University of Zurich

Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 220

Asymmetric Information in Frictional Markets for Liquidity: Collateralized Credit vs Asset Sale

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Revised version, November 2020

Asymmetric Information in Frictional Markets for Liquidity: Collateralized Credit vs Asset Sale*

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November 17, 2020

Abstract

This paper studies (non-)equivalence of collateralized credit and asset sale for information-sensitive assets in over-the-counter markets. A signaling game refined by the undefeated equilibrium endogenizes the choice between pooling and separating offers and addresses the payment puzzle. The results show that non-equivalence depends on lenders' commitment power. Despite information frictions, first-best consumption can occur for collateralized credit, but not for asset sales, with endogenous haircuts and over-collateralization characterizing the terms of trade. The general equilibrium determines asset prices and provides policy recommendations including open market operations.

JEL Classification: D82, E44, G12, G21

Keywords: Assets, Liquidity, Asymmetric Information, Collateral, Undefeated Equilibrium

*First and foremost, I would like to thank Aleksander Berentsen and Guillaume Rocheteau for their helpful comments and tremendous support during this project. Special thanks also go to Randall Wright, David Andolfatto, Christopher Waller, Fernando Martin, Don Schlagenhauf, Francesca Carapella, Lukas Altermatt, Mohammed Ait Lahcen, and Romina Ruprecht for the helpful discussions. Moreover, the paper has benefited from participants of the 'Brownbag Seminar Series' at the Federal Reserve Bank of St. Louis, the 'Annual Congress of the Swiss Society of Economics and Statistics' at the University of Lausanne, the 'Young Swiss Economists Meeting' at ETH Zurich, the 'Global Macroeconomics Workshop' at NYU Abu Dhabi, and the 'Summer Workshop on Money, Banking, Payments, and Finance 2016' at the Federal Reserve Bank of Chicago. All errors are mine.

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1 Introduction

Whenever an individual, a firm, or an institution is in need of liquidity, it has two options to acquire funding in the short run: collateralized credit or selling assets. Theoretically, the two settlement strategies are considered to be allocation-equivalent, referring to the *payment puzzle* raised by Lagos (2011). Nonetheless, with approximately 84% of all transactions in over-the-counter (OTC) derivatives being collateralized, accounting for a daily trading volume of roughly 3.8 trillion USD (Copeland et al. (2014b)), demand for collateralized credit is high, challenging equivalence in these markets in the recent years.[1]

One crucial difference to centralized trading platforms is that OTC markets allow for a larger array of asset classes including corporate bonds, collateralized debt obligations, as well as mortgage- and asset-backed securities. While formerly perceived as information-insensitive, some of these asset classes switched to being information-sensitive at the onset of the Great Recession, introducing new information frictions in OTC markets. An immediate consequence was reduced liquidity, manifested through increased haircuts and over-collateralization, as outlined by Gorton and Metrick (2012*a*,*b*), Dang et al. (2013), and Gorton and Ordoñez (2014).[2] Apart from conventional policy interventions, to alleviate the inefficiencies associated with private information, central banks reacted with open market operations, exchanging distressed assets for fiat money or risk-free bonds.

Animated by the payment puzzle and the recent events, this paper studies bilateral exchange in decentralized markets to assess whether information frictions can generate non-equivalence of collateralized credit and asset sale and under what conditions. The baseline environment corresponds to a variation of Rocheteau (2011), extended by a broader equilibrium notion to allow for both pooling and separating offers. There are two fixed types of agents, consumers and producers, bilaterally exchanging perfectly divisible information-sensitive assets for consumption goods in a decentralized market. Incentive-feasible settlement strategies involve collateralized credit and asset sale. The bilateral nature of the meeting allows for explicit game-theoretic foundations, where a signaling game refined by the *undefeated equilibrium* by Mailath et al. (1993) characterizes the terms of trade. By maximizing the surplus of consumers holding high-quality assets, the refinement endogenously selects between pooling and separating equilibria, eliminating Pareto inefficiencies.

[1] Further estimates including reverse repos are available in Copeland et al. (2012) and Gorton and Metrick (2012*b*).

[2] As shown by Gorton and Metrick (2012*b*), average haircuts for nine asset-backed security and corporate debt classes rose from zero to approx. 50% between 2007 and 2009.

The results show that equivalence of collateralized credit and asset sale is guaranteed for separating equilibria, but not for pooling equilibria. The reason is intuitive. In a separating equilibrium, consumers signal their true asset quality via *asset retention*, eliminating information frictions and allowing credit obligations to be priced adequately. Consequently, consumers and producers are indifferent between repayment and default, suggesting equivalence of collateralized credit and asset sale. In a pooling equilibrium, on the other hand, the payment puzzle hinges on commitment and enforcement frictions. Namely, the producer's commitment to return the collateral upon the consumer's repayment of the agreed-upon credit obligation. If a producer is committed, allocations for asset sale and collateralized credit are non-equivalent. While a sale requires the asset to be priced at the expected (market) value, an asset used as collateral is priced below market value to eliminate a low-quality consumer's incentive for strategic default. Over-collateralization is the result. If a producer is not committed, on the other hand, the equivalence of collateralized credit and asset sale is restored due to the producer's incentive to default. Anticipating a producer will renege on returning high-quality collateral priced below market value, a consumer's best response is to offer a lower quantity of assets, suggesting pricing at the expected value. Since at said price low-quality borrowers default on their credit obligation, so will producers, rendering allocations equivalent to an asset sale.

Welfare depends on commitment frictions, the aggregate asset supply, return heterogeneity, and the distribution of asset qualities in the economy. If a producer can commit, there exists a threshold asset supply above which a pooling collateralized credit is the dominant strategy. If assets are sufficiently abundant, despite information frictions, consumption is first-best. Once the asset supply is below the threshold, consumers choose to sell their assets at a pooling price, analog to a fire sale. In that context, it is worth pointing out that if a producer is committed, a separating equilibrium is dominated at all times, given the cost incurred through asset retention triggered by a binding incentive compatibility constraint. If a producer cannot commit, however, first-best consumption is not attainable due to the equivalence of credit and sale. In said scenario, the offer selection, i.e., the choice between a pooling and a separating offer depends on the distribution of asset qualities in the economy. If high-return assets are scarce, the equilibrium is separating, while an abundance of high-return assets renders a pooling offer the dominant strategy. The vitality of commitment and enforcement frictions motivates institutions like central counterparties, public-record keeping, and punishment schemes, and sheds new light on the lender's side in secured credit transactions.

