RAND Journal of Economics
Vol. 34, No. 3, Autumn 2003
pp. 568–580

Procurement auctions and unit-price contracts

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In competitive procurement auctions, bids often have the form of unit-price contracts (UPCs). We show that optimal bidding behavior in UPC auctions is typically nonmonotonic and therefore may lead to inefficient allocations. However, UPC auctions may still be desirable for the buyer when compared to efficient mechanisms such as the first-price auction. In a UPC auction, low types are subsidized, and the resulting stronger competition reduces the winning bidder's informational rent, which overcompensates the efficiency loss.

1. Introduction

■ Tender auctions involving unit-price contracts (UPCs) constitute a form of mechanism that is widely used in public- and private-sector procurement. Examples include highway contracting (Stark, 1974), pipeline construction (Diekmann, Mayer, and Stark, 1982), procurement in the area of defense (Samuelson, 1983), and typically any large-scale governmental procurement in internationally supported developing countries (World Bank, 2000). Moreover, similar settings on the sales side, such as timber auctions (Athey and Levin, 2001) and the marketing of publishing rights for books (McAfee and McMillan, 1986), can readily be reinterpreted as unit-price contracting.[1]

A UPC procurement auction works as follows: Initially, the buyer estimates the quantities of the respective input factors that will be needed to accomplish a given task. Then the buyer publicly announces her estimates and asks contractors in the market to submit bids that specify a separate unit price for every input factor. From each incoming offer, the buyer calculates the estimated total costs, or the "score" of this offer. Then the firm with the lowest score wins the contract, which means that this firm carries out the works, and the buyer is obliged to pay for the input factors as needed.

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This article is a significantly extended version of the first essay in Fieseler (1999). Presentations of the material contained herein have been given at Stony Brook and Tilburg in 1997, at Toulouse in 2001, and at Magdeburg and Zurich in 2002. The article benefited substantially from the thoughtful comments made by the Editor and by two anonymous referees. We are especially grateful to our mentors Martin Hellwig and Benny Moldovanu, for their continuous guidance and unrelenting support. For valuable comments and discussions we also thank Susan Athey, Josef Falkinger, Leonardo Felli, Hans-Peter Grüner, David Kennedy, Thomas Kittsteiner, Preston McAfee, Georg Nöldeke, Jozsef Sakovics, Patrick Schmitz, Giancarlo Spagnolo, Konrad Stahl, Robert Stark, and Jan Vleugels.

[1] For a limited period, a form of unit-price bidding was also used in U.S. offshore and gas lease auctions (see Porter, 1995).

To get a flavor of the mechanics underlying unit-price contracting, consider the following example. A buyer estimates that the project under consideration requires 3 units of labor and 1 unit of materials. She calls for bids from interested contractors. In response to the tender, contractor A submits prices \$5 for labor and \$4 for materials. Then the buyer evaluates this bid in terms of estimated costs as \$19. Another contractor, B, submits prices \$4 for labor and \$8 for materials. This makes a cost estimate of \$20, which exceeds A's score. So the buyer would choose firm A, which promises to be less expensive. If, however, it turns out that contractor A needs an overtime of, say, 1 unit of labor, then the buyer pays a total of \$24, which is more than the payment to a timely firm B.

In this article we consider a simple model of a UPC auction with two input factors, referred to as materials and labor, and assume that there is uncertainty about the number of hours of labor needed to complete a prespecified project task. Two main results are derived. First, when contractors bid strategically in a UPC auction, then the most inefficient type wins the contract with strictly positive probability, and some less efficient contractors beat some more efficient contractors. This result reflects the general perception that in unit-price auctions, "lowest total bid need not mean lowest project cost" (see Stark, 1974, p. 380). Moreover, this is an auction-theoretic innovation vis-à-vis traditional models where bidding strategies and winning probabilities are in general monotonic in the bidder's type.

The nonmonotonicity of the equilibrium bidding strategies is a consequence of the fact that two forces determine the competitiveness of a bidder in a UPC auction: The first and more usual one is productive efficiency that allows low-cost bidders to submit low bids. This force, however, will sometimes be outweighed by another, which is the bidders' ability to create a gap between bid and payment. Specifically, this will be the case for bidders whose costs are structured very differently from the scoring rule. Those bidders will benefit from the UPC auction format and become more competitive.[2]

Our second main result says that although the above-mentioned nonmonotonicity may lead to inefficient allocations *ex post,* this will nevertheless be desirable *ex ante* for a payment-minimizing principal. The reason is that the UPC auction, by exploiting *ex post* observable information that is not used in standard auction formats such as the first-price auction, offers a subsidy to contractors with inferior production technologies. This subsidy increases competition for contractors with superior technologies, who therefore have to price more aggressively. When parameters are chosen appropriately, this leads to lower informational rents on the part of the bidders and therefore to lower expected contract outlays for the buyer.[3]

Given the described mechanism, the unsupervised contractor has an incentive to extend the project volume and thereby her profit margin by claiming that the project requires additional input quantities, e.g., more time (see Scherer, 1964, and Williamson, 1967). It is therefore natural to assume that the principal will deter excessive claims by monitoring, for example, in the form of random audits. Given that firms may in reality be excluded from an auction if they lack the reputational standard (see Burnett and Wampler, 1998), this may already be an effective instrument to reduce agency costs in repeated settings. In Section 6, we show with a simple example that the basic predictions of this article are unchanged in the presence of moral hazard even when the loss of reputation is not taken into account by the contractors. For most of the article, however, we will abstract from this complication and assume that the principal can closely monitor the project at negligible costs, so that agency problems will not occur.

