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Voting with Public Information

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Abstract

We study the effect of public information on collective decision-making in committees, where members can have both common and conflicting interests. In the presence of public information, the simple and efficient vote-your-signal strategy profile no longer constitutes an equilibrium under the commonly-used simultaneous voting rules, while the intuitive but inefficient follow-the-expert strategy profile almost always does. Although more information may be aggregated if agents are able to coordinate on more sophisticated equilibria, inefficiency can persist even in large elections if the provision of public information introduces general correlation between the signals observed by the agents. We propose simple voting procedures that can indirectly implement the outcomes of optimal anonymous and ex post incentive compatible mechanisms with public information. The proposed voting procedures also have additional advantages when there is a concern for strategic disclosure of public information.

Keywords: strategic voting, collective decision-making, public information, committee design, optimal voting rule, information disclosure.
JEL classification: D72, D82.

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1 Introduction

A common argument for voting mechanisms is that they help aggregate the information that agents in a committee privately hold, and thus lead to better decisions compared to the case of a single decision-maker. Indeed, in a setting of collective decision-making where agents have purely common interests, the celebrated Condorcet Jury Theorem (CJT) suggests that the simple majority rule can lead to the first-best outcome if agents truthfully convey their private information through their votes (Condorcet, [1785] 1994). However, Kawamura and Vlaseros (2017) (henceforth KV) make the interesting observation that, as long as there exists a public signal that can be commonly observed by all agents and that is superior to each of their private signals, a vote-your-private-information strategy profile will not constitute an equilibrium under the simple majority rule, even though this would have been the case if the public signal were absent. What's worse, the presence of public information opens the possibility for agents to coordinate on an equilibrium in which everyone just votes according to whatever the public signal suggests. Clearly, in such an equilibrium, the private information of the committee members is completely disregarded. This can be very inefficient since public information is rarely perfect and the total private information possessed by the committee is often more valuable in determining the optimal collective decision. Experimentally, KV find that a large proportion of subjects in the laboratory behave quite consistently with what the inefficient equilibrium would predict. Consequently, the outcome of the collective decision almost always coincides with that in the inefficient equilibrium.

This observation is highly relevant, because it should be clear that the access to both private and public information for the voters is the rule rather than the exception: in business, members of the board of directors receive (or even ask) advice from the advisory board of the company; in a court, an expert witness states his/her testimony in front of all members of the jury; the Central Committee of the Communist Party of China, which has only seven members, often invites renowned scholars in the relevant fields to give short presentations when important decisions that affect the well-being of more than 1.3 billion people are needed to be made. If in the end only the public information counts, why should we bother to use the voting mechanism in the first place? This issue is even more alarming if we take into account that in reality, the party that provides the relevant public information is often strategic and self-interested as well.

With these practical concerns in mind, we first take KV's observation one step further in this paper. We study the effect of public information in a richer setting where agents have both common and conflicting interests: while agents share the common goal of making a collective decision that will match the state, they may have different payoffs from the different types of decision errors that could occur. We show that the presence of public information can have a profound impact on the agents' voting behavior. In particular, it significantly limits the existence of the *informative voting equilibrium*, in which every agent simply casts her vote in accordance with her private information: If the public information is superior to each agent's private information and the voting threshold is fixed (which is the case for the simple majority rule), the informative voting equilibrium does not exist for *any* preference profile of the agents.[1] To make things worse, the presence of public information introduces the intuitive but inefficient *obedient voting equilibrium*, which robustly exists under different voting rules. In the obedient voting equilibrium, agents always support the alternative suggested by the public information and, hence, the public information is the only determinant of the final decision outcome. We later show that a self-interested party who controls the provision of public information may exploit its influential effect by strategically disclosing (withholding) good (bad) news about his favored alternative.

The inefficient outcome of the obedient voting equilibrium echos the common concern that public information, especially expert opinions, may have excessive influence on decision making.[2] In theory, if agents are sophisticated enough to coordinate on equilibria that entail mixed and/or asymmetric strategy profiles, then the committee's decision may still incorporate both the private and the public information. However, as we argue in Section 4.1, the concern of public information being detrimental should be far from being resolved by this theoretical possibility. In particular, by extending the baseline model and considering more generally how the provision of public information introduces correlation between the signals privately observed by the agents, we are able to show that informational inefficiency can persist even in large

[1]Even if the public information is less accurate than the private information, the set of preference profiles that allow for informative voting under *some* voting rule with a fixed threshold is strictly smaller than it would be in the absence of the public information. For example, if the public information is just slightly less precise than each agent's private information, under the simple majority rule the informative voting equilibrium exists only if all agents are sufficiently unbiased *ex ante* (see Corollary 2).

[2]For instance, because of the concern that their testimonies will have too much influence upon the jury, in the US court rules are set to prevent expert witnesses from "usurping the province of the jury" (Tanay, 2010).

elections, no matter how sophisticated the equilibria played by the agents are.

We then study the design of optimal voting mechanisms in environments with public information. We first introduce a class of more flexible voting rules that we call the *contingent k-voting rules*. Under a contingent k-voting rule, the number of votes required for the committee to select an alternative will depend on the content of the public information: For example, if a job candidate is supported by an exceptionally strong recommendation letter, the committee may consider requiring less votes to approve the hire of this candidate. We show that for any *anonymous* and *ex post incentive compatible* direct mechanism that is optimal, there exists an equivalent contingent k-voting rule. Specifically, by sustaining informative voting as an equilibrium (or *implementing* informative voting), the equivalent contingent k-voting rule achieves the same informational efficiency as the optimal anonymous and ex post incentive compatible mechanisms. Therefore, in the search for optimal mechanisms it is without loss to focus on contingent k-voting rules that can implement informative voting.

A contingent k-voting rule incorporates the public information by letting its voting threshold be contingent on the realization of the public signal. It also incorporates the private information of the agents if it is *responsive*, which requires that the agents' votes can always make a difference on the final decision, regardless of the realization of the public signal. We show that it is often optimal to use a responsive contingent k-voting rule to implement informative voting. Moreover, the informative voting equilibria sustained by the responsive contingent k-voting rules are asymptotically efficient, in the sense that the ex ante probability of the collective decision being matched to the state becomes arbitrary close to 1 as the size of the committee increases. In other words, we obtain a version of the CJT in a voting environment with both private and public information.

Within a setting where agents have purely common interests, which is mostly studied in the literature, we demonstrate that the first-best informational efficiency can always be achieved by using a specific contingent k-voting rule, the *contingent majority rule*, under which the informative voting equilibrium is guaranteed to exist. In particular, we show that given all the information that is available to the committee, the probability of the collective decision being matched to the state is maximized in the informative voting equilibrium under the contingent majority rule. In other words, the contingent majority rule aggregates *both* the private *and* the public information efficiently.

To strengthen the applicability of our results, we further introduce a simple two-stage voting mechanism that can *equivalently implement* the informative voting equilibrium under the contingent k-voting rules. In the first stage of this voting mechanism, agents vote to select the voting threshold that will be used. In the second stage, they proceed to vote about which collective decision to take by using the voting rule that they agreed on. We argue that this two-stage voting mechanism is practically appealing because its procedure is deterministic and independent of the informational details of the environment.

Finally, we show, perhaps to one's surprise, that using voting procedures that incorporate the public information can actually have additional advantages when there is a concern for strategic disclosure of public information. Intuitively, the use of the contingent k-voting rules or the above two-stage voting mechanism makes it possible for the agents to rationally commit to informative voting, independent of the disclosure policy of the public information. Thus, even a self-interested party may find it optimal to always publicly communicate the information it receives to the agents, given that its message will not directly affect the agents' voting behavior but will indirectly increase the accuracy of the collective decision.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the model. In Section 4 we show how the presence of public information can lead to inefficient information aggregation. We study in Section 5 the design of optimal voting mechanisms with public information. In Section 6, we analyze settings where the provision of public information is strategically determined by a self-interested information controller. Finally, Section 7 concludes. All proofs are contained in the Appendix.

2 Related Literature

There is an extensive literature on strategic voting starting with the seminal paper of Austen-Smith and Banks (1996). Many of the papers in this literature study how informational efficiency of various voting mechanisms is affected by the agents' strategic behavior (see, e.g., Feddersen and Pesendorfer (1997) and Duggan and Martinelli (2001) on simultaneous voting rules, and Dekel and Piccione (2000) on sequential voting rules). Among all of them, the most closely related paper besides KV is actually Austen-Smith and Banks (1996). Specifically, they notice that whenever the voters do not have an extremely biased prior, the informative voting equilibrium will exist under some simultaneous voting rule with a fixed voting threshold value (p. 38, Lemma 2). However, our paper shows that if we explicitly take into account how agents' prior is shaped by public information, then the simultaneous voting rules commonly used in practice may no longer suffice to incentivize agents to truthfully reveal their private information via their votes. As another connection to our paper, Section 2 of Austen-Smith and Banks (1996) extends their analysis to a case where agents have access to both private and (exogenous) public information. They conclude that in such a setting, sincere voting, which is equivalent to obedient voting in our model whenever the public information is more precise than each agent's private information, cannot be both informative and rational (p. 42, Theorem 3). In contrast, we address the related but distinct question of whether informative voting can be rational under some simultaneous voting rule when it is not required to be sincere. Our model and focus are also quite different from the few other papers that study the effect of public information in a voting environment (e.g., Gersbach, 2000; Taylor and Yildirim, 2010; Tanner, 2014).

Several papers study the effect of pre-voting deliberation (e.g., Coughlan, 2000; Austen-Smith and Feddersen, 2006; Gerardi and Yariv, 2007). In these models, agents can communicate their private information before the vote takes place, thus public information *endogenously* arises. Our model differs from them in two main aspects. First, in the models with deliberation, conflicts between an agent's private information and the public information usually do not matter because the former has already been incorporated in the latter. In our model, however, such conflicts have a direct and profound effect on agents' provision of private information, which can lead to a severe loss of informational efficiency. Second, unlike in the obedient voting equilibrium in the current paper, in these models it is actually socially efficient for the agents to always follow the public information, conditional on their private information being credibly revealed in the deliberation stage.[3]

Finally, there is a third strand of literature on committee design and optimal voting rules

[3]Buechel and Mechtenberg (2016) is a recent exception that shows that pre-voting communication can actually impede efficient information aggregation within a common-interest setting. They consider a network model in which agents are heterogeneously informed, and each informed agent can privately make a voting recommendation to the uninformed agents that are connected to her. They show that if the network structure is too centralized around a few informed agents, majority voting may lead to inefficient information aggregation. Compared to their paper, we focus on the public communication between a (strategic or non-strategic) information controller and a group of homogeneously informed agents.

with strategic agents.[4] For example, Persico (2004) studies the optimal size and threshold value for simultaneous voting rules when agents' private information is endogenous. Subsequently, Gershkov and Szentes (2009) show that when information is costly, the optimal direct mechanism can actually be implemented by a random, sequential reporting/voting scheme, which suggests in general that the use of more flexible voting rules can be welfare-enhancing. This insight is also shared by Gersbach (2004, 2009, 2017), who shows that allowing the voting rule to depend on the proposal to be determined may yield efficient outcomes for classic social choice problems such as provision of public projects and division of limited resources among agents. More recently, Gershkov et al. (forthcoming) show that in an environment where agents have single-crossing preferences, a successive voting rule with a descending threshold achieves the highest utilitarian efficiency among all anonymous, unanimous and dominant strategy incentive-compatible mechanisms. Our paper contributes to this literature by showing that when relevant public information is salient in the strategic environment being considered, the voting rules should also be more carefully and flexibly designed in order to achieve a more efficient outcome.

3 The Model

3.1 Players, actions and payoffs

Consider a committee of n members (agents) indexed by $i \in \mathcal{I} \equiv \{1, ..., n\}$. We assume n is odd and $n \geq 3$. Agents need to make a collective decision $d \in \mathcal{D} \equiv \{0, 1\}$ over a binary set of alternatives. For concreteness, one could think of a setting in which a board of directors is choosing between two business proposals.

Each agent can cast a vote to support one of the alternatives. We denote $v_i = 1$ if agent i votes in favor of the decision $d = 1$, and $v_i = 0$ otherwise. A voting profile of the agents is denoted by $v = (v_1, ..., v_n) \in \mathcal{V} \equiv \{0, 1\}^n$. For the moment, we restrict our attention to a class of collective decision rules $g^k : \mathcal{V} \to \mathcal{D}$ called k-voting rules, which are arguably most commonly used in practice. Formally, if we set the alternative associated with $d = 0$ as the default option, under the voting rule g^k the alternative associated with $d = 1$ will be chosen if and only if there

[4] See Nitzan and Paroush (1982) and Ben-Yashar and Nitzan (2014) for the design of optimal collective decision rules with non-strategic agents.

are at least $k \in \{1,..,n\}$ votes in favor of it: $g^k(v) = 1$ if $\sum_{i=1}^n v_i \geq k$, and $g^k(v) = 0$ otherwise. Each k-voting rule is uniquely characterized by its threshold value k. In particular, the simple majority rule is given by $k = (n+1)/2$.