Lastly, the challenges for trade created by private information motivate policy interventions. In particular, open market operations exchanging information-sensitive assets for risk-free bonds, analog to the large scale asset purchases conducted during the Great Recession. To begin with, following Tirole (2012), Chiu and Koeppl (2016), and Madison (2019), the results show that timing matters. While interventions prior to a private information shock allow eliminating information frictions, an ex-post intervention has no such effect, since, at this stage, information frictions characterize the terms of trade. Furthermore, the size of the intervention depends on the (non-)equivalence of credit and sale, and thus commitment frictions. If producers can commit, to restore first-best consumption, not all information-sensitive assets need to be replaced. Instead, the supply of bonds needs to be such that a combination of bonds and information-sensitive assets allows for first-best consumption. Interestingly, different to Rocheteau (2011), the co-existence of information-sensitive assets and risk-free bonds does not result in a pecking-order, since, despite a haircut, the producer's commitment to return the collateral after receiving the agreed-upon credit obligation eliminates the incentive to retain high-quality assets and spend risk-free bonds first.

1.1 Related Literature

There exists a broad literature analyzing collateralized credit. Among the first to show that collateral allows agents to overcome solvency concerns was Bester (1985), later revisited by Kiyotaki and Moore (1997) in a search environment subject to commitment and enforcement frictions. Flannery (1996), in turn, argues that while market participants may know about the solvency of the respective counterparty, they may not be aware of the quality of individual assets in their portfolio. An idea closely related to this paper, with the crucial difference, that in Flannery (1996) private credit collapses due to an Akerlof (1970) lemons problem, promoting a role for alternative payment systems such as a government discount window.

In the monetary search literature, a first distinction under full information was provided by Berentsen and Waller (2011). They conclude that any allocation in an economy with asset sales can be replicated by an economy with collateralized credit, but not vice versa. Monnet and Narajabad (2017), in turn, suggest non-equivalence relying on a borrower's uncertainty regarding the value of holding the security in the future. As liquidity constrained agents may end up needing the asset for an upcoming consumption opportunity before being able to re-acquire it on a financial market, collateralized credit is the preferred choice. Awaya et al. (2020) revisit this logic and show that if agents have no option to re-acquire the collateral prior to the next trade, the endogenous borrowing limit can in fact be larger than the intrinsic value of the asset. Parlatore (2019) studies the trade-off between collateralized credit and asset sales under uncertainty regarding the outcome of a risky project. In her model, assets entail a liquidity and a collateral premium, where an increase in liquidity increases the liquidity premium of assets, while the collateral premium declines, suggesting non-equivalence. Tomura (2016) explains the demand for collateralized credit with a hold-up problem introduced through a deadline on retrieving cash. An urgent need to liquidate bonds on an over-the-counter market weakens the bargaining power of the selling party and disincentivizes the acquisition of that bond in the first place. A repurchase agreement subject to a haircut, however, allows an agent to circumvent this inefficiency. Last but not least, Gottardi et al. (2019) focus on the aspect of re-hypothecation to explain the rise of repurchase agreements in the last years. An analysis of said trade-off in the presence of information frictions, however, remains absent so far.

The promoted non-equivalence also borrows elements from the literature on asset prices and private information, pioneered by Akerlof (1970), as well as the literature on security design, precisely Boot and Thakor (1993) and Demarzo and Duffie (1999). A first application to a search and matching environment was provided by Velde et al. (1999), studying the effects of information frictions on asset prices using pooling equilibria. Separating equilibria relying on the Cho and Kreps (1987) Intuitive Criterion, in turn, were studied by Nosal and Wallace (2007), Guerrieri et al. (2010), Rocheteau (2011), and Madison (2019), among others. With the goal to endogenize the selection of pooling and separating equilibria, a first application of the Mailath et al. (1993) undefeated equilibrium, using a Shi (1995) and Trejos and Wright (1995) second-generation monetary search model, was provided by Li and Rocheteau (2008), followed by Bajaj (2018). Their results show that the decision between a pooling and a separating equilibrium depends on the distribution of asset qualities in the economy, whereas the higher the probability of encountering a lemon, the more likely a separating equilibrium. Gorton and Ordoñez (2014) follow a similar logic. However, given their dynamic setup, the switch from pooling to separating does not only depend on the distribution of asset qualities but also the length of the credit boom.

Lastly, the paper relates to the existing literature on policy interventions in the presence of information frictions. The closest papers studying open market operations in a random matching model with private information are Chiu and Koeppl (2016) and Madison (2019), showing that trade can be resurrected by one-time asset purchases exchanging information-sensitive assets for risk-free bonds. Abstaining from bilateral exchange, equivalent policy recommendations are provided by Tirole (2012) and Philippon and Skreta (2012) in a static framework with competitive markets. Policy interventions in a dynamic competitive market, on the other hand, are studied by Fuchs and Skrzypacz (2015) and Bolton et al. (2011), allowing them to address the optimal timing of open market operations, confirming the necessity for early interventions, as suggested in the paper at hand.

Regarding the organization of the paper, Section 2 presents the environment, while Section 3 outlines the bargaining game and the undefeated equilibrium. Section 4 studies an altered environment with increased commitment frictions. The general equilibrium is characterized in Section 5. Optimal policy including open market operations is discussed in Section 6. Lastly, Section 7 concludes.

2 Environment

The environment is based on the unified search-theoretic framework established by Lagos and Wright (2005) and the extension introduced by Rocheteau (2011). Time is discrete, starts at $t = 0$, and continues forever. Each period is divided into two subperiods: a centralized market (CM) and a decentralized market (DM). The discount factor across periods is $\beta \in (0, 1)$, where $\beta = (1 + r)^{-1}$ and r is the rate of time preference. There is a unit measure of two distinct types of agents, consumers and producers, named after their preferences in the DM, and denoted by the subscripts c and p. While both types can produce and consume in the CM, in the DM, consumers want to consume but cannot produce and producers can produce, but do not want to consume, generating gains from trade. Their respective period utilities are:

$$U_c = u(q) + z - h, \tag{1}$$

$$U_p = -c(q) + z - h, \tag{2}$$

with q being the non-durable consumption good in the DM, z the non-durable consumption good in the CM, and h hours worked in the CM. For tractability, the period utility is separable across subperiods. Utility of consumption in the DM, $u(q)$, is twice continuously differentiable, strictly increasing in q, and concave, $u'(q) > 0 > u''(q)$, with $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. The disutility of production, $c(q) = q$, is linear. Efficiency requires $q = q^*$, where q^* solves $u'(q^*) = 1$. The production technology in the CM is linear with labor being the only input. In the following, the timing of events, as visualized in Figure 1, is discussed in detail.

