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# Informality and the Long Run Phillips Curve

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# Informality and the Long Run Phillips Curve

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**Abstract**

This paper studies the implications of informality for the long run relationship between inflation, output and unemployment in developing economies. I present a monetary dynamic general equilibrium model with search frictions in both labor and goods markets and where informality is an equilibrium outcome. Policies that lead to a larger informal sector result in an upward shift in both the money demand relation and the Beveridge curve. In contrast, financial development reduces informality and shifts both the Beveridge curve and the money demand relation downwards. An increase in the long run inflation rate affects unemployment through two channels: On the one hand, higher inflation reduces the surplus of monetary trades which lowers firms' profits, job creation and increases unemployment. On the other hand, it shifts firms' hiring decision from high separation and cash intensive informal jobs to low separation formal jobs which reduces unemployment. I calibrate the model to the Brazilian economy and find that the existence of a large informal sector significantly dampens the long run effects of monetary policy on unemployment and output. This result points to the importance of accounting for informality in the conduct of monetary policy in developing economies.

**Keywords:** informality; Phillips curve; inflation; unemployment; search and matching.

**JEL classification:** E26, E41, J64, H26, O17.

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# 1 Introduction

The long-run relationship between inflation and unemployment in advanced economies has been studied extensively. Several authors found compelling evidence against a vertical long run Philips curve (Karanassou et al., 2003; Beyer and Farmer, 2007; Schreiber and Wolters, 2007; Berentsen et al., 2011). In contrast, there are no studies so far on the shape of the long run Philips curve (LRPC, henceforth) in developing economies. Not only inflation levels are chronically high in many developing economies, but the latter also exhibit certain characteristics mostly absent in advanced economies such as strong informality, weak labor institutions and low levels of financial development which might affect the long run relationship between money and output or unemployment. This work contributes to the literature by studying the impact of informality on the shape of the LRPC.[1] Informality is one of the main issues facing developing economies. The informal sector represents on average 32.5% of official GDP with some countries above 50% in Latin America and Africa (Medina and Schneider, 2017). To address this issue, I introduce informality into a monetary dynamic general equilibrium model with frictional labor and goods markets. This model combines a two-sector version of the labor search model of Mortensen and Pissarides (1994) with the New Monetarist model of Lagos and Wright (2005). Frictions in the labor market result in equilibrium unemployment and informality while frictions in the goods market provide micro-foundations that insure a transaction role for money. I develop a novel mechanism which links firms' hiring decision in the labor market to the choice of payment instruments in the frictional goods market. The resulting theoretical framework captures the main stylized facts reported in the literature (Batini et al., 2010).

In this framework, the existence of informality is an equilibrium outcome as formal and informal activities present different implications for workers and firms. On the one hand, choosing a formal work arrangement allows firms to sell the resulting output against credit. However, it requires the payment of taxes to the government. On the other hand, hiring workers informally allows firms to avoid taxes but precludes the use of credit in selling their production and exposes them to government monitoring and hence a higher job destruction rate. The size of the informal economy results endogenously from the optimal decision of agents faced with these trade-offs.

I show that policies leading to a higher share of the informal sector in the economy result

[1] Informality is defined here as market-based economic activities which are hidden from the government in order to avoid costly regulation; e.g. taxation, labor protection, etc. This definition excludes home production and criminal activities. See Medina and Schneider (2017) for a related definition.

in an increase in both unemployment and money demand. In other words, an increase in the ratio of informal to formal firms shifts both the Beveridge curve and the money demand relation upwards. In contrast, financial development, modeled through a wider availability of credit in formal transactions, reduces informality and shifts both the Beveridge curve and the money demand relation downwards. Monetary policy that increases the long run inflation target affects unemployment through Friedman's real balances channel (Friedman, 1977): A higher growth rate of the money supply increases inflation and the nominal interest rate and reduces real balances held for transaction in the goods market. This in turn lowers consumption and with it firms profits, reduces job creation and increases unemployment. The presence of informality results in an additional effect: as the informal sector is more cash intensive, an increase in inflation shifts the creation of jobs from the informal to the formal sector. Since formal jobs have a lower separation rate, this labor reallocation reduces steady state unemployment. I calibrate the model to the Brazilian economy and find that informality significantly dampens the long run effects of monetary policy on unemployment and output. This dampening effect depends in particular on credit availability, a proxy for financial development, and the difference between formal and informal jobs separation rates, a proxy for labor protection and government monitoring. This result implies that central banks should take into consideration the size of the informal sector in the conduct of monetary policy and in particular when setting long-run policy objectives.

The rest of the paper is organized as follows: in the next section I present a brief review of the literature. The theoretical model is presented in section 3 and the equilibrium solution is characterized in section 4. Section 5 summarizes the main theoretical results of the paper. The calibration procedure and the numerical results are presented in section 6. Section 7 concludes.

## 2 Related literature

Measuring informality faces considerable methodological challenges to the extent that agents operating within the informal economy are relentlessly trying to conceal any traces of their activities. To overcome these challenges, economists resort to indirect measurement methods using electricity consumption, money demand and cash transactions, mismatches in national accounts or household surveys. Schneider et al. (2011) provide an extensive survey of this literature.

This paper is related to the literature on money and informality. Koreshkova (2006) presents a cash-in-advance model that relates the size of the informal sector to the trade-off between inflation and seigniorage income on the one hand and the tax income on the other hand. She shows that inflation works to smooth the tax burden between the formal and informal parts of the economy. Although tax evasion is the main motive for informality in my model, I focus my analysis exclusively on long run monetary policy issues and abstract from public finance considerations. Gomis-Porqueras et al. (2014) introduce informality to the Lagos and Wright (2005) model by allowing agents to avoid taxes on part of their income through the use of cash transactions. They derive a model-based measure of informality and produce country-level estimates which tend to be on the lower range of the reduced-form estimates found in the literature. Compared to their work, I add a frictional labor market and derive a model-based measure of the informal economy which extends their measure with labor market considerations. Bittencourt et al. (2014) use a monetary overlapping generations model with endogenous tax evasion to study the effect of financial development and inflation on the size of the informal economy. They find that a lower level of financial development provides agents with a higher incentive in participating in tax evasion activities. I also integrate the financial development dimension and obtain a similar result. However, I push the analysis further to see how financial development matters for the LRPC in the presence of informality.

In addition to the previous papers, there exist a strand in the informality literature which has focused on analyzing short run monetary policy based on the New Keynesian framework. For example, Castillo and Montoro (2008) extend the basic New Keynesian model with a dual labor market with search frictions and find that the share of informal labor increases during periods of high demand. Mattesini and Rossi (2009) assume instead a unionized formal labor market with rigid wages along a competitive informal labor market. They show that the larger the size of the formal sector the stronger the impact of shocks on the economy and the stronger should be the reaction of monetary policy to these shocks. In a similar model augmented with financial frictions, Batini et al. (2011) show that simple monetary policy rules that respond only to observed aggregate inflation and formal sector output perform poorly. These papers point to the role of the informal sector as a buffer that weakens the effect of demand and productivity shocks on the economy. Although I reach a somewhat similar conclusion for the long run, the mechanisms driving the shock-absorbing role of informality in the short and long run are very different. In particular, monetary transmission in this literature relies on nominal frictions to generate a role for monetary policy. Monetary transmission in my model operates instead through the cost of holding real balances for transaction purposes.

# 3 Model

I consider a setting with discrete time and infinite horizon. In every period, three markets take place sequentially: a decentralized labor market à la Mortensen and Pissarides (1994), called LM, a decentralized goods market following Kiyotaki and Wright (1993) and Lagos and Wright (2005), called DM, and a centralized Walrasian market, called CM. In the CM market, trade is a frictionless process and agents can buy or sell the numéraire good at the equilibrium market price. In the LM and DM, agents must search for matching opportunities and use bargaining to share the match surplus.

Two types of agents live infinitely in this model, firms and households, indexed by $f$ and $h$ respectively and each of measure 1.[2] Households work, buy and consume goods. Firms maximize profits by hiring workers to produce goods and then sell the goods to households. Agents discount between periods using the same time preference rate $\beta$. There is no discounting between markets within the period.

An agent (firm or worker) can be in one of three states depending on the type of work arrangement they are taking part in: formal employment, $e$, informal employment, $i$ and unemployment, $u$. I define the value functions $U$, $V$ and $W$ for the LM, DM and CM respectively. These value functions depend on the agent's type $t \in \{f, h\}$, on their current employment status $j \in \{e, i, u\}$, on their real money holdings $z$ as well as on the loans $\ell$ they extended or received.

Firms and households meet in the LM and try to form bilateral work relationships. I assume random matching based on a matching function, $\mathcal{M} = \mathcal{M}(u, v)$, where the number of matches is a function of $u$, the measure of unemployed workers, and $v$, the measure of posted vacancies. The matching function describes the number of new matches resulting from contacts between unemployed workers and firms seeking to fill open vacancies. As is standard in the labor search literature, $\mathcal{M}$ is increasing, concave and homogeneous of degree one. On the one hand, a firm with a vacancy finds a worker with probability $\alpha_f = \mathcal{M}(u, v)/v = \mathcal{M}(1/\theta, 1)$ where $\theta = v/u$ is the

[2]In what follows, the terms *household*, *worker* and *buyer* are used interchangeably. One could also think of a household as being composed of one worker sent to the LM and one buyer sent to the DM. With unit measures of households and firms, the probability that the buyer meets the firm employing his kin worker is 0.

labor market tightness. On the other hand, an unemployed worker finds a job with probability $\alpha_h = \mathcal{M}(u,v)/u = \mathcal{M}(1,\theta)$. Firms and workers take the aggregate matching probabilities as given.

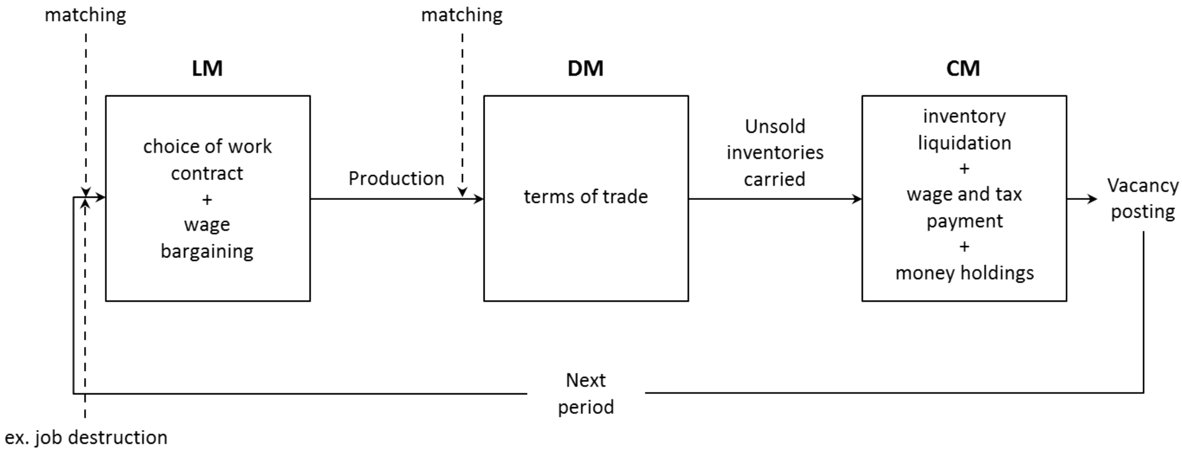



Figure 1: Timeline

All workers and firms are ex-ante identical. Unmatched firms can enter the next period's LM by posting a generic job vacancy at the end of the CM. Once an unmatched firm meets a worker the idiosyncratic productivity of the match, $\varepsilon$, is revealed. This productivity is match specific and reflects the quality of the match. It is drawn from a distribution $F(\varepsilon)$ with bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$. Based on $\varepsilon$, the firm and the worker decide whether to engage in a formal or an informal work relationship.[3] I define $n_e$ as the measure of formally employed workers and $n_i$ as the measure of informally employed workers. Since each firm corresponds to one job, these measures are also those of formal and informal firms in the economy.

Both formal and informal firms produce $y + \varepsilon$ units of a good that is storable within the period. The produced good can be sold to households either in the DM or the CM. Selling a quantity $q$ of the good in the DM costs $c(q)$. The remaining inventories are sold in the CM at price $p$ normalized to 1.

Wages are negotiated in the LM and paid in the following CM in terms of the general good. Firms employing formal workers are subject to a lump-sum tax $\tau$ which reflects the costs of formal employment.

Formal and informal jobs are destroyed with probabilities $\delta_e$ and $\delta_i$ respectively. I assume $\delta_i \geq \delta_e$. The higher separation rate of informal jobs can be rationalized through government

[3]Without loss of generality, I assume $\underline{\varepsilon}$ is high enough such that the value of a match is always positive. This simplifies the model greatly by ruling out post-match separation.

monitoring or weak enforcement of informal contracts.

Next, firms (sellers) and households (buyers) enter the DM where they can trade the search good $q$ pairwise. Matching in the DM is based on a random matching function, $\mathcal{N} = \mathcal{N}(B, S)$, where $B$ and $S$ are the measures of active buyers and sellers respectively. $\mathcal{N}$ is increasing, concave and homogeneous of degree one. On the one hand, all households take part in the DM as buyers provided they are matched hence $B = 1$. On the other hand, a firm can take part in the DM market as a seller only if it is has goods to sell. This is only possible if the firm has managed to recruit a worker in the LM which implies $S = 1 - u$.

A seller meets a buyer with probability $\sigma_f = \mathcal{N}(B, S)/S$. In the same manner, a buyer meets a seller with probability $\sigma_h = \mathcal{N}(B, S)/B$. Since there are formal and informal firms operating in this economy, buyers can be randomly matched with either type of firms. With probability $\frac{n_e}{1-u}$ the encountered seller is a formal firm and with probability $\frac{n_i}{1-u}$ an informal firm. To simplify notation, I define the unconditional probability of meeting a formal seller as $\sigma_e = \sigma_h \frac{n_e}{1-u}$ and the unconditional probability of meeting an informal seller $\sigma_i = \sigma_h \frac{n_i}{1-u}$. Notice that the LM and DM are related not only through the measure of active firms but also through the ratio of informal to formal firms. Indeed, for a given measure of active firms, an increase in the share of formal (informal) employment in the labor market increases the probability for buyers to meet a formal (informal) firm.

The search good $q$ is produced by firms and only households want to consume it during the DM such that there is no double coincidence of wants. Commitment is limited, bilateral meetings are anonymous and agents cannot store the general good in the CM market to use it in subsequent periods as a medium of exchange. I assume the existence of an imperfect record keeping technology which makes contract enforcement by the government possible. Acquiring the record keeping technology is costless but the enforcement of the recorded financial obligations requires compliance with government regulation. Only firms employing formal labor and paying taxes can benefit from government's enforcement of their contracts. In particular, formal firms can offer loans $\ell$ to buyers to be repaid in terms of the general good in the subsequent CM.[4] I assume that credit is available only with probability $\eta$. This insures the co-existence of credit and money in formal DM transactions.[5] Moreover, government punishment is arbitrarily harsh such that default is not an option for buyers. Informal firms in contrast cannot enforce their contracts

[4]These loans can be seen as a form of differed payment or supplier credit.

[5]See Rocheteau and Nosal (2017) for other approaches resulting in the coexistence of credit and money.

and as a consequence resort to money as the only mean of exchange for their transactions in the DM.

The CM is a frictionless Walrasian market where the general good is traded at the equilibrium price $p$ normalized to 1. In the CM, firms liquidate what remains of their production, post vacancies, pay wages and taxes and distribute their profits as dividends to households. The latter buy and consume the general good, repay their loans and decide how much money to take to the next period.

