The Effects of Trade and Endogenous Technological Change on the College Wage Gap and Unemployment

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This version 01.03.2013

Abstract

Skill biased technological change and international trade have emerged as alternative explanations for the increasing college wage gap in the US since the 1980s. Most models neglect the effect of international trade on technological progress and hence understate the effects of trade on wage inequality. We develop a model in which bilateral trade increases the wage gap in two trading countries simultaneously, and thus is consistent with the global increase of inequality during the era of trade liberalization. Furthermore, in the model the wage gap increases due to a disproportional change of high skilled wages relative to low skilled wages, which as well is consistent with empirical findings. Lastly, we match the behavior of unemployment rates of high and low skilled workers in the US. For a calibrated model we show that trade opening of the US to Japan in the 1980s increased the wage gap in the US by about 14% and is consistent with the decreasing unemployment rates in the US, while at the same time the relative price of high skilled to low skilled intensive goods increased only modestly.

JEL classification : F16, J2, J31, O31

Key Words : Trade liberalization, R&D, Endogenous technological change, Skill biased technological change, Search unemployment, College wage gap

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I would like to thank Carolina Silva, Mariano Bosch, Gonzague Vannoorenberghe, Tobias Seidel and Peter Egger for their valuable comments during the development of this paper. Financial support by the Swiss National Science Foundation (SNSF) is gratefully acknowledged.

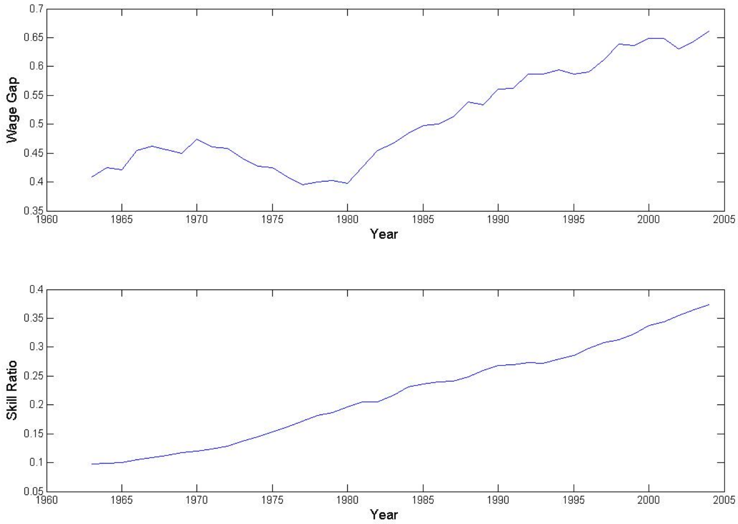

Figure 1: College wage gap (Autor, Katz and Kearny, 2008) and skill ratio - supply of workers with at least a college degree relative to the supply of workers without a college degree (US Census Bureau, Current Population Survey, 2010).

1 Introduction

A large literature documents a rise of wage differentials by education, by occupation, by age and by experience groups in the United States (US) since the late 1970s. While the college wage gap increased since the 1980s, the unemployment rate declined during the same period.[1] These two developments coincide with an increase of the share of skilled workers in the population, technological progress as well as US trade volume, see Figures 1 and 2.

The two most prominent explanations for the rising college wage gap are skill biased technological change (SBTC) and trade liberalization. In the 1990s the SBTC explanation was dominant, especially due to the analysis of Katz and Murphy (1992) and Autor et al. (2008). Their analysis led to many studies about skill biased technological change, especially in the form of information technology (IT), see Krueger (1993) and Jorgenson (2001). Still, Card and DiNardo (2002) critically remarked that, whenever the changes in the relative wages are not fully explained by changes in the relative skill supply, it is claimed that skill biased technological change has caused the opposing wage development. Hence, SBTC is a residual explanation for the increasing wage gap which is hard to quantify empirically.

[1]Throughout this paper the notion wage gap refers to the relative wage of workers with at least a college degree (high skilled) and workers with only a high school diploma (low skilled), w_h/w_l.

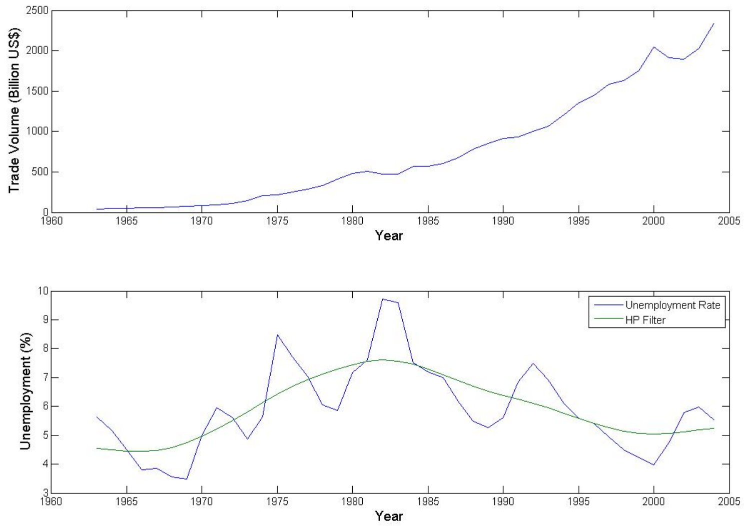

Figure 2: Trade volume (WTO Statistical Database, 2010) and unemployment rate (U.S. Bureau of Labor Statistics, 2010).

On the other hand, the strong increase of international trade suggests that trade has an important contribution to the increasing inequality in the US. Autor et al. (2013) find that the strong increase of trade with China in since the 1990 negatively effected the labor market outcomes of workers in the manufacturing sector in form of lower wages and lower employment rates. Trade theory would suggest such negative outcomes if countries with very different endowments of (high and low skilled) labor or comparative advantages trade. Most of the US trading partner in the 1980s were developed countries and very similar in their industry structure and hence common trade theory did a poor job to explain the increasing wage gap, see the review of Kurokawa (2012). Heckscher-Ohlin trade models rely on changes in the relative price of high skilled and low skilled intensive goods to explain changes in the relative wage of high skilled and low skilled workers (Stolper-Samuelson theorem). But Lawrence and Slaugther (1993) found a small decline of the relative price of high skilled and low skilled intensive goods while Sachs and Shatz (1996) find that the relative price increased slightly after trade opening in the 1980s. They concluded that trade liberalization cannot explain the increasing wage gap as the relative price change was too small.

Moore and Ranjan (2005) used changes in the unemployment rates of high skilled and low skilled workers to distinguish trade effects and SBTC effects on the wage gap. They found that both trade liberalization and SBTC increased the wage gap, but only SBTC was consistent with the decreasing unemployment rates of high and low skilled workers. Instead of seeing trade and technological change as exclusive alternatives, Dinopoulos and Segerstrom (1999), Acemoglu (1998) and Acemoglu (2003) allow trade to have an effect on technology. Still these models were not able to explain the simultaneous increase of inequality in two trading countries as found, for example, by Verhoogen (2008) for Mexico and the US.

Especially (in the public opinion) the trade liberalization with Japan in the 1980s was seen as a major factor for rising wage inequality in the US. Standard trade theory models have difficulties to provide an explanation for the increasing inequality once two very similar countries trade. We develop a model that combines trade induced technological change and imperfect labor markets based on the concepts of Acemoglu (2003) and Moore and Ranjan (2005). Our model is consistent with many empirical findings and hence withstands common criticism of trade explanations. First, the model suggests that the wage gap in the US increased due to a disproportional increase of the wage of high skilled workers. Second, the model indicates that the unemployment rates for high and low skilled workers decreased after trade liberalization. Third, inequality rises simultaneously in two trading countries in response to trade libearlization. Lastly, the relative price of high and low skilled intensive goods does not need to change significantly in order to explain the increasing wage gap.

