

ENDOGENOUS SPILLOVERS AND INCENTIVES TO INNOVATE

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Abstract:

We present a new approach to endogenizing technological spillovers. Firms choose levels of a cost-reducing innovation from a continuum before they engage in competition for each other's R&D-employees. Successful bids for the competitor's employee then result in higher levels of cost reduction. Finally, firms enter product market competition. We apply the approach to the long-standing debate on the effects of the mode of competition on innovation incentives. We show that incentives to acquire spillovers are stronger and incentives to prevent spillovers are weaker under quantity competition than under price competition. As a result, for a wide range of parameters, price competition gives stronger innovation incentives than quantity competition.

Keywords: Innovation Incentives, Spillovers, Product Market Competition

Endogenous Spillovers and Incentives to Innovate

1. Introduction

In both empirical and theoretical studies of the innovation process, the importance of knowledge spillovers has often been emphasized. Some authors have focused on the potential adverse consequences for innovation incentives (d'Aspremont and Jacquemin 1988, Kamien et al. 1992, Suzumura 1992, Henriques 1990, De Bondt et. al. 1992, Leahy and Neary 1997), others have examined the ambiguous effects of spillovers on economic growth (Romer 1986 and 1990, Aghion and Howitt 1992, Grossman and Helpman 1991). Recently, the effects of spillovers on agglomeration patterns have been analyzed (Baldwin et al. 1998).

This literature has used the convenient simplification that spillover levels are exogenous, that is, if one firm achieves a cost reduction, other firms receive a fixed proportion of this cost reduction through spillovers, which cannot be influenced by either party. Only recently have some authors tried to cope with the fact that the spillover level is endogenous (see Katsoulacas and Ulph 1998, Gersbach and Schmutzler 1997, Fosfuri et al. 1998, Roende 1998, Poyago-Theotoky 1999). Most of these papers, however, deal only with a firm's decision whether or not to allow spillovers to competitors. We start from Gersbach and Schmutzler (1997), where firms can not only engage in (costly) activities designed at preventing spillovers to competitors, but also in activities designed at obtaining spillovers from competitors. In their setting, technological spillovers depend on the ability of firms to attract other firms' R&D employees and to prevent their own R&D employees from leaving the firm. We examine a game in which firms choose the levels of a cost-reducing innovation from a continuum before they engage in competition for each other's R&D-employees in the second stage.¹ In this stage, firms make wage offers for the own employee and the competitors. Whether a firm can obtain spillovers by poaching the competitor's employee depends on the relative attractiveness of the contracts offered by the original employer and his competitors. Successful bids for the competitor's employee result in higher levels of cost reduction. Finally, firms enter product market competition, given the cost structure determined by innovation and spillover levels. Our main contributions are as follows:

¹ Gersbach and Schmutzler (1997) only consider 0-1 decisions ("innovate" versus "do not innovate").

First, we characterize necessary and sufficient conditions for equilibria in which one-way and two-way technological spillovers occur for given levels of innovation activity. We show how the extent of spillovers depends on the mode of competition.

Second, we use these results to characterize the subgame perfect equilibria of the innovation game. Most importantly, we show that for Cournot competition the only equilibrium for sufficiently high innovation costs has both firms investing and bilateral spillovers taking place. For Bertrand competition, there is an unilateral equilibrium where one firm invests and there are no spillovers.

Finally, we use these results to show that taking the endogeneity of spillovers into account affects familiar results on the incentives for innovation in a systematic way. We illustrate this point by taking up the long-standing debate on the effects of the mode of competition on innovation incentives. In both IO and growth models, various authors have investigated how tough competition (à la Bertrand) and soft competition (à la Cournot) differ in this respect. Most of these authors (Brander and Spencer 1983, Delbono and Denicolò 1990, Bester and Petrakis 1993) have argued in a world without spillovers. Qiu (1997) has made the important point that, with exogenous spillovers, incentives for innovation become weaker in the Bertrand case than in the Cournot case.² In this paper, we show that, with endogenous spillovers, this argument becomes weaker and, in many cases, is reversed. The reason is that incentives to acquire spillovers are stronger and incentives to prevent spillovers are weaker under quantity competition. It turns out that under Bertrand competition, the possibility of spillovers does not reduce incentives to produce knowledge. As a result, an innovating firm has to worry less about spillovers under price competition than under quantity competition, and, consequently, for wide ranges of parameters, price competition gives stronger innovation incentives than quantity competition. Our results suggest that weakening product market competition in order to spur innovations is hard to justify when spillovers are endogenous.

The paper is organized as follows. In Section 2, we present a three-stage duopoly model. Firms first choose innovation levels, then compete for knowledge by making wage offers to each other's R&D employees. Finally, they compete on the product market. In Section 3, we give general conditions for the extent of spillovers, assuming given innovation levels. Section 4 applies these results to the discussion of relative innovation incentives in the Cournot and

² Another related paper with spillovers is Aghion et al. (1997).

Bertrand cases. In section 5 we discuss the robustness of the findings and potential extensions. In section 6 we summarize our results and offer extensions.

2. The Model

We consider a three-stage game. There are two firms, $i = 1, 2$. Initially, firms have constant marginal costs c . In period 1, they can carry out an innovation that reduces marginal costs by x_i . Following Qiu (1997), innovation costs are $k(x_i) = vx_i^2$, where $v > 0$.

To carry out the innovation, each firm has to hire an R&D employee. In period 2, firms bid for each other's R&D employees: firm i offers wages w_{ij} , $j \neq i$ for firm j 's R&D employee and w_{ii} for their own R&D employee. An R&D employee from firm j is assumed to switch firms if $w_{ij} > w_{jj}$, otherwise he continues to work for his own firm. To attract the employee from firm j , firm i needs to offer a wage which is higher than w_{jj} by the smallest possible currency unit.

We assume that wage contracts offered to R&D employees can be conditioned on the knowledge of both R&D-employees and thus on the relative performance of the R&D employees. Thus, on the one hand, the wage offer depends on the knowledge of the R&D-person himself, on the other hand, it depends on the knowledge of the other firm's employee. While the first element is not problematic, the second element requires that firms can observe and verify the knowledge when they make their wage bids or, more plausibly, when employees have accepted wage contracts and enter firms. In Gersbach and Schmutzler (1997), we discuss in detail how this assumption can be justified.