Finally, I adopt the following convention regarding money. I define real balances $z = m/p = \phi m$ where $\phi$ is the price of money in terms of the general good. The aggregate quantity of money in the economy is given by $M$ and grows at rate $\gamma = M'/M$. I focus on stationary monetary equilibria where the real value of money supply is strictly positive and constant over time such that $\phi M = \phi' M'$. This implies that inflation is completely determined by the growth rate of money supply; i.e. $p'/p = \phi/\phi' = \gamma$. Unless otherwise specified, I assume that $\gamma > \beta$ such that the economy is away from the Friedman rule.[6]

## 3.1 Households

At the beginning of the period, the value for a worker of entering the LM unemployed is

$$U_u^h(z) = \alpha_h \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^h(\varepsilon, z), V_i^h(\varepsilon, z)\right\} \, \mathrm{d}F(\varepsilon) + (1-\alpha_h)V_u^h(z).$$

With probability $\alpha_h$, an unemployed worker is matched with a firm and the match productivity $\varepsilon$ is then revealed. Depending on $\varepsilon$ the worker is hired formally with value $V_e^h$ or informally with value $V_i^h$. With probability $1-\alpha_h$, the unemployed worker is not matched and enters the DM with value $V_u^h$. The value function of an employed worker starting the period with idiosyncratic match productivity $\varepsilon$ and real money balances $z$ is given by

$$U_j^h(\varepsilon, z) = (1-\delta_j)V_j^h(\varepsilon, z) + \delta_j V_u^h(z), \quad j \in \{e, i\};$$

which states that a formal (informal) worker might lose his current job with probability $\delta_e$ ($\delta_i$). In this case, he enters the DM unemployed and can only be matched in next period's LM.

[6]One interpretation would be that the government relies on seigniorage income to finance part of its expenditures. Another one would be that the government is unable to withdraw money by taxing households in the CM. For a discussion of incentive-feasible deflation see Andolfatto (2013).

In the DM, a buyer (household) is randomly matched with a firm to buy $q$ units of the search good. Consuming $q$ provides utility $v(q)$ for the buyer with $v(0) = 0$, $v' > 0, v'' < 0$ and $v'(0) = +\infty$. The value function of a buyer entering the DM with employment status $j \in \{e, i, u\}$ is

$$V_j^h(\varepsilon, z) = \sigma_e \eta \left( v(q_c) + W_j^h(z - \phi d, \ell) \right) + \sigma_e (1 - \eta) \left( v(q_m) + W_j^h(z - \phi d) \right) \\ + \sigma_i \left( v(q_m) + W_j^h(z - \phi d) \right) + (1 - \sigma_e - \sigma_i) W_j^h(z)$$

The buyer is matched with a formal firms with probability $\sigma_e$. With probability $\eta$, the buyer can use both money $d$ and credit $\ell$ to buy quantity $q_c$. Otherwise, with probability $1 - \eta$ the buyer can only use money in which case he buys quantity $q_m$. With probability $\sigma_i$, the buyer meets an informal firm in which case he is limited to the use of money and acquires the same quantity $q_m$. With probability $1 - \sigma_e - \sigma_i$, the buyer is not matched in the DM and carries her money holdings to the CM.

When an unemployed worker enters the CM, he solves the following maximization problem:

$$W_u^h = \max_{x, z'} \left\{ x + b + \beta U_u^h(z') \right\},$$

subject to

$$x + \gamma z' = z - \ell + \Delta + T\,;$$

where $x$ is consumption, $b$ is the flow value of unemployment, $\Delta$ are profits distributed by firms and $T$ are lump sum transfers by the government. $z$ are the real balances that the household brings with it from last period and $z' = \phi' m' = \phi m' / \gamma$, is the amount of real balances carried to the next period. Any loans $\ell$ that the household obtained in the previous DM are settled in the CM. I assume that utility in the CM is linear as in the standard labor search model (Pissarides, 2000).

Inserting the budget constraint in the objective function and making use of the linearity of the value function in $z$, I get:

$$W_u^h(z, \ell) = b + \Delta + T + z - \ell + \max_{z'} \left\{ -\gamma z' + \beta U_u^h(z') \right\}$$

As we can see, the choice of $z'$ is independent of $z$.[7] We will see later that it is also independent of the household's employment status.

Employed workers entering the CM face the following maximization problems:

$$W_j^h(\varepsilon, z, \ell) = w_j(\varepsilon) + \Delta + T + z - \ell + \max_{z'} \left\{ -\gamma z' + \beta U_j^h(\varepsilon, z') \right\}, \; j \in \{e, i\}.$$

Using again the linearity of $W_j^h$ in $z$ and $\ell$, $V_j^h$ can be written as

$$V_j^h(\varepsilon, z) = \sigma_e \eta \left( v(q_c) - \phi d - \ell \right) + \sigma_e (1-\eta) \left( v(q_m) - \phi d \right) + \sigma_i \left( v(q_m) - \phi d \right) + z + W_j^h(\varepsilon, 0), \; j \in \{e, i\},$$

where $W_j^h(\varepsilon, 0)$ is the value function of a household entering the CM with no money holdings and no outstanding loans. In order to simplify notation, I drop $z$ and $\ell$ from the arguments of the value function in the absence of money or loans. Plugging this in the expression for $U_j^h$ and inserting its next period expression into $W_j^h$ I get the following recursive formulations:

$$W_u^h(z, \ell) = b + \Delta + T + z - \ell + Z + \beta \left[ \alpha_h \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max \left\{ W_e^h(\varepsilon'), W_i^h(\varepsilon') \right\} \, \mathrm{d}F(\varepsilon') + (1 - \alpha_h) W_u^h \right] \tag{1}$$

$$W_e^h(\varepsilon, z, \ell) = w_e(\varepsilon) + \Delta + T + z - \ell + Z + \beta \delta_e W_u^h + \beta(1 - \delta_e) W_e^h(\varepsilon)$$
$$W_i^h(\varepsilon, z, \ell) = w_i(\varepsilon) + \Delta + T + z - \ell + Z + \beta \delta_i W_u^h + \beta(1 - \delta_i) W_i^h(\varepsilon)$$

where

$$Z \equiv \max_{z'} \left\{ (\beta - \gamma) z' + \beta \left( \sigma_e \eta (v(q_c') - \phi' d' - \ell') + \sigma_e (1 - \eta)(v(q_m') - \phi' d') + \sigma_i (v(q_m') - \phi' d') \right) \right\}$$

determines DM consumption given the amount of real money balances households carry to the next period. We can see that a household's money demand is independent of its LM outcome.[8]

[7] This is a standard result and follows from the assumption of (quasi-)linear utility (Lagos and Wright, 2005).
[8] I assume $b$ is high enough such that consumption is always positive given the optimal choice of real balances.

### 3.2 Firms

The value of a firm entering the LM with a vacancy is

$$U_u^f = \alpha_f \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^f(\varepsilon), V_i^f(\varepsilon)\right\} \, \mathrm{d}F(\varepsilon) + (1-\alpha_f)V_u^f$$

with probability $\alpha_f$ the firm is matched with a worker. The match productivity level $\varepsilon$ is then revealed and depending on it the firm and worker decide whether to enter a formal work contract with value $V_e^f(\varepsilon)$ or an informal work arrangement with value $V_i^f(\varepsilon)$. With probability $1-\alpha_f$ the firm is not matched and gets the continuation value $V_u^f$.

The value of a firm entering the LM with match productivity $\varepsilon$ is:

$$U_j^f(\varepsilon) = (1-\delta_j)V_j^f(\varepsilon) + \delta_j V_u^f, \; j \in \{e, i\},$$

where $\delta_j$ is the probability of job separation.

Matched firms produce quantity $y+\varepsilon$ of the general good. Firms take their output to the DM and CM for sale. At the beginning of the DM, an active firm is matched with probability $\sigma_f$ to a buyer, supplies him with quantity $q$ of the search good at cost $c(q)$ in terms of the general good with $c' > 0$ and $c'' \geq 0$ against payment $\phi d$. Formal firms can offer the buyer a loan $\ell$ with probability $\eta$. The remaining inventories $y+\varepsilon-c(q)$ are carried to the CM and sold at price $p$ normalized to 1. With probability $1-\sigma_f$, the firm is not matched and carries all of its output to the CM. This leaves us with the following value functions at the beginning of the DM:

$$V_e^f(\varepsilon) = \sigma_f \left(\eta W_e^f(y+\varepsilon-c(q_c), \phi d, \ell) + (1-\eta)W_e^f(y+\varepsilon-c(q_m), \phi d)\right) + (1-\sigma_f)W_e^f(y+\varepsilon)$$

for formal firms and

$$V_i^f(\varepsilon) = \sigma_f W_i^f\left(y+\varepsilon-c(q_m), \phi d\right) + (1-\sigma_f)W_i^f(y+\varepsilon)$$

for informal firms.

The value of entering the CM with productivity $\varepsilon$ carrying inventory $x$, real balances $z$ and loans $\ell$ is

$$W_e^f(\varepsilon, x, z, \ell) = x - w_e(\varepsilon) - \tau + z + \ell + \beta U_e^f(\varepsilon)$$

for formal firms and

$$W_i^f(\varepsilon, x, z) = x - w_i(\varepsilon) + z + \beta U_i^f(\varepsilon)$$

for informal firms. Since holding money is costly and firms have no use for it they do carry none to the next period. Firms employing formal workers have to pay a lump-sum tax in the CM. One can think of $\tau$ as a proxy for the fiscal burden on formal firms and which informal firms are able to avoid.

As in the DM, a firm without a worker has nothing to sell and thus cannot take part in the CM. However, it can decide to enter the market by posting a vacancy at the end of the CM with the value

$$W_u^f = \max\left\{0, -k + \beta U_u^f\right\},$$

where $k$ is the cost paid in the CM for posting a vacancy. Assuming free entry implies

$$0 = -k + \beta U_u^f$$

which can be written as

$$k = \beta\alpha_f \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^f(\varepsilon), V_i^f(\varepsilon)\right\}\ \mathrm{d}F(\varepsilon). \tag{2}$$

The left-hand side of equation (2) represents the cost for a firm of posting a vacancy while the right-hand side represents its discounted expected profits. As $\alpha_f(\theta)$ is decreasing in $\theta$, firms enter the LM up to the point where the two are equalized and the benefits from entry are exhausted; i.e. $V_u^f = W_u^f = 0$.

Using the linearity of $W$, I rewrite $V_e^f$ as

$$V_e^f(\varepsilon) = \sigma_f\eta(\phi d + \ell - c(q_c)) + \sigma_f(1-\eta)(\phi d - c(q_m)) + y + \varepsilon - w_e(\varepsilon) - \tau + \beta U_e^f(\varepsilon)$$

Inserting the expression for $U_e^f$ and using again $V_u^f = 0$, I get

$$V_e^f(\varepsilon) = R_e(\varepsilon) - w_e(\varepsilon) - \tau + \beta(1-\delta_e)V_e^f(\varepsilon)$$

where

$$R_e(\varepsilon) = y + \varepsilon + \sigma_f\eta(\phi d_c + \ell - c(q_c)) + \sigma_f(1-\eta)(\phi d_m - c(q_m))$$

is the current period revenue of a formal firm. In the same way, I write the value of an informal firm entering the CM as:

$$V_i^f(\varepsilon) = R_i(\varepsilon) - w_i(\varepsilon) + \beta(1-\delta_i)V_i^f(\varepsilon)$$

where

$$R_i(\varepsilon) = y + \varepsilon + \sigma_f(\phi d_m - c(q_m))$$

represents the current period revenues for an informal firm.

### 3.3 Government

Government prints money at a rate $\gamma$ and collects lump sum tax $T$ and payroll tax $\tau$ which are used to finance some public spending $G$. The government budget constraint can be written as

$$G = T + \tau n_e + (\gamma - 1)\phi M,$$

where $\tau$ is levied on all formal firms, of measure $n_e$. The last right hand side term represents seigniorage income. I assume that $G$ adjusts to balance the budget.

## 4 Equilibrium

In the following, I solve the problems facing agents in each market separately and then put everything together to solve for the general equilibrium of the model. I start first with the problem facing households and firms in goods markets and solve for the terms of trade which determine the total job surplus. Given that, firms and workers of each type bargain over the wage that splits the surplus. Finally, firms and workers jointly decide which job type to choose given each type's expected surplus and wage.

### 4.1 Real balances and terms of trade in the DM

The terms of trade in the DM are determined using the proportional bargaining solution due to Kalai (1977).[9] I assume that the bargaining power of buyers $\varphi$ is the same for both types of

[9] I use the proportional bargaining solution instead of the generalized Nash solution in order to avoid the non-monotonicty of the latter. I provide the Nash solution in the appendix. For a discussion of different pricing mechanisms in monetary economies see for example Rocheteau and Wright (2005).

DM matches. The proportional solution implies that each party receives a constant share of the total surplus $v(q) - c(q)$ proportional to their bargaining power. For the buyer this share is

$$v(q_m) - \phi d_m = \varphi(v(q_m) - c(q_m))$$

and for the firm it is

$$\phi d_m - c(q_m) = (1 - \varphi)(v(q_m) - c(q_m))$$

where $\varphi \in [0, 1]$ is the buyer's bargaining power and $(q_m, \phi d_m)$ are the terms of trade that solve

$$\max_{q_m, d_m} \; v(q_m) - \phi d_m$$

subject to the seller's participation constraint

$$\phi d_m - c(q_m) \geq (1 - \varphi)(v(q_m) - c(q_m)) \tag{3}$$

and the real balance and feasibility constraints

$$\phi d_m \leq z \tag{4}$$

$$c(q_m) \leq y + \varepsilon \tag{5}$$

The seller's participation constraint (3) states that he will require at least the share of the total surplus resulting from his bargaining power $1 - \varphi$ to participate in the trade. Since the buyer always gets a positive utility from consuming more he will offer the seller just enough to make him participate. As a consequence, the seller's participation constraint will always be binding. The real balance constraint (4) states that the buyer cannot spend more money than he is carrying and the feasibility constraint (5) states that the firm cannot sell more goods than it produced in the LM. Since I assume $\gamma > \beta$ carrying money across periods is costly and therefore households don't have an incentive to carry more money than they intend to spend in the DM market. This makes the real balance constraint always binding. Finally, I assume $y$ is high enough such that the feasibility constraint (5) is never binding. The problem simplifies to

$$\max_{q_m} \; v(q_m) - c(q_m)$$

subject to

$$(1-\varphi)v(q_m) + \varphi c(q_m) = z$$

The resulting proportional bargaining solution is a pair $(q_m, d_m)$ that satisfies $q_m = g^{-1}(z)$ and $\phi d_m = z$ with

$$g(q_m) = \varphi c(q_m) + (1-\varphi)v(q_m).$$

Notice that $\partial q_m/\partial z = \partial g^{-1}(z)/\partial z = 1/g'(q_m) \geq 0$ meaning that more money holdings increases $q_m$.