As far as we can determine, our results are novel. There are only two contributions that we found to be closely related. Athey and Levin (2001) model a U.S. Forest Service sales auction

[2] This relates our analysis to the literature on countervailing incentives. See Lewis and Sappington (1989) and Cramton, Gibbons, and Klemperer (1987). For example, in the latter article, low types become sellers with a high probability and hence have an incentive to overstate their value, while high types tend to understate their value. Nevertheless, equilibrium strategies are monotone. We are grateful to Preston McAfee for directing our attention to this connection.

[3] For other approaches in which *ex post* observable information improves the performance of a mechanism over that of the first-price auction, see Hansen (1985) and Riley (1988).

in which the auctioneer and the bidders have incomplete information about the proportion in which two species of timber will be found in the tract that is offered for purchase. Bidders submit separate prices for each species. The tract is then sold to the bidder that will make the highest total payment under the assumption that the realized proportion conforms with the auctioneer's estimate. It is shown in particular that the equilibrium bid function is U-shaped in the bidder's estimate. The first and obvious difference to the present study is that Athey and Levin consider a common-value setting. As a consequence, efficiency is not an issue. Another distinction, which is more of a technical nature, is that in their model, either all bidders will have signals above or all bidders will have signals below the auctioneer's estimate; in our model, a bidder will typically face opponents faster and slower than the scoring rule.

A second related article is Samuelson (1986), who assumes that contractors are risk averse and face uncertainty about their own production costs. He shows that UPC-style auctions may induce an inefficient contractor to submit a bid that implies an insurance against cost uncertainty. As a consequence, this bidding institution may imply lower expected procurement costs for a risk-neutral buyer than alternative auction formats that impose uniform risk sharing. Note that Samuelson's ranking result is a consequence of risk considerations, which are absent from our formal analysis.

The rest of the article is organized as follows. In Section 2 we present the model. In Section 3 we show that the UPC auction typically leads to nonmonotonic bidding strategies. In Section 4 we characterize equilibrium bidding. We compare the performance of the UPC auction with other auction formats in Section 5. The possibility of moral hazard is considered in Section 6. Section 7 briefly summarizes the main findings and discusses factors that may affect the choice of UPC auctions in practice. The Appendix contains the proof of our payment-ranking result.

2. The model

■ Each of n risk-neutral contractors (or firms) needs 1 unit of materials and h_i hours to complete a prespecified project or task. The type h_i of each contractor i is private information to the firm and is drawn initially from a distribution with continuous density $f(h)$, which is assumed to be strictly positive on its support $[\underline{h}, \overline{h}] \subseteq (0, \infty)$. Bidder types are drawn independently.[4] After the project is completed, the actual number of hours h required is observable and verifiable.

We assume that input factor supply for the contractors is perfectly elastic, with prices for materials $c_M > 0$ and for labor $c_L > 0$. Hence, the total (expected) cost of completing the project for a contractor of type h is $C(h) = c_M + c_L h$, which is strictly increasing in the type.[5]

When the buyer uses a UPC auction, a contractor is selected and paid as follows: First, the buyer announces the *scoring weight* $s \in [\underline{h}, \overline{h}]$. Then, each contractor submits a bid, which is a pair of unit prices $p = (p_M, p_L)$, consisting of a price $p_M \geq 0$ per unit of materials and a price $p_L \geq 0$ per hour of labor. Incoming bids are evaluated on the basis of the *score* $S(p) = p_M + p_L s$. The contract is assigned to the contractor whose bid has the lowest score, where ties are broken randomly. After completion of the project, the employed contractor obtains the payment $P(p, h) = p_M + p_L h$. Thus, if a bidder of type h bidding p wins the auction, she obtains a conditional payoff $\pi(p, h) = P(p, h) - C(h)$. When bidders use bidding functions $\{p^i(\cdot)\}_{i=1,\dots,n}$, we may write the *probability of winning* for a player i of type h_i as[6]

$$Q(p, h_i) = \text{Prob}\left\{S(p^i(h_i)) \leq S(p^j(h_j)) \quad \text{for all } j \neq i\right\}. \tag{1}$$

Then, expected profits for a bidder are $\Pi(p, h) = Q(p, h)\pi(p, h)$.

[4] This seems a plausible assumption for procurement auctions, where the choice of contractor is understood to be a decisive determinant for the project's performance (see Burnett and Wampler, 1998).

[5] Although we assume linear costs throughout the article, one can check that the given proofs depend essentially on the assumption that average costs are strictly decreasing.

[6] Strictly speaking, for equation (1) to hold it is necessary that the measure of the set of types h submitting bids with the same score S be zero. The extension to the general case is straightforward and therefore omitted.

3. Nonmonotonic bidding

■ When unit-price contracting is used in practice, one often observes a peculiar pattern in the composition of the price list. The list of a seller typically contains prices that are below expected average costs for the respective input factor, while other inputs appear overpriced. Sometimes prices even drop to zero. This feature is known as "unbalancing the tender," or "loading the bill rates," or simply "skewed bidding." In our simple model with risk-neutral agents, strategic bid distortion assumes the following extreme form.

We refer to types $h < s$ as *fast*, and to all other types as *slow*.

Proposition 1 (one-sided bidding). In a UPC auction, fast types submit labor-included bids (p_L = 0), and slow types submit materials-included bids ($p_M = 0$). In fact, any alternative bid is either equivalent or dominated. In particular, the score is given by

$$S(p) = \begin{cases} p_M & \text{if } h < s \\ p_L s & \text{if } h \geq s. \end{cases} \tag{2}$$

Proof. Let $h \in [\underline{h}, \overline{h}]$. For a given total bid S, the composition problem is max $p_M + p_L h$ subject to $p_M + p_L s = S$ and $p_M, p_L \geq 0$. The conclusion is apparent. *Q.E.D.*

The above proposition should be intuitively obvious. If a contractor knows that the buyer's time estimate s is too low, then she will submit a higher price for labor and adapt the price for materials such that the score remains unaffected. Similarly, if a contractor knows that the buyer's time estimate is too high, then she will reduce the price for labor and compensate by increasing the price for materials.