If an R&D employee moves to firm i , this firm obtains a further cost reduction x_j thanks to knowledge spillovers, so its marginal production costs are $c_i = c - x_i - x_j$. Hence, we assume that the cost reductions are complementary. Also, the knowledge necessary to reduce costs is completely transferable to other firms; that is, if spillovers arise, they are perfect: if a firm can motivate the R&D person of the other firms to move, this employee will be able to replicate the original cost reduction in his new firm.³

³ For a robustness discussion, see section 5.

We also assume that knowledge can be duplicated within: If firm i loses an employee to the competitor after investing x_i , its costs remain at $c - x_i$.⁴

If firm i does not obtain the services of employee j , production costs remain at $c_i = c - x_i$. In period 3, product market competition takes place. We shall suppose the two firms produce homogeneous goods. The inverse market demand is given by

$$(1) \quad p = a - q_i - q_j; \quad i, j = 1, 2; \quad i \neq j; a > 0$$

Throughout the paper we assume that marginal innovation costs are sufficiently high.

$$\mathbf{A1:} \quad v > \max \left\{ \frac{2a - c}{9c}, \frac{a - c}{2c} \right\}.^5$$

This assumption will later be seen to imply that $c > x_i + x_j$ for equilibrium choices of investment levels, which is necessary and sufficient for positive marginal costs. We consider both price and quantity competition. In either case, a unique equilibrium of the period 3 subgame exists for each cost vector (c_i, c_j) determined in period 1 and 2. We denote the resulting *product market profits* for firm i as $\pi(c - c_i, c - c_j)$. We denote by $\pi^n(c - c_i, c - c_j)$ the *net profit* of firm i in period 2, defined as $\pi(c - c_i, c - c_j)$ minus wages paid to the R&D persons who will be employed.

The assumptions of perfect spillovers and homogenous goods simplify the analysis. They are also chosen because Qiu (1997) has shown that Cournot competition is more likely to give stronger innovation incentives when goods are close substitutes and exogenous spillovers are high. With our assumptions we can make the point that, even in a setting that satisfies these characteristics in the best possible way, the opposite result arises for endogenous spillovers.

3. The Spillover Game

We now consider the spillover game, that is, the subgame starting with first-period investment levels x_i, x_j . We say that the resulting spillovers are bilateral if each firm acquires the services of the other firm's employee, unilateral if only one firm acquires the services of the other

⁴ Clearly, an investing firm has an incentive to secure that the knowledge of R&D projects is codified and distributed within the firm, so that it does not depend on the future services of the knowledge-bearing employee.

⁵ A discussion of the case of v violating A1 will also be the subject of section 5.

firm's employee. We characterize the conditions under which bilateral spillovers or no spillovers occur. Here and in the following, we assume that the firms do not play weakly dominated strategies.

To simplify the exposition, we shall always neglect the smallest currency unit. Hence, if two wages are identical in equilibrium, it is understood that the firm that wins the wage bid offers the equilibrium wage plus the smallest currency unit.

Proposition 1:

(a) *For an equilibrium with bilateral spillovers, the following condition is necessary:*

$$(a1) \quad \pi(x_i + x_j, x_i + x_j) - \pi(x_i, x_i + x_j) \geq \pi(x_i + x_j, x_i) - \pi(x_i + x_j, x_i + x_j); \quad i, j = 1, 2; i \neq j$$

(a1) and (a2) below are sufficient for at least one equilibrium with bilateral spillovers to exist.

$$(a2) \quad \pi(x_i + x_j, x_i + x_j) + \pi(x_i + x_j, x_j) \geq \pi(x_i, x_j) + \pi(x_i + x_j, x_i); \quad i, j = 1, 2; i \neq j$$

If (a1) and (a2) hold, there is a Pareto-dominant equilibrium among these bilateral spillover equilibria, with wages $w_{ji} = w_{ii} = \pi(x_i + x_j, x_j) - \pi(x_i + x_j, x_i + x_j)$ and net profits $\pi^n = 2\pi(x_i + x_j, x_i + x_j) - \pi(x_i + x_j, x_i)$ for $i, j = 1, 2; j \neq i$.

(b) *An equilibrium with unilateral spillovers from firm 2 to firm 1 exists if and only if the following conditions hold*

$$(b1) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2)$$

$$(b2) \quad \pi(x_1 + x_2, x_2) \geq 2\pi(x_1 + x_2, x_1 + x_2) - \pi(x_2, x_1 + x_2)$$

$$(b3) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_2) \geq \pi(x_2, x_1) - \pi(x_2, x_1 + x_2)$$

$$(b4) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq \pi(x_1 + x_2, x_1 + x_2) - 2\pi(x_2, x_1 + x_2) + \pi(x_2, x_1)$$

$$(b5) \quad 2\pi(x_1 + x_2, x_2) - \pi(x_1 + x_2, x_1 + x_2) - \pi(x_1, x_2) \geq \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2)$$

Usually, there are multiple equilibrium wages.

(c) *A necessary condition for an equilibrium without spillovers is*

$$(c1) \quad \pi(x_i, x_j) - \pi(x_i, x_i + x_j) \geq \pi(x_i + x_j, x_i) - \pi(x_j, x_i), \quad i, j = 1, 2; i \neq j.$$

The proof is given in the appendix. Essentially, the proof tests whether firms can increase profits by moving to another spillover regime. Consider for instance the case of bilateral spillovers, as in (a). The left-hand side of (a1) is the loss in product market profits that firm i

would obtain if it did not poach the competitor's employee, thus ending up without a worker. The right-hand side describes firm j 's willingness to pay for its own employee, starting from bilateral spillovers. The condition thus makes sure that, starting from a situation where each firm has poached the competitor's employee, the willingness to pay for keeping the competitor's employee is higher than the willingness to pay for regaining the services of one's own employee.

Proposition 1 indicates that even within a spillover regime equilibria are usually not unique in terms of wages because of a standard coordination problem. Consider case (a) again. In the proposed equilibrium, each firm pays the minimum wage that is necessary to pay for the competitor's employee, assuming that the competitor does not pay more than the value of the worker. In principle, each firm i would be willing to pay as much as the left hand side of (a1). Any combination of wages between the left-hand side and the right-hand side of (a1) that are identical for both firms could also constitute a Nash-equilibrium. Even though firms offer higher wages than their willingness to pay to prevent their employee from leaving to the competitor, they do not actually have to pay, as the competitor obtains his services. However, as wages are unnecessarily high from the firms' point of view, the equilibrium is Pareto-inefficient.

