Entrepreneurial Finance, Home Equity, and Monetary Policy

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Abstract

We model entrepreneurial finance using a combination of fiat money, traditional bank loans, and home equity loans. The banking sector is over-the-counter, where bargaining determines the pass-through from the nominal interest rate to the bank lending rate, characterizing the transmission channel of monetary policy. The results show that the strength of this channel depends on the combination of nominal and real assets used to finance investments, and thus declines in the extent to which housing is accepted as collateral. A calibration to the U.S. economy supports the theoretical results and provides novel insights on entrepreneurial finance between 2000 and 2016.

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1 Introduction

It is well documented that apart from housing services, private real estate is commonly used as collateral to secure home equity loans (HELs) when credit markets are imperfect.\(^1\) While a fair share of these loans are used to finance consumption (Greenspan and Kennedy, 2007), they further find high demand among entrepreneurs financing capital expenditures. According to the Survey of Consumer Finances, between 2001 and 2016, 12% of entrepreneurs financed expenditures through HELs, accounting for a 52% higher average use among self-employed relative to households working for someone else. The corresponding median amount borrowed exceeds the latter group by 74%, unveiling a remarkable feature of the U.S. economy.\(^2\) Namely, the synchronization of the House Price Index (HPI) and the entrepreneurial sector, as illustrated in Figures 1a and 1b. An immediate insight is the strong correlation between the HPI, the number of firms with 1-4 employees, and their capital expenditures, triggering the following question: Do HELs account for these strong correlations throughout the whole time-period between 2000 and 2016?

![Figure 1: House Prices and Entrepreneurial Activity, 2000-2014](image)

Figures 2a and 2b shed light. Between 2000 and 2006, the total amount of HELs/CPI more than doubled.\(^3\) With the crash of the housing market in 2007, house prices plummeted and entrepreneurs faced tighter borrowing constraints, triggering a steep decline in HELs.\(^4\) However, despite a recovery of the HPI in 2012, HELs continued to fall, suggesting a decrease in the acceptability of housing as collateral and thus a paradigm shift in entrepreneurial finance. Consequentially, entrepreneurs had to rely on different financing channels such as savings

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\(^1\) According to Iacoviello (2011), home equity accounts for nearly 50% of a household’s net wealth and is amongst the most valuable asset an average household holds.

\(^2\) Appendix G contains details on data sources and construction.

\(^3\) Reinhart and Rogoff (2009) describe this phenomenon as households turning their homes into “ATM machines”. While Duca et al. (2011) associate the rise of such loans to financial innovations, Dugan (2008) attributes the increased supply to relaxed underwriting standards.

\(^4\) Schweitzer and Shane (2010) estimated that, in 2010, lending to small businesses was 24.5 billion US$ below where it would have been had the housing market not crashed.
and traditional bank loans. The low interest rate environment since 2008 supports this hypothesis. Two questions arise: (i) What accounts for the recovery in capital expenditures after 2008? (ii) What role did monetary policy play and what is the transmission mechanism?

Figure 2: Home Equity Loans and Interest Rates, 2000-2016

To answer these questions, we construct a monetary search model in which entrepreneurs finance idiosyncratic investment opportunities using a combination of internal (retained earnings) and external (bank loans) financing, drawing from Rocheteau et al. (2018). Since we are interested in the role of home equity, we introduce a housing market, where private real estate has two roles: consumption and saving. While the former captures the fact that entrepreneurs draw utility from living in their houses, the latter underpins its role as a medium of exchange, i.e., its ability to facilitate trade when credit markets are imperfect. Given the inherent frictions of the model including a lack of commitment and record-keeping, fiat money is essentially beneficial and bank loans need to be collateralized using private real estate (HELs) and claims on future output (traditional bank loans). The banking sector is over-the-counter and the terms of trade are determined by bilateral bargaining between banks and entrepreneurs. In doing so, we rely on the observations of Mora (2014), relating the observed dispersion in loan rates to differences in banks’ bargaining power.

Contrary to many models in the literature involving the coexistence of money and assets, fiat money has two roles in our environment. First, it serves as insurance against tightened borrowing constraints in the banking sector, and second, it is used as a strategic device, allowing entrepreneurs to obtain more favorable terms of trade when bargaining with a bank. This multiplier makes money and real estate imperfect substitutes, directly affecting the pass-through of the nominal interest rate to the bank lending rate.

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5 The former is in line with Bates et al. (2009), Sánchez and Yurdagul (2013), and Campello (2015) on corporate liquidity management, where fiat money serves as an insurance against the risk of idiosyncratic financing opportunities.
The established model provides novel insights on the transmission of monetary policy. The results show that an increase in the nominal interest rate decreases aggregate entrepreneurial investment. The size of the effect, however, depends on the composition of internal and external financing, i.e. the combination of fiat money, traditional bank loans, and HELs used. If housing is sufficiently pledgeable, the demand for fiat money is low, weakening the transmission channel of monetary policy. If, however, housing is barely accepted as collateral and capital is primarily financed through fiat money and traditional bank loans, the transmission channel is strong.

To study the implications of the theoretical results on the period of interest, 2000-2016, we calibrate the model to the U.S. economy. In doing so, we focus on the semi-elasticity of aggregate entrepreneurial investment in response to a change in the nominal interest rate, capturing the strength of the transmission of monetary policy. Varying the pledgeability of housing allows to account for changes in the composition of internal and external finance. The calibrated results confirm the aforementioned theoretical outcomes. For example, at a nominal interest rate of 0.05, the semi-elasticity of investment varies from $-6.8$ to 0 as the pledgeability of housing goes from 0 to 1, capturing the sensitivity of the pass-through with regards to the composition of financing. More intuitively, the larger the share of investments financed by fiat money, the stronger the effects of a change in monetary policy.

Being aware of our results and the observations in Figures 1-2, we conclude that part of the recovery of entrepreneurs’ capital expenditures stems from the low nominal interest rate environment implemented in the years after the financial crisis, suggesting a shift from HELs to traditional bank loans and fiat money. This is in line with Berentsen et al. (2012), who were among the first to study the effect of monetary policy on innovation and growth, showing that in periods of high inflation, the innovation sector relies more on external finance.

1.1 Related Literature

This paper is deeply founded in the literature on monetary search-theory and markets with frictions, as surveyed in Rocheteau and Nosal (2017) and Lagos et al. (2017). The first to apply the theoretical toolkit of this literature to study corporate finance and monetary policy, and the paper most closely related to ours, were Rocheteau et al. (2018). We build on this framework by introducing a housing market, where private real estate has two roles: consumption and saving. The latter allows entrepreneurs the use of home-equity to secure bank loans, capturing the relationship between the housing market and entrepreneurial in-
Since the Great Recession, a vast literature on the use of home equity to secure bilateral credit transactions emerged, whereas distinction is made between consumption and investment loans. The former are studied by He et al. (2015), incorporating HELs into a Lagos and Wright (2005) environment. The endogenously arising liquidity premium on housing generates dynamics in the value of real estate, explaining parts of the housing boom experienced in the early 2000s. Using similar toolkits, Branch et al. (2016) provide an application to unemployment by endogenizing the construction sector. Their results show that financial innovations that raise the acceptability of homes as collateral increase house prices and reduce unemployment. Among the first to abstain from consumption loans were Liu et al. (2013), introducing land as collateral in firms’ credit constraints using a DSGE model, showing how co-movements of land prices and business investments propagate macroeconomic fluctuations. While the provided results apply for firms of all sizes, a more detailed application to entrepreneurs is provided by Decker (2015). Within a heterogeneous agent DSGE model with housing and entrepreneurship, his results show that while recessions accompanied by a housing crash can explain the decline in entrepreneurial activity experienced in the early 2000s, a broader financial crisis would have no such effects. To further explain the recent synchronization of house prices and entrepreneurial activity, Lim (2018) develops an occupational choice model incorporating home equity loans. His results are in line with Decker (2015) and the paper at hand, showing that a rise in house prices increases entrepreneurial investment. However, a role for monetary policy remains absent. Our paper combines these components, allowing for an analysis of entrepreneurial finance, home equity, and monetary policy.