Following the same logic, the terms of trade in credit matches satisfy

$$\max_{q_c} \quad v(q_c) - c(q_c)$$

subject to

$$(1-\varphi)v(q_c) + \varphi c(q_c) = z + \ell.$$

Since buyers in credit matches don't face a borrowing limit, they will consume the first best quantity that solves

$$v'(q_c) = c'(q_c) \tag{6}$$

and is independent of their real balances. As a consequence, the proportional bargaining solution in credit matches is a triplet $(q_c, d_c, \ell)$ that satisfies the above first order condition, $\phi d_c = z$ and $\ell = g(q_c) - z$ where

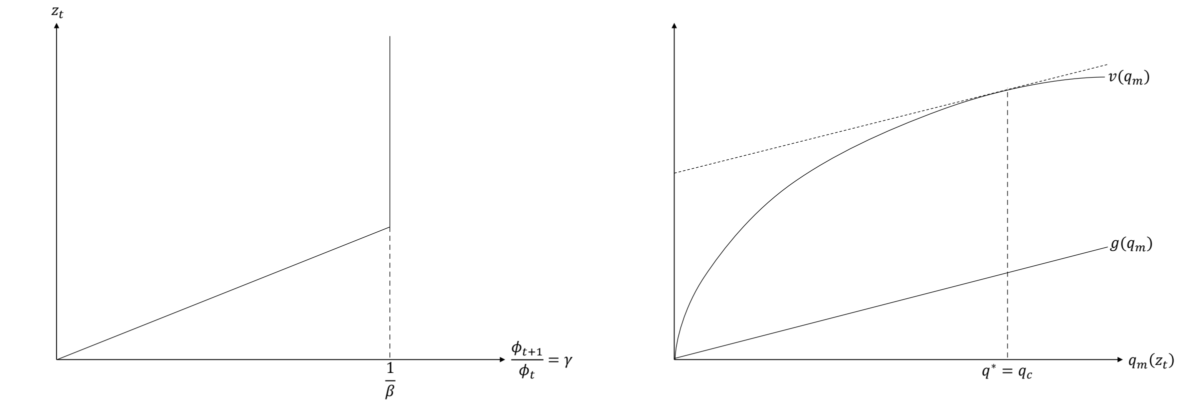
$$g(q_c) = \varphi c(q_c) + (1-\varphi)v(q_c).$$

In particular, we have the following useful lemma:

**Lemma 1** *Consumption in credit matches $q_c$ corresponds always to the first best allocation.*

This is useful since in solving for the equilibrium we can focus on consumption in monetary matches $q_m$. Intuitively, buyers will spend whatever money they have and top up with a real loan $\ell$ sufficient enough to consume the first best. Notice that away from the Friedman rule $q_m < q_c$ always holds which offers formal firms a higher expected trading surplus in the DM compared to informal firms provided they can use credit; i.e. $\eta > 0$.

Given the terms of trade in the DM derived above, households decide on the optimal amount




(a) Real balances as a function of money growth (b) DM consumption as function of money balances

Figure 2: Real money balances and terms of trade in monetary DM matches

of money holdings to carry from the CM by solving the problem

$$\max_z \quad (\beta-\gamma)z + \beta\left[\sigma_e\eta\left(v(q_c)-g(q_c)\right) + (\sigma_e(1-\eta)+\sigma_i)\left(v(g^{-1}(z))-z\right)\right]$$

Due to the independence of $q_c$ from $z$, the above problem can be simplified to

$$\max_z \quad (\beta-\gamma-\beta\sigma_e(1-\eta)-\beta\sigma_i)z + \beta\left(\sigma_e(1-\eta)+\sigma_i\right)v(g^{-1}(z))$$

Assuming an interior solution, the first order condition is:

$$\frac{v'(q_m)}{g'(q_m)} = \frac{\gamma-\beta}{\beta(\sigma_e(1-\eta)+\sigma_i)} + 1$$

Using the Fisher equation $\frac{\gamma}{\beta} = 1+i$, I get the following expression:

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_e(1-\eta)+\sigma_i} + 1 \tag{7}$$

where $i$ is the nominal interest rate and $\sigma_e$ and $\sigma_i$ depend on the measure of formal and informal firms in the LM. The left panel of figure 2 depicts the optimal quantity of money holdings as an increasing function of the return on money. The right panel depicts consumption in DM monetary matches as a function of the quantity of real balances held by buyers. Notice that $q^*$ is obtained in monetary matches only when the Friedman rule $\gamma \to \frac{1}{\beta}$ is satisfied, that is when the opportunity cost of holding money across periods $i \to 0$.

## 4.2 Wage bargaining

The choice of the optimal work arrangement, i.e. the one offering the highest present discounted value, depends on the idiosyncratic productivity of the match $\varepsilon$. When $\varepsilon$ is revealed at the beginning of the LM, the matched firm and worker are each faced with two decisions: first, which type of work arrangement to choose given that keeping the match is worthwhile and second, what should the wage be. The first decision determines the total match surplus and the second determines the way the total surplus will be split. The two decisions combined determine the surplus of each party. I start by solving the wage bargaining problem for each job type.

The match surplus is defined as the sum of net gains for the firm and the worker from being matched

$$S_e(\varepsilon) = V_e^f(\varepsilon) - V_u^f + V_e^h(\varepsilon, z, \ell) - V_u^h(z, \ell) = V_e^f(\varepsilon) + W_e^h(\varepsilon) - W_u^h$$

for formal jobs and

$$S_i(\varepsilon) = V_i^f(\varepsilon) - V_u^f + V_i^h(\varepsilon, z) - V_u^h(z) = V_i^f(\varepsilon) + W_i^h(\varepsilon) - W_u^h$$

for informal jobs. Since money holdings and employment decisions are independent as showed above, it follows that the match surplus for both types is not affected by the money held by the firm and worker.

In what follows, I use Nash bargaining with termination threat points to decide the wage when a firm and a worker are matched. This leads to the sharing rules

$$\omega_e V_e^f(\varepsilon) = (1 - \omega_e)\left(W_e^h(\varepsilon) - W_u^h\right) \tag{8}$$

for formal jobs and

$$\omega_i V_i^f(\varepsilon) = (1 - \omega_i)\left(W_i^h(\varepsilon) - W_u^h\right) \tag{9}$$

for informal jobs where $\omega_e$ and $\omega_i$ are the bargaining power of formal and informal workers respectively. Notice that in the presence of a proportional wage tax $\tau_w$, the sharing rule in formal jobs changes to

$$\omega_e V_e^f(\varepsilon) = (1 + \tau_w)(1 - \omega_e)\left(W_e^h(\varepsilon) - W_u^h\right) \tag{10}$$

In this case identical Nash bargaining power for workers in both types of jobs is not enough to guarantee identical surplus sharing rules. The marginal payroll tax reduces the match surplus and creates a wedge which distorts the way the surplus is split between the worker and the firm. Furthermore, since $\tau_w$ is proportional to the wage, the worker and the firm might find it optimal to reduce the wage in order to increase the total surplus of the match. The presence of a wage tax results also in a difference in sharing rules under formal and informal jobs which creates a discontinuity in the way the surplus is shared. This rises the possibility that for the same productivity level the firm and worker disagree on what is the optimal choice of contract. Intuitively, going from an informal to a formal contract for the same productivity level might simultaneously change the total surplus and decrease the share of one of the parties. To rule out the possibility of disagreement, one can assume that

$$\omega_i = \frac{\omega_e}{1+\tau(1-\omega_e)}$$

resulting in the surplus being shared in the same way independently of the choice of the work contract as suggested by Bosch and Esteban-Pretel (2012). This implies $\omega_i \leq \omega_e$, in line with the empirical fact that formal workers are usually more organized and enjoy more legal protection compared to informal workers. Since my focus here is on monetary issues, I abstract from these considerations by ruling out the proportional wage tax and setting $\omega_e = \omega_i = \omega$.

Using the model's equations and the bargaining solutions (10) and (9), I derive the wage equation for formal jobs

$$w_e(\varepsilon) = \omega\left(R_e(\varepsilon) - \tau\right) - (1-\omega)\left(\Delta + T + Z - (1-\beta)W_u^h\right)$$

and informal jobs

$$w_i(\varepsilon) = \omega R_i(\varepsilon) - (1-\omega)\left(\Delta + T + Z - (1-\beta)W_u^h\right).$$

Next, I combine the surplus sharing rules and the free entry condition (2) to rewrite equation (1) as:

$$W_u^h = \frac{b + \Delta + T + Z + \frac{\omega}{1-\omega}\theta k}{1-\beta}$$

which I use to simplify the wage equations above to

$$w_e(\varepsilon) = \omega\left(R_e(\varepsilon) + \theta k - \tau\right) + (1-\omega)b \tag{11}$$

and

$$w_i(\varepsilon) = \omega\left(R_i(\varepsilon) + \theta k\right) + (1-\omega)b. \tag{12}$$

where the wage in both sectors is a convex combination of the firm's surplus generated by the job and the flow value of unemployment. $\theta k$ represents the economies on search costs that the firm realizes by hiring the worker.

### 4.3 Hiring decision

Given the wage bargaining solution above, I solve for the optimal work arrangement as a function of the match productivity $\varepsilon$. Assuming non-stochastic stationary equilibria, I use the wage equations (11) and (12) to rewrite the two value functions as

$$V_e^f(\varepsilon) = \frac{(1-\omega)\left(R_e(\varepsilon) - \tau - b\right) - \omega\theta k}{1 - \beta(1-\delta_e)}$$

$$V_i^f(\varepsilon) = \frac{(1-\omega)\left(R_i(\varepsilon) - b\right) - \omega\theta k}{1 - \beta(1-\delta_i)}$$

which are strictly increasing in $\varepsilon$. This strict monotonicity result leads to the following proposition:

**Proposition 1** *For some parameter values there exists a productivity threshold $\tilde{\varepsilon}$ above which matched firms and workers will choose a formal work arrangement and below which matched firms and workers will choose an informal work arrangement.*

The proof of this proposition is in appendix B.1. It follows from the strict monotonicity of the value functions in $\varepsilon$ and in particular the fact that the household's optimal money holdings are independent of its LM status. Proposition 1 is illustrated in figure 3. In the segment $[\tilde{\varepsilon}, \overline{\varepsilon}]$ of the support of $F(\varepsilon)$, the formal firm's value $V_e^f(\varepsilon)$ lies above the informal firm's value $V_i^f(\varepsilon)$ while in the segment $[\underline{\varepsilon}, \tilde{\varepsilon}]$ this order is reversed. In what follows I focus on parameter values for which the threshold $\tilde{\varepsilon}$ satisfies the condition $\tilde{\varepsilon} \in [\underline{\varepsilon}, \overline{\varepsilon}]$.

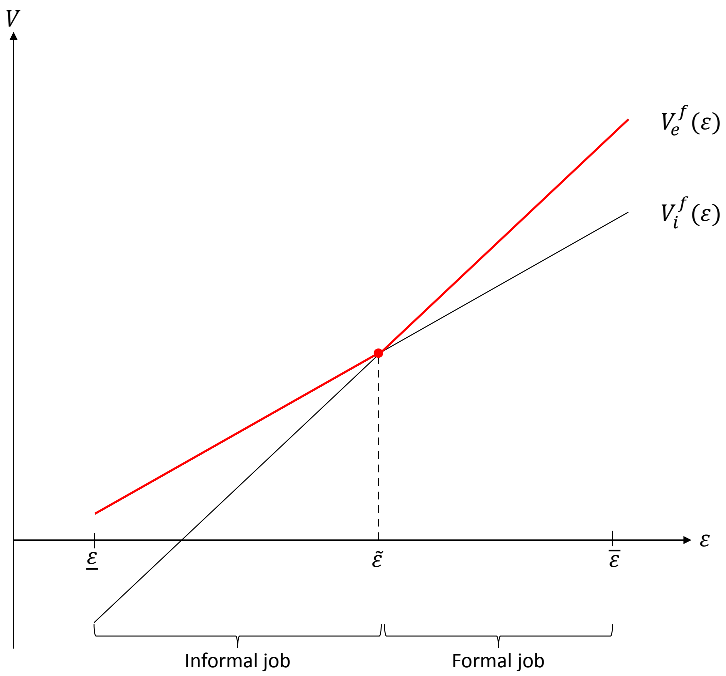



Figure 3: Productivity threshold and hiring decisions.

Formally, $\tilde{\varepsilon}$ is the productivity level for which the matched firm and worker are indifferent between an informal and a formal work contract such that

$$V_e^f(\tilde{\varepsilon}) = V_i^f(\tilde{\varepsilon}) \tag{13}$$

where

$$V_e^f(\varepsilon) = \frac{(1-\omega)\left(y+\varepsilon+\sigma_f\eta(g(q_c)-c(q_c))+\sigma_f(1-\eta)(g(q_m)-c(q_m))-\tau-b\right)-\omega\theta k}{1-\beta(1-\delta_e)}$$

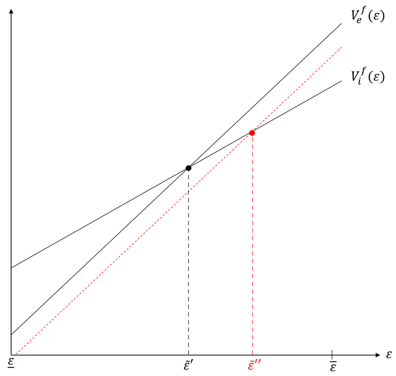
$$V_i^f(\varepsilon) = \frac{(1-\omega)\left(y+\varepsilon+\sigma_f(g(q_m)-c(q_m))-b\right)-\omega\theta k}{1-\beta(1-\delta_i)}.$$

From equation (13) I get the expression

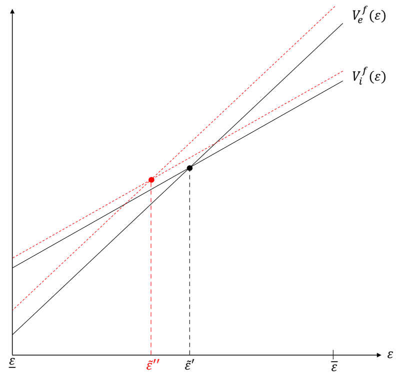
$$\tilde{\varepsilon} = b+\frac{\omega}{1-\omega}\theta k-y-\sigma_f(g(q_m)-c(q_m))-\frac{1-\beta(1-\delta_i)}{\beta(\delta_i-\delta_e)}\left[\sigma_f\eta((g(q_c)-c(q_c))-(g(q_m)-c(q_m)))-\tau\right] \tag{14}$$

which depicts the productivity threshold $\tilde{\varepsilon}$ as a function of $\theta$, $u$, $q_m$, $q_c$ and the model parameters.

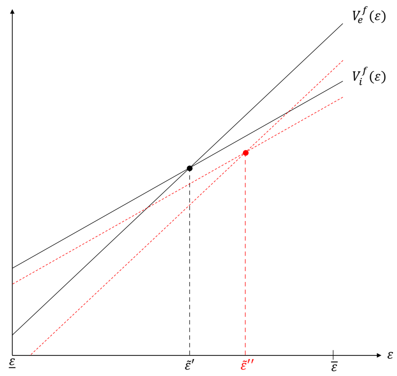
**Lemma 2** *The productivity threshold $\tilde{\varepsilon}$ is increasing in taxes, entry costs and unemployment* *benefits and decreasing in aggregate productivity.*



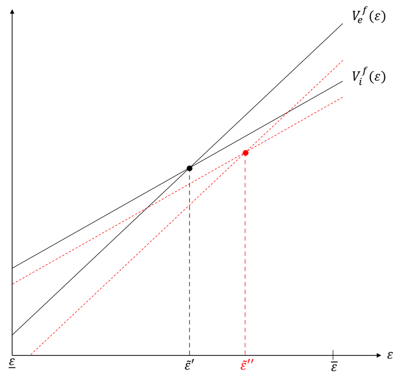

(a) Increase in $\tau$.




(b) Increase in $y$.




(c) Increase in $k$.




(d) Increase in $b$.

Figure 4: Effect of policy parameters on $\tilde{\varepsilon}$.

The proof is presented in appendix B.2. These results are depicted in figure 4. An increase in taxes, everything else being equal, reduces the profit from a formal job which shifts $V_e^f$ downwards and increases $\tilde{\varepsilon}$. An increase in the aggregate productivity $y$ shifts upwards $V_e^f$ by $1/(1-\beta(1-\delta_e))$ and $V_i^f$ by $1/(1-\beta(1-\delta_i))$. Since $\delta_i \geq \delta_e$, the increase in $V_i^f$ is lower and $\tilde{\varepsilon}$ will decrease. The intuition behind this is straightforward: Informal jobs have a shorter expected duration which means an increase in productivity will translate into a lower present discounted value of profits compared to formal jobs and hence firms will shift their hiring decision from informal to formal employment. An increase in $k$ makes the outside option of firms less attractive which improves workers bargaining power and puts upward pressure on wages. This in turn reduces firms profits. However the loss is smaller for informal jobs with a shorter duration which means $V_i^f$ shifts downward less than $V_e^f$ and hence $\tilde{\varepsilon}$ increases. The same logic applies to an increase in the unemployment flow value $b$ which increases the outside option of workers and pushes wages upwards. Informal jobs again are less affected because of their shorter duration.

