Does Globalization Create Superstars?
A Simple Theory of Managerial Wages

Hans Gersbach
CER-ETH - Center of Economic Research
at ETH Zurich and CEPR
Zürichbergstrasse 18
8092 Zurich, Switzerland
hgersbach@ethz.ch

Armin Schmutzler
Department of Economics,
University of Zurich and CEPR
Blümlisalpstrasse 10
8006 Zurich, Switzerland
armin.schmutzler@econ.uzh.ch

This version: March 2014

Abstract
To examine the impact of globalization on managerial remuneration, we consider a matching model where firms compete both in the product market and in the managerial market. We show that globalization, i.e., the simultaneous integration of product markets and managerial pools, leads to an increase in the heterogeneity of managerial salaries. Typically, while the most able managers obtain a wage increase, less able managers are faced with a reduction in wages. Hence our model is consistent with the increasing heterogeneity of CEO remuneration that has been observed in the last few decades.

JEL Classification: D43, F15, J31, L13

Key Words: Globalization, manager remuneration, superstars

*We are grateful to an associate editor and to two anonymous referees, Marc Lickes, Stephan Imhof, Noemi Hummel, and Marc Melitz and to participants at the meetings of EARIE (2013) and Verein für Socialpolitik (2013) for helpful comments.
1 Introduction

The salaries of top managers have recently received considerable public attention. According to Murphy and Zabojnik (2004), the average base salaries and bonuses of Forbes 800 CEOs increased from 700,000 U.S. dollars in 1970 to more than 2.2 million dollars in 2002.\footnote{The figures are in 2002 dollars. The Forbes 800 list contains all companies ranked in the top 500 by assets, income, market capitalization, or revenues. Typically, there are about 800 companies on the list.} This effect is even larger when stock options are taken into account. The rapid rise in CEO pay over the last few decades has been confirmed with more recent data by Frydman and Jenter (2010). Such figures cause particular concern when they are related to ordinary wages. The ratio between CEO cash compensation and average pay for production workers in the U.S. climbed from 25:1 in 1980 to 90:1 in 2000 (Murphy and Zabojnik, 2004). It is hardly surprising that this particular aspect of income redistribution has been highly controversial. Shareholders, labor unions, politicians, and mass media have criticized both the level of managerial incomes and the tenuous connection between pay and performance. The discussion is by no means confined to the United States, as surveys in the Economist (Economist, 2007) and Llense (2010) testify. In the United Kingdom, for instance, the discussions have led to the introduction of transparency rules for managerial pay (Severin, 2003).

Given the amount of public attention and the substantial academic research this area has attracted, it is surprising that the causes of the recent salary increases are still imperfectly understood. In line with popular opinion, Bebchuk and Fried (2004) attribute increasing managerial wages to managerial power. Shareholders, so their argument goes, have limited control over the wage-setting process, and the board often gives in to the interests of CEOs.

Murphy and Zabojnik (2004) take issue with such explanations. Without necessarily denying the existence of managerial rent-seeking activities, they argue that an explanation for recent salary increases on this basis would also require an increase in managerial power, which they find unconvincing. Instead, they propose the idea that the changes reflect an increasing demand for general, rather than firm-specific, managerial skills, “perhaps as a result of the steady progress in economics, management
science, accounting, finance and other disciplines which, if mastered by a CEO, can substantially improve his ability to manage a company” (Murphy and Zabojnik, 2004, p.193). This results in an increasing tendency for outside hiring, and the resulting competition for managers drives up wages. Gabaix and Landier (2006) argue that the increase in managerial pay can be attributed to an increase in firm size.

In this paper, we provide a possible alternative explanation of recent trends that relates to informal arguments that are often advanced in popular accounts of the subject. Some observers regard increasing managerial wages as a by-product of globalization (see the discussion in Llense 2010, Frydman and Jenter 2010). We therefore examine the relation between globalization and managerial wages. In particular, we investigate how the simultaneous integration of product markets and managerial markets affects wages.

We consider a matching model where firms compete both in the product market and in the managerial market. In the product market they interact as oligopolists. In the managerial labor market they compete for the services of managers with heterogeneous abilities, which result in different marginal cost levels of the firms they manage. Globalization refers to the simultaneous replacement of national markets by one integrated market with (i) higher demand, (ii) a larger number of firms, and (iii) a larger pool of managers.

Because channels (i)-(iii) can potentially have countervailing effects on wages, the impact of globalization on managerial remuneration is subtle. Nevertheless, we obtain a robust prediction about the effects of globalization on the distribution of managerial wages. Globalization leads to an increase in the heterogeneity of managerial salaries. Whenever there is a wage increase for some manager, then any more able manager will also face a wage increase. More generally, the difference between post-integration wages and pre-integration wages is increasing in ability. Hence, our model can explain the increasing heterogeneity of CEO remuneration that has been observed in the last few decades.

However, our model does not predict an increase in the average wage levels of managers without additional parameter restrictions. The reduction in wages for less competent

\[ It \text{ is also possible that no managers or all managers (except for the least efficient one) receive a wage increase.} \]
managers may well offset the wage increases of the most competent managers. Nevertheless, our approach is consistent with the idea that globalization lies behind the increasing wages of top executives. Empirical results on average managerial salaries typically refer to the average within a fairly small group of top managers. As our model predicts pay rises for the best-paid managers due to globalization, these averages should also be expected to rise.

It is crucial for our results that the equilibrium wage differences between more and less competent managers reflect the differences in gross profits (that is, profits before subtracting managerial wages) between more and less efficient firms. Understanding the effects of globalization on managerial wages therefore boils down to understanding how efficiency differences translate into profit differences. Intuitively, the more intense competition induced by globalization increases the payoff for being more efficient in the sense that the profit difference between the most efficient firm and its less efficient competitors necessarily increases. In the concluding section, we comment on the robustness of our results within a broader set of theories of managerial compensation.

The paper is organized as follows: Section 2 introduces the general version of the model with symmetric firms. In Section 3, we characterize the equilibrium of this model. Section 4 analyzes the effects of globalization in the symmetric model within the linear Cournot framework. Section 5 shows that the general characterization of the equilibrium and the wage effects in the Cournot model are robust to the introduction of asymmetric firms. It also demonstrates that assortative matching arises, so that managers are matched to those firms that can make the most of their abilities. Section 6 presents some other extensions of the model that demonstrate the robustness of the argument that globalization tends to lead to higher wages for the more able managers, but not necessarily for the less able managers. In particular, we show that the argument works for a standard model of differentiated price competition, and that it is robust to the possibility of exit under globalization. In Section 7, we place the paper in the context of several strands of literature. Section 8 concludes.

---

3 See footnote 1.
2 The General Symmetric Model

The model consists of a wage-setting stage followed by an application stage and a product-market stage. The symmetric firms $i = 1, \ldots, I$ compete for managers $m = 1, \ldots, M$ with $M \geq I$. Each firm first hires a manager. Ability differences between managers are reflected in the marginal costs $c_m$ of a firm that employs manager $m$. We index the managers by quality, that is,

$$c_1 \leq c_2 \leq \ldots \leq c_M.$$

Manager 1 has the highest quality and can achieve the lowest marginal cost. Manager $M$ has the lowest quality. We assume that at least two managers have different quality levels.

At the wage-setting stage, all firms simultaneously make non-negative wage offers to all managers. We denote the offer of firm $i$ to manager $m$ as $w_{im}$.

In the application stage, after having observed the wage bids, managers decide which offer to accept. Their payoff is the wage received from their employer. Outside options are normalized to zero. In the first round of the application stage, each manager accepts the highest offer. If several firms have offered the most attractive wage to a manager $m$, he will select the firm with the lowest index. We call this the first tie-breaking rule. If only one manager accepts an offer from firm $i$, he will be employed. If two or more managers accept the offer, the firm will select one of them. As a second tie-breaking rule, we assume that a firm chooses the most competent manager if it is indifferent among several managers.\(^4\) In the second round of the application stage, the procedure is repeated with the rejected managers and the firms who have not yet filled their vacancies. The application process continues until each manager is either employed by a firm or rejected by all firms.

In the product-market stage, the $I$ firms engage in oligopolistic competition, with marginal costs $c_i$ given by the outcome of the application stage. We first provide an equilibrium characterization result which holds for a wide class of static oligopoly models. To

\(^4\)This second tie-breaking rule can be dispensed with by formulating the matching process as a dynamic game where firms approach managers in decreasing order of ability. In such a model Proposition 1 can still be derived, albeit with higher notational complexity.
obtain comparative statics with respect to globalization, we then specialize to Cournot competition with homogeneous goods and an inverse demand function $p = a - bx$, where $x$ is aggregate output, $p$ is the price, and $a$, $b$ are two positive numbers. In Section 6.4, we show that our main comparative statics result also applies to a standard model of price competition with differentiated goods.

We make the following symmetry assumption concerning profits.

**Assumption 1:** *For any given number of firms that are active in the market, the profit of firm $i$ (gross of managerial wages) is fully determined by the ability of the manager it employs and by the vector of abilities of the managers employed by the remaining $I - 1$ firms; it remains unchanged if the assignment of these remaining $I - 1$ managers to competitors is changed by a permutation.*

Thus, the gross profit of the firm is independent of its own identity and also independent of how the remaining managers are allocated to firms. It does matter, however, which managers are active in the market. Assumption 1 is general enough to include homogeneous or differentiated Cournot competition or price competition, but not localized competition à la Salop or Hotelling.\(^5\)

According to Proposition 1 below, it will be sufficient to consider gross profits in situations where only the $I$ managers with the highest abilities are employed. Thus, using Assumption 1, we write $\Pi_i(m)$ for the gross profits of firm $i$ (where managerial wages are not deducted) if it employs manager $m$ and the best $I$ managers are employed by some firm in the industry. Finally, net profits (or payoffs) of firm $i$ are defined as $\Pi_i(m) - w_{im}$.