As in Acemoglu (2003) research and development (R&D) firms respond to changes in market size and the price of high and low skilled intensive intermediate goods. Trade liberalization increases the demand for intermediate goods and hence the incentives to innovate, which induces technological progress. Search frictions in the labor market break the direct link between marginal productivity and wages. Trade induced technological change does not only change the relative productivity, but as well the labor market tightness and hence the unemployment rates.[2] In contrast to Acemoglu (2003) we allow for changes in the relative price, but skill biased technological change will have only a small price effect, as the imperfect labor markets absorb some of the price effect. This means that trade liberalization is associated with smaller price changes and hence reconciles better with the evidence found by Lawrence and Slaugther (1993)

[2] A recent strand of literature analyses the effects of international trade and labor market imperfections, but this literature focuses on between firm wage differentials of ex ante homogenous workers, see Egger and Kreickemeier (2009), Helpman et al. (2010), Felbermayr et al. (2011a), Felbermayr et al. (2011b) and Egger et al. (2011). This paper is more concerned with wage differentials between different types of workers (high skilled vs. low skilled workers) working in different sectors (high skill and low skill intensive sectors).

and Sachs and Shatz (1996). In contrast to Moore and Ranjan (2005), trade does not only have a price effect, but it increases the (absolute) productivity levels of high and low skilled workers. As in Pissarides and Vallanti (2007), productivity growth leads to a higher job creation and hence to lower unemployment rates. Allowing trade to affect technology can be an explanation for the increasing wage gap and decreasing unemployment rates of high and low skilled workers.

Furthermore, the model permits considering different scenarios of intellectual property rights (IPR). R&D firms react differently if two trading countries respect intellectual property rights or if one country immitates the technology of the other country. In the first case we have a market size and a price effect, that leads to increasing wage gaps in the two trading countries and to decreasing unemployment rates. In the second case, the wage gap still increases, but the unemployment rate of the low skilled workers increases in the country that is skill abundant. Hence, the model refines the results of Moore and Ranjan (2005).

We calibrate the model to match the US unemployment rate and wage gap and analyse the effect of trade liberalization with Japan in the 1980s. We find that complete free trade between Japan and the US in the 1980s would have increased the wage gap in the US by about 14%, while the relative price of high skilled relative to low skilled intensive goods woulf have increased only by 4%.

The reminder of the paper is organized as follows. Section 2 describes the basic model. The equilibrium is discussed in detail in Section 3. Section 4 introduces international trade. Section 5 presents some numerical solutions. Finally, Section 6 discusses the main findings and concludes.

2 Model

2.1 Production

The economy involves high and low skilled indivduals. All individuals have equal lifetime preferences that depend on their consumption of a final good c. The utility function is given by

$$u^j = \int_{\tau=0}^{\infty} c_\tau^j \exp^{-\rho\tau} d\tau \qquad \text{for } j = D, F, \tag{1}$$

where ρ is the discount factor, τ is a time index and $j = D, F$ refers to one of the two countries, Domestic or fForeign. Indices will be omitted when ever it does not lead to any confusion. In each period the final good c is produced by a constant elasticity of substitution (CES) technology using a high skilled intensive intermediate good, c_h, and a low skilled intermediate good, c_l, so that

$$c = \left(\omega c_l^{\frac{\epsilon-1}{\epsilon}} + (1-\omega) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where he parameter ϵ defines the elasticity of substitution between the two goods and ω is a share parameter. The subscript $i = h, l$ indicates high and low skilled variables. Labor is not necessary for the production of the final good.

In each country the two intermediate goods are produced separately by a high and a low skilled intensive (representative) firm using the corresponding kind of local labor, N_h and N_l, and technology, A_h and A_l, respectively. Each intermediate good uses its own specific factor (machines), x_i, which is complementary to the sector specific labor, i.e., skill intensive machines can only be used by high skilled workers and similarly for the low skilled intensive sector. The intermediate good firm decides about the optimal machine and labor usage and takes prices and the technology level as given. Intermediate goods are produced with a Cobb-Douglas production function with labor augmenting technology

$$y_i = A_i^{\beta} x_i^{1-\beta} N_i^{\beta} \qquad \text{for } i = h, l, \tag{3}$$

where $\beta \in (0,1)$ is a common technology parameter. N_i gives the high and low skilled labor employed in the corresponding sector, which is different from total i-type labor supply, $\overline{N}_i$. Employed labor is given by $N_i = (1 - u_i)\overline{N}_i$, where u_i is the sector and skill type specific unemployment rate.

2.2 Prices

With competitive intermediate goods markets the relative price, $\overline{p} = p_h/p_l$, is determined by equation (2). In the optimum, the marginal rate of substitution between the two intermediate goods has to be equal to the relative price such that

$$\overline{p} = \frac{p_h}{p_l} = \frac{\left(\frac{\partial c}{\partial c_h}\right)}{\left(\frac{\partial c}{\partial c_l}\right)} = \frac{1-\omega}{\omega}\left(\frac{c_h}{c_l}\right)^{-\frac{1}{\epsilon}}. \tag{4}$$

The price of the final consumption good, p_c, depends on the prices of both inputs. For a CES production function the price is given by

$$p_c = (\omega^{\epsilon} p_l^{1-\epsilon} + (1-\omega)^{\epsilon} p_h^{1-\epsilon})^{\frac{1}{1-\epsilon}}. \tag{5}$$

The real prices of intermediate goods, $\overline{p}_i$, in terms of the relative price are derived using the above expression for the final good price

$$\overline{p}_h = \frac{p_h}{p_c} = \frac{\overline{p}}{(\omega^{\epsilon} + (1-\omega)^{\epsilon}\overline{p}^{1-\epsilon}))^{\frac{1}{1-\epsilon}}} \qquad and \qquad \overline{p}_l = \frac{p_l}{p_c} = \frac{1}{(\omega^{\epsilon} + (1-\omega)^{\epsilon}\overline{p}^{1-\epsilon}))^{\frac{1}{1-\epsilon}}} \tag{6}$$

The price of the high skilled good increases with the relative price while the price of the low skilled good decreases, $\frac{\partial \overline{p}_h}{\partial \overline{p}} > 0$ and $\frac{\partial \overline{p}_l}{\partial \overline{p}} < 0$.

2.3 Machine Demand and R&D

High and low skilled intensive R&D firms produce h-type and l-type machines under monopolistic competition. Intermediate good firms rent the machines in each period. Profits of the R&D are invested in research of new technologies. The skill specific technology is embedded in the machine. When an intermediate good firm rents a machine it produces with the corresponding technology level. For simplicity, only the newest vintage can be rented.[3]

The intermediate good firms take the rental price, χ_i, and the technology level of the machines as given and maximize their profits with respect to the machine usage, x_i.[4]

$$\begin{aligned} \underset{x_i}{\text{maximize}} \quad & \pi_i = y_i \overline{p}_i - \chi_i x_i - N_i w_i \\ \text{subject to} \quad & y_i = A_i^{\beta} x_i^{1-\beta} N_i^{\beta} \end{aligned} \tag{7}$$

Wages and capital costs are in final good prices. This yields machine demand as a function of the rental price, the real intermediate good price, the technology level and the employed labor.

$$x_i = \left(\frac{(1-\beta)\overline{p}_i}{\chi_i} \right)^{\frac{1}{\beta}} A_i N_i \tag{8}$$

The machines are produced by the R&D firm that will gain monopoly rents. As the

[3]Otherwise, if the technology level drops for some reasons, intermediate good firms would want to rent older machines, as they have a higher technology level and productivity. This assumption excludes this possibility.