### 4.4 Steady state equilibrium

Given $\tilde{\varepsilon}$ and $\theta$, the measures of formal employment $n_e$, informal employment $n_i$ and unemployment $u$ evolve according to

$$\begin{aligned} u' &= (1-\alpha_h(\theta))u + \delta_e n_e + \delta_i n_i; \\ n_e' &= (1-\delta_e)n_e + \alpha_h(\theta)(1-F(\tilde{\varepsilon}))u; \\ n_i' &= (1-\delta_i)n_i + \alpha_h(\theta)F(\tilde{\varepsilon})u, \end{aligned}$$

subject to the condition $u + n_e + n_i = 1$.

The steady state of the model implies a constant real value of money supply

$$\phi M = \phi' M'$$

and equal in and out-flows in the labor market such that

$$u = \frac{\delta_e n_e + \delta_i n_i}{\alpha_h}; \quad n_e = \frac{\alpha_h(\theta)(1-F(\tilde{\varepsilon}))u}{\delta_e}; \quad n_i = \frac{\alpha_h(\theta)F(\tilde{\varepsilon})u}{\delta_i}.$$

From the system of equations above, I solve for $u$ as a function of $\theta$ and $\tilde{\varepsilon}$:

$$u = \frac{\delta_e + \delta_i \rho(\tilde{\varepsilon})}{(1+\rho(\tilde{\varepsilon}))\alpha_h(\theta) + \delta_e + \delta_i \rho(\tilde{\varepsilon})} \tag{15}$$

where

$$\rho(\tilde{\varepsilon}) \equiv \frac{n_i}{n_e} = \frac{\delta_e}{\delta_i}\frac{F(\tilde{\varepsilon})}{1-F(\tilde{\varepsilon})} \tag{16}$$

is the steady state ratio of informal to formal employment. $\rho(\tilde{\varepsilon})$ is a function of the job separation rates, the match productivity distribution $F$ and the informality threshold $\tilde{\varepsilon}$.

Using $\tilde{\varepsilon}$ I restate equation (2) as the job creation (JC) condition

$$\frac{k}{\beta \alpha_f(\theta)} = \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} V_e^f(\varepsilon)\, \mathrm{d}F(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} V_i^f(\varepsilon)\, \mathrm{d}F(\varepsilon). \tag{17}$$

which determines firms' entry and the LM tightness $\theta$ and where $\tilde{\varepsilon}$ satisfies the productivity threshold condition (13).

Finally I use $\rho(\tilde{\varepsilon})$ and the definitions

$$\sigma_e = \sigma_h \frac{n_e}{n_e + n_i} \quad \text{and} \quad \sigma_i = \sigma_h \frac{n_i}{n_e + n_i}$$

to rewrite equation (7) as

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_h(u)\left(1 - \frac{\eta}{1+\rho(\tilde{\varepsilon})}\right)} + 1 \tag{18}$$

where $\rho(\tilde{\varepsilon})$ is given by (16) and $\sigma_h = \mathcal{N}(1, 1-u)$ and hence is a function of both $\theta$ and $\tilde{\varepsilon}$ through the BC (15).

In what follows I call equation (18) the Money Demand (MD) equation because it determines the demand for real balances $z$ and the quantity traded $q_m$ as a function of the nominal interest rate $i$, the availability of credit $\eta$ and the level and composition of employment.[10]

Finally, under the assumption that each household holds a share of a market portfolio composed of all active firms in the economy, the equilibrium dividend income $\Delta$ is equal to aggregate profits $\Pi$:

$$\Pi = n_e \int_{\varepsilon_e}^{\bar{\varepsilon}} R_e(\varepsilon) - w_e(\varepsilon) - \tau \; \mathrm{d}F(\varepsilon) + n_i \int_{\varepsilon_i}^{\varepsilon_e} R_i(\varepsilon) - w_i(\varepsilon) \; \mathrm{d}F(\varepsilon) - u\theta k.$$

where the last term on the right hand side represents the cost of posting vacancies.

The steady state equilibrium of this economy can be defined as follows:

**Definition 1** *A steady-state equilibrium in this economy is defined as: (i) a productivity threshold* $\tilde{\varepsilon}$*, (i) a level of LM tightness* $\theta$*, (iii) a level of unemployment* $u$ *, (iv) and quantities* $\{q_m, q_c\}$ *traded in the DM, which together satisfy*

- *Optimal consumption in credit and monetary DM matches* (6) *and* (18)*;*
- *The Job Creation equation* (17)*;*
- *The informality threshold equation* (13)*;*
- *The Beveridge curve* (15)*.*

[10] In this model, as in Mortensen and Pissarides (1994), talking about firms or employment is the same since each firm employs a single employee. Because the production function exhibits constant returns to scale the number of workers per firm is irrelevant for the results.

### 4.5 Equilibrium properties

As usual in monetary models, equilibrium is not unique. Since fiat money has no fundamental value and its liquidity value depends on expectations of agents, there is always a non-monetary equilibrium where agents don't value money because they expect others not to do so. There could also be multiple monetary equilibria because of the strategic complementarity between the entry decision of firms and the money holdings of households.

Given DM quantities $\{q_c, q_m\}$ and a productivity threshold $\tilde{\varepsilon}$, the JC equation (17) results in a unique $\theta$ as stated in the following proposition:

**Lemma 3** *In the JC equation* (17)*, the cost of entry (LHS) is increasing in* $\theta$ *while the profits from entry (RHS) are decreasing in* $\theta$*. Hence there is a unique steady state* $\theta$ *that satisfies equation* (17) *for a given* $\{\tilde{\varepsilon}, u, q_c, q_m\}$*.*

Lemma 3 is a standard result in labor search models. Its proof is shown in appendix B.3. The intuition behind it is the following: The cost of entry is increasing in $\theta$ since posted vacancies take a longer time to be filled when $\theta$ increases. This results from the congestion externality of the LM's matching function. The profits from entry are decreasing in $\theta$ since a tighter market increases the outside option of workers which pushes wages higher and reduces the net expected profit for firms. This is the thick market externality of the LM matching function. As a consequence, there is a unique $\theta$ that satisfies equation (17) for a given $\{q_c, q_m, \tilde{\varepsilon}\}$. Next I turn to equilibrium in the DM.

**Lemma 4** *Given* $\{\tilde{\varepsilon}, \theta, u, q_c\}$*, there is a unique equilibrium in the DM. In particular, there is a unique quantity consumed in monetary matches* $q_m$ *that satisfies equation* (18)*.*

The proof is presented in appendix B.4.

Figure 5 depicts the MD and JC curves in the $(\theta, q_m)$ space. The JC curve results from equation (17) and depicts $\theta$ as a function of $q_m$. The JC curve is increasing in $q_m$ as the more real balances firms expect households to bring to the DM the more output and expected profits will be and the more firms will enter, hence a higher $\theta$. The concavity of the JC curve reflects the decreasing marginal return of the matching function. The MD curve results from equation (18) and depicts $q_m$ as an increasing function of $\theta$. The transmission from LM to DM works here through the DM matching probability. Increasing $\theta$ increases the frequency of meeting a

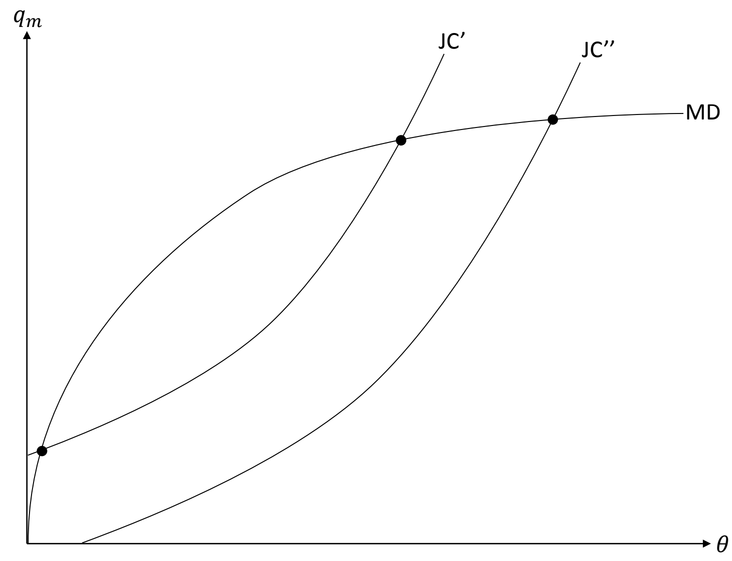
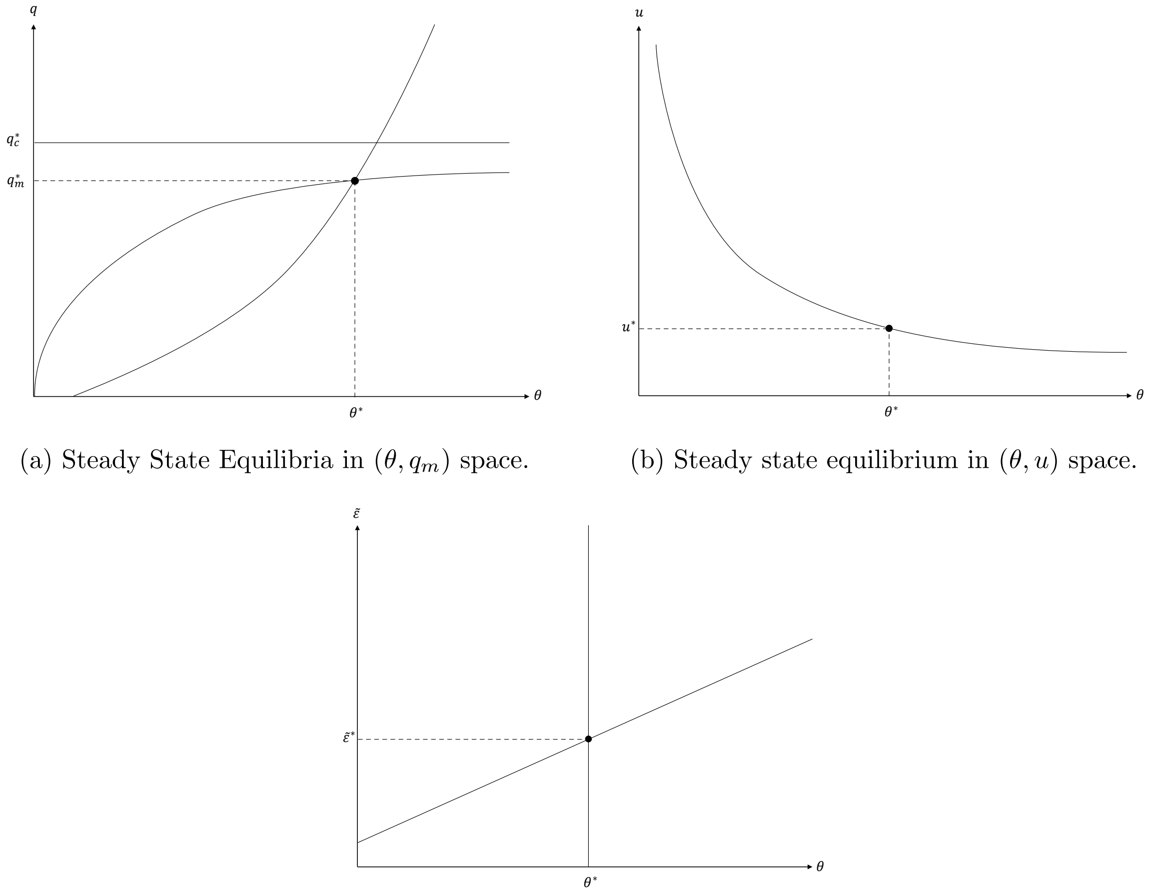



Figure 5: Steady State Equilibria in $(\theta, q_m)$ space

firm in the DM and hence the return on holding money; money will be put to use with a higher probability. As a consequence, as $\theta$ increases $q_m$ increases as well. Again, the shape of the DM curve reflects the concavity of the DM matching function.

Depending on where the two curves intersect, there could be one or two monetary equilibria. For example, the curves JC' and MD intersect in two points: a high and a low monetary equilibrium. In the high equilibrium, households expect more frequent trading in the DM because of a higher level of firms entry. For that reason, households bring a higher amount of money holdings and end up consuming a higher quantity of the DM good. At the same time, firms expect a high level of demand in the DM and hence higher trading surplus which results in higher firms entry and job creation. The intuition is reversed for the low equilibrium. In the presence of credit, which is the case here, the JC curve will intersect with the horizontal axis instead of the vertical one. This is because even when $q_m = 0$, some firms still enter the LM to earn the expected profit from credit matches.

Figure 6 depicts the general equilibrium of the model as the fixed point resulting from the intersection of curves representing the equilibrium conditions. The intersection of the JC and MD curves in panel (a) returns the values of $q_m$ and $\theta$. $q_c$ is determined independently from other variables. Through the Beveridge curve (BC) in panel (b) $\theta$ determines the level of unemployment $u$. As will be discussed below, the shape of the BC is also determined by $\tilde{\varepsilon}$. Finally, in panel (c) the JC equation (17) intersects with the informality threshold equation (13) at the equilibrium levels of $\theta$ and $\tilde{\varepsilon}$.




(a) Steady State Equilibria in $(\theta, q_m)$ space.

(b) Steady state equilibrium in $(\theta, u)$ space.

(c) Steady state equilibrium in $(\theta, \tilde{\varepsilon})$ space.

Figure 6: Steady state equilibrium.

# 5 Theoretical results and comparative statics

The model presented in the previous section offers a relatively simple and tractable framework to think about money, inflation and the labor market in the presence of informality. Although the model is rich enough to discuss issues of fiscal policy or labor market institutions, I focus here on monetary policy and in particular on the effect of informality on the long-run relationship between inflation and unemployment.

### 5.1 Informality and money demand

The MD equation (18) is an extension of the standard equation present in most monetary models and in particular those with search and matching frictions (Lagos and Wright, 2005; Berentsen et al., 2011) to economies with an informal sector. Depending on parameter values, three interesting cases arise:

**Case 1: Pure monetary equilibria.** This case occurs when either $\eta = 0$ or $\tilde{\varepsilon} \to \bar{\varepsilon}$ (or both) holds. $\eta = 0$ corresponds to equilibria where credit is not available in the formal sector while $\tilde{\varepsilon} \to \bar{\varepsilon}$ corresponds to equilibria where all firms are informal and again credit is not available. For both parameter values, the MD equation collapses to

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_h(u)} + 1$$

where $q_m$ depends only on unemployment and the nominal interest rate. This case corresponds to Berentsen et al. (2011). An increase in $i$ increases the opportunity cost of holding money and hence reduces $z$, the amount of real balances households carry across periods. An increase in $u$ decreases the DM matching probability $\sigma_h$ for households which lowers the return on money and real balances $z$. A decrease in $z$ results in a lower quantity exchanged in DM monetary matches $q_m$.

**Case 2: Pure credit equilibria.** Away from the Friedman rule, this case occurs when $\eta \to 1$ and $\tilde{\varepsilon} \to \underline{\varepsilon}$; i.e. all DM formal transactions use credit and there is no informal sector. It follows that $q_m \to 0$. The demand for money goes to zero as all transactions are conducted using credit. Because credit is costless in this economy, the resulting allocation in the DM is socially efficient.

