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Abstract

Transaction costs are omnipresent in markets yet are often omitted in economic models. We show that their presence can fundamentally alter incentives and welfare in markets in which the price equates supply and demand. We categorize transaction costs into two types. Asymptotically uninfluenceable transaction costs—such as fixed and price fees—preserve the key asymptotic properties of markets without transaction costs, namely strategyproofness, efficiency, and robustness to misspecified beliefs and to aggregate uncertainty. In contrast, influenceable transaction costs—such as spread fees—lead to complex strategic behavior (which we call price guessing) and may result in severe market failure. In our analysis of optimal design we focus on transaction costs that are fees collected by a platform as revenue. We show how optimal design depends on the traders' beliefs. In particular, with common prior beliefs, any asymptotically uninfluenceable fee schedule can be scaled to be optimal, while purely influenceable fee schedules lead to zero revenue. Our insights extend beyond markets equalizing demand and supply.

Keywords: Transaction Costs, Markets, Demand and Supply, Incentives, Efficiency, Robustness.

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1 Introduction

There is almost no trade without transaction costs such as taxes, commissions, fees, or transportation and packaging costs. Any difference between what the buyer pays and what the seller receives is a transaction cost and their importance is long-established (Coase, 1960; Demsetz, 1968). The questions we shall address in this paper is how transaction costs affect traders' incentives and resulting welfare and efficiency properties of markets?

Surprisingly, these questions received relatively little attention because transaction costs are often omitted in the strategic analyses of markets.[1] Is the omission of transaction costs affecting market analyses? We show that the answer differs across different cost structures. For some costs, such as fixed transaction costs and price fees, the omission does not substantially affect the strategic properties of the market but nevertheless reduces efficiency. For other costs, such as spread fees, strategic behavior is fundamentally altered and may result in market failure.[2]

A *fixed fee* charged to a trader depends only on whether the trader participates in trade. Examples range from handling fees that are often related to overhead costs, to transaction costs related to packaging and shipping. A *price fee* is a percentage of the price. Examples range from stamp duties set by governments, Tobin taxes as levied in Sweden and Latin America, the 'buyer's premium' charged by art auction houses, to 'service fees' or 'final value fees' as charged by Airbnb, eBay, Uber and Lyft, etc. A *spread fee* is a percentage of the difference between a trader's bid or ask and the market clearing price (that is often unknown to the trader unknown). Examples range from commissions charged by intermediaries such as car dealers, limit orders on stock markets, to markets where trader's pay their bid (e.g., Priceline.com).

This paper contributes to our understanding of the effect of transaction costs on incentives, strategic behavior, and market outcomes. Allowing general continuous and monotonic transactions costs, we consider a market in which the price equates supply and demand. The underlying market mechanism is known as a Double Auction (DA).[3] In the absence of transaction costs, in large DAs the gains from misreporting have been shown to vanish and the resulting outcome to be efficient (cf. Rustichini et al. 1994; Cripps and Swinkels 2006). We characterize optimal strategic behavior and categorize transaction costs into two types, *asymptotically uninfluenceable transaction costs* that preserve the latter desirable properties—asymptotic truthfulness and efficiency—and *influenceable transaction costs* that do not preserve them. We also analyze the robustness of our findings to market

[1] Below we discuss the notable exceptions among the studies of Double Auctions which equate revealed demand and supply: the analysis of efficiency under fixed fees in Tatur (2005), market entry under fixed fees in Marra (2019), and platform revenues with fixed and price fees in Chen and Zhang (2020).

[2] Our analysis does not hinge on whether these transaction costs cover the cost of trading infrastructure or additional services (such as transport or insurance).

[3] Notably, stock exchanges, including the New York Stock Exchange, run opening auctions at the start of each trading day to equate supply and demand. Their mechanism closely resembles a Double Auction with transaction costs. During a trading day stock exchanges run quasi-continuous markets, which can be thought off as open-bid Double Auctions in contrast to the standard (sealed-bid) Double Auction.

participants having misspecified beliefs and aggregate uncertainty.

A transaction cost is *asymptotically uninfluenceable* if, conditional on a market participant trading in the market, the participant's impact on the transaction cost they pay vanishes as the market grows large; the transaction cost is *influenceable* if the cost depends on the trading participant's actions even in the limit. Price fees are examples of asymptotically uninfluenceable transaction costs as, in the limit, the market participants impact on the fee vanishes (and, relatedly, all participants who trade pay the same fee). Spread fees are examples of influenceable transaction costs as, in the limit, the spread and hence the fee paid depends on the trading participant's action. Not surprisingly, under asymptotically uninfluenceable transaction costs, the traders behave similarly to traders in markets with no transaction costs and they are approximately *truthful* in large markets. In contrast, influenceable transaction costs distort incentives fundamentally, and, asymptotically, lead to what we call *price-guessing* behavior whereby traders bid close to estimated market prices in order to try to minimize their transaction cost.

Asymptotically uninfluenceable transaction costs lead to some unavoidable welfare losses in finite markets that are due to strategic behavior and possible direct loss due to unprofitability of trades whose surplus is insufficient to cover the cost. Because truthfulness emerges in the limit, in large markets the outcomes are not much affected when the transaction costs are small; and the same obtains even when agents have misspecified beliefs.

In contrast, in large markets, influenceable transaction costs lead to no loss due to strategic behavior, but again may lead to a direct loss as described above. However, even slight belief misspecification often leads to substantive market failure. The risk of market failure occurs for all influenceable transaction costs, and the degree of inefficiency does not vanish with decreasing size of the transaction cost.

In our analysis of optimal design we focus on transaction costs that are fees collected by a platform as revenue. Considering an objective function that incorporates traders' welfare and a platform, we show how optimal design depends on the traders' beliefs. With common prior beliefs, any asymptotically uninfluenceable fee schedule can be scaled to be optimal, while purely influenceable fee schedules lead to zero revenue. For some heterogeneous prior beliefs, purely influenceable fee schedules can strictly outperform any asymptotically uninfluenceable fee schedules.

Finally, we discuss how our insights remain valid in any market organization in which the participants believe that they have no influence on market prices.

Related literature

The idea that trade occurs at the price that equates revealed supply and demand goes back many centuries and is at the core of economics until today (cf. Smith 1776; Hosseini 1995). In finite markets, the Double Auction (DA) is the standard mechanism to compute the allocation and the market price.[4] Strategic behavior has consequently been widely studied. Prominently, Myerson and Satterthwaite (1983) showed that for finite markets with incomplete information there generally exists no budget-balanced, incentive-compatible, and individually rational mechanism that is Pareto efficient.[5]

Without transaction costs it has been shown that the incentive to misrepresent becomes arbitrarily small in large markets (Roberts and Postlewaite 1976). For the DA-mechanism in finite markets, Rustichini et al. (1994) and Cripps and Swinkels (2006) show that market participants have incentives to be increasingly truthful, which results in asymptotic efficiency; any given participant's influence on demand or supply—and therefore the market clearing price—vanishes. Rustichini et al. (1994) established this key insight for the DA mechanism with independent private values (cf. Satterthwaite and Williams 1989b). Their work assumes existence of symmetric equilibria, which was later established by Fudenberg et al. (2007) under correlated but conditionally independent private values.[6] Reny and Perry (2006) extend those findings to continuum markets à la Aumann (1964).[7] Moreover, Azevedo and Budish (2019) show that DAs are also strategy-proof in the large, that is, truthfulness is approximately optimal against any action distribution in large finite markets.

In the presence of transaction costs we know much less about strategic behavior. One notable exception is the treatment of constant transaction costs in Tatur (2005). Chen and Zhang (2020) study revenues in linear equilibria of DAs with transaction costs; they allow transaction costs to depend on the size of individual trade but not on price, bid-ask spread, nor other parameters of the market schemes. Marra (2019) studies market entry in DAs with fixed transaction costs. Noussair et al. (1998) provides experimental evidence that fixed transaction costs lead to efficiency loss. Fixed transaction costs have also been the focus in the finance literature on limit order books (Colliard and Foucault 2012, Foucault et al. 2013, Malinova and Park 2015).[8] Where this literature focuses on specific (fixed) transaction costs, we look at transaction costs more generally and our classification has no counterpart in the literature. Our general incentive, efficiency, and robustness results are also new.

As we allow for general belief structures, our analysis also contributes to the burgeoning literature on market behavior under belief misspecifications, a topic of interest since Ledyard (1978) and Wilson (1987), which has received substantial recent interest such as in Bergemann and Morris (2005).[9]

[4] See Friedman and Rust (1993) for a survey of the DA as a market mechanism in history, theory and practice.

[5] The impossibility hinges on the quasilinearity of the preferences, which we also assume; see Garratt and Pycia (2016).

[6] They also generalized the convergence results of Rustichini et al. (1994). Earlier work on equilibrium existence in DAs includes Chatterjee and Samuelson (1983), Wilson (1985), Leininger et al. (1989), Satterthwaite and Williams (1989a), Williams (1991), and Cripps and Swinkels (2006). See also Jackson and Swinkels (2005) who studied equilibrium existence in a broad class of private value auctions that includes DAs.

[7] To define the DA-mechanism in continuum markets, Reny and Perry (2006) impose continuity and monotonicity assumptions on demand and supply to guarantee the existence of a unique market-clearing price. For our analysis, we will use the DA-mechanism for finite and infinite markets introduced in Jantschgi et al. (2022).

[8] See also Shi et al. (2013) who study a numerical model of marketplace competition with transaction costs.

[9] See also Chung and Ely (2007), Chassang (2013), Bergemann et al. (2015), Carroll (2015), Wolitzky (2016),

The main thrust of this literature is that robustness to misspecification requires the mechanism to be simple. In the context of Walrasian markets, the impact of heterogeneous, misspecified, beliefs has been analyzed e.g., by Harrison and Kreps (1978) and Eyster and Piccione (2013).[10] Our new angle is the analysis of the consequences of belief misspecification on the efficiency of markets with transaction costs, and how market robustness is critically a function also of fee type.

Outline

In Section 2 we provide an example that covers all subsequent, general results. Section 3 formally introduces the market model, mechanism and information structure. Section 4 introduces key concepts to understand traders' incentives to then analyse optimal behavior in Section 5. Section 6 then studies welfare and performance implications and optimal design. Finally, Section 7 abstracts our findings and concludes.

2 Example

In the example, we consider a special case of our general model. We assume there is a continuum of traders on each side of the market. One of the main results of our paper is that there are two qualitatively different categories of transaction costs. In the example, we focus on two common transaction costs that are representative of these categories: price fees and spread fees. We make the exposition parallel to the structure of the general results so that the reader can easily read it as both a preview and an illustration of the general results.

Model (cf. Section 3)

The market (cf. Section 3.1)**.** We consider a two-sided infinite market with a unit mass of buyers and sellers who are interested in either buying or selling an indivisible good. *Types*, giving the *gross value* of the item to a trader i, are uniformly distributed with $t_i \in T = [1, 2]$. The *utility* of each trader is the sum of the gross value of the object (if they have it) and their money holdings, normalized such that a trader who does not participate in the mechanism has utility 0.

The mechanism (cf. Section 3.2)**.** Every trader i submits an *action* $a_i(t_i) \in \mathbb{R}^{\geq 0}$ representing a buyer's bid and a seller's ask. Given all actions, the *double auction* selects subsets of buyers and sellers involved in trade at a unique *market price* P^*. The market price is selected to balance demand and supply, which are the total mass of buyers and sellers, who, given their actions, weakly prefer trading over not trading at that price. Additionally, every trader involved in trade has to pay a *transaction cost*. In the example, we consider our representative transaction costs, price and spread

Carroll (2017), Madarász and Prat (2017), Li (2017), Boergers and Li (2019), Pycia and Troyan (2019).

[10]See also, e.g., Heidhues et al. (2018) who study overconfidence in markets, and de Clippel and Rozen (2018) who study the misperception of tastes.

fees. A *price fee* is given by a fixed percentage $\phi \in [0,1]$ of the market price and a *spread fee* is given by a fixed percentage $\phi \in [0,1]$ of the spread between the action of a trader and the market price.

Beliefs and aggregate uncertainty (cf. Section 3.3)**.** We assume that traders know the market mechanism, but have incomplete information about the market environment, that is the distribution of gross values and market behavior of other traders. Both market sides may have incorrect and heterogeneous *beliefs*, and *aggregate uncertainty*. We work with traders' beliefs over actions. In an infinite market—as considered in the example—this simplifies to considering beliefs directly over the market price. Suppose that all buyers believe the market price to be $\beta \in [1,2]$ and all sellers believe it to be $\sigma \in [1,2]$. We say that beliefs have a *common prior*, if $\beta = \sigma$. Otherwise, we call them *heterogeneous prior beliefs*. Traders might be uncertain about the market price and believe that it is distributed according to a *Beta*-distribution over $[1,2]$, with mean equal to β respectively σ.

Key Concepts (cf. Section 4)

Truthfulness (cf. Section 4.1)**.** In a double auction without transaction costs bidding one's gross value is the only action that (1) never results in a loss, (2) dominates all less aggressive actions (that is higher for the buyer and lower for the seller), and (3) is not dominated by any more aggressive action. If transaction costs are due, bidding one's gross value may no longer satisfy these properties. We define the *net value*, t_b^Φ of a buyer with gross value t_b as the largest action satisfying (1)-(3). In analogy, for a seller with gross value t_s, the net value t_s^Φ is the smallest action satisfying (1)-(3). With no transaction costs, the net value is the gross value, and motivated by this we say that bidding is *truthful* if the trader bids their net value. To illustrate the concepts of net values and truthfulness, let us consider price and spread fees. With price fees, for a buyer with gross value t_b, the net value is $t_b^\Phi = t_b/(1+\phi)$ and for a seller with gross value t_s, the net value is $t_b^\Phi = t_s/(1-\phi)$. With positive price fees, trading at the market price equal to gross value results in negative utility while trading at the price equal to net value results in the utility of 0. With spread fees, the net values are equal to the gross values, that is, $t_b^\Phi = t_b$ and $t_s^\Phi = t_s$. A trader is indifferent between trading and not trading if the market price is equal to their gross value.

Predictability of trade (cf. Section 4.2)**.** Without uncertainty, a buyer believes to trade, if their bid is above the market price. Similarly, a seller believes to trade, if their action is below the market price. If their action is equal to the market price they believe to be involved in *tie-breaking* and trade with some probability. In the presence of uncertainty, the probability to be involved in trade is a continuous function of a trader's action. Decreasing the *aggressiveness* of an action, that is the distance to truthfulness, increases the probability of being involved in trade.

Profitability of trade (cf. Section 4.3)**.** In an infinite market, a trader cannot influence the market price and hence also a price fee is independent of a trader's action. In contrast, a spread fee is

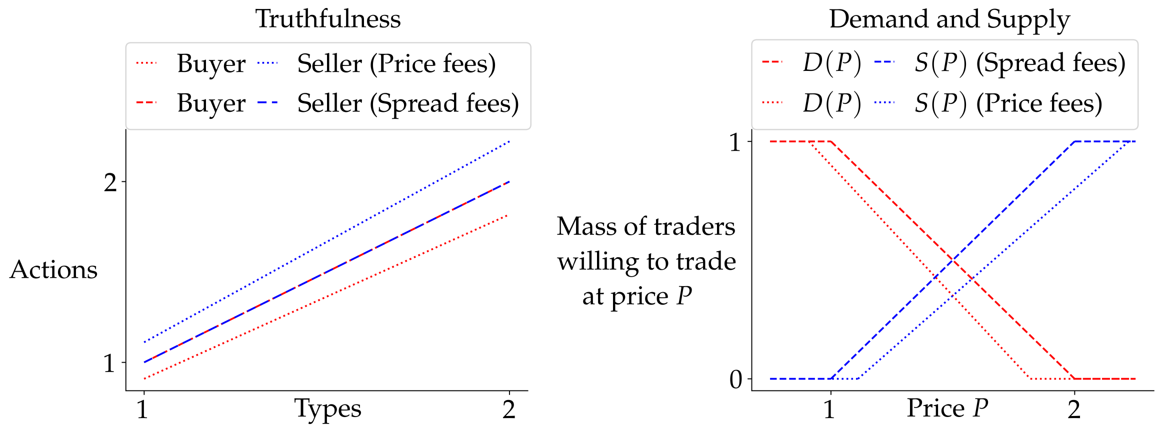


Figure 1: *Left.* Truthful strategy profiles for a 10% price and any spread fee. *Right.* Demand and supply functions, if traders act truthfully, again with a 10% price and any spread fee.

directly influenced by the action of a trader and decreases, if a trader reports a more aggressive action that is closer to the market price. As a general analysis shows, a trader's influence on their transaction cost or its lack plays a crucial role in determining their optimal strategy.

Trader's behavior (cf. Section 5)

Optimal behavior maximizes the expected utility of a trader given their beliefs by finding the right amount of aggressiveness to balance probability and profitability of trade. In the absence of tie-breaking, optimal strategies exist. With tie-breaking, existence of optimal strategies depends on the nature of the transaction cost.

Truthfulness is optimal for price fees (cf. Section 5.1)**.** As a trader cannot influence their payment, in order to maximize expected utility, it is optimal to maximize trading probability as long as the involvement in trade is individually rational. This is achieved by a trader truthfully bidding his net value. Note that truthfulness is independent of beliefs and uncertainty.

Price-guessing is optimal for spread fees (cf. Section 5.2)**.** In the absence of uncertainty, it is optimal to bid the market price, if this is individually rational given a trader's gross value and there is no tie-breaking. We call this behavior *price-guessing*. If there is uncertainty or tie-breaking, the trade-off between decreasing the transaction cost and increasing the probability of trade is non-trivial and depends on beliefs. However, if the uncertainty is sufficiently small, the incentive on the former outweighs the latter and it is optimal to bid close to the estimated market price. Note that price-guessing crucially depends on beliefs and uncertainty.

Figure 2: *Left.* Best responses as a function of the gross value, for a 10% price fee. *Right.* Best responses as a function of the gross value, for a 100% spread fee with deterministic beliefs $\beta = \sigma = 1.5$ without uncertainty (solid lines) and uncertain beliefs according to $Beta(5, 5)$ (dotted lines). For comparison, the diagonal line coincides with reporting the gross value. For price fees, the best responses coincide with the net values. For spread fees, best responses constitute price-guessing for 'in-the-market' gross values and truthfulness otherwise, if there is no uncertainty. Uncertainty diminishes price-guessing.

Market performance and design (cf. Section 6)

Suppose that the fees are collected by a market platform (as opposed to, for example, transportation costs). Then a social planner evaluates market outcomes using standard performance metrics. If the social planner can design the fee structure, what is the optimal choice?

Market Performance (cf. Section 6.1)**.** The *trading volume* Tv is the mass of traders, who are involved in trade. The *trading excess* Ex measures for the two market sides the difference in mass of traders, who are willing to trade at the market price. The *trader's welfare* W is the utility of all traders involved in trade. The *platform's revenue* R is the total amount of collected fees. Their sum is called the *gains of trade*. We distinguish between *realized*, *net*, and *gross gains of trade*, write G, G^{net}, and G^{gross}, depending on whether trader's use some action profile, or report their net or gross values. The *total loss* is the difference $L = G^{gross} - G$, which measures how much gains of trade are lost due to transaction cost considerations and strategic behavior. We split it up into $L = L_d + L_s$, where $L_d = G^{gross} - G^{net}$ is the *direct loss* due to transaction cost constraints and $L_s = G^{net} - G$ is the *strategy-induced loss*. G^{gross} can then be decomposed into welfare, revenue, and loss: $G^{gross} = W + R + L_d + L_s$.

Optimal design (cf. Section 6.2)**.** Suppose that the social planner setting the fee schedule is *revenue-maximizing*. In Section 6, we will consider more general class of objective functions that are induced by social planner's that care about the trader's welfare as well. We will show that price fees can be optimally scaled independent of traders' beliefs. For spread fees, optimal design depends on the trader's beliefs. For common prior beliefs, any spread fee leads to zero revenue due to price-guessing. For some heterogeneous prior beliefs, spread fees can strictly outperform price fees, while for others, they lead to complete market failure.

Optimal price fees are independent of beliefs (cf. Section 6.2.1)**.** Independent of beliefs and uncertainty, truthfulness is optimal. The market price does not depend on the symmetric fee parameter ϕ and is equal to $P^* = 3/2$. The trading volume $Tv = (1 - 3\phi)/2$ decreases linearly in ϕ with maximal trading volume without price fees equal to $1/2$ and full market failure occurring at $\phi = 1/3$. Trading excess is equal to 0, so no tie-breaking is needed. The gross gains of trade are $G^{gross} = 1/4$ and the realized gains of trade are equal to the net gains of trade $= (1 - 9\phi^2)/4$. There is no strategy-induced loss, as traders report truthfully. The direct loss is equal to $9\phi^2/4$, which is strictly increasing in the fee parameter. Welfare is equal to $W = (1 - 6\phi - 9\phi^2)/4$ and revenue is equal to $R = (3\phi - 9\phi^2)/4$. Revenue is maximized at $\phi = 1/6$, where individuals' fee payments and market volume are balanced. At this point, 25% of the gross gains of trade are lost, 50% are revenue and 25% remain as welfare to the traders. The second column of Figure 3 shows the decomposition of the gross gains of trade as a function of the fee parameter ϕ.

Optimal spread fees depend on beliefs (cf. Section 6.2.2)**.** Optimal behavior in the presence of spread fees depends on beliefs and uncertainty. Without uncertainty, price-guessing is optimal. With uncertainty, traders might deviate from price-guessing: Traders with profitable gross values are less aggressive, while traders with gross value close to the true market price might submit actions that are more aggressive. We show that depending on the beliefs β and σ about the market price, market outcomes range from full efficiency (with different decomposition of the gross gains of trade into welfare and revenue) to complete market failure. Note that inefficiency is only due to strategic behavior, as spread fees do not lead to a direct loss. Furthermore, depending on the beliefs, uncertainty can either improve or worsen the market outcome, both from traders and the market maker's perspective. To illustrate the range of possibilities, we analyze five different belief scenarios:

1. **Calibrated beliefs** ($\beta = \sigma = 3/2$)**.** The market is fully efficient. There is no revenue, as there is no bid-ask spread for traders involved in trade. Uncertainty leads to a strategy-induced loss and some revenue.

2. **Homogeneous bias** ($\beta = \sigma \neq 1.5$)**.** The market is not fully efficient. The strategy-induced loss is increasing in the distance between $\beta = \sigma$ and $3/2$. Similar to calibrated beliefs, there is no revenue. Uncertainty diminishes the strategy-induced loss and leads to positive revenue.

3. **Conservative bias** ($\beta \geq 1.5 \geq \sigma$)**.** The market is fully efficient. The revenue decrease, if traders act more aggressive, and β and σ approach $3/2$. Uncertainty decreases the revenue and adds a strategy-induced loss.

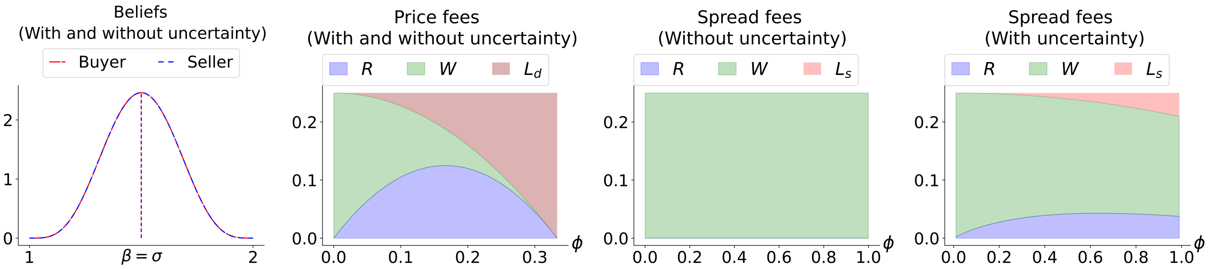
4. **Aggressive bias** ($\sigma \geq 1.5 \geq \beta$)**.** Complete market failure occurs. There is no trade, leading

to zero revenue and surplus. Uncertainty lessens this effect, as traders are less aggressive, leading to trade, and hence some revenue and surplus.

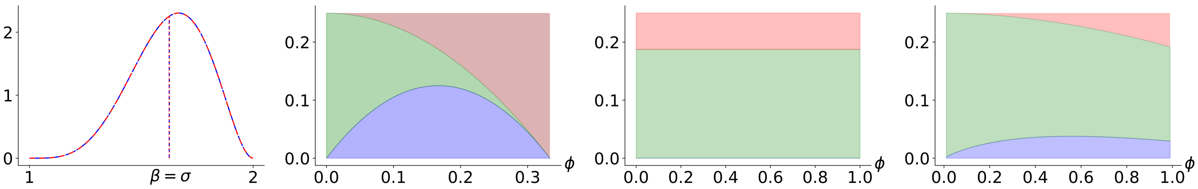
5. **Mixed bias ($1.5 \geq \beta \geq \sigma$).**[11] The market is not fully efficient. The loss is increasing in σ, more aggressive price-guessing by sellers leads to an efficiency loss. The revenue depends on the spread $\beta - \sigma$ and is generated entirely by buyers. Uncertainty leads to greater revenue and less strategy-induced loss.