[4]Labor demand is not derived straightforwardly as search frictions exist. Hence, the firm considers N_i as the given employment level at this point. The optimal employment level will be determined later within the search model.

demand is iso-elastic, the common expression of the Lerner index is used to determine the optimal monopoly mark up of the firm:

$$-\frac{1}{\epsilon_{x_i}} = \frac{\chi_i - MC}{\chi_i} \Rightarrow \beta = \frac{\chi_i - (1-\beta)^2}{\chi_i}.$$

The elasticity of demand is $\epsilon_{x_i} = -\frac{1}{\beta}$. The R&D firm's marginal costs (MC) are constant and fixed to $(1-\beta)^2$, hence the monopoly price for each type of machine is constant at

$$\chi_i = (1-\beta). \tag{9}$$

Replace the monopoly price, χ_i, in equation (8) to obtain machine demand of an intermediate producer as

$$x_i = \overline{p}_i^{\frac{1}{\beta}} A_i N_i. \tag{10}$$

Machine demand increases with the employed labor, the intermediate good price and the technology level. Substituting the demand in the intermediate good production function yields

$$y_i = p_i^{\frac{1-\beta}{\beta}} A_i N_i \qquad \text{for } i = h, l. \tag{11}$$

R&D firms invests profits in research, which determintes the technology level, A_i. The technology production function for each R&D firm is given by

$$A_i = z_i^{\mu} q_i^{1-\mu}, \tag{12}$$

where z_i is the research effort in final good units, $q_i > 0$ is a scale parameter and $\mu \in (0,1)$ is a production coefficient. Technology production has diminishing returns to research effort. Higher levels of technology are more costly to achieve.

R&D firms make zero profits after considering the constant markup price from equation (9) and the demand function in equation (10). On the other hand, R&D firms take the intermediate good price and the employment level as given. The zero profit condition is given by

$$\pi_i^{RD} = 0 \quad \rightarrow \quad \chi_i x_i - MC x_i - z_i \beta(1-\beta) = \beta(1-\beta)\overline{p}_i^{\frac{1}{\beta}} A_i N_i - z_i \mu \beta(1-\beta) = 0$$

where $A_i = z_i^{\mu} q_i^{1-\mu}$ and research effort costs, z_i, are scaled by the expression $\beta(1-\beta)$

to simplify notation. The optimal research effort is

$$z_i = q_i(\overline{p}_i^{\frac{1}{\beta}} N_i)^{\frac{1}{1-\mu}} \tag{13}$$

which implies a technology level of

$$A_i = q_i(\overline{p}_i^{\frac{1}{\beta}} N_i)^{\frac{\mu}{1-\mu}} \tag{14}$$

The higher is the intermediate good price, the more profitable is the production of the intermediate good firm and, hence, the higher is factor demand for machines and workers. Higher machine demand increases the profits of R&D firms and the technology level.

For a country in autarky, the consumption of high and low skill intensive goods equals the local production. Considering the technology level, the relative price can be written as a function of the labor supply ratio:

$$\begin{aligned} \overline{p} &= \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon} \frac{y_h}{y_l}\right)^{-\frac{1}{\epsilon}} = \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon} \overline{p}^{\frac{1-\beta}{\beta}} \frac{A_h N_h}{A_l N_l}\right)^{-\frac{1}{\epsilon}} \\ &= \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon} \overline{p}^{\frac{(1-\mu)(1-\beta)+\mu}{(1-\mu)\beta}} \left(\frac{q_h}{q_l}\right) \left(\frac{N_h}{N_l}\right)^{\frac{1}{1-\mu}}\right)^{-\frac{1}{\epsilon}}. \end{aligned} \tag{15}$$

Solving for $\overline{p}$ yields

$$\overline{p} = \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon} \left(\frac{q_h}{q_l}\right) \left(\frac{N_h}{N_l}\right)^{\frac{1}{1-\mu}}\right)^{-\frac{\beta(1-\mu)}{(1-\mu)(\beta\epsilon+(1-\beta))+\mu}}. \tag{16}$$

A higher employment in the skill intensive sector increases the supply of skill intensive intermediate goods and, hence, reduces its price.

2.4 Labor Markets

This section introduces a search model along the lines of Pissarides (2000). The model allows to determine wages and unemployment rates in each sector. Firms will only create a vacant position if it is profitable. Workers will only accept a job offer if the wage paid is higher than their reservation wage. Exogenous shocks destroy filled positions. Unemployment exists as it takes time for the firm and the worker to form a match. We explicitly derive all equations in in the Appendix.

We use two symmetric matching functions for the high and low skill intensive sector.

$$M(v_i\overline{N}_i; u_i\overline{N}_i) = kv_i^{\gamma}u_i^{1-\gamma}\overline{N}_i = k\theta_i^{\gamma}u_i\overline{N}_i \qquad \text{for } i = h, l. \tag{17}$$

$\theta_i = \frac{v_i}{u_i}$ reflects the labor market tightness, where v_i is the vacancy rate and u_i is the unemployment rate in sector i, $\gamma \in (0, 1)$ is a matching coefficient and k is a scale parameter.

Following Pissarides (2000), the equilibrium unemployment rate is determined as

$$u_i = \frac{\psi}{\psi + k\theta_i^{\gamma}}, \tag{18}$$

where ψ is the exogenous job destruction rate.

2.4.1 Firms

For the wage determination, firms consider the value of a filled and a vacant position. F_i represents the discounted value of a vacancy in a firm and J_i the present discounted value of a filled position in a firm.

A vacant position is an asset for the firm. If capital markets are perfect, the valuation of this asset will be ρF_i and equal to the expected gains from filling a position less the recruitment costs, δ: $k\theta_i^{\gamma-1}(J_i - F_i) - \delta$. Note that $k\theta^{\gamma-1}$ is the probablity of filling a vacancy, $J - F$ are the flow profits of a filled vacancy and δ are the inital costs of creating a vacancy or recruitment costs. In the equilibrium we have

$$\rho F_i = k\theta_i^{\gamma-1}(J_i - F_i) - \delta. \tag{19}$$

Similarly, a filled position has a value for the firm, which is equal to marginal profits of an additionally employed worker plus discounted expected profits until the match is resolved. The instantaneous marginal profits are calculated by substracting the wage, w_i, and the marginal costs of machines, r_i, from the marginal revenues gained from employing a worker, t_i. We derive the marginal revenues from employing a worker using the profit maximization problem of each intermediate good producer, equation (7). The intermediate producer takes the technology level and prices as given and so that marginal revenues can be derived using equation (11):

$$t_i = \left(\frac{\partial y_i}{\partial N_i}\right)\overline{p}_i = \overline{p}_i^{\frac{1}{\beta}} A_i. \tag{20}$$

The capital costs are calculated as $\chi_i x_i$ using equations (9) and (10), hence the marginal rental costs are

$$r_i = \frac{\partial(\chi_i x_i)}{\partial N_i} = \overline{p}_i^{\frac{1}{\beta}} A_i(1-\beta). \tag{21}$$

We can write the value of a filled position as

$$\rho J_i = t_i - w_i - r_i + \psi(F_i - J_i), \tag{22}$$

where the right hand side are instantaneous profits of an additional employed worker, $t_i - w_i - r_i$, plus the expected profits from the match in the future, $\psi(F_i - J_i)$, where ψ is the exogenous job destruction rate. If the job destruction rate is zero, no unemployment exists in the model. Hence, equation (22) would simplify to $\rho J_i = t_i - w_i - r_i$. In this setting the value of an additionally filled position, J_i, would be zero and the above equation would correspond to the first order condition with respect to labor in the firm's maximization problem in expression (7).

