extend this framework, allowing for demand shocks (in addition to cost shocks) as further potential determinants of firms’ export behavior. These papers stick to homothetic preferences; hence, arbitrage cannot arise. This is different from our paper where non-homotheticities and arbitrage incentives play a central role and no exogenous firm heterogeneity is required to generate a trade participation margin.

The remainder of the paper is organized as follows. In the next section, we present the basic assumptions and discuss the autarky equilibrium. We apply our basic framework in Section 3 to study trade patterns and trade gains in a two-country setting. In Section 4 the analysis is extended to many rich and poor countries. Section 5 introduces general preferences and shows that arbitrage equilibria arise even when consumers respond both along the extensive and intensive margin of consumption. Section 6 presents empirical evidence on the impact of per-capita income on export zeros in firm-level and disaggregate trade data. Section 7 concludes.

2 Autarky

We start by presenting the autarky equilibrium. The economy is populated by \( P \) identical households. Each household is endowed with \( L \) units of labor, the only production factor. Labor is perfectly mobile within countries and immobile across countries. The labor market is competitive and the wage is \( W \). Production requires a fixed labor input \( F \) to set up a new firm and a variable labor input \( 1/a \) to produce one unit of output, the same for all firms. Producing good \( j \) in quantity \( q(j) \) thus requires a total labor input of \( F + q(j)/a \).

Consumers. Households spend their income on a continuum of differentiated goods. We assume that goods are indivisible and a given product \( j \) yields positive utility only for the first unit and zero utility for any additional units.\(^3\) Thus consumption is a binary choice: either you buy or you don’t buy. Let \( x(j) \) denote an indicator that takes value 1 if good \( j \) is purchased and value 0 if not. Then utility takes the simple form

\[
U = \int_0^\infty x(j)dj, \quad \text{where } x(j) \in \{0, 1\}. \tag{1}
\]

\(^3\)Preferences of this type were used, inter alia, by Murphy, Shleifer and Vishny (1989) to study demand composition and technology choices, by Matsuyama (2000) to explore non-homotheticities in Ricardian trade, and by Foellmi and Zweimüller (2006) to look at inequality and growth.
Notice that utility is additively separable and that the various goods enter symmetrically. Hence the household’s utility is given by the number of consumed goods.

Consider a household with income \( y \) who chooses among (a measure of) \( N \) goods supplied at prices \( \{ p(j) \} \).\(^4\) The problem is to choose \( \{ x(j) \} \) to maximize the objective function (1) subject to the budget constraint \( \int_0^N p(j) x(j) dj = y \). Denoting \( \lambda \) as the household’s marginal utility of income, the first order condition can be written as

\[
\begin{align*}
x(j) &= 1 \text{ if } 1 \geq \lambda p(j) \\
x(j) &= 0 \text{ if } 1 < \lambda p(j).
\end{align*}
\]

Rewriting this condition as \( 1/\lambda \geq p(j) \) yields the simple rule that the household will purchase good \( j \) if its willingness to pay \( 1/\lambda \) does not fall short of the price \( p(j) \).\(^5\) The resulting demand curve, depicted in Figure 1, is a step function which coincides with the vertical axis for \( p(j) > 1/\lambda \) and equals unity for prices \( p(j) \leq 1/\lambda \).

By symmetry, the household’s willingness to pay is the same for all goods and equal to the inverse of \( \lambda \), which itself is determined by the household’s income and product prices. Intuitively, the demand curve shifts up when the income of the consumer increases (\( \lambda \) falls) and shifts down when the price level of all other goods increases (\( \lambda \) rises).

It is interesting to note the difference between consumption choices under these “0-1” preferences and the standard CES-case. With 0-1 preferences, the household chooses how many goods to buy, while there is no choice about the consumed quantity.\(^6\) In constrast, a household has a choice with CES preferences about the quantities of the supplied goods, but finds it optimal to consume all varieties in positive amounts. This is because Inada conditions imply an infinite reservation price. In other words, 0-1 preferences shift the focus to the extensive margin of consumption, while CES preferences focus entirely on the intensive margin. It is important to note, however, that our central results below do not depend on the 0-1 assumption. In fact, we will show below that more general preferences – which allow for both the extensive and the intensive margin of consumption – generate results that are qualitatively similar to those derived in the 0-1 case.

\(^4\)Notice that the integral in (1) runs from zero to infinity. While preferences are defined over an infinitely large measure of potential goods, the number of goods actually supplied is limited by firm entry, i.e. only a subset of potentially producible goods can be purchased at a finite price.

\(^5\)Strictly speaking, the condition \( 1 \geq \lambda p(j) \) is necessary but not sufficient for \( c(j) = 1 \) and the condition \( 1 < \lambda p(j) \) is sufficient but not necessary for \( c(j) = 0 \). This is because purchasing all goods for which \( 1 = \lambda p(j) \) may not be feasible given the consumer’s budget. For when \( N \) different goods are supplied at the same price \( p \) but \( y < pN \) the consumer randomly selects which particular good will be purchased or not purchased. This case, however, never emerges in the general equilibrium.

\(^6\)The discussion here rules out the case where incomes could be larger than \( pN \), meaning that the consumer is subject to rationing (i.e. he would want to purchase more goods than are actually available at the available prices). While this could be a problem in principle, it will never occur in the equilibrium of the model.
Equilibrium. Since both firms and households are identical, the equilibrium is symmetric. Similar to the standard monopolistic competition model, the information on other firms’ prices is summarized in the shadow price $\lambda$. Hence, the pricing decision of a monopolistic firm depends only on $\lambda$. Moreover, the value of $\lambda$ is unaffected by the firm’s own price because a single firm is of measure zero.

**Lemma 1** There is a single price $p = 1/\lambda$ in all markets and all goods are purchased by all consumers.

**Proof.** Aggregate demand for good $j$ is a function of $\lambda$ only. Consequently, the pricing decision of a monopolistic firm depends on the value of $\lambda$ and not directly on the prices set by competitors in other markets. Thus, it is profit maximising to set $p(j) = 1/\lambda$ as long as $1/\lambda$ exceeds marginal costs. To prove the second part of the Lemma, assume to the contrary that only a fraction $\nu$ of consumers purchases the product at price $p(j) = p = 1/\lambda$. However, this cannot be an equilibrium, as the firm could undercut the price slightly and sell to all consumers.

Each monopolistic firm faces a demand curve as depicted in Figure 1. It will charge a price equal to the representative consumer’s willingness to pay $p = 1/\lambda$ and sell output of quantity 1 to each of the $P$ households. Without loss of generality, we choose labor as the numéraire, $W = 1$. Two conditions characterize the autarky equilibrium. The first is the zero-profit condition, ensuring that operating profits cover the entry costs but do not exceed them to deter further entry. Entry costs are $FW = F$ and operating profits are $[p - W/a]P = [p - 1/a]P$. The zero-profit condition can be written as $p = (aF + P)/aP$. This implies a mark-up $\mu$ – a ratio of price over marginal cost – equal to

$$\mu = \frac{aF + P}{P}.$$ 

Notice that technology parameters $a$ and $F$ and the market size parameter $P$ determine the mark-up.\(^7\) We will show below that the mark-up is a crucial channel through which non-homothetic preferences affect patterns of trade and the international division of labor.

The second equilibrium condition is a resource constraint ensuring that there is full employment $PL = FN + PN/a$. From this latter equation, equilibrium product diversity (both in production and consumption) in the decentralized equilibrium is given by

$$N = \frac{aP}{aF + P}L.$$ 

\(^7\)Notice the difference between the 0-1 outcome and the standard CES-case. With 0-1 preferences, the mark-up depends on technology and market size parameters. With CES, the mark-up is determined by the elasticity of substitution between differentiated goods; it is independent of technology and market size. In fact, the variable mark-ups arising with 0-1 preferences will drive many of our results below.
3 Trade between a rich and a poor country

Let us now consider a world economy where a rich and a poor country trade with each other. We denote variables of the rich country with superscript \( R \) and those of the poor country with superscript \( P \). To highlight the relative importance of differences in per-capita incomes and population sizes, we let the two countries differ along both dimensions, hence \( L^R > L^P \) and \( P^R \geq P^P \). We assume trade is costly and of the standard iceberg type: for each unit sold to a particular destination, \( \tau > 1 \) units have to be shipped and \( \tau - 1 \) units are lost during transportation.

3.1 Full trade equilibrium

We will show below that if the income gap between the two countries is small, all goods are traded internationally. In such a full trade equilibrium, a firm’s optimal price for a differentiated product in country \( i = R, P \) equals the households’ willingnesses to pay (see Figure 1), hence we have \( p^R = 1/\lambda^R \) and \( p^P = 1/\lambda^P \). Since country \( R \) is wealthier than country \( P \), we have \( \lambda^R < \lambda^P \) and \( p^R > p^P \). By symmetry, the prices of imported and home-produced goods are identical within each country.

Solving for the full trade equilibrium is straightforward. Consider the resource constraint in the rich country. When \( N^R \) firms enter, \( N^R F \) labor units are employed to set up these firms; \( N^R P^R / a \) labor units are employed in the production to serve the home market; and \( N^R P^P \tau / a \). Since each of the \( P^R \) households supplies \( L^R \) units of labor inelastically, the resource constraint is

\[
P^R L^R = N^R F + N^R (P^R + \tau P^P) / a.
\]

Similarly, for the poor country. Solving for \( N^i \) (\( i = R, P \)) lets us determine the number of active firms in the two countries

\[
N^i = \frac{aP^i}{aF + (P^i + \tau P^{-i})L^i}, \tag{2}
\]

(where \(-i = P \) if \( i = R \) and vice versa).

Now consider the zero-profit conditions in the two countries. An internationally active firm from country \( i \) generates total revenues equal to \( p^R P^R + p^P P^P \) and has total costs \( W^i [F + (P^i + \tau P^{-i})/a] \). Using the zero-profit conditions of the two countries lets us calculate relative wages

\[
\omega = \frac{W^P}{W^R} = \frac{aF + \tau P^P + P^R}{aF + P^P + \tau P^R}. \tag{3}
\]

When the two countries differ in population size, wages (per efficiency unit of labor) are higher in the larger country. Why are wages higher in larger countries? The reason is that labor is more productive in a larger country. To see this, consider the amount of labor needed by a firm in country \( i \) to serve the world market. When country \( R \) is larger than country \( P \), firms in country \( R \) need less labor to serve the world market because there are less iceberg losses.

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8While \( \omega \) measures relative wages per efficiency unit of labor, \( \omega L^P / L^R \) measures relative per-capita incomes. In principle, \( \omega L^P / L^R > 1 \) is possible, so that country \( P \) (with the lower labor endowment) has the higher per-capita income. We show below that this can happen only in a full trade equilibrium but not in an arbitrage equilibrium. The latter case is the interesting one in the present context.
during transportation, and this is reflected exactly in relative wages. There are two cases in which wages are equalized: (i) $\tau = 1$. When there are no trade costs, the productivity effect of country size vanishes. (ii) $p^P = \mathcal{P}^R$. When the two countries are of equal size, productivity differences vanish because iceberg losses become equally large. Note further that $\tau^{-1} < \omega < \tau$. When the poor country becomes extremely large, iceberg losses as a percentage of total costs become negligible, $\omega \to \tau$. Similarly, when the rich country becomes extremely large, $\omega \to \tau^{-1}$.

Finally, let us calculate prices and mark-ups in the respective export destination. The budget constraint of a household in country $i$ is $W_i L^i = p_i N^R + N^P$. Combining the zero-profit condition with these budget restrictions and the number of firms lets us express the price in country $i$ as

$$p^i = W^i L^i \frac{a F + \mathcal{P}^R + \tau P^P}{a \mathcal{P}^L L^R + a \omega P^P L^P}, \quad i = R, P.$$  

By symmetry, prices for the various goods are identical within each country, irrespective of whether they are produced at home or abroad. Consequently, imported goods generate a lower mark-up than locally produced goods because exporters cannot pass trade costs through to consumers.\footnote{This is different from CES preferences, where transportation costs are more than passed through to prices as exporters charge a fixed mark-up on marginal costs (including transportation). Notice that limited cost pass-through has been documented in a large body of empirical evidence.}

Marginal costs are $W^i / a$ when the product is sold in the home market and $\tau W^i / a$ when the product is sold in the foreign market. Hence mark-ups (prices over marginal costs) are $\mu^D = p^i a / W^i$ in the domestic market and $\mu^X = p^i a / (W^i \tau)$ in the export market.