A table with the full analysis of market characteristics is relegated to the Appendix. The third and fourth column of Figure 3 show the decomposition of the true gains of trade as a function of the fee parameter ϕ for examples of the five belief scenarios with or without aggregate uncertainty. The optimal design for a revenue-maximizing social planner crucially depends on beliefs and uncertainty. First, consider the absence of aggregate uncertainty. If traders have homogeneous prior beliefs, so either calibrated beliefs or a homogeneous bias, revenue is zero regardless of the fee percentage. In that case, it is optimal to not charge any spread fee and avoid price-guessing, which would lead to the fully efficient market outcome. If the beliefs are such that there is a spread, e.g., a conservative or a mixed bias, it is optimal for revenue maximization to charge a 100% spread fee, as price-guessing does not depend on the fee parameter. In the presence of aggregate uncertainty, the optimal fee percentage is given via a non-trivial optimization problem that can be solved analytically.

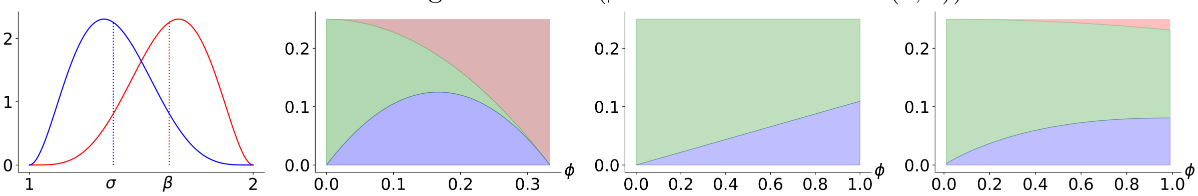
[11] The case $\beta \geq \sigma \geq 1.5$ is analogous.



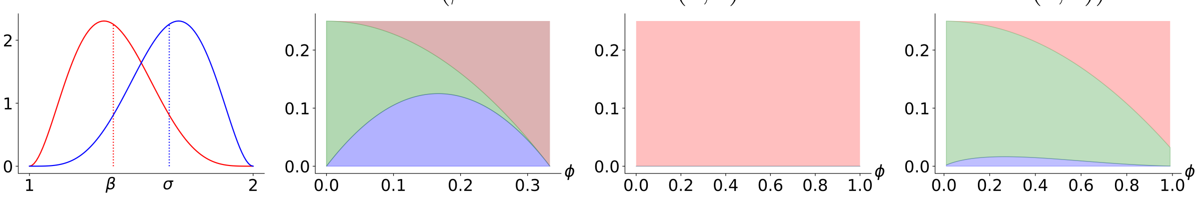
1^{st} row: Calibrated beliefs ($\beta = \sigma = 1.5 \sim Beta(5,5)$)



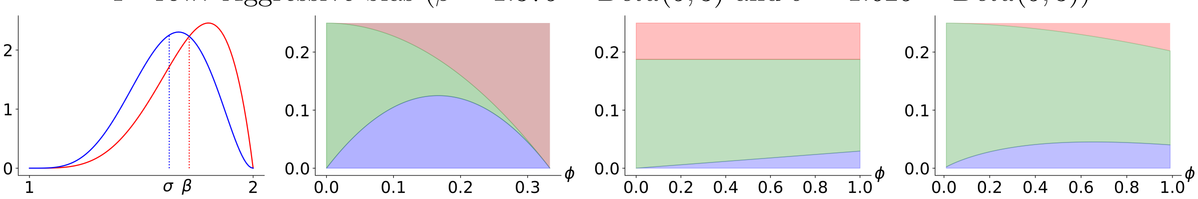
2^{nd} row: Homogeneous bias ($\beta = \sigma = 1.625 \sim Beta(5,3)$)



3^{rd} row: Conservative bias ($\beta = 1.625 \sim Beta(5,3)$ and $\sigma = 1.375 \sim Beta(3,5)$)



4^{th} row: Aggressive bias ($\beta = 1.375 \sim Beta(5,3)$ and $\sigma = 1.625 \sim Beta(5,3)$)



5^{th} row: Mixed bias ($\beta \approx 1.715 \sim Beta(5,2)$ and $\sigma = 1.625 \sim Beta(5,3)$)

Figure 3: Decomposition of the gross gains of trade $G^{gross} = 0.25$ of an infinite uniform market into total revenue R (blue), welfare W (green), direct loss L_d (dark-red) and strategy-induced loss L_s (light-red) as a function of price (2^{nd} column, independent of uncertainty) or spread fee ϕ (3^{rd} column without uncertainty and 4^{th} column with uncertainty), if traders best respond to their beliefs. The first column in each row shows the beliefs as indicated in the sub-captions.

3 The model

3.1 The market

We study a market in which traders play one of two roles: sellers sell and buyers buy a commodity. $\mathcal{B}$ denotes the set of buyers and $\mathcal{S}$ denotes the set of sellers. Each seller s has one unit to sell and each buyer b has single-unit demand. We allow both the finite case, with m buyers $\mathcal{B} = \{1, 2, ..., m\}$ and n sellers $\mathcal{S} = \{1, 2, ..., n\}$, and the infinite case, with $\mathcal{B} \subset \mathbb{R}$ and $\mathcal{S} \subset \mathbb{R}$ being two compact intervals.

We denote by μ_B and μ_S the counting measure (in the finite case) or the Lebesgue measure (in the infinite case) on the sets $\mathcal{B}$ and $\mathcal{S}$. Let $R = \frac{\mu_S(\mathcal{S})}{\mu_B(\mathcal{B})}$.

Our focus is on large finite and on infinite markets. We say that a property $\mathcal{P}$ holds *in sufficiently large finite markets*, if there exist $m, n \geq 1$ such that $\mathcal{P}$ holds in any finite market with at least m buyers and n sellers. If the property also holds in infinite markets, we say that it holds *in sufficiently large markets*.

Every trader $i \in \mathcal{B} \cup \mathcal{S}$ has a *type* $t_i \in T = [\underline{t}, \overline{t}] \subset \mathbb{R}^{\geq 0}$ giving valuation, reservation price or gross value. We assume that the distribution of types are absolutely continuous with probability densities f_B^t and f_S^t that are continuous and strictly positive on their support T, which we call the *type space*. Let (F_B^t, F_S^t) be the corresponding pairs of cumulative distribution functions of types. In finite markets, we assume that traders' types are independent random variables that are identically distributed according to (f_B^t, f_S^t) for each of the two market sides.[12] Given the random variables $t_b^1, ..., t_b^m$ and $t_s^1, ..., t_s^n$, we consider the *random empirical measures* on the sets of types $\mu_B^t = \sum_{j=0}^m \delta_{t_b^j}$ and $\mu_S^t = \sum_{k=0}^n \delta_{t_s^k}$. Letting n and m tend to infinity, normalized versions of μ_B^t and μ_S^t converge uniformly to measures with densities f_B^t and f_S^t; for details see Vapnik and Chervonenkis (2015). In an infinite market, we scale these measures by $\mu_B(\mathcal{B})$ and $\mu_S(\mathcal{S})$ to achieve the market ratio $R = {}^{\mu_S(\mathcal{S})}/_{\mu_B(\mathcal{B})}$ and we denote these measures again by μ_B^t and μ_S^t. Given realizations of types in finite markets and distributions of types in infinite markets, let $t_B : \mathcal{B} \to T$ and $t_S : \mathcal{S} \to T$ denote the functions assigning each trader their type.[13] The type distributions μ_B^t and μ_S^t are then the push-forward measures of μ_B and μ_S via the functions t_B and t_S, i.e. $\mu_B^t(\cdot) = \mu_B(t_B^{-1}(\cdot))$ and $\mu_S^t(\cdot) = \mu_S(t_S^{-1}(\cdot))$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the probability space describing the randomness of sampling type distributions. Denote by $\mathbb{E}[\cdot]$ the *expectation* with respect to the probability measure $\mathbb{P}$. We write $t = (t_i, t_{\text{-}i})$, where t_i is trader i's type and $t_{\text{-}i}$ is the type distribution of all traders excluding trader i. In finite markets, t is obtained by adding a point mass at t_i to $t_{\text{-}i}$. In infinite markets, single traders do not change the type profile.

Every trader i submits an *action* $a_i \in \mathbb{R}^{\geq 0}$ representing a buyer's *bid* and a seller's *ask*. Denote by $a_B : \mathcal{B} \to A_B$ with $a_B(b) = a_b$ and by $a_S : \mathcal{S} \to A_S$ with $a_S(s) = a_s$ Borel-functions that assign an action for each trader. Let the *action distributions* μ_B^a and μ_S^a be two induced σ-additive and

[12] This is a common assumption in the literature, c.f. Rustichini et al. (1994); Azevedo and Budish (2019).

[13] Given the continuity assumptions on assumptions on f_B^t and f_S^t, t_B and t_S are Borel functions in finite and infinite markets.

finite measures on $\mathbb{R}^{\geq 0}$ with support in the *action spaces* $A_B = [\underline{a}_B, \overline{a}_B]$ and $A_S = [\underline{a}_S, \overline{a}_S]$. That is, $\mu_B^a(\cdot) = \mu_B(a_B^{-1}(\cdot))$ and $\mu_S^a(\cdot) = \mu_S(a_S^{-1}(\cdot))$. Write $a = (a_i, a_{\text{-}i})$, where a_i is trader i's action and $a_{\text{-}i}$ is the action distribution of all traders excluding trader i. In finite markets a is obtained by adding a point mass to $a_{\text{-}i}$. In infinite markets, single traders do not influence the action profile. We will sometimes consider *strategies* $a_i : T \to A_i$, where $a_i(t_i)$ specifies the action given i's type. Given type distributions t, strategies of traders induce action distributions a as the push-forward measure of the type distributions.

We compare actions with respect to their *aggressiveness*, which refers to the amount of a bid's (or ask's) misrepresentation: A buyer's bid a_b^1 is *(strictly) more aggressive* than a_b^2, write $\underset{(\succ)}{\succcurlyeq}$, if $a_b^1 \underset{(>)}{\geq} a_b^2$ and similarly a seller's offer a_s^1 is *(strictly) more aggressive* than a_s^2, write $\underset{(\succ)}{\succcurlyeq}$, if $a_s^1 \underset{(<)}{\leq} a_s^2$.

The *utility* of each trader is the sum of the gross value of the object (if they have it) and their money holdings, normalized such that a trader who does not trade has utility 0. A buyer b involved in trade makes a payment, $P_b(a_b, a_{\text{-}b})$, in order to obtain an item and their resulting utility is $u_b(t_b, a_b, a_{\text{-}b}) = t_b - P_b(a_b, a_{\text{-}b})$. Similarly, a seller s involved in trade receives a payment, $P_s(a_s, a_{\text{-}s})$, for their item and their utility is $u_s(t_s, a_s, a_{\text{-}s}) = P_s(a_s, a_{\text{-}s}) - t_s$.

3.2 Double auction with transaction costs

Demand $D(P)$ and *supply* $S(P)$ at a price $P \geq 0$ are defined as $D(P) = \mu_B(\mathcal{B}_{\geq}(P))$ and $S(P) = \mu_S(\mathcal{S}_{\leq}(P))$, where $\mathcal{B}_{\geq}(P) = \{b \in \mathcal{B} : a_b \geq P\}$ and $\mathcal{S}_{\leq}(P) = \{s \in \mathcal{S} : a_s \leq P\}$. $\mathcal{B}_{>}(P), \mathcal{B}_{=}(P), \mathcal{S}_{<}(P)$, and $\mathcal{S}_{=}(P)$ are defined analogous.

A *double auction with transaction costs* takes as given a pricing parameter $k \in [0, 1]$ and maps action profiles a into *market outcomes* that equilibrate demand and supply. Such a market outcome consists of

- A market price $P^*(a)$. The market price is set as

$$P^*(a) = k \cdot \min \mathcal{P}^{MC}(a) + (1-k) \cdot \max \mathcal{P}^{MC}(a),$$

  where $\mathcal{P}^{MC}(a)$ is the set of *market clearing prices* that equilibrate demand and supply.[14]

- An allocation $A^*(a) = \mathcal{B}^*(a) \cup \mathcal{S}^*(a)$ identifying subsets of traders $\mathcal{B}^*(a) \subset \mathcal{B}$ and $\mathcal{S}^*(a) \subset \mathcal{S}$ involved in trade. Given $P^*(a)$, the allocation is:

$$\mathcal{S}^*(a) = \mathcal{S}_{<}(P^*(a)) \cup \tilde{\mathcal{S}}(a) \text{ and } \mathcal{B}^*(a) = \mathcal{B}_{>}(P^*(a)) \cup \tilde{\mathcal{B}}(a),$$

  where $\tilde{\mathcal{B}}(a) \subset \mathcal{B}_{=}(P^*(a))$ (respectively $\tilde{\mathcal{S}}(a) \subset \mathcal{S}_{=}(P^*(a))$) are uniform random sets selecting

[14] Analytic properties of demand and supply, as well as a detailed account of market-clearing prices are formulated in Appendix A.1, and proven for the k-DA without transaction costs in finite and infinite markets in Jantschgi et al. (2022).

players to balance trade in case there is trading excess.[15]

- *Transaction costs* $\Phi(a) = \Phi_i(a)_{i\in\mathcal{B}^*\cup\mathcal{S}^*} = (\Phi_i(a), \Phi_{-i}(a))$ for all active traders.[16]

The payments of traders $i \in \mathcal{B}^* \cup \mathcal{S}^*$ are determined by price $P^*(a_i, a_{\text{-}i})$ and transaction cost $\Phi_i(a_i, a_{\text{-}i})$. The payment a buyer $b \in \mathcal{B}^*$ makes is $P_b = P^*(a_b, a_{\text{-}b}) + \Phi_b(a_b, a_{\text{-}b})$ and the payment a seller $s \in \mathcal{S}^*$ receives is $P_s = P^*(a_s, a_{\text{-}s}) - \Phi_s(a_s, a_{\text{-}s})$. We assume that the payments $P_i(a_i, a_{\text{-}i})$ are continuous and increasing in a_i. Hence, for a buyer bidding more aggressively leads to a lower payment and for a seller bidding more aggressively leads to a higher payment. In Appendix A.3.3, we prove that the function $a_i \mapsto P^*(a_i, a_{\text{-}i})$ is continuous and increasing in a_i. Therefore, a sufficient condition for the monotonicity and continuity of the payment is that the transaction cost is continuous and increasing. The payments of traders $i \notin \mathcal{B}^* \cup \mathcal{S}^*$ are normalized to 0; these traders do not participate in trade.

Commonly observed transaction cost structures result in payments that are continuous and increasing. Examples include *constant* fees, *price* fees, and *spread* fees. Those fee structures are defined as follows. A transaction cost Φ_i is a *constant fee* if $\Phi_i(a) = c_i$ for some constant $c_i \geq 0$. A transaction cost Φ_i is a *price fee* if $\Phi_i(a) = \phi_i P^*(a)$ for some constant $\phi_i \in [0, 1]$. A transaction cost Φ_i is a *spread fee* if $\Phi_i(a) = \phi_i |P^*(a) - a_i|$ for some constant $\phi_i \in [0, 1]$. An interesting case is the price fee for the seller, the payment received by the seller is increasing when the seller bids more aggressively despite the fee paid by the seller being also increasing.[17]

3.3 Beliefs

We assume that traders commonly know the market mechanism, but allow the traders to have incomplete information regarding the market environment. In general, traders may have heterogeneous priors and incorrect beliefs.

Trader i has beliefs about the number of traders, the distribution of their gross values, and their market behavior. Denote by $R_i = {}^{\mu_S(\mathcal{S}_i)}/_{\mu_B(\mathcal{B}_i)}$ the ratio of the number of sellers to buyers. It is common in the literature to assume correct beliefs about the number of traders and their gross value distribution. In this common prior belief setting it is then standard to study symmetric equilibrium strategies, see Rustichini et al. (1994). In an equilibrium, the traders' beliefs over fundamentals then induce their beliefs over other traders' actions. In a more recent strand of literature, e.g., the work by Azevedo and Budish (2019) on Strategy-proofness in the Large, best response behavior to arbitrary action distributions is studied, not only those induced by common prior knowledge and perfectly rational play. With some analytical assumptions, beliefs over actions incorporate the

[15] That is for all $b \in \mathcal{B}_=(P^*(a))$ it holds that $\mathbb{P}[b \in \tilde{\mathcal{B}}(a)] \equiv const$ (respectively for all $s \in \mathcal{S}_=(P^*(a))$ it holds that $\mathbb{P}[s \in \tilde{\mathcal{S}}(a)] \equiv const$). See Appendix A.2 for details regarding the allocation and tie-breaking.

[16] Whenever the dependence on the action profile is clear, we write P^*, $\mathcal{B}^*$ and $\mathcal{S}^*$. When focusing on a single trader with action a_i, we write, e.g., $P^*(a_i, a_{\text{-}i})$.

[17] If $\phi_i = 0$ or $c_i = 0$, the setting simplifies to the classical DA without transaction costs. Further, for spread fees, if $\phi_i = 1$ a trader's payment is equal to their bid/ask.

classical model of traders having beliefs about type distributions and strategies of other traders.[18] Results on best response behavior can therefore be translated to symmetric Bayesian Nash equilibria. We adopt this line of thought and work directly with beliefs over actions, as we will also study the influence of misspecified beliefs on market performance in Section 6.

The distribution of actions of other traders is assumed to be absolutely continuous with probability densities $f^a_{B,i}$ and $f^a_{S,i}$ that are continuous and strictly positive on their supports. Let $\underline{a}_{B,i} = \min\{a_b : f^a_{B,i}(a_b) > 0\}$, $\overline{a}_{B,i} = \max\{a_b : f^a_{B,i}(a_b) > 0\}$, $\underline{a}_{S,i} = \min\{a_s : f^a_{S,i}(a_s) > 0\}$, $\overline{a}_{S,i} = \max\{a_s : f^a_{S,i}(a_s) > 0\}$. We assume that $\overline{a}_{S,i} \geq \overline{a}_{B,i} > \underline{a}_{S,i} \geq \underline{a}_{B,i}$; that is, the action spaces intersect. We also assume that trader i's net value (see Section 4.1) satisfies $t^\Phi_i \in (\underline{a}_{S,i}, \overline{a}_{B,i})$; that is a trader i believes that when being truthful, traders on both market sides will submit both less and more aggressive actions with positive probability.

In finite markets, we impose two additional assumptions. First, we assume that other traders' actions are independent random variables, identically distributed for each of the two market sides. Second, we assume that the supports of distribution of actions of other traders are convex, that is, $A_{B,i} = [\underline{a}_{B,i}, \overline{a}_{B,i}]$ and $A_{S,i} = [\underline{a}_{S,i}, \overline{a}_{S,i}]$. Let $(F^a_{B,i}, F^a_{S,i})$ be the pair of corresponding C^1 distribution functions. Realizations of these random variables induce random empirical action distributions μ^a_B and μ^a_S.

In infinite markets, we allow trader i to believe in any action distribution μ^a_B and μ^a_S. One class of absolutely continuous action distributions is obtained by viewing infinite markets as the limit of finite markets. Letting n and m tend to infinity, the random empirical probability measures converge uniformly to measures with densities $f^a_{B,i}$ and $f^a_{S,i}$; for details see Vapnik and Chervonenkis (2015). Scaling these measures by $\mu_B(\mathcal{B}_i)$ and $\mu_S(\mathcal{S}_i)$ results in deterministic beliefs about absolutely continuous action distributions in infinite markets.

Given the beliefs of trader i, let $(\Omega_{\text{-}i}, \mathcal{F}_{\text{-}i}, \mathbb{P}_{\text{-}i})$ be the probability space describing the randomness of the action distribution $a_{\text{-}i}$ and tie-breaking. Denote by $\mathbb{E}_{\text{-}i}[\cdot]$ the *expectation* with respect to the probability measure $\mathbb{P}_{\text{-}i}$.

Let the *belief system* $\mathfrak{B}$ be the collection of all traders beliefs. $\mathfrak{B}$ is thus a mapping from the set of traders $\mathcal{B} \cup \mathcal{S}$ into the space of beliefs. We say that a belief system $\mathfrak{B}$ has a *common prior*, if all traders' beliefs lead to the same critical value (will be introduced in Section 4.2). An example of a common prior belief system is that all traders have exactly the same beliefs. Moreover, we say that a common prior belief system is *calibrated*, if the traders' belief of the critical value coincides with the (true) critical value induced by the type distributions. If a belief system does not have a common prior, we say that it has a *heterogeneous prior*. Section 6, we will assume that for heterogeneous prior belief systems, traders on the same market side and with the same type have the same belief.

To evaluate the robustness of our findings we allow that traders are uncertain about their beliefs

[18] If trader i believes that types are distributed according to (F^t_B, F^t_S) and all traders use a symmetric strategy profile (a_B, a_S), where both strategies are strictly increasing C^1-functions, then actions are distributed according to $F^t_B(a_B^{-1}(\cdot))$ on $A_{B,i}$ and $F^t_S(a_S^{-1}(\cdot))$ on $A_{S,i}$.

in infinite markets. We give a detailed definition of *aggregate uncertainty* in Appendix A.9. In the main text, after each result, we will state qualitatively how they extend to markets with aggregate uncertainty. The formal results are again relegated to Appendix A.9.

4 Key Concepts

In this section we introduce three key concepts which will allow to analyse optimal behavior. First, we give a definition of what it means to not be loss-making ex-post in the presence of transaction costs. The second is concerned with a trader's ability to estimate their probability of trade. Third, we introduce the key distinction between influenceable and asymptotically uninfluenceable transaction costs and their relationship to the profitability of trade.

4.1 Truthfulness

Without transaction costs, if trader i bids their gross value ($a_i(t_i) = t_i$), they maximize the probability to be involved in trade, conditional on guaranteeing ex-post individual rationality. An action a_i is ex-post individually rational, if for all $a_{\text{-}i}$ it holds that $u_i(t_i, a_i, a_{\text{-}i}) \geq 0$. Such behavior is often called *truthful* because a trader reveals their type. Buyers prefer not to trade at market prices above their gross value, and sellers prefer not to trade at market prices below their gross value. Indeed, bidding gross values represents the maximal bids that constitute undominated actions for buyers, and similarly the minimal asks that constitute undominated actions for sellers. We say that an action a_i^1 dominates an action a_i^2, if for all $a_{\text{-}i}$ it holds that $u_i(t_i, a_i^1, a_{\text{-}i}) \geq u_i(t_i, a_i^2, a_{\text{-}i})$.

In the presence of transaction costs, actions may have to be more aggressive than gross values in order to guarantee ex-post individual rationality, and bidding gross values may be dominated. For some transaction costs, e.g., for constant and price fees, bidding ones gross value would result in negative utility when the market price is equal to the gross value. Taking transaction costs into account, we define a buyer's *net value*, t_b^Φ, as the supremum of the set of undominated and ex-post individually rationality. Similarly, we define a seller's *net value*, t_s^Φ, as the infimum of the latter set. If the net value does not exist the trader has no action guaranteeing ex-post individually rational actions (see Appendix A.4 for pathological examples, where the net value does not exist).

In the presence of transaction costs, we say that a trader is *truthful* if they bid their net value. Recall, that without transaction costs the net value is the gross value. Moreover, we say that an action a_i is *(strictly) individually rational*, if it is (strictly) smaller than the net value for buyers and (strictly) greater than the net value for sellers.

Next, we show that for an important class of transaction costs (that includes constant, price and spread fees), the net value exists and is analytically well-behaved. Consider that transaction cost only depends on the action of a trader and the market price, that is $\Phi_i(a_i, a_{\text{-}i}) = \Phi_i(a_i, P^*(a_i, a_{\text{-}i}))$. Suppose that $P^* \mapsto P_i(a_i, P^*)$ is increasing, $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing and both are continuous. We call such transaction costs *regular*.

Define the sets of gross values that allow for profitable trade, $T_b^+ = \{t_b : \exists a_b : t_b - a_b - \Phi_b(a_b, a_b) > 0\}$ and $T_s^+ = \{t_s : \exists a_s : a_s - t_s - \Phi_s(a_s, a_s) > 0\}$.

Proposition 1 (Existence of net values)**.** *Consider regular transaction costs. For $t_i \in T_i^+$, the net value exists and it is undominated and ex-post individually rational. It is continuous and strictly increasing in the gross value and given by the unique solution of the equation $t_b - x - \Phi_b(x, x) = 0$ for a buyer and $x - t_s - \Phi_s(x, x) = 0$ for a seller.*

Proof details are relegated to Appendix B.1. According to Proposition 1, for such transaction costs the net value is the unique action, at which a trader is indifferent between trading and not trading, when the market price is equal to their action. For constant, price and spread fees, this characterization allows to express the net value as a function of the gross value and the fee parameter.

Corollary 2 (Net values for constant fees, price, and spread fees)**.** *For constant fees, the net value shifts the gross value, that is, $t_b^\Phi = \max(0, t_b - c_b)$ and $t_s^\Phi = t_s + c_s$. Similarly, for price fees the net value scales the gross value, that is, $t_b^\Phi = {}^{t_b}/_{1+\phi_b}$ and $t_s^\Phi = {}^{t_s}/_{1-\phi_s}$. By contrast, for spread fees the gross value equals the net value.*

Proof details are relegated to Appendix B.2. To exclude pathological scenarios we will assume that the net value exists, is strictly increasing, and continuous in the gross value (thus, including the regular transaction costs considered in Proposition 1).