In the following, we concentrate on the Pareto-dominant equilibria. The Pareto selection criterion is equivalent to the criterion that no firm ends up paying higher wages for employees than its own differential profit from keeping or hiring the employee. Uniqueness of equilibria in the remainder of the paper refers to the set of equilibria that are neither Pareto-dominated nor involve playing weakly dominated strategies. It will turn out below that the proposed selection criterion biases the outcome in favor of the Cournot case, but that nevertheless investment incentives are stronger under Bertrand than under Cournot.

In the next section we will apply proposition 1 to price and quantity competition. We will show that, for Cournot competition, only bilateral spillovers occur if innovations are not too large. Under Bertrand competition, only unilateral spillovers can occur. We shall use this result to derive the subgame perfect equilibrium of the innovation game which provides a comparison of innovation incentives under Bertrand and Cournot competition when technological spillovers are endogenized.

4. The Nature of Competition and Innovation Incentives

The effects of the mode of competition on innovation incentives have been the subject of a long-standing debate. In both IO and growth models, various authors have investigated how tough competition (à la Bertrand) and soft competition (à la Cournot) differ in this respect. Most of these papers ignore spillovers.⁶ Qiu (1997) considered exogenous spillovers.⁷ We complement this discussion by showing how endogenizing technological spillovers along the lines of section 3 changes the results. We will show that, with endogenous spillovers, Bertrand competition yields higher innovation incentives than Cournot competition for sufficiently high innovation costs, which differs from the case of exogenous spillovers.

4.1. Spillovers in the Cournot Case

We first apply proposition 1 to the Cournot case. Straightforward applications of the standard result that $\pi(c_i, c_j) = (a - 2c_i + c_j)^2 / 9$ can be used to characterize the second-period equilibria. Here and in the following, we shall use the notation $\alpha \equiv a - c$. We first treat the cases of no spillovers and bilateral spillovers, respectively.

Corollary 1: *A subgame equilibrium without spillovers cannot exist if $x_1 > 0$ and $x_2 > 0$.*

Corollary 2: *In the Cournot case, the following statements hold:*

- (a) *An equilibrium with bilateral spillovers exists if and only if $2\alpha \geq 3x_i - 2x_j$ ($i, j = 1, 2; i \neq j$).*
- (b) *If $2\alpha > 3x_i - 2x_j$, spillovers are bilateral for every pure strategy equilibrium.*
- (c) *In a bilateral spillover equilibrium, net profits are*

$$\pi^n = \left(\frac{2(\alpha + x_i + x_j)^2}{9} \right) - \left(\frac{(\alpha + 2x_j + x_i)^2}{9} \right) = \frac{\alpha^2 + 2\alpha x_i + x_i^2 - 2x_j^2}{9}.$$

The proofs are given in the appendix.

We now use our results to delineate parameter regions for which there is a subgame perfect equilibrium of the full game with bilateral spillovers. For the purpose of the next result, we strengthen assumption A1 for the Cournot case.

⁶ See Brander and Spencer 1983, Delbono and Denicolò 1990, Bester and Petrakis 1993, Aghion et al. 1997.

⁷ See also the discussions in Bonnanno and Haworth 1998 and Gerowski 1995.

A2: v is so large that it is never profitable to deviate to $x_1 = c$ or to $x_1 = c - \frac{\alpha}{9v-1}$ from an investment level of $x_1 = \frac{\alpha}{9v-1}$ if the competitor has chosen the same investment level.

The assumption, which can be stated in terms of the exogenous parameters a, c, v as well,⁸ makes sure that we can disregard deviations that result in zero production costs.

Proposition 2:

Suppose A2 holds and $v > 4/9$. Then, there is a subgame perfect equilibrium in pure strategies with bilateral spillovers in the Cournot case. This equilibrium is unique within the bilateral spillover regime⁹. In this equilibrium,

$$(3) \quad x_1^c = x_2^c = \frac{\alpha}{9v-1} \equiv \gamma. \text{ Equilibrium payoffs are } \frac{\alpha^2 - \gamma^2 + 2\alpha\gamma - 9v\gamma^2}{9}$$

Proof: see appendix.

For values of v below $4/9$ the analysis becomes more complex. The proof shows that the parameter restriction guarantees that there are only two types of feasible deviations from the equilibrium. First, investments may be increased, resulting in unilateral inward spillovers. Second, investments might be reduced to 0, with no spillovers. For $v < 4/9$, the method used in the proof to show that upward deviations are not profitable would have to be modified. For $v < 5/18$, the proof shows that additional downward deviations to investment levels resulting in asymmetric spillovers might arise. In the proof, it also turns out that the bilateral spillover regime definitely does not extend below $v = 2/9$: for lower values, deviations to zero investment, with no spillovers in the second stage, will be profitable.

The focus on sufficient conditions in the proof of proposition 2 suggests that the same pure strategy equilibrium might exist for slightly lower values than $v=4/9$ if the other assumptions hold as well. But as we shall discuss in section 5, mixed strategy equilibria will prevail for lower values. Finally, the analysis in section 5 will show that, for very low values of v (contradicting A1), equilibria will result where both firms choose the highest possible investment level c .

⁸ For instance, a sufficient condition is $81v^3c^2 - 54v^2ac - vc^2 + 6vac + 4va^2 + \frac{4}{9}ac - \frac{2}{9}c^2 - \frac{2}{3}a^2 > 0$.

⁹ Uniqueness in general can be established for $v > \frac{1}{3} + \frac{1}{9}\sqrt{2}$.

4.2. Spillovers under Bertrand Competition

We now consider the case of price competition. In this case, firm i will obtain product market profits of

$$(4) \quad \pi(c - c_i, c - c_j) = \max\{(c_j - c_i)(\alpha - c_j), 0\}$$

provided that c_j is not greater than firm i 's monopoly price. For linear demand, this condition amounts to

$$(5) \quad c - c_i \leq \alpha + 2(c - c_j).$$

Proposition 3:

Suppose firms are labeled so that $x_1 \geq x_2$. Also suppose (5) holds for $i=1, j=2$ and $x_i = c - c_i, x_j = c - c_j$. In the Bertrand case, there exists an equilibrium involving unilateral spillovers. For this equilibrium, the following conditions hold:

$$(6) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq w_{11} + w_{12} = \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2) \text{ and}$$

$$(7) \quad w_{12} \leq \pi(x_1 + x_2, x_2) - \pi(x_1, x_2)$$

Net profits are

$$(8) \quad \pi(x_1 + x_2, x_2) - \pi(x_1 + x_2, x_1) \geq 0 \text{ for firm 1, } 0 \text{ for firm 2.}^{10}$$

If $x_1 > x_2$, the spillovers flow from firm 2 to firm 1; if $x_1 = x_2$, the direction is indeterminate.