In the equilbrium the value of a vacancy has to be zero in equilibrium, $F_i = 0$, otherwise firms would like to create more or less vacancies and the unemployment rate would not be in its steady state. We use the fact that $F_i = 0$ to combine (19) and (22) to obtain the following free entry condition:

$$k\theta_i^{\gamma-1}(t_i - r_i - w_i) = k\theta_i^{\gamma-1}(\beta\overline{p}_i^{\frac{1}{\beta}} A_i - w_i) = \delta(\psi + \rho). \tag{23}$$

where we have substituted equations (20) and (21) in equation (22) to receive the second equality.

2.4.2 Workers

Workers accept any job that pays a higher wage than their reservation wage. The present discounted value of being unemployed is equal to the social benefits, b, plus the expected gain from finding a job. On the other hand, the present discounted value of being employed is equal to the wage plus the expected loss when a match is destroyed.

These considerations lead to the following two standard Bellman equations in the Pissarides model.

$$\rho U_i = b + k\theta_i^{\gamma}(W_i - U_i) \tag{24}$$

$$\rho W_i = w_i + \psi(U_i - W_i), \tag{25}$$

where U_i is the present discounted value of unemployment and W_i is the present discounted value of employment. The worker receives social benefits, b, if unemployed. $k\theta_i^{\gamma}$ gives the rate at which workers find a job and ψ the rate at which workers lose their job.

Note that all exogenous parameters, such as recruitment costs, δ, or social benefits, b, and scale parameter, k, are the same for high and low skilled workers and firms. Different values of the exogenous parameters for the high and low skill intensive sector do not change the qualitative results of the model. For example higher recruitment costs for high skilled workers increase the high skilled unemployment rate and decreases the high skilled wages and consequently the wage gap. This is aligned with the behavior of unemployment and wages in a search unemployment models. Nevertheless, this paper focuses less on the labor market institutions, but rather on the interaction of trade, labor markets and technological progress. See Weiss and Garloff (2009) for an analyse of the behavior of SBTC and unemployment with different institutions in detail.

2.4.3 Wage Bargaining

Wages are determined by Nash bargaining over the profits of a filled position. The parameter η defines the bargaining power of workers and firms. A higher η gives more weight to the workers and $\eta = 0.5$ implies symmetric bargaining.

$$w_i = \arg\max (W_i - U_i)^{\eta}(J_i - F_i)^{1-\eta} \tag{26}$$

The first order condition for equation (26) is

$$W_i - U_i = \frac{\eta}{1-\eta}(J_i - F_i) \tag{27}$$

From this expression, the wage equation can be derived analogously to Pissarides (2000).[5]

[5] The complete mathematical derivation is shown in the appendix.

$$w_i = (1-\eta)b + \eta(\beta\overline{p}_i^{1/\beta}A_i + \delta\theta_i). \tag{28}$$

Combining the wage equation (28) and the free entry condition (23) obtains an implicit function of the labor market thightness θ_i:

$$(1-\eta)[\beta\overline{p}_i^{1/\beta}A_i - b] - \eta\delta\theta_i - \frac{\delta(\rho+\psi)}{k\theta_i^{\gamma-1}} = 0. \tag{29}$$

As in common search models the labor market thightness is a key variable as it defines the equilibrium of the model. Equation (29) gives the equilibrium condition for each sector in the model. After substituting equations (6), (4) and (14) the labor market tightness is a function of the labor supply. The relative price depends on the labor market tightness of both sectors, thus the implicit functions in equation (29) have to hold simultaneously for both sectors. Once the labor market tightness for each sector is determined for a given labor supply, all other variables can be determined.

3 Equilibrium

For a country in autarky the equilibrium is defined by

$$(1-\eta)[\beta\overline{p}_i^{\frac{1}{\beta}}A_i - b] - \eta\delta\theta_i - \frac{\delta(\rho+\psi)}{k\theta_i^{\gamma-1}} = 0 \qquad \text{for } i = h, l \tag{30}$$

where

$$\overline{p} = \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon}\left(\frac{q_h}{q_l}\right)\left(\frac{(1-u_h)\overline{N}_h}{(1-u_l)\overline{N}_l}\right)^{\frac{1}{1-\mu}}\right)^{-\frac{\beta(1-\mu)}{(1-\mu)(\beta\epsilon+(1-\beta))+\mu}}$$

$$\overline{p}_h = \frac{\overline{p}}{(\omega^\epsilon + (1-\omega)^\epsilon\overline{p}^{1-\epsilon}))^{\frac{1}{1-\epsilon}}}$$

$$\overline{p}_l = \frac{1}{(\omega^\epsilon + (1-\omega)^\epsilon\overline{p}^{1-\epsilon}))^{\frac{1}{1-\epsilon}}}$$

$$A_i = q_i(\overline{p}_i^{\frac{1}{\beta}}(1-u_i)\overline{N}_i)^{\frac{\mu}{1-\mu}}$$

$$u_i = \frac{\psi}{\psi + k\theta_i^\gamma}$$

Although the system has no closed form solution, comparative statics help to understand the mechanics of the model. Consider the prices and technology levels as exogenous, the labor market tightness is an increasing function of technology, A_i, and the real intermediate good prices, $\overline{p}_i$.

$$\frac{\partial \theta_i}{\partial A_i} > 0 \qquad\qquad \frac{\partial \theta_i}{\partial \overline{p}_i} > 0$$

By equation (18) a higher θ_i decreases the unemployment rate. Thus, higher intermediate good prices and technological progress reduce the unemployment, as both factors make the employment of more workers more profitable for an intermediate producer.

Considering technology and prices as endogenous, we find

Proposition 1: *For a given θ_l (θ_h), an increase in the labor supply $\overline{N}_h$ ($\overline{N}_l$), will raise the labor market tightness θ_h (θ_l) and reduce the unemployment rate u_h (u_l).*

Assume that the labor market tightness of the other sector is constant, then the labor market tightness of each sector is increasing with its labor supply. This implies that the unemployment rate in each sector is a decreasing function of the respective labor supply, since

$$\frac{\partial \theta_i}{\partial \overline{N}_i} > 0 \qquad i = h, l$$

The decreasing unemployment rate is driven by the increasing technology, which can be interpreted as a "capitalization" effect, i.e., recruitment costs become less and less important and new matches are formed easier. This is consistent with the empirical evidence for a negative relationship between the unemployment rate and labor productivity, see Pissarides and Vallanti (2007).

A high skilled labor supply shock will have two opposing effects on the technology level. First, it reduces the relative price by increasing the production of the high skilled good. A decreasing intermediate good price will diminish the profit incentives of the high skilled skilled complementary R&D firm, which will lead to a lower technology level. On the other hand, the demand for high skilled machines will increase as more high skilled workers are employed. This in turn increases the profits of the high skilled complementary R&D firm and the technology level. The net effect of the negative price change and the positive labor market size effect on the relative technology level can be evaluated by using equation (14) and replacing the relative price by equation (16).

$$\frac{A_h}{A_l} = \left(\frac{q_h}{q_l}\right)\left(\overline{p}^{\frac{1}{\beta}}\frac{N_h}{N_l}\right)^{\frac{\mu}{1-\mu}} = \kappa\left(\frac{q_h}{q_l}\right)\left(\frac{(1-u_h)\overline{N}_h}{(1-u_l)\overline{N}_l}\right)^{\frac{\mu\beta(\epsilon-1)}{(1-\mu)(\beta\epsilon+1-\beta)+\mu}} \tag{31}$$

where $\kappa = \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon}\left(\frac{q_h}{q_l}\right)^{1-\mu}\right)^{\frac{-(1-\mu)}{(1-\mu)(\epsilon\beta+1-\beta)+\mu}}$

The relative technology increases in $\overline{N}_h$ and decreases in $\overline{N}_l$ for given unemployment rates if $\epsilon > 1$. Hence, technological change will be skill biased if the skill ratio increases.