The state of the world θ is drawn from a binary set $\Theta \equiv \{0,1\}$ with equal probability.[5] In the context of the board of directors and business proposals, one could think of θ as the uncertain (relative) quality of the two proposals, where $\theta = 1$ means the proposal associated with $d = 1$ is of higher prospective revenue, while the other is better if $\theta = 0$. We assume agent i's utility function $u_i : \mathcal{D} \times \Theta \to \mathbb{R}$ takes the following form (see also Coughlan, 2000; Kojima and Takagi, 2010; Iaryczower and Shum, 2012):

$$u_i(d,\theta) = \begin{cases} 0 & \text{if } d = \theta, \\ -q_i & \text{if } d = 1, \theta = 0, \\ -(1-q_i) & \text{if } d = 0, \theta = 1, \end{cases}$$

where $q_i \in [0,1]$. In words, we assume the agents in the committee have a common interest in matching the collective decision to the state (i.e., choosing the proposal of higher quality), and we normalize the payoff of successfully choosing $d = \theta$ to zero. However, we allow the agents' payoffs to differ when committing different types of decision errors. We also allow these differences to be heterogeneous across agents. Each agent's utility function is uniquely characterized by the parameter q_i, and the preference profile $\mathbf{q} = (q_i)_{i \in \mathcal{I}}$ is common knowledge among the agents. We interpret q_i as a measure of how biased agent i is towards the default option *ex ante*: If $q_i = 1/2$, agent i is unbiased and indifferent between the two alternatives; if $q_i < 1/2$, agent i is inclined to choose $d = 1$; similarly, $q_i > 1/2$ implies that agent i would prefer $d = 0$ if there is no further information to be revealed. In addition, if $q_i \neq q_j$, the two agents i and j may strictly prefer different alternatives even when they have exactly the same information. Hence, we interpret $q_i \neq q_j$ as a potential conflict of interest between the two agents. We refer to the case where $q_i = 1/2 \; \forall i \in \mathcal{I}$ as the setting where agents have purely common interests.

[5]The assumptions that the prior probability of θ is uniform and that the accuracy of the agents' private signals is state-independent (see Section 3.2) are mainly made for the convenience of exposition. Most of our analysis can be straightforwardly extended beyond the current setting. See, for example, how we prove Proposition 1 in Section 4 more generally in the Appendix without the above two assumptions.

Note that, given the above specification of payoffs, if agent i assigns a posterior probability $\pi \in [0,1]$ to the event $\theta = 1$, she would prefer $d = 1$ over $d = 0$ if and only if $\pi \geq q_i$, that is, whenever the evidence of the state being 1 is sufficiently strong.

3.2 Information structure and timing

Before casting their votes, each agent privately receives an i.i.d. signal $s_i \in S_i \equiv \{0,1\}$, which is drawn according to the conditional probability distribution $\Pr(s_i = 1|\theta = 1) = \Pr(s_i = 0|\theta = 0) = \alpha \in (1/2, 1)$. We denote $s = (s_1, ..., s_n) \in S \equiv \Pi_{i=1}^n S_i$ as the agents' (private) signal profile. In addition to their private signals, all agents commonly observe a public signal $s_p \in S_p \equiv \{0,1\}$, which is independently drawn from the conditional probability distribution $\Pr(s_p = 1|\theta = 1) = \Pr(s_p = 0|\theta = 0) = \beta \in [1/2, 1)$. We choose to model public information as an additional conditionally independent signal mainly because it has a clear interpretation, especially when considering committees of moderate sizes: In the context of the board of directors and business proposals, for example, one can think of the public signal as the opinion expressed by the advisory board to all directors before the vote takes place. If $\beta > \alpha$, we can further interpret the public signal as the advice provided to the committee by some external *expert*. In addition, this modeling assumption allows us to conveniently extend our analysis to settings where the disclosure of public information is strategically determined by a biased party (see Section 6). We will discuss an alternative way to model public information when considering large elections in Section 4.1.

For later use, we define a measure of (relative) informativeness of the public signal:

$$r \equiv \frac{\ln \beta - \ln(1-\beta)}{\ln \alpha - \ln(1-\alpha)}. \tag{3.1}$$

For given α and β the value of r is uniquely determined, and we will say that the public signal is *r-times as informative* as a private signal. For example, if $\alpha = 0.6$, then $\beta = 0.55, 0.69, 0.77$ correspond to the cases where the public signal is 0.5-, 2- and 3-times as informative as a private signal, respectively. Intuitively, the measure r tells us how many private signals of opposite realization would counter-balance the informational effect of the public signal.

The timing of the voting game is as follows. First, Natures draws θ. After that, each agent observes her private signal and, in addition, the public signal. Agents then cast their votes, and the collective decision d is determined according to the voting profile and the voting rule.

Finally, the state is revealed and agents collect their payoffs.

3.3 Strategies and equilibrium

In the voting game, a strategy of agent i is a mapping $\sigma_i : S_i \times S_p \longrightarrow [0,1]$, where $\sigma_i(s_i, s_p)$ denotes the probability that agent i will vote $v_i = 1$ when observing (s_i, s_p). We will frequently refer to the following two types of (pure) voting strategies (see also KV):

Definition 1. *A strategy is* ***informative*** *if* $\sigma_i(s_i, s_p) = s_i$, $\forall s_i \in S_i, s_p \in S_p$.

Definition 2. *A strategy is* ***obedient*** *if* $\sigma_i(s_i, s_p) = s_p$, $\forall s_i \in S_i, s_p \in S_p$.

The informative strategy is interesting because it is simple and allows the agent to fully convey her private information via her vote. In addition, as we will show in Section 5, the outcomes of optimal voting mechanisms with public information can be indirectly implemented by voting procedures that incentivize the agents to play the informative strategy. The obedient strategy is interesting because it is also simple and it can be very appealing in a context where the public signal is considered as a recommendation from someone supposed to be an expert on the issue. The downside of this "follow-the-expert" strategy is that it entirely disregards the agent's private information, which is also informative about the state.[6]

We call a Bayes-Nash equilibrium in which all agents play the informative strategy an *informative voting equilibrium* (IVE). Similarly, a Bayes-Nash equilibrium in which all agents play the obedient strategy will be called an *obedient voting equilibrium* (OVE). For a given preference profile $\mathbf{q}$, if there exists a k-voting rule under which the IVE exists, we say that such a preference profile allows for the existence of the informative voting equilibrium or simply allows for informative voting.

In the absence of public information, if $q_i \in [1-\alpha, \alpha]$ $\forall i \in \mathcal{I}$, it is easy to check that under the simple majority rule the IVE exists and the CJT holds. If all agents are highly biased towards one of the alternatives, we may still be able to sustain informative voting as an equilibrium by using a threshold value different from $(n+1)/2$. For example, if $q_i \in [\alpha, \alpha^3/(\alpha^3+(1-\alpha)^3)]$ $\forall i \in \mathcal{I}$, one can show that the IVE still exists in a voting game with the super-majority rule $k = (n+3)/2$,

[6] Nevertheless, provided it exists, the equilibrium in which all agents play the obedient strategy maximizes the predicted accuracy of the collective decision among all symmetric equilibria in which the agents use a private-information-independent voting strategy (i.e., $\sigma_i(0, s_p) = \sigma_i(1, s_p)$ $\forall s_p \in S_p$).

and the CJT continues to hold as n becomes sufficiently large (Laslier and Weibull, 2013). In fact, in all the above-mentioned cases the informative voting strategy profile also constitutes an *ex post Nash equilibrium* (Cremer and McLean, 1985), since no agent would ever have a strict incentive to revise her vote even if she could observe the whole voting profile. However, as shown in the next section, the set of preferences that allow for informative voting may shrink drastically in the presence of public information.

4 Inefficient Information Aggregation

To see how the presence of a public signal could affect the equilibrium outcome of the voting game, we first provide a necessary and sufficient condition for the existence of the informative voting equilibrium under any given k-voting rule:

Proposition 1. *Given a k-voting rule, the informative voting equilibrium exists if and only if*

$$\forall i \in \mathcal{I},\ q_i \in \left[\frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2}\frac{1-\beta}{\beta}}, \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k-n}\frac{\beta}{1-\beta}} \right]. \tag{4.1}$$

In the Appendix, we prove a more general version of Proposition 1 which allows the prior probability of the state to be non-uniform and the accuracy of the private signals to be state-dependent. By doing so, we generalize a similar result obtained by Wit (1998) for common-interest voting games with majority rule.

To understand Proposition 1, first note that under a given k-voting rule, an agent is pivotal only when there are exactly $k-1$ other agents who vote in favor of the decision $d=1$, while the remaining $n-k$ agents choose to support the decision $d=0$. Second, if agent i prefers to vote according to her private signal even when it conflicts with the public signal, she will also prefer to do so when the two signals agree. Assuming all other agents $j \neq i$ follow the informative voting strategy, for a given k-voting rule, the left (right) endpoint of the interval in (4.1) is the posterior probability that a Bayesian agent i will assign to the event $\theta = 1$ conditional on $s_i = 0, s_p = 1$ ($s_i = 1, s_p = 0$) and being pivotal. Since a rational agent cares only about the cases in which she is decisive about the final voting outcome, we can conclude that all q_i lying between the above two posterior probabilities is a necessary and sufficient condition for the existence of the informative voting equilibrium under the given k-voting rule.

KV observe that if the public signal is more accurate than each of the private signals ($\beta > \alpha$), informative voting for agents who have purely common interests cannot constitute an equilibrium under the majority rule. The next two corollaries, which follow Proposition 1 immediately, generalize this important observation to arbitrary precision of the public signal, the whole class of k-voting rules, and a much larger set of preferences.

Corollary 1. *Suppose $\beta > \alpha$. For any threshold value k and any preference profile $(q_i)_{i\in\mathcal{I}}$, the informative voting equilibrium does not exist.*

Corollary 2. *Suppose $\beta \leq \alpha$. The informative voting equilibrium does not exist under any k-voting rule if there exist $i, j \in \{1, .., n\}$ such that $q_i < 1/\left(1 + \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right)$ and $q_j > 1/\left(1 + \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$.*

In words, Corollary 1 confirms that whenever the public signal is strictly more precise than each of the private signals, it is impossible to obtain the informative voting equilibrium under any k-voting rule.[7] Meanwhile, Corollary 2 implies that even if the public signal is less accurate, it is still hard to guarantee the existence of the informative voting equilibrium as long as there are two or more agents who are sufficiently biased toward different alternatives ex ante. Note that the required bias becomes arbitrarily small when β is close to α.

The intuition behind both corollaries can be understood via the following simple example of three agents with heterogeneous preferences, such that $q_1 = 1-\alpha$, $q_2 = 1/2$ and $q_3 = \alpha$. Assume that the collective decision is made according to the majority rule ($k = 2$). In the absence of public information, one can check that informative voting constitutes an equilibrium, even though the first and third agents are biased toward different alternatives ex ante. Suppose now agents also observe a public signal that is more informative than each of their private signals. If the unbiased agent 2 assumes that the other two agents will vote informatively, she could infer that the only situation in which she is pivotal is when agent 1 and 3 receive conflicting signals, but this implies that the others' private signals are collectively uninformative about the state.

[7] In general, in this case the strategy profile that the agents would vote for some alternative if and only if it is supported by both private and public signals (while the other alternative is always chosen whenever the two signals disagree) does *not* constitute an equilibrium either. This would be the case, for example, if $\forall i \in \mathcal{I}, q_i \in [1-\alpha, \alpha]$ and the simple majority rule is used. This is because that whenever an alternative is supported by the more precise public signal, then conditional on all other agents would vote for that alternative if and only if it is also supported by their own private signals, an agents whose private signal disagrees with the public signal would then have the incentive to deviate from the proposed strategy.

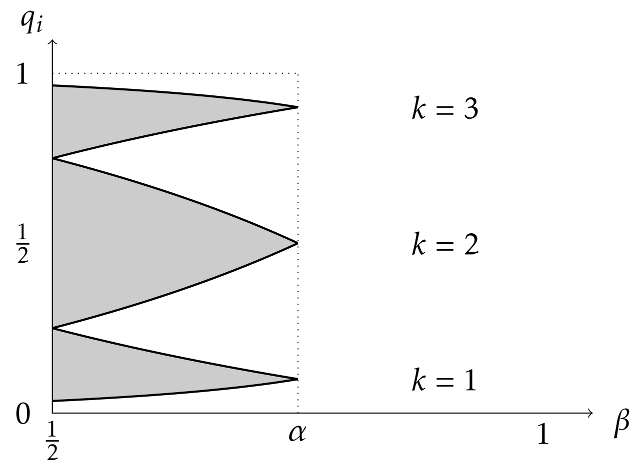

Figure 1: The graphs of the correspondences $Q^{\alpha,k}(\beta)$ given $n = 3, \alpha = 0.75$.

Hence, in this case, agent 2 would make her voting decision by comparing the observed public signal and her own private signal, and simply follows the public one because of its higher precision. Conversely, suppose the public signal is less informative than the private signals. While it is now rational for the unbiased agent 2 to vote informatively (assuming the other two agents do so as well), this is not the case for the two biased agents. For example, agent 1 will still strictly prefer to choose $v_1 = 1$ if $s_1 = 0$ and $s_p = 1$, even when she assumes that the other two agents are voting informatively. This is because the public signal, albeit less informative, is still in favor of her preferred alternative. Moreover, this problem cannot be resolved by using the unilateral ($k = 1$) or unanimity rule ($k = 3$) instead. For example, suppose all three agents are unbiased and the public signal is just slightly more informative than the private signals. While adopting the unanimity rule can successfully encourage agents to vote informatively whenever $s_p = 0$, it provides even stronger incentives for the agents to disregard their private information whenever $s_p = 1$.