We can summarize the properties of a full trade equilibrium as follows: (i) $N^P / N^R = \omega \mathcal{P}^L L^P / (\mathcal{P}^R L^R)$, i.e. differences in aggregate GDP lead to proportional differences in produced varieties; (ii) $p^P / p^R = \omega L^P / L^R$, i.e. differences in per-capita incomes generate proportional differences in prices; and (iii) $\mu^D / \mu^D = \mu^X / \mu^X = L^P / L^R < 1$, i.e. differences in per-capita endowments lead to proportional differences in mark-ups.

Which country gains more from trade? In the full trade equilibrium, all firms sell to all households worldwide, so consumption and welfare levels are equalized across rich and poor countries. Gains from trade are higher for the country with lower product variety under autarky. Product variety in autarky is $N^i = a \mathcal{P}^L L^i / (a F + \mathcal{P}^i)$. The country with a smaller population $\mathcal{P}^i$ and/or lower per-capita endowment (lower $L^i$) gains more from trade.

### 3.2 “Arbitrage” equilibrium with non-traded goods

Full trade ceases to be an equilibrium when per-capita income differences $\omega L^P / L^R$ become large. The reason is a threat of arbitrage. Consider a U.S. firm that sells its product both in the U.S. and in China. Suppose the firm charges a price in China that equals the Chinese households’ willingness to pay $p^P = 1 / \lambda^P$ and a price in the U.S. that equals the U.S. households’ willingness to pay $p^R = 1 / \lambda^R$. If the difference between $1 / \lambda^P$ and $1 / \lambda^R$ is large, arbitrage opportunities emerge. Arbitrageurs can purchase the good cheaply on the Chinese market, ship it back to the U.S., and underbid the producer on the U.S. market. A threat of arbitrage also concerns Chinese firms which both produce for the local market and export to
the U.S.. When these firm charge too high prices in the U.S., arbitrage traders purchase the cheap products in China and parallel export them to the U.S..

Firms anticipate this arbitrage opportunity and adjust their pricing behavior accordingly. Notice that the threat of parallel trade only constrains firms operating on the world market. Firms that abstain from selling the product in the poor country and focus exclusively on the market of the rich country do not face such a threat. Adopting this latter strategy implies a smaller market but lets firms exploit the rich households’ high willingness to pay. In equilibrium, firms are indifferent between the two strategies. Notice that concentrating sales exclusively on the rich market country is, in principle, an option both for producers in the rich and in the poor country. In equilibrium, however, only by rich-country producers adopt this strategy. While total revenues are independent of the producer’s location, total costs are not. To serve households in the rich country, country-R producers face marginal costs $W^R$, while country-P exporters face marginal costs $W^R \omega \tau / a$ (they have to bear transportation costs). Since $\omega \tau > 1$, country-P producers have a competitive disadvantage in serving the rich country even when the poor country has lower wages $\omega < 1$.

An arbitrage equilibrium looks as follows. A subset of rich-country producers sells their product exclusively in the rich country, while the remaining rich-country producers sell their product both in the rich and in the poor country. All poor-country producers sell their product worldwide. To see why this is an equilibrium, consider the alternative situation in which all rich-country producers trade their products internationally. If all firms charged a price that prevents arbitrage, all goods would be priced below rich households’ willingness to pay. In that case, however, rich households do not spend all their income, generating an infinitely large willingness to pay for additional products. This would incentivize country-R firms to sell their product only on their home market. Thus, in equilibrium both types of firms will exist and the fraction of firms selling exclusively on the local market is determined endogenously.

We are now ready to solve for the arbitrage equilibrium. Denote the price in the rich country of traded and non-traded goods by $p^R_R$ and $p^R_P$, respectively. The price of non-traded goods is $p^R_P = 1 / \lambda^P$. Anticipating the threat of parallel trade, the price of traded goods may not exceed and exactly equals the price in the poor country (plus trade costs), $p^R_P = \tau / \lambda^P$, in equilibrium. The price of a product in the poor country is still given by $p^P = 1 / \lambda^P$. The following lemma proofs that this is a Nash equilibrium.

**Lemma 2** In an arbitrage equilibrium, firms that sell their product in both countries (i) set $p^P = 1 / \lambda^P$ in country $P$ and $p^R_R = \tau p^P$ in country $R$, and (ii) sell to all households in both countries.

**Proof.** (i) Assume $1 / \lambda^P$ exceeds marginal costs of exporting. In that case, the profit maximization problem of an exporting firm reduces to maximize total revenue $P^P p^P(j) + P^R p^R(j)$ s.t. $\tau p^P(j) \geq p^R(j)$ and $p^R(j) \leq 1 / \lambda^i$. Applying Lemma 1, it is profit maximizing to set $p^R(j) = 1 / \lambda^i$ if $\tau / \lambda^P \geq \lambda^R$ (full trade equilibrium). If $\tau / \lambda^P < \lambda^R$, the arbitrage constraint is binding $\tau p^P(j) = p^R(j) = p^R_R$ and revenues are maximized when $p^P(j) = 1 / \lambda^P$. (ii) Assume to the contrary that only a fraction $\nu$ of consumers purchases the product at price $p^P(j) = 1 / \lambda^P$. 

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As in Lemma 1, this cannot be an equilibrium, as the firm would lower \( p^P(j) \) and \( p^R(j) \) slightly and gain the whole market in the poor country.

The zero-profit condition for an internationally active country-\( i \) producer is

\[
p^R_N p^P + p^P p^P = W^i \left[ F + (P^i + \tau P^{-i})/a \right].
\]

These firms’ total revenues do not depend on the location of production, but the required labor input depends on location. Differences in population sizes generate differences in (total) transport costs, and relative wages equalize these differences. From the zero-profit conditions we see that relative wages \( \omega \) are still given by equation (3). The zero-profit conditions also let us derive the prices for the various products. Using \( p^R_T = \tau p^P \), we get

\[
p^R_T = \frac{\tau a F + \tau P^P + \tau P^P}{\tau P^R + p^P} \quad \text{and} \quad p^P = \frac{1}{a} \frac{a F + \tau P^R + \tau P^P}{\tau P^R + p^P},
\]

where we have set \( W^R = 1 \). (We use this normalization throughout the paper.) The zero-profit condition for an exclusive rich-country producer is

\[
p^R_N p^R = F + \tau P^R/a,
\]

from which we calculate the equilibrium price of a non-traded variety

\[
p^R_N = \frac{a F + \tau P^R}{a P^R}.
\]

Notice that, due to the arbitrage constraint on exporters’ pricing behavior, prices do not depend on \( L^P \) and \( L^R \). This is quite different from the full-trade equilibrium, where price differences reflect differences in per-capita endowments.

The resource constraint in country \( P \) is the same as that in the full trade equilibrium, so \( N^P \) is still given by (2). The resource constraint in country \( R \) is now different, however, because there are traded and non-traded products. Denoting the range of traded and non-traded goods produced in the rich country by \( N^R_T \) and \( N^R_N \), respectively, the resource constraint of country \( R \) is given by

\[
N^R_T p^P p^P = N^P p^R p^R \quad \text{and} \quad \frac{p^R_T}{p^P} = \tau.
\]

Together with the trade balance condition \( N^R_T p^P p^P = N^P p^R p^R \) and the terms of trade \( p^R_T/p^P = \tau \) we get

\[
N^R_T = \frac{a P^R}{a F + \tau P^R + p^P} \tau L^P, \quad \text{and} \quad N^R_N = \frac{a P^R}{a F + P^R} \left( L^R - \tau \omega L^P \right).
\]

### 3.3 Per-capita income, population size, and patterns of trade

The conditions under which the threat of parallel trade becomes binding and the economy switches from a full trade to a partial trade equilibrium are straightforward. In a full trade equilibrium, relative prices equal relative per-capita incomes \( p^P/p^R = \omega L^P/L^R \). In that case, differences in willingnesses to pay must be small enough, \( \lambda^P/\lambda^R \leq \tau \), so that the threat of parallel trade is not binding. In contrast, when differences in willingnesses to pay become large, \( \lambda^P/\lambda^R > \tau \), the parallel trade constraint kicks in. This happens when

\[
\frac{\omega L^P}{L^R} > \tau^{-1}.
\]
In other words, a full trade equilibrium emerges when per-capita incomes are similar, while an arbitrage equilibrium emerges when the gap in per-capita incomes is large.

We first look at a full trade equilibrium. Let us highlight how the volume and structure of international trade depend on relative per-capita endowments $L_P/L_R$. Let us define “trade intensity” $\phi$ as the ratio between the value of world trade and world GDP. In a full trade equilibrium the value of world trade is given by $p_R N^P p^R + p_P N^R p^P$ while world income is $L^R p^R + \omega L^P p^P$. Trade intensity is given by

$$\phi = \frac{2L^R p^R \cdot \omega L^P p^P}{(L^R p^R + \omega L^P p^P)^2}$$

When all goods are traded, the relative size of aggregate GDP matters for trade intensity. When GDP differs strongly across the two countries, trade intensity is small as most world production takes place in the large country and most of this production is also consumed in this country. Trade intensity is maximized when the two countries are of exactly equal size.

We can now state the following proposition

**Proposition 1** Assume per-capita incomes are similar, $\omega L_P/L_R \in [\tau^{-1}, 1]$. a) All goods are traded. b) Trade intensity $\phi$ increases with both the per-capita endowment $L_P$ and population size $P^P$ if $\omega L_P P^P < L^R p^R$. c) The impact on $\phi$ of $P^P$ is stronger than the one of $L_P$. d) A trade liberalization increases trade intensity if $\omega L_P P^P < L^R p^R$.

**PROOF.** See Appendix A.

Now consider an arbitrage equilibrium. Here the situation is quite different. In that case, the value of traded goods is $p_R^P N^P p^R + p_P^P N^R p^P$ (while world income still is $L^R p^R + \omega L^P p^P$). Using equations (2) and (5) we calculate the trade intensity in an arbitrage equilibrium.

$$\phi = \frac{2 \tau}{\tau + (P^P/P^R)} \cdot \frac{\omega L^P p^P}{L^R p^R + \omega L^P p^P} \quad (7)$$

Equation (7) shows that per-capita incomes differences and differences in population sizes affect trade intensity in different ways. Consider first the impact of a given change in per-capita income of country $P$. The above expression for $\phi$ reveals that a higher per-capita income of the poor country unambiguously increases the intensity of trade. This is reminiscent of the Linder-hypothesis (Linder 1961) postulating that a higher similarity in per-capita incomes is associated with higher trade between trading partners. The intuition for this result is straightforward. When $L_P$ increases by 10 percent, the range of exported goods increases by 10 percent while prices remain unchanged. Hence the aggregate value of trade $p_R^P N^P p^R + p_P^P N^R p^P$ increases by 10 percent as well. In contrast, increasing $L^P$ by 10 percent (while leaving $L^R$ unchanged) increases world GDP by less than 10 percent. Trade intensity, the ratio between world trade and world GDP, thus rises unambiguously.

Now consider a change in population-size of country $P$. It turns out that a change in $P^P$ has a smaller effect on trade intensity than an increase in relative per-capita incomes that
increases GDP by the same magnitude, i.e. we have \( \partial \log \phi / \partial \log P < \partial \log \phi / \partial \log L \). This can be seen from looking at the volume of world trade which is equal to \( 2p^P N^R_T p^P \). An increase in \( P \) has a direct and an indirect effect on world trade. The direct effect increases trade in proportion to country \( P \)'s population. The indirect effect lowers per-capita imports. Notice that imports per capita in country-\( P \) are equal to \( p^P N^R_T \). An increase in \( P \) has a direct and an indirect effect on world trade. The direct effect increases trade in proportion to country \( P \)'s population. The indirect effect lowers per-capita imports. Notice that imports per capita in country-\( P \) are equal to \( p^P N^R_T \). An increase in \( P \) has a direct and an indirect effect on world trade. The direct effect increases trade in proportion to country \( P \)'s population. 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under which a full trade equilibrium emerges. In this sense, a larger population in the poor country fosters trade.\textsuperscript{11}

\textbf{Figure 2}

\subsection*{3.5 Welfare effects of a trade liberalization}

We proceed by studying welfare implications and the gains from trade. In particular, we are interested in how bilateral trade liberalizations affect welfare and the distribution of trade gains between rich and poor countries. A trade liberalization is modeled as a reduction in iceberg transportation costs $\tau$. We then let trade costs vary across countries and discuss unilateral liberalizations.