Proposition 2: *For a given labor market tightness in the two sectors, an increase in the labor supply $\overline{N}_h$ ($\overline{N}_l$) raises (reduces) the relative technology if $\epsilon > 1$. An increasing skill ratio implies skill biased technological change.*

The wage in each sector depends on the sectoral technology level, the intermediate good price and the labor market tightness, as can be seen from equation (28). An increase in the skill supply will have a positive impact on the labor market tightness and the technology, but at the same time the intermediate good price will decline. Which effect dominates is not clear a priori.

4 International Trade

Assume that trade occurs only in intermediate goods between two countries. Trade in intermediate goods equalizes the relative prices in the two economies. As all individuals have the same preferences and face the same prices after trade opening, they have the same relative demand of high and low skilled goods. Hence, the post-trade relative price, $\overline{p}^T$, satisfies[6]

$$\overline{p}^T = \frac{p_h}{p_l} = \frac{1-\omega}{\omega}\left(\frac{y_h^D + y_h^F}{y_l^D + y_l^F}\right)^{-\frac{1}{\epsilon}}, \tag{32}$$

where the post-trade prices for high and low skilled goods are still given by the equation (6) when using the above post-trade relative price.

Intellectual property rights (IPRs) are crucial in the model. We distinguish two cases. First, intellectual property rights are only enforced in the domestic country and not in the foreign country. This implies that R&D activities only take place in the domestic country and the foreign country copies the technology and machines developed domestically. This case can be interpreted as trade with technology imitation.

[6] The superscript T indicates specific trade variables.

In the second case, IPRs are enforced in the foreign country after trade opening. As the R&D firm has a monopoly, only one R&D firm for each sector rents machines to intermediate good firms in both countries.[7]

4.1 Technology Imitation - IPR Violation

Assume that the foreign developing country is a develoiping economy and has no research sector and copies the existing technology and machines of the developed domestic country. Consequently, the domestic R&D sector does not take into account the machine demand of the foreign country. The technology in both countries will be given by equation (14), but the domestic R&D firm considers the post-trade price. Trade liberalization with an imitating country has a price effect but no market size effect for technology.

$$A_i^T = q_i(\overline{p}_i^{T\frac{1}{\beta}} N_i^D)^{\frac{\mu}{1-\mu}} \tag{33}$$

Proposition 2 still holds after trade liberalization, i.e., if the skill ratio in the domestic country increases, technological progress is skill biased, and

$$\frac{A_h^T}{A_l^T} = \left(\frac{q_h}{q_l}\right)\left(\overline{p}^{T\frac{1}{\beta}}\frac{(1-u_h)\overline{N}_h}{(1-u_l)\overline{N}_l}\right)^{\frac{\mu}{1-\mu}}. \tag{34}$$

Trade liberalization induces skill biased technological change if the skill ratio of the domestic country is higher than that of the foreign country. To see this, substitute the post-trade technology level from equation (33) in the production function of the domestic and foreign countries. Considering the relationship for the relative price, $\overline{p}^T$, as given in equation (32) and using that the trade in intermediate goods equalizes the prices in the two economies. The relative world market price is a function of the labor supply of the two trading countries:

$$\overline{p}^T = \left(\left(\frac{1-\omega}{\omega}\right)^{-\epsilon} \overline{p}^{T\frac{(1-\mu)(1-\beta)+\mu}{(1-\mu)\beta}} \left(\frac{q_h}{q_l}\right)\left(\frac{N_h^D}{N_l^D}\right)^{\frac{1}{1-\mu}}\right)^{-\frac{1}{\epsilon}} \left(\frac{1+\frac{N_h^F}{N_h^D}}{1+\frac{N_l^F}{N_l^D}}\right)^{-\frac{1}{\epsilon}}. \tag{35}$$

Solving the above equation for $\overline{p}^T$ yields an analogous expression to equation (16):

[7] As the technology and machines are produced without labor, it does not matter in which country the R&D firm is located. Once IPR are enforced in the two countries, only one R&D firm has the cutting edge patents and hence will cover all the demand for machines.

$$\overline{p}^T = \kappa^{\frac{1}{\beta}} \left(\left(\frac{N_h^D}{N_l^D}\right)^{\frac{\mu}{1-\mu}} \left(\frac{N_h^D + N_h^F}{N_l^D + N_l^F}\right) \right)^{-\frac{\beta(1-\mu)}{(1-\mu)(\epsilon\beta+1-\beta)+\mu}} . \quad (36)$$

The relative price will increase in the domestic country if it starts trading with a country that is scarcer in high skilled labor, i.e., $\frac{N_h^D}{N_l^D} > \frac{N_h^F}{N_l^F}$. The post-trade relative price increase will be greater, the higher is the difference between the skill ratios of the two countries.

Trade liberalization with an imitating country with a lower skill ratio leads to an increasing domestic wage gap. First, the price for high skill intensive goods increases and the price for low skill intensive goods decreases. Second, technological progress is biased towards skilled workers. Either of those benefits the wages of high skilled workers.

Proposition 3: *For given high and low skilled unemployment rates trade opening to a country with a lower skill ratio that does not respect IPR will increase the domestic relative price and induces domestic skill biased technological change. The domestic wage gap will increase in response.*

The echnology level and the relative price are changing after trade liberalization with an imitating country, the expression for the labor market tightness has to be adapted. The post-trade relative price depends on the employment levels in both countries and hence the implicit functions for the labor market tightness for the two sectors in the two countries have to be satisfied simultaneously in equilibrium:

$$(1-\eta)[\beta\overline{p}_i T \frac{1}{\beta} A_i^T - b] - \eta\delta\theta_i^j - \frac{\delta(\rho+\psi)}{k(\theta_i^j)^{\gamma-1}} = 0 \qquad \text{for } j = D, F \text{ and for } i = h, l, \quad (37)$$

where $\overline{p}_i^T$ is given by equation (6) substituting the relative world market price $\overline{p}^T$. The expression for the unemployment rate, u_i, is unchanged and given by equation (18). The labor market tightness has to be the same in both countries after trade liberalization as all exogenous parameters in the matching function, the prices and the technology levels are the same everywhere.

4.2 No Technology Imitation - IPR Enforcement

Trading under IPR enforcement is very similar to a labor supply shock. The domestic R&D firm considers the demand of foreign intermediate good firms for machines. In this setting trade opening implies that only one R&D firm exists in both countries which considers the demand of both countries for machines. The zero profit condition is given by

$$\pi_i^{RD} = 0 \quad \rightarrow \quad \chi_i(x_i^D + x_i^F) - MC(x_i^D + x_i^F) - z_i\beta(1-\beta) = 0$$

where $A_i = z_i^{\mu} q_i^{1-\mu}$.

Since the technology levels and prices are the same in both countries after trade liberalization, the above equality yields the technology level after trade liberalization with an IPR respecting country:

$$A_i^T = q_i(p_i^{\frac{1}{\beta}}(N_i^D + N_i^F))^{\frac{\mu}{1-\mu}} = q_i(p_i^{\frac{1}{\beta}}((1-u_i^D)\overline{N}_i^D + (1-u_i^F)\overline{N}_i^F))^{\frac{\mu}{1-\mu}}. \qquad (38)$$

The expression for the technology level has to be substituted in the equilibrium condition in equation (37), which again has to hold for the two sectors in both countries.

A special and simple case is noteworthy. If the foreign IPR enforcing country and the domestic country are completely symmetric, trade opening will not change the relative price for given unemployment rates and only a market size effect for technology can be observed. The market size effect will be different to the effect from Acemoglu (2003) as changes in technology will change the labor market thightness and hence unemployment rates and wages will change.