Figure 1 interprets the above results graphically. Suppose for a given k-voting rule, an agent i with q_i will find it optimal to play the informative voting strategy when assuming that all other agents $j \neq i$ are voting informatively. Let $Q^{\alpha,k}(\beta) \subseteq [0,1]$ denote the set of all such q_i, for given k, α and β. For a preference profile $\mathbf{q}$, the informative voting equilibrium exists under a given k-voting rule if and only if $q_i \in Q^{\alpha,k}(\beta)$, $\forall i \in \mathcal{I}$. For fixed parameter values $n = 3$ and $\alpha = 0.75$, the top, middle, and bottom part of the gray area in Figure 1 corresponds to the graph of $Q^{\alpha,3}(\beta)$, $Q^{\alpha,2}(\beta)$ and $Q^{\alpha,1}(\beta)$, respectively.[8] As the precision of the public signal increases, the size of each $Q^{\alpha,k}(\beta)$ decreases. In particular, when $\beta > \alpha$, $Q^{\alpha,k}(\beta) = \emptyset, \forall k = 1,2,3$.

Besides shrinking the set of preference profiles that allow for informative voting, the presence of the public signal also opens the possibility for the agents to coordinate on the obedient voting equilibrium. In fact, when $1 < k < n$, the OVE always exists.[9] Clearly, the OVE can be highly inefficient, especially when the public signal is less accurate or just moderately more accurate than each of the private signals.[10] As a numerical example, suppose that $n = 7$, $\alpha = 0.6$ and the simple majority rule is used. By introducing a public signal that is twice as informative as each agent's private signal (i.e., $\beta = 0.69$), the probability of reaching a correct decision can actually decrease (from 0.71 to 0.69) if the agents are induced to switch to the OVE from the IVE. In contrast, if instead we enlarge the size of the committee by two, then the predicted accuracy will increase to 0.73 provided that the agents continue to coordinate on the IVE.[11]

4.1 Discussion

The informative voting equilibrium is desirable because it can aggregate potentially a large amount of private information by asking the agents to play a simple and intuitive strategy that requires little coordination. However, as pointed out by Feddersen and Pesendorfer (1997), from a game-theoretic point of view the non-existence of the IVE does not necessarily imply a failure of information aggregation. For example, suppose that $\beta > \alpha$, $1 < k < n$ and $q_i \neq q_j$ for some $i, j \in \mathcal{I}$. In this case, the IVE does not exist and the OVE is the most efficient equilibrium among the ones that are symmetric (with respect to both the agents and the realization of the public signal), even if the heterogeneous preference profile $\mathbf{q}$ would have allowed for informative voting in the absence of the public signal. However, one cannot generally exclude the existence of asymmetric equilibria, which may efficiently incorporate both public and private information in a more sophisticated way (e.g., by asking agents to play idiosyncratic mixed

[8] For every $\alpha \in (1/2, 1]$, $Q^{\alpha,k}(1/2)$ corresponds to the set of preferences that allow for informative voting under the given k-voting rule when the public signal is absent.

[9] For $k = 1$, the OVE exists if $\forall i \in \mathcal{I}$, $q_i \geq 1/\left(1 + \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$. For $k = n$, the OVE exists if $\forall i \in \mathcal{I}$, $q_i \leq 1/\left(1 + \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$.

[10] For such inefficient use of private information to arise as an equilibrium outcome, it suffices to have more than $n_0 \equiv \max\{k+1, n-k+1\}$ agents who follow the obedient voting strategy.

[11] Suppose that the IVE exists given the preference profile of the original seven-member committee. Then, the IVE also exists after the size of the committee is increased if the preferences of the new members do not exaggerate the initial maximal degree of conflict of interest in the committee (see Proposition 4).

strategies).[12]

Despite the above theoretical possibility, we argue that there are still important reasons for the concern of public information being detrimental. First, the coordination required from the agents in asymmetric equilibria (especially the ones in mixed strategies) can be highly sophisticated. In particular, with asymmetric strategy profiles it can be extremely cognitively challenging for the agents to draw statistical inferences from pivotality. As Esponda and Vespa (2014) demonstrate in their voting experiment, human subjects often have difficulties extracting information from hypothetically pivotal events even when they are playing with computers that are programmed to play symmetric strategies. This makes the prediction of asymmetric equilbria rather unappealing. In contrast, the OVE requires very little sophisticated coordination from the agents. This may be an important reason for the OVE to be an attractive focal point, especially when the IVE does not exist. In fact, for the important benchmark case of unbiased agents, KV present strong experimental evidence showing that a large proportion of voters tend to follow the public signal instead of their private signals much more frequently than other equilibria would predict.[13] Consequently, the collective decisions coincided with what the public signal suggested most of the time. This confirms empirically that the presence of a public signal can indeed lead to a substantial welfare loss.

Second, although in the current setting asymmetric equilibria may indeed lead to information aggregation as in Feddersen and Pesendorfer (1997), such a result would *not* be robust once we relax the assumption on how the public and the private information are being observed by the agents. This would be the case, for example, if we instead assume that each agent only observes a correlated signal $\hat{s}_i = s_i + s_p$. To illustrate this more formally, first note that so far we have modeled public information as an additional conditionally independent signal that is perfectly observed by all agents. This is equivalent to the assumption that each agent privately observes a correlated signal $\hat{s}_i = s_i + \eta s_p$ with $\eta > 1$, as the agents can perfectly back out the signal profile (s_i, s_p) from the realization of $\hat{s}_i$. This assumption fits into applications with committees of moderate size (e.g. boards of directors, hiring committees, juries), since typically

[12] Note that we cannot use the general results from Feddersen and Pesendorfer (1997) to conclude that information will be perfectly aggregated as the size of committee increases either. This is because Feddersen and Pesendorfer (1997) assume that all observed signals are conditionally independent between agents.

[13] In their setting, in addition to the obedient voting equilibrium KV also identify a symmetric equilibrium in which the agents play mixed strategies whenever the public signal disagrees with their private ones, and an asymmetric equilibrium in which only a small subset of the agents vote obediently.

in these scenarios not only the public information itself but also its source is clear (e.g., the expert invited to the board meeting, the reference letters submitted to the hiring committees, the witnesses testify in the court). However, this assumption may not capture very well what happen in large elections (i.e., large size committees), where information often comes from multiple sources and is transmitted in a more decentralized way. In that case, an agent may find it difficult to tell for sure what is publicly known from what is her private knowledge. Nevertheless, this can be captured by our more general model with correlated signals: Letting $\eta = 1$, an agent becomes uncertain about what is publicly known when she observes $\hat{s}_i = 1$.[14] Despite this, assuming $\beta \geq \alpha$, from an individual agent's point of view the correlated signal $\hat{s}_i$ is actually more precise than the independent signal s_i, since the expected conditional variances satisfy $\mathbb{E}[\text{Var}(\theta|\hat{s}_i)] < \mathbb{E}[\text{Var}(\theta|s_i)]$.

Having the above-mentioned general setting in mind, let us fix an arbitrary sequence of preference profiles $\{\mathbf{q}^n = (q_1, ..., q_n)\}_{n\in\mathbb{N}}$. We say that $\{\sigma^{k_n}\}_{n\in\mathbb{N}}$ is a sequence of equilibria *induced by* a sequence of k-voting rules $\{g^{k_n}\}_{n\in\mathbb{N}}$ if $\forall\ n \in \mathbb{N}$, σ^{k_n} constitutes an equilibrium under the voting rule g^{k_n}. We say that a sequence of k-voting rules $\{g^{k_n}\}_{n\in\mathbb{N}}$ *aggregates information asymptotically* if (i) it admits a subsequence $\{g^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ such that $\lim_{\tau\to\infty} k_{n(\tau)}/n(\tau) = \kappa \in [0,1]$, and (ii) $\{g^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ induces a sequence of equilibria in which the probability of reaching a correct decision goes to one. The next result shows that even in large elections, making agents' observations correlated by providing public information can have profound ramifications for information aggregation.

Proposition 2. *Suppose that each agent only observes a correlated signal $\hat{s}_i = s_i + s_p$. For any sequence of preference profiles, there exists no sequence of k-voting rules that aggregates information asymptotically.*

In sum, unlike endowing voters with better private information, introducing public information may actually worsen the quality of the collective decision. This is similar to one of the most striking findings in the global games literature, namely the heterogeneous effect of public and private information. For instance, in a highly influential paper, Morris and Shin (2002) show that in a setting where agents' actions are strategic complements, additional pub-

[14] While the case $\eta = 1$ may seem to be non-generic, this is largely due to the binary structure of the signals in our model. If, for example, both s_i and s_p are continuously distributed over $[0,1]$, then no η can guarantee that an agent will always be able to perfectly back out both s_i and s_p from the signal $\hat{s}_i = s_i + \eta s_p$.

lic information can have negative social value. Although agents in the current setting have no intrinsic motive of coordination, our results suggest similarly that the conventional wisdom that additional information is always beneficial for decision-makers should be carefully examined.[15]

5 Optimal Voting Mechanisms

In this section, we study the design of optimal voting mechanisms with public information. We will show that the outcomes of the optimal mechanisms can be indirectly implemented by simple voting procedures that incorporate the public information appropriately. For clarity of exposition, we will maintain the assumption that the public signal is exogenous throughout this section. We will illustrate in Section 6 that our new voting procedures have also additional advantages when strategic information disclosure is a non-negligible concern.

By the revelation principle, we consider only direct mechanisms $f : S \times S_p \to [0,1]$. The interpretation is that for every signal profile $(s, s_p) \in S \times S_p$ the mechanism specifies the probability $f(s, s_p)$ that the alternative associated with $d = 1$ will be chosen. We start by introducing several definitions.

Definition 3. *A mechanism f is **anonymous** if $\forall s_p \in S_p$ and $\forall s, s' \in S$ such that s is a permutation of s', $f(s, s_p) = f(s', s_p)$.*

Definition 4. *A mechanism f is **ex post incentive compatible** if $\forall s_p \in S_p$, $\forall s_{-i} \in S_{-i}$, $\forall s_i, s_i' \in S_i$ and $\forall i \in \mathcal{I}$, $\mathbb{E}\Big[u_i(f(s_i, s_{-i}, s_p), \theta) | s_i, s_{-i}, s_p\Big] \geq \mathbb{E}\Big[u_i(f(s_i', s_{-i}, s_p), \theta) | s_i, s_{-i}, s_p\Big]$.*

Definition 5. *A mechanism f is **responsive** (to private information) if $\forall s_p \in S_p$, there exist $s, s' \in S$ such that $f(s, s_p) \neq f(s', s_p)$.*

Anonymity requires the mechanism to treat every agent's report equally. It is a common constraint imposed on voting mechanisms. The notion of ex post incentive compatibility (EPIC) requires every agent to prefer truth-telling at every signal profile (s, s_p) if all the other

[15] The non-beneficial effect of public information also resembles the finding from the rational herding literature (e.g., Banerjee, 1992; Bikhchandani et al., 1992). In the models studied in this literature, public information arises endogenously as observed actions taken by previous agents. However, agents who arrive in the future need not be able to fully learn about the state from public observables, as herds or information cascades may arise in equilibrium.

agents also report truthfully. Similar to the role of dominant-strategy incentive compatibility in private-value environments, EPIC guarantees robust behavior of agents in interdependent-value environments (Bergemann and Morris, 2005). Trivially, an anonymous and ex post incentive compatible (A-EPIC) mechanism exists: The mechanism f_o with $f_o(s, s_p) = s_p \forall (s, s_p) \in S \times S_p$ satisfies both anonymity and ex post incentive compatibility. However, f_o is not a responsive mechanism as it makes no use of the agents' reports. By matching the realization of the public signal, it replicates the outcome of the obedient voting equilibrium discussed in Section 4. We are interested in finding an optimal A-EPIC mechanism, i.e., one that maximizes the probability of the collective decision being matched to the state among all A-EPIC mechanisms.

We next introduce a new class of voting rules $g^{k_0,k_1} : \mathcal{V} \to \mathcal{D}$ that we call *contingent k-voting rules*, which can be obtained by adjusting the standard k-voting rules in an intuitive way. In particular, the threshold values in such voting rules will be no longer fixed but a function of the realization of the public signal:

$$k_{s_p} = \begin{cases} k_0 & \text{if } s_p = 0, \\ k_1 & \text{if } s_p = 1, \end{cases} \tag{5.1}$$

where $k_0, k_1 \in \{0, 1, ..., n+1\}$. Any standard k-voting rule amounts to a special case of the contingent k-voting rules with $k_0 = k_1 \in \{1, ..., n\}$. We say that a contingent k-voting rule g^{k_0,k_1} is *responsive* if $k_0, k_1 \notin \{0, n+1\}$. We also say that the voting rule g^{k_0,k_1} *implements* informative voting if it can sustain the informative voting strategy profile as a Bayes-Nash equilibrium in the corresponding voting game. Finally, the voting rule g^{k_0,k_1} is said to be *equivalent* to a A-EPIC mechanism f if, for every realization of the signals (s, s_p), the probability of reaching the correct decision is the same in both the informative voting equilibrium sustained by g^{k_0,k_1} and the truth-telling equilibrium sustained by f. Our next result states that in the search for optimal A-EPIC mechanisms, it is without loss to focus on contingent k-voting rules that can implement informative voting.