Finally, any pure strategy equilibrium involves unilateral spillovers and satisfies (6)-(8), if no weakly dominated strategies are played.

Proof: see appendix.

Using this result, we can immediately analyze the equilibrium structure of the full game, provided marginal innovation costs are not too low.

Proposition 4:

¹⁰ In particular, if $x_1 = x_2$, both firms earn zero profits.

Suppose A1 holds and $v \geq 1/4 + \sqrt{3/8}$. Then, except for relabeling of firms, there exists exactly one equilibrium in the Bertrand case. In this equilibrium,

$$x_1^B = \frac{\alpha}{2v}; x_2^B = 0.$$

Proof: see appendix. Again, a brief discussion of smaller parameter values will be the subject of section 5.

4.3. Comparison of Innovation Incentives

Putting together propositions 2 and 4, it immediately follows that innovation incentives are stronger for price competition than for quantity competition, provided innovation costs are sufficiently high.

Proposition 5:

Suppose $v \geq 1/4 + \sqrt{3/8}$ and that A1 and A2 hold. Then, for suitable choice of firm indexes,

$$x_1^B = \frac{\alpha}{2v} > 2 \frac{\alpha}{9v-1} = 2x_1^c = 2x_2^c > x_2^B = 0.$$

Thus, total investment is higher for Bertrand competition than for Cournot competition.

This result is the main conclusion of our analysis. The relationship between total innovation incentives under Bertrand and Cournot for different values of v are illustrated in figure 1 for $\alpha = 10$.

Intuitively, for Bertrand competition, having the same costs as the competitor is not preferable to having higher costs, whereas having lower costs is preferable to having identical costs. Therefore, when only one firm has innovated incentives to acquire spillovers are low relative to the incentives to prevent them, and the fear of spillovers does not reduce innovation incentives. As this logic does not apply to Cournot competition, ignoring the endogeneity of spillovers overstates the innovation incentives in the Cournot case relative to the Bertrand case. With endogenous spillovers, therefore, innovation incentives may be stronger for Bertrand competition, even when, for exogenous spillovers, the opposite would be true.

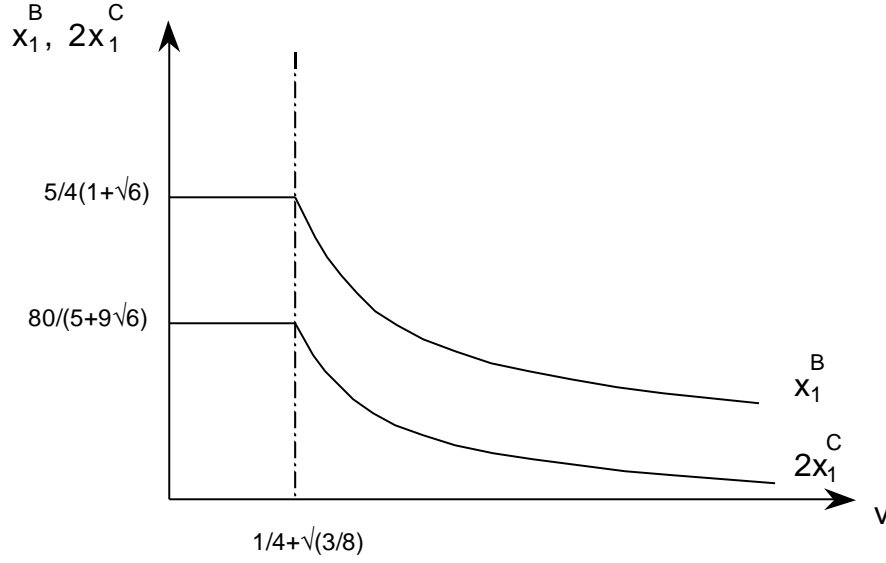


Figure 1: Relationship between innovation incentives under Bertrand and Cournot competition

Note that this result does not depend on our use of Pareto-dominance as selection criterion. Pareto dominated equilibria, that do not rely on weakly dominated strategies, only exist in the Cournot case. If firms coordinate on such an equilibrium in the second stage, this reduces investment incentives even further, thus strengthening the result that innovation incentives are higher for Bertrand than for Cournot competition.

Finally, in the parameter regime described by proposition 5, investment incentives are always higher for Bertrand competition than for Cournot competition, independent of the market size parameter α . Nevertheless, increasing α has an effect on the difference between investment incentives. An immediate corollary of proposition 5 is that increasing market size strengthens investment more in the Bertrand case than in the Cournot case.

Corollary 3: Suppose $v \geq 1/4 + \sqrt{3/8}$ and that A1 and A2 hold. Then,

$$\frac{\partial(x_1^B + x_2^B)}{\partial\alpha} > \frac{\partial(x_1^C + x_2^C)}{\partial\alpha}.$$

5. Robustness

We have shown that for $v > 1/4 + \sqrt{3/8}$, R&D expenditures are higher under Bertrand competition than under Cournot competition. We will explore in this section what happens for lower values of v . We also explain intuitively how this parameter enters our arguments.

The lower bound on v matters for two reasons. First, assumption A1 ensures that marginal production costs cannot become negative under Cournot competition $\left(v > \frac{2a-c}{9c}\right)$ and under Bertrand competition $\left(v > \frac{\alpha}{2c}\right)$. These two assumptions are combined in A1. For sufficiently small a , and c sufficiently close to a , both conditions, however, can be fulfilled by smaller numerical values than $v = 1/4 + \sqrt{3/8}$. Second, the equilibria which are used for our comparisons may not exist for lower values of v .

5.1 Very Low Innovation Costs

We first consider what happens for very small values of v , where A1 is violated. Instead, we impose the constraint $x_i, x_j \leq c$ directly. Then, it is intuitive that there is an equilibrium under Cournot competition where both firms choose the maximal investment level $x_i = c$. The resulting profits are $\pi(c, c) - vc^2 = a^2/9 - vc^2$. Any unilateral reduction of investments would reduce product market profits from $a^2/9$ by an amount that is independent of v . To compensate for this profit reduction, one might attempt to obtain spillovers from the competitor, but again this would lead to positive wage costs that are independent of v . The innovation cost reductions from choosing x_i below c , however, go to zero as v does. Therefore, both firms choose $x_i = c$ under Cournot competition for very small values of v .