4.2 Predictability of trade

Consider trader i's *probability of trading*, $\mathbb{P}_{\text{-}i}\left[i \in A^*(a_i, a_{\text{-}i})\right]$. In finite markets and infinite markets with aggregate uncertainty, the function $a_i \mapsto \mathbb{P}_{\text{-}i}\left[i \in A^*(a_i, a_{\text{-}i})\right]$ is continuous and can be expressed in terms of $F_{S,i}^a$ and $F_{B,i}^a$.[19] In infinite markets without aggregate uncertainty, trader i believes that the market price is deterministic and equal to the unique solution of the equation $\mu_S(\mathcal{S})F_{S,i}^a(\cdot) = \mu_B(\mathcal{B})(1 - F_{B,i}^a(\cdot))$. Call this solution the *critical value* P_i^∞.[20] The probability of trading is equal to 1, if trader i's action is less aggressive than P_i^∞. If their action is equal to P_i^∞ they believe to be involved in tie-breaking and trade with some probability between 0 and 1. If their action is more aggressive, trader i believes that they are not involved in trade.

The critical value is also of central importance for the study of trading probabilities in large finite markets. Given trader i's beliefs about others' behaviors, they can compute the market price with increasing accuracy as the market grows. With increasing numbers of traders on both market sides the variance of the realized market price decreases and it converges to the critical value. The probability of trading then converges to a step function at the critical value P_i^∞.

[19]This is proven in Appendix A.3.2 and Appendix A.7 (see Equations (28) and (29)).
[20]Existence and uniqueness are proven in Appendix B.3.

Proposition 3 (Predictability of trade)**.** *Consider trader i with action a_i. For every $\epsilon > 0$, in sufficiently large markets, the probability of trade for i is (1) bounded from below by $1-\epsilon$ if a_i is strictly less aggressive than the critical value P_i^∞ and (2) bounded from above by ϵ if a_i is strictly more aggressive than the critical value P_i^∞.*

In the omitted case, when $a_i = P_i^\infty$, the trading probability in finite markets is determined by the action distributions and lies strictly between 0 and 1.[21] This results remains true, if trader's have sufficiently small uncertainty about the market, see Appendix A.9.

Proof Outline. Growing market size in finite markets is formalized with respect to a single parameter. Consider a sequence of strictly increasing market sizes $(m(l), n(l))_{l\in\mathbb{N}}$ with $m(l), n(l) = \Theta(l)$ and $|R - \frac{n(l)}{m(l)}| = \mathcal{O}(l^{-1})$ for $R \in (0,\infty)$.[22] A buyer b is involved in trade, if their action a_b is greater (or equal, if they win tie-breaking) than at least $m(l)$ actions of other traders, that is $\mathbb{P}_{\text{-}b}\left[b \in A^*(a_b, a_{\text{-}b})\right] = \mathbb{P}_{\text{-}b}\left[a_b \geq a_{\text{-}b}^{m(l)}\right]$. The probability that the action of any other buyer and seller is below a_b is $p_{a_b} = F_{B,b}(a_b)$ and $q_{a_b} = F_{S,b}(a_b)$. If $X_i^{p_{a_b}}$ and $X_j^{q_{a_b}}$ are Bernoulli random variables with parameters p_{a_b} and q_{a_b}, then the total number of traders with actions below a_b has the same distribution as the sum $S_l^{a_b} = \sum_{i=1}^{m(l)-1} X_i^{p_{a_b}} + \sum_{j=1}^{n(l)} X_j^{q_{a_b}}$. It follows that $\mathbb{P}_{\text{-}b}\left[b \in A^*(a_b, a_{\text{-}b})\right] = \mathbb{P}[S_l^{a_b} \geq m(l)] = 1 - \mathbb{P}[S_l^{a_b} \leq m(l)-1]$. By the Berry-Esseen Theorem (Tyurin, 2012) an appropriately normalized version of $S_l^{a_b}$ converges in distribution to a standard normal random variable with CDF Φ. We show that there exists a sequence $(A_{a_b}(l))_{l\in\mathbb{N}} = \Theta(\sqrt{l})$ with $|\mathbb{P}[S_l^{a_b} \leq m(l)-1] - \Phi(A_{a_b}(l))| \in \mathcal{O}(l^{-\frac{1}{2}})$. For $a_b \prec P_b^\infty$ we show for sufficiently large l that $A_{a_b}(l) < 0$, which yields that $A_{a_b}(l) \in \Theta(-\sqrt{l})$. Using a concentration inequality for a standard Gaussian random variable gives $\Phi(A_{a_b}(l)) \in \mathcal{O}(e^{-l})$. It therefore holds that $\mathbb{P}[S_l^{a_b} \leq m(l)-1] = \mathcal{O}(l^{-\frac{1}{2}})$. The statement for $a_b \succ P_b^\infty$ and for sellers can be derived analogously. In infinite markets, the statement follows directly from the model. Proof details are relegated to Appendix B.4. □

We sometimes focus on *in-the-market* gross values that are gross values t_i such that $t_i^\Phi \prec P_i^\infty$. Traders with such gross values are able to submit individually rational actions that make them likely to be involved in trade when the market is sufficiently large. By contrast, for an *out-of-the-market* trader, that is, one with gross value $t_i^\Phi \succ P_i^\infty$, the probability of trade, when acting individually rationally, vanishes in large markets.

4.3 Profitability of trade

We now turn to the *expected utility conditional on trading.* Write $\mathbb{E}_{\text{-}i}\left[\cdot | i \in A^*(a_i, a_{\text{-}i})\right]$ for the conditional expectation of trader i given their beliefs. Recall, that we assume that payments are

[21] E.g., for uniform action distributions and equally many buyers and sellers, the trading probability is independent of the market size and equal to $\frac{1}{2}$; we provide more details in the proof of point 2 of Theorem 10.

[22] If there exists a parameter l, such that for every $l' \geq l$ Proposition 3 holds in markets with $m(l')$ buyers and $n(l')$ sellers, then the statement also holds in sufficiently large finite markets.

monotone in the aggressiveness of one's action. Further, payments are composed of the market price and a transaction cost. For the former, it is known from Rustichini et al. (1994), that in large markets traders have vanishing influence on the market price. On the other hand, this is not necessarily the case for transaction costs. To this end, a classification of transaction costs into two broad classes turns out to be useful.

Definition (Asymptotically uninfluenceable vs. influenceable transaction costs)**.** Two actions a_i^1 and a_i^2, such that a_i^1 is less aggressive than a_i^2 and both are less aggressive than the critical value, that is $a_i^1 \prec a_i^2 \prec P_i^\infty$, lead to *asymptotically different transaction costs*, if there exists $\epsilon > 0$ such that in sufficiently large markets

$$\mathbb{E}_{\text{-}i}\left[\Phi_i(a_i^1, a_{\text{-}i}) | i \in A^*\left(a_i^1, a_{\text{-}i}\right)\right] - \mathbb{E}_{\text{-}i}\left[\Phi_i(a_i^2, a_{\text{-}i}) | i \in A^*\left(a_i^2, a_{\text{-}i}\right)\right] \geq \epsilon. \tag{1}$$

Otherwise, the two actions lead to *asymptotically equal transaction costs*. Transaction costs Φ_i are *influenceable* if every two such actions $a_i^1 \prec a_i^2 \prec P_i^\infty$ lead to asymptotically different transaction costs. Transaction costs Φ_i are *asymptotically uninfluenceable* if for every $\epsilon > 0$ in sufficiently large markets without aggregate uncertainty

$$\sup_{a_i^1 \prec a_i^2 \prec P_i^\infty} \mathbb{E}_{\text{-}i}\left[\Phi_i\left(a_i^1, a_{\text{-}i}\right) | i \in A^*\left(a_i^1, a_{\text{-}i}\right)\right] - \mathbb{E}_{\text{-}i}\left[\Phi_i\left(a_i^2, a_{\text{-}i}\right) | i \in A^*\left(a_i^2, a_{\text{-}i}\right)\right] \leq \epsilon. \tag{2}$$

In infinite markets, the definitions simplify, as there is no randomness due to sampling. Influenceability is then equivalent to the map $a_i \mapsto \Phi_i(a_i, a_{\text{-}i})$ being strictly increasing for buyers and strictly decreasing for sellers. Uninfluenceability is equivalent to the map $a_i \mapsto \Phi_i(a_i, a_{\text{-}i})$ being constant. For regular transaction costs that only depend on the trader's action and the market price, this implies that in infinite markets, uninfluenceable transaction costs are a function of the market price, i.e. $\Phi_i(P^*)$, while for influenceable transaction costs, the function $a_i \mapsto \Phi_i(a_i, P^*)$ is again strictly monotone.[23]

An influenceable transaction cost might still include an asymptotically uninfluenceable part, e.g., the sum of a price and spread fee. We say that a regular influenceable transaction cost $\Phi_i(a_i, P^*)$ is purely influenceable , if it holds that $\Phi_i(P^*, P^*) = 0$. Note that for purely influenceable transaction costs the net value equal the gross value. Spread fees are an example of purely influenceable transaction costs. For regular transaction costs, it is possible to decompose any transaction cost into an asymptotically uninfluenceable and purely influenceable part .

Lemma 4 (Decomposition of regular transaction costs.)**.** *A regular influenceable transaction cost can be written as the sum of an asymptotically uninfluenceable transaction cost and a purely influenceable transaction cost.*

[23]These monotonicity conditions can be equivalently stated for the payment function $a_i \mapsto P_i(a_i, P^*)$.

Proof details are relegated to Appendix B.5. Moreover, the two types are not mutually exclusive, as one can construct transaction costs that are asymptotically uninfluenceable in some price regions and influenceable at others. However, focusing on these two cases (rather than on hybrids) allows us to study the key strategic differences that in fact yield completely opposing behavior. In particular, the two canonical examples of transaction costs, price and spread fees, fall under the two definitions: Price fees are asymptotically uninfluenceable , and spread fees are influenceable .

5 Trader's behavior

Best responses maximize individual expected utility given beliefs. The maximization finds the right amount of aggressiveness, balancing the opposing forces of increasing the probability of trade versus increasing the utility when trading.[24] Given trader i's beliefs and gross value t_i, an action a_i is an ϵ-*best response* if $\mathbb{E}_{\text{-}i}\left[u_i\left(t_i, a_i, a_{\text{-}i}\right)\right] \geq \sup_{a_i' \in \mathbb{R}} \mathbb{E}_{\text{-}i}\left[u_i\left(t_i, a_i', a_{\text{-}i}\right)\right] - \epsilon$. For $\epsilon = 0$ a_i is a *best response.*

The analysis of best responses includes the special case of *symmetric Bayesian Nash equilibria.* If all buyers use the same strictly increasing and continuous strategy a_B and all sellers use the same strictly increasing and continuous strategy a_S, call (a_B, a_S) a *symmetric strategy profile.* Given type distributions, the corresponding action distributions are given by $\mu_B^a(\cdot) = \mu_B\left(t_B^{-1}(a_B^{-1}(\cdot))\right)$ and $\mu_S^a(\cdot) = \mu_S\left(t_S^{-1}(a_S^{-1}(\cdot))\right)$. Assume that beliefs over action distributions originate from beliefs over gross value distributions and over the symmetric strategy profiles of the other traders (a_B, a_S). If, for every trader and every gross value, the action specified by these strategies are ϵ-best responses, then the strategy profile constitutes a symmetric ϵ-Bayesian Nash equilibrium.[25]

Proposition 5 (Existence of best responses)**.** *Suppose the market environment is finite or, if not, tie-breaking is a probability zero event. Then a best response exists for trader i.*

In infinite markets the no-tie-breaking assumption matters. In its absence, a best response might not exist for a trader i with $t_i \prec P_i^\infty$. This is the case, for example, when spread fees are charged. Under spread fees, it is not optimal to bid P_i^∞ (or more aggressive) due to the risk of loosing out on trading. But for any less aggressive bid, bidding slightly more aggressively would lead to a higher payoff. The results also extends to aggregate uncertainty, see Appendix A.9.

Proof Outline. We show that a best response is necessarily located in a compact action space. Given the continuity assumption of the payment, it follows that the expected utility is continuous in the action a_i and therefore attains a maximum by the Extreme Value Theorem. Proof details are relegated to Appendix B.6. □

[24] A detailed analysis of this trade-off for price and spread fees in finite markets via first order conditions can be found in Appendix A.6.

[25] Therefore all of the results that we shall present in this paper about best responses directly apply to the study of symmetric Bayesian Nash equilibria.

The following theorem is a first indication that transaction costs have significant strategic consequences.

Theorem 6 (Asymptotically equal transaction costs)**.** *Let T^* be the set of gross values of trader i at which bidding the critical value is strictly individually rational. If trader i is best responding, then the expected transaction costs of any two types $t, t' \in T^*$ are asymptotically equal.*

For uninfluenceable transaction costs this result holds by definition. For influenceable transaction costs, the result is non-trivial and will be useful in later analyses (see Section 5.2). This result qualitatively extends to markets with sufficiently small aggregate uncertainty, see Appendix A.9.

Proof Outline. Assume that two actions $a_i^1 \prec a_i^2 \prec P_i^\infty$ lead to asymptotically different transaction costs. We show that in sufficiently large markets, a trader can increase their expected utility, when switching from action a_i^1 to a_i^2, proving that a_i^1 is not a best response. Formally, as $a_i^1 \prec a_i^2 \prec P_i^\infty$, Proposition 3 yields that for every $\epsilon_1 > 0$, $\mathbb{P}_{\text{-}i}\left[i \in A^*(a_i^1, a_{\text{-}i})\right], \mathbb{P}_{\text{-}i}\left[i \in A^*(a_i^2, a_{\text{-}i})\right] \geq 1 - \epsilon_1$ in sufficiently large markets. The difference in trading probability between a_i^1 and a_i^2 is then upper bounded by ϵ_1. If ϵ_1 is sufficiently small, the loss in trading probability and possible influence on the market price is compensated by a decrease in expected transaction cost by at least some $\epsilon_2 > 0$ because transactions are assumed to be asymptotically different. For sufficiently small ϵ_1, the difference in expected utility between actions a_i^1 and a_i^2 is negative, if the market is sufficiently large, proving that a_i^1 is indeed not a best response. Proof details are relegated to Appendix B.7. $\square$

5.1 Truthfulness is approximately optimal with uninfluenceable transaction costs

Strategic misrepresentation is driven by the incentive to influence market price and transaction cost. Reporting truthfully maximizes one's trading probability, while remaining individually rational. In large markets, the influence on the market price is vanishing 'faster' than the influence on one's trading probability, which is what drives the asymptotic truthfulness result in the literature, see Rustichini et al. (1994). Therefore, if the influence on one's own transaction cost is also vanishing 'fast' enough, then it is close to optimal to maximize one's trading probability by reporting truthfully. This is the case for uninfluenceable transaction costs, such as constant or price fee.

Theorem 7 (In large markets with uninfluenceable transaction costs truthfulness is an approximate best response)**.** *If the transaction cost is uninfluenceable and trader i's best response is uniformly bounded away from the critical value P_i^∞, then for every $\epsilon > 0$, in sufficiently large markets, truthfulness is an ϵ-best response.*

In infinite markets, the presence of aggregate uncertainty strenghtens this result, as truthfulness is then the unique best response, see Appendix A.9.

Proof Outline. Consider a best response a_i of trader i. If $a_i \prec t_i^\Phi$, then t_i^Φ is a best response by weak domination. Suppose now that $a_i \succ t_i^\Phi$. By assumption, there exists $\delta > 0$, such that in sufficiently large markets, (i) $a_i \prec P_i^\infty - \delta$ or (ii) $a_i \succ P_i^\infty + \delta$ holds. If (i) holds, then Proposition 3 implies that $\mathbb{P}_{\text{-}i}\left[i \in A^*(a_i, a_{\text{-}i})\right]$ converges to zero as the market gets large. Therefore for all $\epsilon > 0$ the expected utility of a_i is then upper bounded by ϵ, which also proves that that the net value is an ϵ-best response, because it leads to a non-negative expected utility. If (ii) holds, consider $\mathbb{E}_{\text{-}i}[u_i(t_i, a_i, a_{\text{-}i})] - \mathbb{E}_{\text{-}i}[u_i(t_i, t_i^\Phi, a_{\text{-}i})]$. We split the difference into two components and show that for every $\forall\epsilon > 0$ both components are less or equal than $\frac{\epsilon}{2}$ if the market is sufficiently large: (a) Difference in expected transaction costs and (b) Terms corresponding to a classical DA without transaction costs. To bound (a), we can use Proposition 3 and uninfluenceability. For (b), we will use that for a DA without transaction costs truthfulness is an ϵ-best response in sufficiently large markets, see Theorem 8.2 with price fees equal to zero. Proof details are relegated to Appendix B.8. $\square$

Price fees. Fixing a specific transaction cost allows sharper results than Theorem 7. In particular, for a price fee, any best response can be explicitly shown to be close to truthful in large finite markets.

Theorem 8 (In large markets with price fees best responses are approximately truthful and truthfulness is an approximate best response)**.** *If the fee is a price fee, then for every $\epsilon > 0$ it holds that (1) in sufficiently large markets truthfulness is an ϵ-best response and (2) in sufficiently large finite markets all best responses are ϵ-truthful.*

In infinite markets, truthfulness is not unique as a best response. Every action $a_i \succ P_i^\infty$ that is individually rational is also a best response. Theorem 8 is robust to aggregate uncertainty in infinite markets, in which case truthfulness is also the unique best response, see Appendix A.9.

Proof Outline. Consider a buyer b. For (2), a best response satisfies the first order condition $\frac{d\mathbb{E}_{\text{-}b}[u_b(t_b, a_b, a_{\text{-}b})]}{da_b} = 0$, see Appendix A.6. Explicit calculations yield that there exists a constant $\kappa > 0$, such that $t_b - (1 + \phi_b)\, a_b \leq \kappa q(n, m)$, with $q(m, n) = \max\left\{\frac{1}{n}\left(1 + \frac{m}{n}\right), \frac{1}{m}\left(1 + \frac{n}{m}\right)\right\} = O(\max(m, n)^{-1})$, from which the statement follows.[26] For (1), we estimate $\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]$, where a_b denotes the best response. This difference is shown to be upper bounded by $-2k(1 + \phi_b)|t_b^\Phi - a_b|$. It follows from (2) that $\forall\delta > 0$ it holds in sufficiently large finite markets that $t_b^\Phi - a_b \leq \delta$. If for a given $\epsilon > 0$, $\delta > 0$ is chosen such that $\delta \leq \frac{\epsilon}{2k(1+\phi_b)}$, it holds that t_b^Φ is ϵ-close to a best response a_b in sufficiently large finite markets. In infinite markets, the expected utility is deterministic and truthfulness is a best response, as the only strategic incentive is to be involved in trade. Proof details are relegated to Appendix B.9. $\square$

[26] A similar proof technique has been used to show that Bayesian Nash equilibria are approximately truthful in DAs without fees, see Rustichini et al. (1994, Theorem 3.1).

5.2 Price-guessing is approximately optimal with influenceable transaction costs

If a trader can influence their transaction cost, then there remains a (non-vanishing) incentive to act strategically in large markets. Moreover, given a trader will almost certainly trade as long as their action meets the required threshold of the critical value, the incentive to influence their transaction cost asymptotically outweighs the concern of loosing out on the deal. Therefore, it is optimal to bid close to the critical value that corresponds to the predicted price, which is why we shall call such behavior *Price-Guessing*. While our analysis only covers the case of a trader for whom bidding the critical value is individually rational, the case of traders for whom it is not is discussed in Proposition 21.

Theorem 9 (In large markets with influenceable transaction costs best responses are close to price guessing)**.** *If the transaction cost is influenceable and bidding the critical value P_i^∞ is strictly individually rational for trader i, then for every $\epsilon > 0$, in sufficiently large finite markets, all best responses of i are in an ϵ-neighbourhood of the critical value P_i^∞.*

This result extend to infinite markets with sufficiently small aggregate uncertainty, see Appendix A.9.

Proof Outline. Consider a buyer with action $a_b > P_b^\infty$. We show that if $a_b - P_b^\infty \geq \epsilon$, then the difference in expected utility from playing a_b versus $P_b^\infty + \frac{\epsilon}{2}$ is strictly negative in sufficiently large markets, proving that a_b is then not a best response. Similar to the proof of Theorem 6, we show that in such markets, the buyer will be involved in trade with high probability with both actions. Using that the transaction cost is influenceable , the decrease of the transaction cost when switching to the more aggressive action $P_b^\infty + \frac{\epsilon}{2}$ outweighs the decrease in trading probability. Proof details are relegated to Appendix B.10. $\square$

Spread fees. As a spread fee depends linearly on a trader's action, it is an example of a influenceable transaction cost. A best response exists given the spread fee is continuous and must be close to the critical value. However, an analogous statement to Theorem 8.2, i.e., the utility at the critical value is close to optimal, is not true in general. We show that there exist markets, such that bidding the critical value is in general not ϵ-optimal in large markets.

Theorem 10 (In large markets with spread fees best responses are close, but not necessarily equal, to the critical value)**.** *If the fee is a strictly positive spread fee, then a best response exists for a trader i in finite and infinite markets without tie-breaking. Further, if bidding the critical value is strictly individually rational, then (1) for every $\epsilon > 0$, in sufficiently large markets, all best responses of i are in an ϵ-neighbourhood of the critical value P_i^∞ and (2) for sufficiently small $\epsilon > 0$, there exist beliefs, such that in sufficiently large finite markets the critical value P_i^∞ is not an ϵ-best response.*

Theorem 10 is robust to small aggregate uncertainty in infinite markets, see Appendix A.9.

Proof Outline. We show that the expected transaction cost is and therefore the expected utility is continuous in a_i. The existence of a best response again follows as in Theorem 8. Consider a buyer b with $t_b^{\Phi} > P_i^{\infty}$. (1) is proven in complete analogy to Theorem 7.1. For (2), consider beliefs such that the number of traders is equal to l for both market sides, where beliefs are uniformly distributed over $A_B = A_S = [0, 1]$. It follows that $P_b^{\infty} = \frac{1}{2}$. We prove that for every $l > 1$ it holds that $\mathbb{P}_{\text{-}b}[b \in \mathcal{B}^*(P_b^{\infty}, a_{\text{-}b})] = \frac{1}{2}$. Therefore, for every bid $a_b > P_b^{\infty}$ and for every $\epsilon > 0$, it follows from Proposition 3 that the buyer can increase their trading probability by $\frac{1}{2} - \epsilon$ when switching from P_b^{∞} to a_b. If a_b is chosen close to P_b^{∞}, then this outweighs the increase in spread fee payment. Proof details are relegated to to Appendix B.11. □

5.3 Best responses and Bayesian Nash equilibria for price versus spread fees

Consider a finite markets with sizes (i) 2×2 (that is, two buyers and two sellers) and (ii) 5×5 in the presence of either a price fee $\phi_i = 0.1$ or a spread fee $\phi_i = 1$, and $k = 0.5$. Figure 4 shows best response strategies (*Top.*) for uniform beliefs over others' actions in $[1, 2]$ and a symmetric Bayesian Nash Equilibrium (*Bottom*) for uniform beliefs over gross values in $[1, 2]$ for price fees (*Left.*) and spread fees (*Right.*).

In line with Theorem 8, optimal strategic behavior converges to truthfulness with growing market size, if price fees are charged. In a small market with two buyers and two sellers traders have an incentive to be more aggressive and misrepresent their net value, as can be measured by the distance between their respective best response (dashed red/blue lines) and the net value (solid black lines). In contrast, and in line with Theorem 8.1, the best responses (dotted red/blue line) in the larger market (5×5) are approaching truth-telling.

Note that in line with Theorem 10, best responses converge towards price-guessing with growing market size if spread fees are charged. In a small market with two buyers and two sellers traders have an incentive to be aggressive and misrepresent their true net value in order to influence the price and reduce their fee payment. In line with implications from Theorem 10, best responses in a larger market with five buyers and sellers (dotted line) do not approach truth-telling, if $t_i \prec P_i^{\infty}$. Instead traders remain aggressive as they aim to reduce their fee payment. In contrast, their influence on the price diminishes which results in traders approximating the critical value P_i^{∞} provided it is individually rational.

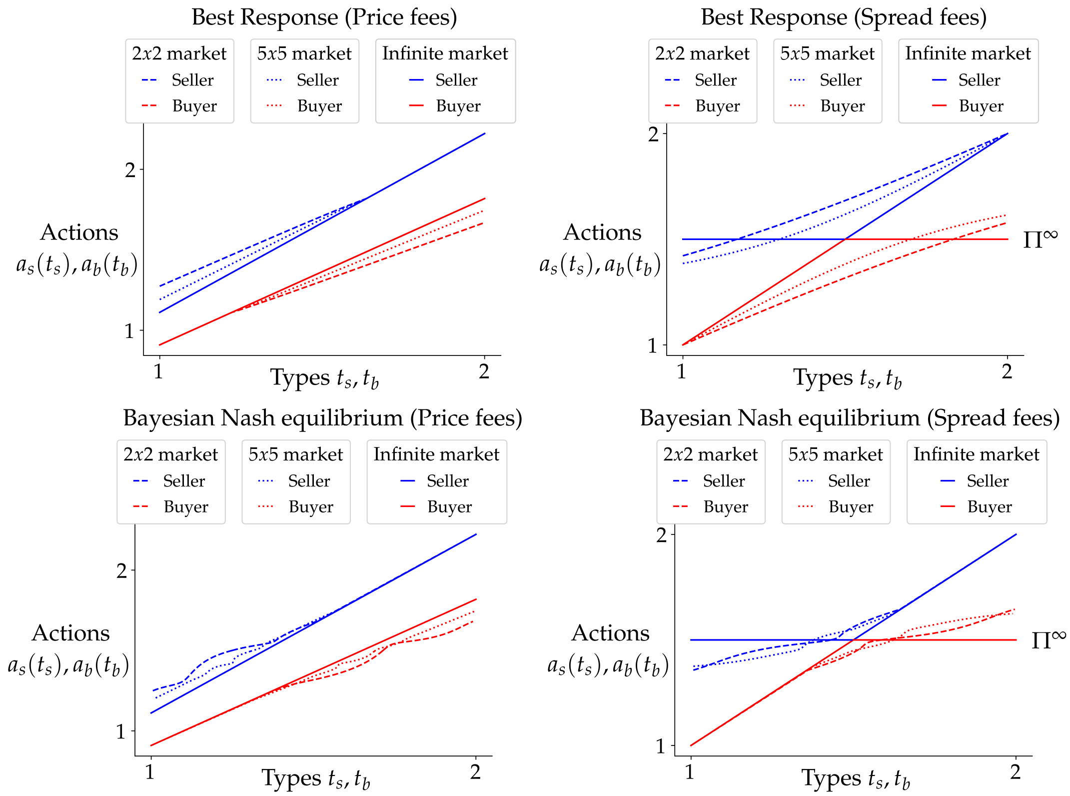


Figure 4: Best responses for uniform beliefs over actions (top) and a symmetric Bayesian Nash equilibrium for uniform beliefs over types (bottom) for buyers (red) and sellers (blue) as functions of their gross value for 2×2 (dashed lines) and 5×5 (dotted lines) markets with price fee $\phi_i = 0.1$ (left) or spread fee $\phi_i = 1$ (right).