Proposition 3. *For every optimal A-EPIC mechanism f, there exists a contingent k-voting rule that is equivalent to f. In addition, the equivalent contingent k-voting rule is responsive if and only if f is responsive.*

Given Proposition 3, the search for optimal mechanisms with public information is reduced to choosing two threshold values $k_0, k_1 \in \{0, 1, ..., n+1\}$. It would be optimal to choose the extreme threshold values $k_0 = n+1$ and $k_1 = 0$, for example, if the degree of conflicts of interests in the committee is so large that the only available A-EPIC mechanisms are the non-responsive ones with $f(s, s_p) = f(s', s_p)$ $\forall s, s' \in S$ and $\forall s_p \in S_p$. While these A-EPIC mechanisms and their equivalent contingent k-voting rules can incorporate the public information, they disregard all the information privately held by the agents. Hence, provided that a responsive A-EPIC mechanism exists, there can be an efficiency gain by using its equivalent and responsive contingent k-voting rule to further incorporate the agents' private information. It also seems intuitive that such an efficiency gain should be increasing in the size of the committee. Hence, one may conjecture that a responsive contingent k-voting rule is optimal when the size of the committee is sufficiently large. To formalize and prove this conjecture, we introduce the notion of *conflict-preserving expansion*: Let $\mathbf{q} = (q_1, ..., q_n)$ be a preference profile with $\bar{q} \equiv \max_{i \in \mathcal{I}} q_i$ and $\underline{q} \equiv \min_{i \in \mathcal{I}} q_i$. We say that a sequence of preference profiles $\{\mathbf{q}^\tau = (\hat{q}_1, ..., \hat{q}_{n+2\tau})\}_{\tau \in \mathbb{N}}$ is a conflict-preserving expansion of $\mathbf{q}$ if $\forall \mathbf{q}^\tau$, $\bar{q}^\tau \equiv \max_{j \in \{1, ..., n+2\tau\}} \hat{q}_j \leq \bar{q}$ and $\underline{q}^\tau \equiv \min_{j \in \{1, ..., n+2\tau\}} \hat{q}_j \geq \underline{q}$. In words, an expansion of the committee is conflict-preserving if it does not exaggerate the initial maximal degree of conflict of interest. We are now ready to state the main result of this section, which demonstrates the optimality of responsive contingent k-voting rules.

Proposition 4. *Suppose that, for a given preference profile $\mathbf{q}$ with $\underline{q}, \bar{q} \in (0,1)$, there exists a responsive contingent k-voting rule g^{k_0,k_1} that implements informative voting. Then, for any conflict-preserving expansion $\{\mathbf{q}^\tau\}_{\tau \in \mathbb{N}}$ of $\mathbf{q}$:*

(i) For each $\mathbf{q}^\tau$, there exists a responsive contingent k-voting rule $g^{k_0^\tau, k_1^\tau}$ that implements informative voting. This contingent k-voting rule is unique if $\bar{q}^\tau \neq \underline{q}^\tau$, and the corresponding threshold values are given by $k_0^\tau = k_0 + \tau$ and $k_1^\tau = k_1 + \tau$.

(ii) There exists τ^*, such that $\forall \tau \geq \tau^*$ there exists a responsive contingent k-voting rule that is equivalent to an optimal A-EPIC mechanism.

(iii) As $\tau \to \infty$, the ex ante probability of the collective decision being matched to the state in the informative voting equilibrium under any responsive contingent k-voting rule becomes arbitrarily close to 1.

We thus obtain a version of the Condorcet Jury Theorem for the contingent k-voting rules in a general voting environment with both private and public information. In particular, Proposition 4 implies that the complete-information outcome can be asymptotically achieved if we incorporate the public information into the voting procedure appropriately. In addition, for finite but large n, no equilibrium under any other A-EPIC mechanism may outperform the informative voting equilibrium under a responsive contingent k-voting rule.

Proposition 4 shows that it is often desirable to use a responsive contingent k-voting rule to implement informative voting. Our next result characterizes when such a practice would be feasible.

Proposition 5. *For a given preference profile* $\mathbf{q}$ *with* $\bar{q}, \underline{q} \in (0,1)$, *there exists a responsive contingent* k*-voting rule that implements informative voting if and only if there exist integers* $k_0, k_1 \in \{1, ..., n\}$ *such that*

$$k_0 \in K_0 \equiv \left[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})\right], \text{ and } k_1 \in K_1 \equiv \left[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})\right],$$

where

$$(\pi_0^0)^{-1}(\underline{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 + r\right), \quad (\pi_1^0)^{-1}(\bar{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + r\right),$$
$$(\pi_0^1)^{-1}(\underline{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 - r\right), \quad (\pi_1^1)^{-1}(\bar{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n - r\right).$$

Importantly, there are cases where allowing the threshold to be contingent on the realization of the public signal is *necessary* for implementing informative voting. To see this, consider a simple example with $n = 5$ and $q_i = 1/2, \forall i \in \mathcal{I}$. If there is no public signal, the informative voting equilibrium exists under the simple majority rule. Now let us introduce a public signal that is twice as informative as a private signal. By Corollary 1, this implies that the informative voting equilibrium no longer exists under any k-voting rule. However, consider the contingent k-voting rule with $k_0 = 4$ and $k_1 = 2$. Suppose all agents $j \neq i$ are voting informatively. If $s_p = 1$, agent i is pivotal only when three of the other agents draw $s_j = 0$ and the remaining one draws $s_j = 1$. Given the above assumption on the informativeness of the public signal, these private signals are collectively uninformative about the state when they are combined with the realization of the public signal. Thus, voting according to her own private signal is a best response for agent i. Similarly, if $s_p = 0$, agent i is pivotal under the contingent k-voting rule only when there are three private signals in favor of $d = 1$ and one in favor of $d = 0$ among all others' private signals. Again, the collective informational effect of all s_j, $j \neq i$, will be exactly counterbalanced by the fact that $s_p = 0$, which makes it optimal for agent i to simply follow her own signal. Intuitively, what we are doing here is to vary the information that agents can infer from pivotality. Under the contingent k-voting rule chosen in the above example, an agent is pivotal when and only when the private signals of the other agents are collectively more against the alternative favored by the public signal. This restores the incentive for agents to vote according to their own signals.

While both $(\pi_0^0)^{-1}$ and $(\pi_0^1)^{-1}$ are strictly increasing in $\bar{q}$, both $(\pi_1^0)^{-1}$ and $(\pi_1^1)^{-1}$ are strictly increasing in $\underline{q}$. Hence, it is possible that both of the intervals K_0 and K_1 contain no integer if $\bar{q}$ is sufficiently larger than $\underline{q}$. Intuitively, if the degree of conflict of interest between the agents is too large, it is very difficult to find a responsive voting rule that ensures the incentive for *all* agents to vote informatively, even if we allow the voting threshold value to be flexibly contingent on the public signal. Nevertheless, we are able to show that for the important limiting cases where agents' preferences are perfectly aligned (e.g., Feddersen and Pesendorfer, 1998; Persico, 2004; Koriyama and Szentes, 2009), there always exists a responsive contingent k-voting rule that implements informative voting, provided that the size of the committee is sufficiently large:

Corollary 3. *Suppose that* $\forall i \in \mathcal{I}, q_i = q \in (0,1)$. *There exists* $\bar{n}(q)$*, such that for each* $n \geq \bar{n}(q)$*, there exists a responsive contingent k-voting rule that implements informative voting.*

5.1 Contingent majority rule

In this subsection, we provide further analysis of optimal voting mechanisms for the setting where agents have purely common interests, i.e., $q_i = 1/2\ \forall i \in \mathcal{I}$. This important benchmark setting has been extensively studied in the literature. Especially, KV show that in this setting if the public signal is r-times as informative as a private signal, where $r \leq (n-1)/2$, then under the simple majority rule there exists an asymmetric equilibrium in which $r^* = \mathbb{N} \cap (r-1, r]$

agents obey the public signal, while the remaining $n - r^*$ agents vote according to their private signals. This r^*-asymmetric equilibrium is shown to be more efficient than both the obedient voting equilibrium and the unique symmetric mixed-strategy equilibrium, as well as all other asymmetric pure-strategy equilibria in the same voting game. In the following, we will show in the same setting that one can always construct a responsive contingent k-voting rule that not only implements informative voting, but also leads to strictly higher efficiency than the r^*-asymmetric equilibrium.

Specifically, consider a contingent k-voting rule with the following threshold values:

$$k_{s_p} = \begin{cases} \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ & \text{if } s_p = 0, \\ \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ & \text{if } s_p = 1, \end{cases}$$

where $[(r-1)/2]^+$ denotes the smallest integer that is larger or equal to $(r-1)/2$. For convenience, we will call this rule the *contingent majority rule*. Note that the contingent majority rule is responsive whenever $r \leq n$. The following result justifies our focus on this particular contingent k-voting rule:

Corollary 4. *Suppose that $r \leq n$. The contingent majority rule implements informative voting if and only if*

$$\forall i \in \mathcal{I},\ q_i \in Q_{cm}^{\alpha}(r) \equiv \left[\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{|r-2[(r-1)/2]^+|-1}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-|r-2[(r-1)/2]^+|+1}} \right]. \tag{5.2}$$

Since $|r - 2[(r-1)/2]^+| \in [0,1]$ for all $r \geq 0$, we always have $1/2 \in Q^{cm}(r)$. Therefore, for the case where all agents are unbiased, one can always use the contingent majority rule to implement informative voting.[16] The next proposition further shows that the informative voting equilibrium under the contingent majority rule achieves the first-best informational efficiency.

Proposition 6. *Given all the information that is available to the committee, the probability of the collective decision being matched to the state is maximized in the informative voting equilibrium under the contingent majority rule.*

[16]The contingent majority rule is also the unique responsive contingent k-voting rule that implements informative voting except when r happens to be is an odd integer.

To gain some intuition, consider a simple example of $n = 5$ and $r = 2$. Assume all agents are unbiased. Imagine that we introduce two additional phantom agents on top of the existing five real agents. These phantom agents are programmed so that they simply vote in line with the public signal. Suppose now the simple majority rule is used to decide which alternative will be chosen. One can easily show that (1) all *real* agents voting informatively constitutes an equilibrium in this game (despite that the public signal observed by the agents is more precise than each of their private ones), (2) the equilibrium outcome is identical to that of the informative voting equilibrium under the contingent majority rule without the phantom agents, and (3) the equilibrium outcome maximizes the probability that the decision will be matched to the state, given all the available information. Intuitively, by allowing the threshold value to be dependent on the public signal and by encouraging agents to vote informatively, the contingent majority rule aggregates both the private and the public information efficiently.

On the contrary, in the r^*-asymmetric equilibrium, inefficiency still prevails because there are r^* agents who always disregard their valuable private information. To see this issue more clearly, consider again the above example. Since in this case we have $r^* = 2 = (n-1)/2$, under the simple majority rule there exists an asymmetric equilibrium in which two agents play the obedient strategy, while the remaining three agents vote informatively. Without loss of generality, assume the first two agents are the obedient voters. Consider the signal profile $s = (1,1,0,1,1)$ and $s_p = 0$. In equilibrium, such a realization of signals will lead to a collective decision $d = 0$. However, from a benevolent social planner's point of view, given all the available information, the welfare maximizing decision should be $d = 1$. Therefore, the r^*-asymmetric equilibrium is strictly less efficient than the first-best.

5.2 Implementation with two-stage voting

The analysis of contingent k-voting rules has highlighted the importance, especially in terms of the potential efficiency gain, of having a more flexible voting procedure that can appropriately incorporate the content of the public information. In practice, however, it might be difficult to implement (or even just prespecify) a voting rule that is contingent on some public information, especially when the source of the relevant public information is ambiguous ex ante. In this subsection, we introduce a simple two-stage voting mechanism that is immune to such concerns.

The voting rule is as follows. After observing the private and the public signals, the agents first vote to select an integer $k \in \{0, 1, ..., n+1\}$. The integer k^* that receives the most votes will be selected, with ties being broken randomly. In the second stage, the agents vote about which collective decision to take according to the voting rule g^{k^*}, i.e., $d = 1$ if and only if $\sum_{i=1}^{n} v_i \geq k^*$. Practically, this two-stage voting procedure is more appealing than the contingent k-voting rules because the procedure itself is deterministic and independent of the informational details of the environment.

Fix a preference profile $\mathbf{q}$, and suppose that there exists a contingent k-voting rule g^{k_0,k_1} that implements informative voting. We say that the above two-stage voting mechanism can *equivalently implement* the informative voting equilibrium under g^{k_0,k_1}, if in the two-stage voting game there exists a Perfect Bayesian Nash equilibrium in which the agents first collectively vote to agree on the threshold value that would have been chosen by g^{k_0,k_1}, and then they vote informatively in the second stage.

Proposition 7. *Suppose that, for a given preference profile* $\mathbf{q}$*, there exists a contingent k-voting rule* g^{k_0,k_1} *that implements informative voting. Then, the two-stage voting mechanism can equivalently implement the informative voting equilibrium under* g^{k_0,k_1}.