**Case 3: Mixed monetary equilibria.** This case is about equilibria where credit and money coexist. This happens for values in the parameter space such that $\tilde{\varepsilon} \in (\underline{\varepsilon}, \bar{\varepsilon})$ and $\eta > 0$ or when $\eta \in (0, 1)$ irrespective of $\tilde{\varepsilon}$. The first parametrization corresponds to the coexistence of the formal and informal sectors with the former having some or all transactions in credit. The second corresponds to economies with some but not all formal transactions in credit and where informality might or might not exist. In this case money demand is given by equation (18). In Mixed Monetary Equilibria with informality, the following result holds:

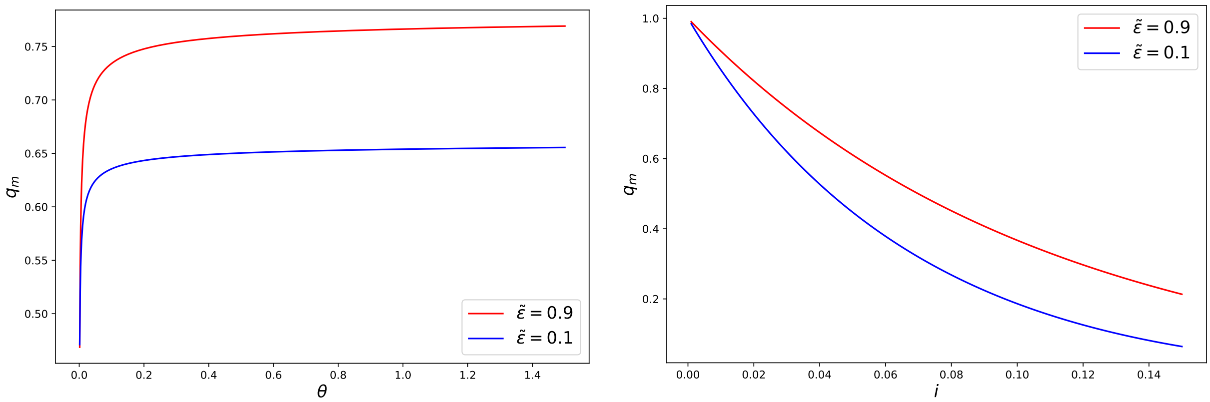



Figure 7: Quantity traded in DM monetary matches and informality.

**Result 1** *Given* $\{\theta, u, q_c\}$*, when credit is available in the formal sector ;i.e.* $\eta > 0$*, an increase in informality increases the demand for money.*

This can be easily seen from equation (18). When $\eta > 0$, an increase in $\rho(\tilde{\varepsilon})$ increases $q_m$ for all levels of $i$. This is simply because an increase in the steady state ratio of informal to formal firms in the DM will increase the frequency of monetary matches for buyers. The latter will respond by carrying more real balances to the DM and hence increase $q_m$ for all levels of $i > 0$. Both panels of figure 7 depict an upward shift in the demand for money when the level of informality increases $\tilde{\varepsilon}$.

## 5.2 Informality and the Beveridge curve

Does the level of informality matter for unemployment? The standard textbook Beveridge Curve (BC) describes the long-run steady state unemployment level as a function of firms entry $\theta$ and the exogenous separation rate (Pissarides, 2000). Firms entry affects unemployment through the number of matches created which is an increasing function of $\theta$. The more firms enter the labor market and post vacancies the higher the number of jobs created and the lower unemployment. In the presence of an informal sector, steady state unemployment not only depends on firms entry decision but also on their hiring decision which determines the relative size of the informal sector. Equation (15) depicts the "informality-augmented" BC as a function of both $\theta$ and the ratio of informal to formal employment $\rho(\tilde{\varepsilon})$ defined by equation (16). However, this dependence holds only when the two types of jobs exhibit different separation rates. By setting $\delta_e = \delta_i = \delta$

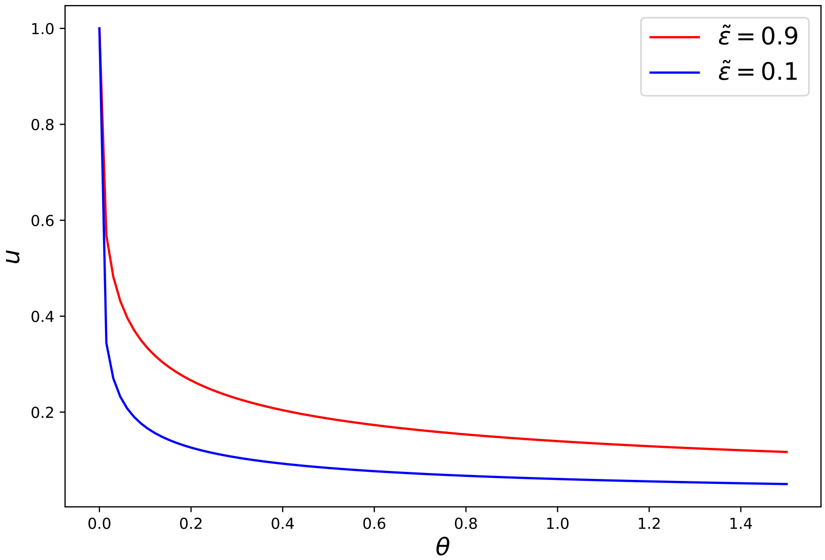



Figure 8: The Beveridge curve for high and low informality.

in equation (15), we revert back to the standard BC

$$u = \frac{\delta}{\alpha_h(\theta) + \delta}$$

where unemployment depends only on $\theta$. When the job separation rates between formal and informal jobs are different, the labor steady state allocation between the two types of jobs matters for the steady state level of unemployment. In particular, we have the following result:

**Result 2** *Given $\{\theta, q_m, q_c\}$, an exogenous increase in informality increases unemployment when $\delta_i > \delta_e$.*

This result follows directly from the BC equation (15) which implies $\frac{\partial u}{\partial \tilde{\varepsilon}} > 0$. Since informal jobs have a higher separation rate; i.e. $\delta_i > \delta_e$, an increase in the ratio of informal to formal jobs $\rho(\tilde{\varepsilon})$, keeping $\theta$ constant, changes the composition of employment from formal jobs to informal jobs and hence increases the steady state unemployment $u$ as seen in figure 8.

### 5.3 Informality and the LRPC

Now I turn to the main question of the paper: how does the presence of informality affect the LRPC? In this model, an increase in inflation affects unemployment by reducing the quantities traded in the DM, which works through the MD equation (18). This is because higher inflation reduces the return on holding money which induces households to carry less real balances across periods. Lower real balances result in a lower $q_m$ thus reducing expected profits for firms.[11] Lower expected profits mean fewer firms entering the LM, fewer jobs created and hence higher unemployment. Because the matching probability in the DM $\sigma_h(\theta)$ depends on firms entry, less active firms in the DM further reduces the return on money amplifying the effects of inflation on unemployment.

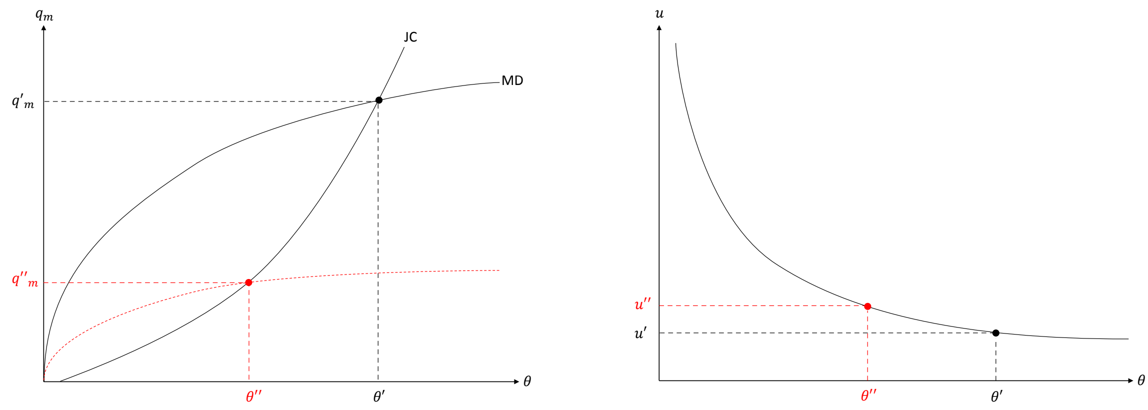



Figure 9: Effect of an increase in $i$ on unemployment without informality.

**Result 3** *Keeping constant $\tilde{\varepsilon}$, an increase in inflation lowers the entry of firms, LM tightness $\theta$ and increases unemployment.*

The first panel in figure 9 depicts the impact of higher inflation in $(\theta, q_m)$ space. An increase in $i$ translates into lower $q_m$ for all levels of $\theta$ resulting into a downward shift of the MD curve. Since $i$ does not enter directly equation (17), the JC curve does not move and the two curves will cross at a lower point $(\theta'', q''_m)$ as shown in figure 9. A lower $\theta$, everything else being equal, results in a higher level of unemployment through the BC equation (15) as illustrated in the second panel of figure 9. This real balances channel of the LRPC is the same as in Berentsen et al. (2011).

In the presence of informality, a second effect is at work. As long as some formal firms are able to offer credit in the DM i.e. $\eta > 0$, a higher inflation tax will affect less the expected profits of formal firms compared to informal ones which will shift job creation from informal to formal sector. This is illustrated in figure 10 where an increase in $i$ shifts both $V_e^f$ and $V_i^f$ downward.

[11] The size of the effect will depend on the trading protocol used to share the DM surplus (Rocheteau and Wright, 2005; Aruoba et al., 2007; Craig and Rocheteau, 2008).

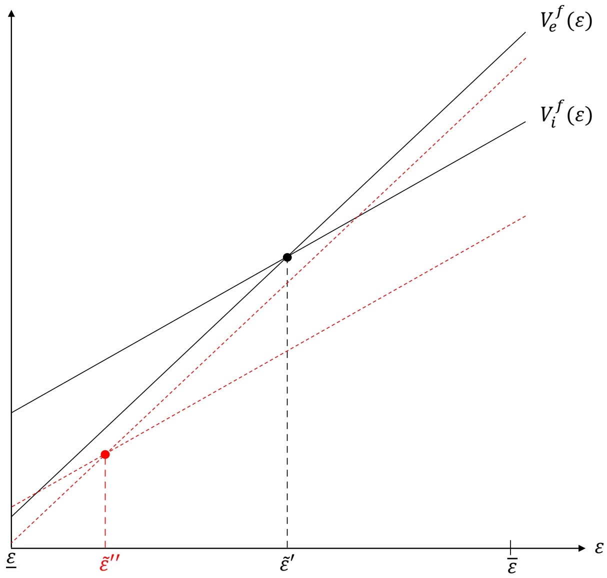



Figure 10: Effect of an increase in $i$ on $\tilde{\varepsilon}$

Since $\frac{\partial q_c}{\partial i} = 0$ from equation (6) and $\frac{\partial q_m}{\partial i} < 0$ from equation (18), it follows that

$$\frac{\partial V_e^f(\varepsilon)}{\partial i} < \frac{\partial V_i^f(\varepsilon)}{\partial i}$$

for all productivity levels $\varepsilon$. In figure 10, this translates into $V_i^f(\varepsilon)$ shifting downward more than $V_e^f(\varepsilon)$ which automatically shifts $\tilde{\varepsilon}$ to the left and lowers $\rho(\tilde{\varepsilon})$, the ratio of informal to formal firms. And we know from result 2 that a decrease in $\rho(\tilde{\varepsilon})$ will result in a downward shift in the BC when $\delta_i > \delta_e$.

**Result 4** *An increase in inflation reduces informality which shifts the Beveridge Curve downward.*

To understand the general equilibrium effect of inflation on unemployment in the presence of informality, we put together the results stated above. This is illustrated in figure 11. The increase in $i$, and the resulting downward shift in the MD curve, lowers LM tightness from point $\theta'$ to $\theta''$. Unemployment level $u'''$ is the level that would occur without a change in the Beveridge Curve. The downward shift in the Beveridge Curve results in an equilibrium level of unemployment $u'' < u'''$, but still higher than the initial level $u'$.

**Result 5** *When $\delta_i > \delta_e$ and $\eta > 0$, the presence of informality dampens the effect of a change in inflation and the nominal interest rate on employment and output.*

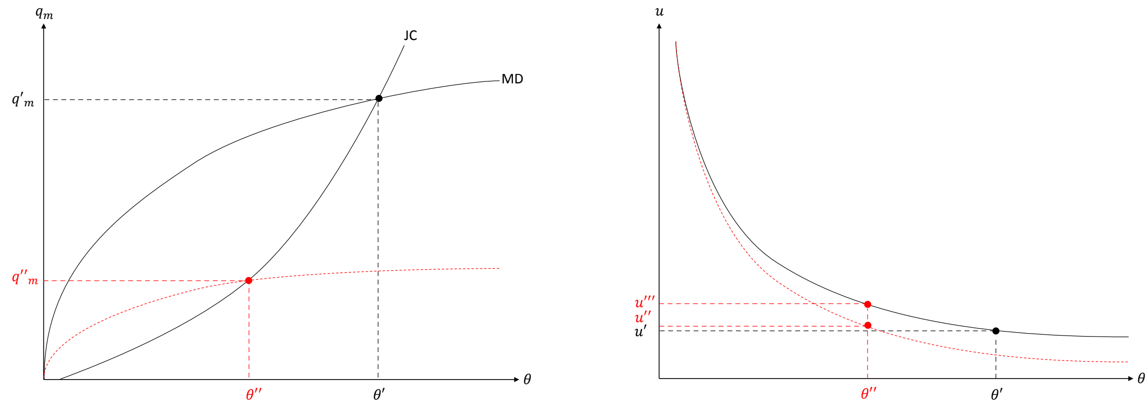



Figure 11: Effect of an increase in $i$ on unemployment with informality.

# 6 Quantitative analysis

To further explore the general equilibrium effect of informality on the LRPC, I calibrate the model to the Brazilian economy and present some numerical results.

## 6.1 Calibration

The model is calibrated using Brazilian data. Brazil has a sizable informal sector, close to the average level in Latin American countries (Perry et al., 2007), and relatively good statistics about informality.[12] Each period in the model corresponds to a quarter. Limited data availability restricts the sample to the period from 1996q1 to 2014q4.

Figure 12 depicts scatter plots of the nominal interest rate and unemployment. I gradually filter the data by removing high frequency fluctuations by applying a stronger HP filter. The data shows clearly a positive relationship between these variables at low frequency in line with the model's implications.

I choose the following functional forms: the utility function in the DM is $v(q) = Aq^{1-a}/(1-a)$. Utility in the CM is linear $U(x) = x$. Firms' cost function in the DM has the form: $c(q) = q$ such that the marginal cost is the same as in the CM. In the absence of data on the productivity distribution of firms, I use a uniform distribution with support $[0, 1]$ for the match productivity $\varepsilon$. The matching functions in the LM and DM are described by $\mathcal{M}(u, v) = \xi u^{1-\sigma} v^{\sigma}$ and $\mathcal{N}(B, S) = BS/(B + S)$ respectively. As argued by Lazaryan and Lubik (2017) and others, in discrete time the LM matching probabilities $\alpha_h$ and $\alpha_f$ can take values above one. Intuitively,

[12] The Brazilian Labor Force Survey covers both formal and informal employment. See for example the discussion in Gerard and Gonzaga (2016).