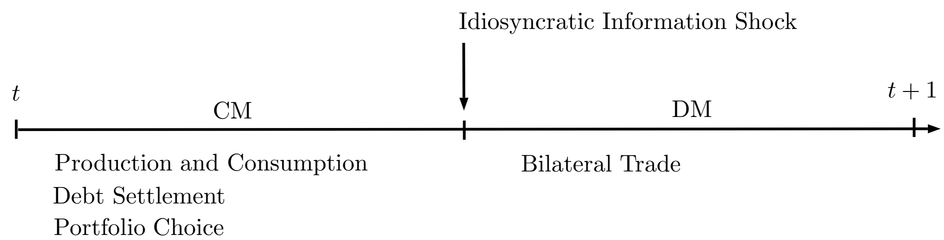

Figure 1: Timing of Events

At the beginning of the CM, each agent is endowed with $A \in \mathbb{R}_+$ units of a perfectly divisible one-period lived real asset. Asset returns in the subsequent CM are stochastic. With probability π, the asset yields a high terminal return, κ^H, while with the complementary probability, $1 - \pi$, the return is low, κ^L. For simplicity and without loss of generality, the expected return is normalized to one, i.e., $R = \pi\kappa^H + (1-\pi)\kappa^L = 1$, implying $0 \leq \kappa^L < 1 < \kappa^H < \infty$. Once asset endowments are received, agents produce and consume the numéraire good, z, adjust their asset holdings for the following DM by trading z for a in the competitive CM, and settle credit obligations from the previous DM.

At the end of the CM, after asset holdings are determined, each agent is privately informed about the idiosyncratic return of her asset, where the realization is common to all assets held by an agent, but independent across agents.[3] Following the idiosyncratic information shock, in the DM, consumers and producers meet bilaterally at random with probability one and remain matched until the beginning of the subsequent CM. There is no record-keeping of transactions in the DM and consumers cannot commit to future actions, eliminating unsecured credit. Given the non-durability of the numéraire good, a medium of exchange, the asset accumulated in the CM, is essential for trade to occur.[4] Two settlement options are feasible: asset sales and collateralized credit, denoted by the calligraphic subscripts $\mathcal{S}$ and $\mathcal{C}$, respectively. In the event of a sale, transactions are settled instantaneously, where a consumer transfers $y_{\mathcal{S}} \in [0, a - y_{\mathcal{C}}]$ units of the asset to the producer in return for

[3]In doing so, the modeling approach follows the 'learning-by-holding' logic of Plantin (2009), arguing that holders of an asset learn quicker about its quality than other investors in the market, creating information frictions.

[4]Earlier work motivating the necessity of a medium exchange in frictional goods markets involves Kocherlakota (1998), Wallace (2001), and Shi (2006).

$q_{\mathcal{S}}$ units of the DM good. In case of collateralized credit, a consumer pledges $y_{\mathcal{C}} \in [0, a - y_{\mathcal{S}}]$ units of the asset in return for $q_{\mathcal{C}}$, and agrees to repurchase the pledged asset in the subsequent CM at a price defined in the DM, where l represents the agreed-upon liability. In case the consumer defaults on her credit obligation, the producer keeps the pledged asset $y_{\mathcal{C}}$. In case the consumer repays, on the other hand, return of the collateral depends on commitment frictions. In the baseline environment, it is assumed that, unlike consumers, producers are committed to return the pledged collateral, while in the altered environment in Section 4, said assumption is relaxed to study an environment in which both parties lack commitment. Viable examples to justify commitment include central counterparties, punishment schemes (e.g. exclusion from the financial sector upon default), and reputation concerns.[5]

3 Bargaining Game

I proceed by backward induction, starting with the agent's problem in the CM. For simplicity, in the DM, I assume that the consumer's asset holdings are common knowledge in the match, but not the return.[6]

The expected utility of a consumer and a producer, $i = \{c, p\}$, entering the CM with a units of the real asset, their corresponding return κ^{χ} with $\chi \in \{L, H\}$, and liabilities from the previous period l are:

$$W_i(a, l; \kappa^{\chi}) = \max_{z,h,a'} z - h + \mathbb{E}V_i(a'; \kappa'^{\chi}) \tag{3}$$

$$\text{s.t. } z + \psi a' = h - l + a\kappa^{\chi} + \psi A, \tag{4}$$

where ψ is the CM price for one unit of the asset denominated in the numéraire good and $\mathbb{E}V_i(a'; \kappa'^{\chi})$ is the expected value of entering the DM with a' units of the asset, where the expectation operator captures the uncertainty about the terminal return, κ'. Hence, an agent finances her end-of-period asset balances, a', her CM consumption, z, and her outstanding credit obligations, l, through inputs of labor, h, and the asset returns, $a\kappa^{\chi}$, realized in the CM. Using the budget constraint to eliminate $z - h$ in the objective function, the value

[5] Prominent work endogenizing commitment using punishment and reputation includes Diamond (1989), Kehoe and Levine (1993), Alvarez and Jermann (2000), and Gu et al. (2013).

[6] This allows to avoid specifying a producer's beliefs regarding the consumer's asset holdings. The assumption is without loss of generality, since a consumer's surplus is monotonically increasing in her asset holdings and hence, if a consumer had the possibility to report her asset holdings prior to the match, there would be an equilibrium in which she would do so truthfully.

function reduces to:

$$W_i(a, l; \kappa^{\chi}) = a\kappa^{\chi} + \psi A - l + \max_{a'}\{-\psi a' + \mathbb{E}V_i(a'; \kappa'^{\chi})\}, \tag{5}$$

where $\mathbb{E}V_{a'} = 1$ is the marginal value of carrying another unit of the real asset into the DM, and $W_a = \kappa^{\chi}$ and $W_l = -1$ are the partial derivatives of $W_i(a, l; \kappa^{\chi})$ with respect to a and l. The usual linearity and independence properties apply, i.e., $W_i(a, l; \kappa^{\chi})$ is linear in wealth and the amount of assets carried into the DM is independent of the current asset holdings when entering the CM.