### 3 Equilibria

We now provide a simple characterization of the symmetric equilibrium in pure strategies of the game.

\(^5\)It is left to future research to find out whether our main results also hold in examples where Assumption 1 is violated.
Proposition 1 There always exist symmetric pure-strategy equilibria in which managers \(1, \ldots, I\) are employed and

\[
\begin{align*}
(i) & \quad w^*_{im} = \Pi_i(m) - \Pi_i(I) \text{ for } m < I \\
(ii) & \quad w^*_{im} = 0 \text{ for } m \geq I. \\
(iii) & \quad \text{All firms obtain net profits } \Pi_i(I).
\end{align*}
\]

All symmetric equilibria must be of this type.

Proof: See Appendix.

We note that the proof does not rely on the specific linear Cournot model we focus on later. Thus Proposition 1 holds for any type of product market competition that satisfies Assumption 1.

According to (i), the wage differentials between managers \(m < I\) reflect the additional gross profit that a firm achieves by replacing a less competent manager with a more competent manager at the expense of some competitor. In the proposed equilibrium, the gross profit increase from hiring better managers is exactly offset by corresponding wage increases. Conversely, lower wages would be offset by losses in gross profits resulting from lower efficiency. By (ii), the marginal manager receives his outside option. Proposition 1 reflects the two-sided competition in the market for managers. Firms compete for managers, which induces them to bid wages up to \(w^*_{im}\). Managers compete by accepting the best offer they can obtain from the firms. (iii) follows because any profit increase from better managers translates completely into higher wages.

For convenience, we have invoked particular tie-breaking rules to resolve indifferences. There are two alternative approaches that lead to an equilibrium as described in Proposition 1. First, managers apply sequentially to all firms among which they are indifferent. The order in which they choose firms is not crucial. Second, managers coordinate on how they apply. For instance if they are indifferent among a set of firms, managers could always select the firm whose index is closest to their own index.\(^6\)

\(^6\)In the context of company worker training and technological spillovers, it has been already observed that equilibrium wages of workers or R&D employees are given by their effects on firms profits (e.g., Gersbach and Schmutzler (2003)).

\(^7\)The equilibrium does not exist if managers randomize among the set of firms among which they are indifferent and no revisions of wage offers by firms and applications decisions of accepted managers.
4 The Impact of Globalization

We now consider the effects of globalization, which we think of as the integration of managerial and/or product markets, resulting in a simultaneous duplication of demand, the number of firms and the managerial pool. We specify the analysis to the linear Cournot model to obtain closed-form solutions for wages, using Proposition 1. In Section 6, we will show that our comparative statics also hold for a standard model of price competition with differentiated goods. Thus, the results are not an artefact of the restriction to strategic substitutes.

4.1 Wages in the Cournot Model

The product market is characterized by a set of \( I \) active firms, inverse demand \( p = a - bx \) \((b > 0)\), and marginal costs \((c_1, \ldots, c_I)\); average costs are \( \bar{c} = \frac{1}{I} \sum_{i=1}^{I} c_i \). The output of an individual firm \( i \) is denoted by \( x_i \) and \( x = \sum_{i} x_i \) is the aggregate output. The following assumption ensures that all firms have positive outputs and profits.

**Assumption 2:** For all \( i \in \{1, \ldots, I\} \)

\[
\frac{a + I\bar{c}}{I + 1} - c_i > 0. \tag{2}
\]

An immediate implication of Assumption 2 is that \( a > \bar{c} \). Moreover, outputs in a Cournot oligopoly are

\[
x_i = \frac{1}{b} \left( \frac{a + I\bar{c}}{I + 1} - c_i \right). \]

The price is

\[
p = \frac{a + I\bar{c}}{I + 1}.
\]

Gross profits of a firm \( i \) that employs a manager \( m \) and thus has marginal costs \( c_i = c_m \) are

\[
\Pi_i(m) = \pi(c_m, \bar{c}) = \frac{1}{b} \left( \frac{a + I\bar{c}}{I + 1} - c_m \right)^2, \tag{3}
\]

are allowed. Then a firm can, for instance, deviate from the candidate equilibrium by setting zero wages for all managers. Subsequently, with positive probability, the firm under consideration can employ a manager \( m < I \) at zero wage as the manager \( m = I \) may be employed in the first round by another firm. As a consequence, the expected payoff is larger than \( \Pi_i(I) \).
where \( \pi(c_m, \overline{\pi}) \) denotes Cournot profits when a firm has marginal costs \( c_m \) and average industry costs are \( \overline{\pi} \). According to Proposition 1 and equation (3), the equilibrium wage of manager \( m \) is given by

\[
 w_m = \pi(c_m, \overline{\pi}) - \pi(c_I, \overline{\pi}) = \frac{1}{b} \frac{(c_I - c_m) ((2a + 2\overline{\pi}I) - (1 + I)(c_m + c_I))}{I + 1}.
\] (4)

Following Proposition 1, we argued that the wage corresponds to the gross profit increase resulting from having a better manager. In (4), this profit increase reflects the own cost reduction and the simultaneous increase in the costs of one competitor.

### 4.2 Market duplication

In the remainder of this section we shall consider integration as the simultaneous addition of symmetric national demands, firms, and manager pools. We shall refer to this type of integration as market duplication. In Section 5, we will address various alternatives.

In the benchmark model of market duplication, we assume that two countries of equal size and with an equal pool of managers integrate. Hence, after integration, instead of two markets with \( I = J \) firms and inverse demand \( p = a - B \cdot x \) \((B > 0)\), we have only one product market with \( I = 2J \) firms, aggregate inverse demand \( p = a - \frac{B}{2} \cdot x \), and two managers of each quality \( c_m \).\(^8\) Equilibrium profits and wages under integration are denoted by \( \Pi_m^G \) and \( w_m^G \), respectively, where \( m \) varies between 1 and \( J \) and each \( m \) stands for two firms that have the same marginal cost \( c_m \). Profits and wages before integration are denoted by \( \Pi_m^A \) and \( w_m^A \), respectively.

Because \( \frac{a + 1}{I + 1} c_m \) is decreasing in \( I \), Assumption 2 is easier to satisfy for \( I = J \) than for \( I = 2J \), that is, survival under autarky is easier than under globalization. Intuitively, while integration increases competition, it also increases demand, but the first effect dominates. Thus, as long as Assumption 2 holds for \( I = 2J \), equilibrium profits and wages under autarky and globalization are given by (3) and (4), where \( I = J \) and \( I = 2J \), respectively.

\(^8\)The demand function results from horizontal addition of the two identical autarky demand functions.
Simple rearrangements show that the effect of integration on the wages of manager \( m \) defined as \( \delta_m := u^G_m - u^A_m \), is

\[
\delta_m = -\frac{1}{B} \frac{(c_J - c_m) ((c_J + c_m) (1 + 3J + 2J^2) - 2a - 4J^2\bar{\sigma} - 6J\bar{\tau})}{2J^2 + 3J + 1}.
\]

This expression can be used to derive various results pertinent to the effects of globalization. We obtain for example:

**Proposition 2** For each parameter constellation, there exists a critical cost level \( c^* \in [c_1, c_J] \) so that integration increases wages for manager \( m \) if and only if his marginal cost is less than \( c^* \), i.e., \( c_m < c^* \). Wages in \((c^*, c_J)\) fall.

**Proof:** See Appendix.

The statement in Proposition 2 includes the possibilities that integration increases the wages of all managers or of none of the managers. It implies that, if the wage of one manager increases as a result of integration, then the same is true for every better manager. As the wage of manager \( J \) is zero before and after integration, an immediate implication is that the wage dispersion, that is, the difference in the wage between manager 1 and manager \( J \), increases whenever the wage of manager 1 increases with integration \((c^* > c_1)\). The wages of managers in the interval \((c^*, c_J)\) decrease.

In addition, several simple observations can be derived.

**Corollary 1** When the cost differences between the managers are small enough, the effect of market duplication on wages is positive for all managers with \( c_m < c_J \).

To see this, note that the numerator of (5) approaches \((c_J - c_m) (2\bar{\tau} - 2a) < 0\) as \( c_m \), \( c_J \) and \( \bar{\sigma} \) become sufficiently similar. This implies the result. Differentiation of (A.2) in the appendix with respect to \( a, \bar{\sigma}, J \) and \( c_J \) immediately yields the next result:

**Corollary 2** \( c^* \) is increasing in market size \( a \) and the average cost level \( \bar{\sigma} \) and decreasing in the number of firms in each country, \( J \), and the cost level \( c_J \) of the least efficient manager.

Corollary 2 implies that, for a given cost distribution of managers (and their corresponding firms), a greater fraction of them will benefit as the initial market size \( a \)
increases. The remaining three statements have to do with changes in the cost distribution. For instance, a sufficient increase in the average cost $\bar{c}$ for unchanged cost $c_j$ of the least efficient manager tends to make managers and their corresponding firms more similar. Consistent with Corollary 1, this increasing similarity means that more managers will benefit from wage increases. The effect of an increase in the cost level of the least efficient manager, $c_j$, for given average cost $\bar{c}$ has the converse interpretation. Finally, an increase in the number of firms $J$ for given levels of average and maximal costs can be interpreted as an increase in competition under autarky. Thus, if the market is initially more competitive, it will require a lower marginal cost for a manager to benefit from market duplication.