6 Market Performance and Design

So far we did not commit to a specific nature of transaction costs. For example, transaction costs could have included shipping costs, taxes, and revenue of a market platform. As we are now interested in welfare metrics, we will assume that all transaction costs are collected by some market platform. Then a social planner evaluates the market outcome by considering the social welfare and platform revenue according to some objective function. If the social planner can design the transaction cost, what is the optimal choice?

The social planner thus chooses the transaction cost (e.g., constant fee, price fee, spread fee) and its scale. For the latter, define the *scaling* of transaction costs Φ; for a two-dimensional parameter $\gamma = (\gamma_B, \gamma_s)$ with $\gamma_B, \gamma_S \geq 0$ the linear γ-scaling of transaction costs Φ are $\Phi_B^\gamma = \gamma_B \cdot \Phi$ and $\Phi_S^\gamma = \gamma_S \cdot \Phi$. For instance, for price and spread fees, γ-scaling linearly scales the fee percentage ϕ_i.

For analytical tractability, we will restrict our analysis to infinite markets with type distributions μ_B^t and μ_S^t and regular transaction costs (Φ_B, Φ_S) that are charged to all buyers and all sellers involved in trade. Recall that those are transaction costs that only depend on a trader's action and the market price. As we work in an infinite market, we will use the term uninfluenceable transaction costs instead of asymptotically uninfluenceable transaction costs.

As in Section 5, traders best respond to their beliefs about the market environment. For uninfluenceable transaction costs we focus on traders that truthfully report their net value. Truthfulness is a best response in infinite markets but other behaviors are also possible. However, we focus on truthfulness, as we show in Theorems 7 and 8, limit best-response behavior in large finite markets approaches truthfulness, and it is the unique best response in infinite markets with aggregate uncertainty, c.f. Appendix A.9. For influenceable transaction costs suppose that traders price-guess. This behavior is the unique best response in infinite markets and the approximate best response in large finite markets, see Theorems 9 and 10. This also holds in markets with sufficiently small aggregate uncertainty, c.f. Appendix A.9. Therefore the results in this section are qualitatively robust to small aggregate uncertainty, but nevertheless may differ, c.f. Section 2 for an analysis of market performance for spread fees in the presence of aggregate uncertainty.

In this section, we will analyze the performance of different transaction costs for fixed belief systems. For uninfluenceable transaction costs, optimal behavior does not depend on traders' beliefs and the analysis of market performance will be independent of the belief system. For influenceable transaction costs, price-guessing does not depend on scaling. Beliefs about the market price are therefore independent of which influenceable transaction cost is analyzed. Based on these two observations, we assume that a change in transaction costs does not affect the traders' belief system.

6.1 Market Performance

The social planner evaluates the market outcome using the following standard performance metrics.

The *traders' welfare* $W = \int_{\mathcal{B}^*} u_b(t_b, a_b, a_{\text{-}b}) d\mu_B(b) + \int_{\mathcal{S}^*} u_s(t_s, a_s, a_{\text{-}s}) d\mu_S(s)$ is the overall utility of all traders involved in trade.[27] The *platform revenue* $R = \int_{\mathcal{B}^*} \Phi_b(a_b, a_{\text{-}b}) d\mu_B(b) + \int_{\mathcal{S}^*} \Phi_s(a_s, a_{\text{-}s}) d\mu_S(s)$ is the total amount of transaction costs that is collected by the market maker. The sum $G = W + R$ are the *realized gains of trade*, and note that $G = \int_{\mathcal{B}^*} (t_b - P^*)\, d\mu_B(b) + \int_{\mathcal{S}^*} (P^* - t_s)\, d\mu_S(s)$. If agents report truthfully in the presence of transaction costs Φ, we denote by G^{net} the *net gains of trade*. If no transaction is charged reporting truthfully thus yields the *gross gains of trade* G^{gross}, where $G^{gross} \geq G^{net} \geq G$ will be shown to hold. The *loss* $L = G^{gross} - G$ measures how much gains of trade are lost due to fee considerations and strategic behavior. It can be split into the *direct loss* $L_\phi = G^{gross} - G^{net}$, that is due to transaction costs, and the *strategy-induced loss* $L_F = G^{net} - G$.

The gross gains of trade are equal to the sum of platform revenue, social welfare and loss, that is $G^{gross} = W + R + L$. We identify *market performance* with the triple (W, R, L). We normalize $G^{gross} = 1$ and hence the set of all performance triples lie on a triangle Δ in a 2-dimensional hyperplane in $\mathbb{R}^3$. We say that a performance triple (W, R, L) is *achievable* for transaction costs Φ

[27] Because best responses are individually rational, W is non-negative.

and belief system $\mathfrak{B}$, if there exists a γ-scaling, such that optimal behavior of all traders leads to that market performance.

6.2 Optimal transaction cost design

Suppose that the social planner aims to maximize a continuous objective function $U : \Delta \to \mathbb{R}$ on performance triples $(W, R, L) \in \Delta$. We will consider objective functions, such that a Pareto improvement of welfare and revenue leads to an increase in utility, that is, for any performance triplet (W, R, L) and for $\alpha, \beta \geq 0$ with $\alpha+\beta \leq L$ it holds that $U\left(W, R, L\right) \leq U\left(W + \alpha, R + \beta, L - (\alpha + \beta)\right)$. In particular, it holds that $U(1, 0, 0) \geq U(0, 0, 1)$, that is the social planner prefers a fully efficient market with zero revenue for the market maker over a fully inefficient market.

We say that transaction costs Φ^1 *(weakly) U-dominate* transaction costs Φ^2 for a class of beliefs, if for any belief system $\mathfrak{B}$ in that class the resulting market performance $U\left(W, R, L\right)$ is (weakly) greater for the former than for the latter. We call transaction costs Φ *U-dominant* for a class of beliefs, if they weakly U-dominate all other transaction costs.

Optimal design of transaction costs depends on the nature of traders' beliefs. The following theorem shows that for common prior beliefs optimal design is possible for any uninfluenceable transaction cost and crucially, independent of the specific belief system. By contrast, for heterogeneous prior belief systems there exists no transaction cost that is always optimal.

Theorem 11 (Optimal Design)**.** *Consider a social planner with objective function U. For the class of common prior beliefs, all uninfluenceable transaction costs can be scaled to be U-dominant. Furthermore, the optimal scaling does not depend on the beliefs. For the class of heterogeneous prior beliefs, there exists no U-dominant transaction cost.*

Notably, for common prior belief systems, the optimal design problem is reduced from the space of all transaction costs to a one-dimensional optimization problem of finding the optimal scaling for any uninfluenceable transaction cost. For some heterogeneous prior beliefs, influenceable transaction costs, even without an uninfluenceable part, can strictly outperform any uninfluenceable transaction cost. However, there also exist heterogeneous prior beliefs, such that any uninfluenceable transaction cost outperforms all influenceable transaction costs, as the latter class would lead to market failure.

The proof is relegated to Appendix B.12. We will omit a proof outline for Theorem 11 as it combines results of the more detailed analysis that follows. Concretely, we analyse uninfluenceable and purely influenceable transaction costs separately, to detail what market performances are achievable and how they depend on the belief system and scaling. Finally, we return to mixed transaction costs to discuss under what circumstances it may be optimal to use those.

6.2.1 Uninfluenceable Transaction Costs

Suppose that the market maker charges uninfluenceable transaction costs Φ. The following proposition characterizes the set of all achievable market performances: First, it is fully specified by the type distributions μ_B^t and μ_S^t, but does not depend on the choice of the uninfluenceable transaction costs, or the traders' belief system. This implies that any market performance achievable with one uninfluenceable transaction cost can be achieved with another provided it is properly scaled. Second, this set is one-dimensional, as scaling is the only way to influence the market performance, which in turn implies that most performances are not achievable with uninfluenceable transaction costs.

Proposition 12 (Equivalence of scaled uninfluenceable transaction costs)**.** *The set of achievable performance triples (W, R, L) is the same for all uninfluenceable transaction costs Φ and belief systems $\mathfrak{B}$. The set is a curve $c_P : [0, 1] \to \Delta$ in the simplex Δ of all performance triples.*

Outline. We prove that for any uninfluenceable transaction cost Φ, the market performance can be represented as a continuous function of the net trading volume V, that is $V \mapsto (W(V), R(V), L(V))$. The revenue is equal to the rectangle with height equal to the net trading volume that fits under the true demand and supply curve. The loss is the area above this rectangle.

Therefore, it suffices to prove that for any $x \in [0, V_{id}]$, there exists a scaling γ, such that the net trading volume is equal to x. The net trading volume is the intersection of net demand and supply. We prove that for a fixed price P, net demand and supply are decreasing as a continuous function of γ. We fix $P = P_{id}^*$ and choose a scaling such that $D(P) = S(P) = x$, which is possible by the Intermediate Value Theorem. Proof details are relegated to Appendix B.13. □

The performance curve c_P has several interesting properties: First, it connects the fully efficient market outcome with zero revenue $(1, 0, 0)$ and the fully inefficient market outcome $(0, 0, 1)$ that corresponds to complete market failure. Second, c_P strictly increasing in the loss and strictly decreasing in the welfare. Therefore, for any level of welfare in $[0, 1]$, there exists a scaling to achieve it. The revenue, as well as the loss, are then uniquely determined by the curve c_P. This implies that positive platform revenue with uninfluenceable transaction costs is directly tied to a positive loss of efficiency.

This has immediate consequences for the optimal design of uninfluenceable transaction costs. The market maker is restricted to a one-dimensional set of achievable performance triples that is fully specified by the type distributions. Given their objective function U, the U-optimal market performance is then achieved by scaling any uninfluenceable transaction cost properly. The belief system of traders does not influence the optimal design.

Corollary 13 (Optimal design of uninfluenceable transaction costs)**.** *Consider a social planner with objective function U. For all uninfluenceable transaction costs Φ, there exists a scaling γ, such that for any belief system $\mathfrak{B}$ the resulting performance triplet (W, R, L) is U-optimal among all achievable market performances.*

Proof Outline. It follows from Proposition 12 that the set of achievable market performances is a compact subset of Δ. Because the objective function U is continuous, it follows from the Extreme Value Theorem that there exists a maximum (W, R, L). We have proven that any achievable market performance is fully specified by the net trading volume and that for any $x \in [0, V_{id}]$, there exists a scaling $\gamma \geq 0$, such that the net trading volume is equal to x. Proof details are relegated to Appendix B.14. □

If the social planner wants to maximize efficiency (that is, minimize the loss), zero transaction costs are optimal. This leads to maximum traders' welfare and zero revenue. For a revenue-maximizing market-maker, there is a non-trivial trade-off between higher transaction costs per trader and trading volume. It follows from the proof of Proposition 12 that the total platform revenue is equal to the area of the rectangle with height equal to the net trading volume that fits under the true demand and supply curve. Maximizing platform revenue is therefore an optimization problem with respect to the net trading volume, for which an optimal solution exists. Once the optimal trading volume is determined, any uninfluenceable transaction cost can be scaled to lead to that trading volume, that is net demand and supply intersect at that height. The horizontal component of the crossing point, that is the market price, determines, how much of the revenue is paid by buyers and sellers. The area to the rectangle left to the market price is paid by sellers, and the area to the right by buyers. With the right scaling – different to buyers and sellers – any market price on the horizontal rectangle can be achieved. Hence, if the total revenue is equal to R, for any $\alpha \in [0, 1]$, there exists a scaling, such that the revenue generated by buyers is $\alpha \cdot R$ and the revenue generated by sellers is $(1 - \alpha) \cdot R$.

One interesting open question is that given type distributions, what is the maximum revenue that can be generated. It was shown in Section 2 that for uniform type distributions, the answer is equal to 0.5. It is therefore impossible to extract the full gains of trade as revenue. Is there a general lower bound for any distribution?

6.2.2 Purely Influenceable Transaction Costs

Suppose that the market makers charges purely influenceable transaction costs Φ. The following proposition characterizes the set of all achievable market performances: For common prior belief systems, platform revenue is always zero and the market maker cannot influence the distribution of welfare and loss. That is, there is a unique achievable market performance that is the same for all purely influenceable transaction costs. Second, even if traders have heterogeneous priors, the loss is again independent of the transaction cost structure. Via scaling, the market maker has some influence on the welfare-revenue distribution.

Proposition 14 (Non-equivalence of scaled influenceable transaction costs)**.** *Suppose the transaction costs are purely influenceable .*

- *For common prior beliefs, there exists a unique achievable market performance (W, R, L) that is the same for all such transaction costs. There is zero revenue, $R = 0$, and, if additionally the belief system is calibrated, then the market is fully efficient, $W = 1$.*
- *For heterogeneous prior beliefs, the set of achievable market performances is a singleton or line-segment with constant loss L that is the same for all such transaction costs. Furthermore, for any $L \in [0, 1]$, there exist a belief systems $\mathfrak{B}$ that lead to loss L.*

Proof Outline. We show that the loss L is fully characterized by the belief system and therefore independent of the scaling of a purely influenceable transaction cost. For common prior beliefs, price guessing leads to market outcomes, where all traders involved in trade submitted an action equal to the realized market price P^*. Therefore, purely influenceable transaction costs lead to zero revenue. If the belief system is calibrated around the true critical value, then the trading volume is maximized and the market is fully efficient. For heterogeneous prior beliefs, scaling of the transaction costs leads to a continuous increase or decrease in revenue. As the loss is fixed, this yields that the set of achievable market performances is a line-segment or singleton. To show that any loss can be realized, we construct belief systems such that the traders with the most profitable gross values are involved in trade with price-guessing. Then, the loss is a continuous function of the trading volume. We prove that any trading volume can be realized with some heterogeneous prior beliefs. Proof details are relegated to Appendix B.15. □

Note that if traders have common prior beliefs, then the market maker has no influence on the market performance (W, R, L) via the choice of the purely influenceable transaction cost. For heterogeneous priors, the market maker might influence the welfare-revenue distribution by the choice of the transaction cost. For the special case of spread fees, any such distribution is achievable. That is, any performance triple (W, R, L) is achievable for some belief system and scaling. This is in stark contrast to uninfluenceable transaction cost, where only a one-dimensional subset of the space of all performance triples is achievable.

Note that for some beliefs (e.g., an aggressive bias as illustrated in Section 2, complete market failure, that is $(0, 0, 1)$ is the only achievable market performance. However, the market maker has the possibility to scale the transaction cost to zero, which is not purely influenceable any more and leads to the fully efficient market with zero revenue, that is $(1, 0, 0)$.

Proposition 14 implies that the optimal design of purely influenceable transaction costs crucially depends on the traders' belief system. For some belief systems, including common prior beliefs, it turns out to be optimal to not charge any purely influenceable transaction costs at all.

Corollary 15 (Optimal design of purely influenceable transaction costs)**.** *Consider a social planner with objective function U and purely influenceable transaction costs Φ.*

- *For common prior beliefs, the U-optimal scaling of Φ is $\gamma = (0, 0)$ with market performance $(1, 0, 0)$.*

- *For heterogeneous prior beliefs, the U-optimal scaling of Φ depends on the belief system. For some such beliefs the scaling $\gamma = (0, 0)$ is again U-optimal.*

Proof Outline. It follows from Proposition 14 that for common prior beliefs, there exists a unique achievable market performance with zero revenue. As the social planner values welfare over loss, it is optimal to not charge any purely influenceable transaction costs with the fully efficient market performance $(1, 0, 0)$. For some heterogeneous prior beliefs, price-guessing will lead to complete market failure, that is $(0, 0, 1)$. In that case the scaling $\gamma = (0, 0)$ is again optimal. Proof details are relegated to Appendix B.16. □

For some belief systems and influenceable transaction costs, in contrast to uninfluenceable transaction costs, it is possible to achieve market outcomes with strictly positive revenue and zero loss, that is $(x, 1 - x, 0)$. Note that for some belief systems and objective functions, a high scaling might be optimal. For example, if spread fees are charged and heterogeneous prior beliefs with strictly positive gains of trade, it is optimal for a revenue-maximizing market maker to scale the fees to 100%. Moreover, for certain beliefs, it is possible to achieve the optimal performance triplet (W, R, L) in the space of all market performances. More formally, for any objective function U, there exist spread fees and beliefs F, such that the corresponding market performance is U-optimal among all market performances. For example, for a revenue-maximizing market maker, there exist belief systems such that the market performance $(0, 1, 0)$ is achievable.

6.2.3 Mixed Transaction Costs

Consider transaction costs that have an uninfluenceable and purely influenceable part. If the uninfluenceable transaction cost is not optimally scaled, there exist common prior beliefs, such that adding a purely influenceable transaction cost can be beneficial. That is, the resulting influenceable transaction cost can outperform the original uninfluenceable one. Note that this is not due to an increase in revenue from the purely influenceable part, as price-guessing leads to zero revenue from this part. However, price-guessing might change the market price and lead to higher revenue from the uninfluenceable part of the transaction cost. The following example illustrates this:

Example 16 (Strategic addition of influenceable transaction costs)**.** *Consider a market environment with a unit mass of traders in $[1, 2]$ and a 10% price fee, such that truthfulness leads to a market price of 1.5 and a trading volume of 0.5. Moreover, assume that types are distributed, such that there is only ϵ-demand in the set $[1.5, 1.8]$. If the common prior beliefs are such that all traders believe that the market price will be equal to 1.8, adding a spread fee changes the strategic behavior from truthfulness to price-guessing and the market price will turn out to be 1.8 instead of 1.5. The trading volume decreases by at most ϵ, but the revenue increases due to a higher market price, even though no revenue is due to the additional spread fee.*

7 Conclusion

We have studied a standard model of trade where the price is set to equate revealed supply and demand and have shown how the presence of transaction costs fundamentally alter incentives and welfare in markets. In particular, we have categorized transaction costs into asymptotically uninfluenceable transaction costs (examples include fixed and price fees) and influenceable transaction costs (examples include spread fees). Uninfluenceable transaction costs don't fundamentally alter strategic incentives and, in large markets, inefficiency only arises from the direct loss that resemble the dead-weight loss of taxation or monopoly power. By contrast influenceable transaction costs starkly alter strategic consideration. Dependent on beliefs and uncertainty total market failure may occur.

Our results remain valid for any mechanism in which:

- A trader's expected utility $\mathbb{E}[u(a)]$, given their action a, can be expressed as the product of the probability of trade $\mathbb{P}[\text{trading given } a]$ and the expected utility conditional on trading $\mathbb{E}[u(a)|\text{trading given } a]$; we assume here that the utility when not trading is zero.

- A buyer's $\mathbb{P}[\text{trading given } a]$ is increasing in a and $\mathbb{E}[u(a)|\text{trading given } a]$ is decreasing in a, while a seller's $\mathbb{P}[\text{trading given } a]$ is decreasing in a and $\mathbb{E}[u(a)|\text{trading given } a]$ is increasing in a.

We say that trade is *predictable* if the probability of trade approaches a 0-1 step-function in the trader's action. Examples include large markets without aggregate uncertainty (c.f. Proposition 3) and posted-price mechanisms. If trade is predictable, the trading probability is de facto a constraint and to maximize utility a trader either chooses not to trade or chooses an action that ensures trade and maximizes $\mathbb{E}[u(a)|\text{trading given } a]$. For the latter maximization our categorization into asymptotically uninfluenceable and influenceable payments is crucial and our analysis of the two categories carries over to this more general setting. Extended in this way, our treatment includes Vickrey mechanisms as an example of the asymptotically uninfluenceable case and the first-price auction as an example of the influenceable case.

Transaction costs have often been overlooked in the strategic analysis of markets. The stark influence that we have uncovered points towards both, empirical and theoretical questions. Given that both asymptotically uninfluenceable and influenceable transaction costs are charged in practice, what explains the choices? May the choice depend on differences in sophistication of traders; for example influenceable transaction costs might be charged in situations where traders have incorrect beliefs or face aggregate uncertainty. Theoretically, this opens the question of optimal information design for a social planner or market maker. Finally, extending our insights to more complex market interactions, where traders are interested in bundles should also be of great interest.

References

Aumann, Robert J. (1964), "Markets with a continuum of traders." *Econometrica*, 32, 39–50.

Azevedo, E.M. and E. Budish (2019), "Strategy-proofness in the Large." *Review of Economic Studies*, 86, 81–116.

Bergemann, D., B. Brooks, and S. Morris (2015), "The limits of price discrimination." *American Economic Review*, 105, 921–57.

Bergemann, D. and S. Morris (2005), "Robust mechanism design." *Econometrica*, 73, 1771–1813.

Boergers, T. and J. Li (2019), "Strategically simple mechanisms." *Econometrica*, 87, 2003–2035.

Carroll, G. (2015), "Robustness and linear contracts." *American Economic Review*, 105, 536–63.

Carroll, G. (2017), "Robustness and separation in multidimensional screening." *Econometrica*, 85, 453–488.

Chassang, S. (2013), "Calibrated incentive contracts." *Econometrica*, 81, 1935–1971.

Chatterjee, K. and W. Samuelson (1983), "Bargaining under incomplete information." *Operations Research*, 31, 835–851.

Chen, D. and A.L. Zhang (2020), "Subsidy schemes in double auctions."

Chung, K.-S. and J. C. Ely (2007), "Limited foundations of dominant-strategy mechanisms." *The Review of Economic Studies*, 74, 447–476.

Coase, R.H. (1960), "The problem of social cost." *The Journal of Law and Economics*, 3, 1–44.

Colliard, J.-E. and T. Foucault (2012), "Trading fees and efficiency in limit order books." *The Review of Financial Studies*, 24, 3389–3421.

Cripps, M.W. and J.M. Swinkels (2006), "Efficiency of large double auctions." *Econometrica*, 74, 47–92.

de Clippel, G. and K. Rozen (2018), "Consumer theory with misperceived tastes." *Working paper*.

Demsetz, H. (1968), "The cost of transacting." *The Quarterly Journal of Economics*, 82, 33–53.

Eyster, E. and M. Piccione (2013), "An approach to asset pricing under complete and diverse perceptions." *Econometrica*, 81, 1483–1506.

Foucault, T., O. Kadan, and E. Kandel (2013), "Liquidity cycles and make/take fees in electronic markets." *The Journal of Finance*, 68, 299–341.

Friedman, D. and J. Rust (1993), *The double auction market: institutions, theories, and evidence.* Westview Press.

Fudenberg, D., M. Mobius, and A. Szeidl (2007), "Existence of equilibrium in large double auctions." *Journal of Economic Theory*, 133, 550 – 567.

Garratt, R. and M. Pycia (2016), "Efficient bilateral trade." *Unpublished Paper, UCLA.[535].*

Harrison, J. M. and D. M. Kreps (1978), "Speculative investors behavior in a stock market with heterogeneous expectations." *Quarterly Journal of Economics*, 92, 323–336.

Heidhues, P., B. Kőszegi, and P. Strack (2018), "Unrealistic expectations and misguided learning." *Econometrica*, 86, 1159–1214.

Hosseini, H. (1995), "Understanding the market mechanism before Adam Smith: economic thought in Medieval Islam." *History of Political Economoy*, 27, 539–561.

Jackson, M.O. and J.M. Swinkels (2005), "Existence of equilibrium in single and double private value auctions." *Econometrica*, 73, 93–139.

Jantschgi, S., H.H. Nax, B.S.R. Pradelski, and M. Pycia (2022), "On Market Prices in Double Auctions." *Working Paper.*

Ledyard, J.O. (1978), "Incentive compatibility and incomplete information." *Journal of Economic Theory*, 18, 171–189.

Leininger, W., P.B. Linhart, and R. Radner (1989), "Equilibria of the sealed-bid mechanism for bargaining with incomplete information." *Journal of Economic Theory*, 48, 63–106.

Li, S. (2017), "Obviously strategy-proof mechanisms." *American Economic Review*, 107, 3257–3287.

Madarász, K. and A. Prat (2017), "Sellers with misspecified models." *The Review of Economic Studies*, 84, 790–815.

Malinova, K. and A. Park (2015), "Subsidizing liquidity: The impact of make/take fees on market quality." *The Journal of Finance*, 70, 509–536.

Marra, M. (2019), "Pricing and fees in auction platforms with two-sided entry." Technical report, Sciences Po.