The intuition behind Proposition 7 is simple: Since the voting threshold k^* is determined by a simple plurality rule, no agent could unilaterally change the voting outcome in the first stage if all other agents agree to choose either k_0 or k_1. But then given that the informative voting strategy profile constitutes an ex post Nash equilibrium under g^{k_0,k_1}, no agent would have the incentive to deviate from informative voting in the second stage either, no matter how she updates her beliefs about the state and other agents' private information after observing the voting outcome of the first stage.

We close this section by noting that the use of the plurality rule for determining the voting threshold is not generally necessary for our result. To see this, suppose, for example, that the agents have purely common interests and the following unanimity rule is used in the first stage: If all agents agree to use some $k \in \{0, 1, ..., n+1\}$, then we let $k^* = k$. Otherwise, the simple majority rule will be used, i.e., $k^* = (n+1)/2$. This alternative two-stage voting mechanism can also equivalently implement the informative voting equilibrium under the contingent majority rule. The reason is that, according to Proposition 6, the expected social welfare is maximized when the voting threshold values of the contingent majority rule are used. Since each agent's interest is perfectly aligned with the social welfare, any deviation in the first stage will only yield a lower expected payoff to an agent.

6 Strategic Information Disclosure

In this section, we drop the assumption that the disclosure of public information is exogenous, and consider it to be strategically determined by a possibly biased *information controller* (e.g., an external expert). As illustrated in Section 4, public information can have a huge influence on the committee's decision when the standard simultaneous voting rules are in use. Taking this into account, a biased controller may only publicly reveal his information to the agents when its content is in support of his favored alternative. For example, an advisory board member who has private interests in the targeted firm may withhold unfavorable information from the directory board when the acquisition decision is being made. In what follows, we will formalize this intuition by extending our baseline model from Section 3. In addition, we will show that using a contingent voting rule adapted from the ones constructed in Section 5 can mitigate the controller's incentive for strategic disclosure and his influence on the voting outcome, which in turn improves the efficiency of the collective decision.[17]

Suppose now that the signal s_p described in Section 3.2 is no longer public by default but can only be observed by an information controller with some probability. Specifically, with probability $\lambda \in (0,1)$, the controller is uninformed and can only send a public message $m = \emptyset$ (remain silent) to the agents. With the complementary probability $1-\lambda$, the controller observes the signal and can decide whether to publicly communicate its content to the agents or not. While we allow the controller to withhold his information, we assume that the signal is hard information and hence cannot be faked. In other words, in the latter case the public message m can only be chosen from the set $\{s_p, \emptyset\}$.

Assume for simplicity that $q_i = 1/2 \; \forall i \in \mathcal{I}$ and that the collective decision is made according to the simple majority rule.[18] Note again that in this case, the informative voting equilibrium exists if no additional public information is available to the committee members. Assume also

[17] The same result will also hold if we use the two-stage voting mechanism (see Section 5.2) instead.

[18] With the general results in Sections 4 and 5, our analysis in the current section can be straightforwardly extended to settings with general preference profiles and voting rules.

that the controller has the same form of utility function as the agents, and his bias parameter is given by $q_c \in [0,1]$. Let $\hat{\lambda} = \max\{0, (\beta-\alpha)/(\beta-(1-\alpha)\}$. The following proposition establishes that if agents update their beliefs sufficiently little upon observing silence (i.e., λ is large enough), a biased information controller may indeed exploit the publicity of his message and reveal his information selectively.

Proposition 8. *Suppose $\lambda \geq \hat{\lambda}$. There exists $\hat{q} \in [1-\beta, 1/2]$ such that if $q_c \leq \hat{q}$ ($q_c \geq 1-\hat{q}$), then there exists a sequential equilibrium in which the controller sends $m = s_p$ if and only if he observes $s_p = 1$ ($s_p = 0$), and the agents vote obediently if $m = s_p$ and informatively if $m = \emptyset$.*

As a numerical example, if $n = 3$, $\alpha = 0.65$ and $\beta = 0.7$, the threshold values are given by $\hat{\lambda} \approx 0.14$ and $\hat{q} \approx 0.48$, respectively. Depending on the relative precision of the signals, the informational efficiency of the committee's decision could be improved if there were more or less information disclosure than that in the equilibrium. For instance, the decision will be more accurate in the above numerical example if the controller always keeps silent and just lets the agents credibly coordinate on informative voting in the voting stage.

Some recent papers look at the question how an information controller can optimally persuade *uninformed* agents by designing the *informational content* of a public signal (e.g., Wang, 2015; Alonso and Câmara, 2016; Bardhi and Guo, 2017). In our model, voters are privately informed and the controller has control over the disclosure of the public signal only. Hence, our environment is notably less favorable for the controller. Nevertheless, Proposition 8 suggests that the strategic incentive of the controller and his impact on the committee's decision still cannot be ignored.[19]

Fortunately, the concern of strategic information disclosure can be mitigated by instead

[19] This result does not necessarily hold, however, if the controller himself is also a member of the committee. This is because other members in the committee may anticipate that the information contained in the controller's message will already be incorporated in his vote. For example, suppose that the controller is agent 1, $q_i = 1/2$ $\forall i = 2, ..., n$, $\beta \geq \alpha$ and the simple majority rule is used. One can check that if $q_c \in [1-\beta, \beta]$ and $r < 2$, then regardless of whether the controller reveals his signal to the other agents or not in the communication stage, it will be incentive compatible for all agents including the controller to vote informatively in the voting stage . This example suggests that the public information emerges from pre-voting deliberations is less likely to threaten the existence of the informative voting equilibrium. We cannot, however, conclude from this that there is no value in pre-voting deliberations, because informative voting per se does not necessarily lead to the efficient outcome under other voting rules (e.g. the unanimity rule).

using a contingent voting rule with the following threshold values:

$$k_m = \begin{cases} \frac{n+1}{2} & \text{if } m = \emptyset, \\ \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ & \text{if } m = 0, \\ \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ & \text{if } m = 1. \end{cases}$$

Proposition 9. *Suppose $r \leq n$ and the proposed contingent voting rule is used. There exists $q^* \in [0, 1-\beta]$ such that*

1. *If $q_c \in [q^*, 1-q^*]$, there exists a sequential equilibrium in which the controller sends $m = s_p$ whenever he is informed, and the agents always vote informatively.*
2. *If $q_c \leq q^*$ ($q_c \geq 1-q^*$) and $\lambda \geq \hat{\lambda}$, there exists a sequential equilibrium in which the controller sends $m = s_p$ if and only if he observes $s_p = 1$ ($s_p = 0$), and the agents always vote informatively.*

By comparing Propositions 8 and 9, we can see that the contingent voting rule has two main advantages over the simple majority rule. First, the contingent voting rule incorporates the informational content in the controller's message appropriately and makes it credible for the agents to coordinate on informative voting in the voting stage. Hence, by the same reasoning as in Proposition 6, the decision selected by the contingent voting rule is most likely to match the state, given all the information that is available to the committee, independent of the controller's disclosure policy and the relative precision of the signals. Second, under the contingent voting rule the controller also has a higher incentive to share his information unconditionally, since he anticipates that his message will not have a direct impact on the agents' voting behavior and will always help increase the accuracy of the committee's decision. Indeed, for the previous numerical example ($n = 3$, $\alpha = 0.65$ and $\beta = 0.7$), we have $q^* \approx 0.23$, which is substantially smaller than $\hat{q}$.

7 Conclusion

This paper makes two main contributions. First, we show in a general setting of collective decision-making that the provision of public information can have a detrimental effect on the efficiency of the committee decision. The inefficient equilibrium outcome is consistent with experimental evidence, and it echos the common concern that expert opinions may have excessive influence on (both individual and collective) decision-making. We believe these results to be of high policy relevance, especially since the immense influence of public information may be strategically exploited by a biased information controller.

Second, we propose simple voting procedures that can indirectly implement the outcomes of optimal anonymous and ex post incentive compatible mechanisms. By appropriately incorporating the public information and providing incentives for the agents to vote informatively, our voting procedures facilitate information aggregation and enhance the accuracy of the collective decision. By reducing the direct effect of public information on the agents' voting behavior, the proposed voting procedures also mitigate the concern of strategic information disclosure.

It should be remarked that our results are not suggesting that experts should be discouraged from providing their expertise to decision makers. For example, besides providing additional information, advice from experts may also help decision makers to better assess the situation based on their private knowledge (i.e., the precision of the private signals α increases). Intuitively, this effect should be beneficial for increasing the probability of reaching the correct decision. Therefore, we would like to highlight that the key message of this paper is that in a voting environment with both private and public information, the voting procedure matters and the optimal voting rule should reflect the content of the public information. For example, if the advisory board indicates that one of the business proposals is more promising than the other, it might be desirable for the board of directors to set up a voting rule that is more in favor of the acceptance of that proposal. The design of optimal mechanisms in more general social choice environments with public information remains an open and important research question.

Appendix: Proofs

Proof of Proposition 1

We prove a more general version of Proposition 1 by allowing the prior distribution of the state to be non-uniform and the accuracy of the private signals to be state-dependent. Specifically, we assume $\Pr(\theta = 0) = 1 - \Pr(\theta = 1) = \pi \in (0, 1)$, and each of the private signals is independently drawn according to the conditional probability distribution characterized by $\Pr(s_i = 0|\theta = 0) = \alpha_0$ and $\Pr(s_i = 1|\theta = 1) = \alpha_1$, where $\alpha_0, \alpha_1 \in (1/2, 1)$. The results in the main text will then follow by letting $\pi = 1/2$ and $\alpha_0 = \alpha_1 = \alpha$.

For every signal profile $s = (s_1, ..., s_n)$, let $m_s = \sum_{i=1}^{n} s_i$. As an auxiliary result, note that conditional on observing the whole profile of private signals and the public signal, the posterior belief $\pi(s, s_p)$ that a Bayesian agent would assign to the event $\theta = 1$ is given by:

$$\begin{aligned}
\pi(s, s_p) =& \frac{\Pr(s, s_p|\theta = 1)\Pr(\theta = 1)}{\Pr(s, s_p|\theta = 1)\Pr(\theta = 1) + \Pr(s, s_p|\theta = 0)\Pr(\theta = 0)} \\
=& \frac{\alpha_1^{m_s}(1-\alpha_1)^{n-m_s}\beta^{\mathbb{1}_{s_p=1}}(1-\beta)^{\mathbb{1}_{s_p=0}}(1-\pi)}{\alpha_1^{m_s}(1-\alpha_1)^{n-m_s}\beta^{\mathbb{1}_{s_p=1}}(1-\beta)^{\mathbb{1}_{s_p=0}}(1-\pi) + (1-\alpha_0)^{m_s}\alpha_0^{n-m_\mathbf{s}}(1-\beta)^{\mathbb{1}_{s_p=1}}\beta^{\mathbb{1}_{s_p=0}}\pi} \\
=& \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{m_s}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-m_s}\left(\frac{1-\beta}{\beta}\right)^{\mathbb{1}_{s_p=1}}\left(\frac{\beta}{1-\beta}\right)^{\mathbb{1}_{s_p=0}}\left(\frac{\pi}{1-\pi}\right)},
\end{aligned}$$

where the first equality follows from Bayes rule and the second equality follow from the independence assumption of the signals.

We will show that under a given k-voting rule g^k, the informative voting equilibrium exists if and only if

$$\forall i \in \mathcal{I},\ q_i \in \left[\frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k-1}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k+1}\left(\frac{1-\beta}{\beta}\right)\left(\frac{\pi}{1-\pi}\right)}, \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k}\left(\frac{\beta}{1-\beta}\right)\left(\frac{\pi}{1-\pi}\right)}\right].$$

Suppose all agents $j \neq i$ play $v_j(s_j, s_p) = s_j$. Firstly, note that if $v_i(1, 0) = 1$ is rational for agent i, so is $v_i(1, 1) = 1$; similarly, if $v_i(0, 1) = 0$ is rational for agent i, so is $v_i(0, 0) = 0$. Hence, we only need to consider the optimality of the informative voting strategy in the cases where $s_i \neq s_p$. Secondly, agent i is decisive when and only when there are $k-1$ agents who observe a positive signal ($s_j = 1$) and each of the remaining $n-k$ agents observes an opposite signal ($s_j = 0$). Therefore, given $s_i = 1, s_p = 0$ and being pivotal, the posterior probability that agent i

assigns to the event $\theta = 1$ is:

$$\pi_1^0 = \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^k \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k} \left(\frac{\beta}{1-\beta}\right)\left(\frac{\pi}{1-\pi}\right)}.$$

Similarly, given $s_i = 0, s_p = 1$ and being pivotal, the posterior probability that agent i assigns to the event $\theta = 1$ is:

$$\pi_0^1 = \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k-1} \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k+1} \left(\frac{1-\beta}{\beta}\right)\left(\frac{\pi}{1-\pi}\right)}.$$

Hence, to have informative voting as an equilibrium, it is both necessary and sufficient to have $\forall i \in \mathcal{I},\ q_i \in [\pi_0^1, \pi_1^0]$. By letting $\pi = 1/2$ and $\alpha_0 = \alpha_1 = \alpha$, we immediately obtain condition (4.1). □