Proposition 1 is consistent with anecdotal evidence that can be found in the applied literature. A particularly illustrative example is found in Gates (1967), who reports on a construction auction in which the winning Connecticut contractor set the unit price for "trench sheeting" to one cent because he anticipated correctly that he would not need this input factor. To give another example, in an examination of public records of highway bidding data, Stark (1974, p. 375) reports a "distinct variation in the unit bids for many items," with the unit price for "clearing and grubbing" in one example varying by a factor of six.

While Proposition 1 predicts all one-sided bids, unbalancing will typically be less pronounced, for the following reasons. First, there may be reputational considerations that make such bids unfeasible. For example, firms may wish to avoid perceived pricing anomalies, and possibly conceal firm-specific pricing policies (see Burnett and Wampler, 1998). Moreover, it is common legal practice that a client may reject unbalanced bids, e.g., because prices are "too extreme."[7] However, as the terms lack precision, these provisions are rarely used (see Stark, 1974). Another reason why bids are not always all one-sided is that bidders may be risk averse. Indeed, the heuristics used in the applied literature weigh the expected benefit from skewing against the risks from having a wrong assessment of the quantity requirements. While all these factors will reduce the extent of unbalancing, we believe that the basic predictions of our model are not likely to be affected, so that we will abstract from these considerations.

Unbalancing may in principle also occur as a consequence of the fact that the construction usually takes time, and the contractor obtains periodic payments that are based on actual rather then estimated quantities. For example, Diekmann, Mayer, and Stark (1982, p. 379) propose the following strategy: "It is advantageous for a contractor to tender relatively high unit prices on items scheduled for early completion and proportionately lower prices for other items. In this way, monies are obtained earlier to help finance later stages of the job." It has even been suggested that manipulations should involve the sequence and time at which items are executed. However, it also has been argued that these aspects of bid distortion may be of minor importance in practice (see Green, 1989).

[7] A reason for such behavior by the auctioneers may be that it reduces the probability of having to deal with contractors that have their back to the wall after having misestimated their factor requirements.

Proposition 1 has the useful consequence that the bid vector $p = (p_{M,} p_L)$ can be determined alone from the score S and from the information whether the corresponding type is fast or slow. It is therefore feasible to assume that bidders choose a total bid $S = S(h) \geq 0$ rather than the price vector p. This reduces the dimension of the action space to one and will mean a significant simplification for the subsequent analysis.[8] We can also see more clearly the payoff consequences of strategic bidding. Indeed, it is clear that slow types will improve upon their score because they bid only $S = sp_L$ but obtain a payment of $P = hp_L \geq S$. In contrast, fast types realize *ex post* precisely their materials-included bid $P = p_M = S$. Thus, when the bidders' strategic behavior is taken into account, then the UPC auction effectively means a subsidy to slow types.

From the above, it follows that a contractor of type h chooses score $S = S(h)$ so as to maximize expected profits $\Pi(S, h) = Q(S)\pi(S, h)$, where

$$\pi(S, h) = \begin{cases} S - C(h) & \text{if } h < s \\ S\dfrac{h}{s} - C(h) & \text{if } h \geq s. \end{cases} \tag{3}$$

This expression reflects the remark made before: Each type of contractor has to pay its cost, but the payment differs between fast and slow contractors. Fast firms obtain their bid, while slow firms are subsidized.

The resulting subsidy to slow contractors is so strong that for these types, productive efficiency becomes irrelevant as a determinant of optimal bidding behavior. Indeed, it turns out that the slower the contractor, the stronger the effect of the subsidy, . This is so because a slow type's profit per hour is $S/s - C(h)/h$, so that her competitive advantage in the auction depends on her *average* costs rather than her total costs. As average costs are decreasing in the type, the effect works in favor of the inefficient contractor. The subsidy thereby turns around the order of winning probabilities from type s onward, so that among slow types, slower contractors have a better chance to win. These considerations are summarized in our first main result.

Theorem 1 (nonmonotonicity). The bidding function is inversely U-shaped. More precisely, in the UPC auction, any best response $S(h)$ is nondecreasing for $h \leq s$ and nonincreasing for $h \geq s$.

Proof. The profit function $\Pi(S, h)$ is supermodular in (S, h) for $h \in [\underline{h}, s]$. Hence, from Topkis' monotonicity theorem (see, for example, Milgrom and Roberts, 1990), any best reply $S(h)$ is nondecreasing on $[\underline{h}, s]$. On the other hand, because average costs are decreasing, the hourly profit function $\Pi(S, h)/h$ is submodular for $h \in [s, \overline{h}]$. Thus, any best response $S(h)$ is nonincreasing on $[s, \overline{h}]$. *Q.E.D.*