Under Bertrand competition, for $v \rightarrow 0$, there is a unilateral spillover equilibrium where one firm will innovate, say firm 1, with $x_1 = c$ and the other firm neither innovates nor obtains spillovers. Therefore, for sufficiently small values of marginal R&D costs, aggregate innovation incentives under Cournot can be higher, but the firm that is active under Bertrand competition produces at the same marginal costs as firms under Cournot competition.

5.2 Intermediate Innovation Costs

For intermediate values of the marginal cost of R&D production, the arguments become more difficult. So far, we have characterized the equilibrium behavior for Cournot for $v > 4/9$, whereas for Bertrand we only treated $v > 1/4 + \sqrt{3/8}$. We now extend the Bertrand analysis to the parameter region $\frac{1}{2} \leq v < \frac{1}{4} + \sqrt{3/8}$ to allow for a more complete comparison with the Cournot case. For Bertrand competition we obtain:

Proposition 6: *Suppose that A1 holds and that $\frac{1}{2} \leq v < \frac{1}{4} + \sqrt{3/8}$. Then, no subgame perfect equilibrium in pure strategies exists.*

The proof is given in the appendix. Clearly, there exist equilibria in mixed strategies for $v \in \left[\frac{1}{2}, \frac{1}{4} + \sqrt{\frac{3}{8}} \right]$ under Bertrand. In any symmetric mixed strategy equilibrium, for each firm there is a positive probability that it will not innovate, because choosing low but positive levels of cost reduction can never be profitable if the other follows symmetric strategies. Therefore, in the range $\frac{1}{2} \geq v > \frac{1}{4} + \sqrt{3/8}$ there exists a positive probability such that no innovation takes place under Bertrand competition. Thus, our clear cut result (proposition 5) does not carry over automatically to lower values of v .¹¹ Also, for $v \leq \frac{4}{9}$, we cannot exclude mixed strategy equilibria under Cournot competition as long as the equilibrium in which both firms choose the maximal investment level $x_i = c$ does not exist. Thus the analysis of innovation incentives below $v = \frac{4}{9}$ will require a comparison of mixed strategy equilibria under Bertrand and Cournot.¹²

6. Extensions and Conclusion

In this paper, we introduced a framework for the analysis of innovation incentives with endogenous spillovers. We showed that, compared to the case of exogenous spillovers, innovation incentives are strengthened in the Bertrand case relative to the Cournot case, implying that for sufficiently high innovation costs, these incentives are stronger for Bertrand.

The results have been derived under the simplifying assumption of homogeneous goods and perfect spillovers. With suitable modifications, our central result still holds with these assumptions relaxed. For instance, as long as product differentiation is not too strong, incentives to obtain spillovers are still fairly low under price competition, and by similar reasoning as above, innovation incentives are hardly affected by the prospect of spillovers when competition is in prices; accordingly, familiar results on the relation between innovation incentives in the Bertrand and Cournot case may be reversed.

¹¹ One can even give examples when expected investment under Bertrand is lower than under Cournot.

¹² The analysis becomes extremely tedious and no comprehensive comparison is available yet.

Our model could be extended in various directions. First, the analysis would be much more complicated in the case that there are $n > 1$ R&D employees in each firm. This would introduce competing effects. On the one hand, it may be hard for firms to avoid spillovers if the knowledge of one employee can substitute the knowledge of others - competitors may only have to attract a small number of employees out of a large group to obtain spillovers. On the other hand, no employee can easily appropriate the information rent because of the competition from other employees. The net effect compared with the present situation is unclear. Compared with the exogenous case, however, it remains true that appropriability is easier to satisfy with endogenous spillovers. Also, because the incentives for firms to acquire spillovers are particularly small in the Bertrand case, we conjecture that it is still true that in this case the endogeneity of spillovers strengthens incentives for innovation relative to the Cournot case. The proof of this general conjecture will be left for future research.

Second, we worked with the joint assumptions that R&D knowledge can be duplicated internally and is transferable. Thus, once an R&D-employee has implemented a cost reduction, costs remain low, even if the knowledge-bearing employee changes the firm, and a firm that successfully lures away R&D-employees from the competitor is assumed to obtain the same cost reduction as the competitor, i.e., knowledge travels with the employee. The duplicability assumption is more palatable for managers than for workers. If a trained worker leaves a firm, the fruits of human capital investments are usually gone without traces.¹³ Incentives for firms to invest in general human capital of workers rather than managers are significantly different compared to the R&D situation examined in the paper.

To understand the effects of assuming that an employee's knowledge is worth less once he is gone, consider Bertrand competition. A firm that has trained a worker not only loses the services of an employee who is poached by a competitor, but in addition the competitor is strengthened. This makes labor turnover particularly undesirable and forces firms to increase the wages to trained workers in order to deter them from leaving, thereby reducing training incentives in the first place, since a firm that has invested in training has no advantage over a firm that has not invested in bidding for trained workers. Workers might obtain all rents from training in this constellation, thus making the training investment sunk costs for firms. As a consequence, there can be an equilibrium where no training investments take place under Bertrand competition, since these investments can never be recovered. The same can happen

¹³ Most likely, a realistic model lies in between the two polar cases discussed in this section.

under Cournot competition, because again a firm that has invested in training has no advantage over a firm that has not when trained workers decide at which firm they want to be employed. However, under Cournot competition another equilibrium can exist, in which both firms invest in worker training. The intuition why firms are willing to invest is as follows: Starting from a situation where both firms have invested, neither would gain much by poaching the competitor's worker, since both firms already have sufficiently high levels of human capital in house.¹⁴ Therefore, to keep workers, firms need to offer wages that are smaller than in the case where only one firm invests in training. As a consequence, the fear of increasing future wage costs by not investing in training can motivate firms to invest in human capital if the other firm invests as well.

To sum up, instead of considering the polar case where knowledge can be duplicated within the firm and is fully transferable to the other firm, one might move to the other polar case where knowledge is transferable to other firms if the employee leaves, but cannot be duplicated within the firm because it is embodied in human capital. This opens up a host of new interesting issues for future research with the possibility of multiple equilibria. Our discussion in this section suggests the results on the comparison between Bertrand and Cournot with respect to investment incentives could be reversed.

¹⁴ An example is available upon request. These considerations require that trained workers are to some extent substitutes or there are sufficiently decreasing marginal returns to human capital.

Appendix:

Proof of Proposition 1:

Note that in any equilibrium $w_{ij} = w_{ji}$ for $i, j = 1, 2; i \neq j$, for otherwise, the firm obtaining the bid could decrease its wage offer and still obtain spillovers.

(a) An equilibrium with bilateral spillovers requires that it is not possible to increase profits by modifying wages such that:

- (i) The firm ends up with no employee.
- (ii) There are no spillovers.
- (iii) The firm ends up with both employees.