5 Numerical Results

The equilibrium conditions for the basic model and the trade model, equation (30) and (37), respectively, have no closed form solution, but they can be solved numerically. We present three different scenarios. First, we consider a country in autarky with a changing skill ratio. Second, we show the effects of trade liberalization with a country that respects IPR. Finally, we investigate the effects of trade liberalization with an IPR violating country.

5.1 Calibration

We calibrate the model to match the wage gap and unemployment rate of the US economy. The values for the exogenous variables presented in Table 1 are in line with the values taken by Pissarides (2007, 2009). For the supply of high skilled and low

Parameter	Value	Description
β	0.66	Cobb-Douglas production coefficient
γ	0.5	Matching parameter
δ	$\in$(0.1,1)	Recruitment costs
ϵ	$\in$(1, 3)	Elasticity of substitution for the final good production
η	0.5	Bargaining power parameter
μ	$\in$(0.25,0.6)	Technology production parameter
ρ	0.004	Discount factor
ψ	0.019	Job destruction rate
b	free	Social benefits
k	free	Unemployment scale parameter
q_h	free	Skilled technology scale parameter
q_l	free	Low skilled technology scale parameter

Table 1: Parameter values following Pissarides (2007, 2009).

skilled workers, we use data from the US Census of Population Educational Attainment from 1963 to 2003, where skilled workers are defined as to have at least a college degree. The skill ratio increases because the supply of high skilled workers increases more strongly than the supply of low skilled workers. The literature suggests an elasticity of substitution in final good production function of $\epsilon \approx 1.5$, see Hamermesh and Grant (1979), Krusell et al. (2000), Katz and Murphy (1992), Autor et al. (2008) or Epifani and Gancia (2008).

A key parameter is the elasticity of substitution, ϵ, in the production function of the final good. We solve the models alternatively for ϵ between 1 and 3 and discuss the effects when needed. The parameter b for social benefits and the recruitments costs δ have to be sufficiently low relative to the wage to ensure a solution. A higher technology production coefficient μ increases the spread between pre- and post-trade wage levels. The technology scale parameter q_i is set to 3.5 for the high and low skilled technology to give a reasonable wage gap.

As all exogenous parameters determining the labor market tightness are the same for the two countries and the post-trade prices and technology levels are the same, also the labor market tightness has to be the same in both countries after trade liberalization. This condition makes the numerical results stable to changes in the starting values for the algorithm.

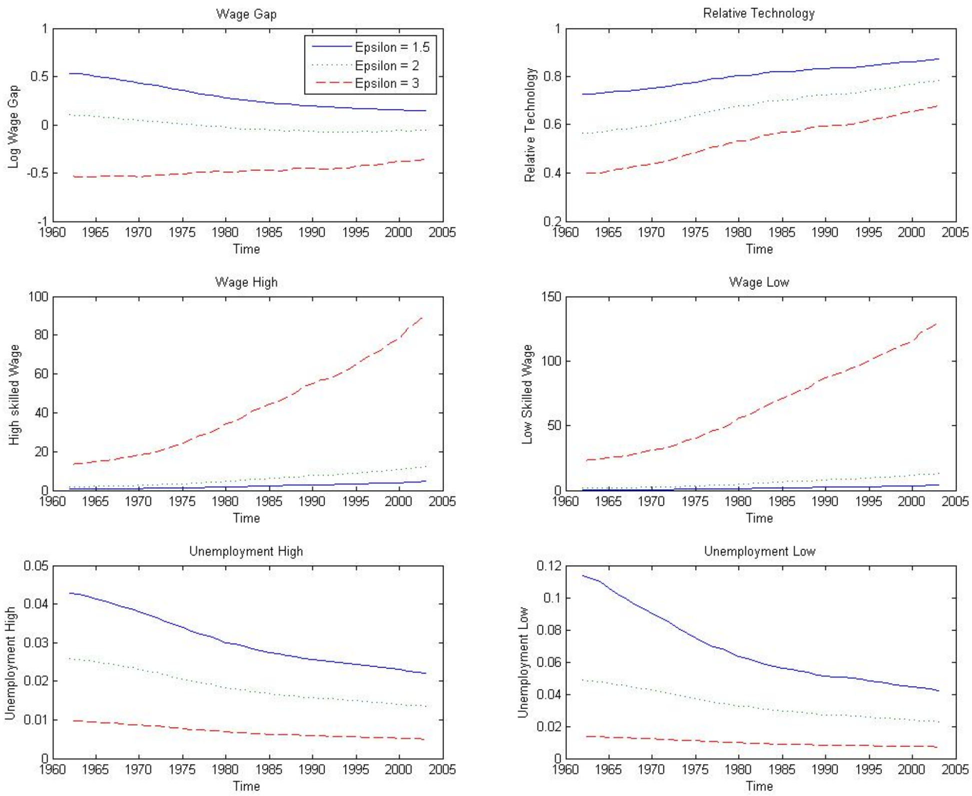

Figure 3: Numerical solution for the US labor supply of high skilled and low skilled workers using various values of ϵ.

5.2 US in Autarky

In this section we show the results for wages, unemployment rates and technology of both skill groups for a country in autarky. The change in the labor supply of the US increased the skill ratio from 0.09 to 0.37 between 1963 and 2003. Figure 3 presents the associated consequences in the calibrated model graphically.

The wage gap decreases for elasticities of substitution in the interval (1, 2.5). For higher values of ϵ the wage gap increaes, so in contrast to Acemoglu (2003) and Moore and Ranjan (2005) an ϵ in excess of 2.5 is needed to explain a slight increase of the wage gap.[8] Still the increasing skill ratio will lead to skill biased technological progress for any $\epsilon > 1$. The model is able to consider the supply of high and low skilled workers separately. Previous models focus just on the skill ratio, which was steadily increasing in the US, but neglect the absolute supply. In the context of the present model, the commonly estimated elasticity of substitution between high and low skill intensive intermediate goods, ϵ, is too low to explain the increasing wage gap in autarky due to

[8]For all other simulations we use $\epsilon = 1.5$, which is the common estimate for the parameter.

an increasing supply of skilled workers.

5.3 Trade of the US with Japan

The case of trade between the US and a foreign IPR respecting country, taking Japan as an example, is shown in Figure 4. The supply of low skilled workers is 70% of the supply in the domestic country (US) and the supply of skilled workers is 60% of the US supply. The lower relative labor supply ensures that the relative price increases after trade liberalization. The values for the foreign country are chosen to match the skill supply and the size of the Japanese population in the 1985 census.

At $t = 20$ the two countries start trading. In response to that, the wage gap increases significantly, unemployment rates decrease and wages increase for both skill groups. Two points are very important. First, the wage inequality rises because the high skilled wages increase faster than low skilled wages. Second, the unemployment rates for both skill groups decrease after trade opening. This illustrates that trade liberalization can increase the wage gap and decrease the unemployment rate of low skilled workers at the same time. Note that the number of vacancies is a forward looking variable, it adapts immediatly to trade opening. This leads to the big jump at $t = 20$, where countries swit ch from autarky to free trade by assumption. If the two countries opened only gradually, the transistion would be smoother.