We also draw on empirical work studying the importance of housing as collateral in entrepreneurial finance. Schmalz et al. (2017) found that higher values of collateral increased the likelihood of becoming an entrepreneur and that those with higher values of collateral took on more debt, started larger firms, and were more likely to remain in the long run. Corradin and Popov (2015) in turn focus on real estate prices in the U.S. and found that a 10% increase in home equity increased the probability of becoming an entrepreneur by 7%. Adelino et al. (2015) estimated that the collateral lending channel could explain 15-25% of employment variation in the U.S. between 2002-2012, showing the value of housing as collateral is directly tied to the formation of new businesses. Black et al. (1996) used data from the UK and found that a 10% increase in the value of home equity increased VAT reg-

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6 Another extension to study how heterogeneity of financial frictions and monopolistic competition influences this the transmission channel was provided by Silva (2017).

7 Complementary results are provided by Justiniano et al. (2015) and Garriga et al. (2018), studying the relationship between the liquidity of real estate and house prices in the United States.
istrations by nearly 5%, suggesting that an increase in the value of housing led to more small business formation. Harding and Rosenthal (2017) estimated that a 20% increase in real home value over two years increased entry into self-employment by 15 percentage points and that self-employed homeowners are more likely to use home equity lines of credit. Abstaining from the value of housing but focusing on its pledgeability, Jensen et al. (2014) found that a reform in Denmark which increased the availability of home equity loans increased entry into entrepreneurship.

The rest of this paper is organized as follows. Section 2 introduces the environment of the established model, derives the value functions in each market, determines the equilibrium of the bargaining game between banks and entrepreneurs, and defines optimal portfolio choices. Monetary policy is discussed in Section 3, followed by a calibration in Section 4. Last but not least, Section 5 concludes.

2 Model

2.1 Environment

Time is discrete, starts at $t = 0$, and continues forever. There are three fixed types of infinitely lived agents, $j \in \{e, s, b\}$: entrepreneurs, capital suppliers, and banks, each of them in a continuum $[0, 1]$. Banks and capital suppliers are risk neutral. The discounted lifetime utility of an entrepreneur is:

$$E \sum_{t=0}^{\infty} \beta^t [c_t + \vartheta(a_t)],$$

where $\beta = (1 + r)^{-1}$ is the discount factor, $r > 0$ the rate of time preference, $c_t \in \mathbb{R}_+$ the consumption of the numéraire good, and $\vartheta(a_t)$ the utility of consuming housing, $a_t \in \mathbb{R}_+$. All entrepreneurs have linear utility over the numéraire good, $c_t$, while $\vartheta(a_t)$ is continuously differentiable, increasing and concave, $\vartheta'(a_t) > 0 > \vartheta''(a_t)$, with $\vartheta(0) = 0$, $\vartheta'(0) = \infty$, and $\vartheta'(\infty) = 0$. Each period is divided into two subperiods, as displayed in Figure 3, whereas the timing of events, starting with the investment market in the first subperiod, is discussed below.

At the beginning of the period, entrepreneurs face idiosyncratic uncertainty in two-dimensions: investment and financing. With probability $\lambda \in [0, 1]$, an entrepreneur encounters an investment opportunity, allowing him to transform $k_t \in \mathbb{R}_+$ units of capital into $f(k_t)$ units of the numéraire good in the subsequent settlement market, whereas the capital function, $f(k_t)$,
is continuously differentiable, increasing and concave, $f'(k_t) > 0 > f''(k_t)$, with $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. Capital fully depreciates after one period, and hence there are no gains from storage.

After realization of that shock, entrepreneurs enter the investment market, where two markets open simultaneously: a competitive capital market and an over-the-counter banking sector. In the former, entrepreneurs trade with capital suppliers to acquire capital, while in the latter, bilateral bank loans can be obtained. To overcome frictions including the entrepreneurs inability to commit to future actions and the fact that transactions are not publicly recorded, a medium of exchange is indispensable for trade in the capital market, as shown by Rocheteau and Wright (2005). Thus, in order to accumulate capital, entrepreneurs rely on a combination of retained earnings (fiat money) and intra-period bank loans, where the latter consist of perfectly divisible and recognizable one-period liabilities (IOUs) issued by banks, i.e. banks can commit. In order to obtain an IOU, entrepreneurs meet with banks in the over-the-counter market, where $\alpha \in [0, 1]$ is the probability that an entrepreneur is eligible for a bank loan. Both the probability of having an investment opportunity, $\lambda$, and the probability of receiving a bank loan, $\alpha$, are independent from each other and independent across agents. Given the lack of record-keeping and the entrepreneur’s inability to commit to future actions, bilateral loans need to be collateralized to be incentive compatible. We consider private real estate (HELs) and claims on future output (traditional bank loans). Settlement takes place in the subsequent settlement market (intra-period), where given the banks’ ability to commit to future actions, redemption of collateral is guaranteed. In case of default on the part of the entrepreneur, the bank keeps the collateral.

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8 IOUs are perfectly recognizable within a period, but can be counterfeited thereafter, which precludes them from circulating across periods.

9 We assume that housing and future output are pledgeable only to banks, as banks are the only agents who have the technology to verify homeownership and recover investments. Thus, there is no trade credit between capital suppliers and entrepreneurs.
In the centralized settlement market, entrepreneurs consume the produced numéraire good, $c_t$, housing services, $\vartheta(a_t)$, settle outstanding credit obligations, $l_t \in \mathbb{R}_+$, and adjust their portfolio consisting of $m_t \in \mathbb{R}_+$ units of perfectly divisible fiat money, and $a_t$ units of housing. There is a central bank managing the supply of fiat money in the economy according to $M_{t+1} = (1 + \tau)M_t$, where $M_t \in \mathbb{R}_+$ denotes the stock at the beginning of period $t$ and $T = \tau M_t$ are lump-sum transfers conducted by the central bank. One unit of money can buy $q_{m,t}$ units of the numéraire good in the settlement market. Since we focus on symmetric and stationary monetary equilibria, where all agents follow identical strategies and real balances are constant over time, i.e., $q_{m,t+1}M_{t+1} = q_{m,t}M_t$, it holds that $M_{t+1}/M_t = q_{m,t}/q_{m,t+1} = \gamma$ with $\gamma$ being the exogenous gross growth rate of the fiat money supply. The aggregate supply of housing, $A \in \mathbb{R}_+$, is fixed with $q_{a,t}$ representing the centralized market price per unit of housing in terms of the numéraire good at time $t$. Housing is durable, infinitely lived, and each unit of housing generates one unit of housing services, $\vartheta(a_t)$, each period, analogue to a Lucas tree. Going forward, we solve the model by backward induction, starting with the settlement market.

2.2 Settlement Market

The respective problem of type $j \in \{e, s, b\}$ entering the settlement market with fiat money, $m$, real estate, $a$, capital, $k$, and outstanding intra-period liabilities, $l$, is:

$$W^e(m, a, k, l) = f(k) + \vartheta(a) + q_m m + q_a a + T - l + \max_{m', a'}\{-q_m m' - q_a a' + \beta V^e(m', a')\},$$

(2)

$$W^s(m, a, k, 0) = q_m m + q_a a + q_k k + T + \max_{m', a'}\{-q_m m' - q_a a' + \beta V^s(m', a')\},$$

(3)

$$W^b(m, a, 0, l) = q_m m + q_a a + T + l + \max_{m', a'}\{-q_m m' - q_a a' + \beta V^b(m', a')\},$$

(4)

where variables evaluated in the following period are denoted with a ‘prime’ and $\beta V^j(m', a')$ is the continuation value. The capital supplier’s proceeds from the capital sales in the investment market are denoted by $q_k k$. According to (2)-(4), agents choose how many numéraire goods to consume and how much money, $m'$, and real estate, $a'$, to carry into the subsequent investment market, in order to maximize their discounted lifetime utility. The corresponding first-order and envelope conditions are:

$$\beta V^j_{m'} = q_m; \quad \beta V^j_{a'} = q_a; \quad W^j_{m} = q_m; \quad W^c_{a} = q_a + \vartheta'(a); \quad W^s_{a} = q_a;$$

$$W^e_l = -1; \quad W^b_l = 1; \quad W^s_k = f'(k); \quad W^e_k = q_k.$$

(5)
Since $W^j(\cdot)$ is linear in current wealth, an agent’s choice of money and housing for the subsequent investment market, $m'$ and $a'$, is independent of the current balances, $m$ and $a$. Furthermore, each type $j \in \{e, s, b\}$ leaves the settlement market with the same amount of money and housing, $m^j$ and $a^j$. Since banks and capital suppliers have no need for fiat money or real estate, $(m^s, a^s) = (m^b, a^b) = (0, 0)$ if money or housing is costly to carry along the period. The optimal portfolio choice of an entrepreneur, $(m^e, a^e)$, in turn, is determined in Section 2.4.