The bargaining game between a consumer and a producer in the DM has the structure of a signaling game, i.e., the informed agent moves first and makes the offer. A strategy for the consumer is to specify an offer, $(q_c, y_{\mathcal{S},c}, y_{\mathcal{C},c}, l_c)$, where $q_c(a; \kappa_c^{\chi}) = q_{\mathcal{S},c}(a; \kappa_c^{\chi}) + q_{\mathcal{C},c}(a; \kappa_c^{\chi})$ is the amount of DM goods received, $y_{\mathcal{S},c}(a; \kappa_c^{\chi})$ the share of assets sold, $y_{\mathcal{C},c}(a; \kappa_c^{\chi})$ the share of assets deposited as collateral, and $l_c(a; \kappa_c^{\chi})$ the credit obligation in period $t+1$, all as a function of the consumer's type, κ_c^{χ}, and her asset holdings, a. In doing so, the transfer of assets is constrained by the agent's current asset holdings, i.e., $y_{\mathcal{S},c} + y_{\mathcal{C},c} \leq a$.[7] Given the offer placed by the consumer, the producer updates her beliefs about the terminal return of the consumer's asset and defines an acceptance rule, $\mathcal{A}_p$, that specifies the set of acceptable offers. Due to bilateral matching and thus $q_c = q_p$, $y_{\mathcal{S},c} = y_{\mathcal{S},p}$, $y_{\mathcal{C},c} = y_{\mathcal{C},p}$, and $l_c = l_p$, I refrain from the subscripts c and p for these variables going forward. Hence, the consumer's payoff in the state κ_c^{χ} is:

$$\begin{aligned} V_c(a; \kappa^{\chi}) &= \big\{u(q) + \beta\big[(1-d)W_c(a - y_{\mathcal{S}}, l; \kappa^{\chi}) + dW_c(a - y_{\mathcal{S}} - y_{\mathcal{C}}, 0; \kappa^{\chi})\big]\big\}\mathbb{I}_{\mathcal{A}_p} \\ &+ \beta W_c(a, 0; \kappa^{\chi})\left(1 - \mathbb{I}_{\mathcal{A}_p}\right), \end{aligned} \tag{6}$$

where $\mathbb{I}_{\mathcal{A}_p}$ is an indicator function equal to one if the consumer's offer is in the producer's set of acceptable offers, i.e., if $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l) \in \mathcal{A}_p$, and zero otherwise. If engaging in a collateralized credit, default is a binary choice variable, $d \in \{0; 1\}$, allowing consumers to default on their credit obligation, where $d = 0$ denotes repayment, and $d = 1$ default. Using the linearity of the agent's value function in the CM, the consumer's payoff in case of trade can be reduced to her match surplus, $S_c(a; \kappa^{\chi}) = \big\{u(q) - \beta\big[y_{\mathcal{S}}\kappa^{\chi} + (1-d)l + dy_{\mathcal{C}}\kappa^{\chi}\big]\big\}\mathbb{I}_{\mathcal{A}_p}$.

[7]This is in line with the lotteries introduced by Berentsen et al. (2002) under indivisibility, where agents in a bilateral trade are able to offer their asset probabilistically.

Similarly, the producer's payoff is given by:

$$\begin{aligned} V_p(a;\kappa^\chi) &= \Big\{-q+\beta\big[(1-d)W_p\left(a+y_{\mathcal{S}},l;\kappa^\chi\right)+dW_p\left(a+y_{\mathcal{S}}+y_{\mathcal{C}},0;\kappa^\chi\right)\big]\Big\}\mathbb{I}_{\mathcal{A}_p} \\ &+ \beta W_p\left(a,0;\kappa^\chi\right)\left(1-\mathbb{I}_{\mathcal{A}_p}\right), \end{aligned} \tag{7}$$

which can be reduced to the match surplus $S_p(a;\kappa^\chi)=\big\{-q+\beta\big[y_{\mathcal{S}}\kappa_c^\chi+(1-d)l+dy_{\mathcal{C}}\kappa_c^\chi\big]\big\}\mathbb{I}_{\mathcal{A}_p}$. In order for a producer to accept the offer made by the consumer, she has to form expectations about the terminal return of the consumer's asset, κ_c^χ. Let $\lambda=\text{Prob}[\kappa_c=\kappa_c^H\mid(q,y_{\mathcal{S}},y_{\mathcal{C}},l)]\in[0,1]$ represent the producer's posterior belief that the consumer's asset is of high quality, $\kappa_c=\kappa_c^H$, conditional on the offer, $(q,y_{\mathcal{S}},y_{\mathcal{C}},l)$, made. The posterior expected return can therefore be formulated as:

$$\mathbb{E}_\lambda[\kappa_c]=\lambda(q,y_{\mathcal{S}},y_{\mathcal{C}},l)\kappa_c^H+[1-\lambda(q,y_{\mathcal{S}},y_{\mathcal{C}},l)]\kappa_c^L, \tag{8}$$

determining the producer's set of acceptable offers, $\mathcal{A}_p(\lambda)\subseteq\mathcal{F}$:

$$\mathcal{A}_p(\lambda)=\Big\{(q,y_{\mathcal{S}},y_{\mathcal{C}},l)\in\mathcal{F}:-q+\beta\big[y_{\mathcal{S}}+dy_{\mathcal{C}}\big]\big[\lambda\kappa_c^H+[1-\lambda]\kappa_c^L\big]+\beta(1-d)l\geq 0\Big\}. \tag{9}$$