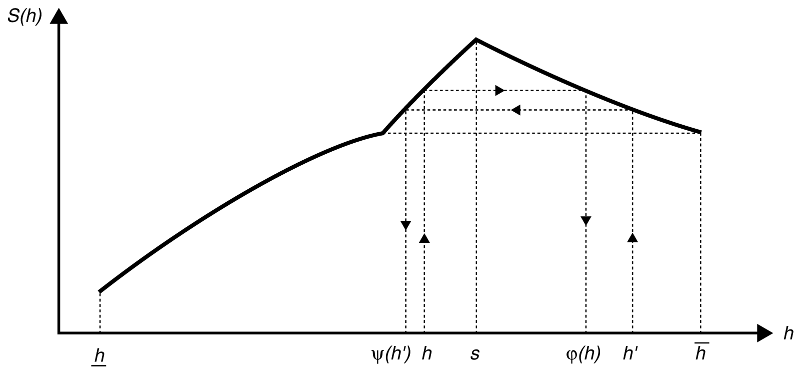
This theorem says that any optimal bidding strategy will have a unimodal shape. The main driver for the nonmonotonicity is that the scoring rule neglects the quantity dimension. In selecting the winner according to the weight s on labor prices, the scoring rule gives too much credit to the slow types, who actually need more labor than s. In paying the winner by the actual amount of labor (instead of s), the auction rewards the slow types for being slow. Given the same unit price for labor, a slow-type bidder would get a higher payment than would a fast winner. Therefore, slow types can profitably lower their bids to compete with fast types.

Figure 1 shows an equilibrium bidding strategy for a uniform-type distribution. In general, the shape of the score function depends on the parameters of the auction. However, we will see later that there are essentially two cases. First, when s lies more on the left in the interval $[\underline{h}, \overline{h}]$, then the bidding function corresponds to the depicted form. In the other case, the bidding function would look like a distorted mirror image of the graph in Figure 1. To avoid clumsy case distinctions, the subsequent exposition will focus on the first case. Proofs will be general, however.

[8] A similar technique has been used by Che (1993).

FIGURE 1

THE EQUILIBRIUM BIDDING FUNCTION



4. Equilibrium analysis

■ There is another helpful consequence of considering the score as the bidder's choice variable. From (3) one can easily see that if a fast type h and a slow type h' satisfy

$$C(h) = C(h')\frac{s}{h'}, \tag{4}$$

then types h and h' have objective functions that are identical up to a positive factor. As a consequence, these two types will choose the same score. This basic insight allows us to define a simple correspondence between slow and fast types, as illustrated in Figure 1. Specifically, for a given fast type h, define $\varphi(h)$ as the unique slow type h' that satisfies (4), if it exists, and as $\overline{h}$ otherwise. Note that because average costs are strictly decreasing, this definition is unambiguous. Analogously, for a given slow type h', define $\psi(h')$ as the unique fast type h that satisfies (4), if it exists, and as $\underline{h}$ otherwise. The working of these maps is illustrated in Figure 1. One can check that the mappings $\varphi(h)$ and $\psi(h)$ are nonincreasing, continuous, piecewise differentiable, and fully determined by the exogenous parameters s, c_M, and c_L. These functions are helpful because they allow determination of the probability of winning from the parameters of the model in spite of the nonmonotonicity of the bidding functions.

To this end, let $S(h)$ be the bidding function in a symmetric equilibrium of the UPC auction. Assume that $S(h)$ is strictly increasing for fast types and strictly decreasing for slow types. It is not difficult to see that under these strict monotonicity assumptions, a fast type h wins against all types between h and $\varphi(h)$ and loses against all other types. Similarly, a slow type h wins against all types between $\psi(h)$ and h and loses against all other types. Thus, for any type h, the probability of winning the contract is

$$Q(h) = \begin{cases} \{F(\varphi(h)) - F(h)\}^{n-1} & \text{if } h < s \\ \{F(h) - F(\psi(h))\}^{n-1} & \text{if } h \geq s, \end{cases} \tag{5}$$

where $F(h)$ is the distribution function over types.

The derivation of the equilibrium is now essentially standard. Given an equilibrium candidate $S(h)$, a necessary condition for equilibrium is that no bidder of type h has an incentive to mimic another type $\hat{h}$. Consider first the case of a fast type h that mimics another fast type $\hat{h}$. Assuming differentiability of $S(h)$ and $Q(h)$ at h for the moment, we have the necessary first-order condition

$$\frac{\partial}{\partial \hat{h}}\{Q(\hat{h})(S(\hat{h}) - C(h))\}\bigg|_{\hat{h}=h} = 0 \tag{6}$$

for any two fast types h and $\hat{h}$. This yields the differential equation

$$\frac{S'(h)}{S(h) - C(h)} = -\frac{Q'(h)}{Q(h)}. \tag{7}$$

When we choose the initial condition $S(s) = C(s)$, this equation has the unique solution

$$S(h) = C(h) + \frac{1}{Q(h)} \int_h^s C'(\eta)Q(\eta)d\eta, \tag{8}$$

which determines the bidding function of a fast type. When actual costs $C(h)$ are replaced by "scaled" average costs $C(h)s/h$ in the above derivation, equation (8) delivers analogously the bidding function for slow types. We show below that, indeed, the solution obtained both is a global maximizer of the bidder's optimization problem and satisfies the strict monotonicity assumptions made before, so that we have found an equilibrium in the UPC auction.

Proposition 2 (equilibrium). Assume that $Q(h)$ is the probability of winning for type h, as defined by (5). Then the following bidding strategies constitute a symmetric equilibrium in the unit-price auction:

$$S(h) = \begin{cases} C(h) + \dfrac{c_L}{Q(h)} \displaystyle\int_h^s Q(\eta)d\eta & \text{for } h < s \\ C(h)\dfrac{s}{h} + \dfrac{c_M}{Q(h)} \displaystyle\int_s^h \dfrac{s}{\eta^2} Q(\eta)d\eta & \text{for } h \geq s. \end{cases} \tag{9}$$