**Definition A.** *An equilibrium in the centralized settlement market is a pair of strategies, $(m', a')$, such that the terms of trade, $(m', a')$, are a solution to the agent $j$’s problems (2), (3), and (4) with $j \in \{e, s, b\}$.*

### 2.3 Investment Market

Starting with the competitive capital market, the capital supplier’s value function, entering the investment market with $m$ units of money and $a$ units of real estate, is:

$$V^s(m, a) = \max_k \{-k + W^s(m, a, k, 0)\}.$$  

(6)

Plugging in (3), it immediately follows that $q_k = 1$ and $W^s = V^s$. Moving on to the bargaining game in the over-the-counter banking sector, the respective value functions of an entrepreneur and a bank entering the investment market with $m$ units of fiat money and $a$ units of real estate are:

$$V^e(m, a) = (1 - \lambda)W^e(m, a, 0, 0) + \lambda[(1 - \alpha)W^e(m, a, k_I, 0) + \alpha W^e(m, a, k_E, l)],$$  

(7)

$$V^b(m, a) = (1 - \lambda)W^b(m, a, 0, 0) + \lambda[(1 - \alpha)W^b(m, a, 0, 0) + \alpha W^b(m, a, 0, l)].$$  

(8)

If an investment opportunity is encountered, entrepreneurs have two options to finance their capital accumulation: immediate settlement with fiat money (internal finance), and secured over-the-counter bank loans (external finance). If no bank loan is available, which occurs with probability $1 - \alpha$, fiat money is the only means of payment, where $k = k_I$ is the amount of capital purchased directly from the supplier, and $l = 0$. With probability $\alpha$, however, the entrepreneur receives a loan from the bank and enters the second stage with capital, $k = k_E$, and liabilities, $l > 0$. The subscripts $I$ and $E$ stand for Internal and External finance, respectively.

The terms of the loan contract, $(k_E, \phi, d, y)$, are determined via proportional bargaining,
where $\phi$ is the bank’s fee, $y \in [0,a]$ the amount of real estate used as collateral, and $d \in [0,m]$ the monetary downpayment. Let $S = S^e + S^b$ be the total surplus to be bargained over:

\[ S^e \equiv W^e(m, a, k_E, l) - W^e(m, a, k_I, 0) = f(k_E) - k_E - \phi - \Delta^e_m(m), \] (9)

\[ S^b \equiv W^b(m, a, 0, l) - W^b(m, a, 0, 0) = \phi, \] (10)

and $\Delta^e_m(m)$ is defined as:

\[ \Delta^e_m(m) = f(k_I) - k_I, \quad \text{with} \quad k_I = \min\{q_m, m^*\}, \] (11)

where $m^*$ solves $f'(m^*) = 1$. Equation (11) represents the entrepreneur’s outside option, i.e., the profits he would earn if he purchased capital directly from the supplier using the real money balances, $q_m m$, carried along the period. It follows that the terms of the loan contract solve:

\[ (k_E, \phi, d, y) \in \arg \max_{k_E, \phi, d, y} f(k_E) - k_E - \phi - \Delta^e_m(m), \] (12)

s.t. $\theta[f(k_E) - k_E - \phi - \Delta^e_m(m)] \geq (1 - \theta)\phi$, \hspace{1cm} (13)

s.t. $l \equiv k_E - q_m d + \phi \leq \chi f(k_E) + \rho q_a y$, \hspace{1cm} (14)

and $d \in [0, m]$ and $y \in [0, a]$, where $\theta \in [0, 1]$ represents the bank’s bargaining power and (13) governs how the total surplus is split proportionally between the bank and the entrepreneur. Equation (14) is the entrepreneur’s liquidity constraint and determines the size of the future credit obligation, $l$, where $\rho q_a y$ is the collateral value of housing and $\chi f(k_E)$ the collateral value of future output. Debt limits are imposed exogenously following Kiyotaki and Moore (1997) with $\chi$ ($\rho$) being the fraction of future output (housing) that is pledgeable.\footnote{One can interpret $\rho$ as a loan-to-value ratio representing various transaction costs and information asymmetries regarding the resale value of a house, and $\chi$ as the parts of the capital input a bank can recover in case the entrepreneur defaults on his credit obligation (scrap value).} Next to future output and housing, the size of the loan, and thus the credit obligation, $l$, depends on the amount of fiat money (internal finance) an entrepreneur uses as a downpayment, $d \in [0, m]$.

**Definition B.** An equilibrium of the bargaining game in the over-the-counter banking sector is a pair of strategies, $(k_E, \phi, d, y)$, such that the terms of trade, $(k_E, \phi, d, y)$, are a solution to the bargaining problem (12).

If the entrepreneur’s liquidity constraint, (14), does not bind, the entrepreneur may achieve
the socially-efficient level of investment, \( k_E = k^* \). Using (13) and solving for \( \phi \) gives:

\[
\phi = \theta[f(k^*) - k^* - \Delta_m^e(m)],
\]

(15)
denoting the bank’s fraction of the total match surplus. Comparative statics show that \( \partial \phi / \partial \Delta_m^e(m) < 0 \), so the fee collected by the bank is decreasing in the value of the entrepreneur’s outside option. Thus, apart from being an insurance against not receiving a bank loan, fiat money incorporates a strategic role in the bargaining game, reducing the bank’s surplus. In the limiting case with \( \chi = 0 \) and \( \rho = 0 \), \( k_E = k_I = k^* \) if \( q_m m \geq k^* \). If, however, \( q_m m < k^* \), then \( k_I < k_E \leq k^* \), where \( k_E \leq k^* \) holds with equality if either output or housing is sufficiently pledgeable. Plugging (15) into (14), we characterize a set of pairs, \((\hat{\chi}, \hat{\rho})\), such that \( k_E = k^* \), with the threshold values:

\[
\hat{\chi} = \frac{\theta[f(k^*) - f(k_I)] + (1 - \theta)(k^* - k_I) - \hat{\rho}q_ay}{f(k^*)},
\]

(16)

\[
\hat{\rho} = \frac{\theta[f(k^*) - f(k_I)] + (1 - \theta)(k^* - k_I) - \hat{\chi}f(k^*)}{q_ay}.
\]

(17)

From (16), \( \hat{\chi} \) is decreasing in \( \hat{\rho} \) and \( \hat{\chi} \to 0 \) as \( \rho \to \rho^* \), where \( \rho^* \) allows entrepreneurs to accumulate \( k^* \) when \( \chi = 0 \) (analogue for \( \chi^* \)). The same, but vice versa, holds for \( \hat{\rho} \) in (17), where the corresponding set is defined as:

\[
A(q_m m) = \left\{ (\hat{\chi}, \hat{\rho}) \in \mathbb{R}^2_+ : \hat{\chi}f(k^*) + \hat{\rho}q_ay \geq \theta[f(k^*) - f(k_I)] + (1 - \theta)(k^* - k_I) \right\},
\]

(18)

with \( \hat{\chi} \leq 1 \) and \( \hat{\rho} \leq 1 \). From (18), with an increase in \( q_m m \), \( A(q_m m) \) converges towards the origin, since there are more combinations of \( \chi \) and \( \rho \) that allow for \( k_E = k^* \). Figure 4 illustrates.

Consider now the case where the entrepreneur’s liquidity constraint, (14), is binding. Solving (13) for \( \phi \) and substituting into (14) with \( d = m \) and \( y = a \) gives:

\[
(1 - \theta)k_E + \theta[f(k_E) - \Delta_m^e(m)] = \chi f(k_E) + \rho q_ay + q_m m,
\]

(19)

which determines \( k_E \). The corresponding comparative statics show that \( \partial k_E / \partial \theta < 0 \), \( \partial k_E / \partial k_I > 0 \), \( \partial k_E / \partial q_ay > 0 \), \( \partial k_E / \partial \rho > 0 \), and \( \partial k_E / \partial \chi > 0 \). Lemma A summarizes.