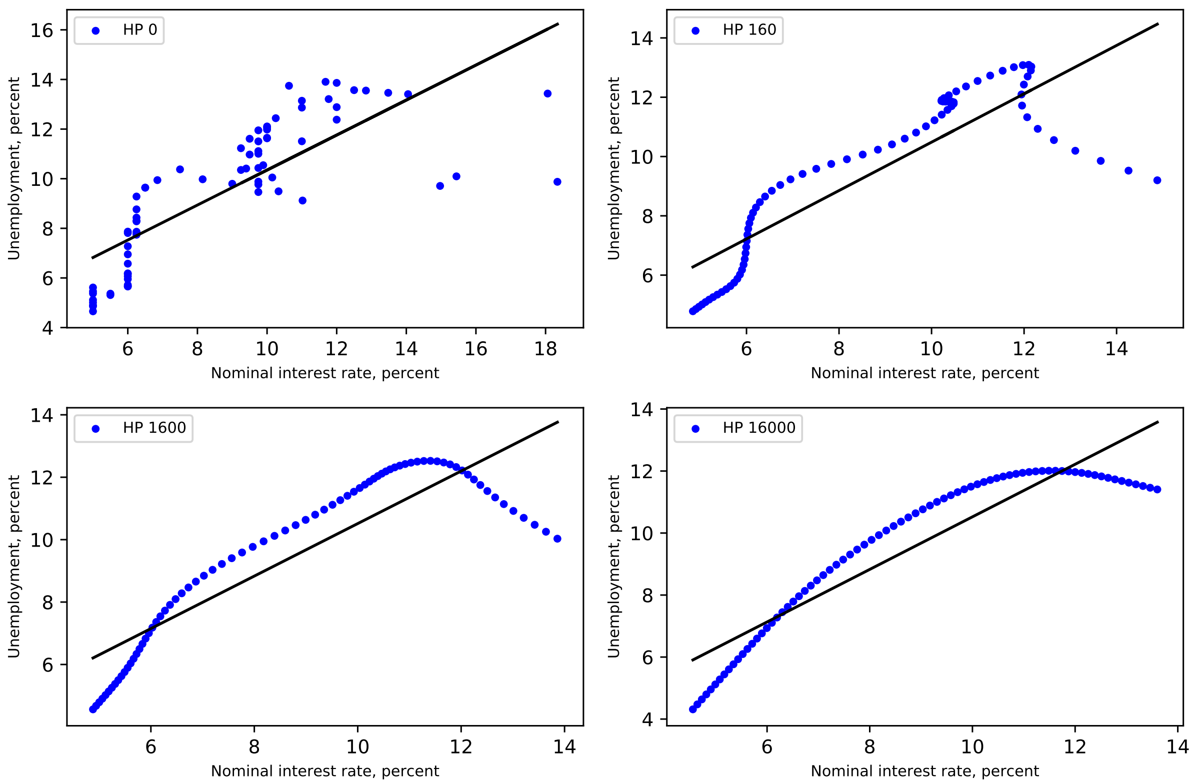



Figure 12: Nominal interest rates and unemployment in Brazil.

if the time period is long enough, everyone can exit unemployment at least once. This violates the idea that the matching in the labor market is probabilistic. Restricting the probabilities to lie on the unit interval requires

$$\left(\frac{1}{\xi}\right)^{1/(\sigma-1)} < \theta < \left(\frac{1}{\xi}\right)^{1/(1-\sigma)}$$

which is a non-empty interval for $\theta$ when $\xi \in (0, 1)$.

I separate the model's parameters into two groups: independent parameters and jointly calibrated parameters. The first group consists of parameters which I directly set to a specific value while the second group is jointly calibrated with the equilibrium solution to match certain targets from the data.

The chosen values for the first group of parameters are listed in table 1. $\beta$ is set such that the real interest in the model matches the average difference between the risk-free nominal interest rate and the rate of inflation in the data.

As discussed by Ljungqvist and Sargent (2017), the calibration of the flow value of unemployment $b$, which captures the value of leisure, unemployment benefits and home production,

Table 1: Independent parameters

| Parameter | Description | Value | Source |
|---|---|---|---|
| $\beta$ | Discount factor | 0.99 | Quarterly data, 1996-2014 |
| $i$ | Nominal interest rate | 0.02 | Quarterly data, 1996-2014 |
| $y$ | General productivity level | 1.00 | Normalization |
| $b$ | Labor disutility | 0.40 | Shimer (2005) |
| $\eta$ | Credit availability in formal DM matches | 0.20 | % Credit card in retail transactions |
| $\delta_e$ | Formal jobs separation rate | 0.03 | Bosch and Esteban-Pretel (2012) |
| $\delta_i$ | Informal jobs separation rate | 0.10 | Bosch and Esteban-Pretel (2012) |
| $\sigma$ | Elasticity of LM matching function | 0.50 | Petrongolo and Pissarides (2001) |
| $\omega$ | LM workers bargaining power | 0.50 | Hosios condition |

is very important for the quantitative performance of the model as it determines the elasticity of the matching function to changes in productivity or other aspects that affect the return on job creation. This is because $b$ determines, mostly, the fraction of resources that the market can allocate to vacancy creation, what the authors call the fundamental surplus fraction. There are several possibilities when it comes to its calibration of the flow value of unemployment $b$ which captures the value of leisure, unemployment benefits and home production. The standard one is to target a given percentage of the average productivity which corresponds in this model to the average productivity in both formal and informal matches. Shimer (2005) sets the value to 40%, Hall and Milgrom (2008) choose 0.71 while Hagedorn and Manovskii (2008) argue for a value of around 96%. Another way is to define $b$ as unemployment insurance benefits and calibrate it such that the average replacement ratio in the model matches the empirical replacement rate. In Brazil, the benefit level ranges between 100% to 187% of the minimum wage which means that the replacement rate is very high for formal workers at the bottom of the wage distribution (Gerard and Gonzaga, 2016). These benefits are only received by unemployed workers and informal workers who were formally employed. Taking this into account will complicate the calibration without much added value. To keep things relatively simple I calibrate $b$ at 40% of the lowest productivity level such that

$$\frac{b}{y + \underline{\varepsilon}} = 0.4.$$

The elasticity of the LM matching $\sigma$ is set to 0.5 as is standard in the literature. The LM bargaining power of workers $\omega$ is set equal to $\sigma$ to satisfy the Hosios efficiency condition. I set $\delta_e$ and $\delta_i$ to match the observed quarterly job separation rates of formal and informal jobs of 3% and 10% respectively (Bosch and Esteban-Pretel, 2012). The share of credit formal matches

$\eta$ is set at 20% which corresponds roughly to the average share of credit card transactions in total consumer spending transactions in Brazil[13].

Once all the independent parameters are set I proceed to jointly calibrate the second group of parameters. This group comprises utility function parameters $A$ and $a$, DM bargaining power $\varphi$, the matching function efficiency $\xi$, the cost of posting vacancies $k$ and the lump-sum tax on formal firms $\tau$. The joint calibration procedure consists in solving for the vector of calibrated parameters $\mathcal{P} = \{A, a, \varphi, k, \xi, \tau\}$ and the equilibrium solution $X = \{\tilde{\varepsilon}, \theta, u, q_m, q_c\}$ which together reduce the distance (squared percentage difference) between the targeted moments $S_{data}$ and the corresponding theoretical moments $S_{model}$ subject to the system of steady state equilibrium equations (6), (14), (15), (17) and (18) as equality constraints and imposing interval bounds $\mathcal{I}$ on the value of some parameters:

$$\min_{\mathcal{P},X} \left(\mathcal{S}_{model}(X;\mathcal{P}) - \mathcal{S}_{data}\right)^2$$
$$\text{s. t.} \quad \mathbf{EC}(X;\mathcal{P}) = 0 \quad ; \quad \mathcal{P} \in \mathcal{I}$$

To specify the theoretical moments to match, I first define how the model maps into data. In terms of output I have

$$\begin{aligned}
Y_{e_{DM}} &= n_e \sigma_f \left[\eta(g(q_c) - c(q_c)) + (1-\eta)(g(q_m) - c(q_m))\right];\\
Y_{i_{DM}} &= n_i \sigma_f (g(q_m) - c(q_m));\\
Y_{e_{CM}} &= n_e \left(y + \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \varepsilon \, \frac{\mathrm{d}F(\varepsilon)}{F(\bar{\varepsilon}) - F(\tilde{\varepsilon})}\right) - (1 - F(\tilde{\varepsilon}))u\theta k;\\
Y_{i_{CM}} &= n_i \left(y + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon \, \frac{\mathrm{d}F(\varepsilon)}{F(\tilde{\varepsilon}) - F(\underline{\varepsilon})}\right) - F(\tilde{\varepsilon})u\theta k,
\end{aligned}$$

where $Y_{e_{DM}}$ and $Y_{i_{DM}}$ are the net aggregate output of formal and informal firms sold in the DM and $Y_{e_{CM}}$ and $Y_{i_{CM}}$ are their net aggregate output sold in the CM respectively. Depending on the country, the real GDP in the data might or might not take into account informal activities (Andrews et al., 2011). In some countries, GDP accounts only for formal activities which is the case for example in the US. In some others, GDP includes formal and some of the informal activities. This is increasingly the case in EU countries. For the purposes of calibration I assume that a share of informal activities is included in Brazil's GDP data. This allows me to define

[13] The figures are readily available at the website of the Associação Brasileira das Empresas de Cartões de Crédito e Serviços.

recorded activities in the model as the total of formal firms output sold in both DM and CM and informal firms' output sold in the CM. $M$ is defined in the model as the amount of cash carried by households to be spent in both formal and informal monetary transactions in the DM. In the data $M$ corresponds to either M1 or sweep-adjusted M1 (Aruoba et al., 2011). As a consequence, I define the size of the informal sector as a share of observed GDP as

$$\frac{Y_i}{Y} = \frac{Y_{i_{DM}} + Y_{i_{CM}}}{Y_{e_{DM}} + Y_{e_{CM}} + Y_{i_{CM}}},$$

and the model-based money demand equation as

$$L(i) = \frac{g(q_m)}{Y_{e_{DM}} + Y_{e_{CM}} + Y_{i_{CM}}},$$

where $L(i)$ depends on $i$ directly through real balances $g(q_m)$ and indirectly through the different GDP measures.

Following Lucas (2000) and Lagos and Wright (2005), $A$ and $a$ are included in the joint-calibration in order to fit the model-based money demand function $L(i) \equiv \frac{M}{pY}$ to the data. The idea is to match two moments: the average real money balances at the average nominal interest rate and the elasticity of money demand to the nominal interest rate.[14] To estimate the interest elasticity of money demand, I use a log-log specification

$$\log M/PY_t = \beta_1 + \beta_2 \log i_t + \nu_t.$$

The OLS estimate of $\beta_2$ is used as a point estimate for the interest elasticity of money demand $\epsilon$:

$$\epsilon = \frac{\partial L(i)}{\partial i} \frac{i}{L(i)}$$

Following Aruoba et al. (2011), I use $\varphi$, the bargaining power of buyers in the DM, to target an average markup in the DM trades of 30%. The markup in monetary transactions is defined as

$$\mu_m = \frac{g(q_m)/q_m}{c'(q_m)} - 1$$

[14] Another way is to solve for the parameter values which minimize the distance between the model-based money demand and the observed money demand for each observed interest rate. However, this procedure is computationally more consuming and doesn't affect much results.

and in credit transactions as

$$\mu_c = \frac{g(q_c)/q_c}{c'(q_c)} - 1.$$

The average markup in DM trades can be written as

$$\mu_{DM} = \frac{n_e(\eta\mu_c + (1-\eta)\mu_m) + n_i\mu_m}{n_e + n_i}.$$

Since the CM is a competitive market, the markup there is 0. Taking the economy wide average, I get

$$\bar{\mu} = \mu_{DM}\frac{Y_{e_{DM}} + Y_{i_{DM}}}{Y}.$$

In addition to the average markup and the money demand moments, I target the unemployment rate and the tax burden on formal firms. The average unemployment rate in Brazil over the period 1996-2014 was around 9.5%. The World Bank's Doing Business reports an average tax rate of 68% of before-tax commercial profits in Brazil in 2016. This measure includes all taxes and mandatory contributions payable by the firm after accounting for allowable deductions and exemptions.[15] In the model, this corresponds to

$$\frac{\tau}{\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} R_e(\varepsilon) - w_e(\varepsilon)\frac{\mathrm{d}F(\varepsilon)}{F(\bar{\varepsilon})-F(\tilde{\varepsilon})}}.$$

Finally, in the absence of data on the tightness of the Brazilian labor market, I calibrate the model such that $\theta$ is normalized to 1. Table 2 summarizes the results of the joint calibration. The model is able to match all of the targets and, in particular, the money demand data as shown in figure 13.

In order to test the validity of the model I compare its calibrated equilibrium solution with empirical statistics not included as targets in the calibration exercise. In table 3, I report the resulting values for the informal sector and informal employment. This relatively simple model is able to explain around half of the reduced-form estimates of the size of the informal sector in Brazil reported in the literature. Notably Medina and Schneider (2017) report an estimated average size of the informal sector in Brazil of 37.63% with a minimum of 32.56% and a maximum of 41.69% over the period 1991-2015 using the MIMIC (multiple indicators-multiple

[15]These taxes include the corporate income tax, all social contributions and labor taxes paid by the employer, property taxes, turnover taxes and other taxes. However, this measure excludes the value-added tax (VAT) which does not affect formal firms profits but is effectively a tax on formal goods consumption.

Table 2: Calibration results

| | | Data | Model |
|---|---|---|---|
| **Jointly Calibrated parameters** | | | |
| $k$ | Cost of vacancy posting | - | 0.952 |
| $\xi$ | LM matching efficiency | - | 0.421 |
| $\tau$ | Tax on formal firms | - | 0.093 |
| $\varphi$ | Bargaining power in DM | - | 0.420 |
| $A$ | Parameter of DM utility | - | 0.755 |
| $a$ | Parameter of DM utility | - | 0.266 |
| **Calibration targets** | | | |
| $u$ | Unemployment | 0.095 | 0.095 |
| $\theta$ | Normalization of LM tightness | - | 1.0 |
| $\frac{\tau}{\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} R_e(\varepsilon)-w_e(\varepsilon)\mathrm{d}F(\varepsilon)}$ | % of taxes in gross profits | 68% | 68% |
| $\mu_{DM}$ | Average markup in retail sector | 30% | 30% |
| $L(\bar{i})$ | Average Money Demand | 0.217 | 0.217 |
| $\epsilon$ | Elasticity of MD to $i$ | -0.398 | -0.398 |
| Sum of squared residuals | | - | 0.000 |

causes) model, a linear latent variables econometric approach. The size of the DM resulting from the calibration is very close to that found by Aruoba et al. (2011) for the US. The share of credit card payments in the total value of DM transactions is 22%, higher than what's observed in the Brazilian retail sector.

Table 3: Model validation

| | | Data | Model |
|---|---|---|---|
| $n_i$ | Informal employment | est. 30% | 18.2% |
| $\frac{Y_i}{Y_e+Y_i}$ | Informal sector | est. 37.6% | 18.7% |
| $\frac{Y_{DM}}{Y_{DM}+Y_{CM}}$ | Size of DM | - | 2.5% |
| $\ell\%$ | Share of credit in retail sales (value) | 15.0% | 22.0% |

### 6.2 Numerical experiments

In what follows I conduct several numerical experiments by changing the policy parameters in the calibrated steady state equilibrium. In order to make the discussion more interesting, I compare the results of the calibrated baseline model to an economy without informality. The latter is computed by setting $\tilde{\varepsilon}$ to 0 and recalibrating the model to match the same empirical targets as before. Since the only difference between the two models is the presence of an additional margin, comparing the two allows me to isolate the new mechanisms brought about by the presence of informality.