Proof. We show first that $S(h)$, as given by (9), satisfies the strict monotonicity properties assumed for the calculation of the winning probability. Assume that $Q(h)$ is defined by (5). Then $Q(h) > 0$ for all $h \neq s$ because the type distribution has full support. Thus, from (9), $S(h)$ is strictly increasing in $[\underline{h}, s]$ and strictly decreasing in $[s, \overline{h}]$. Hence, $Q(h)$ as defined by (5) corresponds indeed to type h's probability of winning. We can now show that (9) describes an equilibrium. We consider first the case where $S(\underline{h}) \leq S(\overline{h})$ (see Figure 1). Note first that it does not pay for a contractor to bid strictly above $S(s)$, because any bid $S \geq S(s)$ implies a zero probability of obtaining the contract and therefore an expected payoff of zero. Similarly, no contractor will submit a bid strictly below $S(\underline{h})$, because any bid $S \leq S(\underline{h})$ wins with certainty, but payment is increasing with S. Consider now a fast type h. Using the explicit form (8), we find

$$\frac{\partial}{\partial \hat{h}} \{Q(\hat{h})(S(\hat{h}) - C(h))\} = (C(\hat{h}) - C(h))Q'(\hat{h}) \tag{10}$$

for all $\hat{h}$ at which $Q(\hat{h})$ and $S(\hat{h})$ are differentiable. As both $Q(\hat{h})$ and $S(\hat{h})$ are continuous and piecewise differentiable, where $Q'(\hat{h}) < 0$ whenever defined, a consideration of the signs of the right-hand side of (10) shows that $\hat{h} = h$ is a global optimum. An analogous consideration shows that no slow type will make a bid S that is made by some other slow type. Does it pay for a slow type h' to make a bid $S(h)$ that corresponds to a fast type $h < \psi(\overline{h})$? No, because otherwise the corresponding fast type $\psi(h') > \underline{h}$ would also find it advantageous to mimic h, which would contradict our previous findings. Thus, the bid function $S(h)$ specified in (9) describes an equilibrium when $S(\underline{h}) \leq S(\overline{h})$. For the case $S(\underline{h}) > S(\overline{h})$, one considers a deviation of a slow type and replaces $C(h)$ by $C(h)s/h$ in (10). The argument is then completely analogous, so that we have proved the assertion. *Q.E.D.*

The equilibrium bids specified by Propositions 1 and 2 allow a simple interpretation. A fast type offers a labor-included contract $p = (p_M, 0)$, where p_M sums up actual costs $C(h)$ and a markup due to the informational asymmetry. Similarly, a slow type offers a materials-included contract $p = (0, p_L)$, where p_L sums up average costs $C(h)/h$ and an informational rent.

The formal expressions in Proposition 2 also show that the two kinks in the bidding function in Figure 1 are characteristic for equilibrium bids. First, the kink at s is due to the one-sided strategic bidding behavior of fast and slow types described in Proposition 1. The second kink at $\psi(\overline{h})$ is due to the fact that all types left of $\psi(\overline{h})$ win the auction against all slower types, while all types slightly right of $\psi(\overline{h})$ win only against a subset of all slower types. In the sequel, we will refer to types $h < \psi(\overline{h})$ as *very fast* and to fast types $h \geq \psi(\overline{h})$ as *moderately fast*.

5. Payment ranking

■ In this section we compare the expected payment for the buyer in the UPC auction with that resulting from standard auction methods such as the first-price sealed-bid auction, the second-price auction, or the English auction. We will show that there exists a choice of the scoring weight s^* such that the principal has strictly lower total contract outlays in a UPC auction in comparison to these alternative auction formats.

The payment comparison is simplified because of the following two facts. First, recall that efficient standard auctions do not make use of *ex post* observable information. Thus, given our informational assumptions, we know from the Revenue Equivalence Theorem (see Myerson, 1981) that all efficient auctions in which only the winner is paid are equivalent in terms of expected payments. Hence, with respect to expected payment, the first-price auction can be used as a representative of a more general class of standard auctions.

Second, note that the first-price auction can be considered as a UPC auction in which the scoring weight s satisfies $s = \overline{h}$. This is because in a UPC auction with $s = \overline{h}$, all types are fast and submit fixed-price offers by one-sidedness, thereby making the payment independent of the *ex post* observation. Hence, the two auction formats are equivalent for $s = \overline{h}$ in terms of expected payments for the buyer.

Given the previous arguments, it suffices to compare the UPC auction and first-price auctions via comparative statics with respect to s. Because of symmetry, expected payments for the buyer equal n times the *ex ante* expected payment to bidder i, which is given by

$$P_i(s) = \int_{\underline{h}}^{s} S(h,s)\, Q(h,s) dF(h) + \int_{s}^{\overline{h}} \frac{h}{s} S(h,s)\, Q(h,s) dF(h). \tag{11}$$

Replacing $S(h, s)$ by the expressions given in Proposition 2 and subsequently changing the order of integration yields the following useful description of the buyer's expected payment to bidder i as

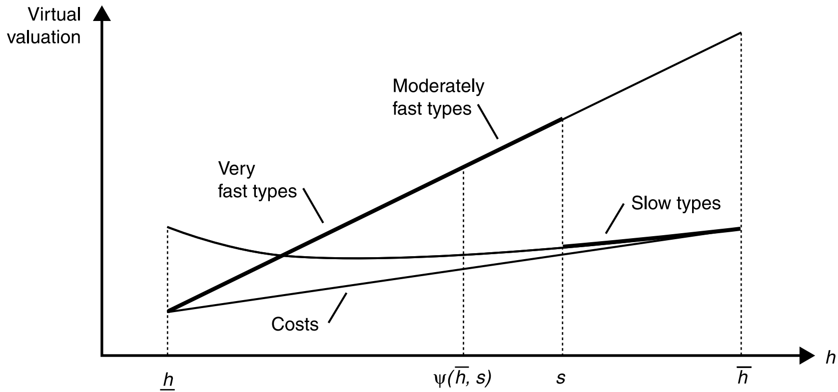
$$\begin{aligned} P_i(s) = &\int_{\underline{h}}^{s} Q(h,s) \left[C(h) + c_L \frac{F(h)}{f(h)} \right] dF(h) \\ &+ \int_{s}^{\overline{h}} Q(h,s) \left[C(h) + \frac{c_M}{h} \frac{E[\eta \mid \eta \geq h]}{h} \frac{1 - F(h)}{f(h)} \right] dF(h), \end{aligned} \tag{12}$$