Thus, as firm i earns net equilibrium profits $\pi(x_i + x_j, x_i + x_j) - w_{ij}$, a wage profile is an equilibrium with bilateral spillovers if and only if for $i, j = 1, 2, i \neq j$.

$$\pi(x_i + x_j, x_i + x_j) - w_{ij} \geq \pi(x_i, x_i + x_j)$$

$$\pi(x_i + x_j, x_i + x_j) - w_{ij} \geq \pi(x_i, x_j) - w_{ii}$$

$$\pi(x_i + x_j, x_i + x_j) - w_{ij} \geq \pi(x_i + x_j, x_j) - w_{ii} - w_{ij}$$

As $w_{ij} = w_{jj}, w_{ii} = w_{ji}$, this is equivalent with

$$(i) \quad \pi(x_i + x_j, x_j) - \pi(x_i + x_j, x_i + x_j) \leq w_{ii} \leq \pi(x_i + x_j, x_i + x_j) - \pi(x_j, x_i + x_j)$$

$$(ii) \quad \pi(x_i + x_j, x_i) - \pi(x_i + x_j, x_i + x_j) \leq w_{ij} \leq \pi(x_i + x_j, x_i + x_j) - \pi(x_i, x_i + x_j)$$

$$(iii) \quad \pi(x_j, x_i) - \pi(x_i + x_j, x_i + x_j) \leq w_{ij} - w_{ii} \leq \pi(x_i + x_j, x_i + x_j) - \pi(x_i, x_j)$$

(i) and (ii) clearly require (a1).

Also, if (a1) and (a2) hold, (iii) is satisfied as well for the wages under consideration.

Clearly, this equilibrium is Pareto-dominant among the bilateral spillover equilibria, as wages cannot be reduced without leaving the regime.

(a) Condition (b1) can be seen to be necessary as follows. In the unilateral spillover

equilibrium, firm 1 gets net profits $\pi(x_1 + x_2, x_2) - w_{11} - w_{12}$. By lowering its wages

sufficiently, it would lose both employees and obtain net payoffs $\pi(x_1, x_1 + x_2)$. This is

not a profitable deviation if $\pi(x_1 + x_2, x_2) - w_{11} - w_{12} \geq \pi(x_1, x_1 + x_2)$. Similarly, the competitor does not want to attract both employees if

$$\pi(x_1 + x_2, x_1) - w_{11} - w_{12} \leq \pi(x_2, x_1 + x_2).$$

Therefore

$$(iv) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq w_{11} + w_{12} \geq \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2)$$

is necessary for an equilibrium with unilateral spillovers, and hence (b1) is. Similarly, we require

$$(v) \quad \pi(x_1 + x_2, x_2) - \pi(x_1 + x_2, x_1 + x_2) \geq w_{11} \geq \pi(x_1 + x_2, x_1 + x_2) - \pi(x_2, x_1 + x_2)$$

$$(vi) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_2) \geq w_{12} \geq \pi(x_2, x_1) - \pi(x_2, x_1 + x_2)$$

and hence (b2) and (b3).

For (v), (vi) to be compatible with (iv), we require that the lowest (highest) values of w_{11} and w_{12} satisfying (v) and (vi) also satisfy the left (right) hand side of (iv). This requirement yields conditions (b4) and (b5). Hence, (b1) – (b5) are necessary for an equilibrium. They are also sufficient, for they imply that (iv) to (vi) can be satisfied simultaneously. Obviously, if (b1)-(b5) hold with inequality, there are multiple equilibria.

(c) A wage profile is an equilibrium without spillovers if and only if for $i, j = 1, 2, i \neq j$

$$\pi(x_i, x_j) - w_{ii} \geq \pi(x_i, x_i + x_j)$$

$$\pi(x_i, x_j) - w_{ii} \geq \pi(x_i + x_j, x_i + x_j) - w_{ij}$$

$$\pi(x_i, x_j) - w_{ii} \geq \pi(x_i + x_j, x_j) - w_{ij} - w_{ii}$$

Equivalently,

$$\pi(x_i, x_j) - \pi(x_i, x_i + x_j) \geq w_{ii} \geq \pi(x_i + x_j, x_i) - \pi(x_j, x_i)$$

$$\pi(x_j, x_i) - \pi(x_j, x_i + x_j) \geq w_{ij} \geq \pi(x_i + x_j, x_j) - \pi(x_i, x_j)$$

$$\pi(x_i, x_j) - \pi(x_i + x_j, x_i + x_j) \geq w_{ii} - w_{ij} \geq \pi(x_i + x_j, x_i + x_j) - \pi(x_j, x_i)$$

The first two conditions require (c1).

Proof of Corollary 1:

Apply Proposition 1c. This gives

$$(\alpha + 2x_i - x_j)^2 + (\alpha - x_i + 2x_j)^2 \geq (\alpha + x_i - x_j)^2 + (\alpha + 2x_j + x_i)^2$$

For $x_i > 0$, simple derivations show this is equivalent with $2\alpha \leq 3x_i - 6x_j$. This condition cannot hold for $i = 1, 2$ $j \neq i$ at the same time, because $3x_i - 6x_j > 0$ implies $3x_j - 6x_i < 0$.

Proof of Corollary 2:

(a) $2\alpha \geq 3x_i - 2x_j$ is condition (a1) of proposition 1. It also implies condition (a2) of proposition 1, because in the Cournot case with linear demand, this condition becomes $2\alpha \geq 3x_i - 6x_j$ for $i, j = 1, 2; j \neq i$. Hence, part (a) follows. By corollary 1, an equilibrium without spillovers does not exist. By comparison of conditions (a1) and (b2) in proposition (1) there can be no equilibrium with unilateral spillovers when an equilibrium with bilateral spillovers exists, completing part (b) of the corollary. Part (c) follows directly from part (a) of proposition 1.

Proof of Proposition 2:

First note that, by corollary 2(a) and 2(b), for the proposed values of x_1^c and x_2^c , and $v > 1/6$ the resulting subgame, there is a unique Pareto-dominant pure strategy equilibrium with bilateral spillovers. By corollary 2(c), therefore, a Nash equilibrium with bilateral spillovers requires

$$x_i^c = \arg \max_{x_i} \left\{ 2 \frac{(\alpha + x_i + x_j)^2}{9} - \frac{(\alpha + 2x_j + x_i)^2}{9} - vx_i^2 \right\}, \text{ which implies } x_1^c = x_2^c = \frac{\alpha}{9v-1}.$$

Note that assumption A2 implies $v > (2a-c)/9c$ and hence $x_1^c + x_2^c < c$, so that marginal production costs are positive.

