ENDOGENOUS TECHNOLOGICAL SPILLOVERS: CAUSES AND CONSEQUENCES

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Abstract:

We develop a new approach to endogenizing technological spillovers. We analyze a game in which firms can first invest into cost-reducing R&D, then compete on the human capital market for their knowledge-bearing employees, and finally enter the product market. If R&D-employees change firms, spillovers arise. We show that technological spillovers are most likely when they increase total industry profits. We use this result to show that innovation incentives are usually stronger for endogenous than for exogenous spillovers and that endogenous spillovers may reverse the result that innovation incentives are stronger under quantity competition than under price competition. Finally, we explore the robustness of our results with respect to contractual incompleteness and the number of R&D-workers.

Keywords: Innovation, Spillovers, Product Market Competition, Contracts
1. Introduction

Knowledge spillovers play a role in various economic contexts. Growth theorists have focused on the importance of knowledge creation and diffusion (Romer 1986, 1990, Aghion and Howitt 1992, Grossman and Helpman 1991). At least since Marshall (1920), spillovers have been identified as a possible source of agglomeration, and hence as potentially important for economic geography.¹ Most of the existing literature on R&D and research joint ventures has emphasized that spillovers drive a wedge between private and social returns, thereby leading to inefficient R&D levels and the need for cooperation (e.g. d'Aspremont and Jacquemin 1988, Kamien, Muller, and Zang 1992, Suzumura 1992).² When spillovers are modeled, they are usually assumed to be exogenous. However, spillovers are a result of decisions made by economic agents: firms have to take costly actions to acquire spillovers, and they may be able to prevent spillovers at some cost. In this paper, we therefore treat spillovers as endogenous, and we explore how this modification affects familiar results on innovation incentives.

The particular route towards endogenization of spillovers that we follow in this paper uses the human capital market. It is widely acknowledged that spillovers arise when employees change jobs and take all their knowledge with them, some of which is not specific to their original firm.³ There are many prominent examples where firms have hired managers, scientists and other employees to obtain knowledge in this fashion. At the heart of a controversy between General Motors and Volkswagen in the nineties was VW’s desire to get access to knowledge of the former GM employee Lopez and his group. Similarly, after German banks moved to London to establish a stronger position in investment banking, they hired a considerable number of employees from competitors to obtain industry-specific knowledge (Economist, January 18, 1997, p. 74). After World War I, DuPont ”... elected to lure experienced German chemists to the United States, with pay packages of up to $25,000 per year - 10 to 15 times their German salaries.” (Hermes 1996, p.61) This move played an important role in the development of the American dyestuff industry, which at the end of World War I had not yet acquired the knowledge necessary to compete against the German firms.

¹ Recently, Martin and Ottaviano (1997) have made progress towards an integration of the growth theory literature and economic geography.

² See Leahy and Neary (1997) for a comprehensive discussion.

³ Two other reasons for spillovers are often discussed: informal communication and inference from other firms’ actions, e.g., exposure to their products (see e.g. Geroski 1995).
These examples are by no means isolated events. The thriving business of headhunting testifies to the importance of the human capital market for the transfer of knowledge between firms. In addition, the possibility of such knowledge transfer is economically relevant even when it does not actually occur. The outside option of leaving a firm can improve an employee’s internal bargaining position, resulting, for example, in wage increases or promotions. Anticipating this, firms may reduce their innovation efforts. Clearly, general legal restrictions (e.g. patent laws) or specific contractual arrangements sometimes limit the extent to which inside knowledge may be used in a new firm. Nevertheless in many circumstances such measures do not have much bite. Also, one of our main goals is to show that, under plausible circumstances, knowledge spillovers through the human capital market might not even occur in cases when they are allowed in principle. We therefore deliberately consider a setting where employees can choose new employers freely, and may use whatever knowledge they have wherever they want.

In this setting, we investigate the incentives for firms to acquire the services of knowledge-bearing employees, and, similarly, the incentives for firms to prevent the departure of employees to other firms. We determine the conditions under which technological spillovers arise endogenously when duopolists compete for knowledge in the market for human capital and for profits in the product market. We analyze a game in which firms can first try to carry out a cost-reducing innovation, an effort which may or may not be successful. Then, they can attempt to obtain additional knowledge spillovers by offering suitable labor contracts to knowledge-bearing R&D employees of other firms; and by offering sufficiently high wages to their own employees, they can prevent knowledge spillovers to competitors. Knowledge spillovers can be asymmetric, if only one firm hires the competitor’s employee, or symmetric, if both firms hire each other’s employees. Finally, firms enter product market competition.

In this framework we first investigate the causes of spillovers. An unsurprising, but useful result is that spillovers are more likely to arise in equilibrium when they increase total industry profits. In addition, we make the following main points. First, we show that firms’ costs of obtaining and preventing spillovers reflect wage increases for knowledge bearing employees. For example, when both firms have innovated successfully and symmetric spillovers occur, R&D employees obtain a wage increase equal to the difference between industry profits under symmetric and asymmetric spillovers.

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4 Related results arise in the licensing literature (Gallini 1984, Katz and Shapiro 1985).
Second and most importantly, we apply our results to understand how endogenizing spillovers affects innovation. Unsurprisingly, innovation incentives tend to be stronger for endogenous spillovers than in the exogenous case, in particular, if product market competition is vigorous or innovations are significant. For Bertrand competition, the costs of preventing spillovers for a firm even turn out to be zero if the competitor has not innovated successfully. Hence, though spillovers are allowed in principle, innovation incentives are the same as if they were excluded by definition.\footnote{Similarly, for significant innovations, i.e., substantial cost reduction opportunities, the costs of preventing spillovers are low, so that innovation incentives are much higher than for exogenous spillovers.} Thus, the focus of public policy towards R&D on the correction of technological externalities by encouraging joint ventures, possibly even at the cost of less competition in product markets, may not always be justified when spillovers are endogenous\footnote{Leahy and Neary (1999) also argue that spillovers may not be a severe problem for R&D incentives, but for different reasons. To acquire spillovers, firms need absorptive capacity, which is developed by own R&D.}. Another main point is that in situations where innovation incentives are stronger for quantity than for price competition under exogenous spillovers, the reverse result may hold with endogenous spillovers.

A first step in the direction of endogenizing spillovers through the human capital market was taken by Pakes and Nitzan \cite{PakesNitzan1983}. They consider the optimal labor contracts for a single firm hiring employees who may be able to use the information acquired during the R&D process in a rival enterprise.\footnote{Also related is a paper by Anton and Yao \cite{AntonYao1995} who consider innovation incentives of firms whose employees might start their own companies.} Unlike Pakes and Nitzan, we consider the strategic interaction between firms. For all our results, the human capital market plays a crucial role. Other approaches to endogenous spillovers which do not consider this market lead to different results. For instance, in Katsoulacos and Ulph \cite{KatsoulacosUlph1998}\footnote{For a similar model, see Poyago-Theotoky \cite{PoyagoTheotoky1999}.}, two firms first choose whether to invest into reducing their marginal production costs. Then they decide whether to share the knowledge obtained in the first period with the opponent. Hence, contrary to our framework, parties that want to obtain spillovers cannot engage in activities to obtain spillovers (e.g. poaching workers). Katsoulacos and Ulph show that technological spillovers will not arise if firms compete in the same industry and their research activities are perfect substitutes: given the other firm’s behavior, a firm can only lose from sharing its secrets. This differs from our model because we allow firms to attract knowledgeable employees from other firms, and we take the costs of preventing spillovers into account.
The paper is organized as follows. In section 2, we introduce the model. In section 3, we analyze the human capital market by establishing the subgame equilibria for given innovation results of firms. In section 4 we apply the results to the most widely known models of competition in product markets. In section 5, we apply our results to innovation incentives. In section 6, we will explore how our results are affected if knowledge is created by a group of R&D persons within firms. Section 7 considers alternative contractual arrangements. In section 8, we discuss the relation to the licensing literature. Section 9 concludes.