where $E[\eta \mid \eta \geq h]$ is the expected value of type η with respect to the distribution $F(\eta)$, conditional on the event that η is larger than or equal to h. The usefulness of this equation comes from the fact that it assigns to each type h a marginal contribution to the principal's expected payment, weighted by the respective type's winning probability. The marginal contribution of type h, referred to as her *virtual valuation*, is the respective expression in the bracket. It differs from actual costs by a term that captures the negative externality that the respective type incurs on the principal by allowing other types to realize a higher informational rent. In Figure 2 we have drawn the virtual valuations for the uniform distribution and the same parameters as before.

Assume that s is close to $\overline{h}$. We wish to demonstrate that increasing s marginally will increase the above integral over the virtual valuations and thereby the principal's expected contract outlays. We start by noting that when we slightly increase s, the probability of winning shifts from slow types to fast types. To see this, consider again Figure 1, and recall that the probability of winning corresponds to the probability weight of the interval between a type and his corresponding type.

FIGURE 2

INFORMATIONAL RENTS IN THE UPC AUCTION



If s shifts upward marginally, then the probability of winning remains constant for very fast types $h \in [\underline{h}, \psi(\overline{h}, s)]$ because the corresponding type $\overline{h}$ does not move. However, the probability increases for moderately fast types $h \in [\psi(\overline{h}, s), s]$ because the corresponding fast type $\varphi(h, s)$ moves to the right. Finally, the probability of winning decreases for slow types $h \in [s, \overline{h}]$ because here also, the corresponding slow type $\psi(h, s)$ moves to the right. In total, therefore, a marginal increase of s shifts weight from slow types to moderately fast types.[9]

The argument is now driven by a useful property of the virtual valuation that is illustrated in Figure 2, and which follows from equation (12): For s sufficiently close to $\overline{h}$, the virtual valuation of *any* moderately fast type is higher than the virtual valuation of *any* slow type. We have shown that when the buyer slightly increases s, he shifts probability from slow types to moderately fast types. Because moderately fast types have higher virtual valuations than slow types, the expected payment for the buyer increases. This holds for all s sufficiently close to $\overline{h}$, so that setting $s = \overline{h}$ is always suboptimal. We have shown the following:

Theorem 2 (superiority of the UPC auction). There always exists a UPC auction that leads to lower expected payments than does the standard first-price auction.

Proof. See the Appendix.

Although the above result proves the existence of a scoring weight s for which the UPC auction beats the first-price auction, the argument used is nonconstructive. To determine the optimal scoring weight s^*, we return to the general case, where it could happen that the most inefficient type submits the lowest bid, and differentiate expression (12) for the expected payment to bidder i. Noting that the winning probability is a continuous function of h, and that a marginal change in s leaves the winning probability of very fast types unaffected, one arrives at the necessary first-order condition

$$\int_{\psi(\overline{h},s)}^{s} Q_s(h, s)\left[C(h) + c_L \frac{F(h)}{f(h)}\right] dF(h)$$
$$= -\int_{s}^{\varphi(\underline{h},s)} Q_s(h, s)\left[C(h) + \frac{c_M}{h}\frac{E[\eta \mid \eta \geq h]}{h}\frac{1 - F(h)}{f(h)}\right] dF(h). \qquad (13)$$

This equation emphasizes the tradeoff of virtual valuations between moderately fast and (analogously defined) moderately slow types (see Figure 2). When (13) holds, then from a marginal

[9] Perhaps contrary to intuition, marginal type s is irrelevant in the analysis of expected payments (because she wins with probability zero).

increase in s, the increase in the aggregate virtual valuation of moderately fast types is equal to the reduction in the aggregate virtual valuation of moderately slow types. Thus, when the expected payment for the buyer is minimized, s^* satisfies (13).

6. Moral hazard

■ As mentioned in the Introduction, unit-price contracting may engender agency problems because the chosen contractor has an incentive to overstate input factor requirements at the buyer's expense. It is clear that this possibility is likely to reduce the benefits of unit-price contracts when compared to, e.g., fixed-price agreements. However, as we now want to show with the help of a simple example, the structure of equilibrium bidding is not necessarily affected.

To this end, assume that in contrast to our previously made assumptions, costs are now unverifiable, and that the contractor, after having invested h hours to complete the project, sends an invoice to the buyer in which she requests payment for $\hat{h}$ hours.[10] Assume also that by the end of the project, the buyer has obtained (unverifiable) private information about the actual number of hours worked. For example, the buyer might have visited the project site a few times or received useful information about the project from industry insiders. For tractability, we assume that the buyer's signal is of the form $\tilde{h} = h + \varepsilon$, where $\varepsilon \geq 0$ is exponentially distributed with mean $1/\lambda$ (so that a higher λ corresponds to a better signal). This implies in particular that the buyer knows with certainty that the actual number of hours worked is not larger than her private signal.[11]

Upon receipt of the invoice, the buyer compares the number of hours claimed, $\hat{h}$, with her private information, $\tilde{h}$. If $\hat{h} > \tilde{h}$, then she must conclude that the contractor has overstated the number of hours. Under this condition, the buyer is assumed to initiate an audit that reveals the contractor's type. For example, industry experts may then be asked to analyze the contractor's production technology in some detail and determine what, given this technology, a "reasonable" termination time for the project would have been. In this case, the contractor receives just the amount she could have charged when being honest and faces a multiple of her hourly charge κp_L as a contractual penalty, where $\kappa \geq 0$. If the number of hours claimed does not exceed the buyer's signal, i.e., if $\hat{h} \leq \tilde{h}$, then the buyer is assumed to accept the invoice and to pay the requested amount without deductions.