To test for subgame perfection, we need to check that it is not worthwhile for firm i to change its investment level so much that firms are not in the bilateral spillover regime in period 2.

First note that, by corollary 1, an equilibrium without spillovers does not exist if both investment levels are positive. If firm i deviates to $x_i = 0$ and there are no spillovers, the

resulting profit is $(\alpha - \gamma)^2/9$, so that the net deviation profit becomes $-\frac{4\alpha\gamma}{9} + \frac{2\gamma}{9} + v\gamma^2$.

This expression is negative if $v > 2/9$. For spillovers to flow from i to j , condition (b2) of proposition 1 would have to hold with i and j replaced and $x_i = \gamma$ and some positive x_j .

However, simple but tedious derivations show that condition (b2) would yield

$$x_i < \frac{1}{2}\alpha \frac{18v-5}{9v-1} < 0.$$

There is a subgame equilibrium with unilateral spillovers from j to i if and only if $x_i \geq 6\alpha v/(9v-1)$. This can be seen because, in the Cournot case with $x_j = \alpha/(9v-1)$ the conditions in proposition 1 (b) are

$$(i) \quad x_i \geq \frac{1}{3}\alpha \frac{18v-5}{9v-1}$$

$$(ii) \quad x_i \geq 6\alpha v/(9v-1)$$

$$(iii) \quad x_i \geq -\frac{1}{10}\alpha \frac{18v-5}{9v-1}$$

$$(iv) \quad 2\alpha x_i + 3\left(\frac{\alpha}{9v-1}\right)^2 \leq 2\alpha\left(\frac{\alpha}{9v-1}\right) + 3x_i^2 + 4x_i\left(\frac{\alpha}{9v-1}\right)$$

$$(v) \quad 2\alpha x_i + 3\left(\frac{\alpha}{9v-1}\right)^2 \leq 2\alpha\left(\frac{\alpha}{9v-1}\right) + 3x_i^2 + 4x_i\left(\frac{\alpha}{9v-1}\right)$$

In the parameter regime under consideration, (i) is implied by (ii), and (iii) always holds.

Condition (iv) and (v), which are equivalent in our case, hold for $v > 4/9$ and

$x_i > 6\alpha v/(9v-1)$. Thus all five conditions hold for $v > 4/9$ as long as (ii) does.

If $x_i \geq 6\alpha v/(9v-1)$ and one of the asymmetric spillover equilibria results, the proof of proposition 1b gives a lower bound of $\pi(x_i + x_j, x_i) - \pi(x_j, x_i + x_j)$ for the wage payments,

and a corresponding upper bound for the deviation profit of

$$\begin{aligned} & \pi(x_i + x_j, x_j) - (\pi(x_i + x_j, x_i) - \pi(x_j, x_i + x_j)) - vx_i^2 = \\ & \frac{\alpha^2}{9} + \frac{4x_i^2}{9} - \frac{2x_i}{9}\left(\frac{\alpha}{9v-1}\right) - \frac{2}{9}\left(\frac{\alpha}{9v-1}\right)^2 - vx_i^2. \end{aligned}$$

The derivative of this expression with respect to x_i is $-\frac{2}{9}\left(\frac{81x_i v^2 - 45vx_i + 4x_i + \alpha}{9v-1}\right)$. The

derivative can be shown to be negative for positive x_i . Hence, the profit is decreasing in x_i in the parameter regime under consideration. Thus, if there is a profitable deviation, it must be to the regime boundary, $x_i = 6\alpha v/(9v-1)$. Inserting this value for x_i gives

$-\frac{\alpha^2(324v^3 - 225v^2 + 30v + 1)}{9(9v-1)^2}$. Subtracting the equilibrium profit gives a net deviation profit of $-\frac{\alpha^2(315v^3 - 127v^2 + 10v + 2)}{9(9v-1)^2}$. This turns out to be negative for all positive values of v .

We now turn to uniqueness. Since marginal investment costs are zero at $x_i = 0$ and the marginal profit increase from investment is positive, no firm will choose $x_i = 0$ in any equilibrium under Cournot competition. Therefore, applying corollary 1, no equilibrium without spillovers can exist. We are left with two constellations. We check whether there may be further equilibrium constellations $0 < x_i \leq x_j$ with bilateral spillovers.

In the bilateral spillover regime corollary 2c.) shows that profits of firm i are given by:

$$\frac{\alpha^2 + 2\alpha x_i + x_i^2 - 2x_j^2}{9} - vx_i^2$$

Since there are no interaction terms between x_i and x_j , the optimal choice of firm i must fulfill $x_i = \alpha/(9v-1)$, no matter what x_j is. The same argument applies for firm j . Therefore, an equilibrium with bilateral spillovers different from the one in the proposition cannot exist.

Proof of Proposition 3:

The proposed equilibrium exists by (i) - (vi) below.

(i) Because $w_{11} + w_{12} = \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2) = \pi(x_1 + x_2, x_1)$ firm 2 does not want to attract both employees: this would leave net payoff unaffected.

(ii) For firm 2, attracting only the other firm's employee would lead to payoffs

$$\pi(x_1 + x_2, x_1 + x_2) - w_{21} \leq 0.$$

- (iii) Avoiding spillovers would lead to net payoffs $\pi(x_2, x_1) - w_{22}$ for firm 2. As $x_2 \leq x_1$, this expression is also non-positive.
- (iv) Firm 1 makes no deviation where both firms obtain spillovers and both firms end up with equal costs, as it would lose its profit.
- (v) Similarly, firm 1 does not let firm 2 have both employees, because of
- $$\pi(x_1 + x_2, x_2) = \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq w_{11} + w_{12}.$$
- (vi) Firm 1 does not refrain from hiring the other firm's employee because, by (7),
- $$\pi(x_1, x_2) \leq \pi(x_1 + x_2, x_2) - w_{12}.$$

Also there is no equilibrium with unilateral spillovers that does not satisfy (6) and (7). By (i), (v) and (vi), the inequalities on wages given in (6) and (7) are necessary. In addition, the equality

$$w_{11} + w_{12} = w_{12} + w_{22} = \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2) \text{ holds,}$$

for otherwise firm 2 would be offering more for the two employees than they are worth, that is, it would play a weakly dominated strategy. There is no equilibrium with bilateral spillovers, as this would lead to zero profits for both firms, and no firm would be willing to pay a positive amount to obtain the competitor's employee. There is no equilibrium without spillovers: Such an equilibrium would require $w_{ii} = w_{ji}$. Supposing that $x_2 < x_1$, firm 2 would only pay $w_{22} = 0$. Increasing w_{12} slightly above zero would increase profits for firm 1.