Proof of Corollary 1

Note that the interval $[\pi_0^1, \pi_1^0]$ as defined in the proof of Proposition 1 is non-empty if and only if

$$\left(\frac{1-\alpha_0}{\alpha_1}\right)^{k-1} \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k+1} \left(\frac{1-\beta}{\beta}\right)\left(\frac{\pi}{1-\pi}\right) \geq \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k} \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k} \left(\frac{\beta}{1-\beta}\right)\left(\frac{\pi}{1-\pi}\right),$$

which is equivalent to

$$\left(\frac{\alpha_0}{1-\alpha_0}\right)\left(\frac{\alpha_1}{1-\alpha_1}\right) \geq \left(\frac{\beta}{1-\beta}\right)^2. \tag{A.1}$$

If the accuracy of the private signals is state-independent, i.e., $\alpha_0 = \alpha_1 = \alpha$, (A.1) is further equivalent to $\alpha \geq \beta$. □

Proof of Corollary 2

Suppose that $\pi = 1/2$, $\alpha_0 = \alpha_1 = \alpha$ and there exists a k-voting rule under which the informative voting equilibrium exists. According to the proof of Proposition 1, the preferences of agents i and j must satisfy

$$q_i, q_j \in \left[\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}}\right]. \tag{A.2}$$

Moreover, (A.2) and $q_i < \frac{1}{1+\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}$ implies

$$\frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2}\frac{1-\beta}{\beta}} < \frac{1}{1+\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \iff k < \frac{n+1}{2}. \tag{A.3}$$

Similarly, (A.2) and $q_j > \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}}$ implies

$$\frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k-n}\frac{\beta}{1-\beta}} > \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}} \iff k > \frac{n+1}{2}. \tag{A.4}$$

Clearly, (A.3) and (A.4) are mutually exclusive. Hence, we can conclude that the informative voting equilibrium does not exist under any k-voting rule. □

Proof of Proposition 2

Given that each agent i observes $\hat{s}_i = s_i + s_p$, we let $\hat{S}_i \equiv \{0,1,2\}$ be each agent's signal space. Therefore, agent i's strategy is now a mapping $\sigma_i : \hat{S}_i \to [0,1]$, where $\sigma_i(\hat{s}_i)$ denotes the probability that agent i will vote $v_i = 1$ when observing $\hat{s}_i \in \hat{S}_i$.

Fix an arbitrary sequence of preference profiles $\{\mathbf{q}^n\}_{n\in\mathbb{N}}$. Suppose, in contradiction, that there exists a sequence of k-voting rules that aggregates information asymptotically, where $\{g^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ and $\{\sigma^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ are the corresponding convergent subsequences of voting rules and equilibria, and $\lim_{\tau\to\infty} k_{n(\tau)}/n(\tau) = \kappa \in [0,1]$. Since for any agent i her posterior belief about the state being 1 is strictly increasing in $\hat{s}_i$, we must have for all $i \in \{1,\dots,n(\tau)\}$ and for all $\tau \in \mathbb{N}$, $\sigma_i^{k_{n(\tau)}}(0) \le \sigma_i^{k_{n(\tau)}}(1) \le \sigma_i^{k_{n(\tau)}}(2)$. Let $Y^\tau \equiv \sum_{i=1}^{n(\tau)} v_i$. For each $\tau \in \mathbb{N}$, we can decompose the conditional expectation of Y^τ as follows:

$$\mathbb{E}\left[Y^\tau \middle| \theta, s_p\right] = \Pr\left(Y^\tau \ge k_{n(\tau)} \middle| \theta, s_p\right)\mathbb{E}\left[Y^\tau \middle| Y^\tau \ge k_{n(\tau)}, \theta, s_p\right] + \Pr\left(Y^\tau < k_{n(\tau)} \middle| \theta, s_p\right)\mathbb{E}\left[Y^\tau \middle| Y^\tau < k_{n(\tau)}, \theta, s_p\right].$$

Now consider the case $\theta = 1$ and $s_p = 0$, which occurs with probability $(1-\beta)/2 > 0$. In this case, along the sequence of the equilibria $\sigma^{k_{n(\tau)}}$ agent i will cast the right vote ($v_i = 1$) with probability $\alpha\sigma_i^{k_{n(\tau)}}(1) + (1-\alpha)\sigma_i^{k_{n(\tau)}}(0)$. Therefore, given the sequences of equilibira and voting rules, we have

$$\mathbb{E}\left[Y^\tau \middle| \theta = 1, s_p = 0\right] = \sum_{i=1}^{n(\tau)} \left[\alpha\sigma_i^{k_{n(\tau)}}(1) + (1-\alpha)\sigma_i^{k_{n(\tau)}}(0)\right].$$

Thus, for the probability of reaching the right decision ($d = 1$) converging to 1 along this path, that is $\Pr\left(Y^\tau \geq k_{n(\tau)} \middle| \theta = 1, s_p = 0\right) \to 1$, we must have

$$\lim_{\tau\to\infty} \mathbb{E}\Big[Y^\tau | \theta = 1, s_p = 0\Big] = \lim_{\tau\to\infty} \mathbb{E}\Big[Y^\tau | Y^\tau \geq k_{n(\tau)}, \theta = 1, s_p = 0\Big].$$

Together with the monotonicity condition $\sigma_i^{k_{n(\tau)}}(0) \leq \sigma_i^{k_{n(\tau)}}(1)$, this further implies that

$$\lim_{\tau\to\infty} \frac{\sum_{i=1}^{n(\tau)} \alpha \sigma_i^{k_{n(\tau)}}(1)}{n(\tau)} \geq \lim_{\tau\to\infty} \frac{k_{n(\tau)}}{n(\tau)} = \kappa. \tag{A.5}$$

Next, consider the case $\theta = 0$ and $s_p = 1$, which also occurs with probability $(1-\beta)/2 > 0$. In this case, along the sequence of the equilibria $\sigma^{k_{n(\tau)}}$ agent i will cast the right vote ($v_i = 0$) with probability $\alpha(1 - \sigma_i^{k_{n(\tau)}}(1)) + (1-\alpha)(1 - \sigma_i^{k_{n(\tau)}}(2))$. Hence, similar the previous case, for the probability of reaching the right decision ($d = 0$) converging to 1 along this path, that is, $\Pr\left(Y^\tau < k_{n(\tau)} \middle| \theta = 0, s_p = 1\right) \to 1$, it is necessary to have

$$\lim_{\tau\to\infty} \frac{\sum_{i=1}^{n(\tau)} \alpha(1 - \sigma_i^{k_{n(\tau)}}(1))}{n(\tau)} \geq 1 - \kappa \Longleftrightarrow \lim_{\tau\to\infty} \frac{\sum_{i=1}^{n(\tau)} \alpha \sigma_i^{k_{n(\tau)}}(1)}{n(\tau)} \leq (\alpha - 1) + \kappa. \tag{A.6}$$

Since $\alpha < 1$, (A.5) and (A.6) cannot hold simultaneously. We thus reach a contradiction. Therefore, ex ante there must be some non-vanishing probability that the committee will reach a wrong decision even its size becomes arbitrarily large. In other words, no sequence of k-voting rules can aggregate information asymptotically. □

Proof of Proposition 3

We first establish a lemma that fully characterizes ex post incentive compatibility.

Lemma A1. *A mechanism f is ex post incentive compatible if and only if $\forall s_{-i} \in S_{-i}$, $\forall s_p \in S_p$ and $\forall i \in \mathcal{I}$,*

(i) $f(1, s_{-i}, s_p) \geq f(0, s_{-i}, s_p)$, *and*

(ii) $\pi(0, s_{-i}, s_p) > q_i$ *or* $\pi(1, s_{-i}, s_p) < q_i \Longrightarrow f(0, s_{-i}, s_p) = f(1, s_{-i}, s_p)$.

Proof of Lemma A1. Given the specification of the agents' payoff functions, the ex post incentive compatibility constraints can be equivalently rewritten as follows:

$$\Big(f(s_i,s_{-i},s_p)-f(s_i',s_{-i},s_p)\Big)\Big(\pi(s_i,s_{-i},s_p)-q_i\Big)\geq 0, \tag{A.7}$$

for all $s_i,s_i'\in S_i$, $s_{-i}\in S_{-i}$, $s_p\in S_p$ and $i\in\mathcal{I}$, where $\pi(s_i,s_{-i},s_p)$ is interpreted as agent i's posterior belief about the event $\theta=1$ after she knows that the actual signal profile is (s_i,s_{-i},s_p). The sufficiency part of the lemma then immediately follows.

Let us now prove the necessity part. For (i), suppose, in contradiction, that there exist some $i\in\mathcal{I}$, $s_{-i}\in S_{-i}$ and $s_p\in S_p$ such that $f(0,s_{-i},s_p)>f(1,s_{-i},s_p)$. By (A.7), we must have

$$\pi(1,s_{-i},s_p)-q_i\leq 0\leq\pi(0,s_{-i},s_p)-q_i,$$

which contradicts to $\pi(1,s_{-i},s_p)>\pi(0,s_{-i},s_p)$. Hence, we have $f(1,s_{-i}s_p)\geq f(0,s_{-i},s_p)\ \forall s_i,s_i'\in S_i$, $s_{-i}\in S_{-i}$, $s_p\in S_p$ and $i\in\mathcal{I}$. To prove (ii), note that together with (A.7) either $\pi(0,s_{-i},s_p)>q_i$ or $\pi(1,s_{-i},s_p)<q_i$ would imply that $f(0,s_{-i},s_p)-f(1,s_{-i},s_p)\geq 0$. By implication (i) of EPIC, we further have $f(0,s_{-i},s_p)=f(1,s_{-1},s_p)$. □

In our setting, it is straightforward to check that anonymity is equivalent to requiring that $\forall s_p\in S_p$ and $\forall s,s'\in S$, $m_s=m_{s'}$ implies $f(s,s_p)=f(s',s_p)$. Therefore, we can use $f(m,s_p)$ to denote the allocation rules for anonymous mechanisms, where m is the number of agents who report $s_i=1$. Since the posterior belief $\pi(s,s_p)$ is also symmetric in every private signal, we also write $\pi(m,s_p)$ as the posterior belief that a Bayesian agent will assign to the event $\theta=1$ when observing a signal profile (s,s_p) with $m_s=m$. Lemma A1 then implies that an anonymous mechanism is ex post incentive compatible if and only if $\forall m\in\mathcal{I}\equiv\{1,\ldots,n\}$ and $\forall s_p\in S_p$, we have (i) $f(m,s_p)$ being non-decreasing in m, and (ii) $f(m,s_p)=f(m-1,s_p)$ if either $\pi(m-1,s_p)>\max_{i\in\mathcal{I}}q_i$ or $\pi(m,s_p)<\min_{i\in\mathcal{I}}q_i$. Hence, for every A-EPIC mechanism f, one can find a partition $\{\mathcal{I}_1^f,\ldots,\mathcal{I}_{L_f}^f\}$ of $\mathcal{I}$, such that for all $m\in\mathcal{I}_\ell^f$ and $m'\in\mathcal{I}_{\ell'}^f$, $m>m'$ if $\ell>\ell'$, and $f(m,s_p)=f(m',s_p)$ if and only if $\ell=\ell'$.

Now consider any optimal A-EPIC mechanism f. Let us construct two threshold values m_0 and m_1 as follows: If $\pi(n,0)<1/2$, let $m_0=n+1$. Otherwise, we consider ℓ_0, the smallest $\ell\in\mathcal{L}^f\equiv\{1,\ldots,L_f\}$ that satisfies $\mathbb{E}[\theta=1|m_s\in\mathcal{I}_\ell^f,s_p=0]\geq 1/2$. We then let m_0 be the smallest element in $\mathcal{I}_{\ell_0}^f$. Similarly, we let $m_1=0$ if $\pi(0,1)>1/2$. Otherwise, we let m_1 be the smallest element in $\mathcal{I}_{\ell_1}^f$, where ℓ_1 is the smallest $\ell\in\mathcal{L}^f$ that satisfies $\mathbb{E}[\theta=1|m_s\in\mathcal{I}_\ell^f,s_p=1]\geq 1/2$, and .

By the optimality of f, we must have $f(m,0)=0$ for all $m<m_0$. Otherwise, we can further

decrease $f(m,0)$ for all $m < m_0$, which strictly increases $\Pr(d = \theta | s_p = 0)$ without violating any of the incentive compatibility constraints. This contradicts to that f being optimal. If $\pi(m_0, 1) > 1/2$, we can use a similar argument to conclude that the optimality of f implies $f(m,0) = 1$ for all $m \geq m_0$. It is, however, possible to have $f(m_0, 0) \in (0,1)$ if $\pi(m_0, 0) = 1/2$ and $\mathcal{I}^f_{\ell_0} = \{m_0\}$. But in this case, note that the optimality of f still implies that $f(m,0) = 1$ for all $m > m_0$. Hence, increasing $f(m_0, 0)$ to 1 will not violate any incentive compatibility constraint and $\Pr(d = \theta | m_s = m_0, s_p = 0)$ will remain unchanged. Therefore, we can assume without loss that in an optimal A-EPIC mechanism, $f(m,0) = 1$ if $m \geq m_0$. Similarly, we can also conclude from the optimality of f that it is without loss to require $f(m,1) = 0$ for all $m < m_1$, and $f(m,1) = 1$ for all $m \geq m_1$.