**Lemma A.** There exists a unique solution to (12) with \( q_m d \in \min\{q_m m, k^*\} \). If \( \chi < \chi^* \) and \( \rho < \rho^* \), there exists an \( m^* \) such that \( k^* > q_m m^* \) and the following is true: If \( m \geq m^* \), the
solution to (12) is:

\[ k_E = k^*, \quad \phi = \theta[f(k^*) - k^* - \Delta^e_m(m)]. \]  

If \( m < m^* \), however, then \((\phi, k_E)\) solves:

\[ \phi = \theta[f(k_E) - k_E - \Delta^e_m(m)], \]

\[ \theta[f(k_E) - k_E - \Delta^e_m(m)] = \chi f(k_E) + \rho q_m - k_E + k_I, \]

and \( k_E \geq k_E\), where \( \chi f'(k_E) = 1 \). Proof in Appendix A.

It is important to note that in equilibrium \( \partial[k_I + \chi f(k_E) + \rho q_m] / \partial k_I > 1 \), and thus by carrying another unit of fiat money along the period, entrepreneurs can increase their accumulated capital by more than one unit. The intuition behind this result is straightforward. An additional unit of fiat money does not only buy the entrepreneur more capital from the supplier, but also signalizes a higher investment to the bank, enabling the entrepreneur to credibly pledge more future output. Lemma B revisits this result and determines its implications for the bank lending rate.

**Lemma B.** The bank lending rate, \( r^b \), defined as the ratio of the fee, \( \phi \), to the loan size,
Figure 5: Bank Lending Rate

\(k_E - k_I\), is given by:

\[
{r^b} = \frac{\phi}{k_E - k_I} = \begin{cases} 
\frac{\theta[f(k^*) - k^* - \Delta^e_m(m)]}{k^* - k_I} & \text{if } m \geq m^*, \\
\frac{\theta[f(k_E) - k_E - \Delta^e_m(m)]}{k_E - k_I} & \text{if } m < m^*, 
\end{cases}
\]

(24)

where, given \(q_m, \chi\) and \(\rho\), \(m^*\) is the minimal amount of fiat money the entrepreneur needs to attain \(k^*\) through bank credit. Proof in Appendix B.

From Lemma B and Figure 5, the bank lending rate is decreasing with \(k_I\), as the entrepreneur faces a more valuable outside option, \(\Delta^e_m(m)\). Hence, the more real money balances an entrepreneur is able to bring into the investment market, i.e., the more capital is financed internally, the lower the real lending rate, \(\partial r^b/\partial k_I < 0\). This pass-through is revisited in more detail in Sections 3 and 4.

### 2.4 General Equilibrium

To solve for the entrepreneurs’ optimal portfolio choice in the settlement market, the following market clearing conditions hold.\(^{11}\)

\[
\int_0^1 a(j) dj = A, \quad \text{and} \quad \int_0^1 m(j) dj = M.
\]

(25)

\(^{11}\)We focus on entrepreneurs, since capital suppliers and banks have no use for fiat money and housing, and thus their portfolio choice in the settlement market can be neglected.
Using the linearity of $W(\cdot)$ to replace $W(\cdot)$ in (7), plugging the resulting equation into (2), and updating yields an entrepreneur’s portfolio choice of money and housing:

$$
\max_{m',a'} \left\{ -\left[ \frac{q_m}{\beta} - q_m' \right] m' - \left[ \frac{q_a}{\beta} - q_a' \right] a' + \vartheta(a') + \lambda \left[ a \Delta_c^e(m', a') + (1 - \alpha) \Delta_m^e(m') \right] \right\}, \quad (26)
$$

where

$$
\Delta_c^e(m, a) = \begin{cases} 
(1 - \theta) \left[ f(k^*) - k^* \right] + \theta \Delta_m^e(m) & \text{if } m \geq m^*, \\
(1 - \chi) f(k_E) - \rho q a - k_I & \text{if } m < m^*, 
\end{cases} \quad (27)
$$

and the terms of trade, $[k_E(m, a), \phi(m, a), d(m, a), y(m, a)]$, are a function of the entrepreneur’s aggregate money and real estate holdings, $m$ and $a$. The first term, $-\left[ \frac{q_m}{\beta} - q_m' \right] m'$, represents the opportunity costs of carrying fiat money across the period, while $-\left( \frac{q_a}{\beta} - q_a' \right) a'$ is the cost of holding real estate.

**Definition C.** An equilibrium in the settlement market is a list of portfolios, terms of trade in the investment market, and aggregate balances, $\{[m(\cdot), a(\cdot)], [k_E(\cdot), \phi(\cdot), d(\cdot), y(\cdot)], M, A\}$ such that:

(i) $[m(\cdot), a(\cdot)]$ is a solution to (26);

(ii) $[k_E(\cdot), \phi(\cdot), d(\cdot), y(\cdot)]$ is a solution to (12);

(iii) $M_{t+1} = (1 + \tau) M_t$ is the law of motion of the fiat money stock;

(iv) $A \in \mathbb{R}_+$ is the total supply of housing in the economy; and

(v) Market clearing conditions, (25), hold.

**Lemma C.** There exists a unique solution to (26) with the equilibrium settlement market prices of fiat money and housing, $q_m$ and $q_a$, corresponding to:

$$
q_m = \beta q_m'[1 + \mathcal{L}_m], \quad (28)
$$

$$
q_a = \beta q_a'[1 + \mathcal{L}_a] + \vartheta'(a)], \quad (29)
$$

with $(\mathcal{L}_m, \mathcal{L}_a) = (0, 0)$ for $k_I = k^*$, and:

$$
\mathcal{L}_m = \begin{cases} 
\lambda [1 - \alpha(1 - \theta)][f'(k_I) - 1] & \text{for } m \geq m^*, \\
\lambda \alpha \left[ \frac{(1 - \chi)f'(k_E)[1 + \theta f'(k_I) - 1]}{1 - \theta - (\chi - \theta)f'(k_E)} - 1 \right] + \lambda (1 - \alpha)[f'(k_I) - 1] & \text{for } m < m^*,
\end{cases} \quad (30)
$$

$$
\mathcal{L}_a = \begin{cases} 
0 & \text{for } m \geq m^*, \\
\lambda \alpha \rho \left[ \frac{(1 - \chi)f'(k_E)}{1 - \theta - (\chi - \theta)f'(k_E)} - 1 \right] & \text{for } m < m^*,
\end{cases}
$$
where \( L_m \) and \( L_a \) correspond to the liquidity premia of money and housing, respectively. Proof in Appendix C.

Consider the three cases from the bargaining game in Section 2.3: \( k_I \geq k^* \), \( m \geq m^* \), and \( m < m^* \). If money is costless to hold, i.e. \( L_m = 0 \), an entrepreneur will accumulate enough fiat money to purchase \( k_I = k^* \) from the supplier in the capital market. As a consequence, housing will be priced at its fundamental value with \( L_a = 0 \). Once we deviate from \( k_I \geq k^* \), however, money becomes costly to hold and entrepreneurs are not able to purchase the socially-efficient amount of capital with only fiat money anymore. As a result, they rely on bank loans. If \( m \geq m^* \), the entrepreneur’s liquidity constraint, (14), is non-binding and an entrepreneur accumulates the socially-efficient amount of capital, \( k^* \). Nonetheless, money incorporates a positive liquidity premium, \( L_m > 0 \), since an additional unit would increase the entrepreneur’s outside option, \( \Delta_e^m(m) \), allowing him to circumvent the loss in surplus from bargaining with the bank. Housing still trades at the fundamental value, i.e., \( L_a = 0 \), since the entrepreneur is not constrained in the amount of collateral carried into the investment market. If \( m < m^* \), however, the entrepreneur’s liquidity constraint, (14), binds and \( k_E < k^* \). As a consequence, the demand for real estate increases and housing incurs an endogenous liquidity premium, \( L_a > 0 \). The costlier fiat money becomes, the larger the premium.

3 Monetary Policy and the Transmission Mechanism

Having determined the bargaining game and the general equilibrium results of the model, this section characterizes optimal monetary policy. In doing so, we start by characterizing the nominal interest rate, followed by an analysis of the pass-through to understand how changes in monetary policy affect the terms of trade in the banking sector. Last but not least, we then analyze the transmission of monetary policy to aggregate investment and lending.