To understand the economic logic behind Proposition 2, it is useful to note that

$$w_m = \int_{c_j}^{c_m} \frac{\partial \pi}{\partial c_i}(c_i, \bar{c}) \, dc_i,$$

so that understanding the effects of globalization on wages boils down to understanding the effect on $\frac{\partial \pi}{\partial c_i}$. Equation (6) reflects the effect of a cost reduction of firm $i$ from $c_j$ to $c_m$ with fixed average industry costs. Expressed differently, the wage reflects the joint effect of a reduction in the firm’s own marginal costs and an increase in one competitor’s costs by the same amount. For Cournot competition, we obtain:

**Proposition 3** $\frac{\partial \pi^G}{\partial c_i} - \frac{\partial \pi^A}{\partial c_i}$, the effect of market duplication on $\frac{\partial \pi}{\partial c_i}$, is decreasing in $c_i$. Thus, if market duplication raises $\frac{\partial \pi}{\partial c_i}$ for a particular firm, it does so for any firm with lower marginal costs. Also, $\frac{\partial \pi^G}{\partial c_i} - \frac{\partial \pi^A}{\partial c_i}$ is positive for firms that have lower than average marginal costs.

**Proof:** See Appendix.

The fact that the effect of integration on $\frac{\partial \pi}{\partial c_i}$ is decreasing in $c_i$ (and thus increasing in ability) lies behind the wage effects of integration. Together with Proposition 3, equation (6) implies that wage dispersion, that is, the difference between the wage of manager 1 and manager $J$, is higher after globalization than before globalization. This immediately implies the statement of Proposition 2 that if the wage effect of integration is positive for any manager, it is positive for any more able manager. The logic of this
argument does not necessarily require Cournot competition; it merely requires that
\[ \frac{\partial \pi^A}{\partial c_i} - \frac{\partial \pi^A}{\partial c_i} \] is decreasing in \( c \).

To understand the effect of globalization on \( \frac{\partial \pi}{\partial c_i} \), let \( Q(c_i, \bar{c}) \) stand for equilibrium outputs and \( M(c_i, \bar{c}) \) for equilibrium margins. From \( \pi(c_i, \bar{c}) = Q(c_i, \bar{c}) \cdot M(c_i, \bar{c}) \) we obtain
\[ \frac{\partial \pi}{\partial c_i} = M \frac{\partial Q}{\partial c_i} + Q \frac{\partial M}{\partial c_i}. \] (7)

Thus, the value of having lower marginal costs (while simultaneously increasing the costs of a competitor) consists of a positive effect on output evaluated at the margin and a positive effect on the margin evaluated at the output level. Market duplication affects three of the four components in (7). It reduces the profit margin \( M \), which makes it less attractive to increase own equilibrium output by having a more able manager. Crucially, the equilibrium output \( Q \) of good managers increases, whereas the equilibrium output of bad managers may fall. This output redistribution explains why, if integration has a positive effect on \( \frac{\partial \pi}{\partial c_i} \) for some firm, then it also has a positive effect on \( \frac{\partial \pi}{\partial c_i} \) for more efficient firms. It thus also explains why, if integration has a positive effect on the wage for some manager, then it also has a positive effect on the wage for some more able manager. Simple calculations show that globalization also increases the impact of lower costs on the equilibrium output \( \left( \frac{\partial Q}{\partial c_i} \right) \). This effect is more valuable for relatively efficient firms (with high margins) than for less efficient firms (with low margins).\(^9\) All told, globalization thus has an ambiguous effect on incentives for improvement \( \frac{\partial \pi}{\partial c_i} \) in general, but the effect tends to be positive for efficient firms and negative for inefficient firms. This observation is the general force behind the positive effect of integration on wage dispersion.

The following result helps to place our findings in the perspective of existing work on the effects of competition on cost-reducing investments.

Proposition 4  (i) Market duplication reduces total gross profits if firms are sufficiently similar.

(ii) If \( c_m < c_{m+1} \), \( \frac{\Pi^C_m}{\Pi^C_{m+1}} > \frac{\Pi^A_m}{\Pi^A_{m+1}} \).

\(^9\) The effect of a lower marginal cost on the margin \( \frac{\partial M}{\partial c_i} \) turns out to be independent of globalization.
Proof: See Appendix.

As to (i), changes of market parameters that reduce total profits have sometimes been used as a reduced-form description of increasing competition (Schmidt 1997). However, it is straightforward to provide examples where market duplication increases profits when firms are very asymmetric.10

As to (ii), Boone (2008) has argued that many standard measures of increasing competition are associated with a positive effect on the ratio between profits of good firms and profits of bad firms. Together, (i) and (ii) confirm that globalization has effects commonly associated with increasing competition.

4.3 Effects on Total Wages

While the preceding analysis supports a positive effect of integration on dispersion, the effect on total wages is unclear. We now introduce a specific example to show that it is not only possible that some managers will lose from globalization, but also that the total wage sum falls. We assume that ability differences are constant:

\[ c_m = c_J - (J - m)\Delta, \ m = 1, \ldots, J, \ \text{for some } \Delta > 0. \]  

\[ (8) \]

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Results</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>J \ Δ</td>
<td>a \ B</td>
<td>c_J</td>
<td>\Pi^A_1</td>
<td>\Pi^G_1</td>
<td>\Pi^A_2</td>
<td>\Pi^G_2</td>
<td>\sum_i \Pi^A_i</td>
<td>\sum_i \Pi^G_i</td>
<td>\sum_i w^A_i</td>
<td>\sum_i w^G_i</td>
</tr>
<tr>
<td>1. 5 10</td>
<td>300 1 100</td>
<td>1.47 1.78</td>
<td>15444</td>
<td>12000</td>
<td>12666</td>
<td>12000</td>
<td>1.66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. 5 10</td>
<td>600 1 100</td>
<td>1.22 1.38</td>
<td>77112</td>
<td>48694</td>
<td>32666</td>
<td>33818</td>
<td>2.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. 5 10</td>
<td>1000 1 100</td>
<td>1.13 1.22</td>
<td>237112</td>
<td>143900</td>
<td>59334</td>
<td>62910</td>
<td>3.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. 5 10</td>
<td>1500 1 100</td>
<td>1.08 1.15</td>
<td>562112</td>
<td>337290</td>
<td>92666</td>
<td>99272</td>
<td>5.30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Numerical results for \( c_m = c_J - (J - m)\Delta \)

In this case, as shown by the results displayed in Table 1, the total wage sum under autarky, \( 2 \sum_i w^A_i \), can be smaller or larger than the total wage sum after globalization, 10

\[ 10 \text{Let } J = 2, \ M \geq 2 \text{ and } p = 1 - x. \ \text{Let } c_1 = 0 \text{ and } c_2 > 0. \ c_2 \leq 1/3. \ \text{Using (3), total gross profits decrease after integration if (and only if) } c_2 \in (c^*, 1/3), \text{ where } c^* \text{ is a critical value of } \frac{1}{10} \sqrt{7} \approx 0.29987. \]
Total wages increase under integration for high values of \( a \); they decrease for low values of \( a \).\[^{11}\]

5 Asymmetric Firms

We now show how our approach can be extended to the case of asymmetric firms. In Subsection 5.1, we provide a general characterization of the equilibrium along the lines of Proposition 1, focusing on the autarky case for notational simplicity. In Subsection 5.2, we show for the Cournot case that the comparative statics of managerial wages still follow the pattern revealed in Section 4: If globalization increases a manager’s wage, the same is true for every more able manager.

5.1 Assignment of Managers to Firms under Autarky

In this section, we continue to assume that the gross profits of a firm that employs a better manager increase. However, the size of the profit increase differs across firms. This is supposed to reflect arbitrary exogenous differences between firms, resulting from technology, location, size etc.

We assume that, in each country, managers can be strictly ordered with respect to abilities. This means that, at any firm, a more able manager achieves lower marginal costs than a less able one. Writing the costs of firm \( i \) if it employs manager \( m \) as \( c_{i,m} \), we thus assume that \( c_{i,m} < c_{i,m+1} \) for all firm types \( i \in \{1, \ldots, J\} \), and manager types \( m \in \{1, \ldots, M\} \).

Similarly, in each country firms can be strictly ordered with respect to how much they benefit from the skills of better managers. Then, the type of a firm and the type of its manager together fully determine its gross profits. Thus, we write \( \Pi_i^a(m) \) to denote the gross profits of a firm of type \( i \) with a manager of type \( m \) under autarky. The above reasoning is summarized in the following assumption.

**Assumption 3:** For firms with indices \( i < j \in \{1, 2, \ldots, J\} \), the following conditions

\[^{11}\]The table also confirms the results of Proposition 4 and the effect of \( a \) on the critical value (Corollary 2).
hold for arbitrary \( m, m' \in \{1, 2, ..., J \} \) such that \( m' < m \):

\[
\Pi^A_i(m') - \Pi^A_i(m) > 0
\]
\[
\Pi^A_i(m') - \Pi^A_j(m') > \Pi^A_j(m).
\]

Thus, the gross profit increase resulting from having a better manager is higher for a firm with lower type.

The remaining assumptions from Section 2 will continue to be maintained.

**Proposition 5** Suppose Assumption 3 holds.

(i) In any equilibrium of the matching game, a firm with type \( i \in \{1, 2, ..., J \} \) employs manager \( m_i = i \).

(ii) In any equilibrium, the wage offers satisfy \( w_{JJ} = 0 \) and

\[
\Pi^A_m(m) - \Pi^A_m(m + 1) \geq w_{mm} - w_{m+1,m+1} \geq \Pi^A_{m+1}(m) - \Pi^A_{m+1}(m + 1).
\]  

(iii) Suppose \( w_{JJ} = 0 \) and (11) holds for \( m \in \{1, ..., J - 1 \} \). Suppose further that

\[
w_{im} = w_{mm} \quad \text{for} \quad i \in \{1, ..., m - 1, m + 1 \},
\]
\[
w_{im} \leq w_{mm} \quad \text{for} \quad i \in \{m + 2, ..., M\}.
\]

Then \( w_{im} \) describes an equilibrium wage profile.