Finally, fix a preference profile $\mathbf{q}$ and consider any optimal A-EPIC mechanism f that is characterized by the two threshold values m_0 and m_1. Consider the contingent k-voting rule g^{k_0,k_1} with $k_0 = m_0$ and $k_1 = m_1$. It is straightforward to check that this g^{k_0,k_1} implements informative voting, since the corresponding direct mechanism f is ex post incentive compatible. The fact that $k_0 = m_0$ and $k_1 = m_1$ makes sure that the informative voting equilibrium under g^{k_0,k_1} will achieve the same $\Pr(d = \theta | s, s_p)$ for every $(s, s_p) \in S \times S_p$ as the truth-telling equilibrium under mechanism f. The construction of k_0 and k_1 also makes it clear that g^{k_0,k_1} is responsive if and only if f is responsive. □

Proof of Proposition 4

To prove (i), fix a conflict-preserving sequence $\{\mathbf{q}^\tau\}_{\tau \in \mathbb{N}}$ and pick any element $\mathbf{q}^\tau$ from it. From Proposition 5, we know that for the preference profile $\mathbf{q}^\tau$, there exists a responsive contingent k-voting rule that implements informative voting if and only if there exists a pair of integers $k_0^\tau, k_1^\tau \in \{1, \dots, n + 2\tau\}$ such that

$$k_0^\tau \in K_0^\tau = \left[(\pi_1^0)_\tau^{-1}(\bar{q}^\tau), (\pi_0^0)_\tau^{-1}(\underline{q}^\tau)\right], \text{ and } k_1^\tau \in K_1^\tau = \left[(\pi_1^1)_\tau^{-1}(\bar{q}^\tau), (\pi_0^1)_\tau^{-1}(\underline{q}^\tau)\right],$$

where

$$(\pi_0^0)_\tau^{-1}(\underline{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + 2 + r\right), \quad (\pi_1^0)_\tau^{-1}(\bar{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + r\right),$$

$$(\pi_0^1)_\tau^{-1}(\underline{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + 2 - r\right), \quad (\pi_1^1)_\tau^{-1}(\bar{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau - r\right).$$

Let $\mathbf{q}^0 \equiv \mathbf{q}$ and suppose that there exists $k_0 \in \{1,...,n\} \cap K_0^0$. Since $\ln\left(\frac{1-\underline{q}}{\underline{q}}\right) \leq \ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right) \leq \ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right) \leq \ln\left(\frac{1-\bar{q}}{\bar{q}}\right)$, we have $k_0+\tau \in \{1,...,n+2\tau\} \cap K_0^\tau$. Similarly, if there exists $k_1 \in \{1,...,n\} \cap K_1^0$, then $k_1+\tau \in \{1,...,n+2\tau\} \cap K_1^\tau$. Moreover, since

$$(\pi_0^0)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^0)_\tau^{-1}(\bar{q}^\tau) = \frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right) - \ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} + 1, \text{and}$$

$$(\pi_0^1)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^1)_\tau^{-1}(\bar{q}^\tau) = \frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right) - \ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} + 1,$$

it is clear that both $(\pi_0^0)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^0)_\tau^{-1}(\bar{q}^\tau)$ and $(\pi_0^1)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^1)_\tau^{-1}(\bar{q}^\tau)$ are strictly less than one if $\underline{q}^\tau < \bar{q}^\tau$. This implies that whenever $\underline{q}^\tau < \bar{q}^\tau$, both the intervals K_0^τ and K_1^τ can contain *at most* one integer. Hence, in this case the contingent k-voting rule that can be used to implement informative voting is unique.[20]

We now proceed to prove (ii). Consider the threshold values

$$k_0^\tau = \left[\frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + r\right)\right]^+, \quad k_1^\tau = \left[\frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau - r\right)\right]^+,$$

where $[x]^+$ denotes the smallest integer that is larger or equal to $x \in \mathbb{R}$. Since for the preference profile $\mathbf{q}^0 = \mathbf{q}$ there exists a responsive contingent-k voting rule that implements informative voting, from our analysis for (i) we can conclude that for every $\tau \in \mathbb{N}$ and preference profile $\mathbf{q}^\tau$, the voting rule $g^{k_0^\tau, k_1^\tau}$ implements informative voting. By Proposition 6, for an efficiency-maximizing social planner who can observe all the $n+2\tau$ private signals and the public signal, it would be optimal to implement $d=1$ if either $s_p = 0$ and there are more than $k_0^{\tau *} = \frac{n+2\tau+1}{2} + \left[\frac{r-1}{2}\right]^+$ private signals equal to 1, or $s_p = 1$ and there are more than $k_1^{\tau *} = \frac{n+2\tau++1}{2} - \left[\frac{r-1}{2}\right]^+$ private signals equal to 1. Otherwise implementing $d=0$ would be optimal.

Now consider the differences

$$\Delta_\tau^0 \equiv \frac{k_0^\tau - k_0^{\tau *}}{n+2\tau} = \frac{\left[\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} + \frac{r-1}{2}\right]^+ - \left[\frac{r-1}{2}\right]^+}{n+2\tau}, \text{ and } \Delta_\tau^1 \equiv \frac{k_1^\tau - k_1^{\tau *}}{n+2\tau} = \frac{\left[\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} - \frac{r+1}{2}\right]^+ + \left[\frac{r-1}{2}\right]^+}{n+2\tau}.$$

[20] The intervals K_0^τ and K_1^τ will contain at least one integer (and at most two) if $\bar{q}^\tau = \underline{q}_\tau = q$. In particular, K_0^τ will contain exactly two integers if and only if $(\pi_1^0)_\tau^{-1}(q)$ is an integer. Similarly, there will be two integers in K_1^τ if and only if $(\pi_1^1)_\tau^{-1}(q)$ is an integer.

Both Δ_τ^0 and Δ_τ^1 are decreasing in τ, and we have $\lim_{\tau\to\infty}\Delta_\tau^0 = \lim_{\tau\to\infty}\Delta_\tau^1 = 0$. This implies that as τ increases and goes to ∞, the ex ante probability that the collective decision made in the informative voting equilibria under $g^{k_0^\tau,k_1^\tau}$ coincides with the social planner's choice is increasing and converges to 1. Finally, for any preference profile $\mathbf{q}^\tau$ in the sequence, whenever a non-responsive contingent k-voting rule $g^{k_0^\tau,k_1^\tau}$ with $\{k_0^\tau, k_1^\tau\}\cap\{0, n+2\tau+1\} \neq \emptyset$ is used all the private information in the committee will be entirely disregarded for at least some realization of the public signal. This implies that the efficiency of the informative voting equilibrium under any non-responsive contingent k-voting rule would be strictly dominated by the social planner's solution.[21] Hence, for sufficiently large τ, it will also be dominated by the informative voting equilibrium under the responsive contingent k-voting rule that we constructed above. As a result, there must exist $\tau^* \in \mathbb{N}$ such that for all $\tau \geq \tau^*$, there exists a responsive contingent k-voting rule that is equivalent to an optimal A-EPIC mechanism.

Finally, we prove (iii). Note that $\forall q \in (0,1)$,

$$\lim_{\tau\to\infty}\frac{(\pi_1^0)_\tau^{-1}(q)}{n+2\tau} = \lim_{\tau\to\infty}\frac{(\pi_0^0)_\tau^{-1}(q)}{n+2\tau} = \lim_{\tau\to\infty}\frac{(\pi_1^1)_\tau^{-1}(q)}{n+2\tau} = \lim_{\tau\to\infty}\frac{(\pi_0^1)_\tau^{-1}(q)}{n+2\tau} = \frac{1}{2}.$$

Hence, after adding sufficiently many members to the committee, the probability that the collective decisions made in the informative voting equilibria under the corresponding responsive contingent k-voting rules coincide with that in the informative voting equilibrium under the simple majority rule becomes arbitrarily close to 1. Since the informative voting equilibrium under the simple majority rule is asymptotically efficient if the agents' private signals are informative ($\alpha > 1/2$), so are the informative voting equilibria under a responsive contingent k-voting rules. □

Proof of Proposition 5

We start from establishing a lemma that characterizes when informative voting can be implemented by a *given* contingent k-voting rule with $k_0, k_1 \in \{1, ..., n\}$, which is in fact a counterpart to Proposition 1.

[21]The assumption of the proposition implies that the public signal would not be so precise that it would be optimal for the social planner to always follow the public signal. Otherwise, for the preference profile $\mathbf{q}$ there cannot be a responsive contingent k-voting rule that implements informative voting.

Lemma A2. *A contingent k-voting rule with* $k_0, k_1 \in \{1,\ldots,n\}$ *implements informative voting if and only if*

$$\forall i \in \mathcal{I},\ q_i \in \left[\max\{\pi_0^0, \pi_0^1\}, \min\{\pi_1^0, \pi_1^1\}\right], \tag{A.8}$$

where

$$\pi_0^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2}\frac{\beta}{1-\beta}}, \quad \pi_1^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n}\frac{\beta}{1-\beta}},$$
$$\pi_0^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2}\frac{1-\beta}{\beta}}, \quad \pi_1^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n}\frac{1-\beta}{\beta}}.$$

Proof of Lemma A2. Suppose $s_p = 1$. Given a responsive contingent k-voting rule g^{k_0,k_1}, the threshold value for choosing $d = 1$ is $k_1 \in \{1,\ldots,n\}$. Assume all agents $j \neq i$ are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent i would assign to the event $\theta = 1$ if $s_i = 0$ or $s_i = 1$ are, respectively:

$$\pi_0^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{k_1-1}\left(\frac{\alpha}{1-\alpha}\right)^{n-k_1+1}\frac{1-\beta}{\beta}} = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2}\frac{1-\beta}{\beta}}$$

and

$$\pi_1^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{k_1}\left(\frac{\alpha}{1-\alpha}\right)^{n-k_1}\frac{1-\beta}{\beta}} = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n}\frac{1-\beta}{\beta}}.$$

Now suppose $s_p = 0$. Under the contingent k-voting rule g^{k_0,k_1}, the threshold value for choosing the decision $d = 1$ is $k_0 \in \{1,\ldots,n\}$. Assume all agents $j \neq i$ are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent i would assign to the event $\theta = 1$ if $s_i = 0$ or $s_i = 1$ are, respectively:

$$\pi_0^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2}\frac{\beta}{1-\beta}} \quad \text{and} \quad \pi_1^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n}\frac{\beta}{1-\beta}}.$$

Hence, the voting rule g^{k_0,k_1} implements informative voting if and only if $\forall i \in \mathcal{I}$, $q_i \geq \max\{\pi_0^0, \pi_0^1\}$ and $q_i \leq \min\{\pi_1^0, \pi_1^1\}$. □

Lemma A2 implies that for the existence of a responsive contingent k-voting rule that implements informative voting, it is necessary and sufficient that there exist $k_0, k_1 \in \{1,\ldots,n\}$ satisfying (A.8). To check whether such integers k_0 and k_1 exist, we first invert the functions π_0^0 and π_1^0 of k_0 and the functions π_0^1 and π_1^1 of k_1 that are defined in the above lemma. This is feasible because all these are strictly increasing functions. We then apply the inverse functions

$(\pi_0^0)^{-1}$ and $(\pi_0^1)^{-1}$ to $\bar{q}$ and $(\pi_1^0)^{-1}$ and $(\pi_1^1)^{-1}$ to $\underline{q}$. It is straightforward to check that if there exist $k_0, k_1 \in \{1,...,n\}$ such that $k_0 \in K_0 \equiv [(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$ and $k_1 \in K_1 \equiv [(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$, then k_0 and k_1 will also satisfy condition (A.8). $\square$.