2. Model

We consider a three-period duopoly model.

2.1. Innovations

In period 1, two risk neutral firms, , decide whether to invest in cost-reducing development activities. Denote the investment of firm by . The firm’s research projects are not necessarily identical. To carry out a project, a firm needs specific human capital. For instance, a car company may want to redesign its cars so that they are easier to manufacture and thus cost less to produce. This requires basic engineering knowledge, design and manufacturing techniques (CAD, CIM), etc. We assume that the firm can hire people possessing the required human capital. For now, we suppose only one R&D person is necessary to undertake the research project. The wage costs for this person are included in the overall project costs.9

With probability , R&D will lead to a reduction of marginal costs.10 For later reference, denote the success probability of one firm conditional on the competitor being successful (non-successful) as . We allow for a non-negative, but not necessarily perfect correlation between the firms’ research efforts. A small correlation can be interpreted as a case where the knowledge needed for R&D is very diffuse: different firms may then try different paths towards cost reduction. Nearly perfect correlation makes sense when the necessary R&D-activities are very clearly defined. Finally, let denote the innovation results of firm : if the firm does not invest or is not successful; characterizes a successful innovation.

9 In section 6, we discuss how the results change if there is more than one R&D person in each firm.
10 Innovation activities can be different. To reduce notation we assume, that the cost reduction is the same in both cases.
2.2. The Market for Human Capital

In period 2, both firms can try to gain additional knowledge by offering suitable wage contracts to the R&D employee of the other firm and can attempt to prevent knowledge spillovers by offering sufficiently high wages to their own R&D employees. The activities on the human capital market determine whether knowledge is transferred between firms. We make the following assumptions.

First, knowledge is usually not completely transferable across firms. Suppose a firm has not invested or that its innovation efforts have not been successful. Then, if it can attract the R&D person of the successful firm, he will be able to replicate a fraction $\gamma \in [0,1]$ of the original cost reduction in his new firm. Second, knowledge can be duplicated within the firm. Once the R&D employee has implemented a cost reduction, costs remain low, even if the employee changes the firm. Therefore, if a firm loses its employee to the competitor, the only disadvantage results from the competitor’s lower costs. Third, the knowledge of the competitor may be a substitute or a complement for a firm’s own knowledge. A successful innovator who recruits the employee of a successful competitor obtains a cost reduction of $\Delta(1+\beta)$, $0 \leq \beta$. $\beta = 0$ characterizes knowledge substitutes, i.e., a successful firm does not gain anything from buying the know-how of the competitor. The greater $\beta$, the stronger are the knowledge complementarities between the two firms.

Assumption 1A: Wages can be conditioned on the innovation success of the R&D employees of both firms.

Assumption 1A implies that firms can write complete labor contracts and thus firms can observe and verify the entire success vector at the beginning of period 2. An alternative and equivalent assumption is that firms observe and can verify the full success vector when product market competition starts, since this guarantees that product market competition takes place under complete information about costs.

11 Clearly, an investing firm has an incentive to secure that the knowledge of R&D projects is codified and distributed within the firm, so that it does not depend on the future services of the knowledge-bearing employee.

12 The importance of complementarities in knowledge production has been pointed out by Griliches (1979) and Gerowski (1995), and recently by Klette (1996), among others. Allowing complementarities partly addresses the point that spillovers are only valuable if the receiving firm engages in R&D activities itself: if $\beta$ is high, investment is required to obtain substantial benefits from spillovers and thus to learn from competitors (see also Cohen and Levinthal 1989, Leahy and Neary 1999).

13 In section 7, we shall consider incomplete labor contracts such that wages can only be conditioned on the success of the firm where an employee is working.
By 1A, if firm $i$’s innovation result is $s_i$, it offers wage contracts $w_i(s_i,s_j)$ depending on $s_i$ and $s_j$ to its own employee, if he continues to work for the firm. Similarly, it offers wage contracts $w_j(s_i,s_j)$, $j \neq i$, to the competitor’s employee. Observe the logic of the notation: the first of the two subscript indices corresponds to the firm that makes the wage offer; the second index stands for the firm for which the employee worked so far. The first and second term in brackets give the innovation result of the firm making the offer and the competitor, respectively. We shall drop the terms in brackets and write $w_i$ and $w_j$ whenever this is unambiguous. We shall then assume that each employee chooses to work for the firm that offers higher wages (for details see section 3.1).

2.3. Product Market Competition

In period 3, firms enter product market competition. We allow different forms of interaction such as quantity, price or quality competition with homogenous or heterogeneous products. We only assume that a unique Nash equilibrium in the product market competition stage exists. We assume complete information about marginal costs of both firms, so that success vectors are common knowledge. The equilibrium depends on these marginal costs and hence on the knowledge firms have obtained, either through innovation activities or through purchasing the know-how which is embodied in human capital. If initial costs are $c_0$ for both firms, product market profits are uniquely determined by $c_0$ and the combination of marginal cost reductions $r_i$ and $r_j$ that both firms have achieved compared to the status quo. We therefore denote the profits of firm $i$ as $\pi_i(r_i,r_j;c_0)$. These are gross profits, including R&D-wages. For firm $i$ the following four possibilities for $r_i$ can arise.

$$r_i = 0$$ if the firm neither innovates successfully itself, nor benefits from spillovers.

$$r_i = \Delta$$ if the firm innovates successfully, and does not benefit from spillovers.

$$r_i = \gamma \Delta$$ if the firm does not innovate successfully itself, but benefits from spillovers.

$$r_i = (1+\beta)\Delta$$ if the firm innovates successfully itself and benefits from spillovers.
In the following, we shall often drop $c_0$ and $\Delta$ and write $\pi_i\left(\frac{r_i}{\Delta}, \frac{r_j}{\Delta}\right) = \Pi_i(r_i, r_j; c_0)$. Hence, we obtain $\pi_i(1, \gamma) = \Pi_i(\Delta, \gamma; c_0)$; $\pi_i(1 + \beta, 1) = \Pi_i((1 + \beta)\Delta, \Delta; c_0)$, etc. We shall always assume that $\Pi_i(r_i, r_j; c_0)$ is non-decreasing in $r_i$ and non-increasing in $r_j$, which is true for most conceivable forms of oligopolistic competition.

2.4. Summary: The Structure of the Game

We summarize the overall game, which consists of three stages.

**Stage 1:** Firms choose $I_i \in \{0, I\}$ R&D-levels.

**Stage 2:** The success vectors become common knowledge Conditional on their own innovation success $s_i$ and the success of their competitors $s_j$, firms simultaneously offer wage contracts $w_i(s_i, s_j)$, $w_j(s_i, s_j)$ to R&D employees. Employees choose to work for the firm that offers the highest wage. Firms implement cost reductions.

**Stage 3:** Firms enter product market competition. Profits $\Pi_i(r_i, r_j; c_0)$ are realized.

3. The Likelihood of Spillovers

We now investigate the determinants of the spillover patterns.

