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Money and Information

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Abstract

This paper investigates the role of fiat money in decentralized markets, where producers have private information about the quality of the goods they supply. Money is divisible, terms of trade are determined endogenously, and agents can finance their consumption with money or with real production. When the fraction of high quality producers in the economy is given, money promotes the production of high-quality goods, which improves the quality mix and welfare unambiguously. When this fraction is endogenous, however, we find that money can be valued even though it decreases welfare relative to the barter equilibrium. The origin of this inefficiency is that money provides consumption insurance to low-quality producers, which can result in a higher fraction of low-quality producers in the monetary equilibrium. Finally, we find that most often agents acquire more information in the monetary equilibrium. Consequently, money is welfare-enhancing because it promotes useful production and exchange, but not because it saves information costs.

Keywords: Money, search, adverse selection, moral hazard.

JEL: E00, D83, E52

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‘‘The recognition that information is imperfect, that obtaining information can be costly, that there are important asymmetries of information, and that the extent of information asymmetries is affected by actions of firms and individuals ... has provided explanations of economic and social phenomena that otherwise would be hard to understand.’’
Joseph Stiglitz (2000)

1 Introduction

This paper studies the role of money in decentralized markets where agents have private information about the quality of the goods they supply. We address three issues that are at the crossroads of the economics of information and the pure theory of money (see the Joseph Stiglitz quotation above). First, we explore how money affects the supply of high-quality goods (the quality mix) when the fraction of high-quality producers in the economy is exogenous. This question is related to Akerlof’s (1970) paper on “lemons” and the adverse selection problem. Second, we endogenize the fraction of high-quality producers and consider how money affects agents’ incentives to engage in opportunistic behavior, i.e., to produce lemons. Along the lines of Williamson and Wright (1994), this analysis addresses the role of money in alleviating the moral hazard problem. Third, we consider how money affects agents’ decision to acquire information. We investigate the validity of Brunner and Meltzer’s (1971) and King and Plosser’s (1986) claim that money is a substitute for information acquisition.¹

To analyze these issues, we build a search model of money along the lines of Shi (1999) and Berentsen and Rocheteau (2000a), where the exchange process and the formation of the terms of trade are made explicit. Money and goods are perfectly divisible, and the terms of trade are determined in bilateral meetings through alternating offer bargaining games. In contrast to most matching models of money, all agents in the market are endowed with money and production opportunities, which allows them to finance their purchases through real production, money, or both. In order to abstract from the double coincidence problem, we assume that a commodity of a given quality provides the same utility to all agents. This setting allows us to focus on private information, which, according to Alchian (1977), is the principal friction underlying the institution of monetary exchange.

We first show how money affects the terms of trade, that is, the quantities produced

¹There is a voluminous literature that studies the functioning of markets with asymmetric information. For a survey, see Stiglitz (1987). There also are a large number of articles that attempt to explain the use of money. There are only a few articles that connect these two fields. Among them are Alchian (1977), Banerjee and Maskin (1996), Brunner and Meltzer (1971), King and Plosser (1986), Trejos (1999), and Williamson and Wright (1994).

and consumed in each meeting. We identify two effects. The first effect is related to the recognizability property of money. With valued money, sellers can ask to be paid with money, an object of universally recognizable quality, instead of with goods of uncertain quality. This possibility crowds out the use of real production as a means to finance consumption. This crowding out of real goods payments by monetary payments is what we call the *recognizability effect* of money.

The second effect is the *insurance effect* of money. With money, buyers consume more relative to what they consume in the barter economy because money disconnects what buyers can buy from how they are perceived. In this sense, money acts as consumption insurance. In particular, this insurance allows low-quality producers to consume even when they are recognized as lemon producers. In contrast, in a barter economy, recognized low-quality producers cannot consume, because low-quality goods are worthless.

The first part of the paper investigates the adverse selection problem. We assume that the fraction of high-quality producers is exogenous. The supply of low- and high-quality goods, however, is endogenous, because the quantities produced and consumed are negotiated between agents in bilateral meetings. The key result of this section is that both an increase in the real value of money and an increase in the level of information improve the quality mix.² We also show that moving from barter to monetary exchange is strictly welfare-improving, because money promotes the production of high-quality goods and reduces the production of lemons. Thus, money is a device to partially overcome the adverse selection problem that arises in barter.

In the second part of the paper, we endogenize the fraction of high-quality producers to study how money affects agents' incentives to become either high- or low-quality producers. We assume that prior to each match all agents choose the quality of the good they will supply and that this decision is irrevocable once the match is formed. The recognizability effect raises the benefit of being a high-quality producer, and therefore induces agents to produce high-quality goods more often. In contrast, the insurance effect increases the benefit of being a low-quality producer. This effect, therefore, induces agents to take more risks, that is, to become lemon producers more often. Consequently, money can exacerbate the *moral hazard problem*.

Which effect dominates depends on the severity of the information problem. If the information problem is severe, that is, if most often agents do not recognize the quality of the goods, then the recognizability effect of money dominates the insurance effect and the fraction of high-quality producers is larger in the monetary equilibrium than in the barter equilibrium. When the level of information increases, the recognizability effect becomes relatively less important and is eventually dominated by the insurance effect. As

²In this sense, money and information are substitutes. The quality mix is the ratio of the production of high-quality goods to the production of all goods.

a consequence, if the information problem is not too severe, the fraction of high-quality producers is smaller in the monetary equilibrium than in the barter equilibrium. Moreover, for some parameter values welfare is strictly lower in the monetary equilibrium than in the barter equilibrium. Finally, if information is abundant, no monetary equilibrium exists, whereas a barter equilibrium exists where all agents produce high-quality goods: there is no need for money in an economy where nobody cheats.

In the third part of the paper, we endogenize the level of information by allowing agents to invest in a costly inspection technology. This investment improves their ability to recognize the quality of the goods supplied in the market. We find that there is always a positive fraction of cheaters in equilibrium. Hence, heterogeneous quality is a natural outcome when the information structure of the economy is endogenized. We show that for most parameter values agents invest more in information in the monetary economy than in the barter economy. Moreover, when information costs are high there is no active barter equilibrium, whereas an active monetary equilibrium always exists. Thus, in contrast to Brunner and Meltzer (1971) and King and Plosser (1986), our model suggests that information acquisition and money are complements.³ The basis for this result is that money by providing consumption insurance increases the return of information, which induces agents to acquire more information than in the barter equilibrium.

Our paper is most closely related to the random-matching models of money of Williamson and Wright (1994), Kim (1996), and Trejos (1999).⁴ Like Williamson and Wright (1994), we consider an environment where in the absence of private information there is a double coincidence of real wants in each meeting. This allows us to abstract from the double coincidence of real wants problem that is most often used to explain money (e.g. Kiyotaki and Wright (1991, 1993)), and to focus on asymmetric information as an explanation for why agents use fiat money. In contrast to Williamson and Wright (1994) and Kim (1996), who consider environments where both money and goods are indivisible and where agents can hold at most one object at a time, we have divisible money, divisible goods, and no inventory restrictions on money holdings. In contrast to Trejos (1999), in our analysis money is perfectly divisible and the cost of cheating is endogenous because it depends on the equilibrium terms of trade. Furthermore, Trejos (1999) rules out barter trades, which implies that money has a welfare-improving role even in the absence of a private information problem. Finally, Banerjee and Maskin (1996) consider the adverse selection problem

³Quoting Brunner and Meltzer (1971, p.799): “For individuals, money is a substitute for investment in information and labor allocated to search. By using money, individuals reduce the amount of information they must acquire, process, and store.”

⁴Private information problems in search models of money have also been studied by Cuadras-Morató (1994), Li (1995), Green and Weber (1996), Haegler (1997), Trejos (1997), and Velde et al. (1999). All these papers, however, concentrate on issues quite different from the ones we treat.

in a Walrasian framework, where each good can be produced in two qualities. They show that the commodity that has the smallest discrepancy between the two qualities will emerge endogenously as the medium of exchange. In this sense, they derive rather than assume the recognizability property of money.

In contrast to most random-matching models of money, our framework does not rely on the indivisibility of money and inventory restrictions. In particular, divisible money allows us to study the effects of changes in the growth rate of the money supply on the quality mix and the incentive to produce high-quality goods. Interestingly, it also greatly simplifies the typology of equilibria. In particular, under the Friedman rule we generically find uniqueness of the monetary equilibrium. Another important difference with respect to the standard search literature is that we allow matched agents to finance their consumption with money, real production, or both, which makes it clear that matched agents face neither an explicit nor an implicit cash-in-advance constraint. Moreover, divisible money breaks the artificial link between the quantity of money and the fraction of buyers. In fact, an important characteristic of the model is that money is neutral but not superneutral.⁵

The remainder of the paper is organized as follows. In Section 2 we present the environment. Section 3 describes the equilibrium and identifies the recognizability effect and the insurance effect of money. Section 4 addresses the adverse selection problem when the fraction of high-quality producers is exogenous. In Section 5, the fraction of lemon producers is endogenized. Section 6 endogenizes the information level in the economy. Section 7 concludes.

⁵In contrast to most search models of money, in our divisible money framework money is neutral, and without search externalities or private information the Friedman rule holds (see Berentsen 2001). Therefore, our model is not subject to Banerjee and Maskin's (1996, p. 960) criticism that "one may run into trouble when basing the theory of money on search frictions while relying on the Walrasian model for one's other macroeconomic intuitions."

2 Environment

We consider a random-matching model with divisible goods and divisible money along the lines of Shi (1999). The economy is populated with a large number of infinitely-lived households, each consisting of a continuum of members of measure one that regard the household's utility as the common objective. In the market, household members attempt to exchange money or their production good for consumption goods. In this attempt household members follow the strategy that has been given to them by their households. After trading, household members return home, where they pool their money holdings. This assumption allows us to abstract from distributional issues and to focus on the effect of private information on the formation of the terms of trades and on the incentive to produce high-quality goods within a representative household framework.⁶

We consider symmetric equilibria only, where all households consume and produce the same quantities. In the following we refer to an arbitrary household as household h . Decision variables of this household are denoted by lowercase letters. Capital letters denote other households' variables, which are taken as given by the representative household h . Because we will only consider the steady state equilibria where all real variables are constant, we most often omit the time index. Nevertheless, because in a steady state nominal variables are not necessarily constant, the index $+1$ refers to a variable at the following period, and the index -1 to the variable at the previous period.

2.1 Technology preferences

There is a continuum of nonstorable goods, where each good can be produced in low or high quality. We assume that the quality of goods is an inspection attribute: the only way to discover the quality of a good is to consume it. Household h has the technology to produce one good, and it derives utility from consuming *all* high-quality goods other than its production good.

Producing q units of a good of high quality yields disutility $c(q) = q$. Producing low-quality goods (lemons) costs nothing. The instantaneous utility of consuming q units of a commodity of high quality is $u(q)$, where $u(q)$ is increasing and twice differentiable, and satisfies $u(0) = 0$, $u'(0) = \infty$, and $u''(q) < 0$. Furthermore, there exists $q^* > 0$ such that $u'(q^*) = 1$. Consuming a lemon generates no utility. Goods cannot be stored, and production is instantaneous. The utility of a household in one period is the sum of the consumption utilities of its members minus their disutility of production. The discount

⁶A family construct of this type was introduced by Lucas (1990). In search models of money it was first used by Shi (1997, 1999).

factor is $\beta \in (0, 1)$.⁷

In sections 3 and 4 we assume that the fraction of household members that produce high quality, denoted by Π , is exogenous and identical across households. Holding Π constant allows us to study the relation between money and the adverse selection problem. In section 5 we endogenize Π to see how money affects this choice.

In addition to the consumption goods, there is also an intrinsically worthless, storable, and fully divisible object called fiat money. At the beginning of each period, each household has m units of money per member.

The chronology of events within a period is as follows. First, the money stock is divided evenly between ex ante identical household members, and each member receives m units of money.⁸ Second, the household members are endowed with the production technology allowing them to produce in the market. Third, they leave the household to search for trading partners. Prior to the matching phase, a fraction Π of all household members receives a technology shock that allows them to produce high-quality goods. The remaining fraction of household members can only produce low-quality goods. Fourth, household members are matched and carry out their exchanges according to the prescribed strategies. Within a period, a member of the household cannot transfer money balances to another member of the same household. After trading, members bring back their receipts of money, and each agent consumes the goods he has bought. At the end of a period, the household receives a lump-sum money transfer τ , which can be negative, and then carries the stock m_{+1} to $t + 1$.

The quantity of money in the economy is assumed to grow at the gross growth rate γ . We restrict γ to be larger than the discount factor β .⁹ The (indirect) marginal utility of money is denoted by ω . It is equal to $\beta V'(m_{+1})$, where $V(m)$ is the steady-state lifetime discounted utility of a household holding m units of money.

2.2 Information

Time is discrete. In each period, household members meet pairwise and at random. We normalize the length of a period so that in each period each household member meets another member. When two traders meet they bargain over the terms of trade. Before we discuss the bargaining game, however, let us determine what information matched traders have and how they use this information to assess the quality of the good produced by their partner.

⁷Throughout the paper, when we present simulations we use the specification $u(q) = \alpha^{-1}q^\alpha$ and $\beta = 0.99$.

⁸Because all household's members have the same level of money holdings, there is no signaling through wealth.

⁹This condition guarantees the existence of a steady state monetary equilibrium.

When matched, the traders receive some information about the quality of the goods produced by their partners that they use to form Bayesian beliefs. In appendix A1 we model this information as a signal that each trader receives prior to bargaining, which is imperfectly correlated with the true characteristic of the good produced by his partner in the match. The signals, which are common knowledge in a match, differ in their reliability taking into account that agents are heterogeneous in their ability to assess the quality of a specific good.¹⁰ We assume that all goods of a given quality (high or low) are ex ante identical in the following sense: They have the same chance that their quality is perceived in a certain way by an individual chosen at random.

Consider agent i who is randomly matched to some partner j . From the point of view of i , the match type is the pair $(\varepsilon_i, \varepsilon_j)$, where $\varepsilon_j \in [0, 1]$ ($\varepsilon_i \in [0, 1]$) denotes the probability that agent j (agent i) attributes to agent i (agent j) to be a high-quality producer. Conditional on the information available to i at the beginning of the period, ε_i and ε_j are two independent random variables. The variable ε_j is distributed according to the cumulative distribution $F_H(\cdot)$ with density $f_H(\cdot)$ if i is a high-quality producer, and according to the cumulative distribution $F_L(\cdot)$ with density $f_L(\cdot)$ if i is a low-quality producer, and ε_i is distributed according to the unconditional distribution $F(\cdot)$ with density $f(\cdot)$ defined as follows:

$$f(x) = \Pi f_H(x) + (1 - \Pi) f_L(x) \quad \forall x \in [0, 1] \quad (1)$$

It is shown in appendix A2 that

$$F_L(x) \geq F_H(x) \quad \forall x \in [0, 1]$$

This first-order stochastic dominance of $F_L(\cdot)$ by $F_H(\cdot)$ reflects the fact that on average agent i will benefit from a better assessment from his partner if he is a high-quality producer rather than a low-quality one. Furthermore, we show in appendix A2 that the density functions $f(\cdot)$ and $f_H(\cdot)$ are related as follows:

$$x f(x) = \Pi f_H(x) \quad \forall x \in [0, 1] \quad (2)$$

Equation (2) is a property of conditional expectations. It states that when making their Bayesian calculations, agents use all relevant information. Equation (2) also implies that $\int_0^1 x dF(x) = \Pi$. Individuals do not make mistakes on average: their beliefs are consistent with the fractions of high-quality producers in the economy.

Equation (2) can be used to express the means of the distributions $F_L(\cdot)$ and $F_H(\cdot)$, denoted by $\mathbb{E}_L[\varepsilon_j]$ and $\mathbb{E}_H[\varepsilon_j]$, as a function of the information that is available to the

¹⁰One way to think about this information structure is related to Alchian (1977), who suggested that each agent is a specialist in some goods in the economy and a novice in others. A specialist in a good recognizes the quality of the good with a high probability, a novice with a low probability only.

agents when they bargain. To see this, multiply each side of equation (2) by x and integrate with respect to x to get

$$\mathbb{E}_\chi[\varepsilon_j] = \theta \mathbf{1}_{\{\chi=H\}} + (1 - \theta) \Pi \quad \chi = H, L \quad (3)$$

where $\mathbf{1}_{\{\chi=H\}}$ is the indicator function that is equal to one if $\chi = H$, and where θ is a function of the variance σ_f^2 of the distribution $F(\cdot)$ that satisfies

$$\theta = \frac{\sigma_f^2}{\Pi(1 - \Pi)} \quad (4)$$

According to (3), the mean of the distribution $F_\chi(\cdot)$ is a weighted mean of the beliefs of informed agents ($\varepsilon_j = 1$ if $\chi = H$, and $\varepsilon_j = 0$ if $\chi = L$) and the beliefs of agents that are completely uninformed (Π). The weight on the beliefs of the informed agents is θ . Therefore, throughout the paper we consider the parameter θ to be a measure of the level of information available in the economy.