**Proof:** See Appendix.

The meaning of the statements in (i) and (ii) is as follows. First, there always is assortative matching, and second, we can find simple upper and lower bounds for the wage differences between adjacent managers that are determined by profit differences. (iii) shows that, whenever the wages paid to the manager obey these bounds, an equilibrium resulting in these wages exists.

Proposition 5 immediately implies that, contrary to the symmetric case, the net profits of different firms are usually not identical. Better firms obtain higher profits. Thus the rents from superior managerial abilities are shared between firms and managers. Uniqueness of the wage offers \( w_{mm}^* \) that are paid out in equilibrium is not guaranteed. However, by using dominance arguments, stronger restrictions than (11) are possible. To see this, we consider a simple example with \( J = 2 \).
We know from Proposition 5 that, in any equilibrium, there is assortative matching and that \( w_{22} = 0 \) and \( \Pi^A_1(1) - \Pi^A_1(2) \geq w_{11} - w_{22} \geq \Pi^A_2(1) - \Pi^A_2(2) \). We will show that any strategy \((w_{21},w_{22})\) of firm 2 with \( w_{22} = 0 \) and \( w_{21} \geq \Pi^A_2(1) - \Pi^A_2(2) \) is weakly dominated by \((w_{21}',w_{22}') = (\Pi^A_2(1) - \Pi^A_2(2),0)\).

To see this, first suppose that \( w_{22} = 0 \) and \( w_{11} \geq w_{21} \). Then manager 2 will be assigned to firm 2, and its payoffs are \( \Pi^A_2((2)) \). Deviation to \((w_{21}',w_{22}')\) yields the same payoffs. If \( w_{22} = 0 \) and \( w_{11} < w_{21} \), then manager 1 will be assigned to firm 2 which obtains payoffs of \( \Pi^A_2((1)) - w_{21} < \Pi^A_2((1)) - (\Pi^A_2((1)) - \Pi^A_2((2))) = \Pi^A_2((2)) \). For the payoff from deviation to \((w_{21}',w_{22}')\), there are two subcases. If \( \Pi^A_2((1)) - \Pi^A_2((2)) > w_{11} \), then firm 2 employs manager 1 and obtains payoffs \( \Pi^A_2((1)) - (\Pi^A_2((1)) - \Pi^A_2((2))) = \Pi^A_2((2)) \). If \( \Pi^A_2((1)) - \Pi^A_2((2)) \leq w_{11} \), then manager 1 will be assigned to firm 1 and firm 2, which employs manager 2, also obtains payoffs of \( \Pi^A_2((2)) \). Thus the proposed strategy of player 2 is weakly dominated. Thus, if player 1 assumes that player 2 does not play weakly dominated strategies he will choose \( w_{11} = w_{11} - w_{22} = \Pi^A_2((1)) - \Pi^A_2((2)) \).

Our model clearly does not capture all important aspects of managerial compensation. In particular, we make no attempt to model agency conflicts between owners and managers. However, the framework is rich enough to generate clear empirical implications. For instance, the properties of the equilibrium in Proposition 5 relate to recent empirical evidence. In particular, Kaplan and Rauch (2010) and Kaplan (2012) suggest that the compensation of CEOs is related to firm performance. More profitable firms pay significantly more than less profitable ones. Consistent with this evidence, Proposition 5 predicts that gross and net profits (or the market value of the firm) and the remuneration of CEOs are positively associated. Nevertheless, even under the assumptions of our models that rule out complications such as agency conflicts, profit differentials do not determine managerial wages uniquely; they only delineate a range of possible wages.

5.2 Integration and Wages

We now turn to the analysis of the effects of globalization on wages. For each of the \( J \) firms in one country, there exists a firm of identical type in the other country. The
same is assumed for managers. Moreover, we have to assume that conditions analogous to Assumption 3 also apply after integration. Moreover, with slight abuse of notation, we have to reinterpret \( w_{im} \) as the wage that a firm of type \( i \) offers to a manager of type \( m \). With these modifications, Proposition 5 also applies to the case of the integrated economy. The statement and proof of the proposition are analogous, so that they hold with \( \Pi_i^A \) replaced by \( \Pi_i^G \).

For the comparative statics, we focus again on the linear Cournot case. Using (3), we obtain,

\[
\Pi_i(m) = \pi(c_{i,m}; \nu) = \frac{1}{b} \left( \frac{a + I^\nu}{I + 1} - c_{i,m} \right)^2. \tag{12}
\]

According to Proposition 5, the equilibrium wage difference between managers \( m \) and \( m + 1 \) is bounded below by \( \Delta w_{m,m+1} \equiv \Pi_{m+1}(m) - \Pi_{m+1}(m + 1) \) and above by \( \Delta w_{m,m+1} \equiv \Pi_m(m) - \Pi_m(m + 1) \). We obtain:

**Proposition 6** Suppose Assumption 3 holds. If globalization increases the lower bound for the equilibrium wage differential between manager \( m \) and manager \( m + 1 \) (\( \Delta w_{m,m+1}^G > \Delta w_{m,m+1}^A \)), it also increases the wage differential between manager \( m - 1 \) and manager \( m \) (\( \Delta w_{m-1,m}^G > \Delta w_{m-1,m}^A \)). An analogous statement holds for the upper bound.

**Proof:** See Appendix.

Proposition 6 is thus the analogue of Proposition 2 for the case of asymmetric firms: If a manager of some given ability benefits from globalization, then so does any manager of higher ability.

### 5.3 Summary

In this section, we have extended our general characterization of the matching equilibrium to the case of asymmetric firms. We have shown that assortative matching will emerge, so that more able managers are hired by firms that benefit more from these managers’ higher abilities. Moreover, for the linear Cournot model, we have seen that if globalization increases the wages of some manager, the same will be true for any more able manager.
6 Extensions

To illustrate the impact of globalization on managerial remuneration in the most transparent way, we have so far focused on the simple case of market duplication. We now consider alternative approaches. All of the extensions are based on the symmetric model of Section 2.

6.1 Biased Integration

To obtain a better understanding of the factors behind the effects of globalization in the Cournot model, we now allow for biased integration, where demand and the number of firms increase by different factors. Specifically, the number of firms after integration is $I = kJ$ ($k \in \mathbb{N}; k \geq 1$), and the demand parameter is $b = \frac{B}{l}$ ($l \in \mathbb{R}; l \geq 1$). We assume that there are more managers than firms before and after integration and that the costs of the marginal manager and the average cost remain at $c_J$ and $\bar{c}$, respectively, so that Proposition 1 and the resulting wage formula (4) can be applied. Then the wage effect of biased integration is positive for a firm $m$ with costs $c_m < c_J$ if and only if

$$\frac{l (2a + 2\bar{c}kJ - (c_m + c_J)(kJ + 1))}{kJ + 1} - \frac{(2a + 2\bar{c}J - (c_m + c_J)(J + 1))}{J + 1} > 0.$$  (13)

This has immediate implications for the polar cases where integration affects only demand or only the number of firms, which we state without proof in the following result.

Proposition 7 (i) If integration corresponds to a pure demand increase ($l > 1, k = 1$), all wages will increase as a result of integration. (ii) If integration corresponds to a pure increase in the number of firms ($l = 1, k > 1$), all wages will fall as a result of integration.

(i) A ceteris paribus reduction in the demand parameter $b$ (multiplication of $b = B$ with $l > 1$) corresponds to an increase in demand resulting from integration that is met exclusively by the firms from one country (for instance, because their managers are so much more competent than the ones in the other country that the firms in the other country immediately disappear after integration). As the increased demand on
the product market translates into an increased demand for managers, this kind of integration has unambiguously positive effects on managerial wages. Moreover, while wage inequality increases in absolute terms, the relative wages remain the same. (ii) At the other extreme, one can isolate the effect of increasing the number of firms $I$ in the market without changes in the demand parameters. Intuitively, this corresponds to unilateral trade liberalization, where firms in one country are exposed to the exports from the other country but obtain no market access themselves. (13) immediately implies that, in this case, the wage effect is negative for all managers. Increasing competition from other firms reduces not only the overall profits of a firm, but also the incremental effect on profits of a better manager.\footnote{The absolute value of the derivative of (13) with respect to $c_m$ increases.}

Going beyond the polar cases of Proposition 7, Figure 1 uses (13) to illustrate arbitrarily biased integration in a specific parameterized example. The figure refers to the parameterizations given in Table 1. Thus, we fix $J = 5$, $B = 1$, $c_s = 100$ and $\Delta = c_{m+1} - c_m = 10$, and we consider demand parameters $a = 300, 600, 1000, \text{ and } 1500$. We delineate those combinations of $k$ and $l$ for which an average manager (marginal cost 80) experiences a wage increase for the respective parameter values. Points
on the dashed line correspond to unbiased integration; \((2, 2)\) is the special case of market duplication. For the specific parameterization, duplication increases average wages for \(a = 300, 600\) and \(1000\). The lightly shaded area in the upper left-hand corner of the figure corresponds to those types of integration \((k, l)\) that increase wages for average managers even when the demand parameter is comparatively small \((a = 300)\). As \(a\) increases, wages increase even for larger values of \(k\) (stronger growth of the number of firms) and lower values of \(l\) (less demand growth). In line with Proposition 7, a pure increase of demand \((l > 1\) and \(k = 1\)) always leads to higher wages, whereas a pure increase of the number of firms \((k > 1\) and \(l = 1\)) always leads to a reduction in wages.\(^{13}\)

To sum up, a bias towards the demand effect \((l\) large, \(k\) small) fosters a positive effect of integration on wages. Moreover, increases in \(a\) also work toward a positive effect of integration.\(^{14}\)

### 6.2 Pure Labor Market Integration

Next, consider the effect of pure labor market integration, which corresponds to an integration of managerial pools without a change in the remaining parameters of the model. Thus, in each country, firms can now make wage offers to all managers in the two countries. Using the logic of Proposition 1, it is straightforward to see that before and after labor market integration the wages are determined by the profit differential that a manager generates relative to the marginal manager \(I\).\(^{15}\) As product markets are unaffected by pure labor market integration, there are no wage effects from pure labor market integration.\(^{16}\)

\(^{13}\)Similar results hold for non-average managers.