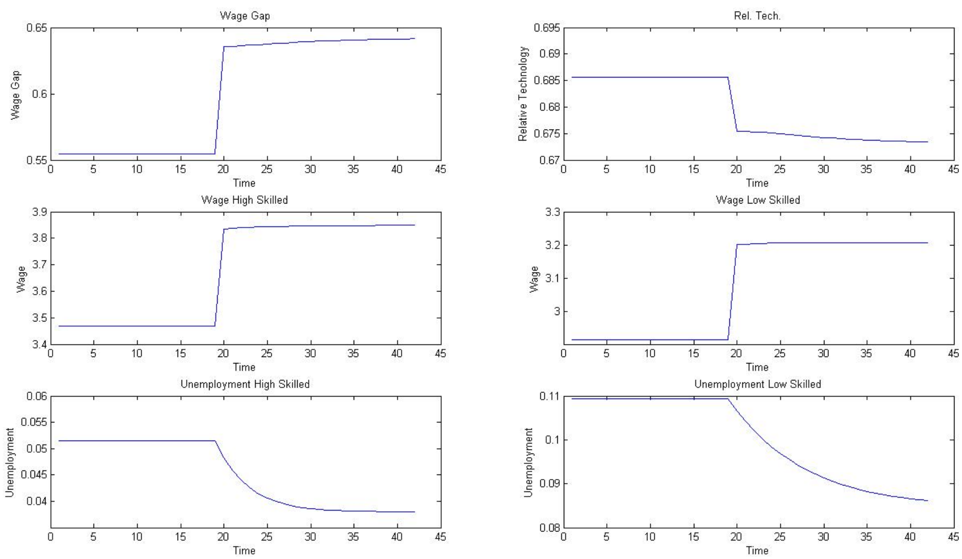

Figure 4: Numerical solution for the domestic country trading with smaller developed country that respects IPR, for example Japan. $\epsilon = 1.5$.

The numerical result can be seen as evidence that increasing trade, especially with Japan, in the 1980s contributed to the development of the wage gap in the US. In this setup a relatively small price change is able to explain a significant increase of the wage gap: the relative price increases by less than 4% while the average wage gap increases by about 14%. These results are consistent with the empirical findings of Lawrence and Slaugther (1993) and Sachs and Shatz (1996).

The results are very similar for the foreign country as all exogenous parameters are the same. This implies that in the foreign country the wage gap increases as well. Figure 5 shows the results graphically. In contrast to the domestic case, the relative technology increases as the domestic country has a higher skill ratio. Also decreasing unemployment rates for high and low skilled workers can be observed in this case.
If we alllow trade to not only alter the relative technology but as well the absolut technology level, bilateral trade can consistently explain the increase of wage inequality in the two trading countries.

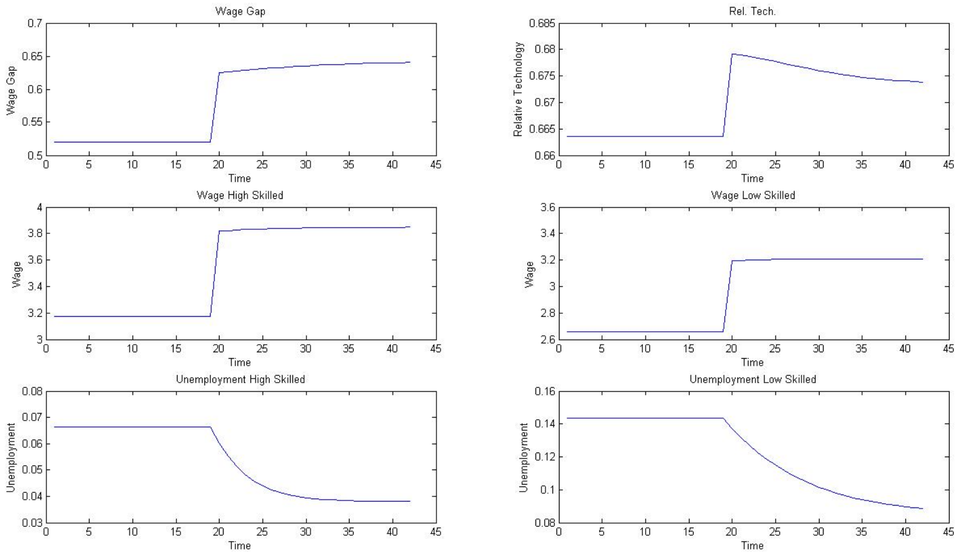

Figure 5: Numerical solution for the foreign country that respects IPR trading with bigger developed country. $\epsilon = 1.5$.

5.4 Trade with a Technology Imitating Country

Figure 6 summarizes the numerical results for trade with a technology imitating foreign country. The supply of high skilled workers in the foreign country is only 25% of the supply in the domestic country and 50% for low skilled workers, taking Mexico as an

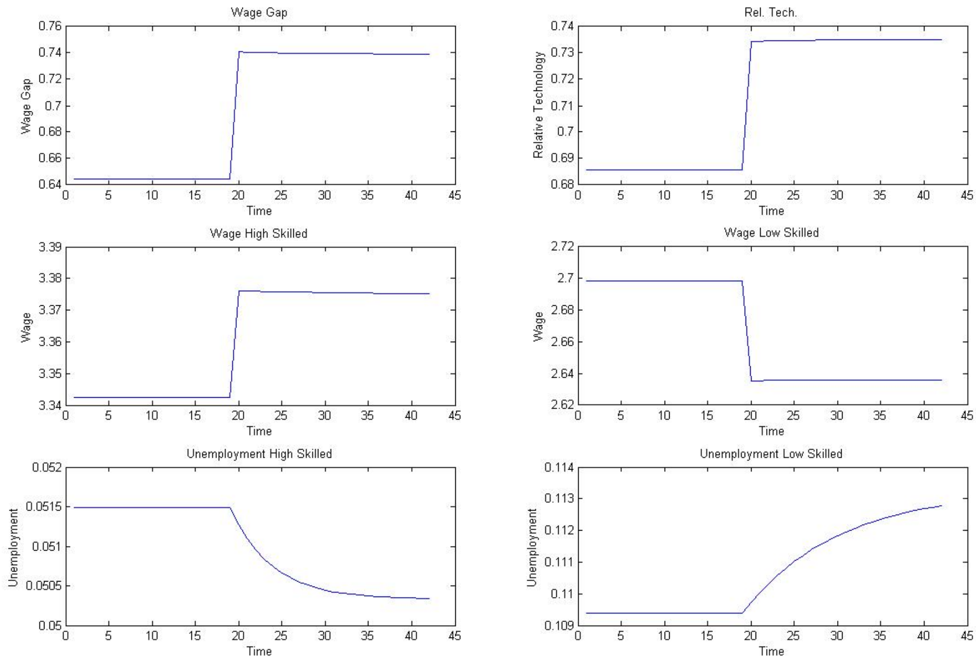

Figure 6: Numerical solution for the domestic country trading with a small developing country (no IPR) with a low skill ratio. $\epsilon = 1.5$

example. This implies that the price change is considerably greater than in the case of trade with a developed country.

Trade liberalization leads to an increasing wage gap and skill biased technological change in the domestic country. The high skilled unemployment rate decreases while the low skilled unemployment rate increases. This behavior is consistent with the findings of Moore and Ranjan (2005) who use the increasing unemployment rate of low skilled workers to identify the effect of trade on the wage gap. Similar to Moore and Ranjan, trade causes the low skilled wages to decrease and the high skilled wages to increase.

6 Concluding Remarks

We analyse the wage gap in the US and its relationship with unemployment rates of high and low skilled workers in the context of trade liberalization and endogenous technological change. We emphasize the importance of absolute technological change due to trade liberalization. Search frictions in the labor market break the direct link between productivity and wages and diminish the impact of trade on relative prices.

A higher supply of a certain type of workers increases the employment of workers of this skill type. This increases the demand for machines which are complementary to this worker type and increases the research effort in R&D to develop the complementary technology. Thus, an increasing skill ratio, due to a higher supply of high skilled workers, always leads to skill biased technological change. The higher technology levels make employment more profitable and reduce unemployment rates. This can be interpreted as a common capitalization effect in search unemployment models.