An information structure satisfying (1) and (2) is used by Williamson and Wright (1994). With probability θ agent i recognizes perfectly the quality of a commodity ($\varepsilon_i = 1$ or $\varepsilon_i = 0$), and with probability $1 - \theta$ he has no information ($\varepsilon_i = \Pi$). Consequently, the distribution of probabilities f_H , f_L , and f take the following forms:¹¹

$$\begin{aligned} f_H(0) &= 0 & f_H(\Pi) &= 1 - \theta & f_H(1) &= \theta \\ f_L(0) &= \theta & f_L(\Pi) &= 1 - \theta & f_L(1) &= 0 \\ f(0) &= \theta(1 - \Pi) & f(\Pi) &= 1 - \theta & f(1) &= \theta\Pi \end{aligned} \quad (5)$$

Throughout the paper we use this information structure when we derive closed form solutions or when we present simulations. It can be checked from (5) that θ satisfies (4).

2.3 Bargaining

Terms of trade are determined in alternating offer bargaining games. In what follows we consider the bargaining between agents i and j . The match type is characterized by the initial beliefs of each player about the quality produced by his partner, that is the match type is $\varepsilon = (\varepsilon_i, \varepsilon_j)$. Consequently, the space of match types is $E = [0, 1]^2$.

2.3.1 Rules of the game¹²

Suppose that it is agent i 's turn to make an offer and that he proposes the terms of trade $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$, where q_ε^b is the quantity of goods produced by agent j and consumed by

¹¹Because the space of beliefs is discrete, $f(\cdot)$, $f_H(\cdot)$, and $f_L(\cdot)$ are not density functions.

¹²We formalize the bargaining game as closely as possible to Berentsen and Rocheteau (2000a). This allows us to use several results of that paper in the following. A more detailed description of the bargaining game in the barter economy is available at <http://www-vwi.unibe.ch/staff/berentsen/aleks.htm>.

agent i , q_ε^s is the quantity of goods delivered by agent i , and x_ε is the quantity of money exchanged. If $x_\varepsilon > 0$, agent i delivers x_ε units of money to agent j , and if $x_\varepsilon < 0$, he receives x_ε units of money. The subscript ε indicates that these quantities will depend on the beliefs $\varepsilon = (\varepsilon_i, \varepsilon_j)$. These beliefs are common knowledge in the match.

Two important features of the bargaining are that (i) bargaining strategies are determined at the household level but are carried out by household members, and (ii) households' strategies depend on the match type ε and on the distribution of their potential bargaining partners' characteristics, which is degenerate in equilibrium. They do not depend on the specific level of money holdings of the partner in the match.¹³

The bargaining proceeds as follows. Each period is divided into an infinite number of subperiods of length Δ where Δ is small. If, in a given subperiod, it is agent i 's turn to make an offer and agent j rejects the offer, in the following subperiod it is agent j 's turn to make a counteroffer. If an offer is refused, the negotiation breaks down with probability $\delta\Delta$ ($\delta > 0$). The possibility of an exogenous breakdown of the negotiation gives an incentive to traders to agree immediately. Furthermore, we assume that the players can always retract their offers and that if an offer is retracted, the game ends.¹⁴ Allowing retractable offers guarantees that no agent is forced to trade after he has recognized his partner is a low-quality producer.

When two randomly chosen agents i and j meet, they do not know whether their partner is a high-quality or a low-quality producer. Hence, the bargaining game is a game with two-sided incomplete information. In the following, we argue that an equilibrium cannot be a separating equilibrium where high- and low-quality producers are recognized by their offers or acceptance rules in the bargaining game. Suppose, to the contrary, that there is a separating equilibrium where at some stage of the game low-quality producers are recognized with certainty. In this equilibrium, they could never trade, because there is no gain from buying a lemon.¹⁵ Consequently, low-quality producers have an incentive to deviate from the strategy that supposedly sustains a separating equilibrium by imitating the strategy of a high-quality producer. Therefore, a separating equilibrium cannot exist.

In the following, therefore, we focus on pooling equilibria. Moreover, we restrict the profile of strategies to be stationary in the following sense. At each information set, a player's strategy only depends on his own belief about his partner's type and the belief of his opponent about his own type. Hence, if in two different informations sets of the game

¹³For a discussion of this assumption, see Berentsen and Rocheteau (2001).

¹⁴We impose the rule that a game ends after a player has retracted his offer for tractability. In general, there is no reason that the players cannot continue to negotiate, since ex-post — after an offer is retracted — they will have an incentive to do so. See Muthoo (1999) for an analysis of retractable offers in bargaining games.

¹⁵Recall that a player can retract his offer at any time, so no trader is ever forced to trade with an agent that is recognized as a low-quality producer.

the beliefs of the two players are equal, and if it is player i 's turn to make an offer, he will make the same offer, and agent j will apply the same acceptance rule.

2.3.2 A refinement of sequential equilibrium

Many sequential pooling equilibria can be sustained, depending on how the players' beliefs after they observe an out of equilibrium move are specified. The reason for this multiplicity is that the concept of sequential equilibrium imposes few restrictions on the formation of the beliefs after an out-of-equilibrium move. Many of these beliefs act as “unreasonable” threats. In order to exclude such beliefs and to attain a unique equilibrium, we impose restrictions regarding the beliefs that agents hold when they observe actions that are not consistent with the equilibrium strategies.¹⁶

The idea of the refinement is that if there is an unexpected move by one player, the responding player evaluates (rationalizes) for which type such a deviation is beneficial if he accepts it. There are two rules: First, we allow for pessimistic conjectures in the following circumstances. If a player deviates with respect to his equilibrium strategies by proposing to supply more goods, and if this proposal lowers the expected payoff of the deviating player if he is a high-quality producer, then the responding agent assumes that the offer is from a low-quality producer with certainty. Second, we do not allow for optimistic conjectures. For all other deviations, the responding agent does not update his belief.

To define and explain the first rule, consider an ε -meeting, and suppose that it is agent i 's turn to make an offer. Denote by $(Q_\varepsilon^b, Q_\varepsilon^s, X_\varepsilon)$ the equilibrium offer — or the offer made by other agents in the same type of meetings — and by $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ the offer by i . Agent j will update his belief after i 's proposal if and only if the following two inequalities are satisfied:

$$\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \Omega < \varepsilon_i u(Q_\varepsilon^b) - Q_\varepsilon^s - X_\varepsilon \Omega \quad (6)$$

$$q_\varepsilon^s > Q_\varepsilon^s \quad (7)$$

Inequality (6) means that agent i 's offer $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ does not increase the surplus of agent i if he is a high-quality producer, and inequality (7) means that agent i proposes to produce more than what others producers propose to supply in the same match types. Inequality (7) captures the fact that the only way for a low-quality producer to improve his situation is by proposing to produce more, because production is costless to him.¹⁷

¹⁶A refinement similar in spirit is described in Osborne and Rubinstein (1990, Section 5.5), or in Cho and Kreps (1987).

¹⁷If producers are constrained to supply a given quantity q_ε^s , both high- and low-quality producers will choose the same values for q_ε^b and x_ε in order to maximize their expected surpluses subject to the acceptance rule of their partner and the constraints on money holdings. Thus, high- and low-quality producers only

Our refinement has the following implications. First, the first rule implies that if an agent has an offer $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ that improves the situation of a high-quality producer compared to $(Q_\varepsilon^b, Q_\varepsilon^s, X_\varepsilon)$, this proposal will not be interpreted as coming from a low-quality producer: He is not punished by the beliefs of his partner. Thus, beliefs that act as “unreasonable” threats are not possible. Second, high-quality producers cannot signal their nature, because their partners never upgrade their initial assessment. This rule comes from our focus on pooling equilibria. Third, low-quality producers always propose to supply Q_ε^s units of their output, and adopt the same acceptance rule than high-quality producers. Fourth, the refinement generates a unique equilibrium. Indeed, under these rules high-quality producers can always maximize their payoff subject to the reservation value of their partner without the risk of being punished because of some “unreasonable” beliefs. Consequently, there is a unique equilibrium strategy for high-quality producers, which also pins down the equilibrium strategies of low-quality producers.

3 Equilibrium

In this section we describe the program of the household when the fraction of high-quality producers Π is exogenous. We assume that $\Pi < 1$, so that there is a positive fraction of cheaters in the economy.¹⁸

3.1 The offers

In the monetary economy each agent holds m units of money when matched. To derive the offers, without loss of generality, we restrict our attention to an $(\varepsilon_i, \varepsilon_j)$ -meeting between member i of household h and some agent j from another household. Remember that from the point of view of agent j the match type is $\varepsilon' = (\varepsilon_j, \varepsilon_i)$.

In the alternating offer game, offers and counteroffers converge to the same limiting proposal when Δ goes to zero. Consequently, the first-mover advantage vanishes when Δ goes to zero.¹⁹ Because of this and because it facilitates the derivation of the envelope condition, we let members of household h make the first offer in all meetings. In equilibrium all households have the same characteristics: as a consequence, first offers of household h are always accepted. Moreover, because the length of time between two consecutive offers differ in the quantity of goods they would like to supply. Because for low-quality producers production is costless, they would like to produce unbounded quantities.

¹⁸A monetary equilibrium with $\Pi = 1$ does not exist, because there is no need for money in an economy where nobody cheats and where in each meeting there is a symmetric double coincidence of real wants.

¹⁹This argument is standard in the bargaining literature. See, for example, Muthoo (1999, chapter 3), Osborne and Rubinstein (1990, chapter 3).

is infinitesimal, the first offers are equal to the counteroffers that would have been made by h 's partners.

Agent i is a high-quality producer. Suppose that i proposes the terms of trade $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$, and denote by Ω the marginal value of money of other households (including j). If agent j accepts the offer $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ and if $x_\varepsilon > 0$, the acquired amount of money x_ε will add to the money balances of j 's household at the beginning of the next period, whose value today is Ωx_ε . If $R_{\varepsilon'}$ denotes the reservation value of a high-quality producer from another household, any optimal offer $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ must make a high-quality producer just indifferent between accepting or rejecting the offer:

$$\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \Omega = R_{\varepsilon'} \quad (8)$$

By stating this optimality condition we take the following facts into account: First, as discussed in the previous section, as long as high-quality producers maximize their expected surplus in the match subject to the acceptance rule of their partner, inequalities (6) and (7) are never satisfied simultaneously, and consequently according to our refinement ε_j remains unchanged. Thus, when a high-quality producer maximizes his own payoff subject to the reservation value $R_{\varepsilon'}$, he can take the belief ε_j as given. Second, agent i is only concerned whether the offer $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ is acceptable by a high-quality producer. Third, the reservation value $R_{\varepsilon'}$ is taken as given by the household of agent i .

Agent i is a low-quality producer. By similar reasoning, an optimal offer $(\tilde{q}_\varepsilon^b, \tilde{q}_\varepsilon^s, \tilde{x}_\varepsilon)$ by a low-quality producer must satisfy

$$\varepsilon_j u(\tilde{q}_\varepsilon^s) - \tilde{q}_\varepsilon^b + \tilde{x}_\varepsilon \Omega = R_{\varepsilon'} \quad (9)$$

In equilibrium, any offer $(\tilde{q}_\varepsilon^b, \tilde{q}_\varepsilon^s, \tilde{x}_\varepsilon)$ that satisfies (9) and which is distinct from $(Q_\varepsilon^b, Q_\varepsilon^s, X_\varepsilon)$ will satisfy (6). Therefore, our refinement implies that low-quality producers cannot propose to produce more than Q_ε^s otherwise they reveal themselves as low-quality producers:

$$\tilde{q}_\varepsilon^s = Q_\varepsilon^s \quad (10)$$

Finally, the reservation utility of agent j if j is a high-quality producer is defined as follows. If it is agent j 's turn to make an offer, he proposes the terms of trade $(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s, X_{\varepsilon'})$. Thus, the reservation value of a high-quality producer when the match type is $\varepsilon' = (\varepsilon_j, \varepsilon_i)$ is given by

$$R_{\varepsilon'} = (1 - \delta\Delta) [\varepsilon_j u(Q_{\varepsilon'}^b) - Q_{\varepsilon'}^s - X_{\varepsilon'} \Omega] \quad (11)$$

Note that $X_{\varepsilon'}$ is a monetary transfer from j to i .

Finally, note that at the equilibrium of this bargaining game, a low-quality producer has no incentive to refuse the equilibrium offer and to delay the agreement in order to make a counteroffer that is identical to those made by high-quality producers.²⁰

3.2 The program of the household

When the household determines the trading strategies, it is subject to two sets of constraints. First, household members cannot spend more money than what they have:

$$x_\varepsilon \leq m \quad \tilde{x}_\varepsilon \leq m \quad \forall \varepsilon \in E \quad (12)$$

Second, household members cannot ask for more money than what their bargaining partner holds:

$$-x_\varepsilon \leq M \quad -\tilde{x}_\varepsilon \leq M \quad \forall \varepsilon \in E \quad (13)$$

Note that (12) and (13) are not cash-in-advance constraints, because in each match agents can also finance their purchases with their own production. As previously mentioned, $\tilde{q}_\varepsilon^s = Q_\varepsilon^s$. Then, a household's trading strategy consists of the terms of trade $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon, \tilde{q}_\varepsilon^b, \tilde{x}_\varepsilon)$ for each $\varepsilon \in E$, and an acceptance rule for each offer $(Q_{\varepsilon'}^b, Q_{\varepsilon'}^s, X_{\varepsilon'})$ by another household. Agent i from household h makes the first offer, which is immediately accepted by j . For each period, the household chooses $\left\{ m_{+1}, (q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon, \tilde{q}_\varepsilon^b, \tilde{x}_\varepsilon)_{\varepsilon \in E} \right\}$ to solve the following dynamic programming problem:²¹

$$\begin{aligned} V(m) = & \max_{(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon, \tilde{q}_\varepsilon^b, \tilde{x}_\varepsilon)_{\varepsilon \in E}, m_{+1}} \left\{ \pi \int_E [\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s] f_H(\varepsilon_j) f(\varepsilon_i) d\varepsilon_i d\varepsilon_j \right. \\ & \left. + (1 - \pi) \int_E \varepsilon_i u(\tilde{q}_\varepsilon^b) f_L(\varepsilon_j) f(\varepsilon_i) d\varepsilon_i d\varepsilon_j + \beta V(m_{+1}) \right\} \end{aligned} \quad (14)$$

²⁰This point is demonstrated explicitly in the case of the barter economy. For more details see our extended description of the bargaining game at <http://www-vwi.unibe.ch/staff/berentsen/aleks.htm>.

²¹Alternatively, the program can be written as follows:

$$\begin{aligned} V(m) = & \max_{(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon, \tilde{q}_\varepsilon^b, \tilde{x}_\varepsilon)_{\varepsilon \in E}, m_{+1}} \left\{ \pi \Pi \int_E [u(q_\varepsilon^b) - q_\varepsilon^s] f_H(\varepsilon_j) f_H(\varepsilon_i) d\varepsilon_i d\varepsilon_j \right. \\ & + \pi(1 - \Pi) \int_E [-q_\varepsilon^s] f_H(\varepsilon_j) f_L(\varepsilon_i) d\varepsilon_i d\varepsilon_j \\ & \left. + (1 - \pi) \Pi \int_E u(\tilde{q}_\varepsilon^b) f_L(\varepsilon_j) f_H(\varepsilon_i) d\varepsilon_i d\varepsilon_j + \beta V(m_{+1}) \right\} \end{aligned}$$

This equation has the following interpretation. In a fraction $\pi\Pi$ of the meetings, both agents in the match are high-quality producers. Then, households' members both enjoy utility of consumption and suffer disutility of production. In a fraction $\pi(1 - \Pi)$ of the meetings, household members are high-quality producers whereas their partners are lemon producers. In this case, households' members only suffer the disutility of production. In a fraction $(1 - \pi)\Pi$ of the meetings, household members are low-quality producers whereas their partners are high-quality producers. In this case, households' members only enjoy utility of production. In the remaining meetings, both traders are low-quality producers.

s.t. (8), (9), (12), (13), and

$$m_{+1} - m = \tau - \pi \int_E x_\varepsilon f_H(\varepsilon_j) f(\varepsilon_i) d\varepsilon_i d\varepsilon_j - (1 - \pi) \int_E \tilde{x}_\varepsilon f_L(\varepsilon_j) f(\varepsilon_i) d\varepsilon_i d\varepsilon_j \quad (15)$$

The variables taken as given in the above problem are the state variable m and other households' choices (the uppercase variables). Moreover, in this section the fraction π of high-quality producers of the household is assumed to be exogenous and equal to Π . The first integral in equation (14) aggregates the net expected utilities of all high-quality members in all meetings, where high-quality producers offer the trades $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$, which are immediately accepted by their trading partners. The second integral aggregates the net expected utilities of all low-quality members where low-quality producers offer the trades $(\tilde{q}_\varepsilon^b, \tilde{q}_\varepsilon^s, \tilde{x}_\varepsilon)$, which are also immediately accepted by their trading partners. Equation (15) specifies the law of motion of the household's money balances. The first term on the right-hand side is the amount of the lump-sum transfer the household receives each period. The second and third terms are the net amounts of money that high-quality and low-quality producers receive in each period.