\textbf{Bilateral trade liberalization.} In a full trade equilibrium, households in both countries purchase all goods produced worldwide. Hence the welfare levels are identical in both countries despite their unequal endowment with productive resources

$$ U^R = U^P = \frac{aL^R P^R}{aF + \tau P^R + \tau P^P} + \frac{a\omega L^P P^P}{aF + \tau P^R + \tau P^P}.$$

Firms’ price setting behavior drives this result. $R$-consumers are willing to pay higher prices than $P$-consumers because their income is higher. In the full trade equilibrium, higher nominal incomes translate one-to-one into higher prices, welfare is therefore identical. To see the mechanism by which welfare is equalized, consider mark-ups in the special case when the two countries are equally large. When $P^P = P^R$, prices are higher in country $R$, while costs are the same for each country. In other words, country-$R$ households bear a larger share of total costs. In this case, the poor country’s welfare is lower under autarky. \textsuperscript{12}

In an arbitrage equilibrium, consumers’ welfare levels in the two countries diverge. Country-$P$ households’ welfare equals $N^R + N^P$, while country-$R$ households’ welfare equals $N^P + N^R + N^R$. Using (2) and (5), these welfare levels are given by

$$ U^P = \frac{aL^P (P^P + \tau P^R)}{aF + \tau P^R + \tau P^P} \quad \text{and} \quad U^R = \frac{aL^R (P^P + \tau P^P)}{aF + \tau P^R + \tau P^P} + \frac{aPID (L^R - \tau L^P)}{aF + \tau P^R}.$$

It is straightforward to verify, that $\partial U^P/\partial \tau > 0$ while $\partial U^R/\partial \tau < 0$. We are now able to state the following proposition.

\textbf{Proposition 4} a) In a full trade equilibrium, welfare levels are equalized. A trade liberalization (a lower $\tau$) increases welfare for both countries. b) In an arbitrage equilibrium, a trade liberalization increases the welfare of country-$R$ households but decreases it for country-$P$ households.

\textsuperscript{11}Notice that there is international trade even when income differences become extremely large and $L^P/L^R$ becomes very small. The range of traded goods approaches zero, however, when $L^P/L^R$ goes to zero.

\textsuperscript{12}This continues to hold when $P^P \neq P^R$. Only when $P^P \ll P^R$, so that $\omega L^P > L^R$, prices become higher in country $P$. In that case, country-$P$ bears the larger share in total costs.
Proof. In text. ■

Proposition 4 shows the crucial role of trade costs for welfare. Unequal countries have different preferred trade barriers (or different preferred degrees of trade liberalizations). Consumers in the rich country are essentially free-traders, whereas consumers in the poor country only want liberalization up to a positive level of trade costs. What is the intuition behind this result? Country-$R$ firms’ pricing behavior provided the explanation. As higher trade costs imply a less tight arbitrage constraint, country-$R$ firms can charge higher prices for traded goods relative to non-traded goods. This induces country-$R$ firms to export rather than sell exclusively to domestic customers. The result is an increase in trade intensity which benefits the poor country. Put differently, poor country households are pro-trade but against a complete trade liberalization because too low a $\tau$ decreases trade and welfare.

Figure 3 shows the welfare responses of changes in $\tau$ graphically. Panel a) is drawn for relatively low per-capita income differences $\omega L^P/L^R > \tau^*$. In that case, an arbitrage equilibrium emerges with low trade costs, while a full trade equilibrium emerges with moderate trade costs. Panel b) is drawn for higher per-capita income differences $\omega L^P/L^R \leq \tau^*$ so that a full trade equilibrium is not feasible. Country-$R$ welfare (the bold graph) is monotonically decreasing in $\tau$ in both panels of Figure 3. Hence the $R$-consumer reaches his maximum welfare when trade costs are at their lowest possible level $\tau = 1$. In contrast, the impact of $\tau$ on country-$P$ welfare (the dotted graph) interacts with per-capita income differences. When these differences are low (panel a), country-$P$ welfare increases in $\tau$ when $\tau < (\omega L^P/L^R)^{-1}$ and decreases in $\tau$ when $\tau \geq (\omega L^P/L^R)^{-1}$. Welfare is maximized at $\tau = (\omega L^P/L^R)^{-1}$ (when the equilibrium switches from a full-trade to an arbitrage equilibrium). When per-capita income differences are large (panel b), country-$P$ welfare decreases monotonically in $\tau$ (full trade is not feasible) and welfare is maximized at $\tau = \tau^*$.

Figure 3

Unilateral trade liberalization. Up to now we have assumed symmetric trade costs across countries. However, policy makers can influence trade costs through tariffs and regulations. This is interesting in the present context because, in an arbitrage equilibrium, the poor country has an incentive to increase trade barriers and relax the arbitrage constraint as this increases the supply or northern varieties and hence welfare in the South.

Now let trade costs differ between countries, with $\tau^i$ denoting iceberg costs for imports into country $i$. While total revenues of exporters are still $p_R^i p_R^R + p_P^i p_P^P$, now total costs do not only vary as a result of unequally large populations but also because of differences in transportation costs, $W^i \left[ F + (\mathcal{P}^i + \tau^{-i} p^{-i})/a \right]$. From the zero-profit condition we derive relative wages $\omega$ as

$$\omega \equiv \frac{W^P}{W^R} = \frac{aF + \tau^P p^P + \mathcal{P}^R}{aF + \mathcal{P}^P + \tau^R p^R},$$

which implies that $(\tau^R)^{-1} < \omega < \tau^P$. Assume that income differences are sufficiently large, $\omega L^P/L^R < \tau^R$, so that an arbitrage equilibrium prevails. To prevent arbitrage, the price of
traded goods in the rich country may not exceed the price in the poor country plus transportation costs, hence firms will charge $p_T^R = \tau^R p_P$ in the rich country.\footnote{To make sure that such an equilibrium exists, we also assume that country-$R$ exporters can charge a price in country $P$ that covers (production plus transportation) costs, $p_P^R > \tau^P/a$. This implies $\tau^F \tau^R < aF/p^R + 1$. If this condition is satisfied also country-$P$ exporters will export, $p_P^R > \tau^R/a$ because $p_P^R = \tau^R p_P$.}

Using zero-profit conditions and resource constraints is it is straightforward to calculate welfare in the two countries as

$$U^P = \frac{aP^P + aR^R p_P R}{aF + \tau^R p_P R} L^R \quad \text{and} \quad U^R = U^P + \frac{aP^R}{aF + \tau^R p_P R} (L^R - \tau^R \omega L^P).$$

Interestingly, a unilateral trade liberalization by the poor country (a fall in $\tau^P$) does not have any effect on poor households, but affects rich households through a fall in $\omega$. Lower costs of exporting to the poor country makes producers in country $R$ more productive, improving their terms of trade while leaving the arbitrage constraint unaffected. This saves resources for country $R$ which are employed to produce non-traded goods. This raises welfare of rich consumers.

In contrast, an unilateral increase in trade barriers into the rich country (a larger $\tau^R$) harms country-$R$ but benefits country-$P$ households. Hence, our model predicts that a poor country has an incentive to levy an export tax. This relaxes the arbitrage constraint and increases the supplied varieties and hence welfare in country $P$.

4 Many rich and poor countries

In an arbitrage equilibrium with two countries, all firms in the poor country are exporters while only a subset of firms in the rich country exports. Moreover, a trade liberalization that relaxes the arbitrage constraint always hurts poor consumers. We now show that these predictions need to be qualified in a multi-country world. The effect of moving from two to many countries can be most easily shown when there are $n$ identical rich countries and $m$ identical poor countries, i.e. a world with a fragmented rich North and a fragmented poor South. As before, we assume that countries differ in per-capita endowments (and population size) but are identical in all other respects.

The general equilibrium has a structure very similar to that of the two-country case. From the zero-profit conditions for internationally active firms, it is straightforward to show that relative wages are now given by

$$\omega \equiv \frac{W^R}{W^P} = \frac{aF + \tau P^{-R} + p^R}{aF + \tau P^{-P} + p^P},$$

where $P^{-R} = (n-1)P^R + mP^P$ and $P^{-P} = nP^R + (m-1)P^P$ are rest-of-the-world populations from the perspective of country $R$ and country $P$, respectively. In full world trade equilibrium, relative prices of southern relative to northern markets are determined by relative per-capita incomes, $p^P/p^R = \omega L^P/L^R$, and the ratio of produced varieties still reflects differences in
aggregate GDP, \( N^P/N^R = \omega L^P P^P/L^R P^R \).

The interesting case is when income differences sufficiently large, so that \( \omega L^P/L^R > \tau^{-1} \). In that case, the arbitrage constraint is binding, limiting trade between the rich North and the poor South. A northern firm now has two options: either export worldwide or export only to other northern countries. Notice that, unlike in the two-country case, all northern firms are now exporters. Firms that export exclusively to the North have a smaller market but can charge higher prices. Firms that export to all countries worldwide set low prices but have the large world market. While large differences in per-capita incomes limit trade across regions, there is full trade within regions. As there are no income differences within a region, all goods produced in that region are also sold to other countries in that region.

The arbitrage equilibrium can now be solved in a straightforward way (for details see Appendix E). We first study how differences in per-capita incomes and population sizes affect trade intensity. It is straightforward to calculate

\[
\phi = 2 \frac{m \omega L^P P^P}{n L^R P^R} \frac{(m-1) P^P + \tau P^R}{m P^P + n \tau P^R} + 2 \frac{(n-1) L^R P^R}{n L^R P^R + m \omega L^P P^P},
\]

which readily reduces to the expression derived in the last section when \( n = m = 1 \). An increase in \( L^P \) increases world trade intensity (and reduces North-North trade with exclusive goods). It can also be shown that a larger population in the South has a weaker effect on trade intensity than a larger per-capita income. Hence with respect to per-capita incomes and populations sizes, the results of the two-country case carry over to the multi-country framework.

In contrast to the two-country case, the effect of a trade liberalization on welfare is now ambiguous. There are two effects. On the one hand, a lower \( \tau \) implies a tighter arbitrage constraint for globally active producers. Lower prices for globally traded products (relative to products exclusively sold in the North) induce former northern world-market producers to concentrate their sales on northern markets only. This reduces trade intensity between the North and the South. On the other hand, a reduction in \( \tau \) stimulates trade within regions. While South-South trade increases less than North-South trade falls (first term of above equation increases in \( \tau \)), North-North trade unambiguously increases (second term decreases in \( \tau \)). A trade liberalization is more likely to stimulate trade if there are more countries with a region. Within-regional trade is more strongly affected in this case and dominates the reduction in North-South trade. A trade liberalization is also more likely to stimulate North-South trade, the larger is the North relative to the South. In that case, North-North trade (which is positively affected) comprises the bulk of world trade. When the North is much larger than the South, positive effects on North-North trade of a trade liberalization dominate negative effects on North-South trade flows.

It turns out that the welfare level of a country-\( P \) household is given by

\[
U^P = m N^P + n N^R_N = \frac{a L^P (m P^P + n P^R)}{a F + \tau P^P + \bar{P} P^P}.
\]

It is straightforward to see that \( \partial U^P/\partial \tau < 0 \) if \( a F < (m-1) P^P (1 + (m/n) (P^P/P^R)) \). This
means that a trade liberalization may raise welfare in country \( P \) and is more likely to do so the higher is \( m^P \). A reduction in \( \tau \) has two opposing effects. The arbitrage channel is still at work and induces northern firms to abstain from selling to southern households. This is harmful for southern welfare. However, a lower \( \tau \) stimulates South-South trade, which has a beneficial effect on southern welfare. Households in the poor country gain from a trade liberalization when there are many poor countries and when poor countries are large. In such a situation, there is a lot to gain from South-South trade because there are many trade barriers and because the southern markets are large.

We summarize the above discussion in the following

**Proposition 5** Assume there are \( m \) identical poor countries and \( n \) identical rich countries, with \( L^P / L^R < (\omega \tau)^{-1} \). a) All northern firms export, but some of them export only to other northern countries; b) A trade liberalization (a lower \( \tau \)) unambiguously increases welfare of rich households. It increases welfare of poor households if \( aF < (m - 1)P^P (1 + (m/n)(P^P/P^R)) \) and decreases it otherwise. The increase in welfare is larger in the North than in the South.