Hence, for a given belief system, $\lambda(q,y_{\mathcal{S}},y_{\mathcal{C}},l)$, in order for a producer to accept an offer made by the consumer, the offer has to yield a non-negative expected surplus. Assuming a tie-breaking rule according to which a producer agrees to any offer that makes her indifferent between accepting and rejecting, the consumer chooses an offer that maximizes her surplus, $S_c(a;\kappa^\chi)$, taking as given the acceptance rule of the producer, (11), incentive-compatibility constraints, (12)-(13), and a feasibility constraint, (14). Therefore, the consumer's problem reduces to:

$$\begin{aligned} S_c(a;\kappa^\chi)=\max_{d\in\{0;1\}}\Bigg[& (1-d) \max_{q,l,y_{\mathcal{S}},y_{\mathcal{C}}}\Big[u(q)-\beta y_{\mathcal{S}}\kappa^\chi-\beta l\Big] \\ + & \; d \max_{q,l,y_{\mathcal{S}},y_{\mathcal{C}}}\Big[u(q)-\beta y_{\mathcal{S}}\kappa^\chi-\beta y_{\mathcal{C}}\kappa^\chi\Big]\Bigg]\mathbb{I}_{\mathcal{A}_p}, \end{aligned} \tag{10}$$

subject to:

$$-q+\beta\big[y_{\mathcal{S}}+dy_{\mathcal{C}}\big]\big[\lambda\kappa^H+[1-\lambda]\kappa^L\big]+\beta(1-d)l\geq 0 \tag{11}$$

$$u(q^H)-\beta(y_{\mathcal{S}}^H+dy_{\mathcal{S}}^H)\kappa^L-\beta(1-d)l^H\leq u(q^L)-\beta(y_{\mathcal{S}}^L+dy_{\mathcal{S}}^L)\kappa^L-\beta(1-d)l^L \tag{12}$$

$$u(q^L)-\beta(y_{\mathcal{S}}^L+dy_{\mathcal{S}}^L)\kappa^H-\beta(1-d)l^L\leq u(q^H)-\beta(y_{\mathcal{S}}^H+dy_{\mathcal{S}}^H)\kappa^H-\beta(1-d)l^H \tag{13}$$

$$a-y_{\mathcal{S}}-y_{\mathcal{C}}\geq 0. \tag{14}$$

I consider Perfect Bayesian Equilibria, where an equilibrium for this bargaining game is a profile of strategies for the consumer and the producer, and a system of beliefs. If $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)$ is an offer made by the consumer, then $\lambda(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)$ is derived from the producer's prior belief according to Bayes' rule. Offers can be separating or pooling. Since without restriction there is no discipline for unreasonable out-of-equilibrium beliefs, the equilibrium is refined by the Mailath et al. (1993) *undefeated equilibrium*, endogenously selecting between separating offers, $(q^L, y^L_{\mathcal{S}}, y^L_{\mathcal{C}}, l^L; a, \kappa^L) \neq (q^H, y^H_{\mathcal{S}}, y^H_{\mathcal{C}}, l^H; a, \kappa^H)$, and a pooling offer, $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^L) = (q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^H) = (\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$. Among the sustainable separating and pooling offers, the refinement chooses the one maximizing the high-type consumer's period surplus, $S_c(a; \kappa^H)$, to eliminate potential Pareto inefficiencies. This includes the choice between collateralized credit and asset sale. Definition A defines:

Definition A. ***(Undefeated Equilibrium)*** *An undefeated equilibrium is a pair of strategies and a belief system,* $\{[q(a;\kappa),\ y_{\mathcal{S}}(a;\kappa), y_{\mathcal{C}}(a;\kappa), l(a;\kappa), d(a;\kappa)], \lambda(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)\}$*, such that the terms of trade are:*

(i) $(q^L, y^L_{\mathcal{S}}, y^L_{\mathcal{C}}, l^L) \neq (q^H, y^H_{\mathcal{S}}, y^H_{\mathcal{C}}, l^H)$ *if* $S_c(a;\kappa^H) > \bar{S}_c(a;\kappa^H)$*, and*

(ii) $(\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$ *if* $\bar{S}_c(a;\kappa^H) > S_c(a;\kappa^H)$,

with $S_c(a;\kappa^H)$ *and* $\bar{S}_c(a;\kappa^H)$ *being the surpluses of a high-type consumer placing a separating or a pooling offer respectively, and the producer's posterior belief is:*

(i) $\lambda = 0$ *for all offers that make the high-type consumer strictly worse off, and are preferred to* $(\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$ *by the low-type consumer; and*

(ii) $\lambda = 1$ *for all offers that make the low-type consumer strictly worse off than* $(q^L, y^L_{\mathcal{S}}, y^L_{\mathcal{C}}, l^L)$.

Out-of-equilibrium offers, $(\tilde{q}, \tilde{y}_{\mathcal{S}}, \tilde{y}_{\mathcal{C}}, \tilde{l})$*, are associated to the low-type consumer, i.e.,* $\lambda = 0$.

Hence, an equilibrium is considered undefeated if it maximizes the high-type consumer's surplus, $S_c(a;\kappa^H)$ or $\bar{S}_c(a;\kappa^H)$, and if there exists no other equilibrium offer such that either the high-type consumer has an incentive to unilaterally deviate to a separating offer, or both types prefer a deviation to a pooling offer. This includes the choice between collateralized credit and asset sale. Incentive-compatible separating and pooling offers are characterized in Proposition A and B respectively, while Proposition C determines the undefeated equilibrium.

3.1 Separating Offers

I start by characterizing the least-inefficient separating offers, $(q^L, y_S^L, y_C^L, l^L) \neq (q^H, y_S^H, y_C^H, l^H)$. Proposition A summarizes. The proof is delegated to Appendix A.

Proposition A. ***(Separating offers)*** *Separating offers,* $(q^L, y_S^L, y_C^L, l^L) \neq (q^H, y_S^H, y_C^H, l^H)$, *solve* (10)-(14). *For* $\kappa^L > 0$, *the DM allocations correspond to:*

$$q_j^L = \min\{q^*, \beta a \kappa^L\}, \tag{15}$$

$$y_j^L = \min\{q_j^L/\beta\kappa^L - y_{-j}^L, a - y_{-j}^L\}, \tag{16}$$

$$l^L = y_C^L \kappa^L, \tag{17}$$

and:

$$q_j^H = \left[\kappa^H/\kappa^L\right]\left[u(q_j^H) - \left[u(q_j^L) - q_j^L\right]\right] \in [0, q_j^L), \tag{18}$$

$$y_j^H = q_i^H/\beta\kappa^H - y_{-j}^H, \tag{19}$$

$$l^H = y_C^H \kappa^H, \tag{20}$$

with $j \in \{S; C\} = \{sale; credit\}$, $d \in \{0; 1\}$, *and* $q_S^L = q_C^L$, $q_S^H = q_C^H$, $y_S^L = y_C^L$, *and* $y_S^H = y_C^H$. *If* $\kappa^L = 0$, *then* $q_j^L = q_j^H = y_j^L = y_j^H = 0$. *Proof in Appendix A.*

Consider first a low-type consumer. In equilibrium, a low-type can do no worse than to reveal her type, since this offer is always acceptable to the producer, independent of her beliefs. At the same time, a low-type cannot do better, since every deviation would require pooling with a high-type's offer, and such out-of-equilibrium offers are ruled out by the binding incentive compatibility constraint, (12). Hence, low-type consumption is efficient if real balances are large enough to compensate the producer for her disutility of production, i.e., if $q^L = \beta a \kappa^L = q^*$. Else, $q^L < q^*$ holds. Taking the low-type's offer as a benchmark, a high-type consumer separates. However, this separation comes at a cost, where (18) uniquely determines $q^H \in [0, q^L)$. Given $q^H < q^L$, and thus $y^H\kappa^H < y^L\kappa^L$, it immediately follows that $y^H < y^L$.[8] In other words, separation takes place through *asset retention*. The amount of assets retained is exactly at the threshold such that the low-type has no incentive to mimic the high-type's offer – an application of Gresham's law suggesting that low-quality assets

[8]Note that under full-information, since $0 \leq \kappa^L < 1 < \kappa^H < \infty$, the high-type would consume a higher quantity than the low-type, $q^H > q^L \geq 0$, and spend a lower fraction of her assets, $y^H < y^L$.

crowd out high-quality assets. In the limiting case with $\kappa^L = 0$, separation breaks down, and $q^H = q^L = y^H = y^L = 0$.

Corollary A. ***(Equivalence)*** *In a separating equilibrium, the allocations for collateralized credit and asset sale are equivalent.*

Proposition A shows that in a separating equilibrium, the allocations for collateralized credit and asset sale are equivalent. By signaling her true type, a consumer eliminates the producer's uncertainty regarding the future return of the provided asset, i.e., $\lambda(q^H, y^H_{\mathcal{S}}, y^H_{\mathcal{C}}, l^H) = 1$ and $\lambda(q^L, y^L_{\mathcal{S}}, y^L_{\mathcal{C}}, l^L) = 0$, allowing future credit obligations, l, to be priced adequately, rendering the consumer's default decision, $d \in \{0; 1\}$, irrelevant.

Lemma A. ***(Comparative statics: separating equilibrium)*** *In a separating equilibrium, for $q^L < q^*$:*

(i) $\partial y^H / \partial(\kappa^H - \kappa^L) < 0$; and

(ii) $\partial y^H / \partial a \in (0, 1)$.

If $q^L = q^*$, then $\partial y^H / \partial a = 0$. Proof in Appendix B.

The first part of Lemma A shows that the amount of high-return assets sold or pledged decreases with an increase in the distance between the asset returns κ^H and κ^L. Hence, the more severe the return heterogeneity, the more binding the incentive compatibility constraint, (12), and thus the lower y^H. For the second part of Lemma A, assessing the response of y^H to a, two cases need to be distinguished. If $q^L < q^*$, an additional unit of the asset raises the low-type's surplus. As a consequence, the high-type's marginal willingness to sell or pledge is positive. However, since she faces a binding constraint and can only use a fraction of each additional asset without generating the incentive for the low-type to mimic her offer, the high-type's marginal willingness to sell is less than one, and hence $\partial y^H / \partial a \in (0, 1)$. If $q^L = q^*$, in turn, an additional unit does not relax the high-type's constraint, and thus her willingness to sell or pledge another unit of the asset is zero, i.e., $\partial y^H / \partial a = 0$.

3.2 Pooling Offer

Having determined the least-inefficient separating offers, I now determine the most-efficient pooling offer, $(\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$. Proposition B summarizes. The proof is delegated to Appendix C.

Proposition B. ***(Pooling offer)*** *A pooling offer, $(\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$, solves* (10)-(14)*. For $\kappa^L > 0$, the DM allocations correspond to:*

$$\bar{q}_{\mathcal{S}} = \beta \bar{y}_{\mathcal{S}} R < q^*, \tag{21}$$

$$\bar{y}_{\mathcal{S}} = \bar{q}_{\mathcal{S}} / \beta R - \bar{y}_{\mathcal{C}}, \tag{22}$$

for an asset sale, $\mathcal{S}$; and:

$$\bar{q}_{\mathcal{C}} = \min\{q^*, \beta \bar{y}_{\mathcal{C}} \kappa^L\}, \tag{23}$$

$$\bar{y}_{\mathcal{C}} = \min\big\{\bar{q}_{\mathcal{C}} / \beta \kappa^L - \bar{y}_{\mathcal{S}}, a - \bar{y}_{\mathcal{S}}\big\}, \tag{24}$$

$$\bar{l} = \bar{y}_{\mathcal{C}} \kappa^L, \tag{25}$$

for a collateralized credit, $\mathcal{C}$, where $d \in \{0; 1\}$, and $\bar{q} = \bar{q}_{\mathcal{S}} + \bar{q}_{\mathcal{C}}$. If $\kappa^L = 0$, then $\bar{q}_{\mathcal{C}} = 0$. Proof in Appendix C.

Proposition B shows that allocations for collateralized credit and asset sales are non-equivalent in a pooling equilibrium. In the event of an asset sale, no information is revealed and the producer's updated belief corresponds to the initial belief, $\lambda(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l) = \pi$, suggesting pricing at the expected value, $R = \pi \kappa^H + (1 - \pi)\kappa^L$. As a consequence, with every unit of the asset sold, a high-type consumer subsidizes the consumption of the low-type consumer. An immediate reaction is reduced consumption, $\bar{q}_{\mathcal{S}} < q^*$, in the DM. Pricing differs in the event of a collateralized credit. If the high-type offers the same quantity as in an asset sale, suggesting asset pricing at the expected value, R, a producer updating her beliefs about the consumer's default decision in (10) understands that every high-type will honor her credit obligation to receive back her high-return asset, while every low-type will default. The consequence is a strictly negative surplus for the producer, $S_p < 0$, violating the acceptance rule $\mathcal{A}_p(\lambda)$ in (11). Hence, to ensure participation of the producer, a haircut, $\mathcal{H} = R - \kappa^L$, pricing every asset at the low return, κ^L, is applied, eliminating a low-type's incentive to default. The result is over-collateralization resulting in first-best consumption, $\bar{q}_{\mathcal{C}} = q^*$, for both low- and high-type consumers if $\beta a \kappa^L \geq q^*$, and $\bar{q}_{\mathcal{C}} < q^*$ if $\beta a \kappa^L < q^*$, equivalent to the low-type consumer's full information offer in (15)-(17).

Corollary B. ***(Non-equivalence)*** *In a pooling equilibrium, the allocations for collateralized credit and asset sale are non-equivalent.*

Given the producer's updated beliefs about the consumer's default decision in (10) and her commitment to return the asset upon repayment of the credit obligation in the subsequent CM, the allocations for asset sale and collateralized credit are non-equivalent in a pooling equilibrium. While an asset sale requires the asset to be priced at the expected value, R, a collateralized credit allows a high-type consumer to mimic the low-type consumer's full-information offer and obtain first-best consumption if in possession of sufficient real balances, understanding that upon repayment of her credit obligation, she can consume the high return of her assets without the necessity to sacrifice consumption in the DM. Given the Intuitive Criterion, an outcome unattainable in the separating equilibrium characterized in Proposition A.

Lemma B. ***(Comparative statics: pooling equilibrium)*** *In a pooling equilibrium, in the event of an asset sale, $\mathcal{S}$:*

(i) $\partial \bar{y}_{\mathcal{S}}/\partial(\kappa^H - \kappa^L) < 0$.

In the event of a collateralized credit, $\mathcal{C}$:

(ii) For $\bar{q}_{\mathcal{C}} = q^*$*:* $\partial \bar{y}_{\mathcal{C}}/\partial \kappa^L < 0 = \partial \bar{y}_{\mathcal{C}}/\partial \kappa^H$.

(iii) For $\bar{q}_{\mathcal{C}} < q^*$*:* $\partial \bar{y}_{\mathcal{C}}/\partial \kappa^L = \partial \bar{y}_{\mathcal{C}}/\partial \kappa^H = 0$.

Proof in Appendix D.

The comparative statics in Lemma B are divided into three parts. The first part (i) considers pooling sale and shows that the amount of assets sold by the high-type consumer decreases with increase in the distance between the asset returns κ^L and κ^H. Part (ii) and (iii), in turn provide comparative statics for collateralized credit, where distinction is made between $\bar{q}_{\mathcal{C}} < q^*$ and $\bar{q}_{\mathcal{C}} = q^*$. If a consumer gets first-best consumption, i.e., $\beta \bar{y}_{\mathcal{C}} \kappa^L = q^*$, an increase in the return of the low-quality asset reduces the amount of assets pledged to the producer, $\partial \bar{y}_{\mathcal{C}}/\partial \kappa^L < 0$. An increase in the return of the high-quality asset, on the other hand, has no effect on $\bar{y}_{\mathcal{C}}$ since the offer corresponds to the full-information offer of a low-type in the separating equilibrium characterized in Proposition A. Lastly, if $\beta\bar{y}_{\mathcal{C}}\kappa^L < q^*$, for intuitive reasons, an increase in both κ^L and κ^H has no effect on the amount of assets pledged, since $\bar{y}_{\mathcal{C}} = a$.

3.3 Undefeated Equilibrium

Having determined the incentive-compatible offers in the DM, showing non-equivalence of collateralized credit and asset sales in a pooling equilibrium, we now revisit the undefeated equilibrium characterized in Definition A, and hence the offer selection maximizing the high-type consumer's surplus. Following the consumer's problem (10)-(14) and the separating and pooling allocations determined in Proposition A and B, the high-type consumer's surpluses are defined as:

$$S_c(a;\kappa^H) = u(q^H) - q^H, \tag{26}$$

$$\bar{S}_{c,\mathcal{C}}(a;\kappa^H) = u(\bar{q}_{\mathcal{C}}) - \bar{q}_{\mathcal{C}}, \tag{27}$$

$$\bar{S}_{c,\mathcal{S}}(a;\kappa^H) = u(\bar{q}_{\mathcal{S}}) - \bar{q}_{\mathcal{S}}\kappa^H/R, \tag{28}$$

where $S_c(a;\kappa^H)$ denotes the high-type consumer's surplus offering a separating offer, $\bar{S}_{c,\mathcal{C}}(a;\kappa^H)$ the high-type consumer's surplus offering a pooling collateralized credit, and $\bar{S}_{c,\mathcal{S}}(a;\kappa^H)$ the high-type consumer's surplus offering a pooling sale, with $(q^H, y^H_{\mathcal{S}}, y^H_{\mathcal{C}}, l^H)$ and $(\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$ defined in (18)-(20) and (21)-(25) respectively.

Proposition C. ***(Undefeated equilibrium)*** *Assume $\kappa^L > 0$. There exists a threshold asset supply, $\tilde{a} \in (0, a^*)$ with $q^* = \beta a^* \kappa^L$, such that:*

(i) If $a > \tilde{a}$, then $\bar{S}_{c,\mathcal{C}}(a;\kappa^H) > \bar{S}_{c,\mathcal{S}}(a;\kappa^H)$ and $\bar{S}_{c,\mathcal{C}}(a;\kappa^H) > S_c(a;\kappa^H)$;

(ii) If $a < \tilde{a}$, then $\bar{S}_{c,\mathcal{S}}(a;\kappa^H) > \bar{S}_{c,\mathcal{C}}(a;\kappa^H)$ and $\bar{S}_{c,\mathcal{S}}(a;\kappa^H) > S_c(a;\kappa^H)$;

(iii) If $a = \tilde{a}$, then $\bar{S}_{c,\mathcal{S}}(a;\kappa^H) = \bar{S}_{c,\mathcal{C}}(a;\kappa^H) > S_c(a;\kappa^H)$,

where $S_c(a;\kappa^H)$ denotes the high-type consumer's surplus offering a separating offer defined in (18)-(20), $\bar{S}_{c,\mathcal{C}}(a;\kappa^H)$ the high-type consumer's surplus offering a pooling collateralized credit defined in (23)-(25), and $\bar{S}_{c,\mathcal{S}}(a;\kappa^H)$ the high-type consumer's surplus offering a pooling sale *defined in* (21)-(22). *If* $\kappa^L = 0$*, a pooling sale is the only sustainable offer* $\forall\, a \in (0, a^*]$. *Proof in Appendix E.*