The contractor's problem is to balance the benefits from the excess claim against the higher probability of an audit. Clearly, under a fixed-price contract, there is no point in overstating hours, so that in this case, $\hat{h} = h$. Assume therefore $p_L > 0$. Then expected revenues for a contractor of type h that claims payment for $\hat{h} \geq h$ hours are given by

$$R(\hat{h}, h) = (1 - e^{-\lambda(\hat{h}-h)})(p_M + (h - \kappa)p_L) + e^{-\lambda(\hat{h}-h)}(p_M + \hat{h}p_L). \tag{14}$$

Since costs are unaffected by the contractor's decision, maximizing expected profits is equivalent to maximizing (14). It is not hard to show that this problem possesses the unique solution $\hat{h} = h+\delta$, where

$$\delta = \max\left(\frac{1}{\lambda} - \kappa, 0\right) \tag{15}$$

is the type-independent optimal excess claim. The expected revenue for contractor h in anticipation of overstating possibilities is therefore

$$\max_{\hat{h}} R(\hat{h}, h) = p_M + (h + \delta')p_L, \tag{16}$$

[10] In an alternative interpretation, payment is made dependent on the verifiable project duration, and the contractor delays the project deliberately. The subsequent analysis is consistent with this interpretation when the delay is costless for the contractor.

[11] Alternative assumptions are surely possible but would make the determination of bidding behavior somewhat involved. Since our intention is only to give an example, we will not try to be more general here.

where

$$\delta' = e^{-\lambda\delta}\delta - (1 - e^{-\lambda\delta})\kappa \geq 0. \tag{17}$$

Thus, under moral hazard, the expected revenues to a bidder of type h from any bid are simply as if her type were $h + \delta'$.

The above exercise allows us to apply the arguments already employed in the basic model of UPC auctions. Specifically, by the logic of one-sided bidding, all types $h < s - \delta'$ submit labor-included contracts, while all other types submit materials-included contracts. Intuitively, those fast types that are close to the scoring weight now choose materials-included contracts, since only this type of contract allows them to exploit the benefits of excess claims. Payoff consequences are as follows. Conditional on winning the contract, expected profits for types $h < s - \delta'$ are $\pi(S, h) = S - C(h)$ as before, while expected profits for types $h \geq s - \delta'$ are now given by

$$\pi(S, h) = S\frac{h + \delta'}{s} - C(h). \tag{18}$$

Thus, the subsidy from unit-price contracting will be stronger and extend to more types in the presence of moral hazard.

Equation (18) implies that the predictions of our model remain valid when the expected benefits from excess claims are not too high. Specifically, when $\delta' < c_M/c_L$, it is not difficult to show that adjusted average costs $C(h)/(h + \delta')$ are strictly decreasing in the type h. Using analogous arguments as before, the bidding function can then be seen to be inversely U-shaped with mode $s - \delta'$. This shows that moral hazard need not necessarily affect the predictions of our model.

7. Concluding remarks

■ In this article, we have analyzed the strategic behavior of profit-maximizing contractors in a stylized model of the UPC auction format. The central insight should be that while the UPC auction will sometimes select a contractor with an inferior production technology, this will nevertheless be beneficial for the auctioneer because contractors with superior technologies will have to bid more aggressively.

A point to note is that UPC auctions are probably not optimal from a general mechanism-design perspective (see, e.g., Riordan and Sappington, 1988). In fact, the buyer could in principle freely exploit the bidders' rents using the following mechanism:[12] Initially, she asks the bidders to reveal their types. Then the supposedly fastest type obtains the contract. *Ex post,* the buyer reimburses the chosen contractor for the verifiable project costs. As no firm can make a profit under these assumptions, the described mechanism would in fact lead to truthful-type revelation, efficiency, and minimal payments for the principal. However, in a more realistic setting, it is much less likely that the bidders' informational rents can be costlessly extracted in such a simple way, if at all.[13]

The above consideration points to the question of why a unit-price contract is used in practice rather than some other, not necessarily optimal, contract that also employs *ex post* cost information. We see four main reasons for this. First, unit-price contracts allow for the possibility of *risk sharing* with respect to exogeneous shocks. Indeed, as mentioned in the Introduction, unit prices can transfer risks due to uncertain factor requirements from the contractor to the buyer (see Samuelson, 1986). Second, the use of unit prices in procurement may help to reduce the *cost of renegotiation,* especially in relationships involving significant uncertainty and complexity (see Bajari and Tadelis, 2001). Third, unit-price contracts are *easy to work with*, because their linear

[12] We are grateful to Patrick Schmitz for pointing this out.

[13] For example, a full extraction of rents may be infeasible in the presence of both moral hazard and limited liability.

structure helps to simplify both bidding and settlement. Fourth, as for many other institutions, there may also be some element of *history dependence* in their use.

Thus, in comparison to alternative auction methods, UPC auctions may lead to more economical procurement, allow for more efficient risk sharing, and reduce the costs of renegotiation without adding complexity, which may explain why they are used so pervasively in practice.