**Proposition A.** *(Nominal Interest Rate)* Define the nominal interest rate as \( i = \gamma/\beta - 1 \) and \( i^* \), where \( i^* \) corresponds to \( m = m^* \). If \( i = 0 \) (the Friedman rule), then \( k_I = k_E = k^* \). If \( 0 < i \leq i^* \), then \( k_I < k_E = k^* \). If \( i > i^* \), then \( k_E < k^* \). Comparative statics involve \( \partial i^*/\partial \rho > 0 \) and \( \partial i^*/\partial \chi > 0 \). Proof in Appendix D.

Proposition A characterizes optimal monetary policy using the general equilibrium results in
In order to study the pass-through of the nominal interest rate to the bank lending rate, we rely on first-order Taylor approximations. Distinction is made between an unconstrained and a constrained equilibrium. In an unconstrained equilibrium, we use a first-order approximation of the equilibrium for \( i \) close to 0, and hence \( k_I \) close to \( k^* \). We take this approach for two reasons. The first is that for \( i \approx 0 \), \( k_I < k^* \), and thus bank credit is essential. The second is that it allows us to derive closed form expressions that are useful for illustrating the mechanisms in the model. To analyze a constrained equilibrium and maintain analytical tractability, we set \( \theta = 0 \) and take a first-order approximation of an equilibrium where \( i \approx i^* \) and thus \( k_E \approx k^* \). While setting the bank’s bargaining power to zero implies that \( r^b = 0 \), we are still able to derive closed form approximations for \( k_I \) and \( k_E \). With this in mind, Proposition B summarizes the pass through from the nominal interest rate, \( i \), to the bank lending rate, \( r^b \).

**Proposition B. (Pass-Through)** For \( i \approx 0 \), the pass-through of the nominal interest rate to the bank lending rate is approximated by:

\[
r^b \approx \frac{\theta i}{2\lambda[1 - \alpha(1 - \theta)]}.
\]

(31)

For \( i \approx i^* \), however, \( r^b = 0 \) since \( \theta = 0 \). Comparative statics involve \( \partial r^b / \partial \lambda < 0 \), \( \partial r^b / \partial \theta > 0 \), and \( \partial r^b / \partial \alpha > 0 \). Proof in Appendix E.

---

A more general analysis with \( \theta > 0 \) can be found in the calibration (Section 4).
For \( i \approx 0 \), equation (31) identifies a positive pass-through from the nominal interest rate to the bank lending rate, \( \partial r^b / \partial i > 0 \), since entrepreneurs rely more on external finance. For \( i \approx i^* \), however, the pass-through cannot be characterized, since we set \( \theta = 0 \) for analytical tractability. Further comparative statics show that an increase in \( \lambda \) weakens the pass-through from the nominal interest to the bank lending rate, while an increase in \( \theta \) or \( \alpha \) strengthens the pass-through. Moreover, a change in \( \rho \) or \( \chi \) has no effect on the pass-through.

With this understanding, we proceed to analyze the transmission of monetary policy to aggregate lending and investment, defined as \( K \equiv \lambda [(1 - \alpha)k_I + \alpha k_E] \) and \( L \equiv \lambda \alpha (k_E - k_I) \), respectively. Let \( \bar{k} = k^* - \chi f(k^*) - \frac{\rho \alpha \theta'(a)}{1 - \beta} \) and \( i^* = \lambda (1 - \alpha)[f'(\bar{k}) - 1] \), where \( \bar{k} \) is defined as the minimum \( k_I \) an entrepreneur needs to obtain \( k_E = k^* \) after having pledged all of his private real estate and claims on future output, and \( i^* \) is the corresponding nominal interest rate. For \( i \approx 0 \) and thus \( k_E \approx k^* \), the transmission of the nominal interest rate to aggregate investment and lending is approximated by:13

\[
K \approx \lambda k^* + \frac{(1 - \alpha)i}{f''(k^*)[1 - \alpha(1 - \theta)]}, \tag{32}
\]

\[
L \approx -\frac{\alpha i}{f''(k^*)[1 - \alpha(1 - \theta)]}. \tag{33}
\]

For \( i - i^* \approx 0 \) and \( \theta = 0 \), however,

\[
k_E - k^* \approx \frac{k_I - \bar{k}}{1 - \mathcal{O}} \approx \frac{i - i^*}{D}, \tag{34}
\]

\[
L \approx \lambda \alpha \left[k^* - \bar{k} - \frac{\mathcal{O}(i - i^*)}{D}\right], \tag{35}
\]

where the determinant \( D > 0 \), and:

\[
\mathcal{O} = \chi + \left(\frac{\beta \rho}{1 - \beta}\right)^2 \left[\frac{\alpha \lambda a \theta'(a) f''(k^*)}{1 - \chi}\right]. \tag{36}
\]

Proposition C summarizes the transmission mechanism.

**Proposition C. (Transmission Mechanism)** For \( \theta = 0 \), \( \chi < \chi^* \), and \( \rho < \rho^* \), transmission of monetary policy to aggregate lending is characterized by the following three regions:

- **A**: \( i \leq i^* \) with \( \partial k_I / \partial i < \partial k_E / \partial i = 0 \), and \( \partial L / \partial i > 0 \),
- **B**: \( i > i^* \) with \( \partial k_I / \partial i < \partial k_E / \partial i < 0 \), and \( \partial L / \partial i > 0 \),
- **C**: \( i > i^* \) with \( \partial k_E / \partial i < \partial k_I / \partial i < 0 \), and \( \partial L / \partial i < 0 \),

13Details to the derivation can be found in Appendix H.
as represented in Figure 7 and equations (32)-(36). Comparative statics show:

\[
\frac{\partial |\partial k_E/\partial i|/\partial \rho}{\partial k_I/\partial i/\partial \rho} = 0 \quad \text{for} \quad i \leq i^*,
\]

\[
\frac{\partial |\partial k_E/\partial i|/\partial \rho}{\partial k_E/\partial i/\partial \rho} < 0 \quad \text{for} \quad i > i^*.
\]

Proof in Appendix F.

Proposition C analyzes the effect of a change in the nominal interest rate on aggregate investment and lending, characterizing three distinct regions. In region A, \( i \leq i^* \), and thus the entrepreneur’s liquidity constraint, (14), does not bind. As a consequence, \( k_I \) decreases with an increase in \( i \), while \( k_E \) remains unaffected. From (33), aggregate lending is increasing in the nominal interest rate, as entrepreneurs rely more on external finance when their money holdings are less valuable. For \( i > i^* \), however, an increase in the nominal interest rate decreases both internally and externally financed capital. Given (34) and (36), the strength of this effect depends on the composition, i.e., the combination of retained earnings, traditional bank loans, and HELs used to purchase \( k_E \). The effect on aggregate lending, i.e., \( \partial L/\partial i \), follows this outcome. Consider first region B, in which entrepreneurs rely extensively on HELs. In this scenario, \( k_I \) will decrease more than \( k_E \) in response to an increase in \( i \), as entrepreneurs successfully hedge against inflation using their real asset when bargaining with the bank. The increased demand for housing and the consequential price appreciation increases, \( L \), and weakens the transmission channel. In contrast, in region C, an increase in the nominal interest rate, \( i \), decreases \( k_E \) more than \( k_I \), implying a decrease in aggregate lending, \( L \). Thus, whenever the pledgeability of housing is limited, entrepreneurs are unable to alleviate the inflation tax and as a result, the appreciation in the house prices does not compensate for the reduction in real money balances and the consequentially worse terms of trade when bargaining with a bank. Further, comparative statics show that for region B
and $C$, an increase in $\rho$ weakens the transmission mechanism, while in region $A$, a change in $\rho$ has no effect on $|\partial k_E/\partial i|$.

4  Calibrated Results

Having characterized the transmission channel analytically, we now complement the previous results with a calibrated version of the model. The key difference to the previous Taylor approximations is that we allow banks to have a positive bargaining power ($\theta > 0$), enabling an analysis of the pass-through in a constrained equilibrium.