Myerson, R.B. and M.A. Satterthwaite (1983), "Efficient mechanisms for bilateral trading." *Journal of Economic Theory*, 29, 265–281.

Noussair, C., S. Robin, and B. Ruffieux (1998), "The effect of transaction costs on double auction markets." *Journal of Economic Behavior & Organization*, 36, 221–233.

Pycia, M. and P. Troyan (2019), "A theory of simplicity in games and mechanism design." In *ACM Conference on Economics and Computation EC'19*.

Reny, P.J. and M. Perry (2006), "Toward a strategic foundation for rational expectations equilibrium." *Econometrica*, 74, 1231–1269.

Roberts, D.J. and A. Postlewaite (1976), "The incentives for price-taking behavior in large exchange economies." *Econometrica: journal of the Econometric Society*, 115–127.

Rustichini, A., M.A. Satterthwaite, and S.R. Williams (1994), "Convergence to efficiency in a simple market with incomplete information." *Econometrica*, 1041–1063.

Satterthwaite, M.A. and S.R. Williams (1989a), "Bilateral trade with the sealed bid k-double auction: Existence and efficiency." *Journal of Economic Theory*, 48, 107 – 133.

Satterthwaite, M.A. and S.R. Williams (1989b), "The rate of convergence to efficiency in the buyer's bid double auction as the market becomes large." *The Review of Economic Studies*, 56, 477–498.

Shi, B., E.H. Gerding, P. Vytelingum, and N.R. Jennings (2013), "An equilibrium analysis of market selection strategies and fee strategies in competing double auction marketplaces." *Autonomous Agents and Multi-Agent Systems*, 26, 245–287.

Smith, A. (1776), *The Wealth of Nations*. reprint: Everyman's Library, 1977.

Tatur, T. (2005), "On the trade off between deficit and inefficiency and the double auction with a fixed transaction fee." *Econometrica*, 73, 517–570.

Tyurin, I.S. (2012), "A refinement of the remainder in the lyapunov theorem." *Theory of Probability and its Applications*, 56.

Vapnik, V.N. and A.Y. Chervonenkis (2015), *On the uniform convergence of relative frequencies of events to their probabilities*, 11–30.

Williams, S.R. (1991), "Existence and convergence of equilibria in the buyer's bid double auction." *The Review of Economic Studies*, 58, 351–374.

Wilson, R. (1985), "Incentive efficiency of double auctions." *Econometrica*, 53, 1101–1115.

Wilson, R. (1987), "Game theoretic approaches to trading processess. in t. bewley, ed., advances in economic theory: Fifth world congress."

Wolitzky, A. (2016), "Mechanism design with maxmin agents: Theory and an application to bilateral trade." *Theoretical Economics*, 11, 971–1004.

A Auxiliary results

A.1 Demand, supply, and market-clearing prices

We clarify how the double auction chooses the market price. For a detailed treatment of the double auction and the proofs of Lemmas 17, 18, and 19 see Jantschgi et al. (2022).

Recall the following notation: For a relation $\mathcal{R} \in \{\geq, >, =, <, \leq\}$, define $\mathcal{B}_{\mathcal{R}}(P) = \{b \in \mathcal{B} : t_b \mathcal{R} P\}$ and $\mathcal{S}_{\mathcal{R}}(P) = \{s \in \mathcal{S} : t_s \mathcal{R} P\}$.

Definition (Demand and supply functions)**.** The *demand* and *supply functions* at price P are defined as $D(P) = \mu_B(\mathcal{B}_{\geq}(P))$ and $S(P) = \mu_S(\mathcal{S}_{\leq}(P))$, that is, by the mass of all traders who weakly prefer trading over not trading at price P.

We next define a special class of action distributions, which arise in infinite markets, e.g., if they are interpreted as the limit of finite markets where actions are modelled as independent random variables. Say that action distributions μ_B^a and μ_S^a are *regular*, if they are equivalent to the Lebesgue-measure on A_B and A_S and their densities f_B and f_S are continuous, that is $\mu_B^a(A) = \int_A f_B(x)dx$ and $\mu_S^a(A) = \int_A f_S(x)dx$ for $A \subset \mathbb{R}$.

Lemma 17 (Analytic properties of demand and supply functions)**.** *The demand function is non-increasing, left-continuous with right limits. The supply function is non-decreasing, right-continuous with left limits. It holds that $D(P+) = \mu_B(\mathcal{B}_{>}(P))$ and $S(P-) = \mu_S(\mathcal{S}_{<}(P))$. If action distributions are regular, then demand is continuous and strictly decreasing on A_B and supply is continuous and strictly increasing on A_S.*

The following concept corresponds to market prices that equilibrate demand and supply.

Definition ((Strong) market clearing prices)**.** P is a market-clearing price if $D(P) \geq S(P)$ and $D(P+) \leq S(P)$ (*type I*) or $S(P) \geq D(P)$ and $S(P-) \leq D(P)$ (*type II*). P is a *strong market-clearing price* if $D(P) = S(P)$. Denote the set of all market-clearing prices by $\mathcal{P}^{MC}$ and the set of all strong market-clearing prices by $\mathcal{P}^{SMC}$.

Using the analytical properties of demand and supply, we can characterize the topology of the set of (strong) market clearing prices.

Lemma 18 (Topology of $\mathcal{P}^{SMC}$ and $\mathcal{P}^{MC}$)**.** *The set $\mathcal{P}^{SMC}$ is a convex subset of T. Every strong market-clearing price is a market-clearing price (of type I and II). The set of market-clearing prices is non-empty, convex and closed. The set $\mathcal{P}^{MC} \setminus \mathcal{P}^{SMC}$ has Lebesgue-measure zero. More precisely, if $\mathcal{P}^{SMC} \neq \emptyset$, then $\mathcal{P}^{MC} = \overline{\mathcal{P}^{SMC}}$, and if $\mathcal{P}^{SMC} = \emptyset$, then $\mathcal{P}^{MC}$ is a singleton.*
If action distributions are continuous, and $\overline{a}_S > \underline{a}_B$, then there exists a unique market clearing price with positive trading volume and $\mathcal{P}^{SMC} = \mathcal{P}^{MC}$.

In finite markets the mechanism described in Section 3 coincides with the classical k-DA (Rustichini et al., 1994), for which an explicit formula for the set of market-clearing prices is given. Let $a^{(m)}$ be the m'th smallest action in the set of all actions a.

Lemma 19. *In finite markets with m buyers and n sellers $\mathcal{P}^{MC} = [a^{(m)}, a^{(m+1)}]$. If $a^{(m)} \neq a^{(m+1)}$, then for every $P \in (a^{(m)}, a^{(m+1)})$ it follows that $P \in \mathcal{P}^{SMC}$.*

A.2 Allocation and tie-breaking

If the double auction results in a strong market-clearing price P^*, that is $D(P^*) = S(P^*)$, then no tie-breaking is needed. The allocation is set as $\mathcal{B}^* = \mathcal{B}_{\geq}(P^*)$ and $\mathcal{S}^* = \mathcal{S}_{\leq}(P^*)$, which *balances trade*, that is $\mu_B(\mathcal{B}^*) = \mu_S(\mathcal{S}^*)$. Therefore, the allocation consists of all traders, who weakly prefer trading over not trading at P^*.

Next, suppose that the double auction results in a market clearing price of type I, which is not a strong market clearing price. Then, $D(P^*) > S(P^*)$ and $D(P^*+) \leq S(P^*)$. Set $\mathcal{S}^* = \mathcal{S}_{\leq}(P^*)$, that is all sellers who, given their action, weakly prefer trading over not trading are involved in trade. Consider the set of all buyers who strictly prefer to trade at P^*, that is $\mathcal{B}_{>}(P^*)$. It follows from Lemma 17 that $D(P^*+) = \mu_B(\mathcal{B}_{>}(P^*))$. Let $x = S(P^*) - \mu_B(\mathcal{B}_{>}(P^*)) \geq 0$ and let $\tilde{\mathcal{B}}$ be a subset of $\mathcal{B}_{=}(P^*)$ with μ_B-measure equal to x. Such a set exists because $D(P^*) = \mu_B(\mathcal{B}_{\geq}(P^*)) = \mu_B(\mathcal{B}_{>}(P^*)) + \mu_B(\mathcal{B}_{=}(P^*)) \geq S(P^*)$ and $D(P^*+) = \mu_B(\mathcal{B}_{>}(P^*)) \leq S(P^*)$. Set $\mathcal{B}^* = \mathcal{B}_{>}(P^*) \cup \tilde{\mathcal{B}}$. That is, all buyers who strictly prefer to trade at P^* are involved in trade, together with a subset of traders with bid equal to P^* that are indifferent in order to balance trade.

Finally, if a market clearing price of type II is chosen, the allocation is set analogously: $\mathcal{B}^* = \mathcal{B}_{\geq}(P^*)$ and $\mathcal{S}^* = \mathcal{S}_{<}(P^*) \cup \tilde{\mathcal{S}}$, where $\tilde{S}$ is a subset of $\mathcal{S}_{=}(P^*)$ that balances trade.

Suppose that $\tilde{B}$ (respectively $\tilde{S}$) are chosen uniformly at random, this ensures fairness. That is, they are random compact sets such that for all $b \in \mathcal{B}_{=}(P^*)$ it holds that $\mathbb{P}[b \in \tilde{\mathcal{B}}] \equiv \mu_B(\tilde{\mathcal{B}})/\mu_B(\mathcal{B})$ (respectively for all $s \in \mathcal{S}_{=}(P^*)$ it holds that $\mathbb{P}[s \in \tilde{\mathcal{S}}] \equiv \mu_S(\tilde{\mathcal{S}})/\mu_S(\mathcal{S})$). The existence of uniform random sets is discussed in Jantschgi et al. (2022).

A.3 Explicit formulas

In this section we derive explicit formulas for some of the concepts introduced in the model in Section 3 that will be used in subsequent proofs. We will sometimes differentiate between finite markets with m buyers and n sellers and infinite markets with market ratio R.

Throughout this section, consider a buyer b with gross value t_b and bid a_b, and a seller s with gross value t_s and ask a_s. Let a denote an action distribution. Recall that in a finite market, $a^{(k)}$ denotes the k'th smallest element in the set of all taken actions.

A.3.1 Involvement in trade

Finite markets. If $a_b < a_{\text{-}b}^{(m)}$, then it is strictly smaller than the $m+1$'st smallest element in the set of all actions a (including a_b) and buyer b is not involved in trade, because their bid is below the market price. If $a_b > a_{\text{-}b}^{(m)}$, then it is at least the $m+1$'st largest element and therefore sufficient to be involved in trade. If $a_b = a_{\text{-}b}^{(m)}$, then the buyer might be subject to tie-breaking.

If $a_s > a_{\text{-}s}^{(m)}$, then it is at least the $m+1$'st smallest element in the set of all actions (including a_s) and seller s is not involved in trade, because their ask was above the market price. If $a_s < a_{\text{-}s}^{(m)}$, then it is at most the m'th smallest action and therefore sufficient to be involved in trade. If $a_s = a_{\text{-}s}^{(m)}$s, then the seller might be subject to tie-breaking.

Infinite markets. If there exists no demand excess, then a buyer is involved in trade, if $a_b \geq P^*(a)$. If $a_b < P^*(a)$, then the buyer is not involved in trade. If there exists demand excess, it is generated by bids at $P^*(a)$. If $a_b > P^*(a)$, then the buyer is involved in trade. If $a_b = P^*(a)$, then the buyer might be subject to tie-breaking.

If there exists no supply excess, then the seller is involved in trade, if $a_s \leq P^*(a)$. If $a_s > P^*(a)$, then the seller is not involved in trade. If there exists supply excess, it is generated by asks at $P^*(a)$. If $a_s < P^*(a)$, then the seller is involved in trade. If $a_s = P^*(a)$, then the seller might be subject to tie-breaking.

We can now express the probability of trade, given the beliefs of a trader.

A.3.2 Trading probabilities given beliefs

Finite markets. Given the belief that actions are random variables with continuous distribution, tie-breaking is a probability zero event in finite markets. It follows from Appendix A.3.1 that

$$\mathbb{P}_{a_{\text{-}b}}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \mathbb{P}_{a_{\text{-}b}}\left[a_b \geq a_{\text{-}b}^{(m)}\right] \quad \text{and} \quad \mathbb{P}_{a_{\text{-}s}}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right] = \mathbb{P}_{a_{\text{-}s}}\left[a_s \leq a_{\text{-}s}^{(m)}\right]. \tag{3}$$

In Appendix A.7, explicit formulas for such probabilities are derived in a more general context (see Equations (28) and (29)).

Infinite markets. If there exists no demand excess at P^*, then

$$\mathbb{P}_{a_{\text{-}b}}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \begin{cases} 1 & a_b \geq P^*(a), \\ 0 & \text{else.} \end{cases} \tag{4}$$

Suppose that there is strictly positive demand excess. That is $\mu_B(\mathcal{B}_{\geq}(P^*(a))) = V(a) + x$ and $\mu_B(\mathcal{B}_{>}(P^*(a))) = V(a) - y$ for $x > 0$ and $y \geq 0$ (see Appendix A.2). Then,

$$\mathbb{P}_{a_{\text{-}b}}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \begin{cases} 1 & a_b > P^*(a), \\ \frac{y}{x+y} & a_b = P^*(a), \\ 0 & \text{else.} \end{cases} . \tag{5}$$

If there exists no supply excess, then

$$\mathbb{P}_{a_{\text{-}s}}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right] = \begin{cases} 1 & a_s \leq P^*(a), \\ 0 & \text{else.} \end{cases} \tag{6}$$

Suppose that there is strictly positive supply excess. That is $\mu_S(\mathcal{S}_{\leq}(P^*(a))) = V(a) + x$ and $\mu_S(\mathcal{S}_{<}(P^*(a))) = V(a) - y$ for $x > 0$ and $y \geq 0$. Then,

$$\mathbb{P}_{a_{\text{-}s}}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right] = \begin{cases} 1 & a_s < P^*(a), \\ \frac{y}{x+y} & a_s = P^*(a), \\ 0 & \text{else.} \end{cases} \tag{7}$$

Note that in the presence of strictly positive trading excess, traders believe that if they are involved in tie-breaking in an infinite market, then they have a fair chance of being involved in trade.

A.3.3 Market Price

Finite markets. Recall that by Lemma 19 $P^*(a) = ka^{(m)} + (1-k)a^{(m+1)}$. Interpreting the market price as a function of a single action yields

$$P^*\left(a_b, a_{\text{-}b}\right) = \begin{cases} (1\text{-}k)a_{\text{-}b}^{(m)} + ka_b & \text{if } a_{\text{-}b}^{(m)} \leq a_b \leq a_{\text{-}b}^{(m+1)}, \\ (1\text{-}k)a_{\text{-}b}^{(m)} + ka_{\text{-}b}^{(m+1)} & \text{else.} \end{cases} \tag{8}$$

$$P^*\left(a_s, a_{\text{-}s}\right) = \begin{cases} (1\text{-}k)a_s + ka_{\text{-}s}^{(m)} & \text{if } a_{\text{-}s}^{(m\text{-}1)} \leq a_s \leq a_{\text{-}s}^{(m)}, \\ (1\text{-}k)a_{\text{-}s}^{(m\text{-}1)} + ka_{\text{-}s}^{(m)} & \text{else.} \end{cases} \tag{9}$$

Note that $P^*\left(a_b, a_{\text{-}b}\right)$ depends only on $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$, and $P^*\left(a_s, a_{\text{-}s}\right)$ depends only on $a_{\text{-}s}^{(m\text{-}1)}$ and $a_{\text{-}s}^{(m)}$. In some proofs, this dependence will be of importance and we will, for example, write $P^*\left(a_b, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}\right)$ instead of $P^*\left(a_b, a_{\text{-}b}\right)$.

In addition, for a trader i, we will in some proofs consider $\tilde{P}^*\left(a_i, a_{\text{-}i}\right)$, which is equal to the market price, if i is involved in trade, and zero otherwise.

Infinite markets. In an infinite market, a single trader cannot influence the market price. It therefore holds for a trader i and for all actions a_i and a_i' that $P^*(a_i, a_{\text{-}i}) = P^*(a_i', a_{\text{-}i})$. By abuse of notation, we will in some proofs write $P^*(a_{\text{-}i})$.

A.3.4 Utility functions

For a buyer the utility of being involved in trade is equal to the difference between their gross value and the market price minus the additional transaction cost:

$$u_b\left(t_b, a_b, a_{\text{-}b}\right)) = \begin{cases} t_b\text{-}P^*\left(a_b, a_{\text{-}b}\right)\text{-}\Phi_b\left(a_b, a_{\text{-}b}\right) & b \in \mathcal{B}^*, \\ 0 & \text{else.} \end{cases} \tag{10}$$

For a seller the utility of being involved in trade is equal to the difference between the market price and their gross value minus the additional transaction cost:

$$u_s\left(t_s, a_s, a_{\text{-}s}\right)) = \begin{cases} P^*\left(a_s, a_{\text{-}s}\right)\text{-}t_s\text{-}\Phi_s\left(a_s, a_{\text{-}s}\right) & s \in \mathcal{S}^*, \\ 0 & \text{else.} \end{cases} \tag{11}$$

Finite markets. Let $\mu_b\left(a_{\text{-}b}\right)$ denote the distribution of $a_{\text{-}b}$ according to the beliefs of trader b. It holds that

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = \\ \int\limits_{\{a_b \geq a_{\text{-}b}^{(m)}\}} \left(t_b\text{-}P^*\left(a_b, a_{\text{-}b}\right)\text{-}\Phi_b\left(a_b, a_{\text{-}b}\right)\right) d\mu_b\left(a_{\text{-}b}\right) = \\ t_b \cdot \mathbb{P}_{\text{-}b}[b \in \mathcal{B}^*(a_b, a_{\text{-}b})] - \int\limits_{[\underline{a}_{S,b}, \overline{a}_{S,b}]^2} \tilde{P^*}\left(a_b, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}\right) d\mu_b(a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}) - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]\end{gathered} \tag{12}$$

Note that both $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$ have support in $[\underline{a}_{S,b}, \overline{a}_{S,b}]$. That is because $a_{\text{-}b}$ consists of $m-1$ bids and n asks. So there must be at least one ask below or equal to $a_{\text{-}b}^{(m)}$.

Let $\mu_s\left(a_{\text{-}s}\right)$ denote the distribution of $a_{\text{-}s}$ according to the beliefs of a seller s. It holds that

$$\begin{gathered}\mathbb{E}_{\text{-}s}\left[u_s(t_s, a_s, a_{\text{-}s})\right] = \\ \int\limits_{\{a_s \leq a_{\text{-}s}^{(m)}\}} \left(P^*\left(a_s, a_{\text{-}s}\right)\text{-}t_s - \Phi_s\left(a_s, a_{\text{-}s}\right)\right) d\mu_s\left(a_{\text{-}s}\right) = \\ \int\limits_{[\underline{a}_{B,s}, \overline{a}_{B,s}]^2} \tilde{P^*}\left(a_s, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right) d\mu_s(a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}) - t_s \cdot \mathbb{P}_{\text{-}s}[s \in \mathcal{S}^*(a_s, a_{\text{-}s})] - \mathbb{E}_{\text{-}s}\left[\Phi_s(a_s, a_{\text{-}s})\right].\end{gathered} \tag{13}$$

Note that both $a_{\text{-}s}^{(m\text{-}1)}$ and $a_{\text{-}s}^{(m)}$ have support in $[\underline{a}_{B,s}, \overline{a}_{B,s}]$.

Infinite markets. The expectation is only concerned with tie-breaking, as both the market price and the transaction cost are deterministic. Therefore,

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = \left(t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(a_b, a_{\text{-}b})\right)\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \tag{14}$$

and

$$\mathbb{E}_{\text{-}s}\left[u_s(t_s, a_s, a_{\text{-}s})\right] = \left(P^*(a_s, a_{\text{-}s}) - t_s - \Phi_s(a_s, a_{\text{-}s})\right)\mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right]. \tag{15}$$

Difference in expected utility for actions a_i^1 and a_i^2 in finite markets In multiple proofs, we will estimate the difference in expected utility in finite markets for two actions a_i^1 and a_i^2. The following lemma yields an upper bound:

Lemma 20. *For bids $a_b^1 > a_b^2$ and for asks $a_s^1 < a_s^2$ it holds that*

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] \leq \\ t_b\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]\right) - \left(\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right]\right).\end{gathered} \tag{16}$$

and

$$\begin{gathered}\mathbb{E}_{\text{-}s}\left[u_s\left(t_s, a_s^1, a_{\text{-}s}\right)\right] - \mathbb{E}_{\text{-}s}\left[u_s\left(t_s, a_s^2, a_{\text{-}s}\right)\right] \\ \leq 2\overline{a}_{B,s}\left(1 - \mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right]\right) - t_s\left(\mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s^1, a_{\text{-}s})\right] - \mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s^2, a_{\text{-}s})\right]\right) \\ - \left(\mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^1, a_{\text{-}s}\right)\right] - \mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^2, a_{\text{-}s}\right)\right]\right).\end{gathered} \tag{17}$$

The proof of this Lemma is relegated to Appendix B.19.

A.4 Discussion of truthfulness for pathological transaction costs

The set of undominated actions might be empty. First, the net value might not exist, as the set of undominated actions can be empty. Consider a seller s with a positive gross value t_s. If a price fee $\phi_s = 1$ is charged, then any involvement in trade results in a loss for the seller. Therefore any action a_s is dominated by a greater action a_s', proving that there does not exist an undominated action.

The net value might be dominated. Second, the net value might be dominated, as the set of undominated actions might be open. Consider a buyer b with positive gross value t_b. If a constant fee $c_b > t_b$ is charged, any involvement in trade results in a loss for a buyer. It is therefore optimal to not be involved in trade. Formally, this would mean to submit a negative action $a_b \in [-\infty, 0)$. But $a_b = 0$ is dominated by any negative action, as a buyer can still be involved in trade, if all other traders submit 0, and the buyer wins tie-breaking. Therefore the supremum of the set of undominated and ex-post individually rational actions is not attained as a maximum.

The maximal undominated action might not be ex-post individually rational. Third, the maximal undominated action might not be ex-post individually rational. Consider a buyer with gross value t_b and a fee Φ_b that is equal to zero, unless for one action distribution $a'_{\text{-}b}$, where the buyer is involved in trade with action t_b and the fee is greater than t_b. The largest undominated action is equal to t_b, as this action dominates all larger actions actions, but is not dominated by smaller actions. But it is not ex-post individually rational, because $u_b(t_b, a_b, a'_{\text{-}b}) < 0$.

A.5 Out-of-the market gross values

We sometimes focus on *in-the-market* gross values that is gross values t_i such that $t_i^\Phi \prec P_i^\infty$. Traders with such gross values are able to submit individually rational actions that make them likely to be involved in trade when the market is sufficiently large. By contrast, for an *out-of-the-market* trader, that is, one with gross value $t_i^\Phi \succ P_i^\infty$, the probability of trade, when acting individually rationally, vanishes in large markets. Observe that bidding the critical value P_i^∞ is individually rational for for in-the-market traders but not for out-of-the-market traders.

Proposition 21 (For out-of-the-market gross values, truthfulness is close to optimal)**.** *If bidding the critical value P_i^∞ is not individually rational for trader i, then for every $\epsilon > 0$, in sufficiently large markets, truthfulness is an ϵ-best response.*

Proof Outline. As $t_i^\Phi \succ P_i^\infty$, the best response for a trader is more aggressive than P_i^∞. But for any such action, it follows from Proposition 3 that the trading probability gets arbitrarily small in sufficiently large markets. Therefore, for any $\epsilon > 0$, the expected utility of a best response is less or equal than ϵ, and, as truthfulness leads to a non-negative expected utility by assumption, it is an ϵ-best response in sufficiently large markets. Proof details are relegated to Appendix B.20. □

A.6 Strategic incentives for price and spread fees

This section contains a detailed discussion of the opposing strategic incentives for price and spread fees in finite markets: (i) Utility when trading, versus (ii) probability of trading.[28]

Recall that a trader i believes that actions are distributed in intervals $A_{B,i} = [\underline{a}_{B,i}, \overline{a}_{B,i}]$ and $A_{S,i} = [\underline{a}_{S,i}, \overline{a}_{B,i}]$ with the assumption that $\overline{a}_{S,i} \geq \overline{a}_{B,i} > t_i^\Phi > \underline{a}_{S,i} \geq \underline{a}_{B,i}$.

Consider a buyer b with action a_b. We can omit the analysis of $a_b > \overline{a}_{B,b}$ and $a_b < \underline{a}_{S,b}$; for the first, such an action is by assumption not individually rational and strictly dominated by t_b^Φ, for the second, any action below $\underline{a}_{S,b}$ has probability of trade equal to 0, because no seller is believed to submit an action below it. Therefore, the expected utility at such a bid is equal to 0. We therefore consider $a_b \in [\underline{a}_{S,b}, \overline{a}_{B,b}]$.

As the market price depends only on a_b, $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$. For ease of notation, let $y = a_{\text{-}b}^{(m)}$ and $z = a_{\text{-}b}^{(m+1)}$ and denote by $e(y, z)$ the joint density of y and z given the beliefs of buyer b.

[28] This section is closely related to methods used in Rustichini et al. (1994) to analyze strategic incentives in k-DAs without transaction costs.