Appendix

■ *Proof of Theorem 2.* The formal argument follows the discussion in Section 5. Define virtual valuations

$$V(h) = C(h) + c_L \frac{F(h)}{f(h)} \tag{A1}$$

$$W(h) = C(h) + \frac{c_M}{h} \frac{E[\eta \mid \eta \geq h]}{h} \frac{1 - F(h)}{f(h)}. \tag{A2}$$

The first derivative of (12) with respect to s is then given by

$$P_i'(s) = \int_{\psi(\overline{h},s)}^{s} Q_s(h, s)V(h)dF(h) + \int_{s}^{\overline{h}} Q_s(h, s)W(h)dF(h), \tag{A1}$$

where we used the facts that $Q(s, s) = 0$, that $Q(h, s)$ is continuous at $\psi(\overline{h}, s)$, and that $Q_s(h, s) = 0$ for $h < \psi(\overline{h}, s)$. It is clear that $V(\overline{h}) > W(\overline{h})$. As $V(\cdot)$, $W(\cdot)$, and $\psi(\overline{h}, \cdot)$ are continuous, there exists an s close to $\overline{h}$ such that $V(h) > W(h')$ for any $h \in [\psi(\overline{h}, s), s]$ and any $h' \in [s, \overline{h}]$. Moreover, as $Q(h, s)$ is the probability of winning for type h, we have

$$\int_{\underline{h}}^{\overline{h}} Q(h, s)dF(h) = \frac{1}{n}, \tag{A4}$$

and therefore

$$\int_{\psi(\overline{h},s)}^{\overline{h}} Q_s(h, s)dF(h) = 0. \tag{A5}$$

Finally, $Q_s(h, s) < 0$ for $h \in [\psi(\overline{h}, s), s]$ and $Q_s(h, s) > 0$ for $h \in [s, \overline{h}]$, so that $P_i'(s) > 0$ for all s sufficiently close to $\overline{h}$. *Q.E.D.*

References

ATHEY, S. AND LEVIN, J. "Information and Competition in U.S. Forest Service Timber Auctions." *Journal of Political Economy*, Vol. 109 (2001), pp. 375–417.

BAJARI, P. AND TADELIS, S. "Incentives Versus Transaction Costs: A Theory of Procurement Contracts." *RAND Journal of Economics*, Vol. 32 (2001), pp. 387–407.

BURNETT, J.E. AND WAMPLER, B.M. "Unit Price Contracts: A Practical Framework for Determining Competitive Bid Prices." *Journal of Applied Business Research,* Vol. 14 (1998), pp. 63–72.

CHE, Y.-K. "Design Competition Through Multidimensional Auctions." *RAND Journal of Economics*, Vol. 24 (1993), pp. 668–680.

CRAMTON, P., GIBBONS, R., AND KLEMPERER, P. "Dissolving a Partnership Efficiently." *Econometrica*, Vol. 55 (1987), pp. 615–632.

DIEKMANN, J.E., MAYER, R.H., AND STARK, R.M. "Coping with Uncertainty in Unit-Price Contracting."*Journal of the Construction Division*, Vol. 108 (1982), pp. 379–389.

FIESELER, K. "Three Essays in the Theory of Mechanisms." Ph.D. dissertation, Department of Economics, University of Mannheim, 1999.

GATES, M. "On Competitive Bidding Strategies for the Construction Industry." *Journal of the Construction Division*, Vol. 93 (1967), pp. 75–95.

GREEN, S.D. "Tendering: Optimization and Rationality." *Construction Management and Economics*, Vol. 7 (1989), pp. 53–63.

HANSEN, R.G. "Auctions with Contingent Payments." *American Economic Review*, Vol. 75 (1985), pp. 862–865.

LEWIS, T.R. AND SAPPINGTON, D.E.M. "Inflexible Rules in Incentive Problems." *American Economic Review*, Vol. 79 (1989), pp. 69–84.

MCAFEE, R.P. AND MCMILLAN, J. "Bidding for Contracts: A Principal-Agent Analysis." *RAND Journal of Economics*, Vol. 17 (1986), pp. 326–338.

MILGROM, P. AND ROBERTS, J. "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities." *Econometrica*, Vol. 58 (1990), pp. 1255–1277.

MYERSON, R.B. "Optimal Auction Design." *Mathematics of Operations Research*, Vol. 6 (1981), pp. 58–73.

PORTER, R.H. "The Role of Information in U.S. Offshore Oil and Gas Lease Auctions." *Econometrica*, Vol. 63 (1995), pp. 1–27.

RILEY, J.G. "Ex Post Information in Auctions."*Review of Economic Studies*, Vol. 55 (1988), pp. 409–429.

RIORDAN, M.H. AND SAPPINGTON, D.E.M. "Optimal Contracts with Public ex post Information." *Journal of Economic Theory,* Vol. 45 (1988), pp. 189–199.

SAMUELSON, W.F. "Competitive Bidding in Defense Contracting." In R. Engelbrecht-Wiggans, M. Shubik, and R.M. Stark, eds., *Auctions, Bidding, and Contracting: Uses and Theory*. New York: New York University Press, 1983.

———. "Bidding for Contracts." *Management Science*, Vol. 32 (1986), pp. 1533–1550.

SCHERER, F.M. "The Theory of Contractual Incentives for Cost Reduction." *Quarterly Journal of Economics*, Vol. 78 (1964), pp. 257–280.

STARK, R.M. "Unbalanced Highway Contract Tendering." *Operational Research Quarterly*, Vol. 25 (1974), pp. 373–388.

WILLIAMSON, O.E. "The Economics of Defense Contracting: Incentives and Performance." In R. McKean, ed., *Issues in Defense Economics*. New York: Free Press, 1967.

WORLD BANK. *Standard Bidding Documents, Procurement of Works*, 2d. ed. Washington, D.C.: World Bank, 2000.