**Proof.** In text. ■

Notice that there is a positive correlation in an arbitrage equilibrium between the probability that a northern firm exports and the per-capita income of a potential importing country. In our stylized model, the probability that a country-\( R \) firm exports to another rich country is 100 percent, while the probability that it exports to a poor country is strictly less than 100 percent. (The prediction that 100 percent of all firms export is clearly an artefact arising from the assumed absence of firm heterogeneity.)

5 **General preferences**

The assumption of 0-1 preferences yields a tractable framework with closed-form solutions. However, this assumption focuses entirely on the extensive margin of consumption. This contrasts with the standard CES case where all adjustments happen along the intensive margin. We go beyond these two polar cases in this section by studying general preferences. We show that the qualitative characteristics of the equilibria under 0-1 preferences carry over to general preferences featuring non-trivial intensive and extensive margins of consumption. In particular, we precisely define the conditions under which an arbitrage equilibrium with non-traded goods exists and also provide a numerical exercise showing that such an equilibrium emerges under a wide range of parameter values.

5.1 **Utility and prices**

Let us go back to the setup of Section 5 with two countries that differ in per-capita income and population size but let household welfare is now be

\[
U = \int_0^\infty v(c(j))dj,
\]
where \( c(j) \) denotes the consumed quantity of good \( j \). It is assumed that the subutility \( v() \) satisfies \( v' > 0, v'' < 0 \) and \( v(0) = 0 \). Beyond these standard assumptions, we make two further assumptions on the function \( v() \): (i) \( v'(0) < \infty \), (ii) \( v''(0) > -\infty \), and (iii) \(-v'(c)/[v''(c)c]\) is decreasing in \( c \). The first assumption implies that reservation prices are finite, generating a non-trivial extensive margin of consumption; the second ensures that an arbitrage equilibrium exists when per-capita income differences are sufficiently high (see below); and the third implies a price elasticity of demand decreasing along the demand curve. Monopolistic pricing leads to \( p = (1 + v''(c)c/v'(c))^{-1} \beta \), where \( \beta \) denotes marginal cost. To simplify notation, we denote the mark-up by \( \mu(c) = (1 + v''(c)c/v'(c))^{-1} \). Assumptions (i)-(iii) imply that \( \mu(0) = 1 \) and \( \mu'(c) > 0 \).\(^{14}\)

How does firms’ price setting behavior change when there are consumer responses along the intensive margin? With 0-1 preferences, the monopoly price equals the representative household’s willingness to pay and does not depend on marginal production costs. With general preferences, however, firms solve the standard profit maximization problem: the price equals marginal costs times a mark-up that depends on the price elasticity of demand. This implies an important difference to the case of 0-1 preferences. With general preferences, there are price differences between imported and domestically produced goods. While symmetric utility implies that importers and local producers within a given location face the same demand curve, marginal costs differ since importers have to bear transportation costs and since wages vary by location. To allow for such differences, we denote by \( p_j^i, c_j^i \) and \( b_j^i \), respectively, the price, quantity and marginal cost of a good produced in country \( j \) and consumed in country \( i \). Unconstrained monopoly pricing implies \( p_j^i = \mu(c_j^i)b_j^i \).

### 5.2 The arbitrage equilibrium

The arbitrage equilibrium features a situation in which (i) only a subset of country-\( R \) producers sell their product worldwide at sufficiently low prices to avoid arbitrage; (ii) the remaining country-\( R \) firms sell their product exclusively in the rich country at the unconstrained monopoly price; (iii) all poor-country producers export their products, also at prices that avoid arbitrage. The discussion in this section focuses on the conditions under which an arbitrage equilibrium exists. (Appendix E provides the full system of equations that characterize such an equilibrium.)

The arbitrage constraints for country-\( R \) and country-\( P \) producers, respectively, are now given by

\[
\frac{1}{\tau} \leq \frac{p_R^R/p_R^P}{p_R^R} \leq \tau \quad \text{and} \quad \frac{1}{\tau} \leq \frac{p_P^P/p_P^R}{p_P^P} \leq \tau.
\]

A necessary condition for the existence of an arbitrage equilibrium is that these constraints are binding, so that \( p_R^R/p_R^P = p_P^P/p_P^R = \tau \). This happens to be the case if the gap in per-

\(^{14}\)\( \mu'(c) > 0 \) follows directly from assumption (iii). To see why \( \mu(0) = 1 \) we use l’Hôpital’s rule \( \lim_{c \to 0} v'(c)c/v(c) = \lim_{c \to 0} (1 + v''(c)c/v'(c)) \). However, \( \lim_{c \to 0} v'(c)c/v(c) = v'(0) \cdot \lim_{c \to 0} c/v(c) = v'(0)/v'(0) = 1 \). This implies \( \lim_{c \to 0} v'(c)c/v(c) = 0 \) and hence \( \lim_{c \to 0} \mu(c) = 1 \). Since the monopolist optimally chooses a price along the elastic part of the demand curve, no further restrictions on the \( \mu(c) \)-function are needed.
capita incomes becomes sufficiently large. As $L^R/L^P$, and hence $c_R^P/c_R^R$, get large the ratio of (unconstrained) monopoly prices eventually exceeds trade costs, or $\mu(c_R^R)/\mu(c_R^P) > \tau^2$. (Recall that $\mu'(c) > 0$.) Notice, however, that a binding arbitrage constraint does not necessarily imply that there are non-traded goods. The reason is that adjustment now does not only occur at the extensive margin but also at the intensive margin. Hence there are full trade equilibria where the arbitrage constraint binds.

To verify the existence of an arbitrage equilibrium with non-traded goods, we look at incentives of country-$R$ firms to sell exclusively on the home market rather than selling their products worldwide. A country-$R$ producer’s profit is given by (to ease notation we write $p_R^R \equiv \tau p$ and $p_R^P \equiv p$)

$$
\pi = \mathcal{P}_R (\tau p - 1/a) c_R^R + \mathcal{P}_P (p - \tau/a) c_R^P.
$$

The corresponding demand curves are given by the first order conditions $v'(c_R^P) = \lambda^R \tau p$ and $v'(c_R^P) = \lambda^P p$ for households in country-$R$ and and country-$P$, respectively. This yields $dc_R^R/dp = (1/p)v'(c_R^R)/v''(c_R^R)$ and $dc_R^P/dp = (1/p)v'(c_R^P)/v''(c_R^P)$. The first order condition of the monopolistic firm’s price setting choice is given by

$$
\frac{\tau p - 1/a}{p} \left(- \frac{v'(c_R^P)}{v''(c_R^P)} \right) + \frac{p - \tau/a}{p} \left(- \frac{v'(c_R^P)}{v''(c_R^P)} \right) \frac{\mathcal{P}_P}{\mathcal{P}_R} = \tau c_R^R + c_R^P \frac{\mathcal{P}_P}{\mathcal{P}_R}. 
$$

To examine whether an arbitrage equilibrium exists, let $L^P$ and therefore $c_R^P$ approach zero, all other exogenous variables (including $\mathcal{P}_P/\mathcal{P}_R$) remain fixed. The first order condition then becomes

$$
\frac{\tau p - 1/a}{\tau p} \left(- \frac{v'(c_R^P)}{v''(c_R^P)c_R^R} \right) + \frac{p - \tau/a}{\tau p} \left(- \frac{v'(c_R^P)}{v''(c_R^P)c_R^R} \right) \frac{\mathcal{P}_P}{\mathcal{P}_R} = 1. 
$$

Now consider the optimal decision of a country-$R$ firm whether to produce exclusively for the home market. Denoting by $p^N$ and $c_R^N$ price and quantity of non-traded goods, the first order condition for exclusive producers is

$$
\frac{p^N - 1/a}{p^N} \left(- \frac{v'(c_R^N)}{v''(c_R^N)c_R^R} \right) = 1. 
$$

When $\tau$ is sufficiently low, so that $p > \tau/a$, comparing the last two equations shows that the price of a non-exporting firm $p^N$ is strictly larger than the price of an exporting firm $\tau p$. (This is because, by assumption, $-v'(0)/v''(0) > 0$.) Since $c_R^P \to 0$ when $L^P \to 0$, export revenues are zero. Hence non-exporters charge higher prices and their profits are larger than those of exporters. This implies that an outcome where all firms export cannot be an equilibrium. We summarize our discussion in

**Proposition 6** There is a critical income gap $\Delta$ such that, for all $L^P/L^R < \Delta$, an equilibrium emerges in which only a subset of goods is traded.

**Proof.** In text. ■
The above proposition implicitly assumes an equilibrium where the two countries trade with each other. This is not a priori clear because the countries may also remain in autarky. The following proposition shows that transportation costs need to fall short of a certain limit to make sure that trade will take place in equilibrium.

**Proposition 7** Denote by $c^R_i$ consumption per variety under autarky in the rich country. There will be trade in equilibrium, if $\tau < \mu(c^R_i)v'(0)/v'(c^R_i)$ where $aF/P^R = c^R_i(\mu(c^R_i) - 1)$.

**Proof.** See Appendix F. ■

An important result we derived under 0-1 preferences holds that population size has a weaker effect than per-capita incomes in determining trade patterns. We now demonstrate that this is also true with general preferences. The previous proposition showed that, starting from a full trade equilibrium, increasing the gap in per-capita incomes will eventually generate an arbitrage equilibrium with non-traded goods. We now show this is not necessarily the case, when we increase relative population size.

To make this point, we proceed as follows. We first observe that, starting from a full trade equilibrium, an increase in $L^P/L^R$ beyond unity eventually leads to a “reversed” arbitrage equilibrium, in which some country-$P$ producers sell only on the domestic market while all country-$R$ producers export. We now show that such a reversed arbitrage equilibrium cannot emerge from a successive increase in $P^P/P^R$ (keeping $L^P/L^R < 1$ constant), because this does not generate price differences sufficiently large to escape a full trade equilibrium. In other words, increasing $P^P/P^R$, we cannot reach a situation where both arbitrage constraints are violated, $\mu(c^P_i) \geq \mu(c^R_i)$ and $\mu(c^P_i) \geq \mu(c^P_j)\tau^2$. To see this, consider the households’ budget constraints

$$aL^R = N_P\mu(c^R_P)c^R_P\tau \omega + N_R\mu(c^R_R)c^R_R,$$

$$aL^P = N_P\mu(c^P_R)c^P_P + N_R\mu(c^R_R)c^P_R\tau/\omega,$$

and take the difference between the two equations. If both arbitrage conditions are violated, the budget constraints can only hold if $\omega > \tau$. However, if $\omega > \tau$ the zero-profit condition is violated in at least one country in a full trade equilibrium (where firms charge the unconstrained monopoly price). In such an equilibrium, the zero-profit condition in country $j$ is given by

$$P^R c^R_j(\mu(c^R_j) - 1)/a + P^P c^P_j(\mu(c^P_j) - 1)\tau/a = F,$$

where $p^j_P > p^j_R$ and $c^j_P < c^j_R$, since country $P$ has higher marginal cost than country $R$, both on the domestic and the export market. However, this implies $c^j_P(\mu(c^j_P) - 1) < c^j_R(\mu(c^j_R) - 1)$ for both $i = P$ and $i = R$, i.e. country $R$-producers make strictly larger profits on both markets. It follows that, when the zero-profit condition holds in country $R$, it must be violated in country $P$.\textsuperscript{15} In sum, we always have $\omega < \tau$ in a full trade equilibrium. But this implies

\textsuperscript{15}Vice versa, if country $P$ is the low-wage country. In that case, we must have $\omega > 1/\tau$ to ensure that both zero-profit conditions can hold simultaneously. Hence we have $\omega \in (1/\tau, \tau)$ in a full trade equilibrium with unconstrained price setting.
that households’ budget constraints continue to hold simultaneously when $P^P/P^R$ gets very large. Thus, unlike with a successive increase in $L^P/L^R$ (beyond unity), it is not possible to reach a “reversed” arbitrage equilibrium. In this sense, the difference in population sizes has a weaker effect on trade patterns than the difference in per-capita endowments. We summarize our discussion in the following

**Proposition 8** Consider a full trade equilibrium with unconstrained price setting. Successive increases in $P^P/P^R$ cannot generate a “reversed” arbitrage equilibrium.