The model is calibrated to U.S. data covering 2000-2016.\footnote{See Appendix G for details on data sources and calculations.} We start by setting the discount factor to $\beta = 0.97$. Our measure for the nominal interest rate is the 3-month T-bill secondary market rate with an average of $i = 0.0163$. The probability to have a bank loan approved, $\alpha$, is 0.80 following the 2003 Survey of Small Business Finances. The pledgeability of future output is calculated as the average ratio of liabilities to assets among small businesses. From the Federal Reserve Flow of Funds Accounts, we calculate $\chi = 0.24$. We estimate the probability of receiving an investment opportunity, $\lambda$, by calculating the average percentage of entrepreneurs who started their business within the last year. Using data from the Survey of Consumer Finances (SCF) between 2001 and 2016, we estimate $\lambda = 0.0628$. To pin down $\theta$, we follow Rocheteau et al. (2018) in targeting the spread between the prime bank rate and the 3-month T-bill rate of 3.25%, i.e. $r^b - i = 0.0325$.\footnote{For $0 < i < r^s$, $r^b$ is given by equation (24) for the case of $m > m^*$. We check that under the parameters in Table 1 and $\rho = 0$, an entrepreneur’s liquidity constraint does not bind for $i < 0.0233$.} Last but not least, we define the functional forms for the entrepreneur’s production function, $f(k) = \nu k^\eta$, and the utility of housing services, $\vartheta(a) = a^{1/2}$, where $\nu = \beta/(1 - \beta)$ is a scaling parameter.\footnote{Recall that the fundamental value of housing is $q_a = \nu^\eta/(1 - \beta)$. Without scaling the production function, say if $f(k) = k^\eta$, entrepreneurs would mechanically be able to obtain $k^*$ through bank loans (as $f'(k^*) = 1$) for $\rho > 0$.}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.97</td>
<td>Annual frequency</td>
</tr>
<tr>
<td>$i$</td>
<td>Nominal interest rate</td>
<td>0.0163</td>
<td>3-month T-bill rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Probability of receiving bank credit</td>
<td>0.80</td>
<td>Loan acceptance rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Probability of receiving an investment opportunity</td>
<td>0.0628</td>
<td>Formation of new businesses</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Bank’s bargaining power</td>
<td>0.162</td>
<td>Spread</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Fraction of future output that is pledgeable</td>
<td>0.24</td>
<td>Asset-to-liability ratio</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Capital share</td>
<td>1/3</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

Table 1: Parameter values

To study the transmission mechanism of monetary policy, we rely on the pass-through rate,
∂r_b/∂i, and the semi-elasticity of aggregate investment and aggregate lending, i.e., their percentage change in response to a one percentage point increase in the nominal interest rate, ∂log(K)/∂i, and ∂log(L)/∂i. By varying the pledgeability of housing, ρ, between 0 and 1, we account for changes in the composition of financing. We focus on these effects for i ∈ [0, 0.06], as this is an empirically relevant range given the values of the 3-month T-bill rate between 2000-2016, whereas the calibration determines i∗ = 0.0233.

Starting with the pass-through from the nominal interest rate to the bank lending rate, Figure 8 displays a positive pass-through rate, ∂r_b/∂i, for all values of ρ, where |∂r_b/∂i|/∂ρ < 0. Hence, an increase in the pledgeability of housing dampens the pass-through rate, confirming the theoretical results in Proposition C. For example, at i ≈ 0.05, a decrease in ρ from 0.4 to 0 increases the pass-through rate from 1.441 to 1.725, an increase of 19.7%, illustrating the sensitivity of the pass-through to the pledgeability of housing.

Figure 8: The pass-through of monetary policy

Focusing now on the transmission to aggregate investment and lending, Figure 9 presents our results. Panel (a) shows the semi-elasticity of aggregate investment as the pledgeability of housing varies from 0 to 1. From Proposition C we know that ∂log(K)/∂i < 0, whereas the magnitude depends on whether the entrepreneur faces a binding liquidity constraint or not. The calibrated results confirm and show that for i < 0.0233, the equilibrium is unconstrained and hence the semi-elasticity independent of ρ, since ∂k_I/i < ∂k_E/i = 0 for i ≤ i∗. For i > i*, however, the liquidity constraint binds, ∂k_E/i < 0, and thus ∂log(K)/∂i depends on ρ. When ρ is lower, the semi-elasticity is larger, and thus the stronger the transmission channel, as seen in the aftermath of the recent global financial crisis. For example, at i ≈ .05, the semi-elasticity varies from -6.8 to -4.7 as ρ varies from 0 to 0.4. In the limiting

---

17 A decomposition of ∂log(K)/∂i < 0 into ∂k_I/i and ∂k_E/i is provided in Figure 10 in Appendix I, confirming the results in Proposition C.
case with $\rho = 1$, $\partial \log(K)/\partial i$ is close to zero.

Panel (b) in turn focuses on the semi-elasticity of aggregate lending, $\partial \log(L)/\partial i$. The results show that the semi-elasticity is positive in case of a non-binding liquidity constraint and negative if the liquidity constraint binds, whereas $|\partial L/\partial i|/\partial \rho < 0$.\(^{(18)}\) Hence, for the parameter values of the sample period, the response of aggregate lending is strictly negative, even for large values of $\rho$ (region $C$ in Proposition C). In an alternative calibration with a sample period of 1958-2007, following Rocheteau et al. (2018), we show that $\partial \log(L)/\partial i > 0$ in a constrained equilibrium for $\rho = 0.4$ and for $i > 0.20$ (region $B$ in Proposition C).\(^{(19)}\)

5 Conclusion

Motivated by the recent financial crisis and the effect of a crash in home prices on entrepreneurial investment, we have introduced housing to serve as consumption and collateral into a model of entrepreneurial finance. The model generates a connection between the demand for housing and money, investment, lending, and monetary policy. We find that the pledgeability of housing has a direct effect on the transmission of monetary policy to entrepreneurial investment and lending. The results show that the composition of external and internal finance, and thus the combination of nominal and real assets used to finance investments, determines the effectiveness of monetary policy. Further we show that an increase in the pledgeability of housing dampens this transmission channel, since it enables agents to alleviate the inflation tax by substituting nominal for real assets.

\(^{(18)}\)The values of $\partial \log L/\partial i$ for low values of $i$ do not appear on the graph because they approach $\infty$ as $i \rightarrow 0$ (as $L = 0$ when $i = 0$ and $k_I = k^*$).

\(^{(19)}\)The results are available upon request.
References


Appendix: Proofs

A. Proof of Lemma A

We show uniqueness in a constrained equilibrium, as \( k_E = k^* \) in an unconstrained equilibrium. We rearrange (23) as:

\[
\theta f(k_E) + (1 - \theta)k_E - \theta \Delta^e_m(m) = \chi f(k_E) + \rho q_a a + q_m m.
\] (37)

If \( k_E = 0 \), the left hand side of (37) is less than zero and the right hand side is greater than zero. As \( k_E \to \infty \), the left hand side increases at rate \( \theta f'(k_E) + (1 - \theta) \) and the right hand side increases at rate \( \chi f'(k_E) \). The left hand side of (37) will eventually surpass the right hand side if \( 1 - \theta > (\chi - \theta) f'(k_E) \), which is true for some \( k_E > 0 \), as \( f'(k_E) \to 0 \) as \( k_E \to \infty \). Thus, there exists a unique \( k_E > 0 \) that satisfies (23).

Second, we establish that \( k_E \in [k_E, k^*] \) where \( \chi f'(k_E) = 1 \). Consider the entrepreneur’s binding liquidity constraint, (14). Solving for the bank’s surplus, \( \phi \), gives

\[
\phi = \chi f(k_E) - k_E + \rho q_a a + q_m m.
\] (38)

From (38), the bank’s surplus is maximized at \( k_E \). Suppose that \( k_E < k_E \). A Pareto improvement can be made by increasing \( k_E \) to \( k_E \), as both the surplus of the bank and entrepreneur are strictly larger at \( k_E \). Thus, \( k_E \) is a lower bound on capital acquired through bank credit.

B. Proof of Lemma B

Consider the case where \( m < m^* \). From (22), the real lending rate is given by

\[
r^b = \frac{\theta[f(k_E) - k_E - \Delta^e_m(m)]}{k_E - k_I}.
\] (39)

Now consider when \( m \geq m^* \). From (21), \( r^b \) is given by (39) with \( k_E = k^* \).