3.1. Statement of the main result

The following cases have to be considered in the second stage: no firm is successful; both firms are successful; only one firm is successful.\(^{14}\) We assume that equilibrium profits depend only on the marginal costs of both firms, not on the firm index. Hence, we write $\pi(1, 0) \equiv \pi_i(1, 0)$; $\pi(0, 1) \equiv \pi_i(0, 1)$, etc. Recall that knowledge is exchanged if a successful R&D person switches the company. Whether this happens depends on the wages offered in both companies. We assume that a successful R&D person remains within the company if the wage offered by his current employer and by the competitor are the same; if the competitor offers a higher wage, the R&D worker accepts his offer. Thus, given the success vector $(s_1, s_2)$, the wage offers $w_{11}(s_1, s_2)$, $w_{12}(s_1, s_2)$, $w_{21}(s_2, s_1)$, $w_{22}(s_2, s_1)$ determine workers’ employment decisions, thus resulting in a unique vector $(r_1, r_2)$ of cost reductions. This cost structure results in product market profits $\Pi_i(r_i, r_j; c_0)$ and net payoffs.

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\(^{14}\) Since investment outlays are sunk, we do not have to distinguish between a firm that has not invested and a firm that has invested but was not successful.
where $\delta_{ij}$ takes values 1 or 0 according to whether firm $i$ sets wages high enough to obtain the services of the worker from firm $j \neq i$ or not, and $\delta_{ii}$ is defined analogously. We distinguish between the following cases.

A. Only one firm, say firm 1, is successful: Spillovers occur if and only if $w_{21}(0,1) > w_{11}(1,0)$.

With spillovers, total industry profits are $\pi(1,\gamma) + \pi(\gamma,1)$. Without spillovers, total industry profits are $\pi(1,0) + \pi(0,1)$.

B. Both firms are successful: Symmetric (or bilateral) technological spillovers occur if $w_{ij}(1,1) > w_{ij}(1,0)$ for $i, j = 1, 2; i \neq j$. If they do, total industry profits are $2\pi(1 + \beta,1 + \beta)$.

Asymmetric (or unilateral) spillovers from firm 1 to firm 2 occur if $w_{21}(1,1) > w_{11}(1,1)$ and $w_{12}(1,1) \leq w_{22}(1,1)$, resulting in total industry profits $\pi(1 + \beta,1) + \pi(1,1 + \beta)$.

In all other cases no spillovers occur, and total industry profits are $2\pi(1,1)$.

Note that, if one firm is successful, admitting spillovers increases total industry profits if $\pi(1,\gamma) + \pi(\gamma,1) > \pi(1,0) + \pi(0,1)$. For two successful firms the corresponding condition is $\pi(1 + \beta,1) + \pi(1,1 + \beta) < 2\pi(1 + \beta,1 + \beta)$. In the following subsections, we establish the following intuitive result.

**Proposition 1**: For any given vector of first-period cost reductions, there always exists a second-period equilibrium that maximizes total industry profits.

Proposition 1 is not very surprising, but it is useful for three reasons. First, we shall see that its derivation yields additional insights about the equilibrium wage structure: it is quite possible that employees can extract the entire surplus from spillovers. Second, we shall use proposition 1 to show how spillover activity depends on parameters such as the extent of product differentiation or the type of competition (Bertrand vs. Cournot). Third and most importantly, we shall use the result to revisit familiar results on innovation incentives.

3.2 Characterization of wages and spillovers

Obviously, firms only want to offer above market wages to employees with valuable knowledge. We normalize wages for employees without such knowledge to zero:
(2) \[ w_i(0,1) = w_i(0,0) = w_j(1,0) = w_j(0,0) = 0. \quad i, j = 1, 2, \ i \neq j. \]

Recall our assumption that an employee remains with his firm if he does not receive a better offer. More precisely, we assume that an R&D person leaves his current firm if the competitor’s offer is higher by an amount of at least \( \varepsilon \). \( \varepsilon \) can be arbitrarily small although we can think of it as the smallest currency unit that distinguishes two wage offers. We obtain:

**Lemma 1**: In any subgame perfect equilibrium: \[ w_i(s_i,s_j) = w_i(s_j,s_i) \] or \[ w_j(s_j,s_i) = w_j(s_i,s_j) - \varepsilon, \quad i, j = 1, 2. \]

Lemma 1 follows directly from the Bertrand feature of wage competition in the second stage. The winner of the wage auction for the successful R&D person \( i \) pays the smallest wage necessary to obtain the services of the R&D-employee. Hence, a firm must match the competitor’s wage to prevent the employee from changing jobs, and it must add \( \varepsilon \) to get the other firm’s employee. No other constellation can arise in equilibrium.

### 3.3. Equilibria when only one Firm is Successful

The following proposition characterizes spillovers when one firm has innovated successfully, while the other firm has failed or has not invested in R&D.

**Proposition 2**: Let 1 be the successful firm, 2 the competitor.

(i) Suppose \( \pi(1,0) - \pi(1,\gamma) < \pi(\gamma,1) - \pi(0,1) \). Then, there are spillovers in equilibrium;\n
\[ \text{wages satisfy} \quad w_{11}(1,0) = w_{21}(0,1) - \varepsilon = \pi(1,0) - \pi(1,\gamma). \] The resulting subgame payoffs (net of wages) are \( \pi(1,\gamma) \) for firm 1, \( \pi(\gamma,1) + \pi(1,\gamma) - \pi(1,0) \) for firm 2.\n
(ii) Suppose \( \pi(1,0) - \pi(1,\gamma) > \pi(\gamma,1) - \pi(0,1) \). Then, there are no spillovers in equilibrium;\n
\[ \text{wages satisfy} \quad w_{11}(1,0) = w_{21}(0,1) = \pi(\gamma,1) - \pi(0,1). \] The resulting subgame payoffs (net of wages) are \( \pi(1,0) - \pi(\gamma,1) + \pi(0,1) \) for firm 1; \( \pi(0,1) \) for firm 2.

The result is proved in the appendix.\(^{16}\) The inequalities in the two propositions have simple interpretations. The benefits for a successful firm from preventing spillovers (not considering wages) are \( \pi(1,0) - \pi(1,\gamma) \); the benefits for the competitor from obtaining spillovers are \( \pi(\gamma,1) - \pi(0,1) \). Hence, whether there will be spillovers depends on whether, ignoring wages,\n
\(^{15}\) A second equilibrium exists with both wages increased by \( \varepsilon \).

\(^{16}\) In the boundary case that \( \pi(0,0) - \pi(0,\gamma) = \pi(\gamma,1) - \pi(0,1) \) both equilibria are possible.
a successful firm gains more from maintaining its lead over the other firm than the unsuccessful firm gains from catching up. The wages necessary to prevent spillovers amount to what an unsuccessful firm could gain from spillovers; the wages necessary to obtain spillovers amount to what the successful firm loses from spillovers.

3.4 Equilibria when both Firms are Successful.

We now examine the subgame that results when both firms have innovated successfully. We must consider three outcomes, symmetric spillovers, asymmetric spillovers, and no spillovers. To simplify our discussion, we make the following standard assumption.