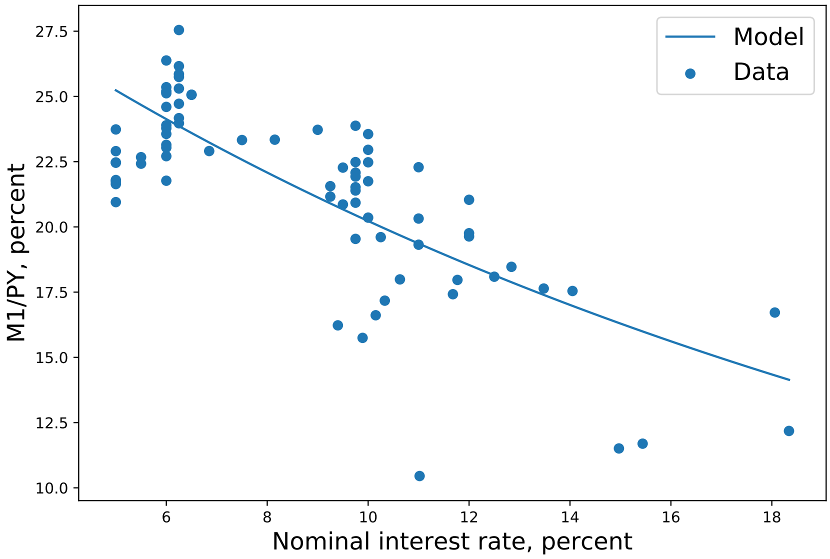
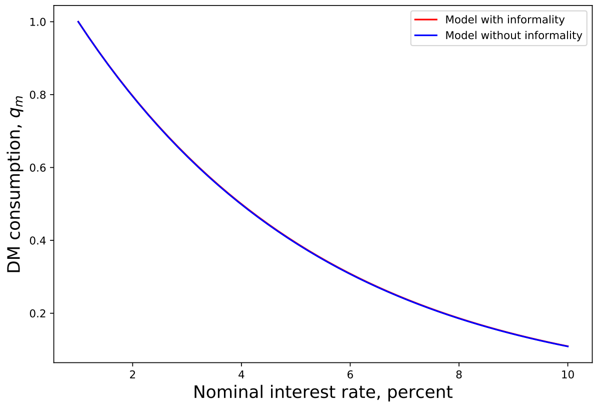



Figure 13: Money demand: model vs. data

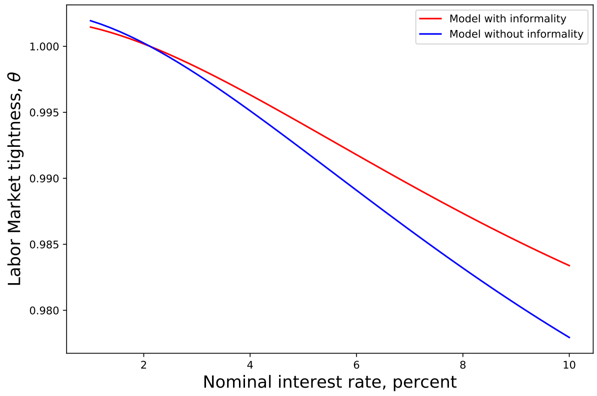
**Effect of changes in inflation:** Figure 14 illustrates the effect of changes in the long run inflation target on some equilibrium quantities. Panel 14d illustrates the main results of the paper. First, the long run Phillips curve is upward sloping in both economies. This is in line with the data as shown in figure 12. Second, the Phillips curve is flatter in the economy with informality compared to the one without informality. This is a result of the reallocation of labor from high separation informal jobs to low separation formal jobs which dampens the effect of changes in inflation and the nominal interest rate on job creation as explained in section 5.3.

The change in $q_m$ is exactly the same in the model with and without informality as seen in panel 14a. This confirms that the long run dampening effect of informality works through shifts in the Beveridge curve.

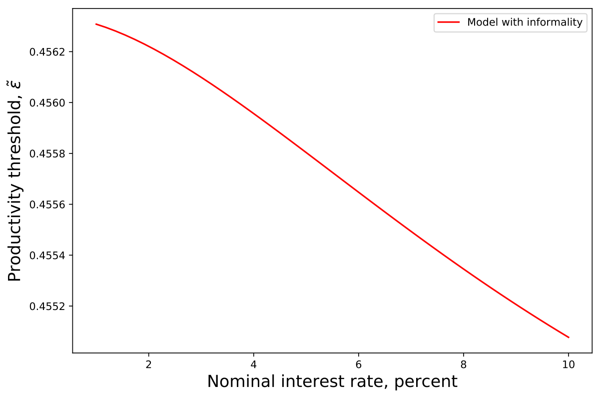
**Effect of changes in taxation:** Now I look at the effect of changes in the tax burden on formal firms on the equilibrium solution. As above, I compare the economy with informality to a similar economy without the presence of an informal sector. The six panels in figure 15 depict the resulting equilibrium allocations. The experiment consists in varying $\tau$ from a tax burden of around 40% to 90% of gross profits of formal firms. In order to facilitate the comparison between the two economies I normalize the results of each economy by the equilibrium values resulting from the first value of $\tau$ on the x-axis. In the model with informality, increasing the tax burden increases money demand and the quantity traded in DM monetary matches while in the model without informality the opposite effect is observed. This is because in the first,



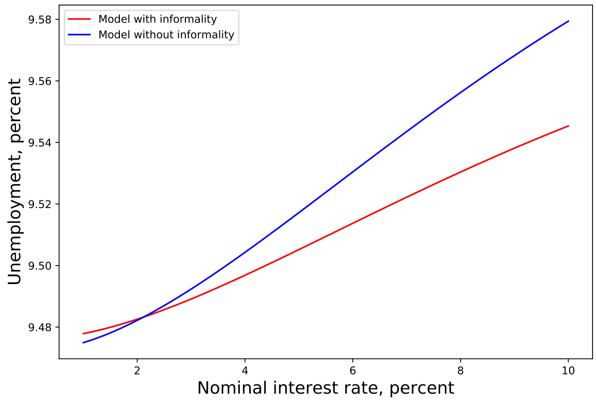

(a) Effect on $q_m$



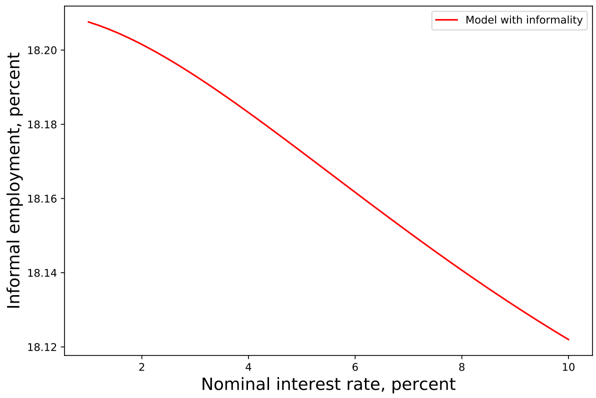

(b) Effect on $\theta$



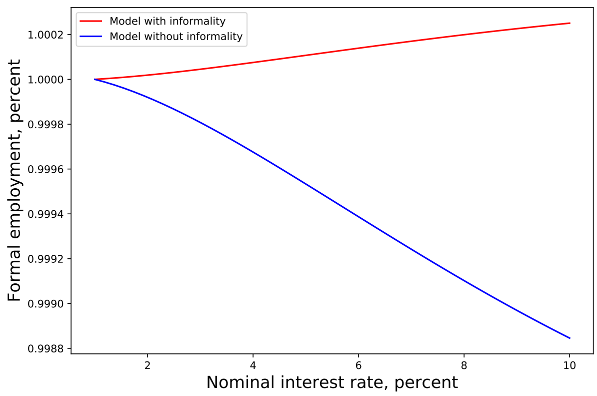

(c) Effect on $\tilde{\varepsilon}$




(d) Effect on $u$



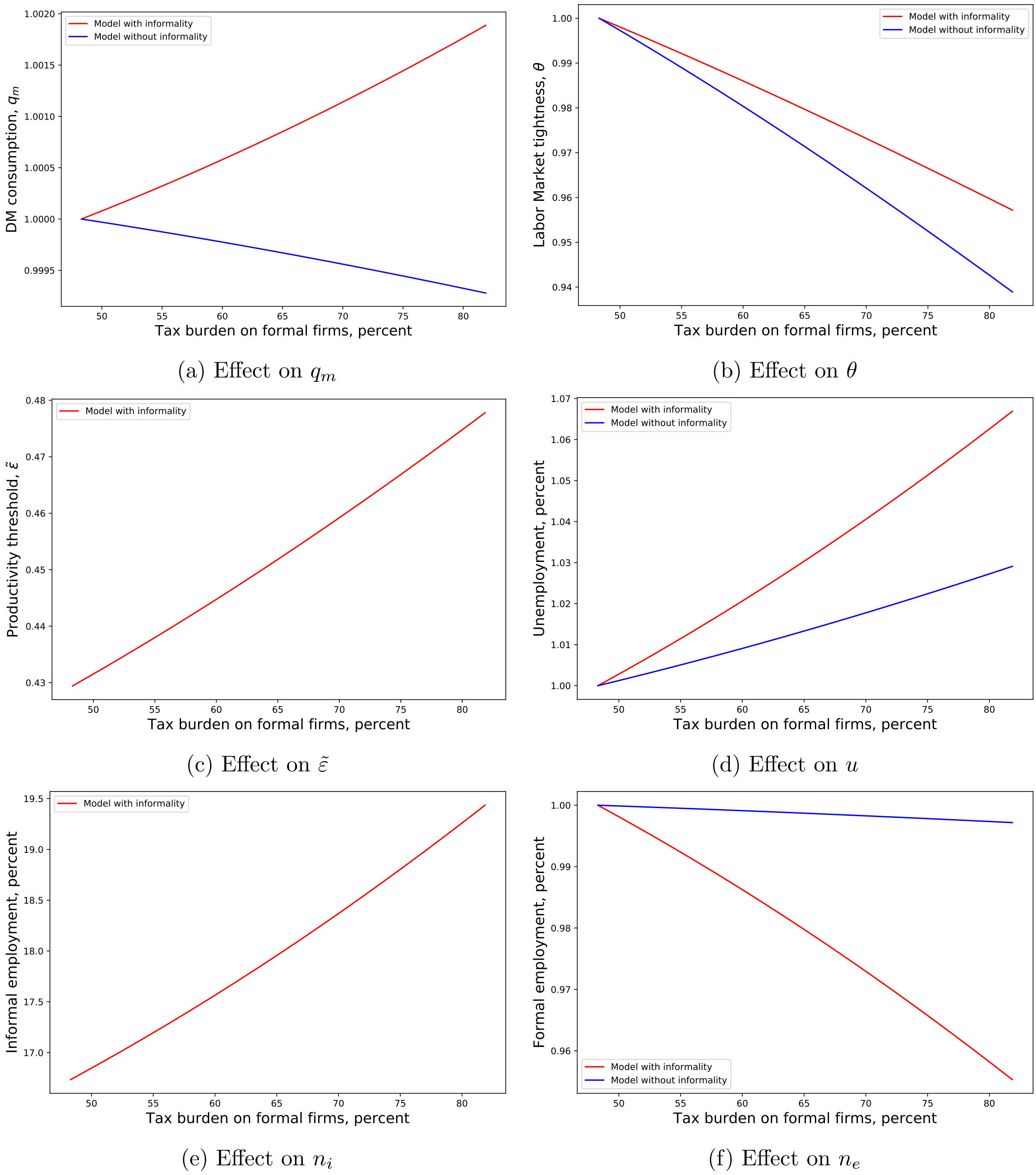

(e) Effect on $n_i$




(f) Effect on $n_e$

Figure 14: Effect of changing $i$ on the model's equilibrium.

an increase in informality drives firms to the informal economy where cash is the only means of payments which results in an increase in the demand for money by households while in the second, the same increase in $\tau$ results in lower profits for firms and less entry which implies less money demand in general. As seen in panel 15b, job creation is lower in both economies, however the impact is stronger in the absence of informality.




(a) Effect on $q_m$

(b) Effect on $\theta$

(c) Effect on $\tilde{\varepsilon}$

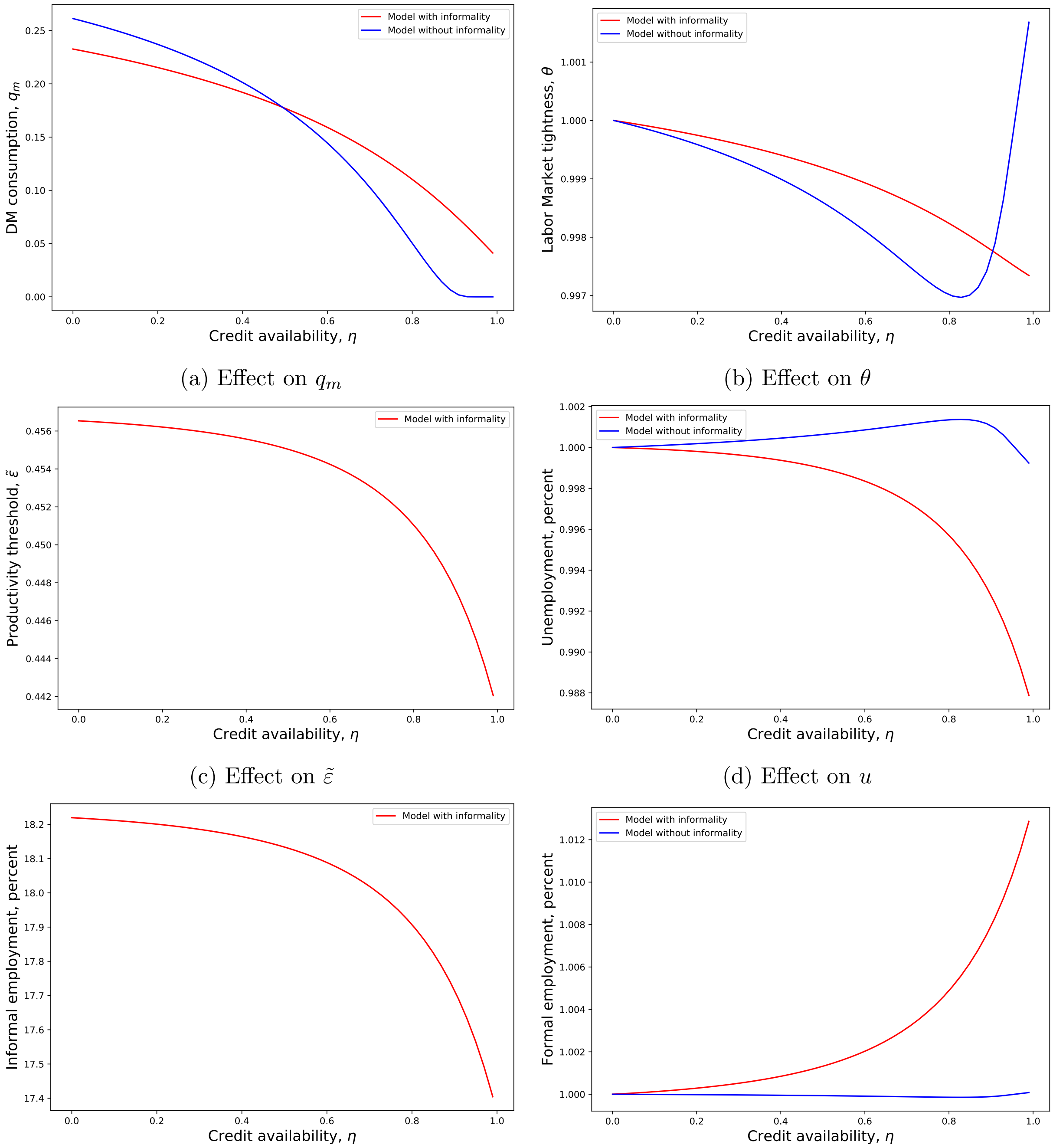
(d) Effect on $u$

(e) Effect on $n_i$

(f) Effect on $n_e$

Figure 15: Effect of changing $\tau$ on the model's equilibrium.

**Effect of changes in credit availability:** Another interesting issue facing developing economies is the impact of financial development. An increase in financial development is likely to reduce the reliance of the economy on immediate settlement using money. Figure 16 depicts the effect of increasing credit availability. Since the trade surplus is higher in credit matches compared to monetary matches, more credit transactions increases the profits of formal firms which increases entry and shifts job creation from the informal to the formal sector. At the same time, a higher




(a) Effect on $q_m$

(b) Effect on $\theta$

(c) Effect on $\tilde{\varepsilon}$

(d) Effect on $u$

(e) Effect on $n_i$

(f) Effect on $n_e$

Figure 16: Effect of changing $\eta$ on the model's equilibrium.

availability of credit depresses money demand and hence trade surplus in monetary matches. This effect reduces entry but also shifts job creation from the informal to the formal sector. In the presence of informality, the first effect clearly dominates since unemployment decreases. The fall in unemployment is amplified by the shift of firms and workers to the lower separation formal jobs as seen in panels 16c, 16e and 16f.