**Proof.** In text.

5.3 A numerical example

To explore the relevance of arbitrage under general preferences, we now provide a numerical example. We assume the utility function belongs to the HARA-class (hyperbolic absolute risk aversion), characterized by $v'(c) = (s - \sigma c)^{1/\sigma}$. We further impose the restriction $s > 0$. This implies $v'(0) = s^{1/\sigma} < \infty$; $v''(0) = -\frac{v'(0)}{s} > -\infty$; and $-v'(c)/[v''(c)c] = s/c - \sigma$ is decreasing in $c$. It therefore satisfies our initial assumptions on the form of $v(c)$.\(^{16}\)

We first calculate the arbitrage frontier as a function of trade costs $\tau$ and relative labor endowment $L^P/L^R$ and draw the corresponding relationship in ($\tau$, $L^P/L^R$)-space for alternative values of $\sigma$ (Figure 4).\(^{17}\) Combinations to the left of this frontier yield an arbitrage equilibrium with non-traded goods. For our quantitative exercise we fix $P^R = P^P = 1$, $s = 1$, $F = 1$ and $a = 2$ and vary the parameter $\sigma$.

Figure 4 shows that with a higher value of $\sigma$ an arbitrage equilibrium becomes more likely. This is quite intuitive. A higher value of $\sigma$ implies that marginal utility declines more strongly in $c$, hence there are larger differences in marginal utility between rich and poor consumers. Hence it becomes more profitable for country-$R$ firms to exclude country-$P$ households and sell only to the rich country.\(^{18}\) Figure 4 shows that there is a large range of parameters that generate an arbitrage equilibrium. (Recall that the median potential trade relations features an income ratio of 1 to 4.) While the set of parameter shrinks when $\sigma$ becomes smaller, but even for $\sigma = 1$ (quadratic subutility), our model generates export zeros for a large number of parameter values.

**Figure 4**

At this point, it is important to note that our model is based on symmetric preferences. While simplifying the analysis considerably, this is restrictive in the present context because

---

\(^{16}\) The HARA class encompasses several frequently used utility functions, such quadratic utility ($\sigma = 1$), Stone-Geary ($\sigma = -1$), CARA ($\sigma \rightarrow 0$), and the CES ($s = 0$ and $\sigma < 0$).

\(^{17}\) The autarky frontier is not shown in the figure. For a given value of $\sigma$, this is a horizontal line at the critical value of $\tau$, below which the two countries engage in trade. Notice that this critical value varies with $\sigma$, but is above lies $\tau \geq 2.5$ for all $\sigma$.

\(^{18}\) In this sense, the value of $\sigma$ captures the relative importance of the extensive margin in consumer choices. Hence our framework provides an alternative way to describe the consumer’s trade off between the intensive and the extensive margin of consumption. This trade-off has been studied in a recent paper by Li (2012), who studies this trade-off by introducing fixed purchasing costs of each new variety but sticking to CES preferences.
symmetry makes it more difficult to generate an extensive margin of consumption. To see why, consider an alternative framework with hierarchical consumption. Assume utility is given by $U = \int \eta(j)v(j)dj$ where $\eta(j)$ denotes an exogenous weighting function, ranking the various goods according to their priority in consumption. Assuming $\eta'(j) < 0$, low-$j$ goods are necessities and high-$j$ goods are luxuries. When the consumption hierarchy is steep ($-\eta'(j)$ is large), necessities generate relatively more utility than luxuries. In that case, consumers are relatively better off purchasing low-$j$ products in large quantities. They will purchase more luxurious products only when their income is sufficiently high. Hence, with hierarchical consumption, the extensive margin becomes more important and export zeros more likely. We conclude that the symmetry assumption provides us with a lower bound for the parameter space that generates an arbitrage equilibrium.

Finally, we take a closer look at the role of trade costs in an arbitrage equilibrium. We assume $L_P/L_R = 1/5$ (close to the median of all potential trade relations) and a preference parameter of $\sigma = 4$ (which guarantees existence of an arbitrage equilibrium over a large range of trade costs, see Figure 4). Just like before, we set $P^R = P^P = 1$, $s = 1$, $F = 1$, and $a = 2$.

Table 1

Lower trade costs lead to a slight increase in the number of goods produced and traded by country-$P$ firms but a substantial reduction in the number of goods produced and traded by country-$R$ firms. The decrease in the mass of goods exported by country-$R$ firms is accommodated by a relatively strong increase in consumption of imported goods by country-$P$ consumers. In contrast, the response along the intensive margin is small in the rich country. As in the basic model with 0-1 preferences, lower trade costs lead to a substantial increase in the number of goods that are produced and consumed only in country $R$ and a corresponding reduction in the fraction of traded goods.

It is also interesting to compare welfare levels in an arbitrage equilibrium to an alternative scenario where the arbitrage channel is shut down (e.g. because parallel trade is prohibited). In the arbitrage equilibrium, only the rich country gains from a trade liberalization while the poor country loses. This resembles the situation under 0-1 preferences. When the arbitrage channel is shut down, both countries gain from a trade liberalization. Unlike in the 0-1 case, welfare levels do not equalize due to consumer-responses along the intensive margin. Moreover, there is a striking difference in welfare levels emerging in a world with and without arbitrage. When the arbitrage channel is shut down, the welfare gap between the rich and the poor country is relatively small ($U^P/U^R > 0.76$ in all columns of Table 1). In contrast, when arbitrage is at work, the welfare gap is substantially higher ($U^P/U^R < 0.41$ in all columns of Table 1). Hence, our example suggests that disregarding arbitrage might lead to potentially large biases in estimating the welfare effects of trade liberalizations.
6 Empirical Evidence

We now examine whether empirical evidence is consistent with the arbitrage hypothesis. First, we discuss U.S. firm-level evidence, drawing on a recent paper by Bernard, Jensen and Schott (2009). Inter alia, this paper provides evidence on how potential trading partners’ per-capita incomes affect the incidence of exports by U.S. firms. Second, we look at disaggregated export flows (from the U.S. and other large exporting countries) to study how a potential trading partner’s per-capita income is associated to the probability that an exporter sells a given product to this country.

Firm-level Evidence. Bernard et al. (2009) provide firm-level evidence consistent with the arbitrage hypothesis. Their study is based on international trade transactions data (covering the universe of shipments of goods into and out of the U.S.). These transactions data are linked to firm-level data from the longitudinal business data base (LBD) for the years 1993 and 2000. Among other evidence, Bernard et al. (2009) provide evidence on the relationship between the export probability of U.S. firms and (potential) importers’ per-capita income. We reproduce their findings in Table 2. Four pieces of evidence emerge from this table.

\begin{table}
\centering
\begin{tabular}{|l|l|l|l|}
\hline
\textbf{Table 2} &  &  &  \\
\hline
First, Bernard et al. (2009) find that U.S. exporters are more likely to export to rich countries than to poor countries. In 1993, 88.3 percent of U.S. exporters sold their product in at least one high-income country, while only 5.2 percent of U.S. exporters sold their product in at least one low-income country. The probabilities of exporting to lower-middle and higher-middle income countries are in between, with 20.5 percent and 21.4 percent, respectively. (China was considered a lower-middle income country in 1993.) The export probabilities to low-income, lower-middle income, and upper-middle income countries increased by 2000, while the export probability to high-income countries remained constant. This descriptive evidence is broadly consistent with the predictions of our many-country model. This model predicts that all rich-country firms export their product to other rich countries, while the export probability to poor countries is strictly less than 100 percent.

The second piece of evidence in Table 2 shows that U.S. exporters to low-income countries tend to be large, while exporters to high-income countries tend to be much smaller. In 1993, U.S. exporters to at least one low-income country employed, on average, 1,863 workers, while exporters to at least one high-income country employed, on average, 293 workers.

\end{table}

\footnote{Bernard et al.’s (2009) study is a descriptive exercise to better understand the trade patterns of U.S. enterprises that are internationally active. They look at various other dimensions of trade (U.S. firms that import, firms that both import and export, the activities of multinational firms versus domestic firms which allows them to study the importance of intra-firm trade versus arm’s-length trade etc.). Our discussion is confined only to U.S. firms, that export because this evidence corresponds to the predictions of our model.}

\footnote{Our Table 1 reproduces the relevant information from Bernard et al.’s (2009) Table 14.8.}

\footnote{The classification into low income, lower-middle income, upper-middle income, and high income follows that of the World Bank.}
corresponding numbers for 2000 are 1,480 and 303, respectively. This evidence is consistent with our many-country framework in which rich-country exporters that sell to poor countries produce for the world market, while rich-country exporters that sell to other rich countries consist of both producers selling exclusively on northern markets and world market producers. Since the former firms are smaller than the latter, our model predicts that exporters to other rich countries are, on average, smaller than exporters to poor countries.

A third (and related) point suggests that U.S. exporters to poor countries sell only a fraction of their output to poor countries, while a substantial share of exports from these firms goes to richer countries. In 1993, only 1.0 percent of total U.S. exports went to poor countries. However, the fraction of all U.S. firms that export to at least one poor country is as high as 5.2 percent and these firms are disproportionately large. This suggests that the value of world exports by these firms will exceed 5.2 percent. This is consistent with our model, which predicts that exporters to poor countries also serve the markets of richer countries.

Bernard et al. (2009) provide a fourth piece of evidence which suggests that a non-negligible fraction of U.S. exporters to rich countries sell their products exclusively to other rich countries and do not sell at all to poorer countries. Column 1 of Table 2 shows that 5.2 percent of U.S. exporters sold their products to at least one low income country in 1993; the respective numbers for lower-middle and higher-middle income countries are 20.5 and 21.4 percent. The result that a large fraction of U.S. firms export exclusively to other high-income countries is consistent with the predictions of the arbitrage model.

Evidence from Disaggregated Trade Flows. While U.S. firm-level evidence is broadly consistent with the predictions of our model, the correlation between export probabilities and per-capita incomes could be spurious, simply reflecting country-group differences in aggregate GDP. To understand the impact of per-capita incomes, it is important to look at per-capita income effects, holding aggregate GDP constant.

We therefore go one step further and look at disaggregate U.S. bilateral export flows. We analyze the following baseline regression

\[ D(i, k) = \alpha_0 + \alpha_1 \ln y(k) + \alpha_2 \ln GDP(k) + X(i, k)\beta + \phi(i) + e(i, k), \]

where \( D(i, k) \) indicates whether the U.S. exports product \( i \) to country \( k \), \( y(k) \) denotes per-capita income of country \( k \), \( GDP(k) \) is aggregate GDP of country \( k \), \( \phi(i) \) is a product-fixed effect, and \( e(i, k) \) is an error term. \( X(i, k) \) is a vector of controls.\(^{24}\)

\(^{22}\)Strictly speaking, this is true if we are willing to assume that larger firms tend to export at least as large a fraction of their output as smaller firms, an assumption which is in line with empirical evidence.

\(^{23}\)Hence at least 52.9 (=100-5.2-20.5-21.4) of all U.S. exporters sold their products exclusively to other rich countries. As many exporter to low-income countries are likely to have sold their products also to lower-middle income and/or upper-middle income countries, the share of firms exporting only to other rich countries is actually much larger than 52.9 percent.

\(^{24}\)The regression presented below include the following controls: log of distance between exporter’s and importer’s capital, dummy for a common border, dummy for importer being an island, dummy for importer being landlocked, dummy for importer and exporter ever having had colonial ties, dummy for currency union between importer and exporter, dummy for importer and exporter sharing a common legal system, dummy for
If arbitrage is relevant, export probabilities depend on per-income of the potential destination, i.e. we should have $\alpha_1 > 0$. This is different from the homothetic model, where the arbitrage channel is not at work. Since only aggregate GDP matters under homotheticity, we should have $\alpha_1 = 0$. Put differently, when $\alpha_1 > 0$, per-capita income has a stronger impact on export probabilities than does population size. To see this, let $Pop(k)$ be country $k$’s population size, so that $\ln Pop(k) = \ln GDP(k) - \ln y(k)$. This lets us write $\alpha_1 \ln y(k) + \alpha_2 \ln GDP(k) = (\alpha_1 + \alpha_2) \ln y(k) + \alpha_2 \ln Pop(k)$. Hence $\alpha_1 > 0$ means that results are consistent with the prediction that per-income is more important than population size in determining whether or not a bilateral export flow exists.