\(^{14}\)Recall the similar statement from Corollary 2 for the unbiased case.

\(^{15}\)This is also true for heterogeneous firms (with the logic of Proposition 5).

\(^{16}\)Note, however, that this argument relies to some extent on the symmetric manager pools in the two countries.
6.3 Exit

We consider an arbitrary situation where only $2K < 2J$ firms can survive after integration. In principle, Proposition 1 still applies in such a situation. However, only the managers of types $m = 1, \ldots, k$ are employed at positive wages. They earn $\Pi^G_i(m) - \Pi^G_i(I) = \Pi^G_i(m)$. The remaining managers are “employed” by the non-producing firms, at wages $0$; all firms earn zero net profits.

Therefore, the wage effect of globalization is $\Pi^G_i(m) - (\Pi^A_i(m) - \Pi^A_i(J))$ for managers of types $m = 1, \ldots, K$ and $- (\Pi^A_i(m) - \Pi^A_i(J)) < 0$ for managers of types $m = K + 1, \ldots, J$. Thus, to obtain a statement analogous to Proposition 2, it suffices to show that if a manager from $m = 1, \ldots, \mu$ benefits from globalization, so will every better manager.

For the Cournot model, let $\overline{c_j} = \sum_{i=1}^J c_i$ and $\overline{c_K} = \sum_{i=1}^K c_i$ be the average cost of active firms before and after integration, respectively. Using (3), we thus obtain:

\[
\begin{align*}
\Pi^A_i(m) &= \frac{1}{B} \left( \frac{a + J\overline{c_j}}{J + 1} - c_m \right)^2; \\
\Pi^A_i(J) &= \frac{1}{B} \left( \frac{a + J\overline{c_j}}{J + 1} - c_J \right)^2; \\
\Pi^G_i(m) &= \frac{2}{B} \left( \frac{a + K\overline{c_K}}{K + 1} - c_m \right)^2.
\end{align*}
\]

Setting $R = \frac{a + K\overline{c_K}}{K + 1}$ and $S = \frac{a + J\overline{c_j}}{J + 1}$, the wage effect is thus

\[
\Pi^G_i(m) - (\Pi^A_i(m) - \Pi^A_i(J)) = \frac{2}{B} (R - c_m)^2 - \left( \frac{1}{B} (S - c_m)^2 - \frac{1}{B} (S - c_J)^2 \right) = \frac{1}{B} \left( c_m^2 + (2S - 4R) c_m + 2R^2 - 2Sc_J + c_J^2 \right). \tag{14}
\]

This expression has at most two zeroes as a function of $c_m$. Further, it is positive for $c_m = c_J$ and negative for $c_m = R \in (c_K, c_J)$. Hence, for $c_m$ in the region between $c_1$ and $R$, $\frac{1}{B} c_m^2 + (2S - 4R) c_m + 2R^2 - 2Sc_J + c_J^2$ has at most one zero and it is declining in $c_m$. Hence, our main argument that globalization increases the heterogeneity of managerial salaries is robust to the possibility of exit under globalization.
6.4 Price Competition

We now show that the comparative statics of managerial wages are robust to the introduction of price competition with differentiated goods. We apply the adaption of the model of Singh-Vives (1984) to \( I > 2 \) asymmetric firms by Ledvina and Sercar (2011). Thus we assume that these firms produce symmetrically differentiated goods with constant marginal costs \((c_1, ..., c_I)\). We first present the model in general and then apply it to integration and autarky, respectively. We assume that there is a representative consumer whose demand for each of the \( I \) varieties is derived from a utility function

\[
U(q) = \alpha \sum_{i=1}^{I} q_i - \frac{1}{2} \left( \beta \sum_{i=1}^{I} q_i^2 + \gamma \sum_{i=1}^{I} \sum_{j=1,j \neq i}^{I} q_i q_j \right). \tag{16}
\]

The inverse demand of a consumer who maximizes \( U(q) - \sum_{i=1}^{I} p_i q_i \) can be derived as

\[
p_i(q) = \frac{\partial U}{\partial q_i} = \alpha - \beta q_i - \gamma \sum_{j=1,j \neq i}^{I} q_j.
\]

Define

\[
a_I = \frac{\alpha}{\beta + (I - 1) \gamma}, \quad b_I = \frac{\beta + (I - 2) \gamma}{(\beta + (I - 1) \gamma)(\beta - \gamma)}, \quad s_I = \frac{\gamma}{(\beta + (I - 1) \gamma)(\beta - \gamma)}
\]

The demand functions for \( i = 1, ..., I \) are then

\[
q_i(p) = a_I - b_I p_i + s_I \sum_{j=1,j \neq i}^{I} p_j.
\]

According to Proposition 2.6 in Ledvina and Sercar (2011), the equilibrium price for firm \( i \) when all firms are viable is

\[
p_i^* = \frac{1}{2b_I + s_I} \left( a_I + s_I \frac{I a_I + b_I I c_I}{2b_I - (I - 1) s_I} + b_I c_i \right). \tag{17}
\]

Setting \( I = 2J, a_I = a_{2J}, b_I = b_{2J} \) and \( s_I = s_{2J} \) in (17) gives the equilibrium prices in the integrated markets.
Under autarky, the preferences of the two representative players are also given by (16) with $I = 2J$. However, the $J$ varieties produced by firms in the other country are prohibitively costly. Using Proposition 1 in Ledvina and Sercar (2011), the equilibrium in each country then corresponds to the previously calculated one in an economy with only $J$ firms, so that the expression $2J$ in each of the coefficients in $a_I$, $b_I$, $c_I$ has to be replaced by $J$.17 Moreover, the coefficients have to be adjusted so that demand in each country is half as large as under globalization: In each of the two countries, there is one representative consumer with the same preferences, but half the income of the representative consumer under globalization. Thus

$$
a^A = \frac{0.5\alpha}{b + (J - 1)\gamma}
$$

$$
b^A = \frac{0.5\beta + 0.5(J - 2)\gamma}{(\beta + (J - 1)\gamma)(\beta - \gamma)}
$$

$$
s^A = \frac{0.5\gamma}{(\beta + (J - 1)\gamma)(\beta - \gamma)}.
$$

We now state the main result of this section.

**Proposition 8** In the model of price competition described above, $w^G - w^A$ is decreasing in $c_i$ for all $J \geq 2$, $\alpha \geq 0$, $\beta > 0$ such that $\gamma \in (0, \beta)$.

The proof is provided in the appendix.

The result confirms the insights from the Cournot model. It immediately shows that, if globalization increases the wages of any manager, it also increases the wages of any better manager. The intuition is the same as for quantity competition: As competition increases, output is redistributed towards the more efficient firms.

## 7 Discussion and Related Literature

In this section, we first place our paper within the literature on managerial wages (Section 7.1). We then explore the similarity between our analysis and recent work on competition and investment and productivity (Section 7.2). Finally, we discuss the paper in the light of previous work on matching and assignment (Section 7.3).
7.1 Managerial Wages

Previous work has dealt with the determinants of managerial wages.\textsuperscript{18} For instance, Gabaix and Landier (2008) provide an empirical analysis of the determinants of managerial wages, using a simple matching model to predict a positive effect of firm size on wages and wage dispersion.\textsuperscript{19} However, they treat “size” in a black box fashion: There are managers of different “talents” $T(m)$ and firms of different “sizes” $S(n)$, where $T(m)$ and $S(n)$ are decreasing functions. Profits are assumed to be $CS(n)\gamma T(m)$, where $C, \gamma > 0$. For “large” firms (small $n$), the partial derivative of the profit function with respect to managerial quality is higher, resulting in a higher wage. While this model provides a useful framework for empirical analysis, the concept of firm size remains vague, and the relation between globalization and firm size (and wages) is not treated.\textsuperscript{20}

Our approach is also complementary to Baranchuk, MacDonald and Yang (2011), who study an agency model with free entry of firms where managers differ in their ability. They show that an increase in industry demand increases both the overall level and skewness of the cross-sectional distribution of managers’ compensation. However, while globalization typically entails an increase in per-firm demand, it is not equivalent to a simple demand shock.\textsuperscript{21}

Contrary to Gabaix and Landier (2008), Subramanian (2013) directly addresses the impact of competition on wages, but he does not deal with wage dispersion, as managers are homogeneous.\textsuperscript{22} Interestingly, he shows that, whereas reductions in entry costs decrease compensation, increases in product substitutability have the opposite effect. Thus, different types of parameter changes which are usually associated with increasing competition have different wage effects, which demonstrates the need for a case-by case