Price fees. The expected utility of a buyer is of the form

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = &\int_{a_b}^{\overline{a}_{S,i}} \int_{\underline{a}_{S,b}}^{a_b} \left(t_b - (1+\phi_b)\left(ka_b + (1-k)\,y\right)\right) e(y,z)dydz + \\ &\int_{\underline{a}_{S,b}}^{a_b} \int_{\underline{a}_{S,b}}^{z} \left(t_b - (1+\phi_b)\left(kz + (1-k)\,y\right)\right) e(y,z)dydz.\end{aligned} \tag{18}$$

The expected utility is continuously differentiable as a function of a_b over the interval $[\underline{a}_{S,b}, \overline{a}_{S,b}]$. Straightforward computation using Leibniz's rule for differentiation under the integral sign yields

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]}{da_b} = \left(t_b - (1+\phi_b)\,a_b\right) f_y(a_b) - (1+\phi_b)\,k\mathbb{P}_{\text{-}b}\left[y \leq a_b \leq z\right], \tag{19}$$

where $f_y(a_b)$ denotes the density function of y. If $a_b \in (\underline{a}_{S,b}, \overline{a}_{S,b})$ maximizes the expected utility, then the first order condition

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]}{da_b} = 0 \tag{20}$$

holds. $f_y(a_b)$ is equal to $\frac{d\mathbb{P}_{\text{-}b}[y \leq a_b]}{da_b}$. A formula for $\mathbb{P}_{\text{-}b}[y \leq a_b]$ is stated in Appendix A.7. Therefore, we can explicitly state the first order condition in terms of distribution and density functions, see Equation (24) below. The first order condition for a seller can be derived in analogy, see Equation (25) below.

Spread fees. The expected utility of a buyer is of the form

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b}\right] = &\int_{a_b}^{\overline{a}_{S,b}} \int_{\underline{a}_{S,b}}^{a_b} \left(t_b - \phi_b a_b - (1-\phi_b)\left(ka_b + (1-k)\,y\right)\right) e(y,z)dydz + \\ &\int_{\underline{a}_{S,b}}^{a_b} \int_{\underline{a}_{S,b}}^{z} \left(t_b - \phi_b a_b - (1-\phi_b)\left(kz + (1-k)\,y\right)\right) e(y,z)dydz.\end{aligned} \tag{21}$$

The expected utility is continuously differentiable as a function of a_b over the interval $[\underline{a}_{S,b}, \overline{a}_{S,b}]$. Straightforward computation using Leibniz's rule for differentiation under the integral sign yields

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b}\right]}{da_b} = \left(t_b - a_b\right) f_y(a_b) - \phi_b\mathbb{P}_{\text{-}b}\left[y \leq a_b\right] - (1-\phi_b)\,k\mathbb{P}_{\text{-}b}\left[y \leq a_b \leq x\right]. \tag{22}$$

where $f_y(a_b)$ denotes the density function of y. If $a_b \in (\underline{a}_{S,b}, \overline{a}_{S,b})$ maximizes the expected utility, then the first order condition

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]}{da_b} = 0 \tag{23}$$

holds. $f_y(a_b)$ is equal to $\frac{d\mathbb{P}_{\text{-}b}[y \leq a_b]}{da_b}$. A formula for $\mathbb{P}_{\text{-}b}[y \leq a_b]$ is stated in Appendix A.7. Therefore, we can explicitly state the first order condition in terms of distribution and density functions, see Equation (24) below. The first order condition for a seller can be derived in analogy, see Equation (25) below.

First Order Conditions To explicitly state the first order conditions, we introduce additional notation: Define $a_{i,j}$ as an action distribution for i buyers and j sellers. In this notation, a as defined in Section 3 corresponds to $a_{m,n}$ and for any buyer b and seller s, $a_{\text{-}b}$ and $a_{\text{-}s}$ correspond to $a_{m-1,n}$ and $a_{m,n-1}$. Denote again by $a_{i,j}^{(l)}$ its l'th smallest element.

We say that an action a_b satisfies the *buyer's first order condition* for gross value t_b if

$$\left.\begin{matrix}(t_b\text{-}\,(1{+}\phi_b)\,a_b)\\ \\ (t_b\text{-}a_b)\end{matrix}\right\}\cdot\left(n\mathbb{P}_{\text{-}b}\left[a_{m\text{-}1,n\text{-}1}^{(m\text{-}1)}\leq a_b\leq a_{m\text{-}1,n\text{-}1}^{(m)}\right]f_{S,b}(a_b){+}\,(m\text{-}1)\,\mathbb{P}_{\text{-}b}\left[a_{m\text{-}2,n}^{(m\text{-}1)}\leq a_b\leq a_{m\text{-}2,n}^{(m)}\right]f_{B,b}(a_b)\right)= \\ \begin{cases}(1+\phi_b)\,k\mathbb{P}_{\text{-}b}\left[a_{m\text{-}1,n\text{-}1}^{(m)}\leq a_b\leq a_{m\text{-}1,n}^{(m+1)}\right] & \text{for price fees}\\ \phi_b\mathbb{P}_{\text{-}b}\left[a_{m,n\text{-}1}^{(m)}\leq a_b\right]+(1\text{-}\phi_b)\,k\mathbb{P}_{\text{-}b}\left[a_{m\text{-}1,n}^{(m)}\leq a_b\leq a_{m\text{-}1,n}^{(m+1)}\right] & \text{for spread fees}\end{cases}. \tag{24}$$

We say that an action a_s satisfies the *seller's first order condition* for gross value t_s if

$$\left.\begin{matrix}((1\text{-}\phi_s)\,a_s\text{-}t_s)\\ \\ (a_s\text{-}t_s)\end{matrix}\right\}\cdot\left((n\text{-}1)\,\mathbb{P}_{\text{-}s}\left[a_{m,n\text{-}2}^{(m\text{-}1)}\leq a_s\leq a_{m,n\text{-}2}^{(m)}\right]f_{S,s}(a){+}m\mathbb{P}_{\text{-}s}\left[a_{m\text{-}1,n\text{-}1}^{(m\text{-}1)}\leq a_s\leq a_{m\text{-}1,n\text{-}1}^{(m)}\right]f_{B,s}(a)\right)= \\ \begin{cases}(1\text{-}\phi_s)\,(1\text{-}k)\,\mathbb{P}_{\text{-}s}\left[a_{m,n\text{-}1}^{(m\text{-}1)}\leq a_s\leq a_{m,n\text{-}1}^{(m)}\right] & \text{for price fees}\\ \phi_s\mathbb{P}_{\text{-}s}\left[a_{m,n\text{-}1}^{(m)}\geq a_s\right]+(1\text{-}\phi_s)\,(1\text{-}k)\,\mathbb{P}_{\text{-}s}\left[a_{m,n\text{-}1}^{(m\text{-}1)}\leq a_s\leq a_{m,n\text{-}1}^{(m)}\right] & \text{for spread fees}\end{cases}. \tag{25}$$

Interpretation of a buyer's first order condition. Despite the extensive and complex form of the condition, it has a natural interpretation: It balances between the probability of trade and the utility when trading.

In particular, an incremental increase Δa_b in a buyer's bid has two opposing effects: If the bid a_b does not include the buyer amongst those who trade, then by increasing it to $a_b + \Delta a_b$, the buyer may surpass other actions and be involved in trade. If the bid a_b is sufficient to include the buyer in trade, then increasing their bid by Δa_b may lead to an increase in market price and their fee.

In Equation (24), the left-hand side of the equation describes the gain from increasing one's trading probability. The sum in brackets times Δa_b is the probability that the buyer enters the set of buyers who trade as they incrementally raise their bid by Δa_b. The first term in the sum is the marginal probability of acquiring an item by passing a seller's offer and the second term is the marginal probability of acquiring an item by passing another buyer's bid. For a price fee the profit from such a trade is between $t_b - (1 + \phi_b)a_b$ and $t_b - (1 + \phi_b)a_b - (1 + \phi_b)\Delta a_b$. Therefore, the marginal expected profit for a buyer who raises their bid is $t_b - (1+\phi_b)a_b$ times the term in the brackets. In analogy, for a spread fee the marginal expected profit for a buyer who raises their bid is $t_b - \phi_b a_b$ times the term in the brackets.

Next, in Equation (24), the right-hand side of the equation describes the buyer's marginal execpted loss from increasing their bid above a_b. $\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m\text{-}1,n} \leq a_b \leq a^{(m+1)}_{m\text{-}1,n}\right]$ is the probability that a buyer who increases their bid by Δa_b increases the market price by $k(1+\phi_b)\Delta a_b$ for a price fee and by $k(1-\phi_b)\Delta a_b$ for a spread fee. Additionally, for a spread fee $\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m\text{-}1,n} \leq a_b\right]$ is the probability that a buyer who increases their bid by Δa_b increases the part of the charged fee depending on their bid by $\phi_b \Delta a_b$.

The interpretation for a seller is symmetric and thus omitted.

A.7 Probabilities in the first order conditions

In this section we derive explicit formulas for the probabilities arising in the first order conditions in Equations (24) and (25), that are also used in the proof of Theorem 8 in Appendix B.9. Instead of deriving expressions for all different probabilities, note that for general n, m, l all of them can be expressed as one of the following three probabilities for different n, m, l: (i) $\mathbb{P}_{\text{-}i}\left[a^{(l)}_{m,n} \leq a_i \leq a^{(l+1)}_{m,n}\right]$, (ii) $\mathbb{P}_{\text{-}i}\left[a^{(l)}_{m,n} \leq a_i\right]$ and (iii) $\mathbb{P}_{\text{-}i}\left[a^{(l)}_{m,n} \geq a_i\right]$.

For (i) it is the probability that action a_i lies between the l'th and $l+1$'st smallest element in a set of m bids and n asks. The probability that another buyer submits an action smaller or equal a_i is $F^a_{B,i}(a_i)$. The probability that a buyer submits an action greater or equal a_i is therefore $1 - F^a_{B,i}(a_i)$. Replace $F^a_{B,i}$ by $F^a_{S,i}$ for sellers. The event that exactly l bids and asks are below a_i can be split up in the following way: Suppose that i buyers and j sellers bid and offer less or equal than a_i. $i+j$ must be equal to l. Assuming that there are m buyers and n sellers in total, this means that exactly $m-i$ buyers and $n-j$ sellers bid and offer more than a_i. Selecting i buyers and j sellers, the probability that exactly $i+j=l$ bids and offers are below or equal to a_i is

$$F^a_{B,i}\left(a_i\right)^i F^a_{S,i}\left(a_i\right)^j \left(1\text{-}F^a_{B,i}\left(a_i\right)\right)^{m\text{-}i} \left(1\text{-}F^a_{S,i}\left(a_i\right)\right)^{n\text{-}j}, \tag{26}$$

because the actions of traders are assumed to be independent. There are $\binom{m}{i}$ possibilities to choose i buyers and $\binom{n}{j}$ possibilities to choose j sellers. Therefore, the total probability that exactly l traders submit below a_i is equal to

$$\mathbb{P}_{\text{-}i}\left[a^{(l)}_{m,n} \leq a_i \leq a^{(l+1)}_{m,n}\right] = \sum_{\substack{i+j=l \\ 0\leq i\leq m \\ 0\leq j\leq n}} \binom{m}{i}\binom{n}{j} F^a_{B,i}\left(a_i\right)^i F^a_{S,i}\left(a_i\right)^j \left(1\text{-}F^a_{B,i}\left(a_i\right)\right)^{m\text{-}i} \left(1\text{-}F^a_{S,i}\left(a_i\right)\right)^{n\text{-}j}. \tag{27}$$

For (ii) it is the probability that a_i is greater than the l'th action. That is, for some $k \in [l, m+n]$ the number of offers below a_i is exactly equal to k. Summing over k yields that

$$\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i\right] = \sum_{k=l}^{n+m} \sum_{\substack{i+j=k \\ 0\leq i\leq m \\ 0\leq j\leq n}} \binom{m}{i}\binom{n}{j} F_{B,i}^{a}\left(a_i\right)^{i} F_{S,i}^{a}\left(a_i\right)^{j} \left(1\text{-}F_{B,i}^{a}\left(a_i\right)\right)^{m\text{-}i} \left(1\text{-}F_{S,i}^{a}\left(a_i\right)\right)^{n\text{-}j}. \tag{28}$$

For (iii), because distributions are assumed to be atomless $\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} = a_i\right] = 0$. It therefore holds that

$$\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \geq a_i\right] = 1 - \mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i\right], \tag{29}$$

which was computed above.

A.8 Market performance in the infinite market with spread fees

Consider the infinite market with type space $T = [1, 2]$, μ_B^t and μ_S^t the Lebesgue-measures from **??**. Assume that a symmetric spread fee, that is, $\phi_b = \phi_s = \phi$ is charged. Best responses divide the population into price-guessers choosing actions at the critical value and price-takers. We suppose all buyers identify the critical value at $\beta \in [1, 2]$, and all sellers at $\sigma \in [1, 2]$. The following table gives different measures describing the outcome in a market with and without fees.

	Case (i)	**Case (ii)**	**Case (iii)**	**Case (iv)**
Buyer strategy $a_B(t_b)$	β if $t_b \geq \beta$ and t_b if $t_b < \beta$			
Seller strategy $a_S(t_s)$	σ if $t_s \leq \sigma$ and t_s if $t_s > \sigma$			
Demand $D(P)$	$2 - P$ if $P \leq \beta$ and 0 if $P > \beta$			
Supply $S(P)$	0 if $P < \sigma$ and $P - 1$ if $P \geq \sigma$			
Market Price P^*	$3/2$	σ	β	$\in (\beta, \sigma)$
Market Volume Q^*	$1/2$	$2 - \sigma$	$\beta - 1$	0
Market Excess Ex^*	0	$2\sigma - 3$	$3 - 2\beta$	0
Max. Gains of Trade G^{gross}	$1/4$			
Gains of Trade	$1/4$	$\frac{3\sigma-\sigma^2-2}{2(\sigma-1)}$	$\frac{3\beta-\beta^2-2}{2(2-\beta)}$	0
Transaction Costs	$\phi\big((2\text{-}\beta)(\beta\text{-}{}^3\!/_2)+\frac{(\beta\text{-}3/2)^2}{2}$ $+(\sigma\text{-}1)({}^3\!/_2\text{-}\sigma)+\frac{(3/2\text{-}\sigma)^2}{2}\big)$	$\phi\big((2\text{-}\beta)(\beta\text{-}\sigma)$ $+\frac{(\beta\text{-}\sigma)^2}{2}\big)$	$\phi\big((1\text{-}\sigma)(\beta\text{-}\sigma)$ $+\frac{(\beta\text{-}\sigma)^2}{2}\big)$	0
Surplus	$G - Tc$	$G - Tc$	$G - Tc$	0
Loss	0	$\frac{2\sigma^2\text{-}5\sigma+3}{4(\sigma\text{-}1)}$	$\frac{2\sigma^2\text{-}7\beta+6}{4(2\text{-}\sigma)}$	${}^1\!/_4$

A.9 Aggregate uncertainty

Consider an infinite market with regular transaction costs. Recall that regular transaction costs in infinite markets only depend on a traders action and the market price. Uninfluenceable regular transaction costs are functions of the market price, that is $\Phi_i(a_i, P^*) = \Phi_i(P^*)$. Examples include constant and price fees. Regular transaction costs are influenceable in infinite markets iff the map $a_i \mapsto \Phi_i(a_i, P^*)$ is strictly increasing for buyers and strictly decreasing for sellers. Spread fees are again an example of influenceable transaction costs.

In this section suppose that trader i is uncertain about the uninfluenceable market price P^*. We assume P^* is a random variable and that the distribution is absolutely continuous with probability density function

$f_{P^*,i}$ that is continuous and strictly positive on its support $[\underline{P_i^*}, \overline{P_i^*}]$ with $\underline{P_i^*} < \overline{P_i^*}$. Denote by $F_{P^*,i}$ the corresponding distribution function. Additionally, trader i also hold individual beliefs about the tie-breaking probability $p_i \in [0,1]$, if a traders' action is equal to P^*. Trader i may be more or less certain about their beliefs, which, for some degree $\delta > 0$, we measure by δ-*aggregate uncertainty* as follows: given $\delta > 0$, there exists a price P_i^*, such that $\mathbb{P}_i\left[P^* \in [P_i^* - \delta, P_i^* + \delta]\right] \geq 1 - \delta$.[29]

Predictability of trade. In general, for a buyer b with action a_b the probability of trading is equal to $1 - F_{P^*,b}(a_b)$. For a selle with ask a_s, it is equal to $F_{P^*,s}(a_s)$. δ-aggregate uncertainty is directly related to the predictability of trade. P_i^* corresponds to the critical value. If trader i submits an action that is strictly less (more) aggressive than the critical value, then for sufficiently small $\delta > 0$, the probability of trading is at least $1 - \delta$ (at most δ). Therefore Proposition 3 directly extends to settings with small uncertainty.

Existence of best responses. Proposition 5 extends to markets with aggregate uncertainty. The same proof method as in Appendix B.6 works. That is, the expected utility is continuous as a function of the action a_i of trader i. As best responses are necessary located in the compact space $[\underline{P_i^*}, \overline{P_i^*}]$, the existence of a maximum follows from the Extreme Value theorem.

Asymptotically equal transaction Costs. Theorem 6 directly extends to settings with sufficiently small δ-aggregate uncertainty. The proof is similar to Proposition 23 below, and will be added.

Uninfluenceable transaction costs. In the presence of aggregate uncertainty, Theorem 7 and Theorem 8 can be strengthened, as truthfulness is the unique best response.

Proposition 22. *Consider an uninfluenceable transaction cost and δ-uncertainty. For every $\delta > 0$, truthfulness is the unique best response.*

The proof is relegated to Appendix B.17.

Influenceable transaction costs. Theorem 9 and Theorem 10 also extend to markets with sufficiently small aggregate uncertainty.

Proposition 23. *Consider an influenceable transaction cost, δ-uncertainty, and assume that for trader i bidding the critical value P_i^∞ is strictly individually rational. Then, if δ is sufficiently small, best responses approximate price-guessing*

The proof is relegated to Appendix B.18.

B Proofs

B.1 Proof of Proposition 1

Proof. Consider a buyer b with gross value $t_b \in T_b^+$. First, we prove that there exists a unique solution to the equation $t_b - x - \Phi_b(x,x) = 0$. Because $t_b \in T_b^+$, there exists an action a_b such that

[29] $\delta = 0$ would describes the case of deterministic beliefs.

$t_b - a_b - F_b(a_b, a_b) > 0$. Furthermore, for $a_b > t_b$, it holds that $t_b - a_b - F_b(a_b, a_b) < 0$. Because the function $x \mapsto t_b - x - F_b(x, x)$ is continuous and strictly decreasing, there exists a unique zero point by the Intermediate Value theorem.

Existence. Next, we show that this solution x is equal to the net value t_b^Φ, by proving that x is undominated, it dominates every larger action a_b, it is ex-post individually rational, and no larger action a_b is ex-post individually rational. Consider $a_b > x$. If $a_{\text{-}b}$ is such that buyer b is not involved in trade with x and a_b, then the utility is equal to 0 for both actions. If $a_{\text{-}b}$ is such that b is involved in trade with both actions, then it follows that $u_b(t_b, x, a_{\text{-}b}) \geq u_b(t_b, a_b, a_{\text{-}b})$, because the fee is monotone. If $a_{\text{-}b}$ is such that b is only involved in trade with a_b, then then the market price $P^*(a_b, a_{\text{-}b})$ is greater or equal than x. It holds that $u_b(t_b, a_b, a_{\text{-}b}) \leq u_b(t_b, P^*(a_b, a_{\text{-}b}), a_{\text{-}b}) = t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(P^*(a_b, a_{\text{-}b}), P^*(a_b, a_{\text{-}b})) \leq t_b - x - \Phi_b(x, x) = 0$. The first inequality follows from the monotonicity of the fee, the second inequality follows, because the map $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing, and the final equality follows from the definition of x. Therefore a_b is dominated by x. Consider $a_b < x$. We show that there exists $a_{\text{-}b}$ such that $u_b(t_b, x, a_{\text{-}b}) > u_b(t_b, a_b, a_{\text{-}b})$. Take $a_{\text{-}b}$, such that buyer b is involved in trade only with x and the market price is strictly less than x. It holds that $u_b(t_b, x, a_{\text{-}b}) = t_b - P^*(x, a_{\text{-}b}) - \Phi_b(x, P^*(x, a_{\text{-}b}) > t_b - x - \Phi_b(x, x) = 0$. The inequality follows from regularity of the fee. Therefore x is not dominated by a_b. To show that x is ex-post individually rational, take any distribution of actions $a_{\text{-}b}$. If buyer b is involved in trade with x, it holds that $P^*(x, a_{\text{-}b}) \leq x$ and therefore $u_b(t_b, x, a_{\text{-}b}) = t_b - P^*(x, a_{\text{-}b}) - \Phi_b(x, P^*(x, a_{\text{-}b})) \geq t_b - x - \Phi_b(x, x) = 0$, where the inequality follows from regularity. Finally, we show that $a_b > x$ is not ex-post individually rational. Take $a_{\text{-}b}$, such that buyer b is involved in trade with a_b and $P^*(a_b, a_{\text{-}b}) > x$. It holds that $u_b(t_b, a_b, a_{\text{-}b}) \leq u_b(t_b, P^*(a_b, a_{\text{-}b}), a_{\text{-}b}) = t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(P^*(a_b, a_{\text{-}b}), P^*(a_b, a_{\text{-}b})) < t_b - x - \Phi_b(x, x) = 0$, where the first inequality follows from monotonicity, and the second one follows, because the map $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing. This finally proves that $x = t_b^\Phi$. Therefore, the net value exists and the supremum is attained as a maximum.

Continuity. It was proven above that the net value exists on T_b^+ and is equal to the unique zero point of the function $x \mapsto t_b - x - \Phi_b(x, x)$. Because this function is strictly increasing and continuous, the zero point continuously depends on the gross value t_b.

Monotonicity. The map $t_b \mapsto t_b - x - \Phi_b(x, x)$ is strictly increasing. Therefore, the zero point of the map $x \mapsto t_b - x - \Phi_b(x, x)$ is strictly increasing in t_b.

The statement for sellers can be proven analogously. □

B.2 Proof of Corollary 2

Proof. Consider a buyer b.

Spread fees. It holds that $\Phi_b(a_b, a_{\text{-}b}) = \phi_b(a_b - P^*(a_b, a_{\text{-}b}) = F_b(a_b, P^*(a_b, a_{\text{-}b})$ with the function $F_b(x, y) = \phi_b(x - y)$. It holds that the map $y \mapsto y + F_b(x, y) = \phi_b x + (1 - \phi_b)y$ is increasing, the map $x \mapsto x + F_b(x, x) = x$ is strictly increasing in y and both are continuous. Therefore spread fees satisfy the conditions of Proposition 1. For any tb, there exists a unique solution of $t_b - t_b^\Phi - F_b(t_b^\Phi, t_b^\Phi) = 0$. It is given by $t_b^\Phi = t_b$, proving that the net value equals the gross value.

Price fees. It holds that $\Phi_b(a_b, a_{\text{-}b}) = \phi_b P^*(a_b, a_{\text{-}b} = F_b(a_b, P^*(a_b, a_{\text{-}b})$ with the function $F_b(x, y) = \phi_b y$. It holds that the maps $y \mapsto y + F_b(x, y) = (1 + \phi_b)y$ and $x \mapsto x + F_b(x, x) = x$ are strictly increasing and continuous. Therefore price fees satisfy the conditions of Proposition 1. The unique solution of $t_b - t_b^\Phi - F_b(t_b^\Phi, t_b^\Phi) = 0$ is given by $t_b^\Phi = \frac{t_b}{1+\phi_b}$, proving that the net value scales the gross value.

Constant fees. It holds that $\Phi_b(a_b, a_{\text{-}b}) = c_b = F_b(a_b, P^*(a_b, a_{\text{-}b})$ with the function $F_b(x, y) = c_b$. It holds that the maps $y \mapsto y + F_b(x, y) = y + c_b$ and $x \mapsto x + F_b(x, x) = x + c_b$ are continuous and strictly increasing in y. Therefore constant fees satisfy the conditions of Proposition 1. There exists a solution to $t_b - t_b^\Phi - F_b(t_b^\Phi, t_b^\Phi) = 0$, if $t_b \geq c_b$. It is given by $t_b^\Phi = t_b - c_b$, proving that the net value shifts the gross value.