We use UN Comtrade data complied by Gaulier and Zignago (2010) containing yearly unidirected bilateral trade flows at the 6-digit-level of the Harmonized System (1992) for the year 2007. We observe 5,018 product categories at the 6-digit level. We look only at consumer goods (according the BEC classification). This leaves us with 1,263 product categories from which we exclude those 11 categories the U.S. did not export in 2007. Our data set includes 135 potential export destinations. Information on per-capita incomes (2005 PPP-adjusted I$) and population sizes are taken from Heston et al. (2006). We exclude all bilateral trade flows with negative quantities and set $D(i, k) = 0$ when the observed quantity falls short of US$ 2,000. We end up with 169,020 potential export flows (1,252 products $\times$ 135 potential importers). 39.1 percent of these potential export flows actually materialized in 2007.

Table 3 shows the results from the empirical model above. The baseline regression is shown in column 1. The coefficient of log per-capita income is 0.085 and highly significant and compares to a coefficient 0.064 of aggregate GDP. In columns 2-4, we show that these results remain unchanged when we exclude small importers; and when we look at a higher levels of aggregation (HS4 and HS2). (We further show in Appendix H that similar results hold in each single year from 1997 to 2007, with coefficients that are very similar to those of Table 2).25

Looking for broader support for the arbitrage hypothesis, Table 4 performs regressions similar to those in Table 3 for large consumer-goods exporters. We consider all countries whose consumer-goods exports exceeded 50 billion US$ in 2007 (China, Germany, USA, France, Japan, Italy, UK, Spain, Netherlands, Belgium-Luxembourg, Canada, Mexico, Switzerland, Korea). Since the arbitrage hypothesis only applies to export flows from a rich exporter to a poorer importer, the evidence displayed in Table 4 is based on export-incidence observations.

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25 The results of Table 2 (as well as Tables 3 and 4) are in line with evidence in Baldwin and Harrigan (2011) and Bernasconi and Würgler (2012). Baldwin and Harrigan (2011) find a significant impact of income-per-worker on export zeros in 10-digit U.S. trade data. In contrast to our analysis, they treat the income-per-worker variable as an additional demand control which is neither important in their theory nor explored systematically in their empirical analysis. The contribution of Bernasconi and Würgler (2012) is primarily empirical and looks at the impact of per-capita incomes and population sizes on the various (intensive, extensive, quality) margins of international trade. Our analysis differs from theirs by its focus on arbitrage.
where the potential importer is poorer than the exporter under consideration. All regressions in Table 4 include exporter fixed-effects and the set of controls listed in the above footnote.

The evidence of Table 4 confirms that destinations’ per-capita incomes significantly affect export probabilities also when we include other export flows by other large exporting nations in our sample. The per-capita income coefficient is significantly positive and similar in size to that in Table 3 where we confine the analysis to U.S. observations only. Moreover, also in this broader sample of (potential) bilateral export flows, the size of the per-capita income coefficient is of a similar order of magnitude as the coefficient of aggregate GDP. This is consistent with the prediction that per-capita income effects are stronger than population-size effects. Columns 2-4 show that the estimates are robust and hold in different samples and across different aggregation levels. (We rerun the baseline regression for each of the 14 countries. It turns out that the point estimates are positive and significant in all countries, except for Mexico. The results are shown in Table A.2 in the Appendix.)

Table 4

7 Conclusions

This paper studies a model of international trade in which an importer’s per-capita income is a primary determinant of export zeros. In particular, we show northern firms’ export probability to a poorer country is the lower the lower the per-capita income of the potential trading partner. Hence our paper emphasizes the role of demand in explaining the extensive margin of trade. This is complementary to standard heterogenous-firm models, emphasizing the role of supply.

The key insight of our analysis is that “export zeros” arise from a threat of international arbitrage. Globally active firms cannot simultaneously set low prices in the South and high prices in the North because this triggers arbitrage opportunities. Northern firms have basically two options to avoid arbitrage: (i) set a sufficiently low price in the North that eliminates arbitrage incentives; or (ii) abstain from exporting to the South to eliminate arbitrage opportunities. These two options involve a trade-off between market size and price: firms that export globally have a large market but need to charge a low price; firms that sell exclusively to northern markets can charge a high price but have a small market. The equilibrium of our model is characterized by Linder-effects, a situation where similarity in per-capita incomes increases trade intensity between two countries. It also generates interesting welfare effects. While rich countries always gain from a trade liberalization, poor countries may lose because lower trade costs tighten the arbitrage constraint. A tighter arbitrage constraint induces more northern firms not to export, thus keeping southern prices high while reducing the menu of goods supplied by northern producers. This harms welfare of households in the South.

The patterns of trade predicted by our model is in line with empirical evidence both in firm-level data and in disaggregate trade data. Firm-level data show that almost all U.S. exporters sell their products to other rich countries, while export probabilities to middle-income and
low-income countries are lower and strongly decreasing in destinations’ per-capita incomes. As predicted by our model, U.S. exporters to poor countries are large, while U.S. exporters to rich countries are small. Disaggregate trade data suggest that the a rich country’s export probability of a HS6-digit product indeed decreases significantly in the destination’s per-capita income. Moreover, a destination’s per-capita income has a much stronger impact on export probabilities than population size. These results are very robust and hold for all large exporting nations.

Our basic model is based on consumption indivisibilities and allows consumer choices only along the extensive margin of consumption. This yields a very tractable model that allows for closed-form solutions. We show, however, that the predictions of the simple model carry over to more general preferences that allow consumers to respond also along the intensive margin. Numerical examples show that arbitrage may arise over a range of parameter constellations. Moreover, we show by numerical examples that the welfare effects of the arbitrage channel may be large. Hence disregarding arbitrage may lead to misleading conclusions concerning the welfare gains from liberalized trade between rich and poor countries.

Our analysis has abstracted from a number of relevant phenomena. In particular, we did not consider a situation where per-capita income differences generate quality differentiation. We also abstracted from any within-country inequality. However these dimensions become potentially important in explaining trade patterns and trade gains when consumers have non-homothetic preferences. (For a recent paper along these lines, see Fajgelbaum et al., 2011.) From an empirical point of view, the challenge is to appropriately disentangle supply and demand effects. This seems particularly relevant for a better understanding of how the emerging markets of China and India (and their rapidly growing per-capita incomes) affect trade patterns and the international division of labor. We think these are interesting topics for future research.
References


A Proof of Proposition 1

Part b). This follows from calculating the derivatives of $\phi$ with respect to $L^P$

$$\frac{\partial \phi}{\partial L^P} = \frac{2L^R P^R \omega P^P}{(L^R P^R + \omega L^P P^P)^2} - \frac{4L^R P^R \omega L^P P^P}{(L^R P^R + \omega L^P P^P)^3} \omega P^P = \frac{\phi}{L^P} \left( 1 - \frac{2\omega L^P P^P}{L^R P^R + \omega L^P P^P} \right),$$

and with respect to $P^P$

$$\frac{\partial \phi}{\partial P^P} = \frac{\phi \left( 1 + \frac{\omega}{P^P} \right)}{P^P} \left( 1 - \frac{2\omega L^P P^P}{L^R P^R + \omega L^P P^P} \right),$$

where $\frac{\partial \omega}{\partial P^P} \frac{P^P}{\omega} > 0$.

Part c). A given increase in $L^P P^P$ has a stronger impact on $\phi$ if it comes from $P^P$ rather than from $L^P$ if $\partial \phi / \partial \log L^P = (\partial \phi / \partial L^P) L^P < \partial \phi / \partial \log P^P = (\partial \phi / \partial P^P) P^P$. This is true since $\frac{\partial \omega}{\partial P^P} \frac{P^P}{\omega} > 0$.

Part d). This follows from the derivative of $\phi$ with respect to $\tau$

$$\frac{\partial \phi}{\partial \tau} = \frac{\phi \partial \omega}{\omega} \left( 1 - \frac{2\omega L^P P^P}{L^R P^R + \omega L^P P^P} \right).$$

B Proof of Proposition 2

Part b) follows from the derivatives of $\phi$ with respect to $L^P$ and $P^P$. It is straightforward to see that $\partial \phi / \partial L^P > 0$. To calculate $\partial \phi / \partial P^P$ we need to take into account that $\omega$ depends on $P^P$

$$\frac{\partial \phi}{\partial P^P} = \frac{-1}{P^R (\tau + P^P / P^R)} \phi + \frac{1 + \frac{\partial \omega}{\partial P^P} \frac{P^P}{\omega}}{L^R P^R + \omega L^P P^P} \phi,$$

hence an increase in $P^P$ increases trade intensity $\phi$ when $P^P$ is small and vice versa.

Part c). We need to show that the volume of trade increases with $P^P$ less than proportionally. The argument in the text was made without considering that $P^P$ increases $\omega$. It remains to show that, taking account of the impact of $P^P$ on $\omega$, an increase in $P^P$ reduces per-capita imports. We have $\partial p^P N^R_f / \partial P^P = \text{sign}(\partial \log(p^P N^R_f) / \partial P^P) < 0$. Calculating $p^P N^R_f$, taking logs and the derivative with respect to $P^P$ reveals that $\partial p^P N^R_f / \partial P^P < 0$ if

$$- \frac{1}{\tau P^R + P^P} + \frac{\tau}{a F + P^R + \tau P^P} - \frac{1}{a F + \tau P^R + P^P} < 0.$$

Multiplying by $a F + P^R + \tau P^P$ yields

$$- a F + P^R + \tau P^P + \frac{\tau(a F + P^R + \tau P^P)}{a F + P^R + \tau P^P} - a F + \frac{P^R + \tau P^P}{a F + \tau P^R + P^P} < 0 \iff - p^P + \frac{\tau - \omega}{a} < 0,$$

which holds true because $p^P > \tau / a$.

Part d). We note that $\text{sign}(\partial \phi / \partial \tau) = \text{sign}(\partial \log \phi / \partial \tau)$. Taking logs of the expression for $\phi$
and the derivative with respect to $\tau$ yields

$$\text{sign}(\partial \phi / \partial \tau) = \text{sign} \left( \frac{1}{\tau} - \frac{1}{\tau + \frac{p_R}{p_P}} + \frac{\omega' (\tau)}{\omega (\tau)} \cdot \frac{\omega (\tau) L_P}{L_R} + \frac{p_R}{p_P} \right) > 0,$$

which, using $\omega (\tau) L_P / L_R < \tau$ and $\omega (\tau) > -1$, implies

$$\text{sign}(\partial \phi / \partial \tau) = \text{sign} \left[ \left( 1 + \frac{\omega' (\tau) \tau}{\omega (\tau)} \right) \left( 1 - \tau + \frac{p_R}{p_P} \right) \right] > 0.$$

### C Proof of Proposition 3

**Part a.** In an arbitrage equilibrium we have $p^P = (a F + \mathcal{P}^R + \tau \mathcal{P}^P) / (a \tau \mathcal{P}^R + a \mathcal{P}^P)$. Country-$R$ firms export if $p^P \geq \tau / a$ or, equivalently, $(a F + \mathcal{P}^R + \tau \mathcal{P}^P) (a \mathcal{P}^R + \mathcal{P}^P)^{-1} \geq \tau$. Solving that latter equation for $\tau$ yields the trade condition. (Notice that, if the trade conditions holds for country-$R$ firms, it also holds for country-$P$ firms, as we have $p^R = \tau p^P > p^P$.)

**Part b.** Under full trade we have $p^P = \omega L_P (a F + \mathcal{P}^R + \tau \mathcal{P}^P) (a \mathcal{P}^R L_R + a \omega \mathcal{P}^R L_P)^{-1} \geq \tau / a$ or $(\omega L_P / L_R) (a F / \mathcal{P}^R + 1) \geq \tau$. But since full trade occurs only when $\omega L_P / L_R \geq 1 / \tau$, the trade condition follows.