**Assumption 2:** \( \pi(1 + \beta, 1 + \beta) > \pi(1,1) \).

Thus firms prefer both having low costs to both having high costs.\(^ {17} \)

**Proposition 3:** Suppose that assumption 2 holds. Then:

(a) A symmetric spillover equilibrium exists if and only if \( \pi(1 + \beta, 1) + \pi(1, 1 + \beta) \leq 2\pi(1 + \beta, 1 + \beta) \). Wages are \( w_{ii} + \varepsilon = w_{ji} = \pi(1 + \beta, 1) - \pi(1 + \beta, 1 + \beta), \ i, j = 1,2, \ i \neq j \).

The resulting net payoffs are \( 2\pi(1 + \beta, 1 + \beta) - \pi(1 + \beta, 1) \) for both firms.

(b) An asymmetric spillover equilibrium exists if and only if \( \pi(1 + \beta, 1) + \pi(1, 1 + \beta) \geq 2\pi(1 + \beta, 1 + \beta) - \varepsilon \). Up to \( \varepsilon \), the wage sum is determined as \( \pi(1 + \beta, 1) - \pi(1, 1 + \beta) \).

Up to \( \varepsilon \), the net payoffs are \( \pi(1, 1 + \beta) \) for both firms.\(^ {18} \)

This result is proved in the appendix. The reasoning for (a) runs as follows. Symmetric spillovers require that neither firm wants to deviate by preventing spillovers to the competitor or by foregoing the possibility of obtaining spillovers. The condition in (a) makes sure such deviations are not profitable, because the profits in the resulting asymmetric situations are relatively low. In situation (b), profits are low for firms with similar costs, but high for asymmetric costs, favoring asymmetric spillovers and thus implying that the firm with both R&D-employees earns higher product market profits than the competitor. The firm with higher costs only accepts this if wages for the two R&D-workers are so high that firms have identical net payoffs (profits minus wages). This condition, however, only determines the sum of the wages for the two workers, not the wage for each individual worker.

\(^ {17} \) The assumption may be violated for quantity competition under extreme conditions on demand elasticities (Shapiro 1989). If the assumption does not hold, a unique equilibrium exists in which no spillovers occur.

\(^ {18} \)
Under assumption 2 an equilibrium in pure strategies with either asymmetric or symmetric spillovers always exists. In the following we will neglect \( \varepsilon \). Thus, the only knife edge case is
\[
\pi(1 + \beta, 1) + \pi(1, 1 + \beta) = 2\pi(1 + \beta, 1 + \beta)
\]
for which both symmetric and asymmetric spillovers occur. For all other cases, the equilibrium is unique with respect to the occurrence of spillovers and net payoffs of firms.

### 3.5 Summary

Together, propositions 2 and 3 imply proposition 1. Thus, if spillovers increase total industry profits, there will be spillovers if one firm is successful and symmetric spillovers if both firms are successful. When spillovers do not increase total industry profits, there will be no spillovers if one firm is successful and asymmetric spillovers if both are successful.

### 4. When do Spillovers Increase Total Industry Profits?

We discuss for which assumptions on product market competition and on \( c_0 \) and \( \Delta \) spillovers increase total industry profits and hence which types of spillovers occur.

#### 4.1. Price competition with homogenous products

Under Bertrand competition with homogenous products, firms’ profits are zero except when one firm has lower marginal costs than the competitor. Hence,
\[
\pi(1 + \beta, 1 + \beta) = \pi(1, 1 + \beta) = \pi(y, 1) = \pi(1, 1) = \pi(0, 1) = \pi(0, 0) = 0.
\]

Thus, spillovers never increase total industry profits, because profits are positive only if a firm has lower marginal costs than its competitor. Using propositions 1 and 2, we immediately get:

**Corollary 1:**

(i) Under Bertrand competition, there are no spillovers if only one firm is successful. If both firms are successful, there are asymmetric spillovers, so that both employees work for one firm.

(ii) If both firms are successful, net profits amount to zero for both firms.

The intuition is as follows. If one firm is successful, the competitor would gain nothing from spillovers; profits would still be zero. Hence, the successful firm’s employee will not obtain a positive wage from the competitor, and spillovers will be prevented at zero costs. If both firms are successful, profits are zero if there are no spillovers or bilateral spillovers. In both cases, an equilibrium would thus require both firms paying zero wages. But then firms could deviate

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18 The equilibrium is not unique with respect to the wages.
by offering low positive wages, resulting in a discontinuous profit increase. Thus, suppose there are asymmetric spillovers, so that one firm obtains positive product market profits and the competitor has zero profits. This can be an equilibrium provided the latter firm has no incentive to hire both employees from the former, that is, the total wage sum equals the product market profit of the firm with two employees, resulting in zero profits for both firms.

With quantity competition, there tends to be greater spillover activity than in the Bertrand case, as the following result shows.

**Corollary 2:** Suppose firms engage in Cournot competition with demand \( x=a-p \), and innovations are non-drastic, that is, a firm cannot become a monopolist if it has all the knowledge from innovations and spillovers.

(i) If one firm is successful, asymmetric spillovers occur for \( \Delta (8-5\gamma) \leq 2(a-c_0) \).

(ii) If both firms are successful, symmetric spillovers occur if \( (3\beta-2)\Delta \leq 2(a-c_0) \).

Asymmetric spillovers occur for \( (3\beta-2)\Delta \geq 2(a-c_0) \).

The result, proved in the appendix, implies that all conceivable spillover possibilities arise for linear Cournot competition. If the cost reduction \( \Delta \) and complementarities \( \beta \) are small, relative to the initial market size \( (a-c_0) \), this favors spillovers. These conditions make sure that a firm cannot come close to a monopoly position by getting ahead of the competitor.

Corollary 1 and 2 imply that with quantity competition spillovers are more likely than with price competition.

5. Implications for Innovation Incentives

Endogenizing spillovers affects several results in the R&D and growth literature, and it has implications for public policy towards R&D and competition. To see this, we formulate the benchmark case of exogenous spillovers. Period 1 is as in section 2, but there is no competition for human capital. Instead, if one firm has innovated successfully, the other firm automatically obtains all possible spillovers. Hence, if both firms are successful, profits are \( \pi(1+\beta,1+\beta) \); if only one firm is successful, profits are \( \pi(1,\gamma) \) and \( \pi(\gamma,1) \), respectively.

For the exogenous and the endogenous case, we define unilateral innovation incentives as the expected additional profit for a firm from innovation, assuming that the competitor does not

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19 Similarly, spillovers can be seen to be more likely for differentiated goods than for homogeneous goods.
innovate successfully. Bilateral innovation incentives are defined as the expected additional profit for a firm from innovation, when the competitor also innovates. Unilateral innovation incentives thus describe the incentive to deviate from the equilibrium (0,0) (net of wages, but gross of the innovation costs $I$). The higher the unilateral incentives, the less likely is a subgame perfect equilibrium where no firm invests. Similarly, the higher the bilateral incentives, the more likely is an equilibrium where both firms invest.

5.1. The Effect of Spillover Endogeneity on Innovation Incentives

It is straightforward to derive the following result.

**Proposition 4:** (i) Unilateral investment incentives are identical in the exogenous case and in the endogenous case if spillovers increase total industry profits. (ii) Unilateral investment incentives are higher in the endogenous case than in the exogenous case if spillovers do not increase total industry profits.

Intuitively, when spillovers increase total industry profits (i), they will not be avoided in the endogenous case. Hence, spillovers will take place just as in the exogenous case, and incentives are the same. As for (ii), even though spillovers are costly to avoid in the endogenous case, incurring these costs is worthwhile: hence, the option to avoid spillovers makes investing firms better off than in the exogenous case.