\textsuperscript{18}See Edmans and Gabaix (2009) for a recent survey.
\textsuperscript{19}Edmans et al. (2012) also use a matching model where they investigate the role of firm size. They consider the effects on wage level and structure (fixed base salary and shares).
\textsuperscript{20}Related to the size explanation, Gayle and Miller (2009) have argued that increasing complexity of managing large firms is reflected in wage increases.
\textsuperscript{21}In addition, it has other, potentially countervailing effects on managerial compensation (see Section 4.2). For instance, it simultaneously reduces margins, which in itself turns out to work against increasing wage spreads. It is therefore not obvious that globalization will have similar effects as a demand increase.
\textsuperscript{22}Contrary to us, he uses a general equilibrium model and managers choose effort levels.
analysis. Hermalin (2005) interprets wage increases as a response to stricter corporate governance rules. Frydman (2007) and Murphy and Zabojnik (2007) argue that general human capital has become increasingly important, thereby increasing the outside options of managers.\textsuperscript{23}

More broadly, our paper is related to the literature on superstars initiated by Rosen (1981), who shows how quality differences between agents lead to more than proportional differences in wages, turning agents with a fairly small quality advantage into “superstars” earning substantially more than the others. Our arguments show that globalization moves the market for managers closer to such a market for superstars.\textsuperscript{24}

\textbf{7.2 Competition, Investment and Productivity}

Contrary to the existing literature, our paper analyzes how the simultaneous increase of demand, number of firms and managers brought about by integration increases wage dispersion. Thereby, it provides a hitherto unobserved link between the analysis of managerial wages and the well-established literature on the relation between competition and firm productivity. Our main result is based on the fact that, for more able managers, integration is more likely to increase the sensitivity of gross profits to marginal costs. This in turn reflects a redistribution of output from relatively bad to relatively good firms as competition increases. This effect leads to increases of wages of high-ability managers relative to those of low-ability managers. The mechanisms underlying the effects of competition and market size on productivity share some common features with our approach.

1. There is a large body of literature on the relation between various types of competition parameters and cost-reducing investments, mainly for ex-ante symmetric

\textsuperscript{23}Expanding on this explanation, Giannetti (2011) argues that the increasing outside opportunities incentivize managers to focus on short-term projects which improve their chances on the job market. Shareholders have to compensate managers sufficiently to reduce these incentives.

\textsuperscript{24}In the context of globalization, such superstar effects have for instance been discussed by Manasse and Turrini (2001), who also argue that globalization increases wage heterogeneity. Their analysis differs from ours in several important respects. First, they consider wage differences between skilled and unskilled workers rather than between managers. Second, the channel through which decreasing trade costs operate is totally different: The increasing wage heterogeneity comes from redistribution of income between exporting and non-exporting firms with different skill intensities.
firms (see, e.g., Vives 2008). Depending on the details of the situation, compe-
tition makes it more or less attractive to get ahead of others. A few contributions
also allow for asymmetric firms. Boone (2000) argues that greater competition
increases investments by leaders and decreases those of laggards, similarly to
Schmutzler (2013). As in the present paper, these different effects on the two
types of firms are partly driven by a redistribution of output from laggards to
leaders, which has positive effects on the investments of leaders, but negative
effects on those of laggards. However, the literature on competition and invest-
ment has not dealt with simultaneous increases in market demand, the number
of firms, and managerial pools.

2. Several papers have treated the effects of increasing market size on the produc-
tivity of firms in the market, where productivity at the level of individual firms
is either endogenous (Raith 2003) or exogenous (Asplund and Nocke 2006 and
Syverson 2004). In these papers, increasing market size also leads to a redis-
tribution of output from laggards to leaders, but the effects of competition are
driven by changes in the number of firms. For example, Raith (2003) considers
the effects on managerial efforts in a model where (ex-ante symmetric) firms com-
pete on the Salop circle. While larger markets increase demand per firm, they
also attract more firms. Nevertheless, the net effect is positive, and innovation
increases. To make the analysis comparable to ours, one should treat the number
of firms as an exogenous parameter and ask, as in our paper, how a simultaneous
proportional increase of market size and the number of firms affects incentives.
While such a change would reduce profits, it would have no effect on cost reduc-
tion incentives. With heterogeneous firms (not treated by Raith 2003), market
duplication would increase cost-reduction incentives for firms that have lower

---

25 In a symmetric firm setting, Schmidt (1997) identifies an inverse U-relation between competition
and equilibrium managerial efforts. Apart from the absence of managerial heterogeneity, the setting
is very different from ours: Competition is an unspecific parameter change that reduces profits and
increases the threat of liquidation, which is assumed to be costly to managers. Competition therefore
makes it less costly for firm owners to induce managerial effort.

26 Moreover, wages in our model reflect the value of cost reductions from attracting a manager that
simultaneously raise rivals’ costs by the same amount (as a competitor loses this manager), whereas
the literature typically considers the effects of individual cost reductions.

27 This follows from Proposition 2 in Raith.
than average costs, whereas it would decrease cost-reduction incentives for firms that have higher than average costs. This would be complementary to our result that market integration changes the wage distribution even if the number of firms is held fixed.

Syverson (2004) analyzes how the density of demand affects productivity dispersion within a market. He argues that higher density of demand attracts more firms into the market, thereby endogenously increasing the degree of substitution between different firms. Thus, less productive firms find it harder to survive, and the equilibrium distribution consists of better and more homogeneous firms. As in our model, the result is driven by an output relocation away from the less productive firms as market conditions become more competitive. In Syverson’s case, the very unproductive firms are driven out of the market completely.

Using a dynamic monopolistic competition model, Asplund and Nocke (2006) ask how increases in market size affect the rate of firm turnover in a dynamic setting with exit and entry, where firms face repeated idiosyncratic productivity shocks. They show that, in markets with large demand, the rate of turnover is higher, resulting in a set of firms that tend to be younger and more productive. As in our model, the ratio between the profits of more productive firms and those of less productive firms increases when competition becomes more intense. However, the difference in the profits for more productive and less productive firms is generally decreasing in competition, whereas in our case, this is not necessarily true for firms that are much more productive.

### 7.3 Matching and Assignment

Our model is essentially an assignment game. When market integration takes place, the assignment game operates on a larger scale than under autarky. In this section, we relate our work in detail to the matching/assignment literature and identify our contribution in this area.

Following the path-breaking contribution to two-sided matching by Gale and Shapley (1962), Shapley and Shubik (1972) wrote the first paper on assignment games (also
called two-sided matching with money or “continuous” matching).\textsuperscript{28} Our treatment of the basic symmetric model of Section 2 differs from the literature on assignment games in three ways. First, on the one side there are firms which are identical with regard to their ability to produce a surplus with a particular manager. This difference is not essential on its own, as the most general stability results on assignment games cover this case.\textsuperscript{29} Second, we focus on the question of how the surplus and its distribution in a stable matching (assignment) change when both market sides are doubled. This question is not discussed in the matching literature (as surveyed by Sönmez and Ünver, 2010, for instance). Third, and most importantly, the surplus produced by a pair consisting of a firm and manager—the gross profit in our context—depends on which other managers have been matched to other firms, as the surplus is determined through competition in the product market after the pairs have been formed. Thus, we have an assignment game with externalities. In this respect, our work is closely related to the small body of literature on matching with externalities, notably Sasaki and Toda (1996), subsequent work by Hafalir (2008), and Mumcu and Saglam (2010). For the assignment game with externalities, it is important to recognize that a deviating pair may need to consider the reactions of the other agents, as such reactions may affect the surplus of the deviating pair because of externalities. Sasaki and Toda (1996) have shown that stable matchings may not exist, unless the deviating pair is extremely pessimistic about the matchings that can arise. In other words, a pair will only destabilize a matching if it is made better off under all conceivable matchings when it makes a pair (see also Mumcu and Saglam, 2010). Moreover, Sasaki and Toda (1996) show that a stable matching may not be Pareto-optimal.

The particular structure of our game allows for sharper results than usual in assignment games with externalities. First, some of the impossibility results of the literature do not hold in our setting. In particular, stable matchings in our set-up exist as long as all previously matched firms and managers are rematched when a pair deviates. Second, every stable matching is Pareto-optimal. The reason is that product market

\textsuperscript{28}Roth and Sotomayor (1990) provide an early survey, Hatfield and Milgrom (2005) develop a unified framework for discrete and continuous matching models, and Sönmez and Ünver (2010) provide a recent survey.

\textsuperscript{29}Moreover, we can extend our results to the case of asymmetric firms (Section 5).
competition imposes a particularly simple type of externality in the assignment game. Specifically, the surplus (profit) of a pair depends only on the set of other managers that are matched, but not on which firm is matched with which manager. In particular, when the \( I \) firms are matched with the best \( I \) managers, the profit of a deviating pair is independent of how the remaining \( I - 1 \) firms and \( I - 1 \) managers are matched. As a consequence, our matchings as established in Proposition 1 are stable, since a deviating pair’s profit is independent of how the remaining pairs are rematched. In addition, all of the stable matchings in our model are Pareto-efficient, since it only matters that the best \( I \) managers are matched, but not with which firm they are matched.

8 Conclusion

In this paper, we have examined how globalization affects the distribution of managerial wages. Our key insight is that globalization increases the heterogeneity of managerial salaries, but not necessarily the overall wage level. Numerous issues deserve further scrutiny. For instance, one could consider localized competition à la Salop and Hotelling. In such cases, profits depend on the entire distribution of managers across firms, which introduces further subtleties. Moreover, incorporating asymmetric information and agency costs, or increasing demand for general rather than firm-specific managerial skills would promise further insights into the structure of managerial remuneration. While such modifications would lead to a richer analysis, it is not evident whether they would affect the main comparative statics implication that globalization leads to increasing managerial wage heterogeneity.