# 7 Conclusion

In this paper, I presented a monetary dynamic general equilibrium model with flexible prices and search frictions in labor and goods markets. Informality in this model results from the optimal choices of firms and households given the frictions they face when deciding how to allocate labor, how much to trade, which means of payment to use, and whether to pay taxes or not. This model provides a rich yet simple framework to understand interactions between the informal sector and monetary policy. A main innovation of this model is to connect informality in both labor and goods markets by limiting buyers to the use of money as a means of payment when trading with firms employing informal workers. Inflation affects unemployment and informality by taxing monetary balances carried by households for transaction purposes. The higher cash intensity of informal transactions makes informal firms and jobs more vulnerable to the inflation tax compared to the formal sector. An increase in informality affects the economy in two ways: it shifts upwards both the Beveridge curve, increasing unemployment for all levels of economic activity; and the money demand curve, increasing real balances for all levels of the nominal interest rate. A main result of the paper is that the presence of informality dampens the long run effects of monetary policy on unemployment and output. This is because monetary policy not only affects job creation and the size of employment but also its composition in terms of formal and informal jobs. An increase in inflation reduces firms profits from job creation which increases unemployment. It also shifts job creation from high separation informal jobs to low separation formal jobs which reduces unemployment. The second effect works as a dampening mechanism that reduces the effects of an increase in inflation and the nominal interest rate on unemployment and output.

I calibrate the model to the Brazilian economy and present some numerical results. The model is able to replicate most of the stylized facts found in the empirical literature on informality. Numerical results show that increasing inflation has a smaller effect on unemployment in the presence of informality compared to a similar economy absent the informality margin. I also explore the effects of changes in taxation and financial development in the presence of informality. In particular, changes in the tax burden result in changes in money demand as increased tax evasion leads to greater use of money. The results presented in this paper point to the importance of understanding and measuring informality for the implementation of monetary policy in developing economies.

This relatively simple model economy presents several testable implications that I leave for future empirical work. On the theory side, one possible research direction would be to study optimal monetary and fiscal policy in the present environment and compare the results to a frictionless economy e.g. Koreshkova (2006). Another direction is to study how the presence of informality affects the effectiveness of short-run stabilization policies such as the ones discussed in Berentsen and Waller (2015).

# Appendix A Nash bargaining in the DM

Here I solve for the terms of trade in the DM using the generalized Nash bargaining solution. I assume that the bargaining power of buyers $\varphi$ is the same for all types of DM matches.

When a buyer is part of a pure monetary match, the bargaining problem can be formulated as follows:

$$\max_{q_m, d_m} \left[v(q_m) - \phi d_m\right]^{\varphi} \left[\phi d_m - c(q_m)\right]^{1-\varphi}$$

subject to:

$$\phi d_m \leq z$$
$$c(q_m) \leq y + \varepsilon$$

Since money is costly, households don't have an incentive to carry more money than they intend to spend in the DM market which makes the first constraint binding. In addition, I assume that the second constraint is never binding which allows us to write the problem as an unconstrained optimization problem. This results in the following first order condition:

$$z = \frac{\varphi v'(q_m)c(q_m) + (1-\varphi)v(q_m)c'(q_m)}{\varphi v'(q_m) + (1-\varphi)c'(q_m)} \equiv g(q_m)$$

As a consequence, the Nash bargaining solution is a pair $(q_m, d)$ that satisfies $q_m = g^{-1}(z)$ and $\phi d = z$. Notice that $\partial q_m / \partial z = \partial g^{-1}(z)/\partial z = 1/g'(q_m) \geq 0$ meaning that more money holdings increases $q$.

Next, I solve for the terms of trade in credit formal matches:

$$\max_{q_c, d, \ell} \left[v(q_c) - \phi d - \ell\right]^{\varphi} \left[\phi d + \ell - c(q_c)\right]^{1-\varphi}$$

subject to:

$$\phi d \leq z$$
$$c(q_c) \leq y + \varepsilon$$

Using the same assumptions as before, the bargaining problem results in the following first order conditions:

$$z + \ell = \frac{\varphi v'(q_c)c(q_c) + (1-\varphi)v(q_c)c'(q_c)}{\varphi v'(q_c) + (1-\varphi)c'(q_c)} \equiv g(q_c)$$
$$z + \ell = (1-\varphi)v(q_c) + \varphi c(q_c)$$

Combining both first order conditions we get the following optimality condition:

$$v'(q_c) = c'(q_c)$$

Hence the optimal solution is $q_c = q^*$ the efficient quantity which solves $v'(q) = c'(q)$. As a consequence, the Nash bargaining solution is a pair $(q_c, g(q_c))$ that satisfies $q_c = q^*$.

# Appendix B Proofs

## B.1 Proof of proposition 1

To ensure that $\exists!\ \tilde{\varepsilon} \in (\underline{\varepsilon}, \bar{\varepsilon})$ such that

- $V_e^f(\varepsilon) - V_i^f(\varepsilon) > 0, \forall \varepsilon \in (\tilde{\varepsilon}, \bar{\varepsilon}]$;
- $V_e^f(\varepsilon) - V_i^f(\varepsilon) < 0, \forall \varepsilon \in [\underline{\varepsilon}, \tilde{\varepsilon})$;
- $V_e^f(\tilde{\varepsilon}) - V_i^f(\tilde{\varepsilon}) = 0$ ,

it is sufficient to have

- $V_e^f(\varepsilon) - V_i^f(\varepsilon)$ strictly increasing in $\varepsilon$;
- $V_e^f(\bar{\varepsilon}) - V_i^f(\bar{\varepsilon}) > 0$;
- $V_e^f(\underline{\varepsilon}) - V_i^f(\underline{\varepsilon}) < 0$.

In what follows I derive the model's parameter values for which the above three conditions hold.

Assuming $0 < \delta_e < 1$, $0 < \delta_i < 1$, $0 < \beta \leq 1$ and $0 \leq \omega < 1$, the first condition

$$\frac{\partial \left( V_e^f(\varepsilon) - V_i^f(\varepsilon) \right)}{\partial \varepsilon} = \frac{(1-\omega)\beta(\delta_i - \delta_e)}{(1-\beta(1-\delta_e))(1-\beta(1-\delta_i))} > 0$$

holds if and only if $\delta_i > \delta_e$.

The second condition simplifies to

$$y+\bar{\varepsilon}-b-\frac{\omega}{1-\omega}\theta k+\sigma_f(g(q_m)-c(q_m))+\frac{1-\beta(1-\delta_i)}{\beta(\delta_i-\delta_e)}\left[\sigma_f\eta((g(q_c)-c(q_c))-(g(q_m)-c(q_m)))-\tau\right]>0$$

Finally, the third condition simplifies to

$$y+\underline{\varepsilon}-b-\frac{\omega}{1-\omega}\theta k+\sigma_f(g(q_m)-c(q_m))+\frac{1-\beta(1-\delta_i)}{\beta(\delta_i-\delta_e)}\left[\sigma_f\eta((g(q_c)-c(q_c))-(g(q_m)-c(q_m)))-\tau\right]<0.$$

### B.2 Proof of lemma 2

It follows from the partial derivatives

$$\frac{\partial\tilde{\varepsilon}}{\partial\tau}>0 \quad ; \quad \frac{\partial\tilde{\varepsilon}}{\partial b}>0 \quad ; \quad \frac{\partial\tilde{\varepsilon}}{\partial k}>0 \quad ; \quad \frac{\partial\tilde{\varepsilon}}{\partial y}<0.$$

### B.3 Proof of lemma 3

From the JC condition (17) we have

$$\frac{k}{\beta\alpha_f(\theta)}=\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}}\frac{(1-\omega)[R_e(\varepsilon)-b-\tau]-\omega\theta k}{1-\beta(1-\delta_e)}\,\mathrm{d}F(\varepsilon)+\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}}\frac{(1-\omega)[R_i(\varepsilon)-b]-\omega\theta k}{1-\beta(1-\delta_i)}\,\mathrm{d}F(\varepsilon)$$

Using Leibniz rule[16] to differentiate the expression above w.r.t. $\theta$ and then using the envelope property of $\tilde{\varepsilon}$ yields

$$-\frac{k}{\beta\alpha_f^2}\frac{\partial\alpha_f}{\partial\theta}=\frac{(1-\omega)}{1-\beta(1-\delta_e)}\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}}\frac{\partial R_e(\varepsilon)}{\partial\theta}-\omega k\,\mathrm{d}F(\varepsilon)+\frac{(1-\omega)}{1-\beta(1-\delta_i)}\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}}\frac{\partial R_i(\varepsilon)}{\partial\theta}-\omega k\,\mathrm{d}F(\varepsilon)$$

It is easy to show that when the Friedman rule ($i=0$) applies we get: $\frac{\partial R_e(\varepsilon)}{\partial\theta}=\frac{\partial R_i(\varepsilon)}{\partial\theta}\leq 0$. From the definition of the matching function, we also have $\dfrac{\partial\alpha_f}{\partial\theta}<0$. Hence, the LHS of the equation above is positive while the RHS is negative. The LHS of the JC equation (17) is increasing in $\theta$ while the RHS is decreasing in $\theta$. Hence $\theta$ is unique.

[16] $\frac{d}{dx}\left(\int_{a(x)}^{b(x)}f(x,t)dt\right)=f(x,b(x))\frac{d}{dx}b(x)-f(x,a(x))\frac{d}{dx}a(x)+\int_{a(x)}^{b(x)}\frac{\partial}{\partial x}f(x,t)dt$ where $-\infty<a(x),\,b(x)<+\infty$

### B.4 Proof of lemma 4

Under the usual assumptions on the DM utility function ($v'(q) > 0, v''(q) < 0, \lim_{q\to 0} v'(q) = +\infty, \lim_{q\to\infty} v'(q) = 0$ ) and cost function ($c'(q) > 0, c''(q) \geq 0$) we have

$$\begin{aligned}
\frac{\partial \frac{v'(q)}{g'(q)}}{\partial q} &= \frac{v''(q)g'(q) - v'(q)g''(q)}{(g'(q))^2} \\
&= \frac{\varphi v''(q)c'(q) - \varphi v'(q)c''(q)}{(g'(q))^2} < 0,
\end{aligned}$$

$$\lim_{q\to 0} \frac{v'(q)}{g'(q)} = \lim_{q\to 0} \frac{v'(q)}{\varphi c'(q) + (1-\varphi)v'(q)} = +\infty > 0 \text{ for } \varphi \in (0,1),$$

$$\lim_{q\to +\infty} \frac{v'(q)}{g'(q)} = \lim_{q\to 0} \frac{v'(q)}{\varphi c'(q) + (1-\varphi)v'(q)} = 0.$$

It follows that the solution to (18) exists and is unique.

## References


David Andolfatto. Incentive-feasible deflation. *Journal of Monetary Economics*, 60(4):383–390, 2013.

Dan Andrews, Aida Caldera Sánchez, and Åsa Johansson. Towards a better understanding of the informal economy. 2011.

S Borağan Aruoba, Guillaume Rocheteau, and Christopher Waller. Bargaining and the value of money. *Journal of Monetary Economics*, 54(8):2636–2655, 2007.

S Borağan Aruoba, Christopher J Waller, and Randall Wright. Money and capital. *Journal of Monetary Economics*, 58(2):98–116, 2011.

Nicoletta Batini, Paul Levine, Young-Bae Kim, and Emanuela Lotti. Informal labour and credit markets: a survey. *IMF Working Papers*, pages 1–41, 2010.

Nicoletta Batini, Paul Levine, Emanuela Lotti, and Bo Yang. Informality, Frictions and Monetary Policy. *School of Economics Discussion Papers*, 711, 2011.

Aleksander Berentsen and Christopher Waller. Optimal stabilization policy with search externalities. *Macroeconomic Dynamics*, 19(3):669–700, 2015.

Aleksander Berentsen, Guido Menzio, and Randall Wright. Inflation and unemployment in the long run. *American Economic Review*, 101(1):371–98, 2011.

Andreas Beyer and Roger EA Farmer. Natural rate doubts. *Journal of Economic Dynamics and Control*, 31(3):797–825, 2007.

Manoel Bittencourt, Rangan Gupta, and Lardo Stander. Tax evasion, financial development and inflation: Theory and empirical evidence. *Journal of Banking & Finance*, 41:194–208, 2014.

Mariano Bosch and Julen Esteban-Pretel. Job creation and job destruction in the presence of informal markets. *Journal of Development Economics*, 98(2):270–286, 2012.

Paul Castillo and Carlos Montoro. Monetary policy in the presence of informal labour markets. *Banco Central de Reserva del Peru, Mimeo*, 2008.

Ben Craig and Guillaume Rocheteau. Inflation and welfare: A search approach. *Journal of Money, Credit and Banking*, 40(1):89–119, 2008.

Milton Friedman. Nobel lecture: inflation and unemployment. *Journal of political economy*, 85 (3):451–472, 1977.

François Gerard and Gustavo Gonzaga. Informal labor and the efficiency cost of social programs: Evidence from the brazilian unemployment insurance program. Technical report, National Bureau of Economic Research, 2016.

Pedro Gomis-Porqueras, Adrian Peralta-Alva, and Christopher Waller. The shadow economy as an equilibrium outcome. *Journal of Economic Dynamics and Control*, 41:1–19, 2014.

Marcus Hagedorn and Iourii Manovskii. The cyclical behavior of equilibrium unemployment and vacancies revisited. *The American Economic Review*, 98(4):1692–1706, 2008.

Robert E Hall and Paul R Milgrom. The limited influence of unemployment on the wage bargain. *The American Economic Review*, 98(4):1653–1674, 2008.

Ehud Kalai. Proportional solutions to bargaining situations: interpersonal utility comparisons. *Econometrica: Journal of the Econometric Society*, pages 1623–1630, 1977.

Marika Karanassou, Hector Sala, and Dennis J Snower. The european phillips curve: does the nairu exist? 2003.

Nobuhiro Kiyotaki and Randall Wright. A search-theoretic approach to monetary economics. *The American Economic Review*, pages 63–77, 1993.

Tatyana A Koreshkova. A quantitative analysis of inflation as a tax on the underground economy. *Journal of Monetary Economics*, 53(4):773–796, 2006.

Ricardo Lagos and Randall Wright. A unified framework for monetary theory and policy analysis. *Journal of political economy*, 113(3):463–484, 2005.

Nika Lazaryan and Thomas Lubik. Global dynamics in a search and matching model of the labor market. Technical report, Federal Reserve Bank of Richmond, 2017.

Lars Ljungqvist and Thomas J Sargent. The fundamental surplus. *American Economic Review*, 107(9):2630–65, 2017.

Robert Lucas. Inflation and welfare. *Econometrica*, 68(2):247–274, 2000.

Fabrizio Mattesini and Lorenza Rossi. Optimal monetary policy in economies with dual labor markets. *Journal of Economic Dynamics and Control*, 33(7):1469–1489, 2009.

Leandro Medina and Friedrich Schneider. Shadow economies around the world: New results for 158 countries over 1991-2015. Technical report, CESifo Working Paper, 2017.

Dale T Mortensen and Christopher A Pissarides. Job creation and job destruction in the theory of unemployment. *The review of economic studies*, 61(3):397–415, 1994.

Guillermo Perry, William Maloney, Omar Arias, Pablo Fajnzylber, Andrew Mason, and Jaime Saavedra. Informality: Exit and exclusion, world bank latin america and caribbean studies. *World Bank, Washington DC*, 2007.

Barbara Petrongolo and Christopher A Pissarides. Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2):390–431, 2001.

Christopher A Pissarides. *Equilibrium Unemployment Theory*, volume 1. The MIT Press, 2000.

Guillaume Rocheteau and Ed Nosal. *Money, Payments, and Liquidity*. MIT Press, 2017.

Guillaume Rocheteau and Randall Wright. Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, pages 175–202, 2005.

Friedrich Schneider, Andreas Buehn, and Claudio E Montenegro. Shadow Economies all over the World: New Estimates for 162 Countries from 1999 to 2007. *Handbook on the shadow economy*, pages 9–77, 2011.

Sven Schreiber and Jürgen Wolters. The long-run phillips curve revisited: Is the nairu framework data-consistent? *Journal of Macroeconomics*, 29(2):355–367, 2007.

Robert Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *The American Economic Review*, 95(1):25–49, 2005.