For bilateral incentives, we obtain the following result.

**Proposition 5:** Suppose $\gamma = 1$. For sufficiently small knowledge complementarities, bilateral incentives are stronger under endogenous spillovers than under exogenous spillovers.

A more precise statement and the proof are given in the appendix. Intuitively, the endogeneity of spillovers has three effects. First, in the endogenous case, a successful innovator tends to appropriate more of the surplus resulting from innovation than in the exogenous case because it has the option to prevent spillovers by incurring wage costs. Second, a firm that has not innovated successfully must pay to obtain spillovers in the endogenous case, whereas they are free in the exogenous case. Both effects make innovation more attractive in the endogenous case. Third, however, a successful firm that wants to benefit from knowledge complementarities will have to incur costs in the endogenous case, must pay a wage to attract a second employee. Thus, unlike in the exogenous case, innovation in itself does not yet
guarantee access to whatever complementary knowledge there is. This effect explains why endogeneity of spillovers may weaken innovation incentives for strong complementarities.\textsuperscript{20}

Homogeneous price competition provides a drastic illustration of the differences between endogenous and exogenous spillovers. By corollary 1, in this case spillovers never occur if only one firm is successful, and they are asymmetric if both firms are successful. Moreover, profits are independent of the possibility of endogenous spillovers and thus spillovers have no effect on innovation incentives. In contrast, (perfect) exogenous spillovers would destroy any profit from innovations and thus any innovation incentives. In particular, therefore, innovation incentives are much stronger in the endogenous case than in the exogenous case.\textsuperscript{21}

These ideas relate to the design of public policy towards R&D and competition. The literature has emphasized that spillovers may lead to underinvestment compared with social welfare maximization. This is obvious for the Bertrand case, where sufficiently high exogenous spillovers destroy any innovation incentives. For the Cournot case, the result has been shown by d’Aspremont and Jacquemin 1988, Suzumura 1992, Leahy and Neary 1997.\textsuperscript{22} Our analysis suggests that underinvestment may not be such a big problem.\textsuperscript{23} Thus the case for allowing R&D cooperation (Spence 1984, d’Aspremont and Jacquemin 1988) or improving patent protection (Gilbert and Shapiro 1990, Klemperer 1990, Gallini 1992) may not be so strong.\textsuperscript{24}

We note in passing that this implication of our approach may be reinforced when innovation costs are not treated as exogenous, but rather as wage payments to R&D-employees. The analysis so far strongly suggests the existence of rents for these employees in the spillover game. Anticipating such future rents, employees may be willing to accept below market wages initially. Therefore, it is not surprising that innovation incentives may be even higher than

\textsuperscript{20} Apart from strong complementarities, one would also require high probabilities that both firms are successful for this effect to matter.

\textsuperscript{21} Similarly, if innovations are significant, i.e., $\Delta$ is high, the costs of preventing spillovers are low, and hence innovation incentives are much stronger than in the exogenous case.

\textsuperscript{22} The result is non-trivial because of the well-known strategic (over-) investment incentive under quantity competition.

\textsuperscript{23} Most closely related to our argument here is the point made by Leahy and Neary (1999). They show that innovation incentives may not be seriously affected by spillovers when “absorptive capacity” is required to make use of spillovers, and this capacity is improved by own R&D.

\textsuperscript{24} However, our analysis does not invalidate another argument that is sometimes brought forward in favor of active R&D-policy, namely that the diffusion of knowledge is socially desirable and that therefore spillovers should be encouraged rather than prevented. In our framework, the appropriability problem is less severe than in a setting with exogenous spillovers, the reason being that firms can avoid spillovers. Hence, if the main problem of R&D-policy is that there are not enough spillovers, then this problem will tend to be worse for endogenous spillovers.
suggested by our analysis if innovation costs consist of R&D-wages that can be below the market level. In the discussion paper, we provide a full analysis of this point.\(^{25}\)

5.2. Product Market Competition and Innovation Incentives

The endogeneity of spillovers also affects the relation between innovation incentives and competition. We restrict ourselves to the case that \(\gamma = 1\), i.e. if spillovers occur, the competitor obtains all the knowledge of the innovating firm. In the appendix we show:

**Proposition 6**: Suppose \(\gamma = 1\).

(a) With exogenous spillovers, Cournot competition always gives stronger unilateral innovation incentives than Bertrand competition.

(b) With endogenous spillovers, Bertrand competition always gives stronger unilateral innovation incentives than Cournot.

Intuitively, with Bertrand competition, spillovers can always be avoided at zero costs in the endogenous case. Hence, incentives are as strong as in the case without spillovers. In the Cournot case, however, spillovers will not be avoided at all if spillovers increase total industry profits, and will be avoided at some costs (wage payments of \(\pi(1,1) - \pi(0,1)\)) if spillovers do not increase total industry profits. Hence, in the Cournot case, innovation incentives are adversely affected by the possibility of spillovers, whereas they remain unaffected in the Bertrand case. Therefore, the result comes at no surprise.

6. Many Employees

Although a full-fledged extension of our analysis to R&D groups with \(N \geq 1\) members is beyond the scope of this paper, we shall sketch some relevant ideas. For simplicity, assume that firms bid sequentially for the employees. Then, the nature of spillovers depends crucially on the distribution of knowledge over this group. An extreme assumption would be that each member possesses indispensable knowledge, without which the knowledge of the other employees has no value (*perfect complementarity*). In this case, our analysis carries over directly, with wage offers interpreted as offers to the entire group.

At the other extreme, suppose each member possesses the entire knowledge of the group (*perfect substitutability*). Then, if one firm is successful, spillovers will only be prevented if

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\(^{25}\) Another technique to increase innovation incentives that is absent in our setting would be to choose the type of innovation strategically towards innovations that are less relevant to the competitor. Such a possibility would lead to incentives to distort the type of R&D.
\( \pi(1,0) - \pi(1,\gamma) > (n-1)(\pi(\gamma,1) - \pi(0,1)) \). Hence, spillovers are much less likely to be prevented and therefore innovation incentives are lower.

An intermediate case occurs when each member possesses knowledge that is mutually exclusive and specific knowledge can be added up to obtain the cost reduction potential of a firm. In Gersbach and Schmutzler (2000), we give sufficient conditions for "no spillovers" and "full spillovers", respectively, where "full spillovers" stands for the case that the entire group changes employers. There, we work with the simplifying assumption that the cost reduction obtained through spillovers depends only on the number of employees that change firms. We obtain a natural generalization of the condition that spillovers arise if they increase total industry profits. Qualitatively, our results on innovation thus generalize to the \( N \) worker case. In particular, spillovers are more likely in the Cournot case than in the Bertrand case, which strengthens innovation incentives in the Bertrand case relative to Cournot.

Summing up, it is more difficult to prevent spillovers with \( N \) workers. How likely spillovers are, depends on the distribution of knowledge within the firm and on product market properties that are similar to those identified before.

7. Alternative Contractual Arrangements

Our assumptions allow a firm to fully condition wages on the other firm’s research success as well as on its own success. It is useful to explore alternatives and most importantly to consider contractual incompleteness in the spirit of the contract theory literature (Hart 1995).

**Assumption 1B:** The wage of firm \( i \) can only be conditioned on the success of its own worker;

To justify this assumption, suppose the firms are asymmetrically informed about the success of their employees at the time they make wage offers, so that they cannot simply refrain from making wage offers to unsuccessful workers. This asymmetric information is, however, resolved before product market competition takes place, and success vectors become common knowledge. The information revealed is, however, not verifiable by third parties.