Proof of Proposition 4:

- (i) First note that assumption A1 implies $\alpha / 2v < c$, so that $x_1^B < c$.
- (ii) There is no equilibrium with $x_1 = x_2 > 0$: both firms would obtain zero profits in the product market; hence they would be better off not investing at all.
- (iii) There is no equilibrium with $x_1 = x_2 = 0$. Both firms obtain zero net profits in the product market; using proposition 3, a small cost reduction x_1 would yield total payoffs $\alpha x_1 - vx_1^2$. For small x_1 , this is positive.

- (iv) There is no equilibrium with $x_1 > x_2 > 0$: firm 2 obtains zero profits in the product market, and hence, subtracting investment costs, negative total payoffs, so it would be better off setting $x_2 = 0$.
- (v) The proposed values of x_i are equilibrium choices. We first note that x_1^B maximizes $\alpha x_1 - v x_1^2$. Moreover, x_1^B is too low for firm 1 to obtain the monopoly profit, since condition (5) $x_1^B \leq \alpha$ is satisfied for $v \geq \frac{1}{2}$. Therefore, $\pi(x_1, 0) = \alpha x_1$ and x_1^B maximizes $\pi(x_1, 0) - v x_1^2$.

Next, we need to check that firm 1 has no incentive to select x_1 so high to obtain monopoly profits. Profits in the monopoly case when $x_1 > a - c$ would amount to

$$\frac{(\alpha + x_1)^2}{4} - v x_1^2. \text{ Maximizing with respect to } x_1 \text{ yields } x_1 = \frac{\alpha}{4v-1} \text{ which is a}$$

contradiction to $x_1 > \alpha$ for $v \geq \frac{1}{2}$. Hence, firm 1 does not want to become a monopolist.

Finally, we have to show that firm 2 cannot set x_2 so high that it obtains a positive profit. We have to distinguish two cases. First, we assume that firm 2 does not become a monopolist by leapfrogging x_1^B . According to proposition 3 firm 2 will obtain spillovers from firm 1, and its total payoff is given by:

$$\pi(x_1^B + x_2, x_1^B) - \pi(x_1^B + x_2, x_2) - v x_2^2 = x_2(\alpha + x_1^B) - x_1^B(\alpha + x_2) - v x_2^2. \text{ The optimal value of } x_2 \text{ obtainable in this way is given by } x_2 = \frac{\alpha}{2v} = x_1^B. \text{ Since}$$

$\pi(x_1^B + x_2, x_1^B) - \pi(x_1^B + x_2, x_2) = 0$ for $x_2 = x_1^B$ firm 2 would have negative total payoff equal to $-v x_2^2$. Thus, firm 2 has no incentive to innovate itself.

Second, we show that firm 2 does not want to set x_2 so high that it will sell at the monopoly price. By (5), it would set the monopoly price if

$$x_2 \geq \alpha + 2x_1^B = \alpha \left(\frac{v+1}{v} \right) = \bar{x}_2. \text{ After choosing } x_2 \text{ so high, it would obtain}$$

$$\pi(x_1^B + x_2, x_1^B) - \pi(x_1^B + x_2, x_2). \pi(x_1^B + x_2, x_1^B) \text{ is the monopoly profit corresponding}$$

to costs of $c - x_1^B - x_2$, i.e. $\frac{(\alpha + x_1^B + x_2)^2}{4}$. $\pi(x_1^B + x_2, x_2)$ is equal to $x_1^B \cdot (\alpha + x_2)$.

Hence, the optimal deviation profit involving monopoly pricing is

$$\max_{x_2 \geq \bar{x}_2} \frac{(\alpha + x_1^B + x_2)^2}{4} - \frac{4x_1^B(\alpha + x_2)}{4} - \frac{4vx_2^2}{4}. \text{ An interior solution requires}$$

$$2(\alpha + x_1^B + x_2) - 4x_1^B - 8vx_2 = 0. \text{ The second-order-condition is fulfilled for } v > \frac{1}{4}.$$

$$\text{We obtain } x_2 = \frac{x_1^B - \alpha}{1 - 4v}.$$

Finally, we show that if this expression is larger than \bar{x}_2 , we obtain a contradiction and hence firm 2 does not set the monopoly price. To this end, note that

$$\frac{x_1^B - \alpha}{1 - 4v} = \frac{\frac{\alpha}{2v} - \alpha}{1 - 4v} = \alpha \left(\frac{1 - 2v}{(1 - 4v)2v} \right). \text{ Hence, } x_2 = \frac{x_1^B - \alpha}{1 - 4v} > \bar{x}_2 \text{ implies}$$

$$\alpha \frac{1 - 2v}{(1 - 4v)2v} > \alpha \left(\frac{v + 1}{v} \right) \text{ or } \frac{1 - 2v}{(1 - 4v)} > 2(v + 1). \text{ For } v > \frac{1}{4}, \text{ this is equivalent to}$$

$$v^2 + \frac{1}{2}v - \frac{1}{8} < 0 \text{ and hence } v < \frac{1}{4} + \sqrt{\frac{3}{8}}, \text{ which is a contradiction to our assumption.}$$

Proof of proposition 6:

Using the same reasoning as in (ii) - (iv) in the proof of proposition 4, x_2^B must be zero in equilibrium. Given $x_2^B = 0$ and using part (v) of the proof of proposition 4, the optimal

response of firm 1 is to set $x_1^B = \frac{\alpha}{2v}$ because $v > \frac{1}{2}$ ensures that firm 1 does not want to

become a monopolist. But if $x_1^B = \frac{\alpha}{2v}$ and $v < \frac{1}{4} + \sqrt{\frac{3}{8}}$, the proof of proposition 4 shows that

firm 2 has an incentive to leapfrog firm 1 by investing $x_2 = \frac{\alpha(1 - 2v)}{(1 - 4v)2v}$ and acting as a

monopolist. Firm 2 sets the monopoly price if $x_2 \geq \alpha + 2x_1^B = \alpha \left(\frac{v + 1}{v} \right) = \bar{x}_2$ which is fulfilled

if $v < \frac{1}{4} + \sqrt{3/8}$. Therefore, $x_2^B = 0$ cannot be an equilibrium choice and no subgame perfect equilibrium in pure strategies exists.

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