For a country in autarky, the wage gap increases with an increasing skill ratio only if the elasticity of substitution between intermediate goods in the production of the final good is sufficiently high. The numerical results in this paper suggest that for the labor supply of the US an elasticity of substitution of at least 2.5 is needed to ensure an increasing wage gap in response to an increasing skill ratio.

Trade liberalization with a country that respects intellectual property rights is consistent with the observed patterns of wages and unemployment rates in the US since the 1980s. First, the wage gap increases due to a disproportional increase of high skilled wages. This squares with the empirical findings of Autor et al. (2008). Second, unemployment rates for both skill types decrease. Third, during the period of trade liberalization in the 1980s inequality increases globally. Lastly, the model predicts that the wage gap increases in both trading countries after trade liberalization, while the associated changes in the relative price are small. This fact is consistent with the empirical findings of Lawrence and Slaugther (1993) and Sachs and Shatz (1996).
On the other hand, the model suggests that the findings of Moore and Ranjan (2005) only hold for trade with a country that imitates technology.

We calibrate the domestic country to match the US in the 1980s and show that trade opening to other developed countries, such as Japan, can explain the increasing wage gap and the decreasing unemployment rates for both skill groups in both countries. In the 1980s the wage gap in the US increased by roughly 14%, at the same time the relative price increased only by 4%.

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A Search Unemployment

This section derives the explicit equations for the search model used in Section 2.4.

Matching functions

$$M(v_i\overline{N}_i; u_i\overline{N}_i) = kv_i^{\gamma}u_i^{1-\gamma}\overline{N}_i = k\theta_i^{\gamma}u_i\overline{N}_i \qquad \text{for } i = h, l. \tag{39}$$

The labor market tightness, θ_i is the ratio of vacancies to unemployed workers, $\frac{v_i}{u_i}$. The rate at which a worker finds a job is defined as

$$\frac{M(v_i\overline{N}_i; u_i\overline{N}_i)}{u_i\overline{N}_i} = k\theta_i^{\gamma} \qquad \text{for } i = h, l, \tag{40}$$

which is increasing in the labor market tightness. The rate at which a vacant position is filled is given by

$$\frac{M(v_i\overline{N}_i; u_i\overline{N}_i)}{v_i\overline{N}_i} = k\theta_i^{\gamma-1} \qquad \text{for } i = h, l, \tag{41}$$

which is decreasing in the labor market tightness.
The flow rate into unemployment per unit of time, $\dot{u}_i$, is given as the rate of exogenously destroyed matches less the rate of workers newly employed.

$$\dot{u}_i = \psi(1 - u_i) - k\theta_i^{\gamma}u_i \qquad \text{for } i = h, l, \tag{42}$$

where the parameter ψ reflects an exogenous break up rate for filled positions. In the equilibrium, $\dot{u}_i = 0$ will be satisfied. This gives the steady state unemployment rate:

$$u_i = \frac{\psi}{\psi + k\theta_i^{\gamma}} \qquad \text{for } i = h, l. \tag{43}$$

A vacant position is an asset for the firm. If capital markets are perfect, the capital costs ρF_i have to be equal to the rate of return on assets. The latter is given as the expected gains from filling a position less the recruitment costs, The expected gains are calucalted using the marginal revenues, t_i and substract the (marginal) rental costs, r_i and the wage w.

$$\rho F_i = k\theta_i^{\gamma-1}(J_i - F_i) - \delta \tag{44}$$

$$\rho J_i = t_i - w_i - r_i + \psi(F_i - J_i) \tag{45}$$

Similarly, a filled position has a capital cost of ρJ_i to the firm. This has to be equal to the current marginal revenues of a worker less the wage and the (marginal) rental costs of machines less the expected loss if the match is destroyed at some point in time.

In equilibrium, all profit opportunities from new jobs are exploited, driving rents from a vacant position to zero. Therefore, the equilibrium condition for the supply of vacant jobs is zero. Given a non-zero discount factor ρ, this is satisfied if $F_i = 0$. This implies that firms can enter and exit the market freely, and

$$J_i = \frac{t_i - w_i - r_i}{\rho + \psi}. \tag{46}$$

After substituting J_i from equation (44) the above equation can be written as

$$k\theta_i^{\gamma-1}(t_i - w_i - r_i) = \delta(\psi + \rho) \tag{47}$$

Workers face a similar problem as firms. U_i is the present discounted value of unemployment and W_i is the present discounted value of employment. Workers receive social benefits b if unemployed. $k\theta_i^{\gamma}$ gives the rate at which workers are employed and ψ the rate at which workers lose their job.

These considerations lead to the following two Bellman equations:

$$\rho U_i = b + k\theta_i^{\gamma}(W_i - U_i) \tag{48}$$

$$\rho W_i = w_i + \psi(U_i - W_i). \tag{49}$$

Note that the permanent income W_i is different from the actual wage rate w_i. This is caused by the risk of unemployment and hence a lower income. It is assumed that the wage will be higher than social benefits, i.e., $w_i > b$, so that an incentive to work exists.

Solving the above equation for W_i yields

$$W_i = \frac{w_i + \psi U_i}{\rho + \psi} \tag{50}$$

To derive the wage as given in equation (28) a common Nash bargaining model is used by way of which

$$w_i = \arg\max (W_i - U_i)^\eta (J_i - F_i)^{1-\eta}. \tag{51}$$

The corresponding first order condition yields

$$W_i - U_i = \frac{\eta}{1-\eta}(J_i - F_i). \tag{52}$$

First, substitute (46) and (50) in (52) and use $F_i = 0$ to obtain

$$w_i = \rho U_i + \eta(t_i - r_i - \rho U_i). \tag{53}$$

From equation (44) and $F_i = 0$ it follows that

$$J_i = \frac{\delta}{k\theta_i^{\gamma-1}}. \tag{54}$$

Replace J_i in equation (52) by equation (44) to obtain

$$W_i - U_i = \frac{\eta}{1-\eta}\left(\frac{\delta}{k\theta_i^{\gamma-1}}\right). \tag{55}$$

Now substitute equation (55) in (48) to derive

$$\rho U_i = b + \frac{\eta}{1-\eta}\delta\theta_i. \tag{56}$$

Use (56) with (53)

$$\begin{aligned} w_i &= b + \tfrac{\eta}{1-\eta}\delta\theta_i + \eta(t_i + r_i - b - \tfrac{\eta}{1-\eta}\delta\theta_i) \\ w_i &= (1-\eta)b + \tfrac{\eta}{1-\eta}\delta\theta_i + \eta t_i - \eta r_i - \tfrac{\eta^2}{1-\eta}\delta\theta_i \end{aligned} \tag{57}$$

which then simplifies to the wage equation as given by equation (28).

$$w_i = (1-\eta)b + \eta(t_i - r_i + \delta\theta_i) \tag{58}$$

The wage depends on three endogenous parameters: marginal revenues, t_i, (marginal) rental costs, r_i, and labor market tightness, θ_i. Substituting the expression for the t_i and r_i from equations (20) and (21) in the equations (58) yields

$$w_i = (1-\eta)b + \eta(\beta\overline{p}_i^{1/\beta} q_i A_i + \delta\theta_i). \tag{59}$$

θ_i can be defined as an implicit function by using equations (58) and (47).

The labor market tightness depends on marginal revenues, t_i, and marginal rental costs, r_i, as well:

$$t_i - r_i - [(1-\eta)b + \eta(t_i - r_i + \delta\theta_i)] - \frac{\delta(\rho+\psi)}{k\theta^{\gamma-1}} = 0. \tag{60}$$

After substituting t_i and r_i from equations (20) and (21), this simplifies to

$$(1-\eta)[\beta\overline{p}_i^{1/\beta} q_i A_i - b] - \eta\delta\theta_i - \frac{\delta(\rho+\psi)}{k\theta_i^{\gamma-1}} = 0. \tag{61}$$