Because of assumption 1B, wages will be denoted as \( w_i(s_i) \) and \( w_j(s_j) \) for \( i=1,2, j \neq i \). This modification has two countervailing effects. To see this, we restrict ourselves to the case \( \beta = 0 \). Clearly, \( w_i(0) = 0 \) must hold in any equilibrium: If a firm’s employee was not successful, there is no need to prevent his departure as he will be of no use to the competitor.
Further, \( w_g(1) = 0 \): Absent complementarities, a firm does not gain from employing another worker. Spillovers arise if only one firm, say firm 1, is successful and \( w_{11}(1) < w_{21}(0) \).

Consider the incentives of the non-successful firm 2 to set wages high enough to attract the other firm’s worker. By setting \( w_{21}(0) > w_{11}(1) \), firm 2 will also get the worker of a non-successful competitor, since \( w_{21}(0) > w_{11}(0) = 0 \). Thus, firm 2 has expected product market profits of \( z\pi(\gamma, 1) + (1 - z)\pi(0, 0) \) after setting wages in this manner.\(^{26}\) Deviating by reducing wages to zero would lead to expected product market profits \( z\pi(0, 1) + (1 - z)\pi(0, 0) \).

The expected benefits from spillovers are thus \( z(\pi(\gamma, 1) - \pi(0, 1)) \). This expression is smaller than for complete contracts (section 3.2.) where we calculated the benefits from spillovers as \( \pi(\gamma, 1) - \pi(0, 1) \): For firm 2, setting wages \( w_{21}(0) > w_{11}(1) \) now only leads to useful knowledge if the competitor actually was successful, which happened with probability \( z \). Similarly, incentives for a successful firm, say firm 1, to set \( w_{11}(1) > w_{21}(0) \) to avoid spillovers can be seen to be \((1 - v)(\pi(1, 0) - \pi(1, \gamma))\).\(^{27}\) This is smaller than \( \pi(1, 0) - \pi(1, \gamma) \), the corresponding amount for complete contracts. Intuitively, a successful firm that sets high wages for its own worker keeps the worker even when the other firm is also successful, in which case spillovers would be of no use to the competitor, and avoiding spillovers would thus be of no use to the successful firm. We obtain a difference of \( v[\pi(1, 0) - \pi(1, \gamma)] \) between the gains from spillovers with complete and with incomplete contracts.

As both incentives to obtain and incentives to avoid spillovers are lower with incomplete contracts, it is unclear a priori whether spillovers become more or less likely. We shall show that spillovers become more (less) likely under incomplete contracts if spillovers decrease (increase) total product market profits, that is, the effect of product market competition on the likelihood of spillovers becomes less pronounced. First, however, note that in general pure strategy equilibria do not exist.

**Proposition 7:** Suppose that IB holds, \( \beta = 0 \) and \( \pi(1, 0) > \pi(1, 1) > \pi(0, 1) \). Then no equilibrium in stage 2 in pure strategies exists.\(^{28}\)

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\(^{26}\) Recall that \( z \) is the probability of success conditional on the competitor being non-successful.

\(^{27}\) Recall that \( v \) is the probability of success conditional on the competitor being successful.

\(^{28}\) Strictly speaking, we require that \((1 - v)(\pi(1, 0) - \pi(1, 1)) > 2\varepsilon \) and \((1 - v)(\pi(1, 1) - \pi(0, 1)) > 2\varepsilon \).
The proof is given in the appendix.\textsuperscript{29} Thus, only mixed strategy equilibria can exist and spillovers occur with some positive probability. Therefore, when profits decrease with spillovers, spillovers become more likely for incomplete contracts, since spillovers never arise for complete contracts. The opposite conclusion obtains if spillovers increase with contracts.

**Corollary 3**: Suppose that $\beta = 0$ and $\pi(1,1) > \pi(0,1)$. Suppose that spillovers decrease (increase) industry profits. Then, the probability of spillovers is higher (lower) with incomplete contracts than with complete contracts.

Rather than considering incomplete contracts, one could also move into the other direction and allow for contracts that are conditioned on additional variables. For instance, contracts could depend on whether other employees are hired or not. With one employee in each firm, the game becomes slightly more complex than before because each firm now makes two wage offers to each worker, and each worker’s optimal choice of firms now depends on the other worker’s decision. However, this modification has no essential effect on equilibria, except that wage offers are now conditioned on hiring decisions. For example, the equilibrium 3(a) would still hold provided that wages are as before if the competitor’s worker accepts the offer, but the own worker leaves, but fall to the willingness-to-pay for the second worker if both workers are employed in one firm. Since firms would not want to hire both employees at the equilibrium wages specified in proposition 3a, the symmetric spillover equilibrium obtains.

The situation is different when each firm employs $N>1$ workers. Consider perfectly substitutable workers. Then a firm can offer wages to the competitor which are only effective if no other employees are hired. Thereby, the firm can avoid paying twice for the same knowledge. Similarly, a firm can offer his own employees wages which are effective only if nobody leaves, since otherwise protecting knowledge is worthless. Such contracts will increase the flexibility of firms to compete for R&D-workers. Since, the possibility of obtaining and protecting knowledge increase simultaneously in this setting, the effects on spillover are ambiguous.\textsuperscript{30}

Another potential extension are performance based wage contracts. For instance, with sales based contracts, wages are partly transformed from fixed costs into marginal costs. Considering the case of one successful firm, a first consequence of such a change is that gross

\textsuperscript{29} If $\pi(1,1) = \pi(0,1)$, as for instance in the Bertrand case, a (unique) equilibrium in pure strategies with incomplete contracts still exists, since a non-successful firm has no incentive to offer any positive wages.

\textsuperscript{30} To examine this case, a complex multi-auction approach is necessary.
product market profits $\pi_i$ become less sensitive to which firm employs the worker, as the cost reductions from employment are partly compensated by wage increases. This, in itself, would decrease both incentives to obtain and incentives to prevent spillovers. Also, industry output and total gross profits for any given cost reduction would tend to decrease because of the higher variable costs, which, other things being equal, would decrease innovation incentives. This, however, has to be weighed against savings from lower fixed wage payments. The net effect on innovation activities is ambiguous.

8. Relation to the Licensing Literature

As mentioned before, our approach bears some similarity with the ex-post licensing literature. Most closely related is Katz and Shapiro (1985). In their paper, a firm that has a patent for a cost-reducing innovation licenses the innovation to a competitor in return for a fixed fee. This happens if and only if industry profits increase if licensing takes place. Of course, this resembles our result that spillovers take place with complete contracts if and only if they lead to an increase in industry profits. However, there are four important differences between the licensing case and our approach.

First, note the distributional issues in the case where industry profits increase. With a license, the beneficiaries of licensing will be the owners of both firms, depending on the license fee. If knowledge is transferred through endogenous spillovers, the innovator loses the benefits to the employees who enjoy a wage increase. Second, as a result of these distributional issues, the possibility of licensing has a substantially different effect on innovation incentives than the possibility of endogenous spillovers. The relation is particularly simple for unilateral incentives: The option of ex post licensing can only increase innovation incentives; the possibility of endogenous spillovers, however, usually decreases innovation incentives. 