The statement for sellers can be proven analogously. □

B.3 Proof that the critical value P_i^∞ exists and is unique

Proof. At the point $\underline{a}_{S,i}$, it holds that $F_{B,i}^a(\underline{a}_{S,i}) < 1$. That is because $F_{B,i}^a$ has a strictly positive density $f_{B,i}^a$ on $[\underline{a}_{B,i}, \overline{a}_{B,i}]$ and $\underline{a}_{S,i} < \overline{a}_{B,i}$ by assumption. Second, it holds that $F_{S,i}^a(\underline{a}_{S,i}) = 0$, because the corresponding density $f_{S,i}^a$ has support $[\underline{a}_{S,i}, \overline{a}_{B,i}]$. Therefore, at $\underline{a}_{S,i}$, it holds that

$$F_{B,i}^a(\underline{a}_{S,i}) + R_i F_{S,i}^a(\underline{a}_{S,i}) < 1. \tag{30}$$

A similar argument yields that at the point $\overline{a}_{B,i}$, it holds that $F_{B,i}^a(\overline{a}_{B,i}) = 1$ and $F_{S,i}^a(\overline{a}_{S,i}) > 0$. This implies that

$$F_{B,i}^a(\overline{a}_{B,i}) + R_i F_{S,i}^a(\overline{a}_{B,i}) > 1. \tag{31}$$

Because $F_{B,i}^a$ and $F_{S,i}^a$ are both continuous, it follows from the Intermediate Value Theorem, that there exists $P_i^\infty \in (\underline{a}_{S,i}, \overline{a}_{B,i})$ with

$$F_{B,i}^a(P_i^\infty) + R_i F_{S,i}^a(P_i^\infty) = 1. \tag{32}$$

Because both $F_{B,i}^a$ and $F_{S,i}^a$ are strictly monotone on $(\underline{a}_{S,i}, \overline{a}_{B,i})$, the uniqueness of P_i^∞ follows. □

B.4 Proof of Proposition Proposition 3

Proof. For trader i, consider a sequence of strictly increasing market sizes $(m(l), n(l))_{l\in\mathbb{N}}$ with $m(l), n(l) = \Theta(l)$ and $|R - \frac{n(l)}{m(l)}| = \mathcal{O}(l^{-1})$ for $R \in (0, \infty)$.[30]

Consider a buyer b. It follows from Appendix A.3 that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \mathbb{P}_{\text{-}b}\left[a_b \geq a_{\text{-}b}^{m(l)}\right]$. This is equal to the probability that at least $m(l)$ actions are below a_b in a sample of actions from $m(l)-1$ buyers and $n(l)$ sellers. Let $p_{a_b} = F_{B,b}(a_b) \in (0,1)$ be the probability that another buyer's bid is below a_b. In analogy, define $q_{a_b} = F_{S,b}(a_b) \in (0,1)$ for sellers. For $i > 0$ let $X_i^{p_{a_b}}$ denote an independent Bernoulli random variable with parameter p_{a_b} and for $j > 0$ let $X_j^{q_{a_b}}$ denote an independent Bernoulli random variable with parameter q_{a_b}. Define

$$S_l^{a_b} = \sum_{i=1}^{m(l)-1} X_i^{p_{a_b}} + \sum_{j=1}^{n(l)} X_j^{q_{a_b}}. \tag{33}$$

$S_l^{a_b}$ has the same distribution as the number of traders in a sample of $m(l)-1$ buyers and $n(l)$ sellers, whose actions are less or equal than a_b. It follows that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \mathbb{P}[S_l^{a_b} \geq m(l)] = 1 - \mathbb{P}[S_l^{a_b} \leq m(l) - 1]. \tag{34}$$

Next, we will show that a properly normalized version of $S_l^{a_b}$ converges in distribution to a standard normal random variable. This follows as an application of the following version of the Berry-Esseen theorem, see Tyurin (2012):

Theorem 24 (Berry-Esseen)**.** *Suppose $X_1, X_2, ...$ is a sequence of independent random variables with (i) $\mu_i = \mathbb{E}[X_i] < \infty$, (ii) $\sigma_i^2 = \mathbb{E}[(X_i - \mu_i)^2] < \infty$ and (iii) $\rho_i = \mathbb{E}[|X_i - \mu_i|^3] < \infty$. Set $r_n = \sum_{i=1}^n \rho_i$, $s_n^2 = \sum_{i=1}^n \sigma_i^2$, $F_n(x) = \mathbb{P}\left[\frac{\sum_{i=1}^n (X_i - \mu_i)}{\sqrt{s_n^2}} \leq x\right]$ and let $\Phi(x)$ be the distribution function of a standard random variable. There exists a constant $C = 0.5591$ such that for all $x \in \mathbb{R}$*

$$|F_n(x) - \Phi(x)| \leq \frac{C r_n}{s_n^3} \tag{35}$$

In order to apply Theorem 24, we rewrite $S_l^{a_b}$ as a single sum of random variables and check all requirements. Define $Y_i^{p_{a_b}} = \sum_{j=0}^{m(i)-m(i-1)} X_{i,j}^{p_{a_b}}$ for $i \leq l-1$ and $Y_l^{p_{a_b}} = \sum_{j=1}^{m(l)-m(l-1)-1} X_{i,j}^{p_{a_b}}$ with $X_{i,j}^{p_{a_b}}$ independent Bernoulli random variables with parameter p_{a_b}. In analogy, define $Y_i^{q_{a_b}} = \sum_{j=1}^{n(i)-n(i-1)} X_{i,j}^{q_{a_b}}$ for $i \leq l$ independent Bernoulli random variables with parameter q_{a_b} and $Z_i^{a_b} =$

[30]This means that both market sides are assumed to have linear growth with respect to a single parameter l, such that neither side of the market dominates the other asymptotically and the ratio of buyers to sellers converges and fluctuates only slightly in finite markets.

$Y_i^{p_{a_b}} + Y_i^{q_{a_b}}$. This yields that in distribution

$$S_l^{a_b} \overset{d}{=} \sum_{i=1}^{l} Z_i^{a_b}. \tag{36}$$

Recall that a Bernoulli random variable with parameter p has expectation p and variance $p(1-p)$. Using linearity of expectation and, because the random variables are independent, linearity of variance, it holds for $i < l$, that the random variables satisfy (i) and (ii) in Theorem 24, i.e.

$$\begin{aligned} \mu_i &= \left(m\left(i\right) - m\left(i-1\right)\right) p_{a_b} + \left(n\left(i\right) - n\left(i-1\right)\right) q_{a_b} < \infty, \\ \sigma_i^2 &= \left(m\left(i\right) - m\left(i-1\right)\right) p_{a_b}\left(1 - p_{a_b}\right) + \left(n\left(i\right) - n\left(i-1\right)\right) q_{a_b}(1 - q_{a_b}) < \infty. \end{aligned} \tag{37}$$

For $i = l$ it holds that

$$\begin{aligned} \mu_l &= \left(m\left(l\right) - m\left(l-1\right) - 1\right) p_{a_b} + \left(n\left(l\right) - n\left(l-1\right)\right) q_{a_b} < \infty, \\ \sigma_l^2 &= \left(m\left(l\right) - m\left(l-1\right) - 1\right) p_{a_b}\left(1 - p_{a_b}\right) + \left(n\left(l\right) - n\left(l-1\right)\right) q_{a_b}\left(1 - q_{a_b}\right) < \infty. \end{aligned} \tag{38}$$

Furthermore, for $i < l$ it holds that

$$\begin{aligned} \rho_i &= \mathbb{E}\left[\left|\sum_{j=0}^{m(i)-m(i-1)} X_{i,j}^{p_{a_b}} + \sum_{j=0}^{n(i)-n(i-1)} X_{i,j}^{q_{a_b}} - \left(m\left(i\right) - m\left(i-1\right)\right) p_{a_b} - \left(n\left(i\right) - n\left(i-1\right)\right) q_{a_b}\right|^3\right] \\ &\leq \left(\left(m\left(i\right) - m\left(i-1\right)\right)\left(1 - p_{a_b}\right) + \left(n\left(i\right) - n\left(i-1\right)\right)\left(1 - q_{a_b}\right)\right)^3 \\ &\leq K < \infty. \end{aligned} \tag{39}$$

The first inequality in Equation (39) holds, because $X_{i,j}^{p_{a_b}} \leq 1$ and $X_{i,j}^{q_{a_b}} \leq 1$ almost surely. The second inequality follows for some finite $K > 0$ from the assumption $\sup_{i\geq 1} m(i) - m(i-1) < \infty$ and $\sup_{i\geq 1} n(i) - n(i-1) < \infty$. In analogy, for $i = l$ it holds that

$$\rho_l \leq K < \infty, \tag{40}$$

which proves that requirement (iii) is fulfilled. Finally, it holds that

$$s_l^2 = (m(l) - 1)p_{a_b}(1 - p_{a_b}) + n(l)q_{a_b}(1 - q_{a_b}). \tag{41}$$

Next, define the sequence $(A_{a_b}(l))_{l\in\mathbb{N}}$ via

$$\begin{aligned} A_{a_b}(l) &= \frac{m\left(l\right) - 1 - \left(\left(m\left(l\right) - 1\right) p_{a_b} + n\left(l\right) q_{a_b}\right)}{\sqrt{\left(m(l) - 1\right) p_{a_b}\left(1 - p_{a_b}\right) + n(l)q_{a_b}\left(1 - q_{a_b}\right)}} \\ &= \sqrt{m(l)}\frac{\left(1 - \frac{1}{m(l)}\right) - \left(\left(1 - \frac{1}{m(l)}\right) p_{a_b} + \frac{n(l)}{m(l)}\right) q_{a_b})}{\sqrt{\left(1 - \frac{1}{m(l)}\right) p_{a_b}\left(1 - p_{a_b}\right) + \frac{n(l)}{m(l)} q_{a_b}\left(1 - q_{a_b}\right)}}. \end{aligned} \tag{42}$$

Theorem 24 now implies that

$$|\mathbb{P}[\leq m(l)-1] - \Phi(A_{a_b}(l))| \leq \frac{Cr_l}{s_l^3} \leq \frac{CKl}{\left(s_l^2\right)^{3/2}} = \mathcal{O}(l^{-\frac{1}{2}}). \tag{43}$$

It follows from Equation (42) that $|A_{a_b}(l)| = \Theta(\sqrt{l})$. We now argue that for $a_b > P_b^\infty$ and sufficiently large l, $A_{a_b}(l) < 0$. This follows, if we show that for sufficiently large l

$$\left(1 - \frac{1}{m(l)}\right) - \left(\left(1 - \frac{1}{m(l)}\right) p_{a_b} + \frac{n(l)}{m(l)} q_{a_b}\right) < 0. \tag{44}$$

Given that a_b is strictly greater than the critical value P_b^∞, there exists $\delta > 0$, such that $p_{a_b} + Rq_{a_b} = 1 + \delta$. By adding and subtracting Rq_{a_b} it follows that Equation (44) is equivalent to

$$1 - \frac{1}{m(l)}(1 - p_{a_b}) - (1 + \delta) + (R - \frac{n(l)}{m(l)})q_{a_b} < 0 \tag{45}$$

and therefore to

$$R - \frac{n(l)}{m(l)} < \frac{1}{q_{a_b}}(\delta + \frac{(1 - p_{a_b})}{m(l)}). \tag{46}$$

Because it is assumed that $|R - \frac{n(l)}{m(l)}| = \mathcal{O}(\frac{1}{l})$, Equation (44) holds for sufficiently large l. This implies that $A_{a_b}(l) = \Theta(-\sqrt{l})$. A standard concentration inequality for a standard Gaussian random variable Z and $x > 0$ using the Chernoff bound gives

$$\mathbb{P}\,|Z| \geq x] \leq 2\exp\left(\frac{-x^2}{2}\right) \tag{47}$$

It follows that

$$\Phi(A_{a_b}(l)) = \mathcal{O}(e^{-l}). \tag{48}$$

Equation (43) therefore implies that $\mathbb{P}[S_l^{a_b} \leq m(l) - 1] = \mathcal{O}(l^{-\frac{1}{2}})$. Recalling Equation (34) finishes the proof. The statements for $a_b < P_b^\infty$ and for sellers can be proven analogously. □

B.5 Proof of Lemma 4

Proof. For a regular transaction cost $\Phi_i(a_i, P^*)$, consider the following two auxiliary transaction costs: $\Phi_i^1(a_i, P^*) = \Phi_i(P^*, P^*)$ and $\Phi_i^2(a_i, P^*) = \Phi_i(a_i, P^*) - \Phi_i^1(a_i, P^*)$. Note that Φ_i^1 depends only on the market price and is therefore uninfluenceable and Φ_i^2 is purely uninfluenceable by construction, that is $\Phi_i^2(P^*, P^*) = 0$. □

B.6 Proof of Proposition 5

Proof. Consider a buyer b with private type t_b.

Finite Markets. As was shown in Equation (12), the expected utility is of the form

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = t_b \cdot \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]. \tag{49}$$

First, we will show that the expected utility is continuous in a_b.[31] The first term $t_b \cdot \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]$ is continuous by Equation (3) and Equation (28). To show that the expected market price is continuous, consider $\mathbb{E}_{\text{-}b}\left[P^*(a''_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a'_b, a_{\text{-}b})\right]$ for two bids $a''_b > a'_b$ as $a''_b - a'_b$ approaches zero. The buyer increases the expected market price when raising their bid if (1) they are involved in trade at a''_b, but not at a'_b or (2) a'_b influences the market price. For (1), the market price is at most a''_b and for (2) the change in market price is at most $a''_b - a'_b$. This implies that

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[P^*(a''_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a'_b, a_{\text{-}b})\right] \leq \\ a''_b\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a''_b, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a'_b, a_{\text{-}b})\right]\right) + (a''_b - a'_b).\end{gathered} \tag{50}$$

The continuity of $\mathbb{E}_{\text{-}b}\left[P^*(\cdot, a_{\text{-}b})\right]$ therefore follows from the continuity of $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(\cdot, a_{\text{-}b})\right]$. For the expected transaction cost, it holds that

$$\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right] = \int_{a_b \geq a_{\text{-}b}^{(m)}} \Phi_b(a_b, a_{\text{-}b}) d\mu(a_{\text{-}b}). \tag{51}$$

By assumption, the map $a_b \mapsto \Phi_b(a_b, a_{\text{-}b})$ is continuous. Therefore Equation (51) implies that the map $a_b \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]$ is continuous as well. Therefore, the expected utility is indeed continuous in a_b.

Every bid $a_b < \underline{a}_{S,b}$ results in zero utility, as the buyer is almost surely not involved in trade. For every bid $a_b > t_b^{\Phi}$, it follows from weak domination ex post that the expected utility for a_b is smaller or equal than for $t_b^{\Phi} \leq t_b$. If $t_b^{\Phi} \leq \underline{a}_{S,b}$, then t_b^{Φ} is a best response with expected utility equal to zero. Otherwise, in order to compute a best response, it is sufficient to consider the interval $\left[\underline{a}_{S,b}, t_b^{\Phi}\right]$. Because the expected utility is a continuous function on this compact set, it follows from the Extreme Value Theorem that the expected utility attains a maximum. Therefore, a best response exists.

Infinite Markets. It was shown in Appendix A.3 that the expected utility is of the form

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = \left(t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(a_b, a_{\text{-}b})\right) \cdot \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]. \tag{52}$$

[31] The same proof strategy for continuity is used in Williams (1991) for the expected utility in a buyer's bid DA without fees in the context of Bayesian Nash equilibria.

In an infinite market, the market price $P^*(a_b, a_{\text{-}b})$ and the fee $\Phi_b(a_b, a_{\text{-}b})$ are deterministic. By assumption, $\Phi_b(a_b, a_{\text{-}b})$ is continuous in the action a_b. By Appendix A.3 it holds that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \begin{cases} 1 & a_b \geq P^*(a) \\ 0 & \text{else} \end{cases}, \tag{53}$$

if there is no tie-breaking. If $t_b^\Phi < P^*(a)$, then buyer b has no undominated action with positive probability of trade. Therefore t_b^Φ is a best response with expected utility equal to zero. If $t_b^\Phi = P^*(a)$, then the only undominated action with positive probability of trade is t_b^Φ. If this results in a strictly positive utility, then it is a best response. If not, then any bid below $P^*(a)$ is a best response. Therefore, consider the case $t_b^\Phi > P^*(a)$. If there is no tie-breaking, then the trading probability is constant and equal to 1 on the compact set $[P^*(a), t_b^\Phi]$. Note that any bid above t_b^Φ is not a best response by weak domination. By similar arguments as before, the expected utility on this interval is equal to $t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(a_b, a_{\text{-}b})$ and therefore a continuous function. The Extreme Value Theorem implies again that the maximum is attained and a best response exists.

The statement for sellers can be proven analogously. □

B.7 Proof of Theorem 6

Proof. Consider a buyer b and two actions $a_b^1 > a_b^2 > P_b^\infty$ that lead to asymptotically different transaction costs. We will prove that in sufficiently large markets a buyer can improve their expected utility when switching from action a_b^1 to a_b^2. This in turn implies that best responses for two different gross values must lead to asymptotically equal transaction costs. Otherwise, there is a buyer with a certain gross value, who has an incentive to change their action in sufficiently large markets to increase their expected utility.

By assumption, there exists $\epsilon > 0$ such that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^1, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}i}\left[\Phi_b(a_b^2, a_{\text{-}b}) | b \in A^*\left(a_b^2, a_{\text{-}b}\right)\right] \geq \epsilon. \tag{54}$$

We will show that in sufficiently large markets a_b^1 cannot be a best response. By contradiction, assume that it was a best response for some gross value t_b. The expected utility $\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right]$ is greater or equal than 0, otherwise it is trivially not a best response. We will prove that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] < 0, \tag{55}$$

which proves that a_b^1 is not a best response in such markets, because a_b^2 increases the expected utility.

Using the law of total expectation, the expected difference in transaction costs can be lower bounded by

$$
\begin{aligned}
&\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right] \\
&= \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^1, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^1, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^2, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^2, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right] \\
&\geq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]\left(\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^1, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^2, a_{\text{-}b}\right)\right]\right)
\end{aligned} \tag{56}
$$

The inequality from the last line follows from the monotonicity of the trading probability, which implies

$$
\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] \geq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]. \tag{57}
$$

It follows from Proposition 3 that for every γ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right] \geq 1 - \gamma$. Combining this with the assumption of asymptotically different transaction costs yields that in sufficiently large markets

$$
\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right] \geq (1-\gamma)\epsilon. \tag{58}
$$

Using Equation (16) in Lemma 20 it holds in sufficiently large markets that

$$
\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] \leq t_b\gamma - (1-\gamma)\epsilon. \tag{59}
$$

If we now choose $\gamma < \frac{\epsilon}{t_b + \epsilon}$, the difference in expected utility is strictly negative, thus contradicting that a_b^1 is a best response. The statement for sellers can be proven analogously. □

B.8 Proof of Theorem 7

Proof. Consider a buyer b with gross value t_b, such that the best response a_b is uniformly bounded away from the critical value. That is, there exists $\delta > 0$, such that in sufficiently large markets either (i) $a_b \leq P_b^\infty - \delta$ or (ii) $a_b \geq P_b^\infty + \delta$. It suffices to prove that for every $\epsilon > 0$ it holds in sufficiently large markets that

$$
\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \geq -\epsilon, \tag{60}
$$

which implies that truthfulness is an ϵ-best response. If it holds that $t_b^\Phi \leq a_b$, it holds that

$$
\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] = \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right], \tag{61}
$$

because t_b^Φ weakly dominates every larger bid and since a_b is a best response, the expected utilities must be equal. Therefore, assume that $t_b^\Phi > a_b$.

If (i) holds, then Proposition 3 implies that for all $\gamma > 0$ $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \gamma$ holds in sufficiently large markets. If $\gamma < \frac{\epsilon}{t_b}$ it follows that

$$
\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \leq t_b\gamma \leq \epsilon. \tag{62}
$$

By assumption it also holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi}, a_{\text{-}b})\right] \geq 0. \tag{63}$$

Combining Equations (62) and (63) yields Equation (60).

If (ii) holds, then

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \geq \\ t_b^{\Phi}\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]\right) - \left(\mathbb{E}_{\text{-}b}\left[P^*(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a_b, a_{\text{-}b})\right]\right) \\ - \left(\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]\right),\end{aligned} \tag{64}$$

because by assumption $t_b^{\Phi} \leq t_b$. It follows from Theorem 8 that for a DA without transaction costs for every $\epsilon_1 > 0$ truthfulness is an ϵ_1-best response in sufficiently large markets. Assume that a buyer has gross value equal to t_b^{Φ}. It therefore holds in sufficiently large markets that for any other bid, i.e., also the best response a_b for gross value t_b

$$\begin{aligned}t_b^{\Phi}\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]\right) - \left(\mathbb{E}_{\text{-}b}\left[P^*(t_b^{\Phi}, a_{\text{-}b})\right] - \right. \\ \left. \mathbb{E}_{\text{-}b}\left[P^*(a_b, a_{\text{-}b})\right]\right) \geq -\epsilon_1.\end{aligned} \tag{65}$$

Using the law of total expectation, the expected difference in transaction costs in Equation (65) is equal to

$$\begin{aligned}&\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right] \\ &= \mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | b \in \mathcal{B}^*\left(t_b^{\Phi}, a_{\text{-}b}\right)\right]\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] - \\ &\quad \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right]\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right].\end{aligned} \tag{66}$$

Because both actions are by assumption greater or equal than $P_b^{\infty} + \delta$, for every $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right], \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \geq 1 - \gamma$. It therefore holds that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] \leq \gamma. \tag{67}$$

This implies that in sufficiently large markets

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right] \leq \\ \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]\left(\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | b \in \mathcal{B}^*\left(t_b^{\Phi}, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right]\right) + \\ \gamma\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | b \in \mathcal{B}^*\left(t_b^{\Phi}, a_{\text{-}b}\right)\right].\end{aligned} \tag{68}$$

Asymptotic uninfluenceability of the transaction costs implies that for every $\epsilon_2 > 0$ the first term in Equation (68) is less or equal than ϵ_2 and for every $\epsilon_3 > 0$ the second term can be chosen to be less or equal than ϵ_3 in sufficiently large markets by choosing $\gamma \leq \frac{\epsilon_3}{\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | A^*\left(b, t_b^{\Phi}\right)\right]}$. If ϵ_1, ϵ_2 and ϵ_3 are chosen such that their sum is less or equal than ϵ, plugging Equations (65) and (68) into

Equation (64) yields that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \geq -(\epsilon_1 + \epsilon_2 + \epsilon_3) \geq -\epsilon, \tag{69}$$

which completes the proof. The statement for sellers can be proven analogously. □

B.9 Proof of Theorem 8

Proof. Consider a buyer b with private type t_b.

Best responses are close to truthfulness (2). We will show that there exists a constant $\kappa > 0$, such that

$$t_b - (1 + \phi_b)\, a_b \leq \kappa q(n, m), \tag{70}$$

with $q(m, n) = \max\left\{\frac{1}{n}\left(1 + \frac{m}{n}\right), \frac{1}{m}\left(1 + \frac{n}{m}\right)\right\} = O(\max(m, n)^{-1})$, from which the statement follows. It was proven in Appendix A.6, that a best response a_b necessarily satisfies the first order condition in Equation (24), which implies the following bound:

$$t_b - (1 + \phi_b)\, a_b \leq \frac{(1 + \phi_b)\, k\mathbb{P}_{\text{-}b}\left[a_{m-1,n}^{(m)} \leq a_b \leq a_{m-1,n}^{(m+1)}\right]}{(m-1)\mathbb{P}_{\text{-}b}\left[a_{m-2,n}^{(m-1)} \leq a_b \leq a_{m-2,n}^{(m)}\right] f_{B,b}(a_b)}. \tag{71}$$

It can be proven analogous to Rustichini et al. (1994, Appendix) that

$$\frac{\mathbb{P}_{\text{-}b}\left[a_{m-1,n}^{(m)} \leq a_b \leq a_{m-1,n}^{(m+1)}\right]}{\mathbb{P}_{\text{-}b}\left[a_{m-2,n}^{(m-1)} \leq a_b \leq a_{m-2,n}^{(m)}\right]} \leq 2\left[F_{B,b}(a_b) + \frac{n}{m}\frac{(1 - F_{B,b}(a_b))\, F_{S,b}(a_b)}{1 - F_{S,b}(a_b)}\right]. \tag{72}$$

Defining

$$\tau_b \equiv 2 \max_{x \in [\underline{a}_{S,b}, \overline{a}_{B,b}]} \left\{\frac{F_{B,b}(x)}{f_{B,b}(x)}, \frac{(1 - F_{B,b}(x))\, F_{S,b}(x)}{f_{B,b}(x)\,(1 - F_{S,b}(x))}\right\} \tag{73}$$

yields that

$$t_b - (1 + \phi_b)\, a_b \leq \frac{\tau_b k\,(1 + \phi_b)}{m - 1}\left[1 + \frac{n}{m}\right]. \tag{74}$$

To obtain the bounds in the theorem, note that $\frac{n}{n-1}$ and $\frac{m}{m+1}$ are both less than 2. Setting $\kappa \equiv 2\tau_b k$ proves the statement for buyers. For a seller s with private type t_s an analogous argument yields

$$(1 - \phi_s)\, a_s - t_s \leq \frac{\tau_s(1\text{-}k)(1 - \phi_s))}{n - 1}\left[1 + \frac{m}{n}\right] \tag{75}$$

for τ_s with

$$\tau_s \equiv 2\max\left\{\frac{1 - F_{S,s}(x)}{f_{S,s}(x)}, \frac{(1 - F_{B,s}(x))\, F_{S,s}(x)}{f_{S,s}(x) F_{B,s}(x)}\right\}. \tag{76}$$

Truthfulness is an ϵ-best response. We start by estimating the difference in utility when a buyer switches from a bid a_b^1 to a smaller bid a_b^2, i.e., $\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^2, a_{\text{-}b})\right]$. The expected utility is not dependent on the entirety of $a_{\text{-}b}$, but only on $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$. We consider all six possible cases for the realizations of $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$ with respect to $a_b^1 > a_b^2$.