3.3 The symmetric steady state equilibrium

In the following we assume that the length of time between an offer and a counteroffer is infinitely small ($\Delta \rightarrow 0$), and we focus on symmetric equilibria where all households make the same offers and adopt the same acceptance rules. Because low- and high-quality producers make the same offers ($(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon) = (\tilde{q}_\varepsilon^b, \tilde{q}_\varepsilon^s, \tilde{x}_\varepsilon) \forall \varepsilon \in E$; see the appendix) and because Π is exogenous, the symmetric steady state equilibrium of this economy corresponds to the equilibrium of the complete information model of Berentsen and Rocheteau (2000a). In the appendix A3 we show that there is a unique steady state monetary equilibrium ($\omega > 0$) and a unique barter equilibrium ($\omega = 0$), and we demonstrate that the terms of trade and the marginal value of money can be determined as follows. First, for a given ω , the terms of trade solve

$$(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon) = \arg \max [\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega] [\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega], \quad \forall \varepsilon \in E \quad (16)$$

s.t. $-m \leq x_\varepsilon \leq m$

Second, for given terms of trade $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$, the marginal value of money satisfies the following envelope condition:

$$\omega_{-1} = \beta \int_E \max [\varepsilon_i u'(q_\varepsilon^b) \omega, \omega] f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j \quad (17)$$

Hence, the model has a very simple structure: The terms of trades correspond to the Nash bargaining solution (16), and the marginal value of money satisfies a standard asset pricing

equation (17), which has the following interpretation. For the household, the value of an additional unit of money received at the end of the previous period is ω_{-1} . In the current period, this unit of money can be either spent or saved. If it is saved, the value of this unit of money from the point of view of the previous period is simply $\beta\omega$. If it is spent in an ε -meeting, the additional utility of consumption is $\varepsilon_i u'(q_\varepsilon^b)\omega$. Indeed, from equations (8) and (9), one additional unit of money buys ω units of real commodity, where each unit provides $\varepsilon_i u'(q_\varepsilon^b)$ additional utility. Accordingly, an additional unit of money is spent if and only if the marginal utility of consumption is larger than the marginal value of money, i.e., if $\varepsilon_i u'(q_\varepsilon^b)\omega > \omega$.

Definition 1 *A monetary steady state equilibrium is a $\omega > 0$ and a set of offers $\{(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)\}_{\varepsilon \in E}$ that satisfy (16) and (17).*

Note that in a monetary steady state equilibrium the quantities q_ε^b and q_ε^s and the real value of money holdings $m\omega$ are stationary. Therefore, if the money supply is not stationary ($\gamma \neq 1$), the marginal value of money ω will not be stationary either. The Nash bargaining solution (16) implies that the terms of trade in an ε -meeting, $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$, satisfy the following two equations:

$$\varepsilon_i \varepsilon_j u'(q_\varepsilon^b) u'(q_\varepsilon^s) = 1 \quad (18)$$

$$\frac{1}{\varepsilon_i u'(q_\varepsilon^b)} = \frac{\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega}{\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega} \quad (19)$$

Furthermore, if neither i nor j is constrained by their money holdings, $q_\varepsilon^b = q_\varepsilon^{b*}$ and $q_\varepsilon^s = q_\varepsilon^{s*}$, where q_ε^{b*} and q_ε^{s*} satisfy $\varepsilon_i u'(q_\varepsilon^{b*}) = 1$ and $\varepsilon_j u'(q_\varepsilon^{s*}) = 1$, respectively. Note that the terms of trade in the barter equilibrium are simply obtained by setting $\omega = 0$ in equation (19). Note further that in any asymmetric meeting ($\varepsilon_i \neq \varepsilon_j$), if $\varepsilon_i > \varepsilon_j$ ($\varepsilon_i < \varepsilon_j$), there is a transfer of money from agent i to agent j (j to i). In contrast, in any symmetric meeting ($\varepsilon_i = \varepsilon_j$) no money is exchanged, because there is no need for compensation, and consequently in symmetric meetings the terms of trade are the same as in the barter equilibrium. Finally, in a monetary equilibrium there is some positive production and consumption in all meetings except when both agents are recognized as lemon producers ($\varepsilon_i = \varepsilon_j = 0$). In contrast, in the barter equilibrium, if one agent is recognized as a low-quality producer (either $\varepsilon_i = 0$ or $\varepsilon_j = 0$), no trade takes place.

Using the fact that $\gamma = \frac{m}{m_{-1}} = \frac{\omega_{-1}}{\omega}$, the envelope condition (17) satisfies

$$\frac{\gamma}{\beta} = \int_E \max [\varepsilon_i u'(q_\varepsilon^b), 1] f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j \quad (20)$$

Note that money is neutral in this model. However, it is not superneutral, because changing the gross growth rate of the money supply γ affects the real value of money holdings $m\omega$

and the terms of trade. Second, for Π and θ given, the Friedman rule $\gamma \rightarrow \beta$ maximizes $m\omega$. Consequently, throughout the paper we will often compare the barter equilibrium with the monetary equilibrium under the Friedman rule, because they are benchmark cases.²²

3.4 Money and terms of trade

In this subsection we consider how money affects the terms of trade, that is, the quantities produced and exchanged in each meeting.

Proposition 1 *Consider a meeting between agents i and j where j is more likely to be a high-quality producer than i ($\varepsilon_i > \varepsilon_j$), and assume that i is constrained by his money holdings. Then $\frac{\partial q_\varepsilon^b}{\partial m\omega} > 0$ and $\frac{\partial q_\varepsilon^s}{\partial m\omega} < 0$.*

Proof: See equations (18) and (19). ■

Proposition 1 considers a meeting between agents i and j with $\varepsilon_i > \varepsilon_j$ where i is constrained by his money holdings.²³ Proposition 1 states that an increase in the real value of money holdings increases the production of those agents who are more likely to produce high-quality goods and decreases the production of those agents who are more likely to produce lemons. Thus, an increase in the real value of money holdings promotes the production of high-quality goods and reduces the production of lemons.

To understand this result, consider first the quantities produced in the barter economy. In the barter equilibrium, equations (18) and (19) imply that $q_\varepsilon^b < q_\varepsilon^s$, that is, agent i , who is more likely to produce lemons than j , produces a larger quantity than j . The origin of this inefficiency is the *quid pro quo* requirement: agent j wants to be compensated for the higher risk of receiving lemons. In a barter economy, the only way to satisfy this requirement is that i produces a large quantity of his good (which is more likely a lemon) and that j produces a small quantity of his good (which is more likely of high quality). In contrast, in the monetary economy under the Friedman rule, equations (18) and (19) imply that $q_\varepsilon^b > q_\varepsilon^s$: that is, agent i , who is more likely to produce lemons than j , produces a smaller quantity than j .

In order to disentangle the effects of money on agents' behavior, we will call agent i the buyer, because he spends money, and agent j the seller, because he receives money (see Proposition 1). This allows us to distinguish an effect on the quantity the buyer supplies and the seller receives (q_ε^s), and an effect on the quantity the seller supplies and the buyer receives (q_ε^b) in the match.

²²At $\gamma = \beta$ there exists a continuum of stationary monetary equilibria with identical terms of trade, which only differ in their stationary value of $m\omega$. By considering the limit when $\beta \rightarrow \gamma$, we select the value of $m\omega$ that is just sufficient to buy the efficient quantity q^* (see Berentsen and Rocheteau 2001).

²³If agent i is not constrained by his money holdings, an increase of the real value of money does not affect production and consumption decisions. It only affects the amount of money transferred.

The recognizability effect of money ($\frac{\partial q_\varepsilon^s}{\partial m\omega} < 0$) In the monetary economy, sellers can ask to be paid with money — an object of universally recognizable quality — instead of with goods of uncertain quality. This possibility reduces the use of real production as a means to finance consumption. Crowding out of payments with real production by monetary payments is what we call the *recognizability effect* of money.

The insurance effect of money ($\frac{\partial q_\varepsilon^b}{\partial m\omega} > 0$) In the monetary economy, buyers can finance their consumption with money, real production, or both. This possibility disconnects what buyers can consume from how they are perceived by their trading partners. In particular, this insurance allows low-quality producers to consume even when they are recognized as lemon producers. In contrast, in a barter economy recognized low-quality producers cannot consume, because they cannot acquire consumption goods. The presence of this “consumption” insurance in the monetary economy is what we call the *insurance effect* of money.

In the following sections we will investigate how the recognizability and the insurance effects of money affect the adverse selection, the moral hazard, and the incentive to acquire information.

4 Money and adverse selection

This section investigates how money affects the quality mix and welfare when the fraction Π of high-quality producers is constant and exogenous.

4.1 Information and the quality mix

The quality mix is defined as the ratio of high-quality output to total output:

$$\phi = \frac{\Pi \int_E q_\varepsilon^s f(\varepsilon_i) f_H(\varepsilon_j) d\varepsilon_i d\varepsilon_j}{\int_E q_\varepsilon^s f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j} \quad (21)$$

We interpret an increase in ϕ as a reduction of the adverse selection problem.

Increasing θ has two effects on the quality mix: First, there is a direct effect because it changes the distribution of the types of meetings.²⁴ Second, there is an indirect effect because a change in θ modifies the real value of money and the terms of trade. In the barter economy there is the direct effect only, which is strictly positive for the Williamson-Wright information structure, i.e., increasing θ increases the quality mix in the barter economy.

²⁴In Section 2.2 we have shown that an increase in θ corresponds to a mean-preserving increase in the spread of the distribution $F(\cdot)$, and this — all other things equal — directly affects ϕ .

In the monetary economy both effects arise. Thus, to analyze the overall effect on ϕ one has to take into account how the real value of money holdings and the terms of trades are affected in all matches when θ changes. Because this cannot be done analytically, we consider the effect of a change in θ by means of simulations for the information structure of Williamson and Wright (1994). Figure 1 displays the fraction of high-quality output ϕ as a function of θ for the Friedman rule (one percent deflation), for price stability, for five and ten percent inflation, and for the barter economy when $\Pi = 0.1$.

A robust feature of all our simulations is that the ratio ϕ is increasing in θ for any value of Π and for any inflation rate. Thus, as expected, if the information problem gets more severe, the quality mix deteriorates. Note that if $\theta = 0$, $\phi = \Pi$: If no information is available, all meetings are of type $\varepsilon = (\Pi, \Pi)$ and the traders exchange the same quantities. Consequently, the ratio of high-quality output to total output is Π . In contrast, if $\theta = 1$, $\phi = 1$: When lemon producers are recognized with certainty, only high-quality goods will be produced, and consequently the ratio of high-quality output to total output is 1.²⁵

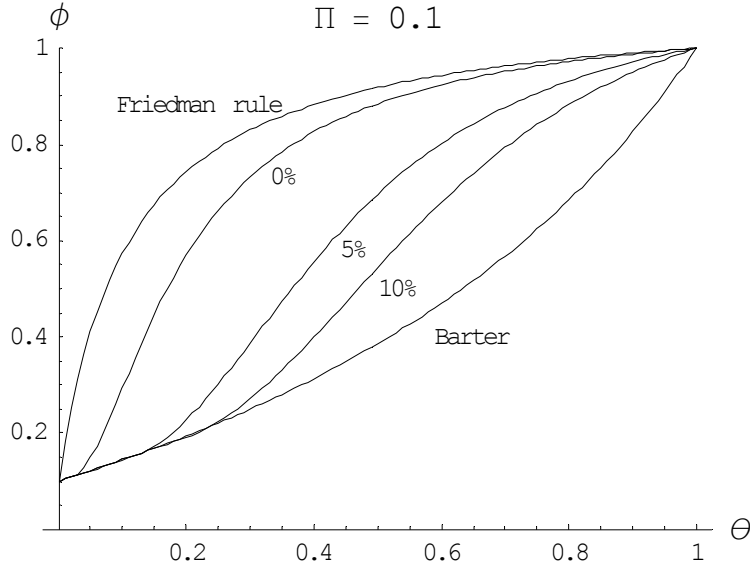


Figure 1: Information and the quality mix.

From Figure 1 one can see that lowering inflation improves the quality mix ϕ . Thus, fighting inflation reduces the adverse selection problem. Our simulations also suggest that the Friedman rule is the optimal monetary policy when the fraction of high-quality producers, Π , is given. Note that the insurance effect of money has a strictly positive

²⁵The envelope condition (20) implies that if $\theta = 0$, $m\omega = 0$. If all agents are uninformed, traders are in the same position when they bargain, and they need no money to compensate each other. In contrast, if $\theta = 1$, agents recognize each other in each match, and consequently $\varepsilon_i = 0$ with probability $1 - \Pi$ and $\varepsilon_i = 1$ with probability Π . There are now asymmetric meetings where agents differ by their willingness to trade.

impact on ϕ because it increases the production of high-quality goods. The recognizability effect has a positive impact too, because it reduces the production of low-quality goods.

To summarize the previous discussion, note that there are two ways to increase the quality mix ϕ : by increasing information for some given inflation rate, which corresponds to a movement along one of the curves in Figure 1, or by decreasing inflation for some given information level, which corresponds to a movement across the curves in Figure 1. Thus, money and information are *substitutes* in the sense that both improve the quality mix.

4.2 Information and welfare

In this section we first compare the outcome of the decentralized economy with the allocation that a social planner would choose. After this we look at how an increase in the level of information θ or a change in the growth rate of the money supply affect welfare.

In appendix A4 we demonstrate that a social planner that maximizes the representative household's welfare chooses the terms of trade such that

$$q_\varepsilon^b = q^* \quad \forall \varepsilon \in E$$

where q^* satisfies $u'(q^*) = 1$. This condition states that welfare is maximized if, in each match, the agents produce and exchange the same quantities q^* . There is no welfare loss or gain when low-quality producers produce, because production of lemons costs nothing and consumption of lemons provides no utility. Consequently, the planner requires everybody to produce as if he were a high-quality producer. Notice that the first best outcome cannot be reached, because under the Friedman rule ($\gamma \rightarrow \beta$) agents exchange the quantities q_ε^{b*} and q_ε^{s*} , where $q_\varepsilon^{b*}, q_\varepsilon^{s*} < q^* \forall \varepsilon$. Thus, in a market with private information, even under the Friedman rule, which is the optimal monetary policy in this environment, agents produce too little relative to what a social planner would dictate.

We now consider by means of simulations how an increase in the level of information as measured by θ affects welfare. Figure 2 displays welfare (a household's lifetime utility) as a function of θ for the Friedman rule (one percent deflation), for five, ten, and fifty percent inflation, and for the barter economy for $\Pi = 0.1$.

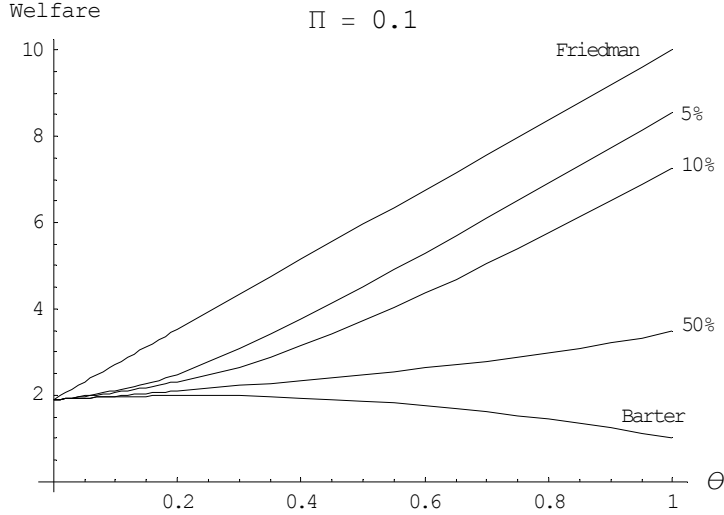


Figure 2: Inflation and welfare.

In a monetary economy, if inflation is not too high, welfare is strictly increasing in the level of information θ (see Figure 2). In contrast, in the barter economy, if α is not too large, welfare is decreasing in θ .²⁶ To see why, note that in the barter economy an increase in θ has two effects. On the one hand, lemon producers are more often recognized and no trade takes place; on the other hand, there are more matches between recognized high-quality producers in which agents produce large quantities for each other. To see this, note that in Figure 2, which is drawn for $\Pi = 0.1$, if $\theta = 1$, only one percent of the meetings generate a trade. In contrast, if $\theta = 0$, in each meeting the traders produce and consume, although the quantities are small relative to the quantities in a match where both traders recognized each other as high-quality producers.

Finally, for any value of θ , the Friedman rule maximizes welfare. Note also that under the Friedman rule, when θ approaches 1, the economy attains the first best: in each match with a high-quality producer, the high-quality producer trades q^* either for money or for q^* units of another high-quality good.

5 Money and moral hazard

In the previous section we have seen that an increase in the real value of money improves the quality mix and welfare. In this section we investigate how money affects households incentives to produce high-quality goods by endogenizing and comparing the fraction of high-quality producers, Π , in the barter and the monetary economy. Throughout this

²⁶Recall that for all simulations we use the utility function $\alpha^{-1}q^\alpha$ that has elasticity of substitution $(1 - \alpha)^{-1}$. If households are not eager to smooth consumption across their members or across time (if α is large), welfare in the barter equilibrium is increasing in θ .

section we consider the information structure of Williamson and Wright (1994), because this allows us to relate in an easy way the endogenous fraction Π to the severity of the information problem, represented by the parameter θ .

5.1 The program of the household

When the representative household h chooses the fraction of its members that are high-quality producers, π , it takes the average decision of other households, Π , as given. By choosing member i to be either a high- or a low-quality producer, the household determines the distribution of probabilities for the beliefs ε_j (ε_j is how i 's production is assessed by i 's trading partner). If i is a high-quality producer, the distribution is given by $f_H(\varepsilon_j)$, and if i is a lemon producer, the distribution is given by $f_L(\varepsilon_j)$. Note that these two distributions depend on Π only, because the choice of π by a household is assumed to be private information.