Third, the number of knowledge bearing employees has a diluting effect that is absent in the licensing literature. As the number of relevant workers increases, the likelihood of knowledge transfer increases. Finally, contractual incompleteness seems to play a greater role in the case of knowledge transfer through the human capital market. Licenses can be conditioned on whether they actually involve useful knowledge, but this is much more difficult with wage contracts for employees. We have shown, however, that with incomplete contracts the role of

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31 Bilateral incentives are less trivial: if the competitor innovates, the chance of obtaining a license ex post may reduce innovation incentives.
product market competition becomes less pronounced in determining whether spillovers take place.

9. Conclusion

We investigated under which conditions spillovers arise endogenously when firms compete in the market for human capital and in the product market. Spillovers between two firms competing in prices are more likely when products are differentiated, when there is quantity rather than price competition, when the two firms are not alone in the market, and when the market is big relative to the size of the cost reduction. Thus, the conditions under which firms compete in the product market determine the extent of technological spillovers.

We saw that our approach has potential policy implications. Not only does our approach suggest that the appropriability problem is weaker than generally believed, it also casts doubts on assertions that sharp price competition reduces innovation incentives. Any stronger policy conclusions should of course be preceded by a thorough robustness discussion. While only future research can establish whether our results can be applied more broadly, we believe that similar results are likely to hold for situations with many employees. Firms will find additional ways to ensure that small number of workers will not be able to take all knowledge to competitors. Therefore, innovation incentives should still remain high if competition is intense.

Our approach to endogenize spillovers may not only be useful for a modified view of how unprotected knowledge affects innovation incentives, but it may also shed light on other issues. For instance, a simple reinterpretation of our approach is that it provides some predictions about the determinants of labor mobility. Applying the results of section 4, we find that qualified labor is more likely to move between firms when innovations are small, when products are differentiated and so on.

There are many useful extensions of our model. For instance, firms might be allowed to hire new workers in the second period, as an alternative to poaching. This would reduce the willingness to pay for the competitor’s employees, suggesting that spillovers would decrease and innovation incentives might increase.\(^{32}\) Also, one might consider the case where private information about costs is not resolved at the beginning of product market competition. In this

\(^{32}\) Another extension is to consider a continuous innovation variable. For such an attempt in the case of perfect complementarity see Gersbach and Schmutzler (2002).
case, the wage competition case would have signaling effects: An employee willing to accept a wage offer by a new firm may reveal that he possesses little knowledge which, in turn makes it less costly for firms to keep employees with knowledge. Therefore innovation incentives might increase if complementarities are weak.

Overall, endogenizing technological spillovers by considering the human capital market is a promising avenue towards understanding knowledge flows and innovation incentives. The work on these issues has just begun.
Appendix: Proofs

Proof of Proposition 2: For the successful firm 1, we only have to show that setting \( w_{11}(1,0) = w_{21}(0,1) \) to avoid spillovers does not increase its profits; that is, 
\[ \pi(1, \gamma) \geq \pi(1,0) - w_{21}(0,1). \] 
This holds because the right hand side of this inequality is equal to \( \pi(1, \gamma) \). For firm 2, which is not successful, setting the wage high enough to attract the other firm’s worker is worthwhile. This follows because \( w_{21}(0,1) = \pi(1,0) - \pi(1, \gamma) \) and \( \pi(\gamma,1) + \pi(1, \gamma) \geq 1 \pi(1,0) + \pi(0,1) \) imply \( \pi(\gamma,1) - w_{21}(0,1) \geq \pi(0,1) \). Similar arguments establish (ii).

Proof of Proposition 3: (a) By lemma 1 and the symmetry of the equilibrium, it suffices to check three deviations for firm \( i \).

(1) Firm \( i \) does not benefit from setting the wage for its own employee high enough to avoid spillovers, but at the same time leaving the wage offer for the other firm’s worker unchanged so that it obtains his services, that is: 
\[ \pi(1 + \beta, 1 + \beta) - w_{ij} \geq \pi(1 + \beta, 1) - w_{ii} - w_{ij}. \] 
Inserting the equilibrium wage, this becomes 
\[ \pi(1 + \beta, 1 + \beta) \geq \pi(1 + \beta, 1 + \beta). \] 

(2) It is not profitable to reduce the wage offer for the other firm’s worker without changing the offer for the own employee, and therefore forego the benefits from spillovers, while letting the other firms reap the benefits of spillovers. This condition requires: 
\[ \pi(1 + \beta, 1 + \beta) - w_{ij} \geq \pi(1 + \beta, 1 + \beta), \] 
Inserting the value of \( w_{ij} \) gives 
\[ \pi(1 + \beta, 1 + \beta) - \pi(1 + \beta, 1 + \beta) \geq \pi(1, 1 + \beta), \] 
which holds by the assumption of the proposition.

(3) It is not profitable to reduce the wage offer for the other firm’s worker and increase the wage offer for the own employee to \( w_{ii} + \epsilon \) so that there will be no spillovers. This holds if 
\[ \pi(1 + \beta, 1 + \beta) - w_{ij} \geq \pi(1,1) - (w_{ii} + \epsilon), \] 
which follows from assumption 2.

Therefore (1) always holds with equality; (2) and (3) hold under the assumption of the proposition and assumption 2. Hence, an equilibrium with symmetric spillovers exists.

(b) In the proposed equilibrium one firm employs both workers. Suppose this is firm 1. Then its best response conditions (BRC) are as follows:

(i) \( \pi(1 + \beta, 1) - w_{12} - w_{11} \geq \pi(1, 1 + \beta) \)

(ii) \( \pi(1 + \beta, 1) - w_{12} \geq \pi(1, 1) \)

(iii) \( \pi(1 + \beta, 1) - w_{11} \geq \pi(1 + \beta, 1 + \beta) \)
For example, the first condition makes sure firm 2 does not deviate by reducing its wage offers to both workers, so that they both work for the competitor. Doing so would lead to wage savings of $w_{11} + w_{12}$. (i) makes sure these wage savings do not exceed the resulting drop in profits. The other conditions follow similarly. The BRC for firm 2 are

$$(iv) \pi(1, 1+\beta) \geq \pi(1+\beta, 1) - w_{12} - (w_{11} + \epsilon)$$

$$(v) \pi(1, 1+\beta) \geq \pi(1, 1) - w_{12}$$

$$(vi) \pi(1, 1+\beta) \geq \pi(1+\beta, 1+\beta) - (w_{11} + \epsilon)$$

For example, consider condition (iv). This makes sure firm 2 does not deviate by paying the minimum wages $w_{11} + \epsilon$ necessary to obtain the services of the other firm’s worker and keep its own worker by matching the competitor’s offer $w_{12}$. (v) and (vi) have similar interpretations. Conditions (i) to (vi) can be summarized as follows:

(i) and (iv) hold if and only if

$$(vii) \pi(1+\beta, 1) - \pi(1, 1+\beta) \geq w_{12} + w_{11} \geq \pi(1+\beta, 1) - \pi(1, 1+\beta) - \epsilon,$$

which implies $w_{12} + w_{11} = \pi(1+\beta, 1) - \pi(1, 1+\beta)$ up to $\epsilon$.