---

30 Note that this holds only for symmetric firms.
31 It would also be possible to construct a non-existence result in our model by imposing particular expectations on deviating pairs. If a deviating pair of manager and firm expected that the single firm is not matched anymore, deviation by a pair becomes extremely attractive, as such deviations reduce the number of active firms in the market and increase profits for all remaining matches.
9 Appendix

9.1 Proof of Proposition 1

Necessary Conditions

We use $\mu$ to denote the index of the manager hired by firm $i$ in equilibrium. We first establish a necessary condition for wages in a symmetric equilibrium. Consider the best-response conditions. Firm $j$ does not want to attract manager $m_i$ by offering a higher wage to $m_i$ than $w_{im_i}$ if

$$\Pi_j(m_j) - w_{jm_j} \geq \Pi_i(m_i) - w_{im_i}.$$  

Firm $i$ will not want to offer a higher wage to manager $m_j$ if

$$\Pi_i(m_i) - w_{im_i} \geq \Pi_j(m_j) - w_{jm_j}.$$  

Together, both inequalities imply $\Pi_i(m_i) - \Pi_j(m_j) = w_{im_i} - w_{jm_j}$. In particular, therefore, using the symmetry condition that $\Pi_i(m_I) = \Pi_I(m_I)$,

$$\Pi_i(m_i) - \Pi_I(m_I) = w_{im_i} - w_{Im_I}. \quad (A.1)$$  

Existence

Next we show that the wages proposed actually constitute an equilibrium. We note that in the proposed equilibrium $w^*_{im_i} = \Pi_i(m_i) - \Pi_I(m_I)$. Given these wage offers, managers are indifferent among all firms and apply first to firm 1, which will select the most competent manager, according to the first and second tie-breaking rules. The procedure is repeated by the other firms until all managers are employed. Firm $i$ will employ manager $i$, i.e. $m_i = i$.

The only reason for a firm to deviate by offering a higher wage would be to employ a more efficient manager $m < I$. However, by the construction of $w^*_{im}$, the required wage increase would exceed the increase in gross profits.

Now consider downward deviations of firm $i$. Suppose the firm $i$ offers smaller (non-negative) wages than $w^*_{im}$ to a subset $S$ of managers ($I \notin S$). We show that the manager with the smallest index in $\{i, \ldots, I\} \setminus S$ will be hired by firm $i$. The downward
deviation has no consequence on the choice of managers for firm 1 to \( i - 1 \). Only managers \( \{ i, ..., I \} \setminus S \) apply at firm \( i \). All the other managers apply at firm \( i + 1 \). As \( I \notin S \), \( \{ i, ..., I \} \setminus S \) is not empty. Hence, the second tie-breaking rule implies that firm \( i \) hires the manager with the smallest index in \( \{ i, ..., I \} \setminus S \). As a consequence, profits remain unchanged and the downward deviation is not profitable.

Finally, a downward deviation where \( w^*_{ij} \) is reduced is impossible, because \( w^*_{ij} = 0 \) is the outside option. So, there are no profitable deviations for firm \( i \).

**Uniqueness**

For uniqueness of symmetric equilibria, it suffices to show that there can be no equilibrium with \( w_{ij} > 0 \). Suppose that an equilibrium with \( w_{ij} > 0 \) exists. By (A.1), the equilibrium wages satisfy

\[
 w^*_{imi} = \Pi_i(m_i) - \Pi_i(m_f) + w_{ij}.
\]

Profits in this candidate equilibrium are given by

\[
 \Pi_i(m_i) - w_{ij}.
\]

Now suppose a firm \( j \) offers wages \( w_{jm} = 0 \) for all \( m \). According to our matching procedure and the tie-breaking rule that managers accept non-negative wages, firm \( j \) would hire manager \( I \) and would obtain profits \( \Pi_i(m_f) \). Hence, the deviation is profitable, so that there can be no equilibrium with \( w_{ij} > 0 \).

**9.2 Proof of Proposition 2**

Using (5), there are exactly two cost levels at which integration has no effect on wages, namely \( c_m = c_J \) and

\[
 c_m = \bar{c} \equiv - \frac{-2a + c_J + 3Jc_J - 4J^2\bar{c} + 2J^2c_J - 6J\bar{c}}{3J + 2J^2 + 1},
\]

provided \( \bar{c} \in [c_1, c_J] \). Next, note that (5) is positive for \( c_m < c_J \) if and only if

\[
 (c_J + c_m) \left( 1 + 3J + 2J^2 \right) - 2a - 4J^2\bar{c} - 6J\bar{c} < 0.
\]
As
\[
\frac{\partial}{\partial c_m} \left( (c_J + c_m) \left( 1 + 3J + 2J^2 \right) - 2a - 4J^2 \tau - 6J \tau \right) = 2J^2 + 3J + 1 > 0,
\]
(A.3) holds if and only if \( c_m < \bar{c} \). The statement of the proposition follows with
\[
c^* = \bar{c} \text{ if } \bar{c} \in [c_1, c_J];
\]
\[
c^* = c_1 \text{ if } \bar{c} < c_1;
\]
\[
c^* = c_J \text{ if } \bar{c} > c_J.
\]

### 9.3 Proof of Proposition 3

Applying Assumption 2 for \( I = 2J \) and all \( i \in \{1, 2, ..., 2J\} \), gross profits are given by (3) with \( I = 2J \) and \( b = B \). Therefore,
\[
\left| \frac{\partial \pi_i^G}{\partial c_i} \right| = \left| \frac{\partial}{\partial c_i} \left( \frac{2}{B} \left( \frac{a + 2J \tau}{2J + 1} - c_i \right)^2 \right) \right| = \frac{4a - 4c_i + 8J \tau - 8Jc_i}{B + 2BJ} \geq 0,
\]
where the positive sign also follows from Assumption 2 for \( I = 2J \). Similarly,
\[
\left| \frac{\partial \pi_i^A}{\partial c_i} \right| = \left| \frac{\partial}{\partial c_i} \left( \frac{1}{B} \left( \frac{a + J \tau}{J + 1} - c_i \right)^2 \right) \right| = \frac{2(a - c_i + \tau J - J c_i)}{B (J + 1)} \geq 0.
\]

We obtain the effect of integration on the marginal effect as the difference between the two expressions above, which yields
\[
\left| \frac{\partial \pi_i^G}{\partial c_i} \right| - \left| \frac{\partial \pi_i^A}{\partial c_i} \right| = \frac{2 \left( a - c_i - 3Jc_i + 2J^2 \tau - 2J^2 c_i + 3J \tau \right)}{B (2J^2 + 3J + 1)}.
\]

Thus
\[
\frac{\partial}{\partial c_i} \left( \left| \frac{\partial \pi_i^G}{\partial c_i} \right| - \left| \frac{\partial \pi_i^A}{\partial c_i} \right| \right) = \frac{-2}{B} < 0.
\]

Moreover \( \left| \frac{\partial \pi_i^G}{\partial c_i} \right| - \left| \frac{\partial \pi_i^A}{\partial c_i} \right| > 0 \) if and only if
\[
c_i < \frac{a + 2J^2 \tau + 3J \tau}{3J + 2J^2 + 1}.
\]

Assumption 2 implies that \( a > \tau \). Hence, the right-hand side of (A.4) is greater than \( \tau \), and this condition holds for \( c_i < \tau \).
9.4 Proof of Proposition 4

(i) When all managers are identical, the difference between a firm’s gross profit before and after integration is

\[
\frac{1}{B} \left( \frac{a + Jc}{J + 1} - c \right)^2 - \frac{2}{B} \left( \frac{a + 2Jc}{2J + 1} - c \right)^2 = \frac{1}{B} \left( a - c \right)^2 \frac{2J^2 - 1}{(2J^2 + 3J + 1)^2} > 0.
\]

Thus, total gross profits decrease as well. Continuity implies the result.

(ii) It suffices to show the corresponding statement for outputs, that is,

\[
\frac{a + Jc_{m+1}}{J+1} - c_m < \frac{a + 2Jc_{m}}{2J+1} - c_{m+1}.
\]

Simple but tedious rearrangements show that this statement is true whenever \(c_m < c_{m+1}\) and \(a > \tau\). The former inequality is the condition of the proposition; the latter inequality is implied by Assumption 2.

9.5 Proof of Proposition 5

(i) Suppose there exists a firm \(i\) that employs a manager with index \(m_i > i\). For this to be an equilibrium, there must be a firm \(j > i\) that employs a manager \(m_j < m_i\). This requires that firm \(j\) does not want to deviate by attracting manager \(m_i\) at a wage just under \(w^*_{i,m_i}\); hence, \(\Pi^A_j(m_j) - w^*_{j,m_j} \geq \Pi^A_i(m_i) - w^*_{i,m_i}\). Similarly, \(\Pi^A_i(m_i) - w^*_{i,m_i} \geq \Pi^A_i((m_j) - w^*_{j,m_j}\). Together, both conditions imply \(\Pi^A_j(m_j) - \Pi^A_i(m_i) \geq \Pi^A_i((m_j) - \Pi^A_i((m_i),\) contradicting Assumption 3. Thus each firm \(i\) employs the manager with index \(m_i = i\).

(ii) We have already shown in (i) that each firm \(i\) must employ the manager with the index \(i\) in equilibrium. In any equilibrium, therefore, \(\Pi^A_i((m) - w_{mm} \geq \Pi^A_i(m + 1) - w_{m+1,m+1}\) and \(\Pi^A_{m+1}(m + 1) - w_{m+1,m+1} \geq \Pi^A_{m+1}(m) - w_{mm}\). Together these conditions imply (11). To see that \(w_{jj} = 0\) must hold, note that, in any equilibrium manager \(J\) is assigned to firm \(J\). If \(w_{jj} > 0\), any wage reduction which gives manager \(J\) at least his reservation value leaves the assignment unaffected, but gives higher net payoffs to firm \(J\).