The chronology of events within a period is as follows. First, the household divides its money holdings evenly across its members. Again, because all the household's members have the same level of money holdings, there is no signaling through wealth. Second, it chooses the probability π at which each of its member will be a high-quality producer in the market. After agents have left the household but before they are matched, they receive a technology shock that lasts one period. The technology shock endows a given member with the high-quality technology with probability π . Third, the members are matched at random with members from other households. When matched, members cannot produce another quality than the one that has been assigned to them by the technology shock.

The derivative \mathcal{D} of the right-hand side of (14) with respect to π equals

$$\begin{aligned} \mathcal{D}(\Pi) = & \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} [\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega] f(\varepsilon_i) f_H(\varepsilon_j) \\ & - \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} [\varepsilon_i u(q_\varepsilon^b) - x_\varepsilon \omega] f(\varepsilon_i) f_L(\varepsilon_j) \end{aligned} \quad (22)$$

The optimal choice of π by the household satisfies

$$\begin{aligned} \pi &= 1 && \text{if } \mathcal{D}(\Pi) > 0 \\ \pi &= 0 && \text{if } \mathcal{D}(\Pi) < 0 \\ \pi &\in [0, 1] && \text{otherwise} \end{aligned} \quad (23)$$

5.2 Symmetric equilibrium

We look for symmetric Nash equilibria where all households choose the same fraction of high-quality producers. The value(s) of Π that sustain a symmetric Nash equilibrium are

defined as follows:

$$\begin{aligned}\Pi &= 1 && \text{if } \mathcal{D}(1) \geq 0 \\ \Pi &= 0 && \text{if } \mathcal{D}(0) \leq 0 \\ \mathcal{D}(\Pi) &= 0 && \text{otherwise}\end{aligned}\tag{24}$$

In the following we call an equilibrium with a positive production of high-quality goods, i.e., $\Pi > 0$, an *active* equilibrium. An active equilibrium can be either a barter or a monetary equilibrium. In a barter equilibrium, $\omega = 0$ in equation (22).

Definition 2 *An active equilibrium is a $\{(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon,)\}_{\varepsilon \in E}$ satisfying (16), an ω satisfying (17), and a $\Pi > 0$ satisfying (24).*

In the following proposition we characterize the barter equilibrium and the monetary equilibrium under the Friedman rule $\gamma \rightarrow \beta$. Note that under the Friedman rule, traders are never constrained by their money holdings.

Proposition 2 *Assume that the fraction of high-quality producers, Π , is endogenous. Then the following is true:*

- (i) *If $\theta = 0$, the unique equilibrium is nonactive.*
- (ii) *For all $\theta > 0$, a nonactive and an active barter equilibrium exist. An active barter equilibrium with $\Pi = 1$ exists iff $\theta \geq \theta_B = \frac{q^*}{u(q^*)} < 1$.*
- (iii) *Under the Friedman rule $\gamma \rightarrow \beta$, there exists an active monetary equilibrium iff $0 < \theta < \theta_M = \frac{2q^*}{u(q^*)+q^*} < 1$. The equilibrium is unique, and the fraction of high-quality producers is strictly increasing in θ .*

Proof: See appendix. ■

According to Proposition 2, there is always a nonactive barter equilibrium: if a household expects all other households to produce lemons, a best response is to choose $\pi = 0$.²⁷ If agents receive no information prior to the bargaining ($\theta = 0$), there is no active equilibrium. For all $\theta > 0$ there is also an active barter equilibrium. Moreover, if the fraction of informed agents is sufficiently large ($\theta \geq \theta_B$), a barter equilibrium exists where all agents produce high-quality goods.

Under the Friedman rule there exists a unique active monetary equilibrium if θ is not too large. If the probability of being recognized is high, then agents have no incentive to cheat and money is valueless. In accordance with intuition, if a monetary equilibrium exists, the fraction of high-quality producers is strictly increasing in the level of information θ .

²⁷When $\theta > 0$, nonactive barter equilibria are unstable in the following sense: If a household anticipates that a small fraction of agents in the market will produce high quality, its best response is to set $\pi = 1$.

Nevertheless, in Trejos (1999) and Williamson and Wright (1994) an increase in information sometimes decrease the fraction of high-quality producers.²⁸

The following proposition ranks the barter and monetary equilibria with respect to the endogenous fraction of high-quality producers in the market.

Proposition 3 *Assume $\gamma \rightarrow \beta$. Then, if θ is close to 0, the fraction of high-quality producers is larger in the monetary equilibrium than in any barter equilibrium. If $\theta \in (\theta_B, \theta_M)$, it is lower in the monetary equilibrium than in the barter equilibrium.*

Proof: See appendix. ■

According to Proposition 3, money has ambiguous effects on the incentives to produce lemons. If the problem of information is severe ($\theta < \theta_B$), the fraction of high-quality producers is larger in a monetary economy. Thus, if information is scarce, valued fiat money disciplines producers. In contrast, if the information problem is not severe ($\theta \in (\theta_B, \theta_M)$), then there is a unique active barter equilibrium where everyone produces high quality, and there is a unique monetary equilibrium where only a fraction of producers produce lemons (see Figure 3). Consequently, if $\theta \in (\theta_B, \theta_M)$, money can be valued even though that it is strictly welfare-decreasing.

5.3 Closed form solutions

In this section we adopt the iso-elasticity utility function $u(q) = \alpha^{-1}q^\alpha$ with $0 < \alpha < 1$, which allows us to derive closed form solutions for the endogenous fraction of high-quality producers in the barter and in the monetary economy under the Friedman rule. The elasticity of substitution of consumption across household members is given by $(1 - \alpha)^{-1}$.²⁹

Proposition 4 *Assume $u(q) = \alpha^{-1}q^\alpha$, $0 < \alpha < 1$, and $\gamma \rightarrow \beta$. Then, for $\theta > 0$, there is a unique active barter equilibrium where the fraction of high-quality producers is*

$$\begin{aligned} \Pi_B &= \left[\frac{\theta(1-\alpha)}{(1-\theta)\alpha} \right]^{\frac{(1-\alpha)(1+\alpha)}{\alpha}} && \text{if } \theta < \theta_B = \alpha \\ &= 1 && \text{otherwise} \end{aligned}$$

²⁸There are more results that differ from Trejos (1999) and Williamson and Wright (1994). First, under the Friedman rule we have an unique monetary equilibrium, while those authors find multiple monetary equilibria. Second, if all agents produce high quality, no monetary equilibrium exists in our model, while there exist monetary equilibria in those papers. In Trejos (1999) this is so because there are also single coincidence meetings. In Williamson and Wright (1994) there is a monetary equilibrium where sellers are indifferent between trading and not trading the good for money.

²⁹This expression is also the intertemporal elasticity of substitution of a member chosen at random.

For all $0 < \theta < \theta_M = \frac{2\alpha}{1+\alpha}$, there is a unique active monetary equilibrium where the fraction of high-quality producers is

$$\Pi_M = \left[\frac{\theta(1-\alpha)}{2\alpha(1-\theta)} \right]^{1-\alpha}$$

Proof: See appendix. ■

Proposition 4 not only confirms Proposition 2, but it also establishes the uniqueness of the active barter equilibrium when $u(q) = \alpha^{-1}q^\alpha$. It is illustrated in Figure 3 for $\alpha = 0.5$. The grey curve labelled Π_B represents the fraction of high-quality producers in the barter economy, and the solid black curve labelled Π_M the fraction of high-quality producers in the monetary economy under the Friedman rule. Note that they are both strictly increasing in the level of information θ until they eventually hit the upper bound $\Pi = 1$.³⁰

Corollary 1 compares the fraction of high-quality producers in the barter and in the monetary equilibrium under the Friedman rule.

Corollary 1 Define $\bar{\theta} = \frac{\alpha}{2\alpha(1-\alpha)+\alpha}$. Then, $0 < \bar{\theta} < \theta_B < \theta_M$. Moreover, the fraction of high-quality producers can be ranked as follows:

If $\theta < \bar{\theta}$, then $\Pi_M > \Pi_B$.

If $\theta \in (\bar{\theta}, \theta_M)$, then $\Pi_M < \Pi_B$.

If $\theta > \theta_M$, then $\Pi_B = 1$ and there is no monetary equilibrium.

Proof: See appendix. ■

According to Corollary 1 (see also Figure 3), there is a threshold $\bar{\theta}$ such that if $\theta < \bar{\theta}$, money increases the incentive to produce high quality, whereas if $\theta > \bar{\theta}$, it increases the incentive to cheat. Note that $\bar{\theta}$ is increasing in α . Thus, money is a less effective device to alleviate the moral hazard problem in an economy, where households have a high aversion to inequalities across members, because the desire to smooth consumption across members already disciplines households.

³⁰Note also that in both the barter and the monetary equilibrium, an increase in α increases the fraction of high-quality producers for small values of θ and reduces the fraction of high-quality producers for large values.

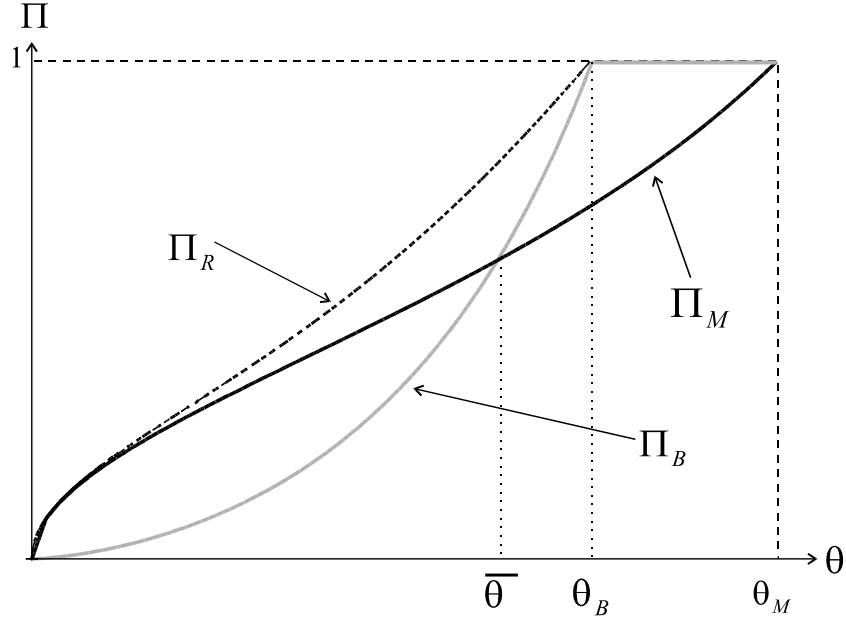


Figure 3: Incentive to produce quality.

5.4 Moral hazard

The key result in the previous subsection is that the fraction of high-quality producers can be smaller in the monetary equilibrium. In this subsection we show that this is so because the *recognizability effect of money* and the *insurance effect of money* have countervailing implications for a household's incentive to cheat.

The recognizability effect The recognizability effect of money raises an agent's incentive to be a high quality producer. In the monetary economy, when a high-quality producer does not recognize his trading partner, he can ask to be paid with money instead of being paid with a good of uncertain quality. In contrast, in the barter economy high-quality producers are always paid with commodities of uncertain quality. Accordingly, the recognizability effect of money improves the gains from trade for high-quality producers, which induces households to choose a larger fraction of high-quality producers.

The insurance effect In contrast, the insurance effect of money raises agents' incentive to engage in opportunistic behavior, because it allows low-quality producers to consume even when they are recognized as lemon producers. This consumption insurance induces households to choose riskier behavior by increasing the fraction of low-quality producers compared to their choices in the barter equilibrium.³¹

³¹In an overlapping generations model where agents have private information about their investment activities, Kitagawa (2001) shows that money can be welfare-decreasing because of a moral hazard problem.

Which effect dominates depends on the severity of the information problem. When θ is low, the recognizability effect dominates and the fraction of high-quality producers is larger in the monetary equilibrium than in the barter equilibrium. As the level of information increases, the recognizability effect fades away, and eventually the insurance effect dominates the recognizability effect, which results in a larger fraction of high-quality producers in the barter equilibrium than in the monetary equilibrium for intermediate values of θ . Consequently, the insurance effect of money exacerbates the moral hazard problem.

In order to isolate the positive effect of money on the incentive to produce high quality, we eliminate the insurance effect by introducing the following trading restriction: households members refuse to trade if they discover that their partner is a low-quality producer.³² Under this rule, low-quality producers cannot benefit from the insurance services of money when they are recognized, because they cannot trade. In appendix A8 we show that in this restricted trade environment, money unambiguously increases the fraction of high-quality producers. Consequently, it is the insurance effect of money that is responsible for the fact that the fraction of lemon producers can be larger in the monetary equilibrium than in the barter equilibrium. This can be seen in Figure 3, where the dotted curve labelled Π_R represents the fraction of high-quality producers in the restricted trade equilibrium under the Friedman rule.

Finally, we have also explored the relationship between inflation, information, and the endogenous fraction of high-quality producers Π by means of simulations.³³ All our simulations suggest that the results we have obtained under the Friedman rule $\gamma \rightarrow \beta$ are robust. For example, we always find that for small value of θ money alleviates the lemon problem, whereas for large values of θ the fraction of cheaters is larger in the monetary equilibrium. One difference, however, is that there are always two monetary equilibria when $\theta \in (\theta_B, \theta_M)$. Note also that when the inflation rate becomes very large, the fraction of high-quality producers in the monetary economy approaches asymptotically the fraction of high-quality producers in the barter economy.

6 Information acquisition

In the previous sections, the information available to the households has been an exogenous parameter. In reality, however, agents have several ways to acquire information about the quality of the commodities they buy. This section analyzes how money affects agents'

Note, however, that the moral hazard problem in Kitagawa (2001) is quite different from the one we treat.

³²We simply assume this restriction here. However, one could imagine a society where people are simply so offended when they discover that an agent is a cheater that they do not want to trade.

³³The simulations are available by request.

incentives to acquire information.

Following Kim (1996), we introduce information acquisition by assuming that at the beginning of each period households have the opportunity to invest in a costly inspection technology, which is equally shared by all household members. Acquiring this technology costs $C(\theta)$.³⁴ It allows each member to recognize the quality of a commodity with probability θ . The acquired inspection technology fully depreciates after one period. Accordingly, households bear the cost $C(\theta)$ in each period. The cost of this inspection technology has the following properties: $C'(\cdot) \geq 0$, $C''(\cdot) > 0$, $C(0) = 0$, $C'(0) = 0$. An explicit form we consider is $C(\theta) = A\frac{\theta^2}{2}$ with $A > 0$.

The choice of θ affects how the members of households h assess the quality produced by their trading partners. Consider member i , and denote by $f(\varepsilon_i, \theta)$ the density function of ε_i , where ε_i is i 's belief about the quality of the good produced by his partner in the match. Denote by $f_\theta(\varepsilon_i, \theta)$ the partial derivative of $f(\varepsilon_i, \theta)$ with respect to θ . Note that beliefs of i 's trading partners only depend on other households' information choices, denoted by Θ . Accordingly, we denote by $f_L(\varepsilon_j; \Theta)$ and $f_H(\varepsilon_j; \Theta)$ the conditional distributions of the beliefs of i 's partners.

6.1 The program of the household

The program of household h is analogous to the program (14) apart from an additional term reflecting the cost of the inspection technology and the fact that the distribution of beliefs of the household depends on its information choice. For simplicity, we assume that the terms of trade, the marginal value of money, and the fraction of high-quality producers are at their equilibrium level that we derived in the previous sections. The choice of information of the household is then given by the following program, where we have taken into account the fact that $\pi = \Pi$. From (2) and (14) we get

$$V(m) = \max_{\theta} \left\{ \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} [\varepsilon_i u(q_\varepsilon^b) - \varepsilon_j q_\varepsilon^s] f(\varepsilon_i; \theta) f(\varepsilon_j; \Theta) - C(\theta) + \beta V(m_{+1}) \right\} \quad (25)$$

$$\text{s.t. } m_{+1} - m = \tau - \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} x_\varepsilon f(\varepsilon_i; \theta) f(\varepsilon_j; \Theta)$$

and (12) and (13)

³⁴In Kim (1996) low-quality producers, high-quality producers, and money holders differ in their investments in the inspection technology.

By differentiating the right-hand side of (25) with respect to θ we obtain the first-order condition that determines the choice of θ . Define the function $I(\theta, \Theta)$ as

$$\mathcal{I}(\theta, \Theta) = \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} [\varepsilon_i u(q_\varepsilon^b) - \varepsilon_j q_\varepsilon^s - x_\varepsilon \omega] f_\theta(\varepsilon_i; \theta) f(\varepsilon_j; \Theta) - C'(\theta) \quad (26)$$

Note that $\mathcal{I}(\theta, \Theta)$ is a decreasing function of θ . Accordingly, household h 's optimal choice of information is

$$\begin{aligned} \theta &= 1 && \text{if } \mathcal{I}(1, \Theta) > 0 \\ \theta &= 0 && \text{if } \mathcal{I}(0, \Theta) < 0 \\ \mathcal{I}(\theta, \Theta) &= 0 && \text{otherwise} \end{aligned} \quad (27)$$

6.2 Symmetric equilibrium

We consider symmetric Nash equilibria where all households choose the same information level and the same fraction of high-quality producers: $\pi = \Pi$ and $\theta = \Theta$. For a given Π , $\Theta \in [0, 1]$ sustains a symmetric Nash equilibrium if and only if

$$\begin{aligned} \mathcal{I}(1, 1) &\geq 0 && \text{if } \Theta = 1 \\ \mathcal{I}(0, 0) &\leq 0 && \text{if } \Theta = 0 \\ \mathcal{I}(\Theta, \Theta) &= 0 && \text{if } \Theta \in]0, 1[\end{aligned} \quad (28)$$

Definition 3 *An active equilibrium is a $\{(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)\}_{\varepsilon \in E}$ satisfying (16), an ω satisfying (17), a $\Pi > 0$ satisfying (24), and a Θ satisfying (28).*

Lemma 1 *In a symmetric equilibrium, households never invest in a perfect information technology ($\Theta < 1$), and there is always a positive fraction of low-quality producers ($\Pi < 1$).*

Proof. See appendix. ■

The intuition of this result is clear. If all agents produce high-quality goods, there is no reason to acquire information; hence, households choose to be uninformed. But if everyone is uninformed, nobody is producing high-quality goods.

Proposition 5 describes the optimal choice of information for the barter and for the monetary economy under the Friedman rule when the fraction of high-quality producers, Π , is given.

Proposition 5 *Assume that the level of information, Θ , is endogenous, and that $u(q) = \alpha^{-1} q^\alpha$ and $C'''(\theta) \geq 0$. Then for given Π the following is true:*

(i) *In the barter economy, the level of information is a function $\Theta_B(\Pi)$ such that $\Theta_B(\Pi) >$*

0 for all $\Pi \in (0, 1)$ and $\Theta_B(0) = \Theta_B(1) = 0$.

(ii) In the monetary economy under the Friedman rule, the level of information is a function $\Theta_M(\Pi)$ such that $\Theta_M(\Pi) > 0$ for all $\Pi \in (0, 1)$, $\Theta_M(0) = 0$, and $\lim_{\Pi \rightarrow 1} \Theta_M(\Pi) = 0$.

(iii) If Π is close to 0, $\Theta_M(\Pi) > \Theta_B(\Pi)$.

Proof. See appendix. ■

Proposition 5 describes the level of information as a function of Π in the barter and in the monetary economy under the Friedman rule. It is illustrated in Figure 4 for the utility function $u(q) = 2\sqrt{q}$ and the cost function $C(\theta) = 0.2\frac{\theta^2}{2}$. The curves labelled Θ_B and Θ_M represent the equilibrium value of θ as a function of Π . In the barter and in the monetary equilibrium, households stop to invest in information if either Π is close to 0 or it is close to 1. The reason for this is that if all agents are uniform (if either all agents are high-quality producers or if all are lemon producers), the return of information is small. In contrast, the heterogeneity among agents, and consequently the return of investments in information, reaches a maximum at some intermediate value of Π .

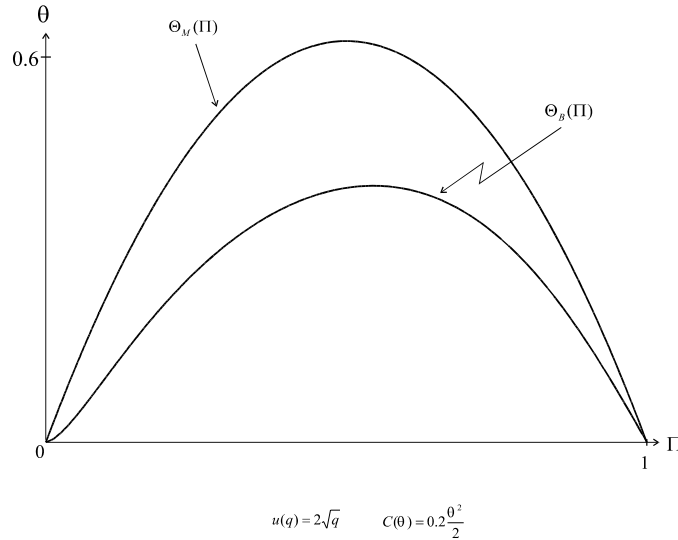


Figure 4: Information acquisition as a function of Π .

Figure 4 suggests that — in contrast to what is claimed in Brunner and Meltzer (1971) and King and Plosser (1986) — for some given Π households invest more in information in the monetary equilibrium than in the barter equilibrium. However, in this respect Figure 4 is partly misleading, because — depending on the specification of the utility function and the values for Π — the opposite can happen if Π is large.³⁵

To improve our understanding of the role of money on the incentive to acquire information, we again look how the *recognizability effect* and the *insurance effect of money* affect the decision to acquire information.

³⁵This can occur if α is large. For instance, if $\alpha = 0.8$, then for large values of Π , $\Theta_B(\Pi) > \Theta_M(\Pi)$.

The recognizability effect Recall that in the monetary economy sellers can ask to be paid with money rather than with a good of uncertain quality. This possibility affects the terms of trade and the incentive to acquire information. For example, consider a high-quality producer (the seller) who is matched with a low-quality producer (the buyer). In the barter economy, for the seller the gain from being informed is the disutility he saves by not producing. In contrast, in the monetary economy the gain is the additional amount of money he receives. Whether the gain from being informed in the monetary economy is larger than the gain in the barter economy depends on the specification of the utility function. If households are eager to smooth consumption across their members or across time (if α is small), the additional units of money received in the monetary economy are more valuable than the saved production cost in the barter economy. The opposite is true if the elasticity of substitution is large (if α is large). The dependence of this effect on the parameter α leads to the ambiguity of the results in Proposition 4.

The insurance effect The “insurance to consume” provides higher incentives to identify the quality of goods. To see this, consider a *recognized* low-quality producer. In the barter economy, he has no benefit of being informed, because he cannot trade, and accordingly information is useless to him. In contrast, in the monetary economy he has a benefit, because information reduces the probability that he spends money for lemons.

Proposition 5 characterizes the barter equilibrium and the monetary equilibrium under the Friedman rule for the quadratic cost function $C(\theta) = A\frac{\theta^2}{2}$ when both Π and Θ are endogenous.

Proposition 6 *Assume that Π and Θ are endogenous, and that $u(q) = \alpha^{-1}q^\alpha$ and $C(\theta) = A\frac{\theta^2}{2}$. Then there is threshold $\bar{A} = (1 - \alpha)^{\frac{1}{\alpha^2}} (1 + \alpha)^{\frac{1}{\alpha^2} - 1}$ such that the following is true:*

- (i) *If $A > \bar{A}$, no active barter equilibrium exists; if $A = \bar{A}$, there exists a unique active barter equilibrium; and if $A < \bar{A}$, two active barter equilibria exist.*
- (ii) *Under the Friedman rule, there exists a unique active monetary equilibrium for all values of A .*

Proof. See appendix. ■

Proposition 6 is illustrated in Figure 5 for $\alpha = 0.5$ (first column) and $\alpha = 0.95$ (second column). It shows the endogenous fraction Π of high-quality producers, the endogenous level of information Θ , and welfare as functions of A for the barter economy (grey curves) and the monetary economy (black curves). The dotted curve represents the critical value \bar{A} .

As stated in Proposition 6, if $A > \bar{A}$, no barter equilibrium exists, whereas if $A < \bar{A}$, two barter equilibria exist. This multiplicity of equilibria reflects a self-fulfilling mechanism that can generate coordination failures. An increase in the cost of the inspection technology

(an increase in A) has different consequences in the two equilibria. In the high equilibrium (the equilibrium with a high Π and a high Θ), it leads to a decrease in the fraction of agents that produce high quality and a decrease in the fraction of informed agents. In the low equilibrium (the equilibrium with a low Π and a low Θ), the opposite happens.

In contrast, in the monetary economy under the Friedman rule, there exists a unique active monetary equilibrium for all values of A . Consequently, when the information problem is sufficiently severe ($A > \bar{A}$), this is the only active equilibrium. This illustrates the strong spillover effect between the incentive to produce high quality and the information choice in the monetary equilibrium. Furthermore, the monetary equilibrium has the same properties as the high barter equilibrium.

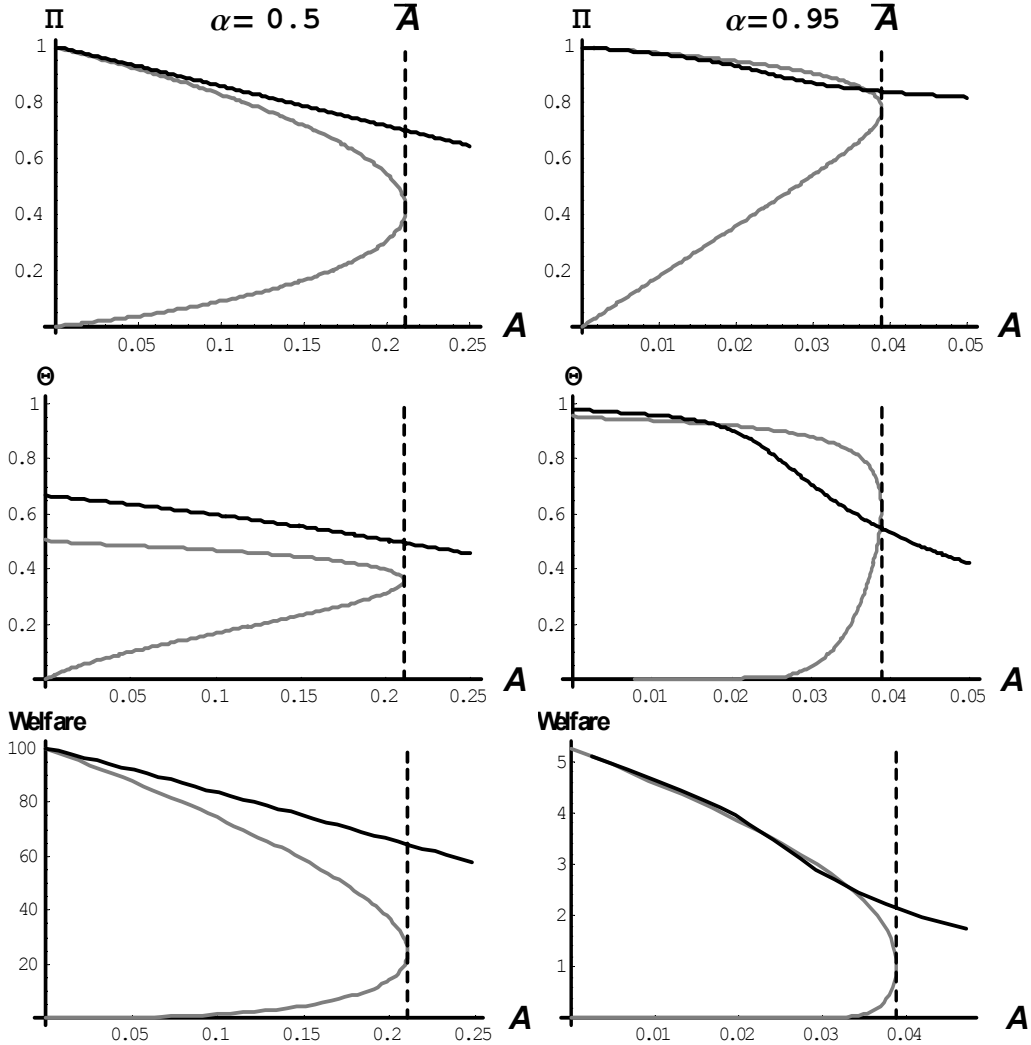


Figure 5: Π , Θ , and welfare as functions of A .

Our simulations suggest that welfare, the fraction of high-quality producers, and the level of information are most often higher in the monetary equilibrium under the Friedman

rule than in the barter equilibrium. This is certainly true for those values of A where no active barter equilibrium exists ($A > \bar{A}$). However, we find that this is also true for $A < \bar{A}$ if α is not too large (our simulations suggest if α is smaller than 0.9). As an illustration, see the case $\alpha = 0.5$ in figure 5. Thus, for most parameter values our model does not support the claim of Brunner and Meltzer (1971), King and Plosser (1986), and Kim (1996) that money and information acquisition are substitutes. Rather, it suggests that they are most often complements. Moreover, our model does not support the widely held belief that the benefit of money comes from its ability to save information costs.

Nonetheless, for large values of α and very small values of A (see the case $\alpha = 0.95$ where $\bar{A} = 0.04$ in Figure 5), our model exhibits results that are in accordance with the previously mentioned papers. Indeed, if α is very large and A very small, (approximately $A = 0.038$), the fraction of high-quality producers and/or the level of information can be smaller, and welfare higher in the monetary equilibrium under the Friedman rule. This suggests that the welfare gain for these parameter values could come from saving information costs.

7 Conclusion

We have investigated the role of fiat money in environments where producers have private information about the quality of the goods they supply. In such environments three issues are at the centre stage: adverse selection, moral hazard, and the incentive to acquire costly information. In order to discuss these issues, we have first studied how money modifies the quantities produced and exchanged in bilateral meetings.

We have identified two effects of money on these quantities. The *recognizability effect of money* states that money crowds out real goods payments. The origin of this effect is that agents prefer to be paid with money – an object of universally recognized quality – rather than with goods of uncertain quality, and this desire gives rise to an endogenous role for money. It is this reduction of uncertainty, that, at least since Menger (1892), has been considered to be an important advantage of monetary exchange over barter. The *insurance effect of money* states that money crowds in consumption. The origin of this effect is that money provides insurance by disconnecting the quantities that agents can buy from how they are assessed by their trading partners. In particular, this insurance allows agents to consume even when they are recognized as low-quality producers.

Our models supports the notion that money is a device for overcoming the adverse selection problem. When the fraction of high-quality producers and the level of information are exogenous, money promotes the production of high-quality goods and reduces the production of lemons. These changes in production increase welfare unambiguously. In contrast, when the fraction of high-quality producers is endogenous, money can be welfare-

decreasing. This is so because while the *recognizability effect of money* raises the benefit of being a high-quality producer, the *insurance effect of money* increases the benefit of being a lemon producer. If the information problem is not severe, the insurance effect dominates the recognizability effect, and consequently the fraction of high-quality producers and welfare are lower in the monetary economy. Thus, valued fiat money sometimes exacerbates the moral hazard problem.

Interestingly, when both the fraction of high-quality producers and the information level are endogenous, the level of information and welfare are most often larger in the monetary economy. The reason for this result is the insurance effect of money. With a valued money, buyers consume more, which raises their incentive to acquire information in order to identify the quality of what they consume. This result does not support the widely held view (Brunner and Meltzer, 1971; King and Plosser, 1986; Kim, 1996) that money has a welfare-improving role by saving information costs.

Divisibility of money has allowed us to study inflation. In general, we find that inflation is welfare-decreasing, because it reduces the real value of money, which adversely affects the quality of the goods produced and exchanged. A money that little value is a less useful device to overcome the adverse selection problem than a highly valued money. Nevertheless, for those parameter values for which money exacerbates the moral hazard problem, it is better to remove it from circulation. Divisible money has also allowed us to focus on the monetary equilibrium under the Friedman rule, for which we have derived several tractable analytical expressions. In particular, under the Friedman rule the active monetary equilibrium is unique.

There are several extensions to this paper that are worth considering. First, it would be interesting to introduce signaling into the model. Wealth (money holdings) could be modelled as a signaling device. Costly advertising of product quality is another device. Second, money is only one possible institution to overcome asymmetric information problems. Intermediaries and middlemen are other institutions whose role is to alleviate those problems. Third, other assets could be viewed as alternative to money. The role of credit in such an economy would be worth studying. Fourth, throughout the paper we have assumed that money is an object whose quality is identifiable by everybody. It would be interesting to see how robust our results are in a model that allows for counterfeiting.

APPENDIX

A1. The information structure.

In this appendix we describe how traders obtain information about their partners and how they form their beliefs. When two traders i and j meet, each agent receives information prior to the bargaining about the quality of the good produced by his partner. This information arrives in the form of two signals s_j^i and s_i^j , where $s_j^i \in \{L, H\}$ ($s_i^j \in \{L, H\}$) is the signal received by agent i (j) about the quality of the good produced by agent j (i). If $s_j^i = H$ ($s_j^i = L$), the signal suggests to i that j is a high (low) quality producer.

Unfortunately, signals are imperfect. If $\chi_i \in \{L, H\}$ denotes the true nature of agent i , then signal reliability (or type) is described by the pair $(\rho_H, \rho_L) \in [0, 1]^2$, where ρ_H and ρ_L are defined as follows:

$$\begin{aligned}\rho_H &= \mathbb{P}[s_i^j = H | \chi_i = H] \\ \rho_L &= \mathbb{P}[s_i^j = H | \chi_i = L]\end{aligned}$$

Thus, ρ_H is the conditional probability that the signal is H when $\chi_i = H$ and ρ_L is the conditional probability that it is H when $\chi_i = L$. We assume that $\rho_H \geq \rho_L$, so that the signals and the true nature of the bargaining partners are positively correlated.

After having received a signal, the traders update their beliefs about their trading partner. The Bayesian beliefs satisfy

$$\mathbb{P}[\chi_i = H | s_i^j = H] = \frac{\rho_H \Pi}{\rho_H \Pi + \rho_L (1 - \Pi)} \geq \Pi \quad (29)$$

$$\mathbb{P}[\chi_i = H | s_i^j = L] = \frac{(1 - \rho_H) \Pi}{(1 - \rho_H) \Pi + (1 - \rho_L) (1 - \Pi)} \leq \Pi \quad (30)$$

If $\rho_H > \rho_L$, the signal reveals some information, i.e., $\mathbb{P}[\chi_i = H | s_i = H] > \Pi$. In contrast, if $\rho_H = \rho_L$, then $\mathbb{P}[\chi_i = H | s_i = H] = \mathbb{P}[\chi_i = H | s_i = L] = \Pi$.

To describe the fact that traders are heterogeneous in their abilities to recognize a certain good, we assume that there are many signals that differ in their reliability. In fact, we assume that there is a distribution of signal types with density $\psi(\rho_H, \rho_L)$. For instance, if $\rho_H = 1$ and $\rho_L = 0$, the trader is an expert in the good produced by his partner, and he is able to recognize its quality with certainty. In contrast, if $\rho_H = \rho_L$, the trader is ignorant about the quality of the good produced by his partner. As shown in appendix A2, the distribution of the signals generates a distribution of beliefs ε_j .

A2. Belief distributions : $F_H(u)$, $F_L(u)$, and $F(u)$

As described in appendix 1, when an individual meets a partner, he receives a signal about the quality of his partner's output. The signal type $\rho = (\rho_H, \rho_L)$ is a random variable

characterized by the density function $\psi(x, y)$ and defined on $\Delta = \{(x, y) \in [0, 1]^2 | x \geq y\}$. We impose the following restriction on the density function:

$$\psi(x, x) = \psi(y, y), \quad \forall x, y \in [0, 1] \quad (31)$$

This restriction permits us to avoid a discontinuity point in the density functions $f_H(\cdot)$, $f_L(\cdot)$, and $f(\cdot)$.

Let $\varepsilon_j \in [0, 1]$ denote the updated belief of player j after he has received a signal about the quality of the output produced by player i :

$$\varepsilon = \hat{\Pi}_H(\rho) \mathbf{1}_{(S_\rho=H)} + \hat{\Pi}_L(\rho) \mathbf{1}_{(S_\rho=L)} \quad (32)$$

where $\rho = (\rho_H, \rho_L)$ is the signal type, $S_\rho \in \{H, L\}$ the signal received by the individual, and $\hat{\Pi}_H(\rho)$ and $\hat{\Pi}_L(\rho)$ the conditional probabilities given by (29) and (30):

$$\begin{aligned} \hat{\Pi}_H(\rho) &= \frac{\rho_H \Pi}{\rho_H \Pi + \rho_L (1 - \Pi)} \\ \hat{\Pi}_L(\rho) &= \frac{(1 - \rho_H) \Pi}{(1 - \rho_H) \Pi + (1 - \rho_L) (1 - \Pi)} \end{aligned}$$

Note that the signal S_ρ and the updated beliefs $\hat{\Pi}_H(\rho)$ and $\hat{\Pi}_L(\rho)$ depend on the signal type ρ , which is a random variable.

1st step. The distribution $F_H(\cdot)$

We take the point of view of player i , and we determine the distribution of beliefs of i 's partners about i 's output. The function $F_H(\cdot)$ describes this cumulative distribution when i is a high-quality producer:

$$F_H(u) = \mathbb{P}[\varepsilon_j < u | \chi_i = H] \quad (33)$$

where χ_i is true type of agent i . Because $S_\rho = H$ implies $\varepsilon_j \geq \Pi$ and $S_\rho = L$ implies $\varepsilon_j \leq \Pi$, it is convenient to distinguish between $u > \Pi$ and $u < \Pi$.

Assume first that $u > \Pi$. Then $\varepsilon_j > u$ is equivalent to $\hat{\Pi}_H(\rho) \mathbf{1}_{(S_\rho=H)} > u$. In this case, it is convenient to compute $1 - F_H(u)$:

$$\begin{aligned} 1 - F_H(u) &= \mathbb{P}[\varepsilon_j > u | \chi_i = H] \\ &= \int \mathbb{P} \left[\hat{\Pi}_H(\rho) \mathbf{1}_{(S_\rho=H)} > u | \rho \in (dx, dy), \chi_i = H \right] \mathbb{P}[\rho \in (dx, dy) | \chi_i = H] \end{aligned}$$

Note that ρ and χ_i are independent random variables. Consequently,

$$\begin{aligned} 1 - F_H(u) &= \int \mathbb{P} \left[\hat{\Pi}_H(x, y) \mathbf{1}_{(S_{x,y}=H)} > u | \chi_i = H \right] \mathbb{P}[\rho \in (dx, dy)] \\ &= \int \mathbb{P}[S_{x,y} = H | \chi_i = H] \mathbf{1}_{A(u)}(x, y) \psi(x, y) dx dy \\ &= \int x \mathbf{1}_{A(u)}(x, y) \psi(x, y) dx dy \end{aligned}$$

where

$$A(u) = \left\{ (x, y) \in \Delta \left| \frac{x\Pi}{x\Pi + y(1-\Pi)} > u \right. \right\}$$

and $\mathbf{1}_{A(u)}(x, y)$ is equal to one if $(x, y) \in A(u)$. Finally, this last equation can be rewritten as follows:

$$1 - F_H(u) = \int_0^1 x \int_0^{\left[\frac{\frac{\Pi}{u} - \Pi}{1 - \Pi}\right]} \psi(x, y) dy dx \quad \forall u > \Pi \quad (34)$$

Assume now that $u < \Pi$. Then $\varepsilon_j < u$ is equivalent to $\widehat{\Pi}_L(\rho) \mathbf{1}_{(S_\rho=L)} < u$. Consequently, we have

$$\begin{aligned} F_H(u) &= \mathbb{P}[\varepsilon_j > u | \chi_i = H] \\ &= \int \mathbb{P} \left[\widehat{\Pi}_L(\rho) \mathbf{1}_{(S_\rho=L)} < u | \rho \in (dx, dy), \chi_i = H \right] \mathbb{P}[\rho \in (dx, dy) | \chi_i = H] \end{aligned}$$

Using the fact that ρ and χ_i are independent, we get

$$\begin{aligned} F_H(u) &= \int \mathbb{P} \left[\widehat{\Pi}_L(x, y) \mathbf{1}_{(S_{x,y}=L)} < u | \chi_i = H \right] \mathbb{P}[\rho \in (dx, dy)] \\ &= \int \mathbb{P} [S_{x,y} = L | \chi_i = H] \mathbf{1}_{B(u)}(x, y) \psi(x, y) dx dy \\ &= \int (1-x) \mathbf{1}_{B(u)}(x, y) \psi(x, y) dx dy \end{aligned}$$

where

$$B(u) = \left\{ (x, y) \in \Delta \left| \frac{(1-x)\Pi}{(1-x)\Pi + (1-y)(1-\Pi)} < u \right. \right\}$$

Finally, we obtain

$$F_H(u) = \int_0^1 (1-x) \int_0^{1 - \frac{(1-x)\Pi}{(1-\Pi)} \frac{(1-u)}{u}} \psi(x, y) dy dx \quad \forall u < \Pi \quad (35)$$

>From (34) and (35) we deduce the density function $f_H(u)$:

$$f_H(u) = \int_0^1 \frac{\Pi}{(1-\Pi)} \left(\frac{x}{u} \right)^2 \psi \left(x, x \left[\frac{\frac{\Pi}{u} - \Pi}{1 - \Pi} \right] \right) dx \quad \forall u > \Pi, \quad (36)$$

$$= \int_0^1 \left(\frac{1-x}{u} \right)^2 \frac{\Pi}{1-\Pi} \psi \left(x, 1 - \frac{(1-x)\Pi}{(1-\Pi)} \frac{(1-u)}{u} \right) dx \quad \forall u < \Pi \quad (37)$$

Under the restriction (31), the density $f_H(\cdot)$ is continuous at $u = \Pi$.

2nd step. The distribution $F_L(\cdot)$

The distribution $F_L(\cdot)$ is the distribution of beliefs of i 's partners about i 's output when i is a low-quality producer:

$$F_L(u) = \mathbb{P}[\varepsilon_j < u | \chi_i = L] \quad (38)$$

Similar reasoning to the above gives

$$1 - F_L(u) = \int_0^1 \int_0^x \left[\frac{\frac{\Pi}{u} - \Pi}{1 - \Pi} \right] y \psi(x, y) dy dx \quad \forall u > \Pi \quad (39)$$

$$F_L(u) = \int_0^1 \int_0^{1 - \frac{(1-x)\Pi}{(1-\Pi)} \frac{(1-u)}{u}} (1-y) \psi(x, y) dy dx \quad \forall u < \Pi \quad (40)$$

Define $\varrho = \left[\frac{1}{u} - 1 \right] \left[\frac{\Pi}{1-\Pi} \right]$. Then, from (39) and (40) the density function $f_L(u)$ satisfies

$$f_L(u) = \varrho \int_0^1 \frac{\Pi}{(1-\Pi)} \left(\frac{x}{u} \right)^2 \psi \left(x, x \left[\frac{\frac{\Pi}{u} - \Pi}{1 - \Pi} \right] \right) dx \quad \forall u > \Pi \quad (41)$$

$$= \varrho \int_0^1 \left(\frac{1-x}{u} \right)^2 \frac{\Pi}{1-\Pi} \psi \left(x, 1 - \frac{(1-x)\Pi}{(1-\Pi)} \frac{(1-u)}{u} \right) dx \quad \forall u < \Pi \quad (42)$$

From (36), (37), (41), and (42) we deduce the following relationship:

$$u(1-\Pi)f_L(u) = (1-u)\Pi f_H(u) \quad \forall u \in [0, 1] \quad (43)$$

Consequently,

$$\begin{aligned} f_H(u) &> f_L(u) & \forall u > \Pi \\ f_H(u) &< f_L(u) & \forall u < \Pi \end{aligned}$$

3rd step. The distribution $F(u)$

The distribution $F(\cdot)$ represents the distribution of i 's beliefs about the quality of a partner chosen at random:

$$\mathbb{P}[\varepsilon_i < u] = \mathbb{P}[\varepsilon_i < u | \chi_j = H] \mathbb{P}[\chi_j = H] + \mathbb{P}[\varepsilon_i < u | \chi_j = L] \mathbb{P}[\chi_j = L]$$

Then

$$F(u) = \Pi F_H(u) + (1-\Pi) F_L(u) \quad (44)$$

and so

$$f(u) = \Pi f_H(u) + (1-\Pi) f_L(u) \quad (45)$$

From (43) and (45), we have

$$u f(u) = \Pi f_H(u) \quad (46)$$

A3. First-order conditions of the household program

Part 1. Terms of trade Denote by λ_ε and $\tilde{\lambda}_\varepsilon$ the multipliers associated with constraints (12). The multipliers associated with constraints (13) will be denoted by η_ε and $\tilde{\eta}_\varepsilon$, respectively. Consider, first, the first-order conditions for the high-quality producers:

$$\varepsilon_i u'(q_\varepsilon^b) = \frac{\lambda_\varepsilon - \eta_\varepsilon + \omega}{\Omega} \quad \forall \varepsilon \in E \quad (47)$$

$$\varepsilon_j u'(q_\varepsilon^s) = \frac{\Omega}{\omega + \lambda_\varepsilon - \eta_\varepsilon} \quad \forall \varepsilon \in E \quad (48)$$

$$\lambda_\varepsilon (m - x_\varepsilon) = 0 \quad \forall \varepsilon \in E \quad (49)$$

$$\eta_\varepsilon (M + x_\varepsilon) = 0 \quad \forall \varepsilon \in E \quad (50)$$

The first-order conditions for the high-quality producers are exactly the same as in Berentsen and Rocheteau (2000a). We therefore do not discuss them here.

Consider, now, the first-order conditions for lemon producers:

$$\varepsilon_i u'(\tilde{q}_\varepsilon^b) = \frac{\tilde{\lambda}_\varepsilon - \tilde{\eta}_\varepsilon + \omega}{\Omega} \quad \forall \varepsilon \in E \quad (51)$$

$$\tilde{\lambda}_\varepsilon (m - \tilde{x}_\varepsilon) = 0 \quad \forall \varepsilon \in E \quad (52)$$

$$\tilde{\eta}_\varepsilon (M + \tilde{x}_\varepsilon) = 0 \quad \forall \varepsilon \in E \quad (53)$$

Note that in equilibrium, for each $\varepsilon \in E$, $(\tilde{q}_\varepsilon^b, \tilde{x}_\varepsilon, \tilde{\lambda}_\varepsilon, \tilde{\eta}_\varepsilon) = (q_\varepsilon^b, x_\varepsilon, \lambda_\varepsilon, \eta_\varepsilon)$. Indeed, under the condition $\tilde{q}_\varepsilon^s = q_\varepsilon^s$, the equations that determine $(q_\varepsilon^b, x_\varepsilon, \lambda_\varepsilon, \eta_\varepsilon)$, i.e., equations (8), (47), (49), and (50), are analogous to the equations that determine $(\tilde{q}_\varepsilon^b, \tilde{x}_\varepsilon, \tilde{\lambda}_\varepsilon, \tilde{\eta}_\varepsilon)$, i.e., (9), (51), (52), and (53). Consequently, low-quality producer make the same offers as high-quality producers.

Equation (18) follows directly from (47) and (48). To derive equation (19) note that in a symmetric equilibrium uppercase variables and lowercase variables are equal: $\omega = \Omega$, $m = M$, and $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon) = (Q_\varepsilon^b, Q_\varepsilon^s, X_\varepsilon)$ for all ε . Then equations (8) and (11) yield

$$\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega = (1 - \delta \Delta) [\varepsilon_j u(q_{\varepsilon'}^b) - q_{\varepsilon'}^s - x_{\varepsilon'} \omega] \quad (54)$$

Equation (54) and an equation that is analogous to (54) but where $\varepsilon = (\varepsilon_i, \varepsilon_j)$ is replaced by $\varepsilon' = (\varepsilon_j, \varepsilon_i)$ determine equation (19). For more details, see Berentsen and Rocheteau (2000a).

Part 2. The envelope condition The envelope condition is

$$\frac{\omega_{-1}}{\beta} = \int_E \varepsilon_j \lambda_\varepsilon + (1 - \varepsilon_j) \tilde{\lambda}_\varepsilon f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j + \omega \quad (55)$$

Taking into account that $\tilde{\lambda}_\varepsilon = \lambda_\varepsilon$, we can rewrite the envelope condition as follows:

$$\frac{\omega_{-1}}{\beta} = \int_E \lambda_\varepsilon f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j + \omega$$

>From (47), the envelope condition becomes

$$\frac{\omega_{-1}}{\beta} = \int_E \max [\varepsilon_i u'(q_\varepsilon^b) \omega - \omega, 0] f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j + \omega$$

and hence

$$\omega_{-1} = \beta \int_E \max [\varepsilon_i u'(q_\varepsilon^b) \omega, \omega] f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j$$

The demonstration of the uniqueness of the barter and the monetary equilibrium is provided by Berentsen and Rocheteau (2000a).

A4. The program of the social planner

We assume that the social planner cannot observe the types of the players in a meeting.³⁶ Consequently, the terms of trade he chooses must satisfy $q_\varepsilon^b = \tilde{q}_\varepsilon^b$. Furthermore, the social planner treats all households in a similar way. Therefore, what an individual must produce if he is in a match of type $(\varepsilon_0, \varepsilon_1)$ equals what he receives when he is in a match of type $(\varepsilon_1, \varepsilon_0)$, i.e., $q_\varepsilon^b = q_{\varepsilon'}^s$.

The social planner maximizes the utility in a period of the representative household. Note that the social planner does not care about the money holdings of the household. Accordingly, the planner solves the following program:

$$\begin{aligned} \max_{q_\varepsilon^b, q_\varepsilon^s, \tilde{q}_\varepsilon^b, \tilde{q}_\varepsilon^s} \quad & \Pi \int_E [\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s] f(\varepsilon_i) f_H(\varepsilon_j) d\varepsilon_i d\varepsilon_j + (1 - \Pi) \int_E \varepsilon_i u(\tilde{q}_\varepsilon^b) f(\varepsilon_i) f_L(\varepsilon_j) d\varepsilon_i d\varepsilon_j \\ \text{s.t.} \quad & q_\varepsilon^b = q_{\varepsilon'}^s \text{ and } q_\varepsilon^b = \tilde{q}_\varepsilon^b \end{aligned}$$

By using the fact that $\Pi f_H(\varepsilon_j) = \varepsilon_j f(\varepsilon_j)$ and $(1 - \Pi) f_L(\varepsilon_L) = (1 - \varepsilon_j) f(\varepsilon_j)$, the program of the social planner can be rewritten as follows:

$$\begin{aligned} \max_{q_\varepsilon^b, q_\varepsilon^s} \quad & \int_E [\varepsilon_i u(q_\varepsilon^b) - \varepsilon_j q_\varepsilon^s] f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j \\ \text{s.t.} \quad & q_\varepsilon^b = q_{\varepsilon'}^s \end{aligned}$$

This program can be rewritten as

$$\max_{q_\varepsilon^b, q_\varepsilon^s} \int_{\{(\varepsilon_i, \varepsilon_j) \in [0,1]^2 | \varepsilon_i > \varepsilon_j\}} [\varepsilon_i u(q_\varepsilon^b) - \varepsilon_i q_\varepsilon^b + \varepsilon_j u(q_\varepsilon^s) - \varepsilon_j q_\varepsilon^s] f(\varepsilon_i) f(\varepsilon_j) d\varepsilon_i d\varepsilon_j$$

³⁶The assumption that the planner cannot recognize the types of the players is not important for this welfare criterion. If he could recognize their types, he could apply the same rule to maximize welfare, because producing lemons cost nothing and consuming them provides no utility.

Differentiating with respect to q_ε^b gives the following first-order conditions:

$$u'(q_\varepsilon^b) = 1 \quad \forall \varepsilon \in [0, 1]^2$$

A5. Proof of Proposition 2

Equation (22) can be rewritten as follows:³⁷

$$\mathcal{D}(\Pi) = \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} \left[\frac{\varepsilon_j}{\Pi} [\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega] - \frac{1 - \varepsilon_j}{1 - \Pi} [\varepsilon_i u(q_\varepsilon^b) - x_\varepsilon \omega] \right] f(\varepsilon_j) f(\varepsilon_i) \quad (56)$$

The proof proceeds in six parts.

Part 1. For all $\theta \geq 0$, a nonactive barter equilibrium exists. If $\Pi = 0$, an agent who is not recognized is perceived as a low-quality producer. Accordingly, beliefs are either 0 or 1. In the barter economy, a trade takes place between agents i and j if and only if $\varepsilon_i = \varepsilon_j = 1$. Thus, according to (56),

$$\mathcal{D}(0) = [u(q_{1,1}^b) - q_{1,1}^s] f_H(1) f(1) - u(q_{1,1}^b) f_L(1) f(1)$$

From (5), if $\Pi = 0$, $f(1) = 0$. Therefore, $\mathcal{D}(0) = 0$. Consequently, if $\Pi = 0$, the best response of any household is to choose any $\pi \in [0, 1]$. Thus, $\pi = \Pi = 0$ is a fixed point of (24).

Part 2. If $\theta = 0$, the unique equilibrium is nonactive. According to (5), if $\theta = 0$, then $f(\Pi) = 1$. Hence, (56) can be rewritten as follows:

$$\mathcal{D}(\Pi) = \begin{cases} -q_{\Pi, \Pi}^s < 0 & \text{if } \Pi > 0 \\ 0 & \text{if } \Pi = 0 \end{cases}$$

Consequently, without arrival of information the only equilibrium is $\Pi = 0$.

Part 3. An active barter equilibrium with $\Pi = 1$ exists iff $\theta \geq \theta_B = \frac{q^*}{u(q^*)}$. In the barter economy, if one of the traders in a match is recognized as a low-quality producer (i.e., $\varepsilon_i = 0$ or $\varepsilon_j = 0$), then no trade takes place (i.e., $q_\varepsilon^b = q_\varepsilon^s = 0$). Hence, using (5), we can rewrite $\mathcal{D}(\Pi)$ as

$$\begin{aligned} \mathcal{D}(\Pi) = & \theta^2 \Pi [u(q_{1,1}^b) - q_{1,1}^s] + (1 - \theta) \theta [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s] \\ & - \theta \Pi (1 - \theta) q_{1,\Pi}^s - (1 - \theta)^2 q_{\Pi,\Pi}^s \end{aligned} \quad (57)$$

An equilibrium where all traders are high-quality producers exists if and only if $\mathcal{D}(1) \geq 0$. Using the fact that $q_{1,1}^b = q_{1,1}^s = q^*$, this condition yields $\theta \geq \theta_B = \frac{q^*}{u(q^*)}$. Note that $\theta_B < 1$.

³⁷Note that because $\frac{1-\varepsilon_j}{1-\Pi} f(\varepsilon_j) = f_L(\varepsilon_j)$ and $\frac{\varepsilon_j}{\Pi} f(\varepsilon_j) = f_H(\varepsilon_j)$, the factor in large brackets is well defined at $\Pi = 0$ and $\Pi = 1$.

Part 4. For all $\theta \in (0, \theta_B)$, an active barter equilibrium with $\Pi < 1$ exists. We first establish the following result:

$$\lim_{x \rightarrow 0} \frac{u'^{-1}\left(\frac{1}{x}\right)}{x} = 0$$

From the strict concavity of the utility function and the fact that $u(0) = 0$, we have $\frac{u(x)}{x} > u'(x)$. Because $u'^{-1}(\cdot)$ is a decreasing function, we have

$$u'^{-1}\left(\frac{u(x)}{x}\right) < x \quad \forall x \quad (58)$$

Multiplying each side of the inequality (58) by $\frac{u(x)}{x}$, we obtain

$$\frac{u(x)}{x} u'^{-1}\left(\frac{u(x)}{x}\right) < u(x) \quad \forall x \quad (59)$$

Define $\varphi(x) \equiv \frac{x}{u(x)}$. We have $\varphi'(x) > 0$ and $\lim_{x \rightarrow 0} \varphi(x) \leq \lim_{x \rightarrow 0} \frac{1}{u'(x)} = 0$. Inequality (59) can be rewritten as follows:

$$\frac{1}{\varphi(x)} u'^{-1}\left(\frac{1}{\varphi(x)}\right) < u(x) \quad \forall x$$

Taking the limit when x approaches 0 and denoting $X = \varphi(x)$ we have

$$\lim_{X \rightarrow 0} \frac{1}{X} u'^{-1}\left(\frac{1}{X}\right) = 0 \quad (60)$$

>From (18) and (19), $q_{\Pi, \Pi}^s$ satisfies $\Pi u'(q_{\Pi, \Pi}^s) = 1$. Hence, $q_{\Pi, \Pi}^s = u'^{-1}\left(\frac{1}{\Pi}\right)$. From (60) we deduce that

$$\lim_{\Pi \rightarrow 0} \frac{q_{\Pi, \Pi}^s}{\Pi} = 0 \quad (61)$$

Since $\omega = 0$ in the barter equilibrium, (19) implies

$$\Pi u(q_{\Pi, 1}^b) - q_{\Pi, 1}^s = \Pi u'(q_{\Pi, 1}^b) [u(q_{\Pi, 1}^s) - q_{\Pi, 1}^b] \quad (62)$$

Consequently, $\mathcal{D}(\Pi)$ given by (57) can be rewritten as

$$\begin{aligned} \mathcal{D}(\Pi) = \Pi \left\{ \theta^2 [u(q_{1, 1}^b) - q_{1, 1}^s] + (1 - \theta) \theta u'(q_{\Pi, 1}^b) [u(q_{\Pi, 1}^s) - q_{\Pi, 1}^b] \right. \\ \left. - \theta (1 - \theta) q_{1, \Pi}^s - (1 - \theta)^2 \frac{q_{\Pi, \Pi}^s}{\Pi} \right\} \end{aligned}$$

The last two terms within the braces go to zero when Π approaches zero. The first term is strictly positive and independent of Π . The limit of the second term when Π approaches zero is a priori indeterminate. Consequently, in the neighborhood of $\Pi = 0$, $\mathcal{D}(\Pi)$ can be approximated by

$$\mathcal{D}(\Pi) \simeq \Pi \left\{ \theta^2 [u(q_{1, 1}^b) - q_{1, 1}^s] + (1 - \theta) \theta u'(q_{\Pi, 1}^b) [u(q_{\Pi, 1}^s) - q_{\Pi, 1}^b] \right\} > 0 \quad \forall \Pi \in]0, \xi]$$

where ξ is arbitrarily close to zero.

Furthermore, for all $\theta < \theta_B$, $\mathcal{D}(1) < 0$. Consequently, we deduce from a continuity argument that for all $\theta < \theta_B$, there is a $\Pi \in]0, 1[$ such that $\mathcal{D}(\Pi) = 0$.

Part 5. Derivation of $\mathcal{D}(\Pi)$ in the monetary equilibrium when $\gamma \rightarrow \beta$. Berentsen and Rocheteau (2000) show that when $\gamma \rightarrow \beta$, the traders produce and exchange the following quantities:

$$\begin{aligned} q_{1,1}^b &= q_{1,1}^s = q_{1,0}^b = q_{0,1}^s = q_{1,\Pi}^b = q_{\Pi,1}^s = q^* \\ q_{\Pi,1}^b &= q_{1,\Pi}^s = q_{\Pi,\Pi}^s = q_{\Pi,\Pi}^b = q_{\Pi,0}^b = q_{0,\Pi}^s = q_{\Pi}^* \\ q_{0,0}^b &= q_{0,0}^s = q_{0,1}^b = q_{1,0}^s = q_{0,\Pi}^b = q_{\Pi,0}^s = q_0^* \end{aligned}$$

where q^* satisfies $u'(q^*) = 1$, q_{Π}^* satisfies $\Pi u'(q_{\Pi}^*) = 1$, and $q_0^* = 0$. Moreover, when $\gamma \rightarrow \beta$, in each meeting agents use a monetary transfer in order to split the total surplus of the match evenly. Hence,

$$x_{\varepsilon}\omega = \frac{\varepsilon_i u(q_{\varepsilon_i}^*) - q_{\varepsilon_j}^* - \varepsilon_j u(q_{\varepsilon_j}^*) + q_{\varepsilon_i}^*}{2} \quad (63)$$

For all $(\varepsilon_i, \varepsilon_j) \in \{(0, \Pi), (\Pi, \Pi), (1, \Pi)\}$ we have

$$\frac{\varepsilon_j}{\Pi} [\varepsilon_i u(q_{\varepsilon}^b) - q_{\varepsilon}^s - x_{\varepsilon}\omega] - \frac{1 - \varepsilon_j}{1 - \Pi} [\varepsilon_i u(q_{\varepsilon}^b) - x_{\varepsilon}\omega] = -q_{\Pi}^* \quad (64)$$

Furthermore, using the fact that $f(0) = \theta(1 - \Pi)$ and $f(1) = \theta\Pi$ (see equation (5)) we have

$$\sum_{\varepsilon_j \in \{0,1\}} \left[\frac{\varepsilon_j}{\Pi} [\varepsilon_i u(q_{\varepsilon}^b) - q_{\varepsilon}^s - x_{\varepsilon}\omega] - \frac{1 - \varepsilon_j}{1 - \Pi} [\varepsilon_i u(q_{\varepsilon}^b) - x_{\varepsilon}\omega] \right] f(\varepsilon_j) = \theta [-q^* + (x_{\varepsilon_i,0}\omega - x_{\varepsilon_i,1}\omega)]$$

Replacing $x_{\varepsilon_i,0}\omega$ and $x_{\varepsilon_i,1}\omega$ by their expression given by (63), we obtain

$$\sum_{\varepsilon_j \in \{0,1\}} \left[\frac{\varepsilon_j}{\Pi} [\varepsilon_i u(q_{\varepsilon}^b) - q_{\varepsilon}^s - x_{\varepsilon}\omega] - \frac{1 - \varepsilon_j}{1 - \Pi} [\varepsilon_i u(q_{\varepsilon}^b) - x_{\varepsilon}\omega] \right] f(\varepsilon_j) = \theta \left(\frac{u(q^*) - q^*}{2} \right) \quad (65)$$

Using (64) and (65), (56) can be rewritten as

$$\mathcal{D}(\Pi) = \theta \left(\frac{u(q^*) - q^*}{2} \right) - (1 - \theta) q_{\Pi}^* \quad (66)$$

Part 6. Monetary equilibrium under the Friedman rule Note, first, that for all $\theta > 0$, we have $\mathcal{D}(0) > 0$, and that $\mathcal{D}(\Pi)$ is strictly decreasing in Π . Consequently, a monetary equilibrium with $\Pi < 1$ exists if and only if $\mathcal{D}(1) < 0$. From (66), this condition yields

$$\theta < \theta_M = \frac{2q^*}{u(q^*) + q^*}$$

Note that $\theta_M < 1$ because $q^* < u(q^*)$. Because $\mathcal{D}(\Pi)$ is strictly decreasing in Π , the monetary equilibrium is unique. Note also that (66) implies that in the monetary equilibrium one has $\frac{\partial q_{\Pi}^*}{\partial \theta} > 0$ and therefore $\frac{\partial \Pi_M}{\partial \theta} > 0$. Finally, recall that if $\Pi = 1$, all meetings are symmetric and consequently money is not valued.

A6. Proof of Proposition 3

We consider the fraction of high-quality producers (Π_B) when θ is close to 0. The condition $\mathcal{D}(\Pi_B) = 0$ can be rewritten as

$$q_{\Pi_B}^* = \theta \left\{ \theta \Pi_B [u(q_{1,1}^b) - q_{1,1}^s] + (1 - \theta) [\Pi_B u(q_{\Pi_B,1}^b) - q_{\Pi_B,1}^s] - \Pi_B (1 - \theta) q_{1,\Pi_B}^s + (\theta + 2) q_{\Pi_B}^* \right\} \quad (67)$$

From (66), the fraction of high-quality producers in the monetary equilibrium (Π_M) satisfies

$$q_{\Pi_M}^* = \theta \left\{ \left(\frac{u(q^*) - q^*}{2} \right) + q_{\Pi_M}^* \right\} \quad (68)$$

From (67) and (68), if θ is close to 0, Π_B and Π_M must also be close to zero. Consequently,

$$\frac{u(q^*) - q^*}{2} + q_{\Pi_M}^* \simeq \frac{u(q^*) - q^*}{2} \gg$$

$$\theta \Pi_B [u(q_{1,1}^b) - q_{1,1}^s] + (1 - \theta) [\Pi_B u(q_{\Pi_B,1}^b) - q_{\Pi_B,1}^s] - \Pi_B (1 - \theta) q_{1,\Pi_B}^s + (\theta + 2) q_{\Pi_B}^* \simeq 0$$

Because q_{Π}^* is increasing in Π , we deduce from (67) and (68) that $\Pi_M > \Pi_B$.

A7. Proof of Proposition 4 and Corollary 1

From equations (18) and (19), one can verify that the terms of trade in the barter equilibrium satisfy

$$q_{1,\Pi}^s = \Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}}, \quad q_{1,\Pi}^b = \Pi^{\frac{1}{(1-\alpha)(1+\alpha)}}, \quad q_{1,1}^s = q_{1,1}^b = q^* = 1, \quad q_{\Pi,\Pi}^s = q_{\Pi,\Pi}^b = q_{\Pi}^* = \Pi^{\frac{1}{1-\alpha}} \quad (69)$$

Substituting these expressions into (57), we obtain

$$\mathcal{D}(\Pi) = \theta^2 \Pi \left(\frac{1-\alpha}{\alpha} \right) + (1-\theta) \theta \Pi^{\frac{1}{(1-\alpha)(1+\alpha)}} \left(\frac{1-\alpha}{\alpha} \right) - \theta (1-\theta) \Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}+1} - (1-\theta)^2 \Pi^{\frac{1}{1-\alpha}} \quad (70)$$

The condition $\mathcal{D}(\Pi) = 0$ can be rewritten as

$$\theta^2 \Pi^{\frac{-\alpha^2}{(1-\alpha)(1+\alpha)}} \left(\frac{1-\alpha}{\alpha} \right) + (1-\theta) \theta \left(\frac{1-\alpha}{\alpha} \right) = \theta (1-\theta) \Pi^{\frac{\alpha}{1+\alpha}} + (1-\theta)^2 \Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}} \quad (71)$$

The LHS of (71) is decreasing in Π whereas the RHS is increasing in Π . Hence, for any θ there is a unique Π satisfying (71). A solution of (71) is given by

$$\Pi_B = \left(\frac{1-\alpha}{\alpha} \frac{\theta}{1-\theta} \right)^{\frac{(1-\alpha)(1+\alpha)}{\alpha}}$$

The expression for Π_M is obtained by replacing q_{Π}^* with $\Pi^{\frac{1}{1-\alpha}}$ in the first-order condition (66). Finally, note that $\Pi_B < \Pi_M$ if and only if:

$$\theta < \bar{\theta} \equiv \frac{\alpha}{\alpha + 2^\alpha (1 - \alpha)}.$$

A8. Restricted Trade Equilibrium

Part 1: Characterization of the restricted trade equilibrium The restricted trade rule specifies that when a trader in a match is recognized as a low-quality producer ($\varepsilon_i = 0$ or $\varepsilon_j = 0$), no trade takes place. The program of the household is given by (14) when all terms with $\varepsilon_i = 0$ or $\varepsilon_j = 0$ are eliminated. The choice Π that sustains a symmetric Nash equilibrium is given by:

$$\begin{aligned} \Pi &= 1 \text{ if } \mathcal{D}_R(1) \geq 0 \\ \Pi &= 0 \text{ if } \mathcal{D}_R(0) \leq 0 \\ \mathcal{D}_R(\Pi) &= 0 \text{ otherwise} \end{aligned}$$

where \mathcal{D}_R is given by (22) when all terms with $\varepsilon_i = 0$ or $\varepsilon_j = 0$ are eliminated:

$$\mathcal{D}_R(\Pi) = [u(q_{1,1}^b) - q_{1,1}^s] \theta^2 \Pi - q_{1,\Pi}^s \theta (1 - \theta) \Pi + [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s - x_{\Pi,1} \omega] (1 - \theta) \theta - q_{\Pi,\Pi}^s (1 - \theta)^2 \quad (72)$$

Let us first determine the necessary and sufficient conditions for an equilibrium without low-quality producers. Using the fact that when $\Pi = 1$ money is not valued, we find that $\mathcal{D}_R(1) = \mathcal{D}_B(1)$. Consequently, a necessary and sufficient condition for an equilibrium without lemon producers is $\theta \geq \theta_B$.

Of course, the marginal value of money is affected by the restricted trade rule. Nonetheless, under the Friedman rule the constraints on money holdings of households' members are not binding and we have

$$\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s - x_{\Pi,1} \omega = \frac{\Pi u(q_{\Pi}^*) - q_{\Pi}^* + u(q^*) - q^*}{2}$$

Consequently, (72) can be rewritten as follows:

$$\mathcal{D}_R(\Pi) = \theta^2 \Pi [u(q^*) - q^*] + \theta (1 - \theta) \left\{ \frac{\Pi u(q_{\Pi}^*) - q_{\Pi}^* + u(q^*) - q^*}{2} \right\} - (1 - \theta) \Pi \theta q_{\Pi}^* - q_{\Pi}^* (1 - \theta)^2 \quad (73)$$

Let us consider equilibria with $\Pi \in (0, 1)$. Then we have $\mathcal{D}_R(\Pi) = 0$. For all $\theta \in (0, \theta_B)$ we have $\mathcal{D}_R(0) > 0$ and $\mathcal{D}_R(1) < 0$. Consequently, an active monetary equilibrium exists.

Part 2: Comparison of the barter equilibrium and the restricted trade equilibrium. According to (57), in the barter equilibrium

$$\mathcal{D}(\Pi) = \theta^2 \Pi [u(q^*) - q^*] + \theta (1 - \theta) [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s] - (1 - \theta) \Pi \theta q_{1,\Pi}^s - (1 - \theta)^2 q_{\Pi}^* \quad (74)$$

To demonstrate that the fraction of high-quality producers in the restricted trade equilibrium is larger than in any barter equilibrium, we can show that $\mathcal{D}_R(\Pi) - \mathcal{D}_B(\Pi) > 0$ for any Π . From (73) and (74), we get

$$\mathcal{D}_R(\Pi) - \mathcal{D}_B(\Pi) = \theta(1-\theta) \left\{ \frac{\Pi u(q_\Pi^*) - q_\Pi^* + u(q^*) - q^*}{2} - [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s] - \Pi q_\Pi^* + \Pi q_{1,\Pi}^s \right\} \quad (75)$$

Berentsen and Rocheteau (2000a) show that in the barter equilibrium in asymmetric matches, $q_{1,\Pi}^s > q_\Pi^*$. Moreover, for all $\Pi \in (0, 1)$, in a $(\Pi, 1)$ -meeting, the buyer surplus is strictly smaller than the seller surplus:

$$\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s < u(q_{\Pi,1}^s) - q_{\Pi,1}^b, \quad \forall \Pi \in (0, 1) \quad (76)$$

>From Berentsen and Rocheteau (2000a), we also know that in the barter economy terms of trade in asymmetric matches are socially inefficient in the sense that the total surplus of the match is not maximized. This implies

$$\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s + u(q_{\Pi,1}^s) - q_{\Pi,1}^b < \Pi u(q_\Pi^*) - q_\Pi^* + u(q^*) - q^* \quad \forall \Pi \in (0, 1) \quad (77)$$

Accordingly, (76) and (77) imply that

$$\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s < \frac{\Pi u(q_\Pi^*) - q_\Pi^* + u(q^*) - q^*}{2} \quad \forall \Pi \in (0, 1)$$

From this we deduce that $\mathcal{D}_R(\Pi) - \mathcal{D}_B(\Pi) > 0$ for all $\Pi \in (0, 1)$.

A9. Proof of Lemma 1

The proof is by contradiction. Assume that everyone produces high quality: $\Pi = 1$. From (5), we find $f(1; \theta) = 1$ and $f_\theta(1; \theta) = 0$. According to (26) and the fact that $f_\theta(\varepsilon_i; \theta) = 0$ for all ε_i , the condition $\mathcal{I}(\theta, \theta) = 0$ can be rewritten as $C'(\theta) = 0$. Consequently, if $\Pi = 1$, households choose to be uninformed ($\theta = 0$). But if $\theta = 0$, the unique equilibrium is nonactive, which contradicts the initial assumption.

Note further that θ cannot be equal to one in equilibrium. Indeed, if $\theta = 1$, households would choose $\Pi = 1$, but then it would be rational for households to be uninformed ($\theta = 0$).

A10. Proof of Proposition 5

Part 1. The barter economy. In the barter equilibrium, there is no trade if one the player is recognized as a low-quality producer. Consequently, $(\varepsilon_i, \varepsilon_j) \in \{\Pi, 1\}^2$. From (26) and the fact that $\omega = 0$, the condition $\mathcal{I}(\theta, \theta) = 0$ can be rewritten as

$$C'(\theta) = \sum_{(\varepsilon_i, \varepsilon_j) \in \{\Pi, 1\}^2} [\varepsilon_i u(q_\varepsilon^b) - \varepsilon_j q_\varepsilon^s] f_\theta(\varepsilon_i; \theta) f(\varepsilon_j; \theta)$$

>From (5), we obtain

$$C'(\theta) = \theta \Pi^2 [u(q^*) - q^*] + (1 - \theta) \Pi [u(q_{1,\Pi}^b) - \Pi q_{1,\Pi}^s] - \theta \Pi [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s] - (1 - \theta) [\Pi u(q_{\Pi}^*) - \Pi q_{\Pi}^*] \quad (78)$$

The LHS of (78) is increasing in θ and is equal to 0 if $\theta = 0$. The RHS of (78) is a linear function of θ . If $\theta = 0$ it is equal to

$$\Pi [u(q_{1,\Pi}^b) - \Pi q_{1,\Pi}^s] - [\Pi u(q_{\Pi}^*) - \Pi q_{\Pi}^*] \quad (79)$$

Under the iso-elasticity specification for the utility function, i.e., $u(q) = \alpha^{-1} q^\alpha$, we have $q_{1,\Pi}^b = \Pi^{\frac{1}{(1-\alpha)(1+\alpha)}}$, $q_{1,\Pi}^s = \Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}}$ and $q_{\Pi}^* = \Pi^{\frac{1}{1-\alpha}}$. The expression (79) can be rewritten as

$$\Pi [u(q_{1,\Pi}^b) - \Pi q_{1,\Pi}^s] - \Pi [u(q_{\Pi}^*) - q_{\Pi}^*] = \Pi \left(\frac{1}{\alpha} - \Pi \right) \left(\Pi^{\frac{\alpha}{1-\alpha^2}} - \Pi^{\frac{\alpha}{1-\alpha}} \right) \geq 0$$

Consequently, (79) is strictly positive for all $\Pi \in]0, 1[$ and is equal to 0 for $\Pi \in \{0, 1\}$.

>From the fact that the RHS of (78) is a linear function of θ that is strictly positive at $\theta = 0$ for all $\Pi \in]0, 1[$, and from the assumption $C'''(\theta) \geq 0$, we deduce that if $C'(1) > \Pi^2 [u(q^*) - q^*] - \Pi [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s]$, there is a unique value of θ on $]0, 1[$ that satisfies (78). If $C'(1) \leq \Pi^2 [u(q^*) - q^*] - \Pi [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s]$, then $\theta = 1$. The function that relates θ to Π is labelled $\Theta_B(\Pi)$. From the previous discussion we have $\Theta_B(\Pi) > 0$ for all $\Pi \in]0, 1[$. Finally, we deduce from (78) that $\Theta_B(0) = \Theta_B(1) = 0$.

Part 2. The monetary economy under the Friedman rule. Berentsen and Rocheteau (2000a) have shown that under the Friedman rule, in each meeting agents transfer money so that the total surplus of each match is split evenly. This implies that:

$$\varepsilon_i u(q_{\varepsilon}^b) - q_{\varepsilon}^s - x_{\varepsilon} \omega = \frac{\varepsilon_i u(q_{\varepsilon_i}^*) - q_{\varepsilon_i}^* + \varepsilon_j u(q_{\varepsilon_j}^*) - q_{\varepsilon_j}^*}{2} \quad (80)$$

According to (26), the condition $\mathcal{I}(\theta, \theta) = 0$ can be rewritten as

$$C'(\theta) = \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} \left\{ (\varepsilon_i u(q_{\varepsilon}^b) - q_{\varepsilon}^s - x_{\varepsilon} \omega) + (1 - \varepsilon_j) q_{\varepsilon_j}^* \right\} f_{\theta}(\varepsilon_i; \theta) f(\varepsilon_j; \theta)$$

Using (80), we have

$$C'(\theta) = \sum_{(\varepsilon_i, \varepsilon_j) \in \{0, \Pi, 1\}^2} \left\{ \frac{\varepsilon_i u(q_{\varepsilon_i}^*) - q_{\varepsilon_i}^* + \varepsilon_j u(q_{\varepsilon_j}^*) - q_{\varepsilon_j}^*}{2} + (1 - \varepsilon_j) q_{\varepsilon_j}^* \right\} f_{\theta}(\varepsilon_i; \theta) f(\varepsilon_j; \theta)$$

After some calculation we obtain

$$C'(\theta) = \frac{\Pi [u(q^*) - q^*] - [\Pi u(q_{\Pi}^*) - \Pi q_{\Pi}^*]}{2} \quad (81)$$

Because of the concavity of the utility function, we have $u(q^*) - u(q_{\Pi}^*) \geq u'(q^*) (q^* - q_{\Pi}^*) = q^* - q_{\Pi}^*$, which implies that $\Pi [u(q^*) - u(q_{\Pi}^*)] \geq \Pi (q^* - q_{\Pi}^*) > \Pi q^* - q_{\Pi}^*$ if $\Pi \in (0, 1)$. The RHS of (81) is strictly positive. Consequently, we can introduce a function $\Theta_M(\Pi)$ that relates θ and Π as follows. If $C'(1) > \frac{\Pi[u(q^*) - q^*] - [\Pi u(q_{\Pi}^*) - q_{\Pi}^*]}{2}$, then $\Theta_M(\Pi)$ is the value of θ that satisfies (81); otherwise $\Theta_M(\Pi) = 1$. From (81), one can check that $\Theta_M(\Pi) > 0$ for all $\Pi \in]0, 1[$, $\Theta_M(0) = 0$, and $\lim_{\Pi \rightarrow 1} \Theta_M(\Pi) = 0$ (if $\Pi = 1$ there is no monetary equilibrium).

Part 3. Ranking of $\Theta_B(\Pi)$ and $\Theta_M(\Pi)$

In the barter economy, the optimal choice of information is given by equation (78), which can be rewritten as follows:

$$\frac{C'(\theta)}{\Pi} = \theta \Pi [u(q^*) - q^*] + (1 - \theta) [u(q_{1,\Pi}^b) - \Pi q_{1,\Pi}^s] - \theta [\Pi u(q_{\Pi,1}^b) - q_{\Pi,1}^s] - (1 - \theta) [u(q_{\Pi}^*) - q_{\Pi}^*] \quad (82)$$

In the monetary economy, the optimal choice of information is given by equation (81) which can be rewritten as follows:

$$\frac{C'(\theta)}{\Pi} = \frac{[u(q^*) - q^*] - [u(q_{\Pi}^*) - \frac{q_{\Pi}^*}{\Pi}]}{2} \quad (83)$$

For Π close to zero, the RHS of (83) is approximately equal to $\frac{[u(q^*) - q^*]}{2} > 0$, whereas the RHS of (82) approaches zero. Consequently, $\Theta_B(\Pi) < \Theta_M(\Pi)$ if Π is close to 0.

A11. Proof of Proposition 6.

Part 1. The barter equilibrium.

According to (69) and (78), in a barter equilibrium, θ satisfies the following equation:

$$A\theta = \theta \frac{(1 - \alpha)}{\alpha} \left[\Pi_B^2 - (\Pi_B)^{\frac{2 - \alpha^2}{1 - \alpha^2}} \right] + (1 - \theta) \left(\frac{1}{\alpha} - \Pi_B \right) \left[(\Pi_B)^{\frac{1 + \alpha - \alpha^2}{1 - \alpha^2}} - (\Pi_B)^{\frac{1}{1 - \alpha}} \right] \quad (84)$$

with

$$\Pi_B = \begin{cases} \left[\frac{\theta(1 - \alpha)}{(1 - \theta)\alpha} \right]^{\frac{(1 - \alpha)(1 + \alpha)}{\alpha}} & \text{if } \theta < \theta_B = \alpha \\ 1 & \text{otherwise} \end{cases}$$

According to lemma 1, $\Pi_B < 1$ in equilibrium. Multiplying each side of (84) by $\frac{\alpha}{\theta(1 - \alpha)}$, we obtain

$$A \frac{\alpha}{(1 - \alpha)} = \left[\Pi_B^2 - (\Pi_B)^{\frac{2 - \alpha^2}{1 - \alpha^2}} \right] + \frac{1}{(\Pi_B)^{\frac{\alpha}{1 - \alpha^2}}} \left(\frac{1}{\alpha} - \Pi_B \right) \left[(\Pi_B)^{\frac{1 + \alpha - \alpha^2}{1 - \alpha^2}} - (\Pi_B)^{\frac{1}{1 - \alpha}} \right]$$

Simplifying the RHS of this last expression, we have

$$A \frac{\alpha}{(1 - \alpha)} = \frac{1}{\alpha} \Pi_B \left[1 - (\Pi_B)^{\frac{\alpha^2}{1 - \alpha^2}} \right] \quad (85)$$

The equilibrium fraction of high-quality producers in the barter economy is a $\Pi_B \in]0, 1[$ that satisfies (85). It can immediately be checked that the RHS of (85) is strictly positive for all $\Pi_B \in]0, 1[$ and is equal to 0 for $\Pi_B \in \{0, 1\}$.

Let Π_B^{\max} the value of $\Pi_B \in]0, 1[$ such that the derivative of RHS of (85) is zero. Π_B^{\max} is unique and is given by

$$\Pi_B^{\max} = (1 - \alpha^2)^{\frac{1-\alpha^2}{\alpha^2}}$$

Consequently, there is a threshold \bar{A} for the information cost such that the following is true: If $A = \bar{A}$, $\Pi_B = \Pi_B^{\max}$. If $A > \bar{A}$, there is no active barter equilibrium. If $A < \bar{A}$, there are two active barter equilibria. The threshold \bar{A} is the value of A that satisfies (85) with $\Pi_B = \Pi_B^{\max}$. Therefore it is equal to

$$\bar{A} = \frac{(1 - \alpha^2)^{\frac{1}{\alpha^2}}}{1 + \alpha} \quad (86)$$

It can be shown that for all $\alpha \in]0, 1[$, $\frac{\partial \bar{A}}{\partial \alpha} < 0$.

Part 2. The monetary equilibrium.

According to (81), if a monetary equilibrium exists, θ must satisfy the following equation:

$$A\theta = \left(\frac{1 - \alpha}{2\alpha} \right) \left[\Pi_M - (\Pi_M)^{\frac{1}{1-\alpha}} \right] \quad (87)$$

with

$$\Pi_M(\theta) = \left[\frac{\theta(1 - \alpha)}{2\alpha(1 - \theta)} \right]^{1-\alpha} \quad \text{if } \theta < \frac{2\alpha}{1 + \alpha} \quad (88)$$

>From (88), we can express θ as:

$$\theta = \frac{2\alpha (\Pi_M)^{\frac{1}{1-\alpha}}}{1 - \alpha + 2\alpha (\Pi_M)^{\frac{1}{1-\alpha}}} \quad (89)$$

Substituting this expression into (87) yields

$$A = \frac{(1 - \alpha)}{(2\alpha)^2} \left[1 - \alpha + 2\alpha (\Pi_M)^{\frac{1}{1-\alpha}} \right] \left[(\Pi_M)^{\frac{-\alpha}{1-\alpha}} - 1 \right] \quad (90)$$

It can be verified that the RHS of (90) is infinite for $\Pi_M = 0$ and is equal to 0 for $\Pi_M = 1$. Consequently, for all $A > 0$, an active monetary equilibrium exists.

We differentiate the RHS of (90) to show that it is strictly decreasing in Π_M :

$$\frac{\partial RHS(90)}{\partial \Pi_M} = \frac{1}{4\alpha} \left\{ 2(1 - \alpha) - 2(\Pi_M)^{\frac{\alpha}{1-\alpha}} - (1 - \alpha)(\Pi_M)^{\frac{-1}{1-\alpha}} \right\}$$

The factor in the braces is equal to $-\infty$ for $\Pi_M = 0$ and to $-(1 + \alpha)$ for $\Pi_M = 1$. For $\Pi_M \in [0, 1]$, that factor reaches a maximum for $\Pi_M = \min \left[\left(\frac{1-\alpha}{2\alpha} \right)^{\frac{1-\alpha}{1+\alpha}}, 1 \right]$. If $\frac{1-\alpha}{2\alpha} < 1$, this maximum is equal to

$$2(1 + \alpha) \left[\frac{(1 - \alpha)}{(1 + \alpha)} - \left(\frac{1 - \alpha}{2\alpha} \right)^{\frac{\alpha}{1+\alpha}} \right] < 0$$

Consequently, $\frac{\partial RHS(90)}{\partial \Pi_M} < 0$ for all $\Pi_M \in [0, 1]$. Therefore, the monetary equilibrium under the Friedman rule is unique.

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