To satisfy (iii)/(vi) and (ii)/(v), we need

$$(viii) \pi(1+\beta, 1) - \pi(1+\beta, 1+\beta) \geq w_{12} \geq \pi(1+\beta, 1+\beta) - \pi(1, 1+\beta) - \epsilon$$

$$(ix) \pi(1+\beta, 1) - \pi(1, 1) \geq w_{12} \geq \pi(1, 1) - \pi(1, 1+\beta)$$

(viii) and (ix) are conditions on wages and profits. The profit condition in (viii) is equivalent to $\pi(1+\beta, 1) + \pi(1, 1+\beta) \geq 2\pi(1+\beta, 1+\beta) - \epsilon$ and thus to the assumption in proposition 3. Similarly, the (ix) requires that $\pi(1+\beta, 1) + \pi(1, 1+\beta) \geq 2\pi(1, 1)$ which is implied by (viii) because of assumption 2.

Finally, wages $w_{11} = \pi(1+\beta, 1+\beta) - \pi(1, 1+\beta)$ and $w_{12} = \pi(1+\beta, 1) - \pi(1+\beta, 1+\beta)$ satisfy the equilibrium conditions (vii) to (ix), using assumption 2 in the latter case. These choices of $w_{11}$ and $w_{12}$ are part of the equilibrium where $w_{22} = w_{12} - \epsilon$; $w_{21} = w_{11}$. Note that there are infinitely many other combinations satisfying (vii) - (ix), but they cannot be distinguished in terms of net payoffs for firms since the sum of the wages paid remains the same.

If $\pi(1+\beta, 1) + \pi(1, 1+\beta) < 2\pi(1+\beta, 1+\beta) - \epsilon$, (viii) is violated, and thus no equilibrium with asymmetric spillovers occurs. If the condition in b) is not fulfilled, case a) occurs. (q.e.d.)
**Proof of Corollary 2:** Let $K = a - c_o + \Delta$. Also, recall in the following that a firm with marginal costs $c_i$ that faces a competitor $c_j$ has profits $(a - 2c_i + c_j)^2 / 9$.

(i) Using proposition 2, if one firm is successful, asymmetric spillovers occur if and only if $\pi(1,0) - \pi(1,1) \leq \pi(\gamma,1) - \pi(0,1)$, i.e., $(K + \Delta)^2 - (K + (1 - \gamma) \Delta)^2 \leq (K + 2(\gamma - 1) \Delta)^2 - (K - 2\Delta)^2$ or equivalently $\Delta(8 - 5\gamma) \leq 2(a - c_o)$.

(ii) By proposition 3, in this case, symmetric spillovers occur for $2\pi(1 + \beta,1 + \beta) \geq \pi(1 + \beta,1) + \pi(1,1 + \beta)$ which, using (5) and (6) yields $2a \geq 2c_o + (3\beta - 2)\Delta$. Asymmetric spillovers occur if $2a \leq 2c_o + (3\beta - 2)\Delta$. (q.e.d.)

**Proof of proposition 5:** We prove the following result:

(i) If spillovers increase total industry profits, bilateral incentives are stronger under endogenous spillovers than under exogenous spillovers if and only if $\pi(1,0) - \pi(1,1) > \pi(1 + \beta,1) - \pi(1 + \beta,1 + \beta)$.

(ii) If spillovers do not increase total industry profits, bilateral incentives are stronger under endogenous spillovers than under exogenous spillovers if and only if $q(1 - v)[\pi(1,0) + \pi(0,1) - 2\pi(1,1)] + qv[\pi(1,1) - \pi(0,1)] > qv[\pi(1 + \beta,1 + \beta) - \pi(1,1 + \beta)]$.

Claim (i) follows immediately from the following considerations. Propositions 2 and 3 show that expected profits are $q[2\pi(1,1) - \pi(1,0)] + (1 - q)\pi(0,0)$ for a non-innovator and $qv[2\pi(1 + \beta,1 + \beta) - \pi(1 + \beta,1)] + q(1 - v)[3\pi(1,1) - \pi(1,0)] + (1 - q)(1 - v)\pi(0,0)$ for an innovator. In the exogenous case, a firm that does no innovate earns $q \pi(1,1) + (1 - q)\pi(0,0)$. An innovator earns $qv\pi(1 + \beta,1 + \beta) + q(1 - v)[2\pi(1,1)] + (1 - q)(1 - v)\pi(0,0)$.

Result (i) now follows by straightforward arithmetics. The proof of (ii) is analogous.

If complementarities are sufficiently small, condition (ii) is fulfilled and incentives under endogenous spillovers are stronger than under exogenous spillovers.
**Proof of proposition 6:**

(a) Incentives for Bertrand are \( q(\pi(1,1) - \pi(0,0)) = 0 \). Incentives for Cournot are
\[
q(\pi(1,1) - \pi(0,0)) = q\left(4(a - c_0)\Delta + 3\Delta^2/9\right) > 0.
\]

(b) In the Bertrand case, proposition 2 implies that there are no spillovers if a firm is successful, and profits for a successful firm are \( \pi(1,0) = q\Delta(a - c_0 + \Delta) \). The equilibrium condition for \((0,0)\) is thus \( q\Delta(a - c_0 + \Delta) < I \).

In the Cournot case, we first consider the case that spillovers increase total industry profits, which requires \( 3\Delta \leq 2(a - c_0) \) by corollary 2. By proposition 1, there will be spillovers if a firm is successful, in which case it obtains \( \pi(1,1) = \frac{K^2}{9} \) with \( K = a - c_0 + \Delta \). Hence, unilateral innovation incentives are:
\[
q(\pi(1,1) - \pi(0,0)) = q\left(K^2 - (K - \Delta)^2/9\right) = q\left(\Delta^2 + 2\Delta(a - c_0)/9\right).
\]
Thus, a firm does not want to innovate if
\[
q(\Delta^2 + 2\Delta(a - c_0)) < 9I.
\]
Therefore, incentives for Bertrand are stronger if and only if \( 9q\Delta(a - c_0 + \Delta) > q(\Delta^2 + 2\Delta(a - c_0)) \), which always holds.

If spillovers do not increase total industry profits a successful firm obtains product market profits \( \pi(1,0) - \pi(1,1) + \pi(0,1) \). Innovation incentives in the Cournot case are therefore
\[
q[\pi(1,0) - \pi(1,1) + \pi(0,1) - \pi(0,0)] = q\left((K + \Delta)^2 - K^2 - (K - 2\Delta)^2 - (K - \Delta)^2/9\right) = 4\Delta^2q/9.
\]
Hence, unilateral incentives are greater for Bertrand if and only if \( 9q\Delta(a - c + \Delta) > 4\Delta^2q \) which always holds.

**Proof of proposition 7:** Suppose there is a pure strategy equilibrium with spillovers whenever only one firm is successful. In such an equilibrium, \( w_{ij}(0) = w_{ij}(1) + \varepsilon \) must hold for \( i, j = 1,2; j \neq i \). Suppose that, e.g., firm 1 is successful. As the proposed equilibrium requires \( w_{21}(1) = 0 \leq w_{11}(1) \) and \( w_{11}(1) < w_{21}(0) \), firm 1’s expected net equilibrium payoffs are
\[
v(\pi(1,1) - w_{11}(1)) + (1-v)\pi(1,1) = \pi(1,1) - vw_{11}(1).
\]
Thus, firm 1’s best response is \( w_{11}(1) = 0 \). Thus, an equilibrium would require \( w_{21}(0) = \varepsilon \). But then firm 1 would have an incentive to set \( w_{11}(1) = 2\varepsilon > w_{21}(0) \) to obtain the benefits \( (1-v)(\pi(1,0) - \pi(1,1)) > 0 \) that are higher than the wage costs. Hence, no equilibrium in pure strategies with spillovers exists. The argument for no-spillover equilibria is similar.
References:


