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Voluntary Disclosure in Asymmetric Contests*

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Abstract This paper studies the incentives for interim voluntary disclosure of verifiable information in probabilistic all-pay contests with two-sided incomplete information. Private information may concern marginal cost, valuations, and ability. Our main result says that, if the contest is *uniformly asymmetric*, then full revelation is the unique perfect Bayesian equilibrium outcome. This is so because the weakest type of the underdog reveals her type in an attempt to moderate the favorite while, similarly, the strongest type of the favorite tries to discourage the underdog—so that the contest unravels. This strong-form disclosure principle is robust with respect to correlation, partitional evidence, randomized disclosures, sequential moves, and continuous type spaces. Moreover, the assumption of uniform asymmetry is not needed when incomplete information is one-sided. However, the principle breaks down when contestants are potentially too similar in strength, possess commitment power, or when information is unverifiable. In fact, cheap talk will always be ignored, even if mediated by a trustworthy third party.

Keywords Asymmetric contests · Incomplete information · Disclosure · Strategic complements and substitutes · Dominance and defiance · Cheap talk

JEL Classification C72 Non-cooperative Games · D74 Conflict, Conflict Resolution, Alliances, Revolutions · D82 Asymmetric & Private Information · J71 Discrimination

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1. Introduction

On February 24, 2014, the concentration of Russian troops along the entire Ukrainian-Russian border became overwhelming. In a major demonstration of force and simultaneous preparation for invasion, the Kremlin had decided to concentrate in the Kyiv, Kharkiv, and Donetsk directions 38 thousand men, 761 armed tanks, 2'200 armored vehicles, 720 artillery systems and multiple rocket launchers, as well as up to 40 attack helicopters, 90 combat support helicopters, and 90 attack aircraft. In the Black Sea, 80 Russian warships were on combat duty. On that same day, the Russian Black Sea Fleet Commander had a conversation with the Ukraine Naval Forces Commander, advocating complete surrender and handing over of the Crimea. And indeed, Ukrainian resistance quickly ebbed away in the days to follow.[1]

In this paper, we extend the standard model of a probabilistic contest (Rosen, 1986; Dixit, 1987) by allowing for pre-play communication of verifiable information (Okuno-Fujiwara et al., 1990; van Zandt and Vives, 2007; Hagenbach et al., 2014). Contestants are assumed to possess private information regarding parameters indicative of their absolute strength in the competition. In general, these parameters may concern marginal cost, valuations, and ability. Then, at a stage preceding the contest, any player may interim, i.e., after having observed her type, choose to disclose that information to her opponent. In this type of framework, we evaluate the incentives of players to voluntarily disclose their private information. Moreover, we characterize the perfect Bayesian equilibrium of the resulting two-stage game. The focus of the analysis lies on contests that are *uniformly asymmetric* in the sense that one of the contestants is, subject to activity, interim always strictly more likely to win than the other. We identify a condition on the primitives of the model that guarantees that the contest is uniformly asymmetric. While restrictive, this condition is consistent with heterogeneity in both valuations (e.g., Amann and Leininger, 1996; Maskin and Riley, 2000) and ability (e.g., O'Keeffe et al., 1984; Meyer, 1992; Franke et al., 2014).[2]

Our main result says that, provided that the contest is uniformly asymmetric, the *only* outcome of the revelation game consistent with the assumption of perfect Bayesian rationality is the one in which all the privately held information is unfolded prior to the contest. Thus, we find

[1] This example has been inspired by the discussion in Lenton (2022) and a recently declassified protocol of the National Security and Defense Council of Ukraine (2016, pp. 10-11).

[2] The formal definition is given in Section 3. While the focus lies on uniformly asymmetric contests, we also delineate the scope of the strong-form disclosure principle in more general contests.

general conditions under which the disclosure principle in the strong form applies to a standard contest setting. This may be of interest because effort choices in probabilistic contests are neither strategic substitutes nor strategic complements, and consequently contests do not satisfy the usual conditions sufficient for the strong-form disclosure principle.[3]

There is a simple intuition for why contestants find it difficult to withhold information in a uniformly asymmetric contest. In view of the high effort expected from a favorite that is left to speculate about the underdog's ability, the weakest type of the underdog has a strict incentive to self-disclose, so as to moderate the opponent. Once the revelation is accounted for, however, the pool of silent types shrinks. Then, the weakest of the remaining types will choose to disclose her private information as well. Thus, there is an unraveling on the underdog's side. But in the resulting contest with one-sided incomplete information, the unraveling continues on the side of the favorite. Indeed, the respective strongest type of the favorite has a strict incentive to self-disclose, so as to discourage the underdog. In the end, full revelation of all private information is inevitable.[4]

Our main result extends in several ways. To start with, strong-form disclosure continues to hold generically when types are correlated. The role of the weakest type of the underdog is then taken by the lowest-bidding type in the contest. Second, the main result extends to partitional information releases and randomized signals. Next, we show that the principle applies likewise if disclosure decisions are taken in a sequential fashion, i.e., with either the favorite or the underdog moving first. Further, we allow for continuous type spaces. Finally, we show that, in the case of one-sided private information, the assumption of uniform asymmetry may be dropped without losing the strong-form disclosure principle. In that case, there is always one extremal type that strictly prefers to self-disclose. The order of the unraveling may then switch hence and forth, in a "bang-bang" fashion, between the remaining weak and strong types of the informed side.

The strong-form disclosure principle is, however, not universally valid. First, if a contest with two-sided incomplete information is not uniformly asymmetric, then it may happen that no type

[3] Instead, given that the best-response function of the favorite is strictly *increasing,* while the best-response function of the underdog is strictly *declining*, asymmetric contests classify, at least under suitable domain restrictions, as games of strategic heterogeneity (Monaco and Sabarwal, 2016; Barthel and Hoffmann, 2019).

[4] Regrettably, this intuition does not translate into a simple proof. This is so because, as it turns out, self-disclosure may lead to *dominant* and *defiant* reactions, i.e., cause some types of the opponent, be it favorite or underdog, to raise their bids.

has an incentive to self-disclose. The reason is that contestants face countervailing incentives. While a relatively efficient type benefits from demoralizing an inefficient opponent, she simultaneously suffers from revealing her information to an opponent of comparable strength. Since the situation may be similar for a relatively inefficient type, full concealment can be an equilibrium. Second, full revelation is not a necessity if contestants possess commitment power. For example, Bayesian persuasion need not lead to full revelation even if the contest would otherwise unravel. Finally, the disclosure principle crucially depends on the assumption that private information is verifiable. In fact, as we show, unverifiable messages are necessarily ignored in any probabilistic contest, even in the presence of a trustworthy mediator.

Regarding welfare, we show that unrestricted communication in probabilistic contests has the potential to lead into a Pareto-inferior outcome for contestants. Still, depending on the objective of the contest organizer, full revelation may be socially desirable and, in particular, the result of optimal information design.

Further illustrations. Our introductory example was taken from the area of military conflict and war. The following examples may serve as additional illustrations.

- *Public enforcement* in the U.S. is characterized by a large disparity between the power of the state prosecutor and the typical criminal defendant (Lynch, 1998). Both parties possess verifiable information (Bibas, 2004). Even when plea bargaining is banned, it is common for the defendant to plead guilty to a subset of the charges (Weninger, 1987). Conversely, the prosecutor releases evidence to induce the defendant to confess (Petegorsky, 2012). As a result, only a small fraction of criminal cases go to trial.

- Implicit or explicit threats are used to *intimidate* whistleblowers (Chassang and Padró I Miguel, 2019) and witnesses (Maynard, 1994).

- In *R&D and patent races*, the frontrunner reveals research results and funding successes to discourage competitors (Baker and Mezzetti, 2005). But also laggards announce new products to influence the outcome of competition in their interest (Robertson et al., 1995).

- In *social conflict*, the use of phenotype "indices" resolves or avoids physical conflicts in dyadic relationships (Hand, 1986). For conflicts arising within dominance or subordination relationships, signals tend to be placating or acquiescent. Within egalitarian or unresolved relationships, however, there are either no signals or signals indicate relative desire for an item on a case-by-case basis.

Related literature. The economics literature has a long tradition of studying incentives for the voluntary disclosure of private information. Seminal contributions by Grossman (1981) and Milgrom (1981) pointed out that, as a consequence of unraveling, sellers will it find hard to withhold verifiable information about the quality of their products. The underlying *disclosure principle* has since shaped the theoretical discussion about the pros and cons of disclosure regulation, as is reflected by a very large body of literature.[5]

Probabilistic contests of incomplete information have been studied for some time. Rosen (1986, fn. 7) still complained that "few analytical results" were available. Early papers include Linster (1993) and Baik and Shogren (1995). The general framework with one-sided and two-sided private valuations is due to Hurley and Shogren (1998a, 1998b). Wärneryd (2003) observed that the uninformed player in a common-value setting is more likely to win than the informed player. Malueg and Yates (2004) analyzed a symmetric two-player Tullock contest with two equally likely and possibly correlated types. Schoonbeek and Winkel (2006) noted that individual types may remain inactive. General results on the existence and uniqueness of Bayesian equilibrium have been obtained by Ewerhart (2014), Einy et al. (2015), and Ewerhart and Quartieri (2020).

The present paper falls into the recent and quickly expanding literature concerned with the disclosure of *verifiable* information in contests.[6] That literature has tended to focus on either ex-ante voluntary disclosure, optimal disclosure policies, or interim voluntary disclosure.[7] Ex-ante voluntary disclosure in probabilistic contests has been studied by Denter et al. (2014), in particular. Assuming a probabilistic contest technology with one-sided incomplete information, they showed that a "laissez-faire" policy regarding the informed player's ex-ante disclosure decision

[5] In addition to the contributions already mentioned, see Verrecchia (1983), Dye (1985), Shin (1994), Seidmann and Winter (1997), Benoît and Dubra (2006), and Giovannoni and Seidmann (2007), for instance. Milgrom (2008) or Dranove and Jin (2010) offer surveys.

[6] Another form of pre-play communication, not considered in the present paper, is the costly signaling of unverifiable information. See, e.g., Katsenos (2010), Slantchev (2010), Fu et al. (2013), Heijnen and Schoonbeek (2017), and M. Yildirim (2017).

[7] Numerous additional research questions, related to learning, feedback, and motivation, for example, arise in the analysis of dynamic contests of incomplete information. Such research questions have been dealt with in papers by Clark (1997), H. Yildirim (2005), Krähmer (2007), Münster (2009), Zhang and Wang (2009), Aoyagi (2010), Ederer (2010), and Goltsman and Mukherjee (2011), for instance.

leads to lower expected lobbying expenditures than a policy of mandatory disclosure.[8] The second topic, optimal disclosure policies in contests, has recently seen a strong development. In particular, effort-maximizing disclosure policies have been characterized by Zhang and Zhou (2016) and Serena (2022) for probabilistic technologies, and by Fu et al. (2014), Chen et al. (2017), and Lu et al. (2018) for deterministic technologies.[9]

The present analysis is mainly concerned, however, with the third topic, i.e., the interim voluntary disclosure in contests. As far as we know, there is only one paper that has dealt with this issue on a comparable level of generality.[10] Specifically, Kovenock et al. (2015) showed that, regardless of whether valuations are private or common, the interim information sharing game followed by an all-pay auction admits a perfect Bayesian equilibrium in which no player ever shares her private information. Instead of the all-pay auction, however, we consider a probabilistic contest. Overall, the review of the literature suggests that the specific research question pursued in the present paper, viz. the analysis of incentives for the interim voluntary disclosure of hard evidence in contests with probabilistic technologies and two-sided incomplete information, has not been addressed in prior work.[11]

The remainder of this paper is structured as follows. Section 2 introduces the set-up. The main result is stated in Section 3. Section 4 outlines the proof of Theorem 1. Section 5 offers extensions, while Section 6 outlines limits to the scope of the disclosure principle. Section 7 concerns efficiency. Section 8 concludes. An Appendix contains all proofs and other material omitted from the body of the paper.

[8] Relatedly, Wu and Zheng (2017) considered a symmetric two-player lottery contest with two equally likely, independently drawn types for each player. In this framework, they showed that ex-ante disclosure decisions lead to information sharing if and only if the two possible type realizations are sufficiently close to each other.

[9] Dubey (2013) studied a set-up with two-sided incomplete information about a binary type distribution and two effort levels. Assuming that abilities are sufficiently dispersed, he showed that incomplete (complete) information engenders more effort if the prize is high (low). Einy et al. (2017) studied the value of public information in Tullock contests with nonlinear costs. Optimal disclosure policies have been analyzed also in models of population uncertainty. See Münster (2006), Myerson and Wärneryd (2006), Lim and Matros (2009), Fu et al. (2011), Feng and Lu (2016), and Fu et al. (2016), among others.

[10] Epstein and Mealem (2013) considered a lottery contest with one-sided incomplete information and characterized the perfect Bayesian equilibrium outcome in the case of two possible type realizations. While they considered also an extension to more than two types, they did not characterize the perfect Bayesian equilibrium in that case.

[11] In general, signals may have multiple audiences. E.g., in Board's (2009) model, a direct benefit from disclosure on the consumer side is balanced by the firm against the cost of tighter competition. The present paper, however, focuses on the informational exchange between contestants in the absence of informational externalities.

2. Set-up

Considered is an interaction over two stages, referred to as revelation stage and contest stage, respectively. The modeling follows the literature on pre-play communication (Okuno-Fujiwara et al., 1990; van Zandt and Vives, 2007; Hagenbach et al., 2014). We will start with the contest stage and continue backwards with the revelation stage.

2.1 The contest stage

Two players (or teams) $i = 1, 2$ exert effort at marginal cost $c_i > 0$ so as to increase their respective odds of winning a contested prize. Player i values winning at V_i, and losing at L_i, where $V_i > L_i$. Contestant i's effort (or bid) is denoted by $x_i \geq 0$. Following Rosen (1986), we assume that player i's probability of winning against $j \neq i$ is given as

$$p_i(x_i, x_j) = \begin{cases} \dfrac{\gamma_i h(x_i)}{\gamma_1 h(x_1) + \gamma_2 h(x_2)} & \text{if } x_1 + x_2 > 0 \\ \gamma_i / (\gamma_1 + \gamma_2) & \text{if } x_1 + x_2 = 0, \end{cases} \tag{1}$$

where $\gamma_i > 0$ denotes i's ability, while $h \equiv h(z)$ is a continuous production function that is twice continuously differentiable at positive bid levels, with $h(0) = 0$, $h' > 0$, and $h'' \leq 0$.[12] Thus, player i's payoff may be written as

$$\Pi_i(x_i, x_j; \theta_i) = p_i(x_i, x_j)V_i + (1 - p_i(x_i, x_j))L_i - c_i x_i, \tag{2}$$

where $\theta_i = (c_i, V_i, L_i, \gamma_i)$ denotes player i's type. This set-up includes, as an important special case, the biased *Tullock contest* (Tullock, 1975; Leininger, 1993; Clark and Riis, 1998), where the production function is given by $h(z) = h^{\text{TUL}}(z; r) \equiv z^r$ for some exogenous $r \in (0, 1]$. The *lottery contest* corresponds to the case $r = 1$.

For convenience, we will assume that each player i's type is independently and discretely distributed—and that it concerns the marginal cost parameter c_i only. As will be explained, the restriction to pure cost types is without loss of generality if either ability is publicly observable or the contest is of the Tullock form. Thus, player i's type is assumed to be drawn from a probability

[12] Relaxing the assumption of a concave production function would take us away from the focus of this paper. In contrast, the extension to player-specific production functions is easily accomplished yet does not yield additional insights.

distribution over the finite set $C_i = \{c_i^1, \ldots, c_i^{K_i}\}$, where $K_i \geq 1$, and

$$\underline{c}_i \equiv c_i^1 < \ldots < c_i^{K_i} \equiv \overline{c}_i \qquad (i \in \{1, 2\}). \tag{3}$$

The symbol $\underline{c}_i$ denotes the most efficient, or strongest type, while $\overline{c}_i$ denotes the least efficient, or weakest type of player i. Next, the ex-ante probability of type c_i^k is denoted by $q_i^k \equiv q_i(c_i^k)$, for $k \in \{1, \ldots, K_i\}$, with $q_i^k > 0$. Moreover, valuations will be normalized so that $V_i = 1$ and $L_i = 0$, for $i \in \{1, 2\}$. We will also write $\Pi_i(x_i, x_j; c_i) = \Pi_i(x_i, x_j; \theta_i)$.

A *bid schedule* for player $i \in \{1, 2\}$ is a mapping $\xi_i : C_i \rightarrow \mathbb{R}_+$. The set of i's bid schedules will be denoted as X_i. A pair of bid schedules $\xi^* = (\xi_1^*, \xi_2^*) \in X_1 \times X_2$ is a *Bayesian Nash equilibrium* if, for any type $c_i \in C_i$ of any player $i \in \{1, 2\}$, the effort level $x_i = \xi_i^*(c_i)$ maximizes type c_i's expected payoff $E_{c_j}[\Pi_i(x_i, \xi_j^*(c_j); c_i)]$, where $E_{c_j}[.]$ denotes the expectation over the realizations of $c_j \in C_j$, with $j \neq i$. Following Schoonbeek and Winkel (2006), a type $c_i \in C_i$ with $\xi_i^*(c_i) > 0$ ($\xi_i^*(c_i) = 0$) will be called *active* (*inactive*). As usual in this type of model, the discontinuity of the payoff functions at the origin implies that both players are necessarily active with positive probability.[13] By the same token, at least one player is active with probability one.

Lemma 1. *The contest stage admits a unique Bayesian Nash equilibrium.*[14]

Special notation will be used in the cases of complete and one-sided incomplete information. If $(c_1, c_2) = (c_1^\circ, c_2^\circ)$ is public information, then i's equilibrium strategy will be written as $x_i^\circ = x_i^\circ(c_1^\circ, c_2^\circ)$. Further, if player i's type $c_i = c_i^\#$ is public, while player j's type, with $j \neq i$, remains uncertain, then equilibrium strategies will be written as $x_i^\# = x_i^\#(c_i^\#)$ for player i and $\xi_j^\# = \xi_j^\#(.; c_i^\#)$ for player j, so that $\xi_j^\#(c_j; c_i^\#)$ is type c_j's equilibrium effort.

2.2 The revelation stage

At a stage preceding the contest, players simultaneously and independently decide whether to disclose their respective type or not. Initially, it will be assumed that private information cannot

[13] To see this, suppose that one player bids zero with probability one. Then, any sufficiently small positive bid is a better response than the zero bid, but any positive bid is suboptimal. Hence, there is no best response.

[14] Given strict concavity of payoff functions, Lemma 1 extends to randomized strategies, i.e., any mixed strategy equilibrium at the contest stage is degenerate and consequently in pure strategies.

be misrepresented. Further, we assume that the decision to self-disclose does not lead to any direct costs.[15]

In response to the observation of verifiable information, prior beliefs are updated according to Bayes' rule whenever possible. One notes that off-equilibrium beliefs may arise, but only in the distinct case where a player chooses to conceal her private information even though the equilibrium strategy entails self-disclosure by all types of that player.[16]

In any case, the contest stage begins with a well-defined posterior *belief* $\mu_i \in \Delta(C_i)$ about each player $i \in \{1, 2\}$, where $\Delta(C_i) = \{\mu_i : C_i \to [0, 1] \text{ s.t. } \sum_{k=1}^{K_i} \mu_i(c_i^k) = 1\}$. Ignoring zero-probability types, a unique Bayesian equilibrium exists by Lemma 1. In particular, the expected continuation payoff from the contest stage is well-defined for any $c_i \in C_i$ and $i \in \{1, 2\}$.[17] A (reduced-form) *perfect Bayesian equilibrium* consists of (i) a set $S_i \subseteq C_i$ of revealing types, for each player $i \in \{1, 2\}$, and (ii) an off-equilibrium belief $\mu_i^0 \in \Delta(C_i)$ for any $i \in \{1, 2\}$ with $S_i = C_i$, such that $E_{c_j}[\Pi_i(x_i^{\#}, \xi_j^{\#}(c_j); c_i)] \geq E_{c_j}[\Pi_i(x_i, \xi_j^*(c_j); c_i)]$, for any $x_i \geq 0$ and $c_i \in S_i$, as well as $E_{c_j}[\Pi_i(\xi_i^*(c_i), \xi_j^*(c_j); c_i)] \geq E_{c_j}[\Pi_i(x_i^{\#}, \xi_j^{\#}(c_j); c_i)]$, for any $c_i \in C_i \backslash S_i$. Here we dropped, for convenience, the reference to prior disclosure decisions in the notation of the equilibrium bids.[18]

3. The unraveling theorem

This section is central to our analysis. We start by defining what we call uniformly asymmetric contests. We then provide a sufficient condition for a contest to be uniformly asymmetric. Finally, we present the main result of this paper.

3.1 Uniformly asymmetric contests

The focus of our analysis lies on probabilistic contests with two-sided incomplete information that satisfy the following definition.

[15] Introducing costs for disclosing information would not change our conclusions, provided those are not too large compared to the benefits of self-disclosure identified below.

[16] A formal account of belief updating is provided in the Appendix.

[17] This is obvious for any $c_i \in C_i$ with $\mu_i(c_i) > 0$. Should, however, a type c_i deviate by not disclosing so that $\mu_i(c_i) = 0$, then there may not be a best response if the thereby deluded opponent plays zero with positive probability. In that case, we replace the continuation payoff by the supremum payoff feasible for c_i.

[18] Type-dependent signal spaces and continuous strategy sets preclude a direct reference to the standard definition of a perfect Bayesian equilibrium in a multi-stage game with observable actions (Fudenberg and Tirole, 1991, p. 331). Otherwise, however, the definition is standard.

Definition 1. *A probabilistic contest of incomplete information will be called* ***uniformly asymmetric*** *if, for any pair of posterior beliefs* $(\mu_1, \mu_2) \in \Delta(C_1) \times \Delta(C_2)$,

(i) all types $c_1 \in \text{supp}(\mu_1)$ *are active at the contest stage; and*

(ii) if all types $c_2 \in \text{supp}(\mu_2)$ *are active as well, then*

$$p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2} > p_2(\xi_2^*(c_2), \xi_1^*(c_1)) \qquad (c_1 \in \text{ supp}(\mu_1); c_2 \in \text{ supp}(\mu_2)). \tag{4}$$

Here, as usual, $\text{supp}(\mu_i) = \{c_i \in C_i : \mu_i(c_i) > 0\}$ denotes the *support* of player i's posterior belief μ_i, for $i \in \{1, 2\}$. Thus, in a uniformly asymmetric contest, two properties hold regardless of posterior beliefs at the contest stage. First, player 1 is active with probability one. Second, provided that player 2 is also active with probability one, player 1 is interim always (i.e., for all type realizations) more likely to win than player 2.[19]

If the contest is of complete information (i.e., if $K_1 = K_2 = 1$), then being uniformly asymmetric is equivalent to what Dixit (1987) called an asymmetric contest. Correspondingly, we will henceforth refer to player 1 alternatively as the *favorite* and to player 2 as the *underdog*.

3.2 A sufficient condition

In this section, we derive a condition on the primitives of the model that is sufficient for a contest to be uniformly asymmetric. While the assumption is strong, it will allow us to capture a very clear and robust intuition.

Assumption 1. *The production function* h *has a bounded curvature* $\underline{\rho}$.[20] *Moreover, the net bias* $\gamma \equiv \gamma_2/\gamma_1$ *satisfies*

$$\gamma < \gamma^* \equiv \frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1} \cdot \begin{cases} \sigma & \text{if } \sigma \leq 1 \\ \sigma^{1/\underline{\rho}} & \text{if } \sigma > 1, \end{cases} \tag{5}$$

where $\sigma = \underline{c}_2/\overline{c}_1$, *and* $\pi_i = \sqrt{\underline{c}_i/\overline{c}_i}$ *for* $i \in \{1, 2\}$.

[19] To understand why the activity of all types of player 2 is presupposed in property (ii) of the definition, it should be noted that an inactive type of player 2 may, in general, dilute the marginal incentives of a strong player 1 so much that the probability ranking (4) could easily break down.

[20] The *curvature* $\underline{\rho} = \underline{\rho}(h)$ corresponds to the smallest ρ for which h is ρ-convex (cf. Anderson and Renault, 2003). In the Tullock case, $\underline{\rho}(h^{\text{TUL}}) = 1/r$. In the lottery case, $r = 1$, and hence $\underline{\rho} = 1$. For background on generalized concavity, see Caplin and Nalebuff (1991a, 1991b).

Assumption 1 is a joint restriction on four parameters, each of which admits an intuitive interpretation.[21] First, $\underline{\rho}$ measures the degree of noise in the contest technology, where a larger value corresponds to more noise. Second, σ captures player 1's *resolve* (Hurley and Shogren, 1998a, 1998b). For example, if $\sigma > 1$, then player 1 is always more efficient than player 2, and σ corresponds to player 1's worst-case relative cost advantage. Third, $\pi_i \in (0, 1]$ reflects the *predictability* of player i's marginal cost, where the maximum value of one corresponds to complete information about c_i. Fourth and finally, the *net bias* γ has an obvious interpretation, where $\gamma < 1$, for example, means that the contest technology is biased against player 2.

When positive, the threshold value γ^* is weakly declining in $\underline{\rho}$, as well as strictly increasing in σ, π_1, and π_2. Thus, for any given net bias, the assumption is more likely to hold when there is less noise, player 1's resolve is larger, or marginal costs are more predictable. In particular, we see that, if Assumption 1 holds for a given contest, changes to the information structure caused by pre-play disclosure decisions cannot invalidate it. For instance, if either C_1 or C_2 is substituted by a nonempty subset, then σ, π_1, and π_2 all rise weakly, so that the cut-off value for the bias, γ^*, likewise rises weakly. Thus, if the assumption holds for type sets C_1 and C_2, then it holds also for any pair of nonempty subsets. A similar remark applies to any updating of beliefs.[22]

In the limit case of complete information and symmetric costs (i.e., $\underline{c}_1 = \bar{c}_1 = \underline{c}_2 = \bar{c}_2$), Assumption 1 says that the technology is biased against player 2 (i.e., $\gamma_2 < \gamma_1$). Further, the case of a biased contest with ex-ante symmetric type distributions (i.e., $\underline{c}_1 = \underline{c}_2 \leq \bar{c}_1 = \bar{c}_2$), as discussed, e.g., by Drugov and Ryvkin (2017), is not generally excluded by Assumption 1.[23]

Clearly, with Assumption 1 in place, player 1 is in a quite strong position relative to player 2. And indeed, as the following result shows, the assumption implies that the contest is uniformly asymmetric.

Lemma 2. (Sufficient condition) *Any incomplete-information contest that satisfies Assumption 1 is uniformly asymmetric.*

[21] The specific form of inequality (5) has been derived from the proof of Lemma 2 below and thus constitutes a sufficient but not necessary condition for the contest to be uniformly asymmetric.

[22] Assumption 1 does not impose any activity conditions. In general, corner solutions are known to be consistent with the existence of a perfect Bayesian equilibrium with no revelation of private information (Okuno-Fujiwara et al., 1990, Ex. 4). In our framework, however, this problem does not occur.

[23] Indeed, in this case, $\gamma^* = \frac{(3\pi-2)\pi^2}{2-\pi}$, with $\pi \equiv \pi_1 = \pi_2 = \sqrt{\sigma}$. For example, for $\pi = 0.8$, we get $\gamma^* = 0.21$. However, as noted by a referee, in that case the additional assumption $\sigma > 4/9$ is needed to fulfill $\gamma < \gamma^*$.

3.3 Main result

We will use the term *full revelation* to characterize the perfect Bayesian equilibrium, or the perfect Bayesian equilibrium outcome, in which all types disclose their private information. The main result of the present paper is the following.

Theorem 1. (Strong-form disclosure principle) *In any uniformly asymmetric contest with pre-play communication of verifiable information, full revelation is the unique perfect Bayesian equilibrium outcome.*

Theorem 1 states that the strong-form disclosure principle applies to any uniformly asymmetric contest.

It is not hard to see that self-disclosure by all types is an equilibrium. Indeed, it suffices to specify off-equilibrium beliefs so that a player that surprises her opponent by concealing her private information is understood to be the *worst-case type*, i.e., the type that no other type would like to masquerade as.[24] In our setting, the worst-case types are the most efficient underdog and the least efficient favorite, respectively. Provided that off-equilibrium beliefs are specified in that skeptical way (Milgrom, 2008), it is optimal for all types to stick to self-disclosure.

Note that uniqueness is claimed for the equilibrium outcome only. To reveal all private information, it suffices that, for each player, all types except one disclose their private information. However, that multiplicity of perfect Bayesian equilibria is trivial since it does not affect the outcome of the contest.

For auctions with interdependent valuations (Benoît and Dubra, 2006; Tan, 2016), incentives to reveal a private signal are typically strongest at the bottom of the signal support of the common-value component. For instance, a bidder in an auction that learns that a purportedly original painting is not authentic has an incentive to share that information with the other bidders. Thus, as in the present analysis, voluntary disclosure is likely to occur whenever it reduces the opponent's incentives for bidding too aggressively.[25]

[24] This useful terminology is borrowed from Seidmann and Winter (1997) and Hagenbach et al. (2014).

[25] We conjecture that allowing for a common-value signal in the present set-up would lead to similar conclusions as the literature on unraveling in auctions has identified.

4. Understanding the unraveling result

This section discusses the mechanics underlying Theorem 1, dealing first with the underdog and, subsequently, with the favorite. The section closes with some discussion.

4.1 Benefits of self-disclosure for the underdog

We focus on the weakest type of the underdog, $\overline{c}_2$, assuming that there are at least two possible type realizations for c_2. Let $\xi^* = (\xi_1^*, \xi_2^*)$ denote the equilibrium at the contest stage resulting if $\overline{c}_2$ does not disclose her type, where the probability of winning and the expected payoff for $\overline{c}_2$ are given by $p_2^*(\overline{c}_2) = E_{c_1}[p_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1))]$ and $\Pi_2^*(\overline{c}_2) = E_{c_1}[\Pi_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1); \overline{c}_2)]$, respectively. Similarly, let $(\xi_1^\#, x_2^\#)$ denote the equilibrium in the contest with one-sided incomplete information that results if $\overline{c}_2$ reveals her type, where $\overline{c}_2$'s probability of winning and expected payoff are given by $p_2^\# = E_{c_1}[p_2(x_2^\#, \xi_1^\#(c_1))]$ and $\Pi_2^\# = E_{c_1}[\Pi_2(x_2^\#, \xi_1^\#(c_1); \overline{c}_2)]$, respectively. The following result summarizes the comparative statics of the equilibrium at the contest stage with respect to $\overline{c}_2$'s disclosure decision.

Proposition 1. (Self-disclosure by the weakest type of the underdog) *Suppose that, in a uniformly asymmetric contest, the underdog has at least two possible type realizations. Then, a unilateral disclosure by the weakest type of the underdog, $\overline{c}_2$,*

(i) induces $\overline{c}_2$ to strictly raise her effort, i.e., $x_2^\# > \xi_2^*(\overline{c}_2)$;

(ii) strictly raises $\overline{c}_2$'s interim probability of winning, i.e., $p_2^\# > p_2^*(\overline{c}_2)$ (even against any given type of player 1); and

(iii) strictly raises $\overline{c}_2$'s expected payoff, i.e., $\Pi_2^\# > \Pi_2^*(\overline{c}_2)$.

Thus, after revealing her relative weakness, the weakest type of the underdog behaves as if gaining confidence. She bids more aggressively and wins with a strictly higher probability. Moreover, the self-disclosure is always strictly beneficial for her.[26]

The proof of Proposition 1 is based on the monotonicity properties of best response mappings in uniformly asymmetric contests. Let $(X_i, \succeq_i)$ denote the set of player i's bid schedules

[26] The conclusions of Proposition 1 are immediate for any type of the underdog that is inactive in ξ^*. Indeed, disclosure is the only way for such types to ensure an active participation, a positive probability of winning, and a positive expected payoff. Thus, Proposition 1 shows that self-disclosure is optimal for $\underline{c}_2$ even if $\xi_2^*(\underline{c}_2) > 0$.

equipped with the product order.[27] Denote by $X_j^* \subseteq X_j$ the set of bid schedules ξ_j for player $j \in \{1,2\}$ that admit a unique maximizer $x_i \equiv \widetilde{\beta}_i(\xi_j; c_i) \in \mathbb{R}_+$ of the expected payoff function $x_i \mapsto E_{c_j}[\Pi_i(x_i, \xi_j(c_j); c_i)]$, for any $c_i \in C_i$ with $i \neq j$. Given $\xi_j \in X_j^*$, the bid schedule $\beta_i(\xi_j) = \widetilde{\beta}_i(\xi_j; \cdot) : C_i \to \mathbb{R}_+$ will be called the *best-response bid schedule* against ξ_j. As shown in the Appendix, the best-response bid schedule $\beta_i(\xi_j)$ is weakly declining in the type for any $\xi_j \in X_j^*$, and strictly so at positive bid levels. Moreover, the thereby defined *best-response mapping* $\beta_i : X_j^* \to X_i$ satisfies monotonicity properties under suitable *domain restrictions*.[28]

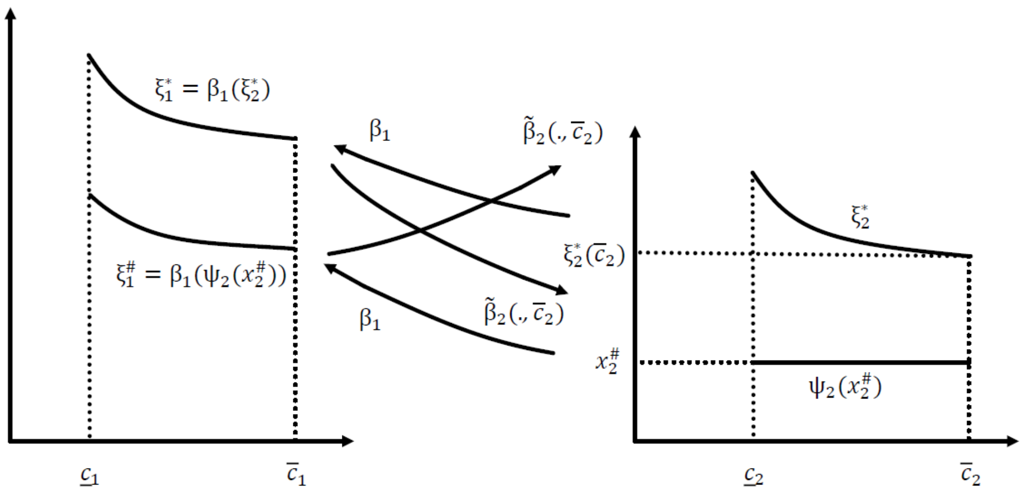


Figure 1. Proof of Proposition 1.

The fact that $\overline{c}_2$ raises her effort after self-disclosure is crucial. To understand this point, suppose that, instead of strictly raising her effort, $\overline{c}_2$ were to weakly lower her effort after disclosure, i.e., $x_2^\# \leq \xi_2^*(\overline{c}_2)$, as illustrated on the right-hand side of Figure 1 for the strict case. Consider now the *flat* bid schedule $\psi_2(x_2^\#) \in X_2$ that prescribes an effort of $x_2^\#$ for each $c_2 \in C_2$. Then, since there are at least two types in C_2, and since the equilibrium bid schedule ξ_2^* is strictly declining at positive bid levels (also recalling that $\xi_2^* \equiv 0$ is not feasible), we get $\xi_2^* \succ \psi_2(x_2^\#)$. From the strict monotonicity of player 1's best-response mapping, checking domain conditions, we therefore obtain $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^\#)) = \xi_1^\#$, as shown on the left-hand side of Figure 1. Applying

[27] Thus, given bid schedules $\xi_i, \widehat{\xi}_i \in X_i$, we write $\xi_i \succeq_i \widehat{\xi}_i$ if $\xi_i(c_i) \geq \widehat{\xi}_i(c_i)$ holds for any $c_i \in C_i$. Further, we will write $\xi_i \succ_i \widehat{\xi}_i$ if $\xi_i \succeq_i \widehat{\xi}_i$ and there is $c_i \in C_i$ such that $\xi_i(c_i) > \widehat{\xi}_i(c_i)$. The subscript i in $\succeq_i$ and $\succ_i$ will be dropped whenever there is no risk of ambiguity.

[28] These properties, which were documented by Dixit (1987) in the case of complete information, are verified in the Appendix. The comparative statics of complete-information contests has been studied by Jensen (2016) and Gama and Rietzke (2017), in particular.

now the strictly declining best-response mapping of $\overline{c}_2$, checking domain conditions also here, one arrives at $\xi_2^*(\overline{c}_2) = \widetilde{\beta}_2(\xi_1^*; \overline{c}_2) < \widetilde{\beta}_2(\xi_1^{\#}; \overline{c}_2) = x_2^{\#}$, which yields the desired contradiction. Thus, the weakest type of the underdog indeed raises her bid after self-disclosure.

Self-disclosure raises also the probability of winning for the weakest type of the underdog. This follows from what we call *Stackelberg monotonicity* in the complete-information model. By this, we mean that an increase of player i's bid, subject to an optimal response by the opponent j, always raises player i's winning probability (and strictly so in the interior). Intuitively, a higher effort is rewarded in terms of a higher winning probability.[29] Applied to the present situation, this says that a Stackelberg-leading underdog that raises her bid from $\xi_2^*(\overline{c}_2)$ to $x_2^{\#}$ strictly raises her probability of winning against any best-responding type c_1. Noting that the equilibrium bid $\xi_1^*(c_1)$ weakly exceeds c_1's best response to $\xi_2^*(\overline{c}_2)$, it follows that indeed, the probability of winning for the weakest type of the underdog against any c_1 rises strictly from her self-disclosure.

In a final step, it is shown that the weakest type of the underdog has a strict incentive to self-disclose. The proof we managed to come up with exploits type $\overline{c}_2$'s first-order condition to rewrite her expected payoff from the contest as a monotone function of ex-post winning probabilities and bids. For instance, in the Tullock contest with parameter r, type $\overline{c}_2$'s equilibrium payoff with and without disclosure may be represented as

$$\Pi_2^{\#}(\overline{c}_2) = E_{c_1}\left[\left(p_2^{\#}(x_2^{\#}, \xi_1^{\#}(c_1))\right)^2\right] + \frac{1-r}{r}\overline{c}_2 x_2^{\#}, \tag{6}$$

$$\Pi_2^*(\overline{c}_2) = E_{c_1}\left[(p_2^*(\xi_2^*(\overline{c}_2), \xi_1^*(c_1)))^2\right] + \frac{1-r}{r}\overline{c}_2 \xi_2^*(\overline{c}_2), \tag{7}$$

respectively. Given parts (i) and (ii) of Proposition 1, this suffices to prove the claim.

4.2 Benefits of self-disclosure for the favorite

Repeated application of Proposition 1 shows that the underdog's side of the contest equilibrium unravels. Let $c_2^{\#}$ denote the commonly known cost type of the underdog. Given one-sided incomplete information, we will now study the incentive of the strongest type of the favorite, $\underline{c}_1$, to disclose her private information.

[29] This property, for which we could not find a suitable reference, may be seen as an analogue of Dixit's (1987, Eq. 8) precommitment result. However, in contrast to that result, Stackelberg monotonicity holds regardless of contestants' relative strengths.

If type $\underline{c}_1$ decides to conceal her private information, then the ensuing contest is one of one-sided incomplete information, with equilibrium efforts $\xi_1^{\#}(\underline{c}_1) \equiv \xi_1^{\#}(\underline{c}_1; c_2^{\#})$ and $x_2^{\#} \equiv x_2^{\#}(c_2^{\#})$. Type $\underline{c}_1$'s probability of winning and expected payoff are consequently given by $p_1^{\#} = p_1(\xi_1^{\#}(\underline{c}_1), x_2^{\#})$ and $\Pi_1^{\#} = \Pi_1(\xi_1^{\#}(\underline{c}_1), x_2^{\#}; \underline{c}_1)$, respectively. If, however, type $\underline{c}_1$ decides to disclose her private information, then the contest is one of complete information, with equilibrium efforts $x_i^{\circ} \equiv x_i^{\circ}(\underline{c}_1, c_2^{\#})$, for $i = 1, 2$. In that case, type $\underline{c}_1$'s probability of winning and expected payoff are given by $p_1^{\circ} = p_1(x_1^{\circ}, x_2^{\circ})$ and $\Pi_1^{\circ} = \Pi_1(x_1^{\circ}, x_2^{\circ}; \underline{c}_1)$, respectively. The following result summarizes the comparative statics of the one-sided incomplete-information contest with respect to a revelation by $\underline{c}_1$.

Proposition 2. (Self-disclosure by the strongest type of the favorite) *Suppose that, in a uniformly asymmetric contest, the type of the underdog is public information, while the favorite has at least two possible type realizations. Then, a unilateral disclosure by the strongest type of the favorite, $\underline{c}_1$,*

(i) induces the underdog to strictly lower her effort, i.e., $x_2^{\circ} < x_2^{\#}$;

(ii) allows $\underline{c}_1$ to strictly lower her effort, i.e., $x_1^{\circ} < \xi_1^{\#}(\underline{c}_1)$;

(iii) strictly raises $\underline{c}_1$'s probability of winning, i.e., $p_1^{\circ} > p_1^{\#}$; and

(iv) strictly raises $\underline{c}_1$'s expected payoff, i.e., $\Pi_1^{\circ} > \Pi_1^{\#}$.

Thus, if the type of the underdog is public, then the self-revelation by the strongest type of the favorite discourages the underdog. As a result, the strongest type of the favorite exerts a lower effort, but still wins with higher probability. While the proof of Proposition 2 employs the same methods that have been used before, the argument is of course much simpler in this case.[30]

An iterated application of Proposition 2 implies that also the favorite's side unravels. Thus, in a uniformly asymmetric contest, full revelation is the only outcome consistent with the assumption of perfect Bayesian rationality. But, as already discussed, self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium, which yields the conclusion of Theorem 1.

[30] Part (ii) of Proposition 2 holds also in the case of two-sided incomplete information. Beyond this observation, however, the analogy to Proposition 1 is incomplete. In fact, we conjecture that parts (iii) and (iv) of Proposition 2 do not generalize to a setting with two-sided incomplete information. Below, we will derive a variant of Proposition 2 that holds even if the contest is not uniformly asymmetric.

4.3 Discussion: Dominance and defiance[31]

As mentioned in the Introduction, the reason why the proof of Theorem 1 is not as straightforward as one might expect is that, in general, a unilateral disclosure of some type may cause some types of the opponent to raise their bids. For intuition, note that there are two countervailing effects. On the one hand, following the self-disclosure by $\overline{c}_2$, say, the favorite's belief collapses, inducing her to lower her bid. On the other hand, $\overline{c}_2$ raises her bid, which induces the favorite to do the same. As a result, the overall effect of the underdog's self-disclosure on the bid of a given type of the favorite is ambiguous. The situation is similar for the underdog who, under two-sided incomplete information, may either drop out, lower her bid, or raise her bid in response to the favorite's self-disclosure. In the Appendix, we illustrate dominant and defiant reactions to self-disclosure using numerical examples and relate those anomalies to general instability properties of probabilistic contests (Wärneryd, 2018).

5. Extensions

In this section, we discuss a variety of extensions of Theorem 1. To keep the exposition as non-technical as possible, most of the formal results and derivations underlying the discussion have been moved to the Appendix.

5.1 Correlated types

The conclusion of Theorem 1 is robust with respect to the introduction of correlation between contestants' types. To fix ideas, suppose that, as a result of *positive* correlation that renders stronger types of the underdog more pessimistic, some type of the underdog other than the weakest type, say $\widehat{c}_2$, submits the lowest bid in the contest. Then, the argument underlying Proposition 1 will go through with $\widehat{c}_2$ replacing $\overline{c}_2$—unless we are in the non-generic scenario in which correlation induces *all* types of the underdog to choose the same bid. Thus, even though the literal conclusions of Proposition 1 may break down with correlated types, straightforward variants of the proposition hold, either for sufficiently small correlation, for negative correlation, or generically. Once the side of the underdog has unraveled, however, the contest is one of one-sided incomplete information, and the argument proceeds as before.

[31] Of course, we acknowledge the crucial role played by emotions in contests (e.g., Kräkel, 2008). Illustrations of dominant and defiant behavior can be found in Caygill (2013).

5.2 Partitional disclosures

In the main analysis, we assumed that disclosure is "all-or-nothing". However, in many cases, contestants may have more control over the information they choose to disclose than what has been assumed so far. In a model with pre-play partitional disclosure of the state space, Hagenbach et al. (2014) identified necessary and sufficient conditions for the existence of a fully revealing sequential equilibrium with "extremal" off-equilibrium beliefs that implements a given Nash equilibrium action profile on and off the equilibrium path. Our main result continues to hold if contestants' message correspondences each contain an *evidence base*. For example, the disclosure decision might alternatively establish an upper (lower) bound for the favorite's (the underdog's) cost parameter. It should be immediate to see that the unraveling argument underlying Theorem 1 extends along these lines to the more general framework of partitional disclosures.[32]

5.3 Randomized revelations

Allowing for randomized revelations does not change the conclusion of Theorem 1. Indeed, a type's randomized decision regarding self-disclosure cannot be more profitable than a pure decision. Therefore, full revelation remains an equilibrium outcome. But we also note that the unraveling is inevitable when players may use randomized revelations because the self-disclosure of the relevant extremal type remains strictly optimal regardless of conditional type distributions.

5.4 Sequential moves

Theorem 1 continues to hold when the revelation stage is replaced by a sequential-move game in which the disclosure decision is made first by the favorite and then, after observation, by the underdog. Intuitively, in any equilibrium in which two or more types of the favorite pool with positive probability in the same information set, the underdog's type will be revealed by Proposition 1. Anticipating this, the strongest type of the favorite strictly prefers to self-disclose instead of pooling with any weaker type. But also in the case where the underdog moves first, we show in the Appendix that full revelation remains the unique perfect Bayesian equilibrium outcome in the lottery contest.[33]

[32] As we further show in the Appendix, hard evidence may also be released if it is only an imperfect signal about the type, provided that the signal is stochastically monotone.

[33] The crucial payoff comparison is derived using a second-moment refinement of Jensen's inequality that is stated and proved in the Appendix as well.

5.5 One-sided incomplete information

If just one of the contestants is privately informed, then the assumption of uniform asymmetry is typically not needed to obtain full revelation as the unique equilibrium outcome.

Theorem 2. (One-sided incomplete information) *Consider a probabilistic contest with one-sided incomplete information and generic types. Then, the conclusion of Theorem 1 continues to hold true.*

While the result holds generally, the intuition for the proof is most transparent in the special case of the Tullock contest. Two observations are important, both of which are derived from the first-order conditions. First, in equilibrium, the expenses of the uninformed contestant, $c_i^\# x_i^\#$, correspond precisely to the expected expenses of the informed contestant, $E_{c_j}[c_j \xi_j^\#(c_j)]$. Second, the type-specific expenses of the informed contestant, $c_j \xi_j^\#(c_j)$, are strictly hump-shaped as a function of the type. Combining these observations, there is always at least one extremal type of the informed contestant, either $\underline{c}_j$ or $\overline{c}_j$, such that making that type *marginally* more likely lowers the expected expenses of the uninformed contestant. Which of the two extremal types has this property depends only on the three parameters $\underline{c}_j$, $\overline{c}_j$, and $c_i^\#$, regardless of the distribution of probabilities. Specifically, the efficient type $\underline{c}_j$ has a strict incentive to self-disclose if $c_i^\# > \sqrt{\underline{c}_j \overline{c}_j}$, while the inefficient type $\overline{c}_j$ has a strict incentive to self-disclose if $c_i^\# < \sqrt{\underline{c}_j \overline{c}_j}$. Thus, replacing the assumption of uniform asymmetry by one-sided incomplete information, there is an additional twist to the logic of the unraveling argument. Rather than following the linear ordering of types on each side of the contest, the unraveling may now follow a "bang-bang" order, in the sense that extremal efficient and extremal inefficient types alternatingly find it strictly optimal to self-disclose conditional on hypothesized prior disclosures.[34]

5.6 Continuous type distributions

Benoît and Dubra (2006) have derived a general unraveling result for auctions and other Bayesian games that allows for multiple players and metric type spaces. That result may be used to extend Theorem 1 to the case of continuous type distributions.[35] Suppose that for $i \in \{1, 2\}$,

[34] Similar to the case of correlated types, a genericity assumption is needed here because different types of the informed player may choose the same bid.

[35] Contests with continuous type distributions have been considered, in particular, by Fey (2008), Ryvkin (2010), Wasser (2013a, 2013b), and Ewerhart (2014).

player i's marginal cost is drawn from an interval $[\underline{c}_i, \overline{c}_i]$, with $0 < \underline{c}_i < \overline{c}_i$, according to some continuous distribution function F_i. Both Definition 1 and Lemma 2 extend to this case, provided that the probability ranking property (4) is required for any pair of cost realizations in the support of players' posterior beliefs. Considering now a uniformly asymmetric lottery contest with independent types, contestants' types are almost surely revealed in any perfect Bayesian equilibrium.

5.7 Private information about valuations and ability

We assumed above that private information concerns marginal cost only. To accommodate more general forms of uncertainty, suppose that player i's private information is instead summarized in the vector $\theta_i = (c_i, V_i, L_i, \gamma_i)$, where the components satisfy the same restrictions as before.

Proposition 3. *Assuming that private information is exclusively about marginal cost is without loss of generality if at least one of the two following conditions is satisfied.*

(i) γ_1 *and* γ_2 *are public information;*

(ii) $h(y) = y^r$ *for some* $r \in (0, 1]$.

Part (i) captures the (well-known) equivalence between uncertainty about marginal cost and valuations. Part (ii) says that, in the case of the Tullock contest, our analysis covers multi-dimensional uncertainty without restriction. This is an important point because, as noted by a referee, in many real-world contests, verifiable evidence may not concern contestants' preferences but instead the likelihood to win.

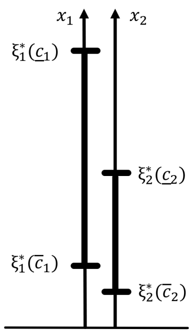
6. Limits of the scope of the disclosure principle

The strong-form disclosure principle is not universally valid in probabilistic contests. As will be shown in this section, the principle may break down if the contest is not uniformly asymmetric or if contestants have commitment power. Moreover, the principle never holds in probabilistic contests if information is unverifiable.

6.1 Contests that are not uniformly asymmetric

The conclusion of Theorem 1 may fail if the contest is not uniformly asymmetric. In line with the intuition provided in the Introduction, we outline below a numerical example of a probabilistic contest that is not uniformly asymmetric and in which the perfect Bayesian equilibrium outcome need not be fully revealing. In view of Theorem 2, any counterexample of this sort necessarily features two-sided asymmetric information.

Example 1. (Countervailing incentives) In an unbiased lottery contest specified by the parameters given in Figure 2, all four types are active, and the strongest type of each player outbids the weakest type of her respective opponent. Thus, the contest is *not* uniformly asymmetric. None of the efficient types, neither $\underline{c}_1$ nor $\underline{c}_2$, has an incentive to self-disclose because that would induce their respective efficient counterpart to bid higher. This would be undesirable because, in a probabilistic contest, *the payoff impact of an increase in the opponent's bid is strongest if the competing bids are in a similar range.* The situation is similar for the inefficient types, $\overline{c}_1$ and $\overline{c}_2$, who likewise do not wish to trigger a higher bid by their respective inefficient counterpart. And indeed, as the data in Figure 2(b) shows, full concealment turns out to be a perfect Bayesian equilibrium in this contest.



(*a*)

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 0.5$	$c_1^2 = 5.5$ $q_1(c_1^2) = 0.5$	$c_2^1 = 2$ $q_2(c_2^1) = 0.9$	$c_2^2 = 6.5$ $q_2(c_2^2) = 0.1$
$\xi_1^*(c_1^1) = 0.2090$ $\Pi_1^*(c_1^1) = 0.4825$	$\xi_1^*(c_1^2) = 0.0357$ $\Pi_1^*(c_1^2) = 0.0926$	$\xi_2^*(c_2^1) = 0.1031$ $\Pi_2^*(c_2^1) = 0.3303$	$\xi_2^*(c_2^2) = 0.0265$ $\Pi_2^*(c_2^2) = 0.0970$
$x_1^\# = 0.2066$ $\Pi_1^\# = 0.4719$	–	$\xi_2^\#(c_2^1) = 0.1147$	$\xi_2^\#(c_2^2) = 0.0000$
–	$x_1^\# = 0.0440$ $\Pi_1^\# = 0.0916$	$\xi_2^\#(c_2^1) = 0.0988$	$\xi_2^\#(c_2^2) = 0.0342$
$\xi_1^\#(c_1^1) = 0.2171$	$\xi_1^\#(c_1^2) = 0.0343$	$x_2^\# = 0.1015$ $\Pi_2^\# = 0.3299$	–
$\xi_1^\#(c_1^1) = 0.1421$	$\xi_1^\#(c_1^2) = 0.0437$	–	$x_2^\# = 0.0294$ $\Pi_2^\# = 0.0956$

(*b*)

Figure 2. Data for Example 1.

Thus, the assumption of uniform asymmetry cannot be easily dropped without losing the strong-form disclosure principle in probabilistic contests with two-sided incomplete information.[36]

6.2 Commitment power and Bayesian persuasion

Self-disclosure of one type need not be in the interest of other types. If, for example, the strongest type of the favorite reveals her private information in a uniformly asymmetric contest, then

[36] While the bid intervals are overlapping in Example 1, it is likewise feasible to construct counterexamples with either nested or symmetric bid intervals. See the Appendix.

the competition for the remaining types of the favorite will typically become tougher. Thus, a contestant may be worse off as a result of voluntary disclosure. Following Kamenica and Gentzkow (2011), one may assume that each contestant possesses commitment power that allows her to follow a communication strategy optimized from an ex-ante perspective. As we show in the Appendix, however, a contestant's optimal Bayesian persuasion strategy may take different forms, including not only full disclosure, but also full concealment, or even the use of randomized signals. For example, given precommitment to a signal, an efficient type of the underdog may occasionally pool with an inefficient type. Conversely, a contestant might seek ways to avoid receiving messages by shutting down communication channels.

6.3 Cheap talk[37]

In Crawford and Sobel's (1982) model of cheap talk, pre-play messages released at the interim stage are assumed to be unverifiable.[38] More generally, in a communication equilibrium (Myerson, 1982), each player reports private information to a mediator in an unverifiable way. Having received the reports, the mediator follows her precommitted instructions and releases a recommendation to each of the players via a bilateral communication channel. Then, based on the recommendation and her type, each player chooses a strategy. For example, the mediator may be precommitted to relay the reports unchanged to the respective other party, which would correspond to cheap talk. In the case of the two-player all-pay auction, Pavlov (2013) identified assumptions under which every communication equilibrium is interim payoff equivalent to the Bayesian equilibrium. The following result is an analogous observation for probabilistic contests.

Theorem 3. (Babbling) *In any communication equilibrium of a probabilistic contest, all recommendations will be ignored.*

Thus, cheap talk in probabilistic contests is necessarily ineffective, even if intermediated by a trustworthy third party. If the contest were a zero-sum game, then this observation would be obvious. Indeed, as noted by Farrell (1985), a player should be very suspicious to make use of information provided by another player with completely opposite preferences. However, probabilistic contests

[37] We are indebted to the Editor for suggesting this interesting extension.

[38] Fey et al. (2007) studied cheap talk in games with two-sided incomplete information and identified a role for complementarity vs. substitutability. While interesting, their results do not resolve our research question.

are not zero-sum, but only strategically zero-sum (Moulin and Vial, 1978). To understand why, one notes that adding the expenses of a player to the other player's payoff function does not change marginal incentives. Therefore, in principle, there might be communication strategies mutually beneficial for both players to select Pareto superior outcomes. However, the Bayesian equilibrium is unique by Lemma 1, which turns out to imply that unverifiable communication is ineffective.[39]

7. Efficiency

What are the welfare implications of information disclosure in probabilistic contests? In the Appendix, we show with the help of an example that, in the absence of commitment power on the part of the contestants, the unraveling may lead into a "disclosure trap." By this, we mean an outcome in which the ex-ante expected payoff for both contestants is strictly lower than under mandatory concealment. Thus, in contrast to the more common situation in which the receiver in a persuasion game, such as an employer, a consumer, or a health insurer, tends to benefit from the unraveling, this need not be the case in a contest. However, many real-world contests have informational and allocational externalities on third parties. It may, therefore, be short-sighted to limit the welfare discussion solely to the expected payoffs of the contestants. For example, if contests are organized to maximize total expected expenses, and if information disclosures lower expected payoffs by tightening the competition, then that may well be desirable from the organizer's point of view.[40]

8. Conclusion

In this paper, we have identified general and robust conditions under which a probabilistic contest with verifiable pre-play communication admits full disclosure as the unique perfect Bayesian equilibrium outcome. Given that the usual assumptions for the uniqueness of the fully revealing

[39] In his analysis of sender-receiver games with two-sided incomplete information, Seidmann (1990, Ex. 1) showed that, even if all types of the sender share the same preferences over pure effort choices by the receiver, the sender types' preferences regarding lotteries over efforts may differ. Even though our model has precisely this property, Theorem 3 shows that equilibria in which receivers react to information are not feasible in probabilistic contests. And indeed, as we checked numerically, the Tullock contest does not satisfy Seidman's condition for $r \in (0, 1]$.

[40] As argued by Denter et al. (2014), Zhang and Zhou (2016), and Serena (2022), contest organizers may be able to influence contestants' beliefs about each other by information design. In the Appendix, we advance theory by characterizing optimal signals under a variety of policy objectives. In particular, we provide conditions under which the *delegation* to an informed contestant is optimal.

equilibrium outcome (Milgrom, 1981; Okuno-Fujiwara et al., 1990; Seidmann and Winter, 1997; van Zandt and Vives, 2007) fail to hold for contests, our results mean an extension of existing theory. In particular, the strong-form disclosure principle is more general than previously perceived. In addition, the analysis has formalized several intuitive concepts for which, to our knowledge, a flexible and all-encompassing framework in the realm of contest theory has been lacking so far. In sum, the analysis sheds further light on the incentives for communication of both verifiable and unverifiable information in competitive situations. This is not only desirable from the perspective of economic theory, but might also facilitate both the mitigation of harmful conflict and the efficient design of real-world contests.[41]

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[41] We did not answer all research questions regarding communication in contests. For example, it would be interesting to extend the analysis to contests with more than two players, to deal with common-value components and interdependent valuations, or to consider some form of dynamics. Tackling such questions seems both interesting and feasible.

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ONLINE APPENDIX

This appendix contains material that has been omitted from the body of the paper. The appendix is organized as follows.

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A. Material omitted from Section 2

Below, we outline the formal details regarding the Bayesian updating as well as the proof of Lemma 1.

A.1 Bayesian updating
Fix a contestant $i \in \{1, 2\}$, and suppose given a set of revealing types, $S_i \subseteq C_i$. Then, there are three scenarios: (i) Suppose first that player i discloses $c_i \in C_i$. Then, player i is believed to be of type c_i with probability one, i.e., $\mu_i(c_i) = 1$. (ii) Next, suppose that player i does not disclose her type, and that player i's decision to not disclose is a possibility on the equilibrium path, i.e., $S_i \subsetneq C_i$. Then, c_i is expected to be in the set-theoretic complement of S_i. Hence, by Bayes' rule, $\mu_i(c_i) = q_i(c_i)/\sum_{c'_i \in C_i \backslash S_i} q_i(c'_i)$ if $c_i \in C_i \backslash S_i$, while $\mu_i(c_i) = 0$ if $c_i \in S_i$. (iii) Finally, suppose that player i does not disclose her type, and that i's decision to not disclose is an off-equilibrium event, i.e., $S_i = C_i$. Then, the belief about player i may be specified by any $\mu_i = \mu_i^0 \in \Delta(C_i)$.

A.2 Proof of Lemma 1
Lemma 1 concerns the existence and uniqueness of the Bayesian equilibrium at the contest stage.

Proof of Lemma 1. This is a special case of a result in Ewerhart and Quartieri (2020). □

Lemma 1 extends to randomized bids. Indeed, since each player is active with positive probability, and payoffs functions are own-bid l.s.c. at the origin, expected payoffs against the opponent's equilibrium strategy are strictly concave over R_+, so that it is suboptimal to randomize strictly.

B. Material omitted from Section 3

This section presents three auxiliary results and the proof of Lemma 2.

B.1 Wärneryd's transformation

The function introduced in the following lemma arises naturally in the first-order conditions.[1]

Lemma B.1 (Wärneryd's transformation) *Let $\Phi(z) = h(z)/h'(z)$, for $z > 0$. Then, the following holds true: (i) $\lim_{z\searrow 0}\Phi(z) = 0$; (ii) $1 \leq \Phi' \leq \underline{\rho}$; (iii) $(d\ln h)/(d\ln\Phi) = 1/\Phi'$; (iv) if $x_i > 0$, then player i's best-response mapping in the complete-information contest is differentiable with*

$$\frac{dx_i}{dx_j} = \frac{\Phi(x_i)}{\Phi(x_j)}\frac{2p_i - 1}{\Phi'(x_i) - 1 + 2p_i}, \tag{B.1}$$

where $i, j \in \{1, 2\}$ with $j \neq i$, and $p_i = p_i(x_i, x_j)$.

Proof. (i) By assumption, h is differentiable in the interior of the strategy space, with h' positive and declining. Hence, $\lim_{z\to 0} h'(z) \in (0, \infty]$. Moreover, by continuity, $\lim_{z\to 0} h(z) = 0$. The claim follows. (ii) Note first that $\Phi' = 1 - (hh''/(h')^2) \geq 1$ by the concavity of h. To see that $\Phi' \leq \underline{\rho}$, take some $\rho > \underline{\rho}$ such that h^ρ is convex. Then, in the interior of the strategy space, $\rho(\rho - 1)h^{\rho-2}(h')^2 + \rho h^{\rho-1}h'' \geq 0$. Recall that $\underline{\rho} \geq 1$. Hence, $\rho > 1$. Dividing by $\rho h^{\rho-2}(h')^2 > 0$, and rearranging, one obtains $\Phi' \leq \rho$. Taking the limit $\rho \to \underline{\rho}$, the claim follows. (iii) A straightforward calculation shows that

$$\frac{d\ln h(z)}{d\ln\Phi(z)} = \left(\frac{dh(z)}{h(z)}\right)\Big/\left(\frac{d\Phi(z)}{\Phi(z)}\right) = \frac{h'(z)dz}{h(z)}\cdot\frac{\Phi(z)}{\Phi'(z)dz} = \frac{1}{\Phi'(z)} \qquad (z > 0), \tag{B.2}$$

as claimed. (iv) The first-order condition characterizing the best response x_i reads $p_i(1 - p_i) = c_i\Phi(x_i)$. Total differentiation delivers $(1 - 2p_i)dp_i = c_i\Phi'(x_i)dx_i$, where

$$dp_i = \frac{p_i(1 - p_i)}{\Phi(x_i)}dx_i - \frac{p_i(1 - p_i)}{\Phi(x_j)}dx_j = c_i dx_i - c_i\frac{\Phi(x_i)}{\Phi(x_j)}dx_j. \tag{B.3}$$

Simplifying, we obtain (B.1). □

B.2 Monotonicity of best-response bid schedules

Best-response bid schedules are monotone declining in marginal cost, and strictly so in the interior.

Lemma B.2 (Monotonicity of best-response bid schedules) *Let $\xi_j \in X_j^*$ and $c_i, \widehat{c}_i \in C_i$ for $i \neq j$ such that $c_i > \widehat{c}_i$. Then, $\widetilde{\beta}_i(\xi_j; c_i) \leq \widetilde{\beta}_i(\xi_j; \widehat{c}_i)$, where the inequality is strict if $\widetilde{\beta}_i(\xi_j; \widehat{c}_i) > 0$.*

Proof. Take a bid schedule $\xi_j \in X_j^*$. The assertion is obvious for $\widetilde{\beta}_i(\xi_j; c_i) = 0$. Suppose instead that $x_i \equiv \widetilde{\beta}_i(\xi_j; c_i) > 0$. Then, from c_i's first-order condition, $\partial E_{c_j}[p_i(x_i, \xi_j(c_j))]/\partial x_i = c_i$. We will first show first that the left-hand side of this equation is strictly declining in x_i. Indeed, because the best-response bid $\widetilde{\beta}_i(\xi_j; c_i)$ exists, there is some $c_j \in C_j$ such that $\xi_j(c_j) > 0$. A straightforward calculation shows, therefore, that

$$\frac{\partial^2 E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i^2} = \frac{\partial}{\partial x_i}E_{c_j}\left[\frac{\gamma_i\gamma_j h'(x_i)h(\xi_j(c_j))}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^2}\right] \tag{B.4}$$

$$= E_{c_j}\left[\frac{\gamma_i\gamma_j h(\xi_j(c_j))\left\{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))h''(x_i) - 2\gamma_i(h'(x_i))^2\right\}}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^3}\right] < 0, \tag{B.5}$$

[1] Cf. Wärneryd (2003) and Inderst et al. (2007).

which proves the claim. There are now two cases. Assume first that $\widehat{x}_i > 0$. For this case, it is claimed that $\widehat{x}_i > x_i$. To provoke a contradiction, suppose that $\widehat{x}_i \leq x_i$. Then, since the marginal probability of winning for player i is strictly declining in i's bid, $\widehat{c}_i = \partial E_{c_j}[p_i(\widehat{x}_i, \xi_j(c_j))]/\partial x_i \geq \partial E_{c_j}[p_i(x_i, \xi_j(c_j))]/\partial x_i = c_i$, in conflict with $\widehat{c}_i < c_i$. Hence, $\widehat{x}_i > x_i$, as claimed. Assume next that $\widehat{x}_i = 0$, i.e., type $\widehat{c}_i$ finds it optimal to respond to ξ_j with a zero effort. But then, clearly, strictly higher marginal costs induce type c_i to do the same, i.e., $x_i = 0$. The lemma follows. □

B.3 Bounds on the bid distributions

From the first-order conditions, we derive upper and lower bounds on active contestants' bid distributions.

Lemma B.3 (Bounds on the bid distributions) *Let $\xi^* = (\xi_1^*, \xi_2^*)$ be a Bayesian equilibrium in an incomplete-information contest such that both players are active with probability one. Then,*

$$\begin{aligned} \gamma_i h(\xi_i^*(\underline{c}_i)) &\leq \frac{1}{\pi_i} \cdot \gamma_i h(\xi_i^*(\overline{c}_i)) + \frac{1-\pi_i}{\pi_i} \cdot \gamma_j h(\xi_j^*(\underline{c}_j)) \quad (i, j \in \{1, 2\},\, j \neq i), & \text{(B.6)} \\ h(\xi_2^*(\overline{c}_2)) &\leq \frac{1}{\widehat{\sigma}} \cdot h(\xi_1^*(\underline{c}_1)), & \text{(B.7)} \end{aligned}$$

where $\widehat{\sigma} = \sigma$ if $\sigma \leq 1$ and *$\widehat{\sigma} = \sigma^{1/\varrho}$ if $\sigma > 1$.*

Proof. Take an arbitrary type $c_i \in C_i$ of player i. Since, by assumption, $\xi_i^*(c_i) > 0$, the necessary first-order condition for type c_i holds, i.e.,

$$E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(c_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(c_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] - c_i = 0, \tag{B.8}$$

where $j \neq i$. To prove the first claim, evaluate (B.8) at $c_i = \overline{c}_i$. Then, making use of Lemma B.2 and the concavity of h, we get

$$\begin{aligned} \overline{c}_i &= E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\overline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}\right)^2\right] & \text{(B.9)} \\ &= E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\overline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \cdot \underbrace{\left(1 + \frac{\gamma_i h(\xi_i^*(\underline{c}_i)) - \gamma_j h(\xi_i^*(\overline{c}_i))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}\right)^2}_{\text{monotone increasing in } c_j}\right] & \text{(B.10)} \\ &\geq E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\overline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2 & \text{(B.11)} \\ &= E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\underline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] \times \underbrace{\left(\frac{h'(\xi_i^*(\overline{c}_i))}{h'(\xi_i^*(\underline{c}_i))}\right)}_{\geq 1} \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2 & \text{(B.12)} \\ &\geq \underline{c}_i \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2. & \text{(B.13)} \end{aligned}$$

Dividing by $\underline{c}_i > 0$, and using $\pi_i = \sqrt{\underline{c}_i/\overline{c}_i}$, we obtain

$$\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \leq \frac{1}{\pi_i}. \tag{B.14}$$

Inequality (B.6) follows. To prove the second claim, one multiplies type c_i's first-order condition (B.8) by $\Phi(\xi_i^*(c_i))$, and subsequently takes expectations. This yields

$$E_{c_i}[c_i \Phi(\xi_i^*(c_i))] = E_{c_1,c_2}\left[\frac{\gamma_1\gamma_2 h(\xi_1^*(c_1))h(\xi_2^*(c_2))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2}\right] \qquad (i = 1, 2), \tag{B.15}$$

where $E_{c_1,c_2}[.]$ denotes the ex-ante expectation. Exploiting the fact that equilibrium bid schedules are monotone declining (by Lemma B.2), and that $\Phi' > 0$, this implies

$$\underline{c}_2 \Phi(\xi_2^*(\overline{c}_2)) \le E_{c_2}[c_2 \Phi(\xi_2^*(c_2))] = E_{c_1}[c_1 \Phi(\xi_1^*(c_1))] \le \overline{c}_1 \Phi(\xi_1^*(\underline{c}_1)), \tag{B.16}$$

or, using that $\Phi(\xi_2^*(\overline{c}_2)) > 0$,

$$\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\overline{c}_2))} \ge \frac{\underline{c}_2}{\overline{c}_1} = \sigma. \tag{B.17}$$

There are two cases. Assume first that $\xi_1^*(\underline{c}_1) \ge \xi_2^*(\overline{c}_2)$. Then, using $\Phi' \le \underline{\rho}$ (see Lemma B.1), we obtain

$$\begin{aligned}
\ln\left(\frac{h(\xi_1^*(\underline{c}_1))}{h(\xi_2^*(\overline{c}_2))}\right) &= \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} d\ln h(z) && \text{(B.18)}\\
&= \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{d\ln h(z)}{d\ln \Phi(z)} d\ln \Phi(z) && \text{(B.19)}\\
&= \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{1}{\Phi'(z)} d\ln \Phi(z) && \text{(B.20)}\\
&\ge \frac{1}{\underline{\rho}} \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} d\ln \Phi(z) && \text{(B.21)}\\
&= \frac{1}{\underline{\rho}} \ln\left(\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\overline{c}_2))}\right). && \text{(B.22)}
\end{aligned}$$

Using (B.17), this implies $h(\xi_2^*(\overline{c}_2)) \le \sigma^{-1/\underline{\rho}} \cdot h(\xi_1^*(\underline{c}_1))$. Assume next that $\xi_1^*(\underline{c}_1) < \xi_2^*(\overline{c}_2)$. Then using $\Phi' \ge 1$ (taken likewise from Lemma B.1) delivers

$$\ln\left(\frac{h(\xi_2^*(\overline{c}_2))}{h(\xi_1^*(\underline{c}_1))}\right) = \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\overline{c}_2)} \frac{d\ln \Phi(z)}{\Phi'(z)} \le \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\overline{c}_2)} d\ln \Phi(z) = \ln\left(\frac{\Phi(\xi_2^*(\overline{c}_2))}{\Phi(\xi_1^*(\underline{c}_1))}\right). \tag{B.23}$$

Hence, in that case, $h(\xi_2^*(\overline{c}_2)) \le \frac{1}{\sigma} \cdot h(\xi_1^*(\underline{c}_1))$. Thus, exploiting that $\underline{\rho} \ge 1$, we see that $h(\xi_2^*(\overline{c}_2)) \le h(\xi_1^*(\underline{c}_1)) \cdot \max\{\sigma^{-1}, \sigma^{-1/\underline{\rho}}\}$. Clearly, this proves (B.7). □

B.4 Proof of Lemma 2

Next, we establish the condition sufficient for uniform asymmetry stated as Lemma 2.

Proof of Lemma 2. Lemma 2 is derived by combining several inequalities, all of which are derived from the first-order conditions necessary for players' bid schedules to be mutual best responses. Property (ii) of Definition 1 will be checked first. Suppose that all types of both players are active. There are two cases.

Case A. Suppose first that $\text{Supp}(\mu_1) = C_1$ *and* $\text{Supp}(\mu_2) = C_2$. We make use of Lemma B.3. Letting $i = 2$ in (B.6) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \le \frac{1}{\pi_2} \cdot \gamma_2 h(\xi_2^*(\overline{c}_2)) + \frac{1 - \pi_2}{\pi_2} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)). \tag{B.24}$$

Combining this with (B.7) delivers

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{ \frac{1}{\pi_2} \cdot \frac{\gamma}{\widehat{\sigma}} + \frac{1-\pi_2}{\pi_2} \right\}}_{\equiv \alpha} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)), \tag{B.25}$$

where $\gamma = \gamma_2/\gamma_1$, as before. Letting $i = 1$ in (B.6), and plugging the result into (B.25) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \alpha \cdot \left\{ \frac{1}{\pi_1} \cdot \gamma_1 h(\xi_1^*(\overline{c}_1)) + \frac{1-\pi_1}{\pi_1} \cdot \gamma_2 h(\xi_2^*(\underline{c}_2)) \right\}. \tag{B.26}$$

To be able to solve for $\gamma_2 h(\xi_2^*(\underline{c}_2))$, we assume for the moment that

$$1 - \alpha \frac{1-\pi_1}{\pi_1} > 0. \tag{B.27}$$

Then, rewriting (B.26), we obtain

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{ \frac{\alpha \cdot \frac{1}{\pi_1}}{1 - \alpha \cdot \frac{1-\pi_1}{\pi_1}} \right\}}_{\equiv \lambda} \cdot \gamma_1 h(\xi_1^*(\overline{c}_1)). \tag{B.28}$$

Thus, $\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \lambda \cdot \gamma_1 h(\xi_1^*(\overline{c}_1))$. We claim that inequality (B.27) holds. Indeed, starting with Assumption 1, we find that

$$\gamma < \frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1} \cdot \widehat{\sigma} \quad \Leftrightarrow \quad \frac{\gamma}{\widehat{\sigma}} + 1 < \frac{2\pi_2}{2-\pi_1} \tag{B.29}$$

$$\Leftrightarrow \quad \underbrace{\frac{(\gamma/\widehat{\sigma}) + 1}{\pi_2}}_{=\alpha+1} < \underbrace{\frac{2}{2-\pi_1}}_{=\frac{\pi_1}{2-\pi_1}+1} \tag{B.30}$$

$$\Leftrightarrow \quad \alpha < \frac{\pi_1}{2-\pi_1} \tag{B.31}$$

$$\Leftrightarrow \quad 1 - \frac{\alpha(1-\pi_1)}{\pi_1} > \frac{\alpha}{\pi_1}. \tag{B.32}$$

Clearly, this implies (B.27). Moreover, it can be readily verified that (B.32) implies $\lambda < 1$. Therefore, $\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \gamma_1 h(\xi_1^*(\overline{c}_1))$. Using the monotonicity of equilibrium bid schedules (Lemma B.2 above), this yields $\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1))$ for any $c_1 \in C_1$ and $c_2 \in C_2$. This proves property (ii) in Definition 1 for the case that all types of both players conceal their private information.

Case B. Supp$(\mu_i) \subsetneq C_i$ *for some player* $i \in \{1, 2\}$. The conclusion remains valid even if not all types conceal. To understand why, note that disclosure by some types means that, in the relevant information set at the contest stage, the sets C_1 and C_2 are replaced by nonempty subsets, respectively. Therefore, player 1's lowest relative resolve $\sigma = \underline{c}_2/\overline{c}_1$ rises weakly. Given that the curvature $\underline{\rho} \geq 1$ stays unchanged, this implies that $\widehat{\sigma}(\sigma, \underline{\rho})$ rises weakly as well. Further, player 1 and 2's predictabilities π_1 and π_2 fall weakly, while the net bias γ stays the same. Therefore, Assumption 1 continues to hold, and the argument detailed under case A goes through as before.

This concludes the proof of property (ii) of Definition 1. It remains to verify property (i) of the definition of uniform asymmetry, i.e., that all types of player 1 are active. Suppose not. Then, all types of player 2 are active. Denote by $\varnothing \neq C_1^* \subsetneq C_1$ the set of active types of player 1, and by $q_1^* = \sum_{c_1 \in C_1^*} q_1(c_1)$ the ex-ante probability that player 1 is active. Then, since any positive bid wins against an inactive type with probability one, the corresponding terms in player 2's first-order condition vanish, so that

$$\sum_{c_1 \in C_1^*} q_1(c_1) \frac{\gamma_2 h'(\xi_2^*(c_2))\gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = c_2 \qquad (c_2 \in C_2). \tag{B.33}$$

In the modified contest, player 1's type set C_1 is replaced by the subset C_1^*, the probability distribution $q_1(.)$ is replaced by $q_1^*(c_1) = q_1(c_1)/q_1^*$, and player 2's type set C_2 is replaced by $C_2/q_1^* = \{c_2/q_1^* | \, c_2 \in C_2\}$. Denote by $\xi_1^*|_{C_1^*}$ the restriction of the mapping $\xi_1^* : C_1 \to \mathbb{R}_+$ to C_1^*, and by $\xi_2^*|_{q_1^*} : \frac{C_2}{q_1^*} \to \mathbb{R}_+$ the bid schedule for player 2 in the modified contest that satisfies $\xi_2^*|_{q_1^*}(\frac{c_2}{q_1^*}) = \xi_2^*(c_2)$ for any $c_2 \in C_2$. We claim that $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is a Bayesian equilibrium in the modified contest. Indeed, quite obviously, the first-order condition of any active type of player 1 holds in the modified contest. Moreover, dividing (B.33) by $q_1^* > 0$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(\xi_2^*(c_2))\gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = \frac{c_2}{q_1^*} \qquad (c_2 \in C_2), \tag{B.34}$$

i.e., also the first-order condition of any type of player 2 holds in the modified contest. Since all types of both players are active in $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ and since, in addition, the expected payoff against a player that is always active is strictly concave in the own bid, this proves the claim, i.e., $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is indeed a Bayesian equilibrium in the modified contest. Next, one notes that, since Assumption 1 holds for the original contest, Assumption 1 holds also for the modified contest (because π_1 and σ rise weakly, while γ, $\underline{\rho}$, and π_2 stay the same). From the first part of the proof, applied to the modified contest, it therefore follows that

$$\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1)) \qquad (c_1 \in C_1^*, c_2 \in C_2). \tag{B.35}$$

Now, by assumption, some types of player 1 remain inactive in the original contest. Since, by Lemma B.2, ξ_1^* is monotone declining, this clearly implies $\xi_1^*(\overline{c}_1) = 0$. Consequently, the marginal productivity at the zero bid level $h'(0) = \lim_{\varepsilon \searrow 0} \frac{h(\varepsilon)}{\varepsilon}$ is finite. Moreover, type $\overline{c}_1$'s marginal payoff at the zero bid level is weakly negative, i.e.,

$$E_{c_2}\left[\frac{\gamma_1 h'(0)}{\gamma_2 h(\xi_2^*(c_2))}\right] \leq \overline{c}_1. \tag{B.36}$$

Plugging (B.35) into (B.36), we see that

$$\frac{h'(0)}{h(\xi_1^*(c_1))} \leq \overline{c}_1 \qquad (c_1 \in C_1^*). \tag{B.37}$$

Moreover, Assumption 1 implies

$$\frac{\gamma_2}{\gamma_1} = \gamma < \underbrace{\frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1}}_{\leq 1} \cdot \underbrace{\widehat{\sigma}(\sigma, \underline{\rho})}_{\leq \sigma} \leq \sigma = \frac{\underline{c}_2}{\overline{c}_1}. \tag{B.38}$$

Multiplying inequality (B.37) by $(\gamma/q_1^*) > 0$, exploiting (B.38), and taking expectations over all $c_1 \in C_1^*$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(0)}{\gamma_1 h(\xi_1^*(c_1))} < \frac{\underline{c}_2}{q_1^*}. \tag{B.39}$$

Thus, in the modified contest, the marginal expected payoff of type $(\underline{c}_2/q_1^*)$ at the zero bid level is strictly negative. But this is impossible given that she is active and her expected payoff against $\xi_1^*|_{C_1^*}$ is strictly concave. The contradiction shows that, indeed, all types of player 1 are active in the original contest. □

C. Material omitted from Section 4

This section contains two auxiliary results, proofs of Propositions 1&2 and Theorem 1, as well as some discussion.

C.1 Best-response monotonicity

We will say that *player 1's domain condition* holds at $(\xi_2; c_1) \in X_2^* \times C_1$ if (i) $\widetilde{\beta}_1(\xi_2; c_1) > 0$, and (ii) $p_1(\widetilde{\beta}_1(\xi_2; c_1), \xi_2(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$. Thus, player 1's domain condition at $(\xi_2; c_1)$ requires that type c_1's best-response bid against ξ_2 is interior, and wins with a probability strictly exceeding one half against any of player 2's types. Similarly, we will say that *player 2's domain condition* holds at $(\widehat{\xi}_1; c_2) \in X_1^* \times C_2$ if (i) $\widetilde{\beta}_2(\widehat{\xi}_1; c_2) > 0$, and (ii) $p_2(\widetilde{\beta}_2(\widehat{\xi}_1; c_2), \widehat{\xi}_1(c_1)) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, player 2's domain condition at $(\widehat{\xi}_1; c_1)$ requires that type c_2's best-response bid against $\widehat{\xi}_1$ is interior, and wins with a probability strictly below one half against any of player 1's types.

Lemma C.1 (Best-response monotonicity)

(i) Let $\xi_2, \widehat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \widehat{\xi}_2$, and let $c_1 \in C_1$. If player 1's domain condition holds at $(\xi_2; c_1)$, then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$. In particular, if player 1's domain condition holds at $(\xi_2; c_1)$ for every $c_1 \in C_1$, then $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$.

(ii) Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \widehat{\xi}_1$, and let $c_2 \in C_2$. If player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$, then $\widetilde{\beta}_2(\xi_1; c_2) < \widetilde{\beta}_2(\widehat{\xi}_1; c_2)$. In particular, if player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$ for every $c_2 \in C_2$, then $\beta_2(\xi_1) \prec \beta_2(\widehat{\xi}_1)$.

Proof. (i) Let $\xi_2, \widehat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \widehat{\xi}_2$, and $c_1 \in C_1$. By assumption, player 1's domain condition holds at $(\xi_2; c_1)$. We wish to show that $x_1 \equiv \widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1) \equiv \widehat{x}_1$. To provoke a contradiction, suppose that $\widehat{x}_1 \geq x_1$. From the domain condition, we have $x_1 > 0$. Therefore, both x_1 and $\widehat{x}_1$ are positive, so that the corresponding first-order conditions imply

$$E_{c_2}\left[\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2}\right] = E_{c_2}\left[\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2}\right] = c_1. \tag{C.1}$$

Fix some $c_2 \in C_2$ for the moment. Letting $x = \gamma_1 h(\widetilde{\beta}_1(\xi_2; c_1))$ and $y = \gamma_2 h(\xi_2(c_2))$, the domain condition implies $x > y$. Clearly, the mapping $y \mapsto y/(x+y)^2$ is strictly increasing over the interval $[0, x]$. Therefore, noting that $\xi_2 \succ \widehat{\xi}_2$ implies $y \geq \widehat{y} \equiv \gamma_2 h(\widehat{\xi}_2(c_2))$, we see that

$$\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2} \geq \frac{\gamma_1 h'(x_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \qquad (c_2 \in C_2), \tag{C.2}$$

with strict inequality for at least one $c_2 \in C_2$. Moreover, from $\widehat{x}_1 \geq x_1$,

$$\frac{\gamma_1 h'(x_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \geq \frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \qquad (c_2 \in C_2). \tag{C.3}$$

Combining (C.2) and (C.3), and subsequently taking expectations , we arrive at

$$E_{c_2}\left[\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2}\right] > E_{c_2}\left[\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2}\right], \tag{C.4}$$

in conflict with (C.1). The contradiction shows that $x_1 > \widehat{x}_1$, as claimed. Moreover, if player 1's domain condition holds for any $c_1 \in C_1$, then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$ for any $c_1 \in C_1$, which indeed implies $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$. (ii) The proof is similar. Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \widehat{\xi}_1$, and $c_2 \in C_2$. By assumption, player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$. Suppose that $x_2 \equiv \widetilde{\beta}_2(\xi_1; c_2) \geq \widetilde{\beta}_2(\widehat{\xi}_1; c_2) \equiv \widehat{x}_2$. Then, from the domain condition, $\widehat{x}_2 > 0$. Hence,

$$E_{c_1}\left[\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2}\right] = E_{c_1}\left[\frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2}\right] = c_2. \tag{C.5}$$

Fix some $c_1 \in C_1$, and let $\widehat{x} = \gamma_2 h(\widetilde{\beta}_2(\widehat{\xi}_1; c_2))$ and $\widehat{y} = \gamma_1 h(\widehat{\xi}_1(c_1))$. By the domain condition, $\widehat{x} < \widehat{y}$. Moreover, the mapping $\widehat{y} \mapsto \widehat{y}/(\widehat{x}+\widehat{y})^2$ is strictly declining for $\widehat{y} \geq \widehat{x}$. Hence, given that $\widehat{\xi}_1 \prec \xi_1$ implies $\widehat{y} \leq y \equiv \gamma_1 h(\xi_1(c_1))$, we see that

$$\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2} \geq \frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(\widehat{x}_2))^2} \qquad (c_1 \in C_1), \tag{C.6}$$

with strict inequality for some $c_1 \in C_1$. Moreover, from $\widehat{x}_2 \leq x_2$,

$$\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_2)) + \gamma_2 h(\widehat{x}_2))^2} \geq \frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_2 h(\xi_1(c_1)) + \gamma_2 h(x_1))^2} \qquad (c_1 \in C_1). \tag{C.7}$$

Combining (C.6) and (C.7), and taking expectations, we arrive at

$$E_{c_1}\left[\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2}\right] > E_{c_1}\left[\frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2}\right], \tag{C.8}$$

in contradiction to (C.5). It follows that, indeed, $\widehat{x}_2 > x_2$. In particular, provided that player 2's domain condition holds for any $c_2 \in C_2$, it follows that $\beta_2(\xi_1) \prec \beta_2(\widehat{\xi}_1)$. This concludes the proof. □

Lemma C.1 shows that the domain conditions are sufficient to ensure that a type's best-response bid and a player's best-response bid schedule, respectively, move in a strictly monotone way to changes in the opponent's bid schedule. For example, in the case of player 1, the best-response bid of type c_1 will strictly rise in response to an increase of player 2's bid schedule. If player 1's domain condition holds at all of her types, then we get a strict order relation even between the best-response bid schedules. Similar comparative statics properties hold for player 2, whose best-response mapping is, however, strictly declining under the assumptions of Lemma C.1. In sum, the contest with two-sided incomplete information exhibits, subject to domain conditions, comparative statics properties analogous to those of the complete-information contest.

C.2 Stackelberg monotonicity

The next auxiliary result establishes monotonicity properties of the complete-information contest.

Lemma C.2 (Stackelberg monotonicity) *Let $x_2 > \widehat{x}_2 \geq 0$ and $c_1 \in C_1$ such that $x_1 = \widetilde{\beta}_1(\psi_2(x_2); c_1)$ and $\widehat{x}_1 = \widetilde{\beta}_1(\psi_2(\widehat{x}_2); c_1)$. If $\widehat{x}_1 > 0$ then, (i) $p_2(x_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$, and (ii) $\Pi_1(x_1, x_2; c_1) < \Pi_1(\widehat{x}_1, \widehat{x}_2; c_1)$.*

Proof. (i) By assumption, $\widehat{x}_1 = \widetilde{\beta}_1(\psi_2(\widehat{x}_2); c_1) > 0$. Therefore, $x_2 > \widehat{x}_2$ implies $p_2(\widehat{x}_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$. Assume first that $x_1 \leq \widehat{x}_1$. Then, clearly, $p_2(x_1, x_2) \geq p_2(\widehat{x}_1, x_2)$ and, hence, $p_2(x_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$, as claimed. Assume next that $x_1 > \widehat{x}_1$. Then, the necessary first-order conditions associated with the respective optimality of $\widehat{x}_1$ and x_1 hold true. As for $\widehat{x}_1$, we find that

$$\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{x}_2)}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{x}_2))^2} = c_1. \tag{C.9}$$

Multiplying by $\gamma h(\widehat{x}_2)/h'(\widehat{x}_1)$, with $\gamma = \gamma_2/\gamma_1$ as before, yields $(p_2(\widehat{x}_1, \widehat{x}_2))^2 = c_1 \gamma h(\widehat{x}_2)/h'(\widehat{x}_1)$. Similarly, one shows that the optimality of x_1 implies $(p_2(x_1, x_2))^2 = c_1\gamma h(x_2)/h'(x_1)$. Recalling that h is strictly increasing and that h' is weakly declining, we see that $(p_2(x_1, x_2))^2 > (p_2(\widehat{x}_1, \widehat{x}_2))^2$. The claim follows. (ii) As a consequence of the envelope theorem,

$$\frac{d\Pi_1(\widetilde{\beta}_1(\psi_2(x_2); c_1), x_2; c_1)}{dx_2} = \left.\frac{\partial\Pi_1(x_1, x_2; c_1)}{\partial x_2}\right|_{x_1=\widetilde{\beta}_1(\psi_2(x_2);c_1)} = -\frac{\gamma_1 h(\widetilde{\beta}_1(\psi_2(x_2); c_1))\gamma_2 h'(x_2)}{(\gamma_1 h(\widetilde{\beta}_1(\psi_2(x_2); c_1)) + \gamma_2 h(x_2))^2} < 0. \quad \text{(C.10)}$$

Thus, player 1 indeed strictly benefits from the lowered effort of player 2. This proves the second claim and, hence, the lemma. □

C.3 Proof of Proposition 1

The following prove establishes the strict incentive of the weakest type of the underdog to self-disclose.

Proof of Proposition 1. The conclusions of Proposition 1 are immediate if $\xi_2^*(\overline{c}_2) = 0$. Suppose that $\xi_2^*(\overline{c}_2) > 0$. Since, by Lemma B.2, the equilibrium bid schedule ξ_2^* is weakly declining, actually all types of player 2 are active in ξ_2^*. Using Lemma B.2 another time, one sees that ξ_2^* is even strictly declining. These observations will be tacitly used below. We now prove the three assertions made in the statement of the proposition. (i) First, it is shown that self-disclosure induces the weakest type of the underdog to strictly raise her bid, i.e., $\xi_2^*(\overline{c}_2) < x_2^\#$. To provoke a contradiction, suppose that $\xi_2^*(\overline{c}_2) \geq x_2^\#$. Then, because ξ_2^* is strictly declining and there are at least two possible type realizations for player 2, we get $\xi_2^* \succ \psi_2(x_2^\#)$. We claim that player 1's domain condition holds at $(\xi_2^*; c_1)$, for any $c_1 \in C_1$. To see this, take some $c_1 \in C_1$. Then, from property (i) of uniform asymmetry, $\widetilde{\beta}_1(\xi_2^*; c_1) = \xi_1^*(c_1) > 0$. Further, since all types of player 2 are active in ξ_2^*, property (ii) of uniform asymmetry implies that $p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$, which proves the claim. We may, therefore, apply Lemma C.1(i) so as to obtain

$$\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^\#)) = \xi_1^\#. \quad \text{(C.11)}$$

Next, it is claimed that player 2's domain condition holds at $(\xi_1^\#; \overline{c}_2)$. Since $(\xi_1^\#(.), x_2^\#)$ is an equilibrium in the contest with one-sided incomplete information, we have $x_2^\# > 0$, i.e., player 2 is active with probability one. Invoking property (ii) of uniform asymmetry shows, therefore, that $p_2(\xi_1^\#(c_1), x_2^\#) < \frac{1}{2}$ holds true for any $c_1 \in C_1$. Since $\widetilde{\beta}_2(\xi_1^\#; \overline{c}_2) = x_2^\#$, this means that $p_2(\xi_1^\#(c_1), \widetilde{\beta}_2(\xi_1^\#; \overline{c}_2)) < \frac{1}{2}$, for any $c_1 \in C_1$. I.e., player 2's domain condition at $(\xi_1^\#; \overline{c}_2)$ is indeed satisfied. Therefore, using relationship (C.11) and Lemma C.1(ii), we see that $\xi_2^*(\overline{c}_2) = \widetilde{\beta}_2(\xi_1^*; \overline{c}_2) < \widetilde{\beta}_2(\xi_1^\#; \overline{c}_2) = x_2^\#$, in contradiction to $\xi_2^*(\overline{c}_2) \geq x_2^\#$. Thus, $\xi_2^*(\overline{c}_2) < x_2^\#$, as claimed. (ii) Next, it is shown that, after disclosure, the probability of winning for the weakest type of the underdog rises strictly, i.e., $p_2^\# = E_{c_1}[p_2(x_2^\#, \xi_1^\#(c_1))] > E_{c_1}[p_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1))] = p_2^*$. In fact, we will prove the somewhat stronger statement

$$p_2(x_2^\#, \xi_1^\#(c_1)) > p_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1)) \qquad (c_1 \in C_1). \quad \text{(C.12)}$$

Take some type $c_1 \in C_1$. It is claimed first that $\widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1) > 0$, as shown in the left diagram of Figure C.1. Indeed, because player 2 is always active in ξ_2^*, the mapping $x_1 \mapsto E_{c_2}[\Pi_1(x_1, \xi_2^*(c_2); c_1)]$ is strictly concave on $\mathbb{R}_+$, and vanishes at $x_1 = 0$. Therefore, the optimality of $\xi_1^*(c_1) > 0$ implies $E_{c_2}[\Pi_1(\xi_1^*(c_1), \xi_2^*(c_2); c_1)] > 0$. But the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$ is everywhere weakly lower than ξ_2^*. Therefore, $E_{c_2}[\Pi_1(\xi_1^*(c_1), \psi_2(\xi_2^*(\overline{c}_2)); c_1)] > 0$, i.e., type c_1 is able to realize a positive payoff against the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$. Since $\xi_2^*(\overline{c}_2) > 0$, it follows that type c_1's best-response bid against $\psi_2(\xi_2^*(\overline{c}_2))$ is positive, as claimed. Next, from the previous step, we know that $x_2^\# > \xi_2^*(\overline{c}_2)$. Invoking Lemma C.2(i), and noting that $\xi_1^\# = \beta_1(\psi_2(x_2^\#))$, it follows that

$$p_2(x_2^\#, \xi_1^\#(c_1)) > p_2(\xi_2^*(\overline{c}_2), \widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1)) \qquad (c_1 \in C_1). \quad \text{(C.13)}$$

Next, comparing the strictly declining equilibrium bid schedule $\xi_2^* = \beta_2(\xi_1^*)$ with the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$, and recalling that there are at least two types, we obtain $\xi_2^* \succ \psi_2(\xi_2^*(\overline{c}_2))$. Moreover, as seen above, all types of player 2 are active. Hence, by property (ii) of uniform asymmetry, $p_1(\xi_1^*(c_1), \xi_2^*(c_2)) > \frac{1}{2}$ for any $c_1 \in C_1$ and any $c_2 \in C_2$, so that via $\widetilde{\beta}_1(\xi_2^*; c_1) = \xi_1^*(c_1)$, player 1's domain condition is seen to hold at $(\xi_2^*; c_1)$, for any

$c_1 \in C_1$. Therefore, by Lemma C.1(i), $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(\xi_2^*(\bar{c}_2)))$, as illustrated in Figure C.1.[2] In particular, $\xi_1^*(c_1) \geq \widetilde{\beta}_1(\psi_2(\xi_2^*(\bar{c}_2)); c_1)$, for any $c_1 \in C_1$.Therefore,

$$p_2(\xi_2^*(\bar{c}_2), \widetilde{\beta}_1(\psi_2(\xi_2^*(\bar{c}_2)); c_1)) \geq p_2(\xi_2^*(\bar{c}_2), \xi_1^*(c_1)) \qquad (c_1 \in C_1). \tag{C.14}$$

Combining (C.13) and (C.14) yields (C.12). In particular, this proves $p_2^{\#} > p_2^*$, as claimed.

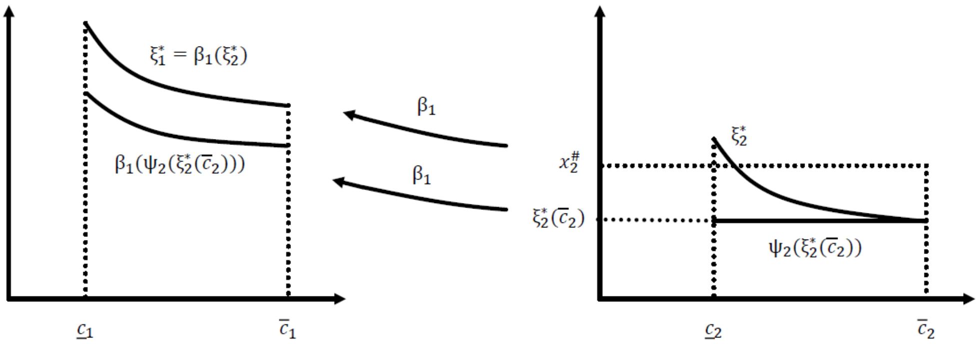


Figure C.1 Proof of Proposition 1(ii).

(iii) Finally, we show that the weakest type of the underdog has a strict incentive to disclose her type. Clearly, the equilibrium effort $x_2^{\#}$ is positive. One can check that type $\bar{c}_2$'s first-order condition is equivalent to

$$E_{c_1}\left[p_2(x_2^{\#}, \xi_1^{\#}(c_1)) - \left(p_2(x_2^{\#}, \xi_1^{\#}(c_1))\right)^2\right] = \bar{c}_2 \Phi(x_2^{\#}). \tag{C.15}$$

Exploiting (C.15), we obtain for type $\bar{c}_2$'s expected payoff from self-disclosure,

$$\Pi_2^{\#} = E_{c_1}\left[\left(p_2(x_2^{\#}, \xi_1^{\#}(c_1))\right)^2\right] + \bar{c}_2\left(\Phi(x_2^{\#}) - x_2^{\#}\right). \tag{C.16}$$

In a completely analogous fashion, we can convince ourselves that concealment grants type $\bar{c}_2$ a payoff of

$$\Pi_2^*(\bar{c}_2) = E_{c_1}\left[(p_2(\xi_2^*(\bar{c}_2), \xi_1^*(c_1)))^2\right] + \bar{c}_2\left(\Phi(\xi_2^*(\bar{c}_2)) - \xi_2^*(\bar{c}_2)\right). \tag{C.17}$$

Now, from (C.12), we see that $E_{c_1}[(p_2(x_2^{\#}, \xi_1^{\#}(c_1)))^2] > E_{c_1}[(p_2(\xi_2^*(\bar{c}_2), \xi_1^*(c_1)))^2]$. Moreover, from Lemma B.1, $\Phi' \geq 1$, so that the mapping $x_2 \mapsto \Phi(x_2) - x_2$ is monotone increasing in x_2. But, as shown above, $\xi_2^*(\bar{c}_2) < x_2^{\#}$. It follows that the weakest type of the underdog has indeed a strict incentive to reveal her type. This proves the final claim and concludes the proof of the proposition. □

C.4 Proof of Proposition 2

Next, we present the proof of Proposition 2, regarding the strict incentive of the strongest type of the favorite to reveal her type, provided the underdog's type is public information.

Proof of Proposition 2. Since x_1° and x_2° are equilibrium efforts under complete information, we have $x_1^\circ > 0$ and $x_2^\circ > 0$. Similarly, one notes that $x_2^{\#} > 0$. Moreover, by property (i) of uniform asymmetry, all types of player 1 are active in $\xi_1^{\#}$, so that by Lemma B.2, the bid schedule $\xi_1^{\#}$ is strictly declining. We now prove the four assertions made in the statement of Proposition 2. (i) It is claimed that $x_2^\circ < x_2^{\#}$. To provoke a contradiction, suppose that $x_2^\circ \geq x_2^{\#}$. Part (ii) of uniform asymmetry implies $p_1(x_1^\circ, x_2^\circ) > \frac{1}{2}$, so that in view of $x_1^\circ = \widetilde{\beta}_1(x_2^\circ; \underline{c}_1)$,

[2] The figure shows an example where $x_2^{\#} < \xi_2^*(\underline{c}_2)$. In general, we may also have that $x_2^{\#} \geq \xi_2^*(\underline{c}_2)$.

player 1's domain condition holds at $(x_2^\circ; \underline{c}_1)$. Hence, by Lemma C.1(i), if even $x_2^\circ > x_2^\#$, then $x_1^\circ = \widetilde{\beta}_1(x_2^\circ; \underline{c}_1) > \widetilde{\beta}_1(x_2^\#; \underline{c}_1) = \xi_1^\#(\underline{c}_1)$. If, however, $x_2^\circ = x_2^\#$, then it is immediate that $x_1^\circ = \xi_1^\#(\underline{c}_1)$. Thus, either way, we arrive at $x_1^\circ \geq \xi_1^\#(\underline{c}_1)$, so that $\psi_1(x_1^\circ) \succeq \psi_1(\xi_1^\#(\underline{c}_1))$. Moreover, given that player 1 has at least two types, and that $\xi_1^\#$ is strictly declining, $\psi_1(\xi_1^\#(\underline{c}_1)) \succ \xi_1^\#$. Hence, $\psi_1(x_1^\circ) \succ \xi_1^\#$. Part (ii) of uniform asymmetry implies that $p_2(\xi_1^\#(c_1), x_2^\#) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, recalling that $x_2^\# = \widetilde{\beta}_2(\xi_1^\#; c_2^\#)$, player 2's domain condition holds at $(\xi_1^\#; c_2^\#)$. Therefore, using Lemma C.1(ii), we arrive at $x_2^\# = \widetilde{\beta}_2(\xi_1^\#; c_2^\#) > \widetilde{\beta}_2(\psi_1(x_1^\circ); c_2^\#) = x_2^\circ$, a contradiction. It follows that $x_2^\circ < x_2^\#$, as claimed. (ii) Next, it is shown that $x_1^\circ < \xi_1^\#(\underline{c}_1)$. From the previous step, we know that $x_2^\# > x_2^\circ$. Via property (ii) of uniform asymmetry, we see that $p_1(\xi_1^\#(\underline{c}_1), x_2^\#) > \frac{1}{2}$. Thus, the domain condition for player 1 holds at $(x_2^\#; \underline{c}_1)$. Lemma C.1(i) implies, therefore, that $\xi_1^\#(\underline{c}_1) = \widetilde{\beta}_1(x_2^\#; \underline{c}_1) > \widetilde{\beta}_1(x_2^\circ; \underline{c}_1) = x_1^\circ$. Thus, the effort of the strongest type of the favorite will indeed be strictly lower after self-disclosure. (iii) Given part (i) above, we have $x_2^\circ < x_2^\#$. Recalling that $x_1^\circ > 0$, Lemma C.2(i) implies $p_2(x_1^\circ, x_2^\circ) < p_2(\xi_1^\#(\underline{c}_1), x_2^\#)$, so that $p_1(x_1^\circ, x_2^\circ) > p_1(\xi_1^\#(\underline{c}_1), x_2^\#)$. Thus, type $\underline{c}_1$ indeed wins with a strictly higher probability after self-disclosure. (iv) The claim that $\Pi_1^\circ > \Pi_1^\#$ follows now directly from Lemma C.2(ii). This completes the proof. □

C.5 Proof of Theorem 1

This subsection combines the auxiliary results to prove our main result.

Proof of Theorem 1. We start by showing that self-disclosure by all types of both players constitutes a perfect Bayesian equilibrium. To this end, we specify off-equilibrium beliefs $\mu_1^0 \in \Delta(C_1)$ and $\mu_2^0 \in \Delta(C_2)$ as follows. The underdog expects a favorite that does not disclose her private information to be of type $c_1 = \overline{c}_1$ with probability one. Thus, $\mu_1^0(c_1) = 1$ if $c_1 = \overline{c}_1$, and $\mu_1^0(c_1) = 0$ otherwise. Similarly, the favorite expects an underdog that does not disclose her private information to be of type $c_2 = \underline{c}_2$ with probability one. Thus, $\mu_2^0(c_2) = 1$ if $c_2 = \underline{c}_2$, and $\mu_2^0(c_2) = 0$ otherwise. To check the equilibrium property, consider first an arbitrary type $c_1 \in C_1$ of the favorite. If c_1 complies with equilibrium self-disclosure, and is matched with some type $c_2 \in C_2$ of the underdog, then c_1 receives a complete-information equilibrium payoff of $\Pi_1^\circ(c_1, c_2) = \Pi_1(\widetilde{\beta}_1(x_2^\circ(c_1, c_2); c_1), x_2^\circ(c_1, c_2); c_1)$. If, however, c_1 chooses to not disclose then, given the off-equilibrium beliefs specified above, an underdog of type c_2 expects the favorite to be of the worst-case type $\overline{c}_1$ and, having revealed her own type c_2, chooses an effort of $x_2^\circ(\overline{c}_1, c_2)$. Responding optimally to type c_2's bid, the deviating favorite of type c_1 chooses an effort of $\widetilde{\beta}_1(x_2^\circ(\overline{c}_1, c_2); c_1)$ at the contest stage, and consequently receives a payoff of $\Pi_1^{\mathrm{dev}}(c_1, c_2) = \Pi_1(\widetilde{\beta}_1(x_2^\circ(\overline{c}_1, c_2); c_1), x_2^\circ(\overline{c}_1, c_2); c_1)$. A straightforward application of Monaco and Sabarwal (2016, Thm. 3) shows that, given Assumption 1, $x_2^\circ(c_1, c_2) \leq x_2^\circ(\overline{c}_1, c_2)$.[3] We claim that $\Pi_1^\circ(c_1, c_2) \geq \Pi_1^{\mathrm{dev}}(c_1, c_2)$. Indeed, if $x_2^\circ(c_1, c_2) < x_2^\circ(\overline{c}_1, c_2)$ then, by Lemma C.2(ii), $\Pi_1^\circ(c_1, c_2) > \Pi_1^{\mathrm{dev}}(c_1, c_2)$. Moreover, if $x_2^\circ(c_1, c_2) = x_2^\circ(\overline{c}_1, c_2)$ then $\Pi_1^\circ(c_1, c_2) = \Pi_1^{\mathrm{dev}}(c_1, c_2)$, which proves the claim. Taking expectations over all $c_2 \in C_2$ yields $E_{c_2}[\Pi_1^\circ(c_1, c_2)] \geq E_{c_2}[\Pi_1^{\mathrm{dev}}(c_1, c_2)]$, for any $c_1 \in C_1$. Hence, a deviation is not profitable for any type $c_1 \in C_1$. On the other hand, if any type of the underdog deviates, and the favorite interprets this as a tactic of the strongest type of the underdog, then one shows in complete analogy that the equilibrium condition holds. It follows that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium. Next, suppose there is a perfect Bayesian equilibrium in which not all private information is revealed. Then, for at least one player $i \in \{1, 2\}$, the set of types concealing their signal, $C_i \backslash S_i$, has at least two elements. By suitably redefining C_1 and C_2, we may assume without loss of generality that all types conceal their types. Suppose first that $K_2 \geq 2$. Then, Proposition 1 implies that the weakest type of the underdog has a strict incentive to unilaterally deviate at the revelation stage, in conflict to the equilibrium assumption. Suppose next that $K_2 = 1$. Then, since there is incomplete information, $K_1 \geq 2$. But, again, this cannot be part of a perfect Bayesian equilibrium by Proposition 2. Thus, either way, we obtain a contradiction, and the claim follows. This proves the theorem. □

C.6 Discussion: Dominance and defiance

To make transparent why analyzing disclosure in probabilistic contests requires new methods, we discuss the comparative statics of the Bayesian equilibrium at the contest stage with respect to changes in the information structure.

First, while self-disclosure by the weakest type of the underdog tends to have an overall moderating effect on the favorite, some types of the favorite may respond by bidding higher.

[3] For a self-contained argument, it suffices to replicate earlier arguments. Indeed, suppose that $x_2^\circ(c_1, c_2) > x_2^\circ(\overline{c}_1, c_2)$. Clearly, all equilibrium efforts are positive under complete information. Therefore, using property (ii) of uniform asymmetry, player 1's domain condition holds at $(x_2^\circ(c_1, c_2); c_1)$, so that, by Lemma C.1(i), $x_1^\circ(c_1, c_2) > x_1^\circ(\overline{c}_1, c_2)$. Moreover, using property (ii) of uniform asymmetry another time, player 2's domain condition is seen to hold at $(x_1^\circ(\overline{c}_1, c_2); c_2)$, so that by Lemma C.1(ii), $x_2^\circ(c_1, c_2) < x_2^\circ(\overline{c}_1, c_2)$, which yields the desired contradiction.

Example C.1 ("Dominant reaction")[4] Table C.1 exhibits data for a uniformly asymmetric contest. As can be seen, after the self-disclosure by $\bar{c}_2 = c_2^2$, the weak type of the favorite, $\bar{c}_1 = c_1^2$, raises her effort.

Player 1		Player 2	
$c_1^1 = 0.1$	$c_1^2 = 0.2$	$c_2^1 = 3.7$	$c_2^2 = 7.8$
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.05$	$q_2(c_2^2) = 0.95$
$\xi_1^*(c_1^1) = 0.1592$	$\xi_1^*(c_1^2) = 0.1042$	$\xi_2^*(c_2^1) = 0.0572$	$\xi_2^*(c_2^2) = 0.0011$
$\vee$	$\wedge$		$\wedge$
$\xi_1^\#(c_1^1) = 0.1495$	$\xi_1^\#(c_1^2) = 0.1051$	–	$x_2^\# = 0.0023$

Table C.1 Equilibrium bids before and after the underdog's self-disclosure.

Despite the non-monotonicity illustrated by the example, the model does impose some structure of the favorite's reaction. First, not all types of the favorite may simultaneously raise their bids in response to the self-disclosure by the weakest type of the underdog. Indeed, this would be incompatible with our earlier conclusion that the weakest type of the underdog necessarily raises her bid. Second, even a dominant reaction of the favorite will never be strong enough to press the probability of winning for the weakest type of the underdog weakly below her probability of winning under concealment.

In analogy to the case just considered, a relatively strong type of the underdog may raise her effort in response to the favorite's attempt to discourage her.

Example C.2 ("Defiant reaction") Data for another uniformly asymmetric contest is shown in Table C.2. In response to the favorite's attempt to discourage the underdog, only the two weaker types of the underdog lower their respective efforts, whereas the strongest type of the underdog raises her effort. In fact, the example illustrates another possibility mentioned in the body of the paper, viz. that a type of the underdog may decide to exert zero effort.

Player 1		Player 2		
$c_1^1 = 0.2$	$c_1^2 = 0.6$	$c_2^1 = 7.5$	$c_2^2 = 11$	$c_2^3 = 11.5$
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.1$	$q_2(c_2^2) = 0.1$	$q_2(c_2^3) = 0.8$
$\xi_1^*(c_1^1) = 0.1195$	$\xi_1^*(c_1^2) = 0.0647$	$\xi_2^*(c_2^1) = 0.0205$	$\xi_2^*(c_2^2) = 0.0031$	$\xi_2^*(c_2^3) = 0.0014$
$\vee$		$\wedge$	$\vee$	$\vee$
$x_1^\# = 0.0872$	–	$\xi_2^\#(c_2^1) = 0.0206$	$\xi_2^\#(c_2^2) = 0.0018$	$\xi_2^\#(c_2^3) = 0.0000$

Table C.2 Equilibrium bids before and after the favorite's self-disclosure.

C.7 Games of strategic heterogeneity

In parameterized games of strategic heterogeneity (Monaco and Sabarwal, 2016; Barthel and Hoffmann, 2019), strategy spaces are multi-dimensional, and payoff functions allow for strategic complements and substitutes at the same time. Under suitable constraints on bids, the incomplete-information contests considered in the present paper would indeed satisfy the definition. Moreover, the monotone comparative statics of the contest stage with respect to changes in the information structure conducted above clearly draws on intuitions suggested by that literature. Quite notably, however, existing conditions do not apply to our model. As Examples C.1 and C.2 have shown, the relevant comparative statics of the Bayesian equilibrium is, in general, monotone for one player only. In contrast, Monaco and Sabarwal's (2016) conditions, like any of the conditions in the literature that we are aware of, imply the monotone comparative statics of the entire equilibrium profile. In fact, the contraction-mapping approach underlying Monaco and Sabarwal's (2016, Thm. 5) result need not go through when the contest is too asymmetric. The problem is that, as noted by Wärneryd (2018) in a different context, the iteration of the best response in an asymmetric contest with complete information need not be a contraction. Indeed, for low bids of the opponent, the best-response function in a probabilistic contest is very steep. The situation is similar under incomplete information. The following numerical example shows that monotone comparative statics results available for games of strategic heterogeneity do not apply to Example C.1.

[4] Unless stated otherwise, all numerical examples are based on the unbiased lottery contest.

Example C.1 (continued) Let $\overline{\beta}_1(\xi_2) = \beta_1(\psi_2(\xi_2(\overline{c}_2)))$ denote player 1's best-response bid schedule against $\psi_2(\xi_2(\overline{c}_2))$, where $\xi_2 \in X_2^*$. Monaco and Sabarwal (2016, Thm. 5) required that $\overline{\beta}_1(\widehat{\xi}_2) \preceq \xi_1^*$, where $\widehat{\xi}_2 = \beta_2(\widehat{\xi}_1)$ and $\widehat{\xi}_1 = \overline{\beta}_1(\xi_2^*)$. A computation shows that $\widehat{\xi}_1(\underline{c}_1) = 0.1016$, $\widehat{\xi}_1(\overline{c}_1) = 0.0715$, and $\widehat{\xi}_2(\overline{c}_2) = 0.0194$. As a result, $\overline{\beta}_1(\widehat{\xi}_2)(\underline{c}_1) = 0.4208 > 0.1592 = \xi_1^*(\underline{c}_1)$ and $\overline{\beta}_1(\widehat{\xi}_2)(\overline{c}_1) = 0.2919 > 0.1042 = \xi_1^*(\overline{c}_1)$. It follows that $\overline{\beta}_1(\widehat{\xi}_2) \succ \xi_1^*$, in conflict with the required condition.

D. Material omitted from Section 5

We go over the extensions discussed in the body of the paper.

D.1 Correlated types

If types are correlated, then each type $c_j \in C_j$ expects to face type c_i with a conditional probability $q_i(c_i^k \,|c_j\,) > 0$ for $k \in \{1, \ldots, K_i\}$. Expected payoffs are conditional expectations, and the definition of Bayesian equilibrium needs to be adapted correspondingly. We start with a particularly clean case in which type distributions are negatively correlated.

Proposition D.1 (Negative correlation) *Suppose that Assumption 1 holds and that the conditional belief $\mu_2(\cdot|\,c_2) \in \Delta(C_1)$ held by the underdog's type c_2 is first-order stochastically decreasing in $c_2 \in C_2$. Then, the conclusion of Theorem 1 continues to hold true.*

Proof. We first show that all information must be revealed in any perfect Bayesian equilibrium. The key point to note is that, even if the underdog's conditional belief $\mu_1(\cdot|\,c_2) \in \Delta(C_1)$ is weakly decreasing in c_2 in the FOSD sense, the underdog's bid schedule remains weakly decreasing globally, as well as strictly decreasing in the interior. Indeed, this follows immediately by combining Lemma B.2 and Lemma C.1, where the underdog's domain condition holds by Lemma 2. Therefore, the proof of Proposition 1, which exploits only the monotonicity properties of the bid schedules and the monotonicity properties of the best-response mappings, extends without change to this more general setting. Thus, the underdog side unravels. For the favorite, correlation now does not matter anymore, i.e., Proposition 2 applies as before. This proves the claim. Next, we show that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium. Even in the presence of arbitrary correlation, this is so provided we keep the specification of off-equilibrium beliefs used in the proof of Theorem 1. The reason is that, under this specification, type-specific payoffs resulting from self-disclosure or unilateral concealment are expected values of complete-information payoffs. Therefore, the correlation does not affect the interim payoff ranking for the contest stage, and the argument proceeds as before. □

The intuition is as follows. The assumption on conditional beliefs means that weaker types of the underdog are more pessimistic, in the sense that they deem stronger types of the favorite more likely. As a result, the best-response bid schedule of the underdog remains strictly declining in the interior, so that the conclusions of the crucial Proposition 1 continue to hold true. Moreover, once the side of the underdog has unraveled, any ex-ante correlation will be resolved, so that full separation obtains as before via Proposition 2.

For similar reasons, the strong-form disclosure principle holds for general forms of correlation provided that the degree of correlation is small enough. For strongly positively correlated types, however, the situation may complicate. In fact, Proposition 1 may break down, as the following example illustrates.

	$c_2^1 = 5.5$	$c_2^2 = 6$
$c_1^1 = 1$	0.45	0.05
$c_1^2 = 1.5$	0.05	0.45

Player 1		Player 2	
$\xi_1^*(c_1^1) = 0.1318$	$\xi_1^*(c_1^2) = 0.1056$	$\xi_2^*(c_2^1) = 0.0242$	$\xi_2^*(c_2^2) = 0.0262$
$\wedge$	$\wedge$		$\vee$
$\xi_1^\#(c_1^1) = 0.1353$	$\xi_1^\#(c_1^2) = 0.1057$	–	$x_2^\# = 0.0260$

Table D.1 Positive correlation.

Example D.1 (**Positive correlation)** Consider the contest specified in Table D.1. Assumption 1 holds in this example. Shown are the equilibrium bids with and without disclosure by the weak type of the underdog. As can be seen, disclosure induces both types of the favorite to bid higher. Hence, the weak type of the underdog does not benefit from disclosing her type.

The logic of the example is that, without disclosure, the strong type of the favorite expects meeting the strong type of the underdog that, as a result of positive correlation, bids lower than the weak type of the underdog. Therefore, with disclosure, the strong type of the favorite raises her bid. In contrast, disclosure does not substantially change the belief of the favorite's weak type, but she expects meeting the weak type with somewhat higher probability, which makes her bid higher.

Despite the fact that Proposition 1 does not hold literally with positive correlation, Theorem 1 is robust if the type distribution is generic, as explained in the body of the paper.

D.2 Noisy signals

It is also of interest to see players' incentives to release noisy signals (not to be confused with randomized revelations). This question is, in general, harder to address. The following result shows that the weakest type of the underdog, provided she is active, has always a strict incentive to send a noisy signal that corresponds to a first-order increase over her type space, provided that her own type will appear more likely.

Proposition D.2 (Noisy signals) *Consider an unbiased lottery contest, and assume that the type $c_1^{\#}$ of player 1 is public, while the type of player 2 is private information. Suppose that $\underline{c}_2 > c_1^{\#}$, and that type $\overline{c}_2$ is active. Then a FOSD shift in the type distribution of player 2 that makes $\overline{c}_2$ strictly more likely induces player 1 to strictly lower her effort $x_1^{\#}$.*

Proof. Before the shift, $x_1^{\#,\text{ before}} = E[\sqrt{c_2}]^2/(c_1^{\#} + E[c_2])^2$. It suffices to prove the claim for a FOSD shift in the type distribution of player 2 that makes $\overline{c}_2$ more likely by a probability $\varepsilon > 0$, and another type $\widehat{c}_2 < \overline{c}_2$ less likely by the same probability. Then, after the shift, we get

$$x_1^{\#,\text{ after}} = \left(\frac{E[\sqrt{c_2}] + \varepsilon(\sqrt{\overline{c}_2} - \sqrt{\widehat{c}_2})}{c_1^{\#} + E[c_2] + \varepsilon(\overline{c}_2 - \widehat{c}_2)} \right)^2, \tag{D.1}$$

Let $\widehat{\varepsilon} = \varepsilon(\sqrt{\overline{c}_2} - \sqrt{\widehat{c}_2})$. Then,

$$x_1^{\#,\text{ after}} = \left(\frac{E[\sqrt{c_2}] + \widehat{\varepsilon}}{c_1^{\#} + E[c_2] + \widehat{\varepsilon}(\sqrt{\overline{c}_2} + \sqrt{\widehat{c}_2})} \right)^2. \tag{D.2}$$

It follows that $x_1^{\#,\text{ after}} < x_1^{\#,\text{ before}}$ holds if and only if

$$c_1^{\#} + E[c_2] < \underbrace{E[\sqrt{c_2 \overline{c}_2}]}_{>E[c_2]} + \underbrace{E[\sqrt{c_2 \widehat{c}_2}]}_{\geq c_1^{\#}}, \tag{D.3}$$

which is a tautology. The claim follows. □

D.3 Sequential moves

The following result shows that the strong-form disclosure principle in probabilistic contests is robust to sequential disclosures

Proposition D.4 (Sequential moves) *Suppose that, instead of moving simultaneously in the revelation stage, players move sequentially. Then, imposing Assumption 1, the conclusion of Theorem 1 continuous to hold true under either of the following conditions:*

(i) The favorite moves first;

(ii) the underdog moves first, and the contest is a lottery.[5]

Proof. (i) If the favorite moves first, the Proposition 1 implies that the Bayesian game beginning with the information set reached by the favorite's decision will unravel on the side of the underdog. Therefore, the highest type of the underdog that uses that decision will strictly prefer to reveal her type by Proposition 2. Thus, the

[5]We conjecture that the assumption that the contest is a lottery is not needed.

game unravels on both sides, as claimed. (ii) Suppose next that the underdog moves first. Focus on the weakest type of the underdog, $\overline{c}_2$. If $\overline{c}_2$'s type is revealed (either because she discloses or because all other types disclose), then the favorite will subsequently reveal her type by Proposition 2. Therefore, the contest stage for $\overline{c}_2$ will be of complete information. In contrast, if $\overline{c}_2$ is not revealed, then the contest will feature incomplete information on the side of the underdog (and possibly on the side of the favorite as well). We, therefore, have to show that $E_{c_1}[\Pi_2^{\circ}(c_1, \overline{c}_2)] > E_{c_1}[\Pi_2^*(\xi_2^*(\overline{c}_2), \xi_1^*(c_1))]$. By Proposition 1, $E_{c_1}[\Pi_2(x_2^{\#}, \xi_1^{\#}(c_1); \overline{c}_2)] > E_{c_1}[\Pi_2^*(\xi_2^*(\overline{c}_2), \xi_1^*(c_1))]$, so that it suffices to prove $E_{c_1}[\Pi_2^{\circ}(c_1, \overline{c}_2)] \geq E_{c_1}[\Pi_2(x_2^{\#}, \xi_1^{\#}(c_1); \overline{c}_2)]$, or equivalently, that

$$E_{c_1}\left[\left(\frac{c_1}{c_1 + \overline{c}_2}\right)^2\right] \geq \frac{E[\sqrt{c_1}]^2 E[c_1]}{(E[c_1] + \overline{c}_2)^2}. \tag{D.4}$$

Noting that this inequality is homogeneous of degree zero, we may assume without loss of generality that $\overline{c}_2 = 1$. But the resulting inequality,

$$E_{c_1}\left[\left(\frac{c_1}{c_1 + 1}\right)^2\right] \geq \frac{E[\sqrt{c_1}]^2 E[c_1]}{(E[c_1] + 1)^2}, \tag{D.5}$$

corresponds to the conclusion of Lemma G.1(ii) with $g(x, y) = x^2 y/(y + 1)^2$. It is straightforward to verify that

$$(d_x \; d_y)\,(H_g(x, y)) \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2yd_x^2}{(y+1)^2} \left(1 + \frac{x(2-y)}{y\,(y+1)} \frac{d_y}{d_x}\right) \left(1 - \frac{x}{y+1} \frac{d_y}{d_x}\right). \tag{D.6}$$

It therefore suffices to show that

$$\left\{x, y \in (0,1), y \geq x^2, d_x > 0, d_y > 0, \frac{d_y}{d_x} < \frac{1-y}{1-x}\right\} \Rightarrow \left\{\frac{x}{y+1} \frac{d_y}{d_x} < 1\right\}. \tag{D.7}$$

The conclusion follows if $\frac{1-y}{1-x} \leq \frac{y+1}{x}$, which is easily seen to be equivalent to $2x \leq 1 + y$, which in turn holds true because $y \geq x^2$. □

D.4 One-sided incomplete information

Suppose that player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in C_2 = \{\underline{c}_2 \equiv c_2^1, \ldots, c_2^{K_2} \equiv \overline{c}_2\}$ is private with $K_2 \geq 2$. By a *marginal piece of evidence*, we mean a K_2-dimensional vector $\delta_2 = (\delta_2^1, \ldots, \delta_2^{K_2})$ such that $\sum_{k=1}^{K_2} \delta_2^k = 0$. The intuition is that δ_2 turns i's prior belief $q_2 \in \Delta(C_2)$ about player 2's type into a nearby posterior $\widetilde{q}_2 \in \Delta(C_2)$ such that $\widetilde{q}_2(c_2^k) = q_2(c_2^k) + \varepsilon \delta_2^k$, where $\varepsilon > 0$ is a small positive number. Given our assumption that all types have a positive ex-ante probability, adding a marginal piece of evidence for small enough $\varepsilon > 0$ will always be feasible in a comparative statics exercise.

Lemma D.1 (Necessary and sufficient conditions) *Suppose that player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in C_2 = \{\underline{c}_2 \equiv c_2^1, \ldots, c_2^{K_2} \equiv \overline{c}_2\}$ is private with $K_2 \geq 2$. Suppose also that all types of player 2 are active. Then, there exists a positive and strictly hump-shaped (which includes the possibility of strictly monotone increasing or strictly monotone decreasing) sequence $(\varphi^1, \ldots, \varphi^{K_2})$ such that, for any marginal piece of evidence δ_2, any type $c_2 \in C_2$ strictly prefers disclosing δ_2 over concealing δ_2 if and only if*

$$\begin{pmatrix} \delta_2^1 \\ \vdots \\ \delta_2^{K_2} \end{pmatrix} \cdot \begin{pmatrix} \varphi^1 \\ \vdots \\ \varphi^{K_2} \end{pmatrix} < 0. \tag{D.8}$$

Proof. We use the shorthand notation $x_2^k \equiv \xi_2^{\#}(c_2^k)$ and $p_2^k \equiv p_2(x_2^k, x_1^{\#})$ for $k \in \{1, \ldots, K_2\}$. We have the first-order conditions

$$\sum_{k=1}^{K_2} \tilde{q}_2^k p_2^k(1-p_2^k) = c_1^\# \Phi(x_1^\#), \tag{D.9}$$

$$p_2^k(1-p_2^k) = c_2^k \Phi(x_2^k) \qquad (k \in \{1,\dots,K_2\}). \tag{D.10}$$

Totally differentiating (D.9) and evaluating at $\varepsilon = 0$ yields

$$\sum_{k=1}^{K_2} q_2^k(1-2p_2^k)dp_2^k + \left\{\sum_{k=1}^{K_2} \delta_2^k p_2^k(1-p_2^k)\right\} d\varepsilon = c_1^\# \Phi'(x_1^\#)dx_1^\#. \tag{D.11}$$

Moreover, for $k \in \{1,\dots,K_2\}$, using (D.10),

$$\begin{aligned} dp_2^k &= \frac{\partial p_2^k}{\partial x_1^\#}dx_1^\# + \frac{\partial p_2^k}{\partial x_2^k}dx_2^k && \text{(D.12)}\\ &= -\frac{p_2^k(1-p_2^k)}{\Phi(x_1^\#)}dx_1^\# + \frac{p_2^k(1-p_2^k)}{\Phi(x_2^k)}dx_2^k && \text{(D.13)}\\ &= c_2^k\left\{-\frac{\Phi(x_2^k)}{\Phi(x_1^\#)}dx_1^\# + dx_2^k\right\}. && \text{(D.14)} \end{aligned}$$

From Lemma B.1(iv),

$$dx_2^k = \frac{\Phi(x_2^k)}{\Phi(x_1^\#)}\frac{2p_2^k-1}{\Phi'(x_2^k)-1+2p_2^k}dx_1^\#, \tag{D.15}$$

so that

$$dp_2^k = -c_2^k\frac{\Phi(x_2^k)}{\Phi(x_1^\#)}\frac{\Phi'(x_2^k)}{\Phi'(x_2^k)-1+2p_2^k}dx_1^\# \qquad (k \in \{1,\dots,K_2\}). \tag{D.16}$$

Using (D.16) to eliminate dp_2^k in (D.11), we obtain

$$\begin{aligned} \left\{\sum_{k=1}^{K_2}\delta_2^k c_2^k \Phi(x_2^k)\right\} d\varepsilon &= \frac{dx_1^\#}{\Phi(x_1^\#)}\left\{c_1^\#\Phi(x_1^\#)\Phi'(x_1^\#) + \sum_{k=1}^{K_2} q_2^k c_2^k \Phi(x_2^k)\frac{(1-2p_2^k)\Phi'(x_2^k)}{\Phi'(x_2^k)-1+2p_2^k}\right\} && \text{(D.17)}\\ &= \frac{dx_1^\#}{\Phi(x_1^\#)}\sum_{k=1}^{K_2} q_2^k c_2^k \Phi(x_2^k)\left\{\Phi'(x_1^\#) + \frac{(1-2p_2^k)\Phi'(x_2^k)}{\Phi'(x_2^k)-1+2p_2^k}\right\}. && \text{(D.18)} \end{aligned}$$

We claim that

$$\Phi'(x_1^\#) + \frac{(1-2p_2^k)\Phi'(x_2^k)}{\Phi'(x_2^k)+2p_2^k-1} > 0 \qquad (k \in \{1,\dots,K_2\}). \tag{D.19}$$

Indeed, this is obvious for $p_2^k \le \frac{1}{2}$ since $\Phi' \ge 1$ by Lemma B.1(ii).[6] On the other hand, if $p_2^k > \frac{1}{2}$, then $2p_2^k - 1 > 0$, and hence,

$$(2p_2^k-1)\cdot\frac{\Phi'(x_2^k)}{\Phi'(x_2^k)+2p_2^k-1} < 2p_2^k - 1 \le 1 \le \Phi'(x_1^\#). \tag{D.20}$$

[6] Note also that $p_2^k > 0$ since $x_2^k > 0$ by assumption.

Hence, claim (D.19) is indeed true. Therefore,

$$\frac{dx_1^{\#}}{d\varepsilon} = \{\text{sth. positive}\}^{-1} \cdot \left\{ \sum_{k=1}^{K_2} \delta_2^k \varphi^k \right\}, \tag{D.21}$$

where $\varphi^k = c_2^k \Phi(x_2^k)$, for $k \in \{1, \ldots, K_2\}$. The strict hump-shape of the sequence $(\varphi^1, \ldots, \varphi^{K_2})$ follows immediately from (D.10) and Lemma B.2. This proves the lemma. □

This lemma offers a necessary and sufficient condition for letting any type $c_2 \in C_2$ prefer to release a marginal piece of evidence δ_2. As will be noted, the condition does not depend on c_2, which means that all types have the same preference for disclosure. Obviously, this is due to the assumption of one-sided asymmetric information, which implies that a decline of $x_1^{\#}$ is equally desirable for all types of player 2.[7] Note also that Lemma D.1 does not require that the contest is uniformly asymmetric. However, the interiority assumption is crucial. Indeed, inactive types may not have a strict incentive to disclose a marginal piece of evidence if it does not change their state of marginalization. Thus, marginalized types (with positive shadow costs) exhibit some inertia with respect to the release of a marginal piece of evidence.

Condition (D.8) gets a simple interpretation in the Tullock case, where it turns out that we may choose $\varphi^k = c_2^k \xi_2^{\#}(c_2^k)$, for $k \in \{1, \ldots, K_2\}$, to be the type-specific *equilibrium costs* (or *expenses*). In an interior equilibrium, these costs indeed exhibit the hump-shape described in Lemma D.1 as a consequence of the first-order condition. Thus, in the Tullock case, a marginal piece of evidence is preferred to be disclosed if, roughly speaking, it makes extremal types (i.e., those with the lowest equilibrium costs) more likely and central types (i.e., those with highest equilibrium costs) less likely. Here as well, the activity assumption is crucial to obtain the conclusion of voluntary disclosure. Note, however, that Lemma D.1 does not admit an unraveling conclusion.

Lemma D.2 *Suppose given generic cost parameters $c_1^{\#} > 0$ and $\overline{c}_2 > \underline{c}_2 > 0$. Then, at least one of the following two statements holds true:*

(i) $c_1^{\#}\Phi(x_1^{\#}) > \underline{c}_2 \Phi(\xi_2^{\#}(\underline{c}_2))$, for all q_2 that assign positive probability to all types in C_2;

(ii) $c_1^{\#}\Phi(x_1^{\#}) > \overline{c}_2 \Phi(\xi_2^{\#}(\overline{c}_2))$, for all q_2 that assign positive probability to all types in C_2.

Proof. As before, we denote by $\widetilde{\beta}_2(\psi_1(x_1); c_2)$ type c_2's best response to the deterministic bid x_1. Let $e_2(x_1; c_2) = c_2 \Phi(\widetilde{\beta}_2(\psi_1(x_1); c_2))$.[8] The relevance of this function stems from the fact that, in equilibrium, $c_1^{\#}\Phi(x_1^{\#}) = E[e_2(x_1^{\#}; c_2)]$. We claim that the two functions $e_2(x_1; \underline{c}_2)$ and $e_2(x_1; \overline{c}_2)$ have the following *single-crossing property*: There exists some threshold value $\widehat{x}_1$ such that $e_2(x_1; \underline{c}_2) \gtreqless e_2(x_1; \overline{c}_2)$ if and only if only if $x_1 \gtreqless \widehat{x}_1$ (in the interval where $x_1 > 0$ and $\widetilde{\beta}_2(\psi_1(x_1); \underline{c}_2) > 0$). To see why the single-crossing property holds, recall first that $\widetilde{\beta}_2(\psi_1(x_1); c_2)$ is strictly hump-shaped in x_1. Therefore, given Lemma B.1(ii), the same is true for $e_2(x_1; c_2)$. The maximum of the function $e_2(x_1; c_2)$ is $\frac{1}{4}$, and that maximum is reached at $x_1 = x_1^{\max}(c_2)$ characterized by $p_2(\widetilde{\beta}_2(\psi_1(x_1); c_2), x_1) = \frac{1}{2}$. As $\widetilde{\beta}_2(\psi_1(x_1); c_2)$ is strictly declining in c_2 in the interior by Lemma B.2, we have that $x_1^{\max}(c_2)$ is strictly declining in c_2. Now, there are three cases. First, in the interval $[x_1^{\max}(\overline{c}_2), x_1^{\max}(\underline{c}_2)]$, the function $e_2(x_1; \overline{c}_2)$ is strictly declining in x_1 when positive, while the function $e_2(x_1; \overline{c}_2)$ is strictly increasing in x_1 when positive. Moreover, both functions are continuous (even differentiable when positive by the implicit function theorem). Hence, there exists a unique $\widehat{x}_1 \in (x_1^{\max}(\overline{c}_2), x_1^{\max}(\underline{c}_2))$ such that the single-crossing property holds in the interval $[x_1^{\max}(\overline{c}_2), x_1^{\max}(\underline{c}_2)]$. Next, in the interval $(0, x_1^{\max}(\overline{c}_2))$, we claim that $e_2(x_1; \underline{c}_2) < e_2(x_1; \overline{c}_2)$. To see why, note that total differentiation of type c_2's first-order condition yields

$$\frac{de_2}{dc_2} = \frac{(1 - 2p_2)\frac{\partial p_2}{\partial x_2}\Phi}{\text{SOC}} > 0, \tag{D.22}$$

where SOC<0 stands for the second derivative of type c_2's payoff function and we dropped the arguments for convenience. Note that $p_2 > \frac{1}{2}$ because $x_1 < x_1^{\max}(\overline{c}_2)$. Thus, lowering c_2 gradually from $\overline{c}_2$ down to $\underline{c}_2$, we indeed find that $e_2(x_1; \underline{c}_2) < e_2(x_1; \overline{c}_2)$. Finally, for values $x_1 > x_1^{\max}(\overline{c}_2)$ such that $\widetilde{\beta}_2(\psi_1(x_1); \underline{c}_2) > 0$, one shows

[7] This contrasts with the case of two-sided asymmetric information, dealt with below, where preferences regarding information release generally differ across types.

[8] In the case of the lottery contest, $e(x_1, c_2)$ is type c_2's expense when using a best response to x_1.

that $e_2(x_1;\underline{c}_2) > e_2(x_1;\overline{c}_2)$. The argument is essentially the same as before, provided that one notes that $p_2 < \frac{1}{2}$ because $x_1 > x_1^{\max}(\overline{c}_2)$. Now, the intersection point of the two functions $e_2(x_1;\underline{c}_2)$ and $e_2(x_1;\overline{c}_2)$ lies either below, on, or above the function $x_1 \mapsto c_1^{\#}\Phi(x_1)$, regardless of the probability distribution q_2. In the first case ("below"), we know from single-crossing that $e_2(x_1;\underline{c}_2) < e_2(x_1;\overline{c}_2)$. Hence, we are in case (i). In the second case ("on"), we know that $e_2(x_1;\underline{c}_2) = e_2(x_1;\overline{c}_2)$, which is a non-generic case, and leads to case (i) if $K_2 \geq 3$. In the third case ("above"), we are in case (ii). The lemma follows. □

Proof of Theorem 2. A type that would be inactive at the contest stage always strictly prefers to reveal her private information. Therefore, one may assume without loss of generality that the equilibrium at the contest stage is interior. But then, the vector $\{c_2^k\Phi(x_2^k)\}_{k=1}^{K_2}$ is strictly hump-shaped as a consequence of the first-order condition $p_2^k(1-p_2^k) = c_2^k\Phi(x_2^k)$, for any $k \in \{1,\ldots,K_2\}$, and the strict declining monotonicity of the bid schedule $\{x_2^k\}_{k=1}^{K_2}$. Now, the "all-or-nothing" disclosure by a type $c_2^k \in C_2$ may be seen as a continuous accumulation of identical pieces of evidence δ_2 with $\delta_2(c_2^k) = 1 - q_2^k$ and $\delta_2(c_2^l) = -q_2^l$ for any $l \in \{1,\ldots,K_2\}$ such that $l \neq k$. Suppose first that $K_2 \geq 3$. Then, by Lemma D.2, there exists an extremal type $c_2^k \in \{\underline{c}_2, \overline{c}_2\}$ such that, for the just defined marginal piece of evidence, the condition in Lemma D.1 is fulfilled. Moreover, this condition remains valid on the entire path. Hence, there is one extremal type that strictly prefers to disclose. Suppose next that $K_2 = 2$. Then the same argument goes through for generic values of $c_1^{\#}$, $\underline{c}_2$, and $\overline{c}_2$. This proves the claim. □

D.5 Continuous types

As a solution concept for the contest stage, we use pure-strategy Nash equilibrium rather than Bayesian equilibrium. The reduced-form definition of perfect Bayesian equilibrium remains unchanged. However, to ensure continuity properties of type-specific payoffs, we restrict attention to the special case of the lottery contest. A formal statement reflecting our discussion in the body of the paper is the following.

Proposition D.4 (Continuous type distributions) *Consider a uniformly asymmetric lottery contest with continuous and independent type distributions. Then, in any perfect Bayesian equilibrium of the contest with pre-play communication of verifiable information, both contestants' types are almost surely revealed at the contest stage.*

Proof. We repeatedly apply Benoît and Dubra (2006, Thm. 1), for which we refer the reader to the original paper. In a first step, we note that type c_2's expected payoff from disclosure,

$$u_2(c_2,c_2,S_1,S_2) \equiv \int_{S_1} \Pi_2(x_1^\circ,x_2^\circ;c_2)dF_1(c_1) + \int_{C_1\backslash S_1} \Pi_2(\xi_1^{\#}(c_1),x_2^{\#};c_2)dF_1(c_1) \tag{D.23}$$

$$= \int_{S_1}\left(\frac{c_1}{c_1+c_2}\right)^2 dF_1(c_1) + \mathrm{pr}\{C_1\backslash S_1\}\frac{E[\sqrt{c_1}|\,c_1\in C_1\backslash S_1]^2E[c_1|\,c_1\in C_1\backslash S_1]}{((E[c_1|\,c_1\in C_1\backslash S_1]+c_2)^2}, \tag{D.24}$$

is continuous in c_2 by Lebesgue's theorem of dominated convergence. Similarly, type c_2's expected payoff from concealment,

$$u_2(c_2,\emptyset,S_1,S_2) \equiv \int_{S_1} \Pi_2(x_1^{\#},\xi_2^{\#}(c_2);c_2)dF_1(c_1) + \int_{C_1\backslash S_1} \Pi_2(\xi_1^*(c_1),\xi_2^*(c_2);c_2)dF_1(c_1) \tag{D.25}$$

$$= \mathrm{pr}\{S_1\}\left(1-\frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\,\widetilde{c}_2\in C_2\backslash S_2]}{c_1+E[\widetilde{c}_2|\,\widetilde{c}_2\in C_2\backslash S_2]}\right)^2 + \int_{C_1\backslash S_1}\left(\frac{\xi_1^*(c_1)}{\xi_1^*(c_1)+\xi_2^*(c_2)} - c_2\xi_2^*(c_2)\right)dF_1(c_1), \tag{D.26}$$

is well-defined by the existence and uniqueness results in Ewerhart (2014, Thm. 3.4 & 4.2). Moreover, the mapping $c_2 \mapsto u_2(c_2,\emptyset,S_1,S_2)$ is continuous because ξ_1^* in the second term does not depend on c_2. By a straightforward generalization of Proposition 1, for any non-degenerate conditional distribution $F_2(\cdot|\,\emptyset,S_2)$, the lowest type $\widehat{c}_2$ in the support of $F_2(\cdot|\,\emptyset,S_2)$ has the property that $u_2(\widehat{c}_2,\widehat{c}_2,S_1,S_2) > u_2(\widehat{c}_2,\emptyset,S_1,S_2)$. By Benoît and Dubra (2006, Thm. 1), the underdog's signal is almost surely known in any perfect Bayesian equilibrium. Next, we consider the decision of the favorite under the assumption that the underdog's type is revealed with probability one. Continuity of the expected payoff functions $u_1(c_1,c_1,S_1,C_2)$ and $u_1(c_1,\emptyset,S_1,C_2)$, defined in analogy to (D.23) and (D.25), may be checked as above. Then, by a straightforward generalization of Proposition 2, almost surely across c_2, for any non-degenerate conditional distribution $F_1(\cdot|\,\emptyset,S_1)$, the highest type $\widehat{c}_1$ in the support of $F_1(\cdot|\,\emptyset,S_1)$ has, typewise across $c_2 \in C_2$, but hence also globally the property that $u_1(\widehat{c}_1, \widehat{c}_1, S_1, C_2) > u_1(\widehat{c}_1, \emptyset, S_1, C_2)$. Applying Benoît and Dubra (2006, Thm. 1) again, we see that also the favorite's type is necessarily almost surely known in any perfect Bayesian equilibrium. This concludes the proof and proves the proposition. □

D.6 Other type of uncertainty

Proposition 3 says that our focus on marginal cost types is essentially without loss of generality. The proof relies on suitable variable substitutions.

Proof of Proposition 3. (i) Suppose first that the ability parameters γ_1 and γ_2 are public information. Then, using the substitution $\widetilde{c}_i = c_i/(V_i - L_i)$, the positive affine transform of type θ_i's payoff function,

$$\frac{\Pi_i(x_i, x_j; \theta_i) - L_i}{V_i - L_i} = \frac{\gamma_i h(x_i)}{\gamma_1 h(x_1) + \gamma_2 h(x_2)} - \frac{c_i}{V_i - L_i} x_i = \Pi_i(x_i, x_j; \widetilde{c}_i), \tag{D.27}$$

is seen to be of the normalized type assumed above. (ii) Suppose, alternatively, that $h(y) = y^r$ for some $r \in (0, 1]$. Then, using the substitution $\widetilde{x}_i = \gamma_i^{-1/r} x_i$, one finds similarly that

$$\frac{\Pi_i(x_i, x_j; \theta_i) - L_i}{V_i - L_i} = \frac{(\gamma_i^{1/r} x_i)^r}{(\gamma_1^{1/r} x_1)^r + (\gamma_2^{1/r} x_2)^r} - \frac{c_i}{V_i - L_i}(\gamma_i^{-1/r} x_i) = \Pi_i(\widetilde{x}_i, \widetilde{x}_j; \widetilde{c}_i). \tag{D.28}$$

This completes the proof. □

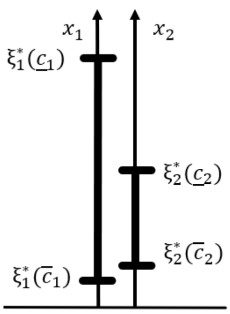
E. Material omitted from Section 6

Section 6 is concerned with limits of the scope of the strong-form disclosure principle. We will discuss contests that are not uniformly asymmetric, Bayesian persuasion, the option to shut down communication, and unverifiable information.

E.1 Contests that are not uniformly asymmetric

In Example 1, bid distributions under mandatory concealment are overlapping. A similar example in which bid distributions are nested is shown below.

Example E.1 (Nested type distributions) The contest specified by the parameters shown in Figure E.1 is not uniformly asymmetric. Assumption 1 does not hold. And indeed, as the data shows, there is a perfect Bayesian equilibrium in which no type reveals her private information.



(*a*)

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 0.7$	$c_1^2 = 7$ $q_1(c_1^2) = 0.3$	$c_2^1 = 2$ $q_2(c_2^1) = 0.8$	$c_2^2 = 5$ $q_2(c_2^2) = 0.2$
$\xi_1^*(c_1^1) = 0.2016$ $\Pi_1^*(c_1^1) = 0.5031$	$\xi_1^*(c_1^2) = 0.0253$ $\Pi_1^*(c_1^2) = 0.0718$	$\xi_2^*(c_2^1) = 0.1019$ $\Pi_2^*(c_2^1) = 0.2717$	$\xi_2^*(c_2^2) = 0.0310$ $\Pi_2^*(c_2^2) = 0.1036$
$x_1^\# = 0.1923$ $\Pi_1^\# = 0.4999$	–	$\xi_2^\#(c_2^1) = 0.1178$	$\xi_2^\#(c_2^2) = 0.0038$
–	$x_1^\# = 0.0270$ $\Pi_1^\# = 0.0703$	$\xi_2^\#(c_2^1) = 0.1178$	$\xi_2^\#(c_2^2) = 0.0038$
$\xi_1^\#(c_1^1) = 0.2144$	$\xi_1^\#(c_1^2) = 0.0208$	$x_2^\# = 0.0968$ $\Pi_2^\# = 0.2712$	–
$\xi_1^\#(c_1^1) = 0.1548$	$\xi_1^\#(c_1^2) = 0.0357$	–	$x_2^\# = 0.0367$ $\Pi_2^\# = 0.1027$

(*b*)

Data for Example E.1

One might conjecture that the strong-form disclosure principle *generally* fails in symmetric contests of incomplete information. The following example shows that this is not the case.

Example E.2 (Symmetric contest) Consider a symmetric lottery contest with two equally likely types, $\overline{c} > \underline{c} > 0$, for each player (Malueg and Yates, 2004). This contest is not uniformly asymmetric.[9] Theorem 2 implies

[9] It suffices to note that, by Lemma 1, the Nash equilibrium after full disclosure is unique, hence symmetric and interior if two equal types are matched.

that one-sided disclosure cannot be an equilibrium. To derive the conditions under which mutual concealment is an equilibrium, suppose that player 2 does not disclose her private information. Then, the efficient type of player 1, $\underline{c}$, has expected payoffs from not disclosing of

$$E_{c_2}[\Pi_1(\xi_1^*(\underline{c}), \xi_2^\#(c_2)); \underline{c}] = \frac{1}{8} + \frac{\overline{c}^2}{2(\underline{c}+\overline{c})^2}. \tag{E.1}$$

If $\underline{c}$, instead, reveals her private information, then there are two cases. If $\overline{c}/\underline{c} \geq 9$, disclosure marginalizes the inefficient type of player 2, i.e, $\xi_2^\#(\underline{c}, \overline{c}) = 0$. As a result, two equally efficient opponents meet with probability one half, so that player 1's expected payoff is $E_{c_2}[\Pi_1(x_1^\#(\underline{c}), \xi_2^\#(c_2; \underline{c})); \underline{c}] = \frac{5}{9}$. Therefore, revealing her type is optimal for the efficient type if $\overline{c}/\underline{c} < \frac{6\sqrt{31}+31}{5} \approx 12.88$. For the inefficient type of player 1, $\overline{c}$, one compares the expected payoff from not disclosing,

$$E_{c_2}[\Pi_1(\xi_1^*(\overline{c}), \xi_2^*(c_2)); \overline{c}] = \frac{1}{8} + \frac{\underline{c}^2}{2(\underline{c}+\overline{c})^2}, \tag{E.2}$$

with the expected payoff in the contest with one-sided private information,

$$E_{c_2}[\Pi_1(x_1^\#(\underline{c}), \xi_2^\#(c_2; \underline{c})); \underline{c}] = \frac{(\sqrt{\underline{c}}+\sqrt{\overline{c}})^2(\underline{c}+\overline{c})}{2(\underline{c}+3\overline{c})^2}, \tag{E.3}$$

which can be easily checked to be always strictly lower. Thus, the strong-form disclosure principle holds in this example if and only if $\overline{c}/\underline{c} < 12.88$.[10]

E.2 Bayesian persuasion

Kamenica and Gentzkow (2011) considered a general setting with one sender and one receiver, and an unknown state of the world, where the sender precommits to a signal about the state of the world. Upon receiving the signal, the receiver rationally updates her belief about the state of the world and takes an action. Depending on whether the commitment power lies with a player or with the social planner, the approach is known as Bayesian persuasion or information design. In this section, we consider the first problem, i.e., the sender is other player.[11]

Consider a lottery contest with one-sided incomplete information, where the type $c_1^\#$ of player 1 is public. Then the expected payoff to an active type c_2 is given by

$$\Pi_2^\#(c_2|\mu_2) = \left(1 - \frac{\sqrt{c_2}E[\mathbf{1}_{\widetilde{c}_2 \text{ active}}\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^\# + E[\mathbf{1}_{\widetilde{c}_2 \text{ active}}\widetilde{c}_2|\mu_2]}\right)^2, \tag{E.4}$$

where $\mathbf{1}_{\widetilde{c}_2 \text{ active}}$ is an indicator variable that equals one (zero) if $\xi_2^\#(\widetilde{c}_2; c_1^\#) > 0$ $(= 0)$ in the contest with one-sided incomplete information and type distribution μ_2 for player 2, and where $E[.|\mu_2]$ denotes the expectation given player 1's belief $\mu_2 \in \Delta(C_2)$ about 2's types at the contest stage.[12] The logic of marginalization is as follows. As the belief μ_2 gives too much weight to strong types, player 1 bids higher which induces weak types of player 2 to bid zero. The condition spelt out in the following lemma ensures that marginalization does not occur for a persuasion model with two types.

[10] Superficially, our main conclusion is just the opposite of the corresponding finding for the all-pay auction. Kovenock et al. (2015, Prop. 5) find that, with either private or common values, the interim information sharing game admits a perfect Bayesian equilibrium in which no firm ever shares its information. As noted by a referee, there is an important caveat here because the literature on all-pay auctions (Kovenock et al., 2015; Tan, 2016) has tended to focus on symmetric contests, while our analysis has focused on asymmetric contests. However, Example E.2 suggests that it is the contest technology that matters. Indeed, the difference in conclusions might mirror a more general fact. While the auction induces a hide-and-seek type of randomized behavior for which keeping secrets seems advisable, the probabilistic contest induces players to think in trade-offs, which may then entail voluntary disclosures of private information to the opponent. More work on the relationship of these two "battle modes" in contests is certainly desirable.

[11] The problem of information design will be considered further below.

[12] The expected payoff of an inactive type is zero.

Lemma E.1 (Interiority condition) *Suppose that $K_2 = 2$ and $c_1^{\#} + \underline{c}_2 > \sqrt{\underline{c}_2\overline{c}_2}$. Then, all types $c_2 \in C_2$ of player 2 are active, regardless of player 1's posterior belief μ_2.*

Proof. Take a posterior belief $\mu_2 \in \Delta(C_2)$. The weakest type $\overline{c}_2 \in C_2$ is active if and only if $\sqrt{\overline{c}_2} < (c_1^{\#} + E[c_2 | \mu_2])/E[\sqrt{c_2} | \mu_2]$, or equivalently, if $m\sqrt{\underline{c}_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2}) < c_1^{\#}$, where $m = \mu_2(\underline{c}_2)$. Letting $m = 1$ yields the condition in the statement of the lemma. □

Now, in the absence of communication, μ_2 simply corresponds to the ex-ante distribution $\{q_2(c_2^k)\}_{k=1}^{K_2}$. Bayesian persuasion allows player 2 to precommit to a signal, which induces a probability distribution $\tau_2 \in \Delta(\Delta(C_2))$ over posterior beliefs $\mu_2 \in \Delta(C_2)$ that is subject to Bayes plausibility

$$\int \mu_2(c_2) d\tau_2(\mu_2) = q_2(c_2) \qquad (c_2 \in C_2). \tag{E.5}$$

Therefore, player 2's problem reads

$$\max_{\tau_2 \text{ s.t. (E.5)}} \int E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2)\right] d\tau_2(\mu_2), \tag{E.6}$$

where $E_{c_2}[.]$ denotes, as before, the expectation with respect to the prior distribution on C_2 given by q_2.

As a general solution of problem (E.6) is beyond the scope of the present analysis, we discuss a simple example with $K_2 = 2$. Then, with precommitment, the signal may lead to a probability distribution τ_2 over two distributions $\mu_2^{\mathrm{A}}, \mu_2^{\mathrm{B}} \in \Delta(C_2)$, with respective probabilities τ_2^A and τ_2^B satisfying

$$\tau_2^{\mathrm{A}}\mu_2^{\mathrm{A}}(c_2) + \tau_2^{\mathrm{B}}\mu_2^{\mathrm{B}}(c_2) = q_2(c_2) \qquad (c_2 \in C_2). \tag{E.7}$$

For instance, in the special case where $\overline{c}_2 > c_1^{\#} > \underline{c}_2$, we might expect that player 2 benefits if, compared to the prior, μ_2^{A} is biased towards $\underline{c}_2$, while μ_2^{B} is biased towards $\overline{c}_2$. Intuitively, the positive effect of overstatement on the weak type's payoff would be combined with the likewise positive effect of understatement on the strong type's payoff.

Proposition E.1 (Bayesian persuasion) *Consider an unbiased lottery contest where player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in \{\underline{c}_2, \overline{c}_2\}$ is private with $\overline{c}_2 > \underline{c}_2$. Suppose also that the interiority assumption of Lemma E.1 holds. Then there exists a threshold value $\chi^* \in [0, 1]$ such that:*

(i) If $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$, then full disclosure is optimal;

(ii) if $c_1^{\#} = \sqrt{\underline{c}_2\overline{c}_2}$, then any signal is optimal;

(iii) if $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$ and $q_2(\underline{c}_2) \leq \chi^*$, then full concealment is optimal;

(iv) if $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$ and $q_2(\underline{c}_2) > \chi^*$, then player uses a randomized signal with posterior beliefs satisfying $\mu_2^{\mathrm{A}}(\underline{c}_2) = \chi^*$ and $\mu_2^{\mathrm{B}}(\underline{c}_2) = 1$.[13]

Proof. The sender (player 2) solves the problem

$$\max_{\tau_2 \text{ s.t. (E.7)}} \tau_2^{\mathrm{A}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{A}})\right] + \tau_2^{\mathrm{B}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{B}})\right]. \tag{E.8}$$

More explicitly, this becomes

$$\max_{\substack{\tau_2^{\mathrm{A}} \equiv 1-\tau_2^{\mathrm{B}} \in [0,1], \\ \mu_2^{\mathrm{A}}(\underline{c}_2) \equiv 1-\mu_2^{\mathrm{A}}(\overline{c}_2) \in [0,1], \\ \mu_2^{\mathrm{B}}(\underline{c}_2) \equiv 1-\mu_2^{\mathrm{B}}(\overline{c}_2) \in [0,1], \\ \text{s.t. (E.7)}}} \tau_2^{\mathrm{A}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{A}})\right] + \tau_2^{\mathrm{B}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{B}})\right]. \tag{E.9}$$

[13] An example illustrating this case is discussed below.

We shart with the case where $c_1^{\#} \geq \sqrt{\underline{c}_2\overline{c}_2}$. By Lemma E.1, both types of player 2 are active. Therefore, the question if player 2 benefits from persuasion (or not) is linked to the strict convexity (or weak concavity) of the function

$$\widehat{\Pi}_2(\mu_2) = q_2(\underline{c}_2)\left(1 - \frac{\sqrt{\underline{c}_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]}\right)^2 + q_2(\overline{c}_2)\left(1 - \frac{\sqrt{\overline{c}_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]}\right)^2, \tag{E.10}$$

where the posterior μ_2 is given as

$$\mu_2 \equiv (\mu_2(\underline{c}_2), \mu_2(\overline{c}_2)) \in \Delta(C_2) = \{(m, 1-m) : 0 \leq m \leq 1\}. \tag{E.11}$$

Let $c_2 \in C_2$. Based on the computation of the first derivative,

$$\frac{\partial}{\partial m}\left(1 - \frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]}\right)^2 = \frac{\partial}{\partial m}\left(1 - \frac{\sqrt{c_2}(m\sqrt{\underline{c}_2} + (1-m)\sqrt{\overline{c}_2})}{c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2}\right)^2 \tag{E.12}$$

$$= 2 \cdot \left(1 - \frac{\sqrt{c_2}(m\sqrt{\underline{c}_2} + (1-m)\sqrt{\overline{c}_2})}{c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2}\right) \cdot \frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^2}, \tag{E.13}$$

we see that the second derivative is given by

$$\frac{\partial^2}{\partial m^2}\left(1 - \frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]}\right)^2 = 2 \cdot \left(\frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^2}\right)^2 \tag{E.14}$$

$$+ 4 \cdot \underbrace{\left(1 - \frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]}\right)}_{>0 \text{ by activity}} \cdot \frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})(\overline{c}_2 - \underline{c}_2)}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^3}.$$

Clearly, the right-hand side of (E.14) is positive (zero) if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$ (if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$) regardless of $c_2 \in C_2$ and $m \in [0, 1]$, which proves parts (i) and (ii). Suppose next that $\sqrt{\underline{c}_2\overline{c}_2} > c_1^{\#}$. Then, combining (E.10) and (E.14), we get

$$\frac{\partial^2\widehat{\Pi}_2(\mu_2)}{\partial m^2} = \frac{2(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})^2(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + E[\widetilde{c}_2|\mu_2])^4} \cdot \left\{\begin{array}{c} E[c_2](c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2}) \\ +2(c_1^{\#} + E[\widetilde{c}_2|\mu_2])E[\sqrt{c_2}](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) \\ -2E[c_2]E[\sqrt{\widetilde{c}_2}|\mu_2](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) \end{array}\right\}. \tag{E.15}$$

Exploiting that $E[\sqrt{c_2}](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) = E[c_2] + \sqrt{\underline{c}_2\overline{c}_2}$ and $E[\sqrt{\widetilde{c}_2}|\mu_2](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) = E[\widetilde{c}_2|\mu_2] + \sqrt{\underline{c}_2\overline{c}_2}$, we see that

$$\frac{\partial^2\widehat{\Pi}_2(\mu_2)}{\partial m^2} = \frac{2(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})^2(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^4} \cdot \left\{\begin{array}{c} (3E[c_2] + 2\sqrt{\underline{c}_2\overline{c}_2})\, c_1^{\#} \\ -\,(3E[c_2] - 2(m\underline{c}_2 + (1-m)\overline{c}_2))\sqrt{\underline{c}_2\overline{c}_2} \end{array}\right\}. \tag{E.16}$$

As the expression in the curly brackets is linear in m, we certainly find a unique cut-off level $m^* \in \mathbb{R}$ such that, if replaced for m in (E.16), renders this term equal to zero. Moreover, $\widehat{\Pi}_2(\mu_2)$ is strictly concave for $m \leq m^*$, and strictly convex for $m \geq m^*$. There are now three cases. Suppose first that $m^* \geq 1$. Then, $\widehat{\Pi}_2(\mu_2)$ is globally strictly concave regardless of $q_2(\underline{c}_2)$, so that full concealment is optimal. In this case, we may set $\chi^* = 1$. Next, suppose that $m^* \in (0, 1)$. Then, taking the convex closure of $\widehat{\Pi}_2(\mu_2)$ over the interval $[0, 1]$, we find a "tangential" point at some $\chi^* \in [0, m^*)$, as illustrated conceptually in Figure E.2 in the case where $m^* \in (0,1)$ and $\chi^* > 0$. (For $\chi^* = 0$, the slope of $\widehat{\Pi}_2(\mu_2)$ at $m = 0$ and the slope of the upper boundary of the convex closure may differ). If $q_2(\underline{c}_2) \leq \chi^*$, then full concealment remains optimal. If $q_2(\underline{c}_2) > \chi^*$, however, player 2's signal randomizes, in response to her type and the randomizing commitment device, between the two signals causing Bayesian posteriors μ_2^{A} with $\mu_2^{\mathrm{A}}(\underline{c}_2) = \chi^*$ and μ_2^{B} with $\mu_2^{\mathrm{B}}(\underline{c}_2) = 1$. Suppose, finally, that $m^* \leq 0$. Then, $\widehat{\Pi}_2(\mu_2)$ is globally strictly convex regardless of $q_2(\underline{c}_2)$, so that full disclosure is optimal. In this case, we may set $\chi^* = 0$. This proves the claim. □

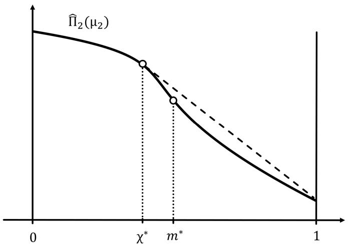


Figure E.2 Bayesian persuasion.

The rough intuition for the underlying effects here is that a stronger uninformed contestant raises her efforts in response to uncertainty, whereas a weaker uninformed contestant lowers her efforts in response to uncertainty (cf. Hurley and Shogren, 1998a). With $c_1^{\#} > \sqrt{\underline{c}_2 \overline{c}_2}$, player 1 is comparably weak, so it makes sense for player 2 to inform player 1. In the knife-edge case where $c_1^{\#} = \sqrt{\underline{c}_2 \overline{c}_2}$, player 1 does not care about player 2's type as each type chooses the same bid level. Hence, any signal is optimal in that case. The situation gets more structured for $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$, where player 1 is comparably strong. In that case, the signal will never be fully informative. Instead, either full concealment is optimal (if $q_2(\underline{c}_2) \leq \chi^*$), or player 2 optimally uses a randomized signal (if $q_2(\underline{c}_2) > \chi^*$). When a randomized signal is used, player 2 reveals her type when strong with a probability τ_2^{B} strictly smaller than one, but never reveals her type when weak. As we have show in the proof of Proposition E.4, player 2's expected payoff $\widehat{\Pi}_2(\mu_2)$ in a contest with posterior μ_2, considered as a function of m, is concave left of some cut-off value m^* and convex right of m^*.

We conclude this subsection by giving an example that illustrates the possibility of a randomizing commitment device.

Example E.3 (Randomization in Bayesian persuasion) Suppose that $c_1^{\#} = 1$, $\underline{c}_2 = 5$, $\overline{c}_2 = 6$, $q_2(\underline{c}_2) = 0.75$, and $q_2(\overline{c}_2) = 0.25$. Then, $m^* = 0.56$ and $\chi^* = 0.32$.

E.3 Shutting down communication

So far, we assumed that, if one player discloses, the other player automatically gets informed, and this is commonly known. But in some situations, it may be possible to publicly commit to ignore any information provided by one's opponent.

Proposition E.2 (The underdog never shuts down communication) *Consider a uniformly asymmetric, unbiased lottery contest. Suppose that the type of the underdog is public information, whereas the favorite has at least two possible type realizations. Then, the underdog's ex-ante expected payoff is strictly higher under full revelation than under mandatory concealment, i.e.,* $\Pi_2^{\mathrm{FR}} > \Pi_2^{\mathrm{MC}}$.

Proof. The underdog's expected profits under mandatory concealment (i.e., the underdog "closes her eyes") and under full revelation (i.e., the underdog "opens her eyes"), respectively, are easily derived as $\Pi_2^{\mathrm{MC}} = E[\sqrt{c_1}]^2 E[c_1]/(E[c_1] + c_2^{\#})^2$ and $\Pi_1^{\mathrm{FR}} = E[c_1^2/(c_1 + c_2^{\#})^2]$. To compare these expressions, we apply Lemma G.1(ii), in which the support of the random variable Y is assumed to be $(0,1)$, with $Y = \sqrt{c_1/c_2^{\#}}$ and $g(x,y) = g_3(x,y) \equiv \frac{x^2 y}{(1+y)^2}$. It suffices to show that, for any $x, y \in (0,1)$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} < \frac{1-y}{1-x}$, the quadratic form

$$(d_x \; d_y)\,(H_{g_3}(x,y)) \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2{d_x}^2}{(1+y)^2} \left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)\left(y + \frac{x(2-y)}{1+y}\frac{d_y}{d_x}\right) \tag{E.17}$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1} \cdot \frac{d_y}{d_x} < \underbrace{\frac{x}{y+1} \cdot \frac{1-y}{1-x}}_{\text{decreasing in } y} \leq \frac{3x}{x^2+1} \cdot \frac{1-x^2}{1-x} = \frac{x^2+x}{x^2+1} < 1. \tag{E.18}$$

This proves the claim. □

Thus, the underdog would never prefer to publicly announce to ignore any information received. The intuitive force behind this result is that the underdog can better target her effort, so that the ex-ante winning probability increases. An analogous result for the favorite is not true, however. Indeed, Example E.3 above shows that the favorite may benefit from committing to ignore any information released by the underdog.

E.4 Unverifiable information

This section presents the proof of Theorem 2. Our derivation draws heavily from Pavlov (2013) who established that communication equilibria and Bayesian equilibria are payoff-equivalent in two-player all-pay auctions.

Proof of Theorem 2. By the existence part of Lemma 1, there is a Bayesian Nash equilibrium of the contest stage, $\xi^* = (\xi_1^*, \xi_2^*) \in X_1 \times X_2$. Thus, for any $i \in \{1,2\}$, $c_i \in C_i$, and $x_i \in [0,1]$, we have

$$E_{c_j}[\Pi_i(\xi_i^*(c_i), \xi_j^*(c_j); c_i)] \geq E_{c_j}[\Pi_i(x_i, \xi_j^*(c_j); c_i)], \tag{E.19}$$

where $j \neq i$. Suppose given a *communication equilibrium*, consisting of a nonempty, finite set of reports R_i as well as a nonempty, finite set of messages M_i for each player $i \in \{1,2\}$, a coordination mechanism $\pi : R_1 \times R_2 \to \Delta(M_1 \times M_2)$, and functions $\rho_i : C_i \to R_i$, $\zeta_i : M_i \times C_i \to [0,1]$, for each player $i \in \{1,2\}$, such that, for all $i \in \{1,2\}$, $c_i \in C_i$, $\widehat{\rho}_i$, and $\widehat{\zeta}_i$,

$$\begin{aligned} &E_{c_j}\left[\sum_{m_1,m_2} \Pi_i(\zeta_i(m_i,c_i), \zeta_j(m_j,c_j)\pi(m_i,m_j|\,\rho_i(c_i), \rho_j(c_j))\right] \\ &\geq E_{c_j}\left[\sum_{m_1,m_2} \Pi_i(\widehat{\zeta}_i(m_i,c_i), \zeta_j(m_j,c_j)\pi(m_i,m_j|\,\widehat{\rho}_i(c_i), \rho_j(c_j))\right]. \end{aligned} \tag{E.20}$$

In particular, inequality (E.20) holds if the deviation $(\widehat{\rho}_i, \widehat{\zeta}_i)$ is given by an uninformative $\widehat{\rho}_i(c_i) \equiv r_i$ (always send the same report $r_i \in R_i$, regardless of type), and $\widehat{\zeta}_i(m_i, c_i) \equiv \xi_i^*(c_i)$.[14] Thus, for all $i \in \{1,2\}$ and $c_i \in C_i$,

$$\begin{aligned} &E_{c_j}\left[\sum_{m_1,m_2} \Pi_i(\zeta_i(m_i,c_i), \zeta_j(m_j,c_j)\pi(m_i,m_j|\,\rho_i(c_i), \rho_j(c_j))\right] \\ &\geq E_{c_j}\left[\sum_{m_1,m_2} \Pi_i(\xi_i^*(c_i), \zeta_j(m_j,c_j)\pi(m_i,m_j|\,r_i, \rho_j(c_j))\right]. \end{aligned} \tag{E.21}$$

Next, replacing x_i by $\zeta_i(m_i, c_i)$ in (E.19), we have for any $i \in \{1,2\}$ and $c_i \in C_i$,

$$E_{c_j}[\Pi_i(\xi_i^*(c_i), \xi_j^*(c_j); c_i)] \geq E_{c_j}[\Pi_i(\zeta_i(m_i,c_i), \xi_j^*(c_j); c_i)]. \tag{E.22}$$

Multiplying by $\pi(m_i, m_j|\,\rho_i(c_i), r_j)$, and summing over all pairs $(m_1, m_2) \in M_1 \times M_2$ yields, for any $i \in \{1,2\}$

[14] Farrell (1985) and Pavlov (2013) assumed that the babbling deviation has all types randomize over reports. Our strategy of proof, where all types send the same report, simplifies the notation but otherwise exploits the same intuition.

and $c_i \in C_i$,

$$E_{c_j}[\Pi_i(\xi_i^*(c_i), \xi_j^*(c_j); c_i)] \geq E_{c_j}\left[\sum_{m_1,m_2} \Pi_i(\zeta_i(m_i, c_i), \xi_j^*(c_j); c_i)\pi(m_i, m_j | \rho_i(c_i), r_j)\right]. \tag{E.23}$$

Adding the two equations (E.21) and (E.23), noting that the cost terms cancel out, and taking expectations with respect to c_i delivers, for any $i \in \{1, 2\}$,

$$E\left[p_i(\xi_i^*(c_i), \xi_j^*(c_j)) + \sum_{m_1,m_2} p_i(\zeta_i(m_i, c_i), \zeta_j(m_j, c_j))\pi(m_i, m_j | \rho_i(c_i), \rho_j(c_j))\right]$$
$$\geq E\left[\begin{array}{c} \sum_{m_1,m_2} p_i(\xi_i^*(c_i), \zeta_j(m_j, c_j))\pi(m_i, m_j | r_i, \rho_j(c_j)) \\ +\sum_{m_1,m_2} p_i(\zeta_i(m_i, c_i), \xi_j^*(c_j))\pi(m_i, m_j | \rho_i(c_i), r_j) \end{array}\right]. \tag{E.24}$$

Adding over players, one arrives at

$$\underbrace{E\left[p_i(\xi_i^*(c_i), \xi_j^*(c_j)) + p_j(\xi_j^*(c_i), \xi_i^*(c_j))\right]}_{=1}$$
$$\underbrace{+E\left[\sum_{m_1,m_2} (p_i(\zeta_i(m_i, c_i), \zeta_j(m_j, c_j)) + p_j(\zeta_j(m_j, c_j), \zeta_i(m_i, c_i)))\pi(m_i, m_j | \rho_i(c_i), \rho_j(c_j))\right]}_{=1}$$
$$\geq \underbrace{E\left[\sum_{m_1,m_2} \left(p_i(\xi_i^*(c_i), \zeta_j(m_j, c_j)) + p_j(\zeta_j(m_j, c_j), \xi_i^*(c_i))\right)\pi(m_i, m_j | r_i, \rho_j(c_j))\right]}_{=1} \tag{E.25}$$
$$+\underbrace{E\left[\sum_{m_1,m_2} (p_i(\zeta_i(m_i, c_i), \xi_j^*(c_j)) + p_j(\xi_j^*(c_j), \zeta_j(m_j, c_j)))\pi(m_i, m_j | \rho_i(c_i), r_j)\right]}_{=1}.$$

Inequality (E.25) is, however, an equality. Hence, given our assumption that all types have a positive probability, inequality (E.23) is an equality for any $i \in \{1, 2\}$ and $c_i \in C_i$. This means that the randomized strategy for player i, in which each type c_i chooses the bid $\zeta_i(m_i, c_i)$ with probability $\sum_{m_j} \pi(m_i, m_j | \rho_i(c_i), r_j)$, is a best response to ξ_j^*. As noted before, however, the best response in a probabilistic contest is a pure strategy and unique. Hence, $\zeta_i(m_i, c_i) = \xi_i^*(c_i)$ for all $i \in \{1, 2\}$, $c_i \in C_i$, and $m_i \in M_i$. Thus, in any communication equilibrium, all recommendations are ignored. □

Notwithstanding Theorem 3, costless unverifiable messages may indeed carry information about types. This, however, is an artefact of the hump-shape of the best-response mapping, as the following example illustrates.

Example E.5 (Irrelevant information) Suppose that $c_1^{\#} = 1$, $\underline{c}_2 = \frac{1}{4}$, and $\overline{c}_2 = 4$. Then, in an unbiased lottery contest,

$$x_1^{\#} = \left(\frac{E[\sqrt{c_2}]}{c_1 + E[c_2]}\right)^2 = \left(\frac{2q + (1-q)\frac{1}{2}}{1 + 4q + (1-q)\frac{1}{4}}\right)^2 = \frac{4}{25}, \tag{E.26}$$

regardless of posterior beliefs. Still, in equilibrium, the receiver would not make use of that information.

F. Material omitted from Section 7

This section elaborates on the welfare implications of communication in contests. We start by presenting the example of the "disclosure trap." Then, we discuss expense maximization. Finally, we deal with the problem of information design.

F.1 The "disclosure trap"

To discuss efficiency, we will compare the equilibrium scenario of full revelation (FR) with the hypothetical benchmark of mandatory concealment (MC). Let $\mathbf{C}^{\mathrm{FR}} = E[c_1 x_1^\circ(c_1, c_2) + c_2 x_2^\circ(c_1, c_2)]$ and $\mathbf{C}^{\mathrm{MC}} = E[c_1 \xi_1^*(c_1) + c_2 \xi_2^*(c_2)]$, respectively, denote total expected costs under full revelation and under mandatory concealment.[15] Further, for $i \in \{1, 2\}$, let $p_i^{\mathrm{FR}} = E\left[p_i(x_1^\circ(c_1, c_2), x_2^\circ(c_1, c_2))\right]$ and $p_i^{\mathrm{MC}} = E\left[p_i(\xi_1^*(c_1), \xi_2^*(c_2))\right]$ denote player i's ex-ante probability of winning under full revelation and under mandatory concealment. Finally, likewise for $i \in \{1, 2\}$, let $\Pi_i^{\mathrm{FR}} = p_i^{\mathrm{FR}} - E[c_i x_i^\circ(c_1, c_2)]$ and $\Pi_i^{\mathrm{MC}} = p_i^{\mathrm{MC}} - E_{c_i}[c_i \xi_i^*(c_i)]$ denote player i's ex-ante expected payoff under full revelation and mandatory concealment.

The following example illustrates the possibility that full revelation may actually be ex-ante undesirable for both contestants.

Example F.1 ("Disclosure trap") The setting specified in Table F.1 satisfies Assumption 1. It can be seen that the unraveling leads the contestants into a strictly Pareto inferior outcome.

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 1$		$c_2^1 = 2$ $q_2(c_2^1) = 0.5$	$c_2^2 = 3$ $q_2(c_2^2) = 0.5$
$x_1^\# = 0.2020$		$\xi_2^\#(c_2^1) = 0.1158$	$\xi_2^\#(c_2^2) = 0.0575$
$x_1^\circ = 0.2222$		$x_2^\circ = 0.1111$	$-$
$x_1^\circ = 0.1875$		$-$	$x_2^\circ = 0.0625$
$\mathbf{C}^{\mathrm{FR}} = 0.4097$	$>$	$\mathbf{C}^{\mathrm{MC}} = 0.4040$	
$p_1^{\mathrm{FR}} = 0.7083 > p_1^{\mathrm{MC}} = 0.7071$		$p_2^{\mathrm{FR}} = 0.2917 < p_2^{\mathrm{MC}} = 0.2929$	
$\Pi_1^{\mathrm{FR}} = 0.5035 < \Pi_1^{\mathrm{MC}} = 0.5050$		$\Pi_2^{\mathrm{FR}} = 0.0868 < \Pi_2^{\mathrm{MC}} = 0.0909$	

Table F.1 Equilibrium bids under full revelation and mandatory concealment.

The example illustrates that the option to disclose verifiable information may be undesirable for a contestant. The reason is an externality that the self-disclosing marginal type imposes on the silent submarginal types. The externality is a virtual one only, because two type realizations of the same contestant never coexist. Notwithstanding, the inability to commit leads to a situation in which the privately informed player loses in expected terms by the unraveling.

F.2 Expense maximization

The following result shows that, even though full revelation need not be in the interest of an informed contestant, a contest organizer maximizing total expected expenses may well find that outcome preferable to full concealment.

Proposition F.1 (Expense maximization) *Consider a uniformly asymmetric, unbiased lottery contest. Suppose that the type of the favorite is public information, whereas the underdog has at least two possible type realizations. Assume also that, under mandatory concealment, all types are active. Then,*

(i) $\mathbf{C}^{\mathrm{FR}} > \mathbf{C}^{\mathrm{MC}}$ *(in both cases, expected costs split evenly between the players);*[16]

(ii) the underdog's (the favorite's) ex-ante probability of winning is strictly lower (strictly higher) under full revelation than under mandatory concealment, i.e., $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$ $(p_1^{\mathrm{FR}} > p_1^{\mathrm{MC}})$*; and*

(iii) the ex-ante payoff for the underdog is strictly lower under full revelation than under mandatory concealment, i.e., $\Pi_2^{\mathrm{FR}} < \Pi_2^{\mathrm{MC}}$.[17]

Proof. (i) Let $c_1^\# \in C_1$ denote the public type of the favorite. For the unbiased lottery contest, an interior equilibrium may be easily derived from the corresponding first-order conditions (Hurley and Shogren, 1998a;

[15] $E[.] = E_{c_1,c_2}[.]$ denotes the ex-ante expectation.

[16] Thus, the effort of the favorite is strictly higher under full revelation than under mandatory concealment. The expected effort of the underdog, however, may either rise or fall, depending on parameters.

[17] The payoff comparison for the favorite is ambiguous, i.e., depending on parameters, it may be that $\Pi_1^{\mathrm{FR}} \geq \Pi_1^{\mathrm{MC}}$, or as in Example E.3, that $\Pi_1^{\mathrm{FR}} < \Pi_1^{\mathrm{MC}}$.

Epstein and Mealem, 2013; Zhang and Zhou, 2016). In our set-up, this yields equilibrium bids $x_1^\# = E[\sqrt{c_2}]^2/(c_1^\# + E[c_2])^2$ for player 1, and $\xi_2^\#(c_2) = \sqrt{x_1^\#/c_2} - x_1^\#$ for any $c_2 \in C_2$, where we dropped the subscript c_2 from the expectation operator. Using these expressions, total expected costs under mandatory concealment are easily derived as $\mathbf{C}^{\mathrm{MC}} = c_1^\# x_1^\# + E[c_2\xi_2^\#(c_2)] = 2c_1^\# E[\sqrt{c_2}]^2/(c_1^\# + E[c_2])^2$. Note that this formula entails, in particular, the complete-information case where c_2 is public as well. Therefore, being an expectation over such complete-information scenarios, total expected costs under full revelation amount to $\mathbf{C}^{\mathrm{FR}} = 2E[c_1^\# c_2/(c_1^\# + c_2)^2]$. To compare the two expressions, we apply Lemma G.1 with $Y = \sqrt{c_2/c_1^\#}$ and $g(x,y) = g_1(x,y) \equiv 2x^2/(1+y)^2$. It suffices to show that, for any $x > 1$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} > \frac{y-1}{x-1}$, the quadratic form

$$(d_x\ d_y)\,(H_{g_1}(x,y))\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{4{d_x}^2}{(1+y)^2}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)\left(1 - \frac{3x}{1+y}\frac{d_y}{d_x}\right) \tag{F.1}$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1}\cdot\frac{d_y}{d_x} > \underbrace{\frac{x}{y+1}\cdot\frac{y-1}{x-1}}_{\text{increasing in } y} \geq \frac{x}{x^2+1}\cdot\frac{x^2-1}{x-1} = \frac{x^2+x}{x^2+1} > 1. \tag{F.2}$$

Clearly then, the right-hand side of (F.1) is positive. This proves the claim. It follows that

$$\mathbf{C}^{\mathrm{FR}} = E\left[\frac{2(c_2/c_1^\#)}{(1+(c_2/c_1^\#))^2}\right] > \frac{2E[\sqrt{c_2/c_1^\#}]^2}{(1+E[c_2/c_1^\#])^2} = \mathbf{C}^{\mathrm{MC}}, \tag{F.3}$$

i.e., total expected costs are indeed strictly higher under full revelation than under mandatory concealment. In particular, given that, by (B.15), expected costs in the lottery contest are the same across contestants, and given that the favorite's type is public, the favorite exerts a higher effort under full revelation than under mandatory concealment. (ii) From the explicit expressions for the equilibrium bids given above, player 1's probability of winning is easily determined as $p_1^{\mathrm{MC}} = E[\sqrt{c_2}]^2/(c_1^\# + E[c_2])$ under mandatory concealment, and by $p_1^{\mathrm{FR}} = [Ec_2/(c_1^\# + c_2)]$ under full revelation. Again, we apply Lemma G.1 for $Y = \sqrt{c_2/c_1^\#}$, using this time the mapping $g(x,y) = g_2(x,y) \equiv x^2/(1+y)$. Suppose that $x > 1$, $y \geq x^2$, $d_x > 0$, and $d_y > 0$. Then, clearly,

$$(d_x\ d_y)\,(H_{g_2}(x,y))\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2{d_x}^2}{1+y}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)^2 \geq 0. \tag{F.4}$$

Moreover, from relationship (F.2), inequality (F.4) is even strict, which implies strict convexity of g_2 along the relevant linear path segment. Thus, we have

$$p_1^{\mathrm{FR}} = E\left[\frac{(c_2/c_1^\#)}{1+(c_2/c_1^\#)}\right] > \frac{E[\sqrt{c_2/c_1^\#}]^2}{1+E[c_2/c_1^\#]} = p_1^{\mathrm{MC}}, \tag{F.5}$$

and, consequently, also $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$. (iii) Since expected costs are equal across players in the lottery contest, ex-ante expected payoffs for the underdog are given by $\Pi_2^{\mathrm{FR}} = p_2^{\mathrm{FR}} - \frac{\mathbf{C}^{\mathrm{FR}}}{2}$ under full revelation, and by $\Pi_2^{\mathrm{MC}} = p_2^{\mathrm{MC}} - \frac{\mathbf{C}^{\mathrm{MC}}}{2}$ under mandatory concealment. As seen above, $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$ and $C^{\mathrm{FR}} > C^{\mathrm{MC}}$. Hence, $\Pi_2^{\mathrm{FR}} < \Pi_2^{\mathrm{MC}}$, as claimed. □

F.3 Information design

Next, we assume that an informed contest designer chooses a signal to maximize some policy objective (Wasser, 2013a; Denter et al., 2014; Zheng and Zhou, 2016). Upon receiving the realization of the signal, the uninformed player updates her belief and the contest takes place. The following result characterizes the optimal signal for three specific policy objectives, viz. maximizing total expected efforts, maximizing total expected payoffs, and minimizing the expected quadratic distance of players' winning probabilities.[18]

Proposition F.2 (Information design) *Consider an unbiased lottery contest where player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in \{\underline{c}_2, \overline{c}_2\}$ is private. Suppose also that the interiority assumption of Lemma E.1 holds. Then, for a contest designer:*

(i) maximizing total expected efforts, full disclosure (full concealment, any signal) is optimal if $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$ (if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$, if $c_1^{\#} = \sqrt{\underline{c}_2\overline{c}_2}$);

(ii) maximizing total expected payoffs, it is optimal to delegate the problem to player 2, i.e., to use the signal characterized in Proposition E.4;

(iii) minimizing $E_{c_2}[(p_1 - p_2)^2]$, it is optimal to use the signal characterized in part (i).

Proof. (i) This result follows immediately from Zhang and Zhou (2016, Prop. 3) by replacing valuations by reciprocals of marginal costs. (ii) In an unbiased lottery contest with incomplete information about marginal costs, the ex-ante expected expenses are identical across players. Moreover, the prize is always assigned to one player. Therefore, maximizing total expected payoffs is equivalent to minimizing player 1's expenses, $c_1^{\#}x_1^{\#}$. However, as shown in the proof of Lemma D.1, all types $c_2 \in C_2$ prefer a strictly lower bid $x_1^{\#}$ over any higher bid. The claim follows. (iii) In an unbiased lottery contest, we have

$$\frac{1}{4}\left\{1 - E_{c_2}\left[\left(p_1(x_1^{\#}, \xi_2^{\#}(c_2)) - p_2(x_1^{\#}, \xi_2^{\#}(c_2))\right)^2\right]\right\} = E_{c_2}\left[p_1(x_1^{\#}, \xi_2^{\#}(c_2))(1 - p_1(x_1^{\#}, \xi_2^{\#}(c_2)))\right] = x_1^{\#}c_1^{\#}. \quad \text{(F.6)}$$

Hence, minimizing the expected quadratic distance between players' winning probabilities is equivalent to maximizing player 1's expenses. But, by the arguments just explained, this is equivalent to the problem considered under part (i). The proposition follows. □

Part (i) says that, to maximize expected efforts, full disclosure is optimal if player 1 is comparably strong (i.e., if $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$), while full concealment is optimal if player 1 is comparably weak (i.e., if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$), with any signal being optimal in the knife-edge case where $c_1^{\#} = \sqrt{\underline{c}_2\overline{c}_2}$. Part (i) is a straightforward reformulation of a well-known result due to Zhang and Zhou (2016).[19] For parts (ii) and (iii), however, we have not found a suitable reference. Part (ii) is a statement about decentralization. Part (iii) may not be too surprising. Indeed, under the assumptions made, minimizing the expected quadratic distance turns out to be equivalent to maximizing total expected efforts.[20]

An interesting policy objective is also the maximization of the expected highest bid. In general, that problem may be difficult. Imposing Assumption 1, however, the problem simplifies. Indeed, since player 1 is known to submit the highest bid, and ex-ante expected costs are identical for both bidders, the problem becomes equivalent to the one considered in part (i) above.

G. Refinement of Jensen's inequality

[18] Still another policy objective, the maximization of the expected highest bid, will be considered below.

[19] In their case, however, private information is about valuations. Zhang and Zhou (2016) also offer an algorithm for solving the case with $K_2 \geq 3$ types. With more than two types, if the uninformed player is strong enough, full disclosure is optimal, otherwise pooling the highest two valuations together and fully separating the others maximizes total efforts. The paper points out the difficulties that arise in a setting with two-sided incomplete information, namely the multi-dimensional state of nature of both contestants' valuations which complicates the persuasion stage, the private information on two sides, where the simplifying step of the analysis of Kamenica and Gentzkow (2011) cannot be applied and lastly, the equilibrium characterization which is in general not available. More recently, some progress on this problem has been made by Serena (2022).

[20] There is an intuitive tension between part (iii) and the discussion of expense maximization. Specifically, in a setting with a comparably strong player 1 in which both Assumption 1 holds and $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$, we find here that the optimal signal entails full disclosure, whereas Proposition F.1(ii) implies mandatory concealment. To understand what is going on, note that Proposition F.2(iii) works with a quadratic distance of probabilities, whereas Proposition F.1(ii) works with ex-ante winning probabilities. Therefore, the policy objective considered here, intuitively speaking, places overproportional weight on the most lopsided encounters, whereas the earlier discussion weights all encounters according to their ex-ante probability of occurrence, which explains the difference in conclusions.

Some of our examples fall into the tractable class of lottery contests with one-sided incomplete information (Hurley and Shogren, 1998a; Zhang and Zhou, 2016). Below, we derive a variant of Jensen's inequality that allows to prove certain payoff inequalities that cannot be easily obtained otherwise. Specifically, these are the observations related to sequentially taken disclosure decisions, the option to shut down communication, and the maximization of expenses. We will state conditions that are sufficient to derive inequalities of the type $E\left[g\left(Y,Y^2\right)\right] > g\left(E[Y],E[Y^2]\right)$ for a function g in two arguments and a nondegenerate random variable $Y>0$ with finite support. As can be seen, the inequality makes use of the second moment of Y, which explains why it can be sharper than Jensen's inequality. The inequality is strict as a result of our assumption that Y is not degenerate.[21]

Assuming that g is twice continuously differentiable, and given $x>0$, $y>0$, $d_x>0$, $d_y>0$, we will say that g is *directionally strictly convex* at (x,y) along (d_x,d_y) if $(d_x\ d_y)\,(H_g(x,y))\,(d_x\ d_y)^T>0$, where $H_g(x,y)$ denotes the Hessian of g, and T denotes transposition.

Lemma G.1 (Jensen's inequality refined) *Suppose that one of the following two conditions holds:*

(i) $Y>1$ with probability one, and g is directionally strictly convex at (x,y) along (d_x,d_y) whenever $y\geq x^2>1$ and $d_y/d_x>(y-1)/(x-1)$.

(ii) $Y\in(0,1)$ with probability one, and g is directionally strictly convex at (x,y) along (d_x,d_y) whenever $1>y\geq x^2$ and $d_y/d_x<(1-y)/(1-x)$.

Then, $E\left[g\left(Y,Y^2\right)\right] > g\left(E[Y],E[Y^2]\right)$.

Proof. (i) By induction. Assume first that Y has precisely two possible realizations $y_1,y_2\in(1,\infty)$. Without loss of generality, $y_1<y_2$. Consider the auxiliary mapping $f:[0,1]\to R^2$ defined through

$$f(t)=(1-t)\begin{pmatrix}y_1\\y_1^2\end{pmatrix}+t\begin{pmatrix}y_2\\y_2^2\end{pmatrix}\qquad(t\in[0,1]),\tag{G.1}$$

as illustrated in Figure G.1(a). By assumption, g is strictly convex along the straight line described by f.[22] In particular, the composed mapping $g\circ f$ is strictly convex. Therefore, if t is considered a random variable that assumes the value $t=0$ with probability $q_1=\mathrm{pr}(Y=y_1)>0$ and the value $t=1$ with probability $q_2=1-q_1=\mathrm{pr}(Y=y_2)>0$, then

$$\begin{aligned}E[g(Y,Y^2)] &= E[g(f(t))] &\text{(G.2)}\\ &> g(f(E[t])) &\text{(G.3)}\\ &= g\left(q_1y_1+(1-q_1)y_2,q_1y_1^2+(1-q_1)y_2^2\right) &\text{(G.4)}\\ &= g([E[Y],E[Y^2]]). &\text{(G.5)}\end{aligned}$$

This proves the claim if Y has two realizations. Suppose that the claim has been shown for $K\geq 2$ realizations, and assume that Y has $K+1$ realizations $y_1<\ldots<y_{K+1}$, with respective probabilities $q_k=\mathrm{pr}(Y=y_k)>0$, where $k=1,\ldots,K+1$. Consider the random variable Y' that attains value y_k, for $k=2,\ldots,K+1$, with probability

$$q_k'=\frac{q_k}{1-q_1}=\frac{q_k}{\sum_{\kappa=2}^{K+1}q_\kappa}.\tag{G.6}$$

Thus, Y' follows a conditional distribution after learning $Y\neq y_1$. In particular, $E[Y]=q_1y_1+(1-q_1)E[Y']$ and $E[Y^2]=q_1y_1^2+(1-q_1)E[(Y')^2]$. Moreover, by the induction hypothesis, $E\left[g\left(Y',(Y')^2\right)\right]>g\left(E[Y'],E[(Y')^2]\right)$. As above, we define an auxiliary mapping

$$\widetilde{f}(t)=(1-t)\begin{pmatrix}y_1\\y_1^2\end{pmatrix}+t\begin{pmatrix}E[Y']\\E[(Y')^2]\end{pmatrix}\qquad(t\in[0,1]).\tag{G.7}$$

[21] For alternative extensions of Jensen's inequality, see Pittenger (1990), Guljaš et al. (1998), and Liao and Berg (2017). However, those results do not render the payoff comparisons mentioned above.

[22] To see this, let $x=(1-t)y_1+ty_2>1$, $y=(1-t)y_1^2+ty_2^2\geq x^2$, $d_x=y_2-y_1>0$, and $d_y=y_2^2-y_1^2>0$. Then, $d_y/d_x=y_2+y_1>y_2+1\geq(y-1)/(x-1)$, so that the precondition in (i) indeed holds true.

Clearly, $E[(Y')^2] > E[Y']^2$. Therefore, as illustrated in Figure G.1(b), the vector that directs from $(y_1, (y_1)^2)$ to $(E[Y'], E[(Y')^2])$ is steeper than the vector that directs from $(y_1, (y_1)^2)$ to $(E[Y'], E[Y']^2)$. Hence, g is strictly convex also along the linear path described by $\widetilde{f}$.[23] Thus, $g \circ \widetilde{f}$ is strictly convex.

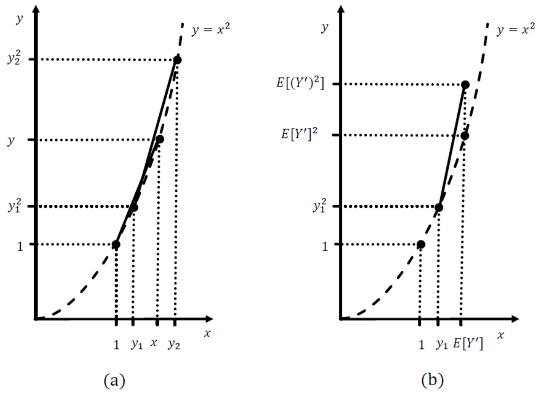


Figure G.1 A refinement of Jensen's inequality.

Therefore, considering t as a random variable that assumes the value $t = 0$ with probability $q_1 = \text{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $1 - q_1 > 0$, the relationships derived above imply

$$\begin{aligned} E[g(Y, Y^2)] &= q_1 g(y_1, y_1^2) + (1 - q_1) E[g(Y', (Y')^2] && \text{(G.8)} \\ &> q_1 g(y_1, y_1^2) + (1 - q_1) g\left(E[Y'], E[(Y')^2]\right) && \text{(G.9)} \\ &= E[g(\widetilde{f}(t))] && \text{(G.10)} \\ &> g(\widetilde{f}(E[t])) && \text{(G.11)} \\ &= g\left(q_1 y_1 + (1 - q_1) E[Y'], q_1 y_1^2 + (1 - q_1) E[(Y')^2]\right) && \text{(G.12)} \\ &= g(E[Y], E[Y^2]). && \text{(G.13)} \end{aligned}$$

Thus, the claim holds for $K + 1$ realizations. This completes the induction, and thereby, the proof of the claim. (ii) The proof is entirely analogous to the one just given and therefore omitted. □

H. Additional references

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[23] Indeed, letting $x = (1-t)y_1 + tE[Y'] > 1$, $y = (1-t)y_1^2 + tE[(Y')^2] > x^2$, $d_x = E[Y'] - y_1 > 0$, and $d_y = E[(Y')^2] - y_1^2 > 0$, we see that $d_y/d_x = (E[(Y')^2] - y_1^2)/(E[Y'] - y_1) > (E[(Y')]^2 - y_1^2)/(E[Y'] - y_1) = E[Y'] + y_1 > 1 + y_1 = (y-1)/(x-1)$, so that the precondition in (i) holds true also in this case.