Proof of Corollary 3

From Proposition 5, we know that for a given preference profile $\mathbf{q}$, there exists a responsive contingent k-voting rule g^{k_0,k_1} that implements informative voting if and only if there exist $k_0 \in \{1,...,n\} \cap K_0$ and $k_0 \in \{1,...,n\} \cap K_1$. When $\bar{q} = \underline{q} = q \in (0,1)$, we have $(\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\bar{q}) = (\pi_0^1)^{-1}(\underline{q}) - (\pi_1^1)^{-1}(\bar{q}) = 1$. Thus, in this case both the intervals K_0 and K_1 will contain *at least* one integer. It remains to show that for sufficiently large n, it is guaranteed that $\{1,...,n\} \cap K_0 \neq \emptyset$ and $\{1,...,n\} \cap K_1 \neq \emptyset$. This is not trivial because the intervals K_0 and K_1 actually also depend on n. Given the remark in footnote 20 and since $(\pi_1^1)^{-1}(q) \leq (\pi_1^0)^{-1}(q)$ and $(\pi_0^1)^{-1}(q) \leq (\pi_1^1)^{-1}(q)$ for all $q \in (0,1)$ and $r \geq 0$, the intersections $\{1,...,n\} \cap K_0$ and $\{1,...,n\} \cap K_1$ are non-empty if

$$(\pi_1^1)^{-1}(q) \geq 0 \Longleftrightarrow \frac{1}{2}\left(\frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n - r\right) \geq 0 \Longleftrightarrow n \geq r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}$$

and

$$(\pi_0^0)^{-1}(q) \leq n+1 \Longleftrightarrow \frac{1}{2}\left(\frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 + r\right) \leq n+1 \Longleftrightarrow n \geq r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}.$$

Let

$$\bar{n}(q) = \left[\max\left\{r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}, r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}\right\}\right]^+.$$

We can now conclude that when agents' preference are perfectly aligned, there exists a threshold value $\bar{n}(q)$, such that for all $n \geq \bar{n}(q)$, there exists a responsive contingent k-voting rule that implements informative voting. $\square$

Proof of Corollary 4

Plugging $k_0 = (n+1)/2 + [(r-1)/2]^+$ in the formulas of π_0^0 and π_1^0 that we obtained in Lemma A2, one can easily verify that for all $r \geq 0$,

$$\max\{\pi_0^0, \pi_1^0\} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{|r-2[(r-1)/2]^+|-1}}.$$

Similarly, with $k_1 = (n+1)/2 - [(r-1)/2]^+$, we have for all $r \geq 0$,

$$\min\{\pi_1^0, \pi_1^1\} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-|r-2[(r-1)/2]^+|+1}}.$$

The result of the corollary thus immediately follows Lemma A2. □

Proof of Proposition 6

Consider a social planner who observes the whole profile of private signals $s = (s_1, ..., s_n)$ and the public signal s_p. Suppose the public signal is r-times more informative than the private signal, where $r \geq 0$. Again we let $m_s = \sum_{i=1}^n s_i$. To maximize the accuracy of his decision, the social planner would choose the following optimal decision rule:

$$d^*(s, s_p) = \begin{cases} 1 & \text{if } m_{\mathbf{s}} - (n - m_{\mathbf{s}}) + r\mathbb{1}_{s_p=1} - r\mathbb{1}_{s_p=0} > 0, \\ \{0, 1\} & \text{if } m_{\mathbf{s}} - (n - m_{\mathbf{s}}) + r\mathbb{1}_{s_p=1} - r\mathbb{1}_{s_p=0} = 0, \\ 0 & \text{if } m_{\mathbf{s}} - (n - m_{\mathbf{s}}) + r\mathbb{1}_{s_p=1} - r\mathbb{1}_{s_p=0} < 0. \end{cases}$$

Under the contingent majority rule, $k(s_p) = \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+$ if $s_p = 1$ and $k(s_p) = \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+$ if $s_p = 0$. First, suppose that $r > n$. In this case, the public signal is so precise that the social planner would find it optimal to always follow it and entirely ignore the private signals, i.e., $d^*(s, s_p) = s_p$ $\forall s \in S$ and $s_p \in S_p$. Meanwhile, we have $k(1) > n$ and $k(0) < 1$, which means that the agents' votes would never count and the contingent majority rule simply replicates the outcome of the obedient voting equilibrium. The statement of the proposition then immediately follows.

Next, suppose $r \leq n$. When $s_p = 1$, in the informative voting equilibrium, $d = 1$ if and only if

$$m_s \geq \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ \iff (n - m_s) - m_s \leq 2\left[\frac{r-1}{2}\right]^+ - 1 \equiv R_1,$$

while when $s_p = 0$, $d = 1$ if and only if

$$m_s \geq \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ \Longleftrightarrow m_s - (n - m_s) \geq 2\left[\frac{r-1}{2}\right]^+ + 1 \equiv R_0.$$

There are four possible scenarios:

1. r is an even integer: $R_1 = r - 1 < r + 1 = R_0$.

2. r is an odd integer: $R_1 = r - 2 < r = R_0$.

3. r is not an integer and $[r]^+$ is even: $R_1 = [r]^+ - 1 < [r]^+ + 1 = R_0$.

4. r is not an integer and $[r]^+$ is odd: $R_1 = [r]^+ - 2 < [r]^+ = R_0$.

Since $|m_s - (n - m_s)|$ is odd, the above four inequalities jointly show that the decision achieved by the contingent majority rule always coincides with the planner's choice. □

Proof of Proposition 7

Consider the following strategy profile of the agents in the two-stage voting game. In the first stage, all agents vote for k_0 if $s_p = 0$, and they all vote for k_1 if $s_p = 1$. In the second stage, if either $s_p = 0$ and $k^* = k_0$, or $s_p = 1$ and $k^* = k_1$, then all agents vote informatively. Since an unilateral deviation from the above first-stage voting strategy will not change the threshold k^* that will be selected to be used in the second stage, we need not specify the agents' contingent strategies for any other case.

We thus need only to check whether the agents indeed have the incentive to vote informatively whenever $(s_p, k^*) \in \{(0, k_0), (1, k_1)\}$. This is the case because the informative voting strategy profile actually constitutes an ex post Nash equilibrium under g^{k_0,k_1}, which implies that no agent would have the incentive to unilaterally deviate from informative voting regardless of how she updates her beliefs after observing the first stage voting outcome. Hence, in the two-stage voting game there must exist a Perfect Bayesian Nash equilibrium in which the agents first vote to agree on choosing either k_0 or k_1 (depends on whether $s_p = 0$ or $s_p = 1$), and then they all vote informatively in the second stage. □

Proof of Proposition 8

Since the signal s_p is only observed to the controller and the vote takes place after the agents receive the message from the controller, we have a dynamic game of incomplete information. We look for sequential equilibria, which require the beliefs and the strategies of the players to be sequentially rational and consistent (Fudenberg and Tirole, 1991). Note that since the biased of the controller is not (directly) payoff-relevant to the agents, we need to keep track of agents' beliefs about the state only.

First, consider the scenario where $m = s_p$ is sent. No agent would have the incentive to deviate from obedient voting given all other agents are following the public signal revealed by the controller. This is always the case regardless of the relative precision of the signals.[22]

Now consider the information set where $m = \emptyset$ is sent and the controller's disclosure policy is to withhold his information if and only if he observes $s_p = 1$. Conditional all other agents are voting informatively, voting informatively is optimal for agent i if and only if

$$\begin{aligned}
\Pr(\theta = 1|s_i = 1, m = \emptyset) &= \frac{\frac{1}{2}\alpha(\lambda + (1-\lambda)(1-\beta))}{\frac{1}{2}\alpha(\lambda + (1-\lambda)(1-\beta)) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)\beta)} \\
&\geq \frac{\frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)\beta)}{\frac{1}{2}\alpha(\lambda + (1-\lambda)(1-\beta)) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)\beta)} \\
&= \Pr(\theta = 0|s_i = 1, m = \emptyset)
\end{aligned}$$

and

$$\begin{aligned}
\Pr(\theta = 0|s_i = 0, m = \emptyset) &= \frac{\frac{1}{2}\alpha(\lambda + (1-\lambda)\beta)}{\frac{1}{2}\alpha(\lambda + (1-\lambda)\beta) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)(1-\beta))} \\
&\geq \frac{\frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)(1-\beta))}{\frac{1}{2}\alpha(\lambda + (1-\lambda)\beta) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)(1-\beta))} \\
&= \Pr(\theta = 1|s_i = 0, m = \emptyset).
\end{aligned}$$

Since $\alpha, \beta \geq 1/2$ and $\lambda > 0$, the second inequality always holds. It can be also checked that the first inequality holds if and only if $(\beta - (1-\alpha))\lambda \geq \beta - \alpha$. Hence, whenever $\lambda \geq \hat{\lambda}$, the informative voting strategy profile and the beliefs that are formed according to Bayes rule are sequentially rational for the agents at the information set $m = \emptyset$. By the same token, if the controller only reveals $s_p = 0$ to the agents, no agent can profitably deviate from the proposed strategy profile

[22] In contrast, as implied by Corollary 1 informative voting does not constitute an equilibrium in these subgames whenever $\beta > \alpha$.

as long as $\lambda \geq \hat{\lambda}$ and beliefs are formed according to Bayes rule.

Given the strategies of the agents, suppose the controller observes $s_p = 1$. By revealing this information to the agents, his expected payoff is given by $U_c^r(1) = -q_c(1-\beta)$. On the other hand, withholding this information from the agents yields him an expected payoff of $U_c^{nr}(1) = -q_c(1-\beta)P - (1-q_c)\beta P$, where

$$P = \sum_{k=\frac{n+1}{2}}^{n} C_n^k (1-\alpha)^k \alpha^{n-k}$$

is the probability that the committee reaches a wrong decision when all agents vote informatively. Similarly, by revealing $s_p = 0$ to the agents, the controller's expected payoff is $U_c^r(0) = -(1-q_c)(1-\beta)$, while concealing this information yields him an expected payoff of $U_c^{nr}(0) = -q_c\beta P - (1-q_c)(1-\beta)P$. Hence, the controller would find it optimal to reveal $s_p = 1$ and withhold $s_p = 0$ if $U_c^r(1) \geq U_c^{nr}(1)$ and $U_c^{nr}(0) \geq U_c^r(0)$, which are equivalent to

$$q_c \leq \hat{q} = \min\left\{\frac{\beta P}{(1-\beta)(1-P)+\beta P}, \frac{(1-\beta)(1-P)}{(1-\beta)(1-P)+\beta P}\right\}.$$

Similarly, revealing $s_p = 0$ and withholding $s_p = 1$ is optimal for the controller if

$$q_c \geq 1-\hat{q} = \max\left\{\frac{\beta P}{(1-\beta)(1-P)+\beta P}, \frac{(1-\beta)(1-P)}{(1-\beta)(1-P)+\beta P}\right\}.$$

The threshold value $\hat{q}$ achieves its supremum at $P = 1-\beta$, which equals to $1/2$. Also, since $\alpha > 1/2$, it is straightforward to check that $P < 1/2$ and, hence, $\hat{q} \geq \min\{\beta, 1-\beta\} = 1-\beta$.

In conclusion, if $\lambda \geq \hat{\lambda}$ and $q_c \leq \hat{q}$ (or $q_c \geq 1-\hat{q}$), the strategy profile stated in the proposition together with the beliefs formed according to Bayes rule constitute a Perfect Bayesian Equilibrium. Since all information sets can be reached with positive probability, it is also a sequential equilibrium. □

Proof of Proposition 9

First, consider the scenario where $m = s_p$ is sent. By the same reasoning as in Corollary 4, no agent would have the incentive to deviate from informative voting given all other agents are voting informatively.

Next, consider the information set where $m = \emptyset$ is sent and suppose the strategy of controller is such that he never withholds information. In this case, the controller's message is not informative at all and given that the agents are unbiased and the voting threshold corresponds to the simple majority rule, the informative voting strategy profile along with the beliefs formed according to Bayes rule are clearly sequentially rational for the agents. Now suppose the controller's strategy is such that he will reveal his information to the agents if and only if $s_p = 1$ (or $s_p = 0$). By Proposition 8, we know that in this case no agent can profitably deviate from informative voting provided that $\lambda \geq \hat{\lambda}$.

Given that the agents will always vote informatively, suppose that the controller observes $s_p = 1$. By withholding the signal, the controller obtains an expected payoff of $U_c^{nr}(1) = -q_c(1-\beta)P - (1-q_c)\beta P$, while revealing yields $U_c^r(1) = -q_c(1-\beta)P' - (1-q_c)\beta\tilde{P}$, where

$$P' = \sum_{k=\frac{n+1}{2}-\left[\frac{r-1}{2}\right]^+}^{n} C_n^k(1-\alpha)^k\alpha^{n-k},\ \tilde{P} = \sum_{k=\frac{n+1}{2}+\left[\frac{r-1}{2}\right]^+}^{n} C_n^k(1-\alpha)^k\alpha^{n-k}.$$

Similarly, by concealing $s_p = 0$ from the agents, the controller's expected payoff is given by $U_c^{nr}(0) = -q_c\beta P - (1-q_c)(1-\beta)P$, while revealing yields him an expected payoff of $U_c^r(0) = -q_c\beta\tilde{P} - (1-q_c)(1-\beta)P'$. Hence, the controller would find it optimal to always share his information with the agents if $U_c^r(1) \geq U_c^{nr}(1)$ and $U_c^r(0) \geq U_c^{nr}(0)$. Note that these inequalities trivially hold for all $q_c \in [0,1]$ if $[(r-1)/2]^+ = 0$ or, equivalently, $\beta \leq \alpha$. Now suppose $[(r-1)/2]^+ \geq 1$. Then, it is straightforward to check that these two inequalities are satisfied if and only if $q_c \in [q^*, 1-q^*]$, where

$$q^* = \frac{(1-\beta)(P'-P)}{\beta(P-\tilde{P}) + (1-\beta)(P'-P)} \geq 0.$$

Since $\alpha \geq 1/2$ and $C_n^k = C_n^{n-k}$, we have

$$\frac{P-\tilde{P}}{P'-P} = \frac{\sum_{k=\frac{n+1}{2}}^{\frac{n+1}{2}+\left[\frac{r-1}{2}\right]^+-1} C_n^k(1-\alpha)^k\alpha^{n-k}}{\sum_{k=\frac{n+1}{2}-\left[\frac{r-1}{2}\right]^+}^{\frac{n+1}{2}-1} C_n^k(1-\alpha)^k\alpha^{n-k}} \leq 1$$

and, hence, $q^* \leq 1-\beta$. Clearly, together with the beliefs formed according to Bayes rule, the proposed strategies for the controller (always share his information) and the agents (always vote informatively) constitute a Perfect Bayesian equilibrium. It is also a sequential equilibrium since all information set can be reached with positive probability in equilibrium. We thus have proven the first part of the proposition. The proof of the second part of the proposition is analogous, and we omit it here to avoid repetition. □

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