(iii) First, we show that, for wage offers satisfying the conditions stated in the proposition, assortative matching arises. To see this, note that the wage offer of firm 1 is
among the highest offers that manager 1 receives. By the first tie-breaking rule manager 1 accepts this offer. Manager 2 is indifferent between the offers of firms 1 and 2, and accepts the offer of firm 2 in the second round. The process goes on until each manager \( m \in \{1, \ldots, J\} \) is matched with the firm with the identical index.

Second, we show that no firm can profitably deviate by attracting a better manager. By Assumption 3, it suffices to show that firm \( i \) cannot benefit from making an acceptable wage offer to manager \( m = i - 1 \). An acceptable offer would require \( w^*_{i,i-1} > w^*_{i-1,i-1} \) because of the first tie-breaking rule. Such a wage would lead to profits \( \Pi^A_i(i-1) - w^*_{i,i-1} < \Pi^A_i(i-1) - w^*_{i-1,i-1} \). As \( \Pi^A_i(i-1) - w^*_{i-1,i-1} \leq \Pi^A_i(i) - w^*_{i,i} \) by (11), attracting a better manager is not a profitable deviation.

Third, firm \( i \) cannot benefit from reducing its wage offer to manager \( i \), while leaving all other wage offers constant. It would end up with manager \( i + 1 \), obtaining net payoffs of \( \Pi^A_i(i+1) - w^*_{i,i+1} \). As \( w^*_{i+1,i+1} = w^*_{i,i+1} \), \( \Pi^A_i(i+1) - w^*_{i,i+1} = \Pi^A_i(i+1) - w^*_{i+1,i+1} \). As the first inequality in (11) requires \( \Pi^A_i(i) - w^*_{i,i} \geq \Pi^A_i(i+1) - w^*_{i+1,i+1} \), the deviation is not profitable.

### 9.6 Proof of Proposition 6

Let \( \Delta w^A_m \) (\( \Delta w^G_m \)) be the minimal wage difference before (after) globalization, given by the right hand side of (11). Equation (12) implies that, for \( \Pi_i = \Pi^A_i, \Pi^G_i \) and \( I = J, 2J \),

\[
\Pi_i(m) - \Pi_i(m+1) = \frac{1}{b} \frac{(c_{i,m} - c_{i,m+1})((c_{i,m} + c_{i,m+1})(1 + I) - 2a - 2\tau I)}{I + 1}. \tag{A.5}
\]

Therefore, using (A.5),

\[
\frac{\Delta w^G_m - \Delta w^A_m}{B} = \frac{2}{2J + 1} \frac{(c_{m+1,m} - c_{m+1,m+1})((c_{m+1,m} + c_{m+1,m+1})(1 + 2J) - 2a - 4\tau J)}{1} - \frac{1}{J + 1} \frac{(c_{m+1,m} - c_{m+1,m+1})((c_{m+1,m} + c_{m+1,m+1})(1 + J) - 2a - 2\tau J)}{B}. \tag{A.6}
\]
From \(c_{m+1, m} < c_{m+1, m+1}\), we obtain that \(\Delta w_m^G > \Delta w_m^A\) if and only if
\[
\frac{2(c_{m+1, m} + c_{m+1, m+1}) (1 + 2J) - 2a - 4\pi J}{2J + 1} < \frac{(c_{m+1, m} + c_{m+1, m+1}) (1 + J) - 2a - 2\pi J}{J + 1}.
\]
(A.7)

After simple rearrangements, this is equivalent to
\[
(c_{m+1, m} + c_{m+1, m+1}) (1 + J) (1 + 2J) - 2(2a + 4\pi J) (J + 1) + (2a + 2\pi J) (2J + 1) < 0.
\]

Similarly, \(\Delta w_{m-1}^G > \Delta w_{m-1}^A\) if and only if
\[
(c_{m, m-1} + c_{m, m}) (1 + J) (1 + 2J) - 2(2a + 4\pi J) (J + 1) + (2a + 2\pi J) (2J + 1) < 0.
\]

The statement for the lower bound now follows because
\[
c_{m+1, m} + c_{m+1, m+1} > c_{m, m-1} + c_{m, m}.
\]

The proof of the statement for the upper bound is analogous.

### 9.7 Price Competition: Proof of Proposition 8

The proof of Proposition 8 relies on the following lemma:

**Lemma 1:** In the model of price competition described in Section 6.4, \(w^G - w^A\) is decreasing in \(c_i\) for all \(J \geq 2\), \(\alpha \geq 0\), \(\beta \geq 0\) and \(\gamma \leq \beta\) if and only if
\[
T(J, \beta, \gamma) \equiv 8(\beta + (J - 2) \gamma + 0.5\gamma^2)(\beta + (2J - 1) \gamma) (\beta + (2J - 2) \gamma) - (\beta + (2J - 2) \gamma) + (\beta + (J - 1) \gamma) + (2\beta + (4J - 3) \gamma)^2 > 0.
\]

We first prove this lemma. For \(a = a^G\), \(b = b^G\), \(s = s^G\) and \(I = 2J\) or \(a = a^A\), \(b = b^A\), \(s = s^A\) and \(I = J\), define
\[
U \equiv a + s \frac{Ia + bI\pi}{2b - (I - 1) s};
\]
\[
V \equiv a (2b + s) + (s (I - 1) - b) \left(a + s \frac{Ia + bI\pi}{2b - (I - 1) s}\right) + sbI\pi.
\]

Using (17), the equilibrium margins and outputs can be calculated as
\[
p_i^* - c_i = \frac{U - (s + b) c_i}{2b + s},
\]
\[
q_i = \frac{1}{2b + s} \left(V - (b^2 + sb) c_i\right).
\]
Equilibrium gross profits of a firm with manager $m$ are
$$\Pi_i(m) = \frac{UV - ((b + s)(V + Ub))c_m + b(b + s)^2c_m}{(2b + s)^2}.$$

By Proposition 1, $w_{im} = \Pi_i(m) - \Pi_i(I)$. Hence,
$$w_{im} = \frac{(c_I - c_m)}{(2b+s)^2} \left((b + s)(V + Ub) - b(b + s)^2(c_m + c_I)\right).$$

Inserting $U$ and $V$ yields
$$w_{im} = (c_I - c_i) \frac{(b+s)}{(2b+s)^2} \left(a(2b+s) + (s(I-1) - b) \left(a + s \frac{1_o + b\varpi}{2b - (I-1)s}\right) + \right)$$
$$sb \varpi + \left(a + s \frac{1_o + b\varpi}{2b - (I-1)s}\right) b - b(b+s)(c_m + c_I)\right).$$

Wages after globalization are higher than before if and only if $w^{G}_{im} > w^{A}_{im}$, which is equivalent with
$$\frac{(b^G + s^G)}{(2b^G + s^G)^2} \left(a^G(2b^G + s^G) + (s^G(2J - 1) - b^G) \left(a^G + s^G \frac{2Ja^G + 2b^G \varpi}{2b^G - (2J-1)s^2}\right) + \right)$$
$$2s^G b^G J \varpi + \left(a^G + s^G \frac{2Ja^G + 2b^G \varpi}{2b^G - (2J-1)s^2}\right) b^G - b^G \left(b^G + s^G\right)(c_m + c_I)\right) >$$
$$\frac{(b^A + s^A)}{(2b^A + s^A)^2} \left(a^A(2b^A + s^A) + (s^A(J-1) - b^A) \left(a^A + s^A \frac{Ja^A + b^A \varpi}{2b^A - (J-1)s^2}\right) + \right)$$
$$s^A b^A J \varpi + \left(a^A + s^A \frac{Ja^A + b^A \varpi}{2b^A - (J-1)s^2}\right) b^A - b^A \left(b^A + s^A\right)(c_m + c_I)\right).$$

This inequality holds if and only if
$$\frac{(2b^A + s^A)^2}{(b^A + s^A)} \frac{(b^G + s^G)}{(2b^G + s^G)^2} b^G \left(b^G + s^G\right)(c_m + c_I) - b^A \left(b^A + s^A\right)(c_m + c_I) < K$$

where $K$ is a term that is independent of $c_m$. The left hand side of this inequality is increasing in $c_m$ if and only if
$$\frac{(2b^A + s^A)^2}{(b^A + s^A)} \frac{(b^G + s^G)}{(2b^G + s^G)^2} b^G - b^A \left(b^A + s^A\right) > 0$$
or, equivalently,
$$(2b^A + s^A)^2 \frac{(b^G + s^G)}{(2b^G + s^G)^2} b^G - b^A \left(b^A + s^A\right)^2 (2b^G + s^G)^2 > 0.$$
Next,

\[
\frac{\partial T}{\partial J}(2; \beta, \gamma) = \gamma \left(24\beta^3 + 100\beta^2\gamma + 114\beta\gamma^2 + 43\gamma^3\right) > 0.
\]

Thus, (SV) holds if \( \frac{\partial T}{\partial J} \) is increasing in \( J \), which is true because

\[
\frac{\partial^2 T}{\partial J^2}(J; \beta, \gamma) = 192J^2\gamma^4 + 288J\beta\gamma^3 - 432J\gamma^4 + 104\beta^2\gamma^2 - 312\beta\gamma^3 + 238\gamma^4 > 0.
\]

This follows because

\[
\frac{\partial^2 T}{\partial J^2}(2; \beta, \gamma) = 104\beta^2\gamma^2 + 264\beta\gamma^3 + 142\gamma^4 > 0
\]

and

\[
\frac{\partial^3 T}{\partial J^3}(J; \beta, \gamma) = \gamma^3(288\beta - 432\gamma + 384J\gamma) > 0.
\]

10 References


37