C. Proof of Lemma C

Taking the first order condition of (26) with respect to \( m' \) gives

\[
q_m = \beta \left\{ q'_m + \lambda \left[ \alpha \frac{\partial \Delta^e(m', a')}{\partial m'} + (1 - \alpha) \frac{\partial \Delta_m(m')}{\partial m'} \right] \right\},
\] (40)
where
\[ \frac{\partial \Delta_m(m')}{\partial m'} = q'_m [f'(k_I) - 1], \] (41)
and
\[ \frac{\partial \Delta^e(m', a')}{\partial m'} = \begin{cases} \theta q'_m [f'(k_I) - 1] & \text{if } m \geq m^*, \\ q'_m \left[ \frac{(1-\chi)f'(k_E)(1+\theta f'(k_I) - 1)}{(1-\theta) - (\chi - \theta)f'(k_E)} - 1 \right] & \text{if } m < m^*. \end{cases} \] (42)
Combining (41) and (42) gives \( L_m \). The first order condition of (26) with respect to \( a' \) is
\[ q_a = \beta \left\{ q'_a + \vartheta'(a') + \lambda \alpha \frac{\partial \Delta^e(m', a')}{\partial a'} \right\}, \] (43)
where
\[ \frac{\partial \Delta^e(m', a')}{\partial a'} = \begin{cases} 0 & \text{if } m \geq m^*, \\ \rho q'_a \left[ \frac{(1-\chi)f'(k_E)}{(1-\theta) + (\chi - \theta)f'(k_E)} \right] & \text{if } m < m^*. \end{cases} \] (44)
Substituting (44) into (43) and rearranging gives \( L_a \). •

D. Proof of Proposition A

The portfolio choice of money and housing can be written as
\[ \max_{k_I, a'} \left\{ -ik_I - [1/\beta - 1]q_a a' + \vartheta'(a') + \lambda \left[ \alpha \Delta^e(k_I, a') + (1 - \alpha)\Delta^e_m(k_I) \right] \right\}, \] (45)
where \( i \equiv \gamma/\beta - 1 \). If \( i = 0 \), then \( k_I = k_E = k^* \) so that \( \partial \Delta^e(k_I, a') / \partial k_I = \partial \Delta^e_m(k_I) / \partial k_I = 0 \). The first order condition of (45) with respect to \( k_I \) gives
\[ i = \lambda [1 - \alpha (1 - \theta)] [f'(k_I) - 1], \] (46)
for \( 0 < i \leq i^* \). It follows that \( k_I < k^* \) to satisfy (46) and that \( \partial k_I / \partial i < 0 \). If \( i > i^* \), \( k_I \) is determined by
\[ i = \lambda \alpha \left[ \frac{(1-\chi)f'(k_E)(1 + \theta f'(k_I) - 1)}{(1-\theta) - (\chi - \theta)f'(k_E)} - 1 \right] + \lambda (1 - \alpha) [f'(k_I) - 1]. \] (47)
If \( i \) increases, (47) is satisfied if and only if the right hand side increases. Since \( f'(k_I) \) is decreasing in \( k_I \), the first and the second term on the right hand side are decreasing in \( k_I \). Thus, \( \frac{\partial k_E}{\partial i} < 0 \). We use this result to show that \( \frac{\partial k_E}{\partial i} < 0 \) and \( k_E < k^* \) for \( m < m^* \). Differentiation of the entrepreneur’s liquidity constraint gives
\[ \frac{\partial k_E}{\partial i} = \left[ \frac{\theta f'(k_I) + (1 - \theta)}{(\theta - \chi)f'(k_E) + (1 - \theta)} \right] \frac{\partial k_E}{\partial i} < 0, \] (48)
as $\chi f'(k_E) < 1$. It follows that $k_E < k^*$ for $i > i^*$.

Next, we show that $\frac{\partial a}{\partial i} \geq 0$. If $m \geq m^*$, then housing is priced at its fundamental value and thus $\frac{\partial a}{\partial i} = 0$. In the case where $m < m^*$, the price of housing is given by

$$q_a = \frac{\beta \vartheta'(a)}{1 - \beta \left( 1 - \lambda \rho \left[ \frac{(1-\chi)f'(k_E)}{(1-\theta)-(\chi-\theta)f'(k_E)} - 1 \right] \right)}.$$  (49)

If $i$ increases, $k_E$ will decrease. From (49), $q_a$ will increase following a decline in $k_E$ and therefore $\frac{\partial a}{\partial i} > 0$ when $m < m^*$.

To determine $\partial i^*/\partial \chi > 0$ and $\partial i^*/\partial \rho > 0$, we rewrite (23) and substitute $k_E = k^*$ (since $i = i^*$) to get:

$$\theta f(\bar{k}) + (1 - \theta)\bar{k} = (\theta - \chi)f(k^*) + (1 - \theta)k^* - \rho q_a a,$$  (50)

where $\bar{k}$ is the amount of internally financed capital to acquire $k^*$ through a bank loan at $i = i^*$. Suppose that $\rho$ or $\chi$ increases. In order for (50) to hold with equality and maintain $k^*$, $\bar{k}$ must decrease. Now consider (46). Rearranging and setting $i = i^*$ gives

$$f'(\bar{k}) = \frac{i^*}{\lambda[1 - \alpha(1 - \theta)]}.$$  (51)

It follows that $i^*$ increases if $\bar{k}$ decreases, $\partial i^*/\partial \chi > 0$, and $\partial i^*/\partial \rho > 0$. 

**E. Proof of Proposition B**

A second-order approximation of $f(k_I) - k_I$ around $k^*$ is given by:

$$f(k_I) - k_I \approx f(k^*) - k^* + \frac{f''(k^*)}{2}(k_I - k^*)^2.$$  (52)

Recall that $\Delta^e_m(m) = f(k_I) - k_I$. Substituting (52) into (24) gives:

$$r^b \approx \frac{\theta f''(k^*) (k_I - k^*)}{2}.$$  (53)

Next, a first-order approximation of $f'(k_I)$ around $k^*$ is given by:

$$f'(k_I) \approx 1 + f''(k^*)(k_I - k^*).$$  (54)
Substituting \( f''(k^*) (k_I - k^*) \approx f'(k_I) - 1 \) into (46) gives:

\[
f''(k^*) (k_I - k^*) \approx \frac{i}{\lambda[1 - \alpha(1 - \theta)]}.
\]  

(55)

Substituting (55) into (53) gives (31).

Last but not least, to determine comparative statics, the strength of the pass-through is given by

\[
\frac{\partial r^b}{\partial i} = \frac{\theta}{2\lambda[1 - \alpha(1 - \theta)]},
\]  

(56)

which is decreasing (increasing) in \( \lambda (\alpha) \) and independent of \( \chi \) and \( \rho \). Rearranging (56) gives

\[
\frac{\partial r^b}{\partial i} = \frac{1}{2\lambda\left(\frac{1}{\theta}(1 - \alpha) + \alpha\right)},
\]  

(57)

which is increasing in \( \theta \). 

**F. Proof of Proposition C**

For \( i \leq i^* \), the economy is in an unconstrained equilibrium (region A in Figure 7) and we refer to the derivations in Appendix H. From equations (66) and (33):

\[
\frac{\partial k_I}{\partial i} = \frac{1}{\lambda f''(k^*)(1 - \alpha(1 - \theta))} < 0,
\]  

(58)

\[
\frac{\partial L}{\partial i} = -\frac{\alpha}{f''(k^*)(1 - \alpha(1 - \theta))} > 0,
\]  

(59)

and \( \partial k_E/\partial i = 0 \) as \( k_E = k^* \). For \( i > i^* \), the economy is in a constrained equilibrium. From equations (34)-(35),

\[
\frac{\partial k_I}{\partial i} = -\frac{1 - O}{D} < 0
\]  

(60)

\[
\frac{\partial k_E}{\partial i} = -\frac{1}{D} < 0
\]  

(61)

\[
\frac{\partial L}{\partial i} = -\lambda\alpha\frac{O}{D} \geq 0,
\]  

(62)

as \( D > 0 \) and \( 1 - O > 0 \). We can see that \( \partial k_E/\partial i < \partial k_I/\partial i < 0 \) and \( \partial L/\partial i < 0 \) if \( O > 0 \). From (36), \( O > 0 \) if \( \chi \) is large relative to \( \rho \), corresponding to region C in Figure 7. If \( O < 0 \), then \( \partial k_I/\partial i < \partial k_E/\partial i < 0 \) and \( \partial L/\partial i > 0 \). From (36), \( O < 0 \) is \( \rho \) is large relative to \( \chi \) (region B in Figure 7).
Consider $\partial|\partial k_E/\partial i|/\partial \rho$. From equation (73),

$$\frac{\partial k_E}{\partial i} = -\frac{1}{D} < 0,$$

(63)

as $D > 0$. In order to show that $|\partial k_E/\partial i|$ is decreasing in $\rho$, we first establish that $\frac{\partial D}{\partial \rho} > 0$.