		$u_b(t_b, a_b^1, a_{\text{-}b})$	$u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}(1+\phi_b)\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}(1+\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}(1+\phi_b)\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0	0

Analogously, we consider the difference in utilities:

		$u_b(t_b, a_b^1, a_{\text{-}b}) - u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	0
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$\text{-}k(1+\phi_b)\left(a_{\text{-}b}^{(m+1)}\text{-}a_b^2\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$\text{-}k(1+\phi_b)\left(a_b^1\text{-}a_b^2\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0

We want to lower bound $\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^2, a_{\text{-}b})\right]$. It is therefore sufficient to lower bound the expression in **II** and **IV**, since they are negative and neglect the positive difference in the other cases. In order to prove truthfulness is close to optimal, consider $a_b^1 = t_b^\Phi$ and $a_b^2 = a_b$ a best response. We show that for any $\epsilon > 0$ it holds in sufficiently large finite markets the difference in expected utility is bounded from below by $-\epsilon$. Because best responses are ϵ-close to truthfulness in sufficiently large finite markets, it holds in such markets that for all $\delta > 0$ $t_b^\Phi - a_b \leq \delta$. Therefore the difference in **II** and **IV** is lower bounded by $-k(1+\phi_b)\delta$. It follows that

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \leq \\ -k(1+\phi_b)\delta\left(\mathbb{P}\left[\mathbf{II}\right] + \mathbb{P}\left[\mathbf{IV}\right]\right) \leq -2k(1+\phi_b)\delta.\end{gathered} \tag{77}$$

If for a given $\epsilon > 0$, $\delta > 0$ is chosen such that $\delta \leq \frac{\epsilon}{2k(1+\phi_b)}$, it holds in sufficiently large finite markets that t_b^Φ is ϵ-close to a best response a_b. In infinite markets, the expected utility is equal to

$$\mathbb{E}[u_b(t_b, a_b, a_{\text{-}b})] = \begin{cases} t_b - (1+\phi_b)P^* & \text{if } a_b \geq P^*, \\ 0 & \text{if } a_b < P^*. \end{cases} \tag{78}$$

If $t_b^\Phi \geq P^*$, then the expected utility is equal to $t_b - (1+\phi_b)P^* > 0$, and therefore a best response. If $t_b^\Phi \leq P^*$, then the expected utility is equal to 0. Because every action $a_b > t_b^\Phi$ is dominated, t_b^Φ is again a best response. Therefore truthfully reporting t_b^Φ is a best response. The statement for sellers can be proven analogously. $\square$

B.10 Proof of Theorem 9

Proof. Consider a buyer b with a gross value t_b, such that $t_b^\Phi > P_b^\infty$. First, we show that in sufficiently large markets an action $a_b^1 < P_b^\infty$ is not a best response. We show that there exists an action $a_b^2 > P_b^\infty$ such that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^2, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] > 0, \tag{79}$$

which implies that a_b^1 is not a best response. Because the net value is by assumption continuous and strictly increasing in the gross value, there exists a gross value $t_b' < t_b$, such that $t_b^\Phi > t_b^{\Phi\prime} > P_b^\infty$. Denote the difference between t_b^Φ and $t_b^{\Phi\prime}$ by $\delta > 0$. It holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi\prime}, a_{\text{-}b})\right] = \mathbb{E}_{\text{-}b}\left[u_b(t_b', t_b^{\Phi\prime}, a_{\text{-}b})\right] + \delta \geq \delta, \tag{80}$$

because the net value is assumed to be ex-post individually rational. Note that this inequality holds for every market size. To prove Equation (98), it therefore suffices to show that for $a_b^1 < P_b^\infty$ it holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] < \delta. \tag{81}$$

We can upper bound the expected utility by neglecting the expected market price and the expected fee and get that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] \leq t_b \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right]. \tag{82}$$

Proposition 3 implies that for any $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \gamma$. If we choose $\gamma < \frac{\delta}{t_b}$, the statement follows. We therefore consider an action a_b that is ϵ-distant to the critical value, that is, there exists $\epsilon > 0$ such that $a_b - P_b^\infty \geq \epsilon$. We will prove that in sufficiently large markets it holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b})\right] < 0, \tag{83}$$

which proves that a_b is not a best response in sufficiently large markets. Therefore, best responses must be ϵ-close, but above the critical value in sufficiently large markets. Using the law of total expectation, the expected difference in transaction cost can be lower bounded by

$$\begin{aligned}
&\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}\right)\right] = \\
&\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] - \\
&\mathbb{E}_{\text{-}b}\left[\Phi_b(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}) | b \in \mathcal{B}^*\left(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b})\right] \geq \\
&\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b})\right] \left(\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right] - \right. \\
&\left.\mathbb{E}_{\text{-}b}\left[\Phi_b(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}) | b \in \mathcal{B}^*\left(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}\right)\right]\right)
\end{aligned} \tag{84}$$

The inequality on the last line holds because the trading probability is monotone, which implies $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \geq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b})\right]$. It follows from Proposition 3 that for every γ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b})\right] \geq 1 - \gamma$. Combining this with the assumption of influenceability of the transaction costs yields that there exists $\delta > 0$ such that it holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}\right)\right] \geq (1-\gamma)\delta. \tag{85}$$

Using Equation (16) from Lemma 20, it therefore holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b}\right)\right] \leq t_b\gamma - (1-\gamma)\delta. \tag{86}$$

If we now choose $\gamma < {}^{\delta}\!/_{t_b + \delta}$, the difference is strictly smaller than 0, which proves that a_b is not a best response in sufficiently large markets.

The statement for sellers can be proven analogously. □

B.11 Proof of Theorem 10

Proof. To prove that best responses are in an ϵ-neighbourhood of the critical value in sufficiently large markets , consider a buyer b with gross value t_b, such that $t_b^{\Phi} > P_b^{\infty}$. It follows analogous to Appendix B.10 that in sufficiently large markets an action $a_b^1 < P_b^{\infty}$ is not a best response. We therefore consider an action $a_b > P_b^{\infty}$. That is, there exists $\epsilon > 0$ such that $a_b - P_b^{\infty} \geq \epsilon$. We will prove that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^{\infty} + {}^{\epsilon}\!/2, a_{\text{-}b})\right] < 0, \tag{87}$$

which proves that a_b is not a best response in such markets. Therefore, best responses must be ϵ-close, but above the critical value in sufficiently large markets. For two bids $a_b^1 > a_b^2$ Lemma 20 implies in the presence of a spread fee that

$$\begin{aligned}&\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right]\\ &\leq \left(t_b - \phi_b a_b^1\right)\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \left(t_b - \phi_b a_b^2\right)\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right].\end{aligned} \tag{88}$$

Now set $a_b^1 = a_b$ and $a_b^2 = P_b^\infty + \epsilon/2$. It follows from Proposition 3 that for any $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right], \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] \geq 1 - \gamma$ and therefore also

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] + \gamma. \tag{89}$$

Combining Equations (88) and (89) implies that it holds in sufficiently large markets that

$$\begin{aligned}&\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right]\\ &\leq -\phi_b(1-\gamma)(a_b - (P_b^\infty + \epsilon/2)) + \gamma\left(t_b - \phi_b a_b\right).\end{aligned} \tag{90}$$

By assumption, it holds that $a_b - (P_b^\infty + \epsilon/2) \geq \epsilon/2$, which yields

$$\begin{aligned}&\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right]\\ &\leq -\phi_b(1-\gamma)\frac{\epsilon}{2} + \gamma\left(t_b - \phi_b a_b\right) \leq -\phi_b(1-\gamma)\frac{\epsilon}{2} + \gamma t_b.\end{aligned} \tag{91}$$

If γ is chosen such that $\gamma < \frac{\phi_b \epsilon}{2t_b + \phi_b \epsilon}$ holds, then in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] < 0, \tag{92}$$

which implies that a_b is not a best response in such markets.

Next, we prove that for sufficiently small $\epsilon > 0$, there exist beliefs, such that the critical value is not an ϵ-best response in sufficiently large finite markets. Consider a buyer b with gross value $t_b^\Phi > P_b^\infty$ in a sequence of market environment with $m(l) = l$, $n(l) = l$, $T = [0, 1]$ and uniformly distributed beliefs over actions for both buyers and sellers. In this case, the critical value P_b^∞ is equal to $\frac{1}{2}$. By assumption, there exists $\epsilon > 0$, such that $t_b = P_b^\infty + \epsilon$ for $\epsilon > 0$. We will show that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/4, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty, a_{\text{-}b})\right] > 0, \tag{93}$$

which proves that P_b^∞ is not a best response. In order to estimate the difference in expected utility for two bids $a_b^1 > a_b^2$, we use a table similar to the one in Appendix B.9:

		$u_b(t_b, a_b^1, a_{\text{-}b})$	$u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	t_b-$\phi_b a_b^1$-(1-ϕ_b) $\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	t_b-$\phi_b a_b^2$-(1-ϕ_b) $\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	t_b-$\phi_b a_b^1$-(1-ϕ_b) $\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	t_b-$\phi_b a_b^2$-(1-ϕ_b) $\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	t_b-$\phi_b a_b^1$-(1-ϕ_b) $\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	t_b-$\phi_b a_b^1$-(1-ϕ_b) $\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	t_b-$\phi_b a_b^2$-(1-ϕ_b) $\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	t_b-$\phi_b a_b^1$-(1-ϕ_b) $\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0	0

Analogously, we consider the difference in utilities:

		$u_b(t_b, a_b^1, a_{\text{-}b}) - u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	$-\phi_b\left(a_b^1 - a_b^2\right)$
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$-\phi_b\left(a_b^1 - a_b^2\right) - k(1-\phi_b)\left(a_{\text{-}b}^{(m+1)} - a_b^2\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b - \phi_b a_b^1 - (1-\phi_b)\left(ka_{\text{-}b}^{(m+1)} + (1\text{-}k)a_{\text{-}b}^{(m)}\right)$
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$-\phi_b\left(a_b^1 - a_b^2\right) - k\left((1-\phi_b)\left(a_b^1 - a_b^2\right)\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b - \phi_b a_b^1 - (1-\phi_b\left(ka_b^1 + (1\text{-}k)a_{\text{-}b}^{(m)}\right)$
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0

In order to obtain a lower bound on the expected difference in utility, we bound all five non-zero terms from below. We set $a_b^1 = P_b^\infty + \epsilon/4$ and $a_b^2 = P_b^\infty$, which implies that there difference is equal to $\epsilon/4$. The expressions in **I**, **II** and **IV** are therefore greater or equal than $-\epsilon/4$. For **III** and **V**, the lower bound $t_b - (P_b^\infty + \epsilon/4) = \frac{3\epsilon}{4}$ holds, because $t_b = P_b^\infty + \epsilon$. Combining these bounds with the probabilities of each event, the following inequality holds:

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/4, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty, a_{\text{-}b})\right] \geq \\ -\frac{\epsilon}{4}\mathbb{P}_{\text{-}b}\left[P_b^\infty \geq a_{\text{-}b}^{(m)}\right] + \frac{3\epsilon}{4}\mathbb{P}_{\text{-}b}\left[P_b^\infty + \epsilon/4 \geq a_{\text{-}b}^{(m)} \geq P_b^\infty\right] = \\ -\frac{\epsilon}{2}\mathbb{P}_{\text{-}b}\left[P_b^\infty \geq a_{\text{-}b}^{(m)}\right] + \frac{3\epsilon}{4}\left(\mathbb{P}_{\text{-}b}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty + \epsilon/4\right] - \mathbb{P}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty\right]\right)\end{gathered} \tag{94}$$

By definition $a_{\text{-}b}^{(m)}$ is the m'th smallest submission in a set of $m-1$ bids and n asks. Since buyer b assumes that those are uniformly distributed and that there are $m(l) = l$ and $n(l) = l$ many buyers and sellers, it follows from order statistics that $a_{\text{-}b}^{(m)} \sim Beta(l, l)$. This distribution is symmetric on $[0, 1]$ for every l and therefore at the critical value $P_b^\infty = \frac{1}{2}$, it holds that $\mathbb{P}_{\text{-}b}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty\right] = \frac{1}{2}$. Furthermore, it follows from Proposition 3 that for any $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty + \epsilon/4\right] \geq 1 - \gamma$. It follows that

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/4, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty, a_{\text{-}b})\right] \geq \\ -\frac{\epsilon}{8} + \frac{3\epsilon}{4}\left(\frac{1}{2} - \gamma\right),\end{gathered} \tag{95}$$

which is positive if γ is chosen to be smaller than $\frac{1}{3}$. The statement for sellers can be proven analogously. □

B.12 Proof of Theorem 11

To be added.

B.13 Proof of Proposition 12

To be added.

B.14 Proof of Corollary 13

To be added.

B.15 Proof of Proposition 14

To be added.

B.16 Proof of Corollary 15

To be added.

B.17 Proof of Proposition 22

Proof. Consider a buyer b with gross value t_b and action a_b. Tie-breaking is a probability zero event. The expected utility is equal to

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] = \int_{\underline{P^*}}^{a_b} (t_b - x - \Phi_b(x))\, f_{P^*}(x)dx. \tag{96}$$

Recall from Proposition 1 that $t_b - t_b^\Phi - \Phi_b(t_b^\Phi) = 0$. By assumption, the map $x \mapsto x + \Phi_b(x)$ is strictly increasing. Therefore, for $x \in [\underline{P^*}, t_b^\Phi)$, the integrand is strictly greater than zero. For $x \in (t_b^\Phi, \overline{P^*}]$, the integrand is strictly negative. Hence, the expected utility is maximized at the unique point $a_b = t_b^\Phi$.[32] The function $a_b \mapsto \mathbb{E}_b[u_b(t_b, a_b, P^*)]$ is continuous, increasing on $[\underline{P^*}_b, t_b^\Phi]$ and decreasing on $[t_b^\Phi, \overline{P^*}_b]$. ϵ-best responses therefore approximate t_b^Φ. As truthfulness is the unique best response a_b, it holds that $E_\Phi = \frac{\mathbb{P}^*_P[b \in \mathcal{B}^*(a_b, P^*)]}{\mathbb{P}^*_P[b \in \mathcal{B}^*(t_b^\Phi, P^*)]} = \frac{\mathbb{P}^*_P[b \in \mathcal{B}^*(t_b^\Phi, P^*)]}{\mathbb{P}^*_P[b \in \mathcal{B}^*(t_b^\Phi, P^*)]} = 1$.

□

[32] Alternatively, this can be proven via the first order condition by differentiating the expected utility using Leibniz's rule and setting the derivative zero.

B.18 Proof of ??

Proof. Consider a buyer b with gross value t_b and action a_b. Suppose that $t_b^\Phi > P_b^*$. Tie-breaking is a probability zero event. The expected utility is equal to

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] = \int_{\underline{P^*}}^{a_b} (t_b - x - \Phi_b(a_b, x))\, f_{P^*}(x) dx. \tag{97}$$

The expected utility is continuous in a_b on $[\underline{P^*}, \overline{P^*}]$ and attains a maximum by the Extreme Value Theorem, which proves the existence of a best response.

First, we show that an action $a_b^1 < P_b^*$ is not a best response. We show that there exists an action $a_b^2 > P_b^*$ such that

$$\mathbb{E}_b\left[u_b(t_b, a_b^2, a_{\text{-}b})\right] - \mathbb{E}_b\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] > 0, \tag{98}$$

which implies that a_b^1 is not a best response. Because the net value is by assumption continuous and strictly increasing in the gross value, there exists a gross value $t_b' < t_b$, such that $t_b^\Phi > t_b^{\Phi\prime} > P_b^\infty$. Denote the difference between t_b^Φ and $t_b^{\Phi\prime}$ by $\delta > 0$. It holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi\prime}, a_{\text{-}b})\right] = \mathbb{E}_{\text{-}b}\left[u_b(t_b', t_b^{\Phi\prime}, a_{\text{-}b})\right] + \delta \geq \delta, \tag{99}$$

because the net value is assumed to be ex-post individually rational.

We therefore consider an action a_b with $a_b - P_b^* \geq \epsilon$ for some $\epsilon > 0$. We will show that if the aggregate uncertainty δ is sufficiently small, then a_b is not a best response, proving that best responses must be ϵ-close to P_b^*. More specifically, we prove that a buyer can increase their expected utility when switching to $P_b^* + \epsilon/2$. For $\delta < \epsilon/2$ it holds that

$$\begin{gathered}
\mathbb{E}_b[u_b(t_b, a_b, P^*)] - \mathbb{E}_b[u_b(t_b, P_b^* + \epsilon/2, P^*)] = \\
\int\limits_{\underline{P^*}}^{a_b} (t_b - x - \Phi_b(a_b, x))\, d\mu_{P^*}(x) - \int\limits_{\underline{P^*}}^{P_b^*+\epsilon/2} (t_b - x - \Phi_b(P_b^\infty + \epsilon/2, x))\, d\mu_{P^*}(x) = \\
\int\limits_{P_b^*+\epsilon/2}^{a_b} (t_b - x)\, d\mu_{P^*}(x) - \left(\int\limits_{\underline{P^*}}^{P_b^*+\epsilon/2} (\Phi_b(a_b, x) - \Phi_b(\epsilon/2, x))\, d\mu_{P^*}(x) + \int\limits_{P_b^*+\epsilon/2}^{a_b} \Phi_b(a_b, x) d\mu_{P^*}(x) \right).
\end{gathered} \tag{100}$$

Note that for any two actions $a_b^1 \geq a_b^2$ there exists a constant $\gamma > 0$, such that for all $P \in [\underline{P^*}, a_b^2]$ it holds that $\Phi_b(a_b^1, P) - \Phi_b(a_b^2, P) \geq \gamma$. That is because the map $a_b \mapsto \Phi_b(a_b, P)$ is strictly increasing on $[\underline{P^*}, a_b]$. Therefore, for fixed actions a_b^1 and a_b^2 the continuous function $P \mapsto \Phi_b(a_b^1, P) - \Phi_b(a_b^2, P)$ is strictly positive on the compact interval $[\underline{P^*}, a_b^2]$ and attains a strictly positive minimum by the Extreme Value theorem. Consider the constant $\gamma > 0$ that corresponds to $a_b^1 = a_b$ and $a_b^2 = P_b^* + \epsilon/2$.

Together with δ-aggregate uncertainty, we get that

$$\int_{\underline{P^*}}^{P_b^*+\epsilon/2} \left(\Phi_b(a_b,x)-\Phi_b(P_b^*+\epsilon/2,x)\right) d\mu_{P^*}(x) \geq (1-\delta)\gamma. \tag{101}$$

Moreover it holds that

$$\int_{P_b^*+\epsilon/2}^{a_b} (t_b-x)\, d\mu_{P^*}(x) \leq \delta t_b \quad \text{and} \quad \int_{P_b^*+\epsilon/2}^{a_b} \Phi_b(a_b,x) d\mu_{P^*}(x) \geq 0. \tag{102}$$

Combining Equations (100) to (102) yields

$$\mathbb{E}_b[u_b(t_b,a_b,P^*)] - \mathbb{E}_b[u_b(t_b,P_b^*+\epsilon/2,P^*)] \leq t_b\delta - (1-\delta)\gamma. \tag{103}$$

If $\delta < \frac{\gamma}{t_b+\gamma}$, then the difference in expected utility is strictly negative, proving that a_b is not a best response. This implies that best responses are ϵ-close to P_b^* if δ is sufficiently small.

□

B.19 Proof of Lemma 20

Proof. Recall that $\tilde{P^*}$ denotes the market price, if a trader is involved in trade, and zero otherwise.

For a buyer b with private type t_b, Equation (12) yields that

$$\begin{gathered}
\mathbb{E}_{\text{-}b}\left[u_b\left(t_b,a_b^1,a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b,a_b^2,a_{\text{-}b}\right)\right] = \\
t_b\left(\mathbb{P}_{\text{-}b}\left[b\in\mathcal{B}^*(a_b^1,a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b\in\mathcal{B}^*(a_b^2,a_{\text{-}b})\right]\right) - \\
\int_{[\underline{a}_{S,b},\overline{a}_{S,b}]^2} \left(\tilde{P^*}\left(a_b^1,a_{\text{-}b}^{(m)},a_{\text{-}b}^{(m+1)}\right) - \tilde{P^*}\left(a_b^2,a_{\text{-}b}^{(m)},a_{\text{-}b}^{(m+1)}\right)\right) d\mu(a_{\text{-}b}^{(m)},a_{\text{-}b}^{(m+1)}) - \\
\left(\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1,a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2,a_{\text{-}b}\right)\right]\right).
\end{gathered} \tag{104}$$

Note that the integral in the difference above is non-negative, because $\tilde{P^*}(a_b,a_{\text{-}b}^{(m)},a_{\text{-}b}^{(m+1)})$ is increasing in a_b for fixed $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$. Equation (16) follows by neglecting the term corresponding to the change in expected market price.

For a seller s with private type t_s, Equation (13) yields

$$\begin{gathered}
\mathbb{E}_{\text{-}s}\left[u_s\left(t_s,a_s^1,a_{\text{-}s}\right)\right] - \mathbb{E}_{\text{-}s}\left[u_s\left(t_s,a_s^2,a_{\text{-}s}\right)\right] = \\
\int_{[\underline{a}_{B,s},\overline{a}_{B,s}]^2} \left(\tilde{M}P\left(a_s^1,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right) - \tilde{P^*}\left(a_s^2,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)\right) d\mu(a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}) - \\
t_s\left(\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^1,a_{\text{-}s})\right] - \mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^2,a_{\text{-}s})\right]\right) - \left(\mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^1,a_{\text{-}s}\right)\right] - \mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^2,a_{\text{-}s}\right)\right]\right).
\end{gathered} \tag{105}$$

$t_s\left(\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^1,a_{\text{-}s})\right] - \mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^2,a_{\text{-}s})\right]\right) \geq 0$ holds, because the trading probability is decreasing for a seller in their ask. To see that the integral in Equation (105) is bounded from above by $2t_s\left(1-\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^2,a_{\text{-}s})\right]\right)$, we split up the integral into all six possible cases for the realizations of and $a_{\text{-}s}^{(m\text{-}1)}$ with respect to $a_s^1 < a_s^2$. which is shown in the following table.[33]

		$\tilde{P^*}\left(a_s^1, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right)$	$\tilde{M}P\left(a_s^2, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right)$
I	$a_{\text{-}s}^{(m)} \geq a_{\text{-}s}^{(m\text{-}1)} \geq a_s^2 \geq a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$
II	$a_{\text{-}s}^{(m)} \geq a_s^2 \geq a_{\text{-}s}^{(m\text{-}1)} \geq a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^2$
III	$a_s^2 \geq a_{\text{-}s}^{(m)} \geq a_{\text{-}s}^{(m\text{-}1)} \geq a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$	0
IV	$a_{\text{-}s}^{(m)} \geq a_s^2 \geq a_s^1 \geq a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^2$
V	$a_s^2 \geq a_{\text{-}s}^{(m)} \geq a_s^1 \geq a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^1$	0
VI	$a_s^2 \geq a_s^1 \geq a_{\text{-}s}^{(m)} \geq a_{\text{-}s}^{(m\text{-}1)}$	0	0

For **I**, **II**, **IV** and **VI**, the difference between $\tilde{P^*}\left(a_s^1, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right)$ and $\tilde{P^*}\left(a_s^2, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right)$ is less or equal than 0. It follows that

$$\begin{aligned}\int_{[\underline{a}_{B,s}, \overline{a}_{B,s}]^2} \left(\tilde{P^*}\left(a_s^1, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right) - \tilde{P^*}\left(a_s^2, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right)\right) d\mu(a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}) \leq \\ \int_{\mathbf{III}} (ka_{\text{-}s}^{(m)} + (1-k)a_{\text{-}s}^{(m\text{-}1)}) d\mu_s^*(a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}) \\ + \int_{\mathbf{V}} (ka_{\text{-}s}^{(m)} + (1-k)a_s^1) d\mu(a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}).\end{aligned} \tag{106}$$

Because both integrands in Equation (106) are less or equal than $\overline{a}_{S,s}$, it follows that

$$\begin{aligned}\int_{[a_s, \underline{s}]^2} \left(\tilde{P^*}\left(a_s^1, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right) - \tilde{P^*}\left(a_s^2, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right)\right) d\mu(a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}) \\ \leq \overline{a}_{S,s}\mathbb{P}[\mathbf{III}] + \overline{a}_{S,s}\mathbb{P}[\mathbf{V}] \\ \leq 2\overline{a}_{S,s}\mathbb{P}[a_{\text{-}s}^{(m)} < a_s^2] = 2\overline{a}_{S,s}\left(1 - \mathbb{P}_{\text{-}s}\left[(s, a_s^2 \in \mathcal{S}^*\right]\right),\end{aligned} \tag{107}$$

which finishes the proof. □

B.20 Proof of Proposition 21

Proof. Consider a buyer b with gross value t_b, such that $t_b^\Phi < P_b^\infty$. A best response a_b with $a_b \leq t_b^\Phi$ must exist. That is because if there is a best response a_b with $a_b > t_b^\Phi$, the expected utilities must be equal, as the net value dominates all larger actions, proving that t_b^Φ is a best response as well. By the monotonicity of the trading probability, it then holds that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^\Phi, a_{\text{-}b})\right]. \tag{108}$$

For all $\gamma > 0$, it holds by Proposition 3 that in sufficiently large markets $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^\Phi, a_{\text{-}b})\right] \leq \gamma$. The expected utility is upper bounded by neglecting the payment of market price and fee, that is

[33] Different to $\tilde{P^*}_b(a_b, y, z)$ it holds that $\tilde{P^*}_s(a_s, y, z)$ is not increasing in a_s for fixed y and z.

the gross value times the probability of trade:

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \leq t_b\gamma. \tag{109}$$

Choose $\gamma \leq \frac{\epsilon}{t_b}$. This implies that $ISLM$, the expected utility of a best response is upper bounded by ϵ. The expected utility of truthfulness is non-negative by assumption. This implies that truthfulness is an ϵ-best response. The statement for sellers can be proven analogously. $\square$