### D Two regions: $n$ rich and $m$ poor countries

In an arbitrage equilibrium, the price of globally traded goods is $p^R_T = \tau p^P$ in the North. Zero profit constraints of globally traded goods are

$$p^P m \mathcal{P}^P + \tau p^P n \mathcal{P}^R = \left( F + \frac{\mathcal{P}^i + \tau \mathcal{P}^{-i}}{a} \right) W^i, \quad i = R, P,$$

where $\mathcal{P}^{-R} = (n - 1) \mathcal{P}^R + m \mathcal{P}^P$ and $\mathcal{P}^{-P} = n \mathcal{P}^R + (m - 1) \mathcal{P}^P$. The prices of globally traded goods can be directly calculated $p^P = (a F + \mathcal{P}^R + \tau \mathcal{P}^{-R}) / (a m \mathcal{P}^P + a \tau n \mathcal{P}^R)$ and $p^R = \tau p^P$. The zero profit conditions for goods exclusively traded in the North are

$$p^R_N \mathcal{P}^R = \left( F + \frac{\mathcal{P}^R + \tau (n - 1) \mathcal{P}^R}{a} \right) W^R,$$

and the price of these goods follows immediately $p^R_N = W^R (a F + \mathcal{P}^R + \tau (n - 1) \mathcal{P}^R) / (a n \mathcal{P}^R)$. From the zero profit conditions of globally traded goods we can calculate relative wages between North and South

$$\omega \equiv \frac{W^P}{W^R} = \frac{a F + \tau \mathcal{P}^{-R} + \mathcal{P}^R}{a F + \mathcal{P}^P + \tau \mathcal{P}^{-P}}.$$
The resource constraints are

\[ L^P P^P = N^P \left( F + \frac{P^P + \tau P^-}{a} \right) \] for a poor country, and

\[ L^R R^R = N^R \left( F + \frac{P^R + \tau P^-}{a} \right) + N^R \left( F + \frac{P^R + \tau (n-1)P^R}{a} \right) \] for a rich country.

Each \( R \)-country imports all goods produced worldwide, while each \( P \)-country imports only a subset of these goods. Hence the aggregate trade balance between the North and the South has to be balanced in equilibrium.\(^{26}\) This implies

\[ \tau N^P P^R = N^R P^P. \]

From the resource constraints and the trade balance condition we get closed-form solutions for \( N_P, N^T_R, \) and \( N^R_N \). This gives welfare of rich and poor households

\[ U^R(\tau) = m N^P + n N^R + n N^R = \frac{a L^P \left( m P^P + \tau n P^R \right)}{a F + P^P + \tau P^- P} + \frac{a \left( L^R - \omega L^P \right) n P^R}{a F + P^R + \tau (n-1)P^R}, \]

and

\[ U^P(\tau) = m N^P + n N^R = \frac{a L^P \left( m P^P + \tau n P^R \right)}{a F + P^P + \tau P^- P}. \]

We see that that \( \partial U^R(\tau) / \partial \tau < 0 \) and \( \partial U^S(\tau) / \partial \tau \leq 0 \) when \( a F < (m-1)P^P(1+mP^P/(nP^R)) \).

It also follows that \( \partial U^R(\tau) / \partial \tau < \partial U^S(\tau) / \partial \tau \), i.e. a trade liberalization benefits the rich country more.

Finally, let us calculate trade intensity. The value of North-North trade is given by

\[ 2(n-1) \left( p^R_N N^R_N + p^R_T N^R_T \right) n P^R = 2(n-1) \left( L^R - \omega L^P \frac{m P^P}{m P^P + \tau n P^R} \right) P^R. \]

The value of South-South trade is

\[ 2(m-1) P^P N_P m P^P = 2(m-1) \frac{m P^P}{m P^P + \tau n P^R} \omega L^P P^P \]

and the value of North-South trade is

\[ 2 m P^P N_P n P^R = 2 m \frac{n \tau P^R}{m P^P + \tau n P^R} \omega L^P P^P. \]

\(^{26}\)Due to the symmetry of our set-up, the volume of bilateral trade is undetermined. One of the Northern countries could produce predominantly (or exclusively) goods that are consumed only in the North, while the other Northern country produces mainly (or exclusively) goods that are consumed worldwide. In that case, the first Northern country runs a trade surplus with the other Northern country and a trade deficit with both Southern countries taken together. Such trade imbalances cannot occur between the Southern countries, since each Southern country consumes all goods the other Southern country produce, meaning that the South-South trade flows are of the same magnitude in either direction. However, each Southern country may run a surplus with one of the Northern countries that is balanced by a deficit with the other Northern country. Notice further that all bilateral trade flows are equalized in a full trade equilibrium since all households in each country consume all goods that are produced worldwide.
This allows us to calculate trade intensity $\phi$, the value of world trade relative to world GDP as

$$\phi = 2 \frac{(n - 1)\left(p_N^R N_R^R + p_N^R N_F^R\right) n P_R + (m - 1) p_P^P N_P^P m P_P + m p_P^P N_P^P n P_R}{n L_R^R P_R + m \omega L_P^P P_P},$$

which, using the above formulas, can be expressed as

$$\phi = 2 \frac{m \omega L_P^P P_P}{n L_R^R P_R + m \omega L_P^P P_P} \left(\frac{(m - 1) P_P + n \tau P_R}{m P_P + n \tau P_R}\right) + 2 \frac{(n - 1) \left(L_R - \tau \omega L_P^P \frac{m P_P}{m P_P + \tau n P_R}\right) P_R}{n L_R^R P_R + m \omega L_P^P P_P}.$$

When $m = 1$ and $n = 1$ we get

$$\phi = 2 \frac{\omega L_P^P P_P}{L_R^R P_R + \omega L_P^P P_P} \tau P_R + \frac{\tau P_R}{n L_R^R P_R + m \omega L_P^P P_P}.$$

Unlike in the arbitrage equilibrium of the two-county case, trade intensity may decrease in $\tau$. This is when a reduction in $\tau$ increases South-South and North-North trade more strongly than it reduces North-South trade.

### E Equilibrium with general preferences

**The arbitrage equilibrium.** Here we state the full system of equations that characterize an arbitrage equilibrium with non-traded goods. *Households* choose consumption levels to maximize utility. This implies marginal rates of substitution

$$\frac{v'(c_R^R)}{v'(c_F^R)} = \frac{p_R^R}{p_F^R}, \quad \frac{v'(c_P^P)}{v'(c_F^P)} = \frac{p_P^P}{p_F^P}, \quad \frac{v'(c_R^N)}{v'(c_F^N)} = \frac{p_R^N}{p_F^N}.$$

*Firms* set prices to maximize profits. Firms that sell exclusively on the home market set the unconstrained monopoly price

$$p_N^R = \mu(c_R^N) \frac{1}{a}.$$

Exporting firms set prices to avoid arbitrage

$$p_R^P = \tau p_P^P, \quad p_R^R = \tau p_R^P.$$
which leads to first-order conditions\textsuperscript{27}

\[
\frac{\tau p^p_P - \omega/a}{p^p_P} \left( -\frac{v'(c^p_P)}{v''(c^p_P)} \right) + \frac{p^p_P - \omega/a}{p^p_P} \left( -\frac{v'(c^p_P)}{v''(c^p_P)} \right) \frac{P^p_P}{\tau P_R} = \tau c^p_R + c^p_R P^p_P,
\]

\[
\frac{\tau p^R_R - 1/a}{\tau p^R_R} \left( -\frac{v'(c^R_R)}{v''(c^R_R)} \right) + \frac{p^R_R - \tau/a}{p^R_R} \left( -\frac{v'(c^R_R)}{v''(c^R_R)} \right) \frac{P^R_R}{\tau P_R} = c^R_R + \tau c^R_R P^R_R.
\]

The resource constraints are

\[
L^p = N_P (F + P^R R^p c^R_R / a + P^p P^p p^p_P / a),
\]

\[
L^R = N^T_R (F + P^R c^R_R / a + P^p P^p p^p_P / a) + N^N_R (F + P^R c^N_R / a),
\]

the trade balance condition is

\[
p^R_R N^T_R P^p c^p_R = p^R_R N_P P^R R^p R^p c^p_P,
\]

and the zero-profit conditions are

\[
P^p c^p_P (p^p_P - \omega/a) + P^R R^p c^R_R (p^R_R - \tau/a) = \omega F.
\]

\[
P^R c^R_R (p^R_R - 1/a) + P^p P^p c^p_R (p^p_P - \tau/a) = F,
\]

\[
P^R c^N_R (p^N_R - 1/a) = F.
\]


**Full trade equilibria.** As mentioned in the main text, a binding arbitrage constraint is a necessary though not sufficient condition for an arbitrage equilibrium with non-traded goods since consumers can now respond also along the intensive margin. There are three types of full trade equilibria: (i) both \(P\)- and \(R\)-firms are price-constrained; (ii) \(P\)-firms are price-constrained while \(R\)-firms set the monopoly price; and (iii) firms in both countries set the monopoly price.\textsuperscript{28}

ad (i). When both firms are price-constrained but all goods are traded, all equations are identical except that \(N^N_R = c^N_R = 0\) and \(p^N_R\) do not exist. The system reduces to 11 equations.

ad (ii). When \(P\)-firms are price constrained but \(R\)-firms are not, we have \(p^R_R = \mu(p^R_R) / a\)

\textsuperscript{27}These conditions derive from maximizing the profit functions for country-\(P\) and country-\(R\) producers, i.e. \(P^p c^p_P (p^p_P - \omega/a) + P^R c^R_R (p^R_R - \tau/a)\) and \(P^R c^R_R (p^R_R - 1/a) + P^p P^p c^p_R (p^p_P - \tau/a)\), subject to the above arbitrage constraints. Moreover, we use the fact that households’ demand functions derive from \(v(c) = \lambda p\) which implies \(\partial c / \partial p = (1/p) v'(c) / v''(c)\).

\textsuperscript{28}Notice that the (unconstrained) price gap of country-\(P\) firms between market \(P\) and market \(R\) is higher than the corresponding price gap for country-\(R\) firms. This is because country-\(P\) firms have low (high) costs and low (high) demand on the home (foreign) market. This is different from the situation of country-\(R\) firms. They have high (low) costs and low (high) demand on the foreign (home) market. This implies that country-\(P\) firms get price-constrained first, and an equilibrium, where country-\(R\) firms are price-constrained - but country-\(P\) firms are not - cannot exist.
and \( p_R^P = \mu(c_R^P) \tau/a \) while \( p_P^P \) and \( p_P^R \) are still determined as above.

ad (iii). When firms in both countries are unconstrained, also \( P \)-firms set the monopoly price \( p_P^P = \mu(c_P^P) \omega/a \) and \( p_P^R = \mu(c_P^R) \omega \tau/a \).

\[ \]

**F Proof of Proposition 7**

We determine the autarky equilibrium and ask under which conditions an entrepreneur has incentives to sell his products abroad. Setting \( W = 1 \), optimal monopolistic pricing implies \( p = \mu(c)/a \). With free entry, profits \( \mathcal{P}^R(p^R_a - 1/a)c_a^R \) must equal set up costs \( F \)

\[ aF/\mathcal{P}^R = (\mu(c_a^R) - 1) c_a^R \]

The equilibrium is symmetric for all firms, hence the resource constraint reads

\[ L^R = N_a^R (F + \mathcal{P}^R c_a^R/a) \]

Solving for \( c_a^R \) and \( N_a^R \), we see that \( c_a^R \) does not depend on \( L^R \). Hence when the two countries differ only in \( L^i \) but have equal populations, intensive consumption levels under autarky are identical between the two countries, \( c_a^R = c_a^P \). Selling one marginal unit abroad at price \( v'(0)/\lambda^P_a \), allows the purchase of \( v'(0)/(\lambda^P_a p_a^P) \) foreign goods. Since \( \lambda^P_a = v'(c_a^P)/p_a^P \) and \( c_a^R = c_a^P \) this is equal to \( v'(0)/v'(c_a^R) > 1 \). Reselling this (new) product at home, yields a price \( v'(0)p_a^R/v'(c_a^R) \) minus trade costs. Hence, this strategy is profitable if \( [v'(0)p_a^R/v'(c_a^R)] \cdot [v'(0)/v'(c_a^R)] > \tau^2 \). Expressing \( p_a^R \) in terms of \( c_a^R \), we get the condition of the Proposition.
Table 1: Consumption and welfare with general preferences (HARA, $\sigma = 4$)