From equation (74),

$$\frac{\partial D}{\partial \rho} = -\lambda(1 - \alpha)(1 - \chi)f''(\bar{k})\frac{\partial \bar{k}}{\partial \rho} + \lambda(1 - \alpha)f''(\bar{k})\frac{\partial \bar{k}}{\partial \rho} \left(\frac{\beta \rho}{1 - \beta}\right)^2 \left[\frac{\alpha \lambda a'\vartheta(a)f''(k^*)}{1 - \chi}\right]$$

$$+ 2\lambda(1 - \alpha)f''(\bar{k}) \left(\frac{\beta \rho}{1 - \beta}\right) \left(\frac{\beta}{1 - \beta}\right) \left[\alpha \lambda a'\vartheta(a)f''(k^*)\right] > 0,$$

(64)

as $f''(\bar{k}) > 0$, $f''(k^*) < 0$ and $\bar{k}/\partial \rho = -\frac{\beta a'\vartheta(a)}{1 - \beta} < 0$. Thus, $\partial|\partial k_E/\partial i|/\partial \rho < 0$. ■
Appendix: Supplementary Material

G. Data sources and construction

We use the S&P/Case-Shiller U.S. National Home Price Index, (FRED series CSUPHINSA, annual average) in Figures 1a and 2a. The data on the number of firms with 1-4 employees in Figure 1a comes from the U.S. Census Bureau. Capital expenditures in Figure 1b is total capital expenditures among non-financial non-corporate businesses in the Federal Reserve Flow of Funds Accounts.\(^{20}\) In Figure 2a, data on home equity loans also comes from the Flow of Funds Accounts, where we measure home equity loans among all sectors and in Q4 of each year.\(^{21}\) We divide capital expenditures and home equity loans by the consumer price index (CPI) (all items) that is published by the Bureau of Labor Statistics. In Figure 2b, data on the 3-month T-bill rate and bank prime loan rate are downloaded directly from FRED using series IDs TB3MS and MPRIME, respectively.

We calculated the extensive margin of home equity debt using the Survey of Consumer Finances (SCF) where we identify respondents as having home equity debt if they owe a positive amount on a home equity line of credit or a home equity loan. The fraction of respondents with home equity debt was calculated using sample weights provided by the SCF.\(^{22}\)

As for the calibration in Section 4, our measure for the nominal interest (bank lending) rate was the 3-month T-bill secondary market (bank prime) rate. Our measure of \(\alpha\), the probability to receive a bank loan, follows from Rocheteau et al. (2018) who found that between \(78 - 90\%\) of respondents in the 2003 Survey of Small Business Finances had their most recent loan application approved. To calculate the pledgeability of output, we first calculated the average of (i) total loans to non-financial non-corporate businesses and (ii) total loans to non-financial non-corporate businesses net total home equity loans. We then divide the amount of loans to non-financial non-corporate businesses by total assets among non-financial non-corporate businesses. Both the data on loans and assets among non-financial non-corporate businesses come from the Flow of Funds Accounts, where we use quantities from Q4 in each year. Last but not least, we estimated the probability of receiving an investment opportunity, \(\lambda\), by using data from the SCF between 2001-2016. Specifically, we calculated the fraction of respondents who started or acquired their business within the last year.\(^{23}\)

\(^{20}\)Entrepreneurs fall within the category of nonfinancial noncorporate businesses in the Flow of Funds Accounts, as small businesses make up nearly 73% of non-corporate employment and 77% of non-corporate receipts (Kobe, 2012).

\(^{21}\)The amount of home equity loans by sector is not available in the Flow of Funds data.

\(^{22}\)Per the recommendation of the Board of Governors, we divide the weights by 5 before carrying out these calculations.

\(^{23}\)As with our previous calculations using the SCF, we used sample weights when carrying out this calculation.
H. Derivation of Equations (32)-(36)

Consider an unconstrained equilibrium. Solving for $k_I - k^*$ from (53) gives:

$$k_I - k^* \approx \frac{2\rho}{\bar{\theta}f''(k^*)}. \quad (65)$$

Substituting (31) into (65) gives

$$k_I \approx k^* + \frac{i}{\lambda f''(k^*)[1 - \alpha(1 - \theta)]}. \quad (66)$$

Plugging $k_I$ from (66) and $k_E \approx k^*$ into $K \equiv \lambda[(1 - \alpha)k_I + \alpha k_E]$ and $L \equiv \lambda\alpha(k_E - k_I)$ gives (32) and (33).

Now consider a constrained equilibrium with $\theta = 0$. The triple $(k_E, k_I, q_a)$ is determined by equations (23), (47), and (49):

$$k_E = \chi f(k_E) + \rho q_a a + k_I, \quad (67)$$

$$i = \lambda \left[ \alpha \frac{f'(k_E) - 1}{1 - \chi f'(k_E)} + (1 - \alpha) \left[ f'(k_I) - 1 \right] \right], \quad (68)$$

$$q_a = \frac{(1 - \chi f'(k_E)) \beta \vartheta'(a)}{(1 - \beta)(1 - \chi f'(k_E)) - \beta \alpha \rho (f'(k_E) - 1)}, \quad (69)$$

Substituting (69) into (67) gives:

$$k_E = \chi f(k_E) + \rho a(1 - \chi f'(k_E)) \beta \vartheta'(a) \frac{(1 - \beta)(1 - \chi f'(k_E)) - \beta \alpha \rho (f'(k_E) - 1) + k_I. \quad (70)$$

A first order approximation of (70) and (68) in the neighborhood $(k_I, k_E) = (\bar{k}, k^*)$ gives:

$$\left( 1 - \chi - \left( \frac{\beta \rho}{1 - \beta} \right)^2 \left[ \alpha \chi \vartheta'(a) f''(k^*) \right] \frac{\chi f''(k^*)}{1 - \chi} \right) (k_E - k^*) \approx (k_I - \bar{k}), \quad (71)$$

$$\chi(1 - \alpha) f''(\bar{k})(k_I - \bar{k}) + \lambda \alpha \frac{f''(k^*)}{1 - \chi} (k_E - k^*) \approx i - i^*. \quad (72)$$
Solving (71) and (72) for \((k_I - \bar{k})\) and \((k_E - k^*)\) gives:

\[
\begin{pmatrix}
    k_I - \bar{k} \\
    k_E - k^*
\end{pmatrix} = \frac{1}{D} \begin{pmatrix}
    \frac{\lambda \alpha f''(k^*)}{1 - \chi} & -(1 - \chi) + \left(\frac{\beta \rho}{1 - \beta}\right)^2 \left[\frac{\alpha \lambda \vartheta'(a) f''(k^*)}{1 - \chi}\right] \\
    -\lambda (1 - \alpha) f''(\bar{k}) & -1
\end{pmatrix} \begin{pmatrix}
    0 \\
    i - i^*
\end{pmatrix},
\]

where

\[
D = -\frac{\lambda \alpha f''(k^*)}{1 - \chi} - \lambda (1 - \alpha) f''(\bar{k}) (1 - \chi) - \left(\frac{\beta \rho}{1 - \beta}\right)^2 \left[\frac{\alpha \lambda \vartheta'(a) f''(k^*)}{1 - \chi}\right] > 0,
\]

gives \(k_I\) and \(k_E\). Finally, using \(L \equiv \lambda \alpha (k_E - k_I)\) and substituting \(k_I\) and \(k_E\) gives (35).

I. Decomposition

In Figure 10, we decompose the effect of changes in the nominal interest rate on aggregate investment into the effects of internally financed capital and externally financed capital.

Figure 10: The transmission of monetary policy (decomposition)