<table>
<thead>
<tr>
<th></th>
<th>$\tau = 1.2$</th>
<th>$\tau = 1.4$</th>
<th>$\tau = 1.6$</th>
<th>$\tau = 1.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Extensive margin</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N^P$</td>
<td>1.651</td>
<td>1.627</td>
<td>1.603</td>
<td>1.580</td>
</tr>
<tr>
<td>$N^R_T$</td>
<td>3.741</td>
<td>4.682</td>
<td>5.800</td>
<td>7.166</td>
</tr>
<tr>
<td>$N^R_N$</td>
<td>5.026</td>
<td>3.990</td>
<td>2.769</td>
<td>1.294</td>
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<tr>
<td><strong>Share of traded goods</strong></td>
<td>0.427</td>
<td>0.540</td>
<td>0.677</td>
<td>0.847</td>
</tr>
<tr>
<td><strong>Intensive margin</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c^P_F$</td>
<td>0.131</td>
<td>0.123</td>
<td>0.115</td>
<td>0.110</td>
</tr>
<tr>
<td>$c^P_R$</td>
<td>0.128</td>
<td>0.115</td>
<td>0.102</td>
<td>0.090</td>
</tr>
<tr>
<td>$c^R_F$</td>
<td>0.243</td>
<td>0.240</td>
<td>0.237</td>
<td>0.234</td>
</tr>
<tr>
<td>$c^R_R$</td>
<td>0.242</td>
<td>0.239</td>
<td>0.235</td>
<td>0.232</td>
</tr>
<tr>
<td>$c^R_N$</td>
<td>0.196</td>
<td>0.196</td>
<td>0.196</td>
<td>0.196</td>
</tr>
<tr>
<td><strong>Welfare (with arbitrage)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U^P$</td>
<td>0.641</td>
<td>0.688</td>
<td>0.733</td>
<td>0.776</td>
</tr>
<tr>
<td>$U^R$</td>
<td>1.923</td>
<td>1.919</td>
<td>1.914</td>
<td>1.908</td>
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<tr>
<td><strong>Welfare (without arbitrage)</strong></td>
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<td></td>
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<tr>
<td>$U^P$</td>
<td>1.502</td>
<td>1.443</td>
<td>1.385</td>
<td>1.272</td>
</tr>
<tr>
<td>$U^R$</td>
<td>1.705</td>
<td>1.685</td>
<td>1.669</td>
<td>1.654</td>
</tr>
</tbody>
</table>

Notes: Simulations based on the following parameter values: HARA preferences with $\sigma = 4$, $F = 4$, $\gamma = 1$, $a = 2$. All entries in the table (except for the last two rows) are solutions to the model described Appendix E under the chosen parameter values. "Welfare without arbitrage" calculates the welfare level of an alternative model in which arbitrage is ruled out (parallel imports prohibited).
<table>
<thead>
<tr>
<th>Income level of destination country</th>
<th>Share of exporters (%)</th>
<th>Employment per firm</th>
<th>Export share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>5.2 7.0</td>
<td>1,863 1,480</td>
<td>1.0 1.0</td>
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<tr>
<td>Lower-middle</td>
<td>20.5 22.7</td>
<td>764 660</td>
<td>10.7 11.1</td>
</tr>
<tr>
<td>Upper-middle</td>
<td>21.4 28.6</td>
<td>766 591</td>
<td>18.9 19.6</td>
</tr>
<tr>
<td>Upper</td>
<td>88.3 85.6</td>
<td>293 303</td>
<td>65.1 68.3</td>
</tr>
</tbody>
</table>

Notes: This table reproduces the relevant parts of Table 14.8 in Bernard et al. (2009). Income levels of U.S. trading partners are according to the 2003 World Bank Income Group classification. The first four columns are based on U.S. firms that export to at least one country in the noted country-income groups. The sums of “Share of exporters” do not equal 100 because firms may appear in more than one row if they trade with countries of more than one type. Columns 5 and 6 report the “Export shares” and do sum to 100 because they sum export flows at the firm-destination country level.
Table 3: Extensive margin of exports, U.S., 2007

<table>
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<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Pop&gt;1million</td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td></td>
<td>HS6</td>
<td>HS6</td>
<td>HS4</td>
<td>HS2</td>
</tr>
<tr>
<td>Mean of dependent variable</td>
<td>0.391</td>
<td>0.399</td>
<td>0.576</td>
<td>0.731</td>
</tr>
<tr>
<td>Log of importer GDP per-capita</td>
<td>0.085***</td>
<td>0.076***</td>
<td>0.087***</td>
<td>0.058***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Log of importer GDP</td>
<td>0.064***</td>
<td>0.079***</td>
<td>0.070***</td>
<td>0.056***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Trade cost indicators</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>HS fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.424</td>
<td>0.440</td>
<td>0.430</td>
<td>0.417</td>
</tr>
<tr>
<td>N</td>
<td>169,020</td>
<td>147,736</td>
<td>42,255</td>
<td>9,045</td>
</tr>
</tbody>
</table>

Notes: Estimates based on a linear probability model, *, **, *** denote statistical significance at the 10%, 5%, 1% level, respectively. Standard errors are clustered on importer level. Sample includes all potential export flows to countries with GDP per capita lower than the U.S.. Year is 2007.
| Table 4: Extensive margin of exports, 14 largest consumer-goods exporters, 2007 |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| (1) All HS6 | (2) Pop>1million HS6 | (3) All HS4 | (4) All HS2 |
| Mean of dependent variable | 0.226 | 0.239 | 0.362 | 0.540 |
| Log of importer GDP per-capita | 0.055*** | 0.050*** | 0.068*** | 0.065*** |
| | (0.008) | (0.011) | (0.010) | (0.011) |
| | [0.003] | [0.004] | [0.004] | [0.004] |
| Log of importer GDP | 0.046*** | 0.055*** | 0.060*** | 0.062*** |
| | (0.006) | (0.007) | (0.007) | (0.006) |
| | [0.002] | [0.003] | [0.002] | [0.002] |
| Trade cost indicators | Yes | Yes | Yes | Yes |
| Exporter fixed effects | Yes | Yes | Yes | Yes |
| HS fixed effects | Yes | Yes | Yes | Yes |
| Adjusted $R^2$ | 0.353 | 0.365 | 0.405 | 0.460 |
| N | 1,980,017 | 1,728,191 | 508,082 | 109,590 |

Notes: Sample is based on exports from countries with consumer goods exports larger than 50 billion USD in 2007: China, Germany, USA, France, Japan, Italy, UK, Spain, Netherlands, Belgium-Luxembourg, Canada, Mexico, Switzerland, Korea. Estimates are based on a linear probability model, *, **, *** denote statistical significance at the 10%, 5%, 1% level, respectively. Standard errors in parentheses are clustered on importer level; standard errors in brackets are clustered on the importer-exporter pair level. Sample includes all potential export flows to countries with GDP per capita lower than the exporter under consideration. Year is 2007.
## Table A.1: Extensive margin of exports, U.S., 1997-2007

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean of log importer GDP</th>
<th>log importer GDP</th>
<th>Adj $R^2$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dep. var.</td>
<td>per-capita</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>coeff.</td>
<td>std.dev.</td>
<td>coeff.</td>
<td>std.dev.</td>
</tr>
<tr>
<td>1997</td>
<td>0.389</td>
<td>0.103***</td>
<td>0.078***</td>
<td>0.452</td>
</tr>
<tr>
<td>1998</td>
<td>0.387</td>
<td>0.102***</td>
<td>0.075***</td>
<td>0.445</td>
</tr>
<tr>
<td>1999</td>
<td>0.384</td>
<td>0.099***</td>
<td>0.075***</td>
<td>0.440</td>
</tr>
<tr>
<td>2000</td>
<td>0.386</td>
<td>0.094***</td>
<td>0.074***</td>
<td>0.430</td>
</tr>
<tr>
<td>2001</td>
<td>0.382</td>
<td>0.093***</td>
<td>0.074***</td>
<td>0.430</td>
</tr>
<tr>
<td>2002</td>
<td>0.372</td>
<td>0.087***</td>
<td>0.074***</td>
<td>0.419</td>
</tr>
<tr>
<td>2003</td>
<td>0.375</td>
<td>0.090***</td>
<td>0.071***</td>
<td>0.420</td>
</tr>
<tr>
<td>2004</td>
<td>0.378</td>
<td>0.091***</td>
<td>0.070***</td>
<td>0.423</td>
</tr>
<tr>
<td>2005</td>
<td>0.391</td>
<td>0.091***</td>
<td>0.069***</td>
<td>0.424</td>
</tr>
<tr>
<td>2006</td>
<td>0.396</td>
<td>0.092***</td>
<td>0.067***</td>
<td>0.430</td>
</tr>
<tr>
<td>2007</td>
<td>0.391</td>
<td>0.085***</td>
<td>0.064***</td>
<td>0.424</td>
</tr>
</tbody>
</table>

Notes: Estimates based on the same specification as in Table 2, column 2 above. Number of observations vary over time because the number of countries poorer than the U.S. and the number of HS6 consumer goods categories exported by U.S. firms may change over time. Estimates are from a linear probability model, *, **, *** denote statistical significance at the 10%, 5%, 1% level, respectively. Standard errors are clustered on importer level.
<table>
<thead>
<tr>
<th>Country</th>
<th>Mean of log importer GDP</th>
<th>Mean of log importer GDP</th>
<th>Adj $R^2$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dep. var.</td>
<td>per-capita</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium-Lux.</td>
<td>0.196</td>
<td>0.039***</td>
<td>(0.011)</td>
<td>0.034***</td>
</tr>
<tr>
<td>Canada</td>
<td>0.126</td>
<td>0.042***</td>
<td>(0.008)</td>
<td>0.033***</td>
</tr>
<tr>
<td>China</td>
<td>0.370</td>
<td>0.060**</td>
<td>(0.026)</td>
<td>0.068***</td>
</tr>
<tr>
<td>France</td>
<td>0.289</td>
<td>0.071***</td>
<td>(0.015)</td>
<td>0.045***</td>
</tr>
<tr>
<td>Germany</td>
<td>0.286</td>
<td>0.058***</td>
<td>(0.013)</td>
<td>0.059***</td>
</tr>
<tr>
<td>Italy</td>
<td>0.283</td>
<td>0.059***</td>
<td>(0.012)</td>
<td>0.056***</td>
</tr>
<tr>
<td>Japan</td>
<td>0.128</td>
<td>0.026***</td>
<td>(0.008)</td>
<td>0.045***</td>
</tr>
<tr>
<td>Korea Rp (South)</td>
<td>0.110</td>
<td>0.022***</td>
<td>(0.007)</td>
<td>0.043***</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.056</td>
<td>0.010*</td>
<td>(0.006)</td>
<td>0.010***</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.228</td>
<td>0.052***</td>
<td>(0.012)</td>
<td>0.038***</td>
</tr>
<tr>
<td>Spain</td>
<td>0.238</td>
<td>0.061***</td>
<td>(0.012)</td>
<td>0.046***</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.176</td>
<td>0.045***</td>
<td>(0.009)</td>
<td>0.046***</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.246</td>
<td>0.051***</td>
<td>(0.012)</td>
<td>0.060***</td>
</tr>
<tr>
<td>USA</td>
<td>0.391</td>
<td>0.092***</td>
<td>(0.014)</td>
<td>0.060***</td>
</tr>
</tbody>
</table>

Notes: Includes countries with consumer goods exports larger than 50 billion USD in 2007. Estimates are based on a linear probability model, specification is identical to the one in Table 2, column 2. *, **, *** denote statistical significance at the 10%, 5%, 1% level, respectively. Standard errors in parentheses are clustered on importer level. Only potential exports flows to countries with GDP per capita lower than the exporter under consideration are included in the sample. Year is 2007.
Figure 1: Demand function
Figure 2: Full trade and arbitrage equilibrium

\[ \tau = \sqrt{\frac{aF}{P^R}} + 1 \]

\[ L^p \]
Figure 3: Welfare and trade costs

\begin{equation}
U^P, U^R
\end{equation}

\begin{equation}
\sqrt{aF/P^R + 1}
\end{equation}

(b)

(a)
Figure 4: The arbitrage frontier with HARA-preferences

Note: The figure is based on the following parameter values: $F = 1$, $\gamma = 1$, $a = 2$. The various graphs show the arbitrage frontier for alternative values of the HARA-preference parameter $\sigma = 1, 2, 4, 10$. The frontier to the very right shows the arbitrage frontier for 0-1 preferences.