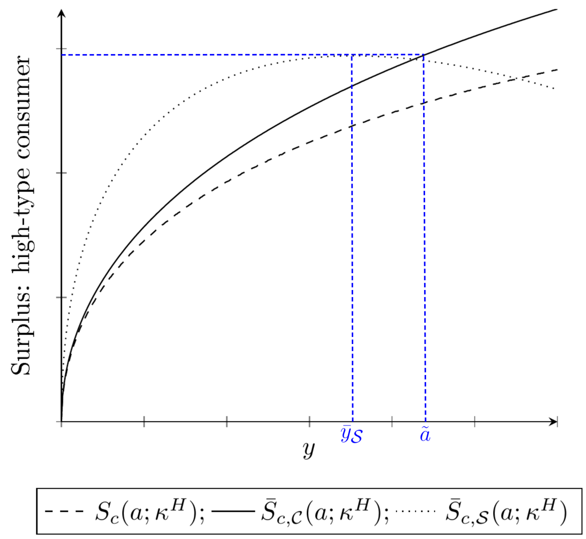

Figure 2: Undefeated equilibrium with $u(q) = 2\sqrt{q}$

The results of the bargaining game show that equilibrium allocations depend on the amount of assets carried along the period, as visualized in Figure 2 displaying the high-type consumer's surpluses offering a separating offer, $S_c(a;\kappa^H)$ (dashed line), a pooling collateralized credit, $\bar{S}_{c,\mathcal{C}}(a;\kappa^H)$ (solid line), and a pooling sale, $\bar{S}_{c,\mathcal{S}}(a;\kappa^H)$ (dotted line) using $u(q) = 2\sqrt{q}$. If assets are relatively plentiful, i.e., if $a > \tilde{a}$, high-type consumers prefer a pooling collateralized credit to selling assets at the expected value, R, since it allows them to consume $\bar{q}_{\mathcal{C}} = \beta a \kappa^L$ without sacrificing the high return, κ^H. Low-type consumers in turn would prefer to sell their assets at the pooling price, R, exploiting the information friction at the cost of the high-type consumers. However, as defined in Definition A, producers know that high-type consumers would never choose to sell assets for $a > \tilde{a}$, and therefore attribute such out-of-equilibrium offers to low-type consumers revealing their type. Hence, for $a > \tilde{a}$, the settlement strategy is unique. Consider now the case in which asset holdings are relatively low. If $a < \tilde{a}$, a fire-sale equilibrium installs itself and high-type consumers prefer outright selling $\bar{y}_{\mathcal{S}} \leq \tilde{a}$ assets at a pooling price rather than engaging in a secured credit contract. Hence, the cost associated with the concomitant over-collateralization offsets the beneficial terms of trade when assets are relatively scarce. In that context, it is also important to note that separating offers are dominated for all ranges of $a \in (0, a^*]$, since the cost of asset retention renders allocations inefficient. Finally, in the limiting case with $\kappa^L = 0$, a pooling sale remains as the only feasible offer.

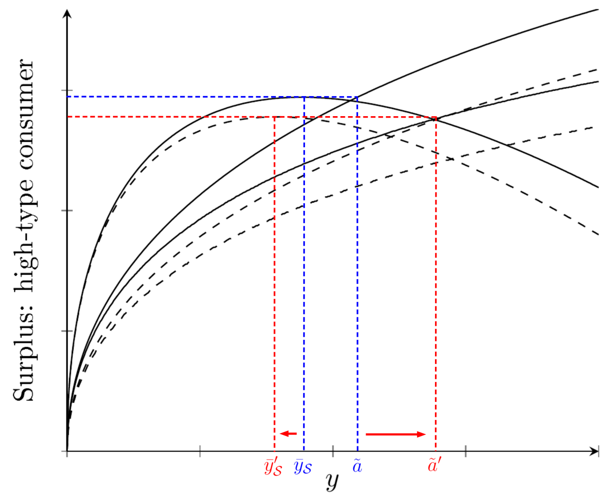

Figure 3: Increase in $\kappa^H - \kappa^L$

Lemma C. ***(Comparative statics)*** *If $\kappa^L > 0$, then $\partial\tilde{a}/\partial(\kappa^H - \kappa^L) > 0$. Proof in Appendix F.*

Relying on the results in Proposition C, Lemma C studies the effect of a change in the distance between the asset returns κ^H and κ^L on the threshold value $\tilde{a}$. The results show that an increase in $\kappa^H - \kappa^L$ increases $\tilde{a}$ for two reasons: First, with a decrease in κ^L, a high-type consumer's surplus offering a secured credit decreases by more than her surplus offering an asset sale, diminishing the superiority of collateralized credit. As a consequence, consumers require higher asset holdings to choose this settlement option over asset sale. Second, an increase in κ^H decreases the high-type consumer's surplus offering an asset sale, as pricing at the expected value and the consequential subsidization of low-type consumers reduces $\bar{y}_{\mathcal{S}}$. The surplus offering a collateralized credit, on the other hand, is not affected by an increase in κ^H since the offer is equivalent to the low-type consumer's full information offer. Figure 3 visualizes the effects. With an increase in $\kappa^H - \kappa^L$, the threshold value $\tilde{a}$ increases to $\tilde{a}'$, while $\bar{y}_{\mathcal{S}}$ decreases to $\bar{y}'_{\mathcal{S}}$ given the results in Lemma B. Hence, if a consumer enters the DM with $a < \tilde{a}'$, she is better off offering $\bar{y}'_{\mathcal{S}} < \tilde{a}'$, while for $a > \tilde{a}'$, a pooling collateralized credit is the dominant strategy.

4 Zero Commitment

Having solved the bargaining game and determined the equilibrium allocations under the assumption that producers can commit, we now alter the environment to an economy with zero commitment.[9] The Proposition D and E summarize the implications for separating and pooling offers, respectively, while Proposition F determines the undefeated equilibrium.

Proposition D. ***(Separating offers without commitment)*** *Even if producers cannot commit to future actions, the separating offers are equivalent to* (15)-(17) *and* (18)-(20) *in Proposition A. Proof in Appendix G.*

The results in Proposition D show that the separating offers, (15)-(17) and (18)-(20), in Proposition A are robust to commitment frictions. Since asset retention allows assets to be priced at their fundamental values, κ^L and κ^H, both consumers and producers have no strict incentive to renege on their promises, rendering the default decision irrelevant. As a consequence, collateralized credit and assets sales remain equivalent in a separating equilibrium, even in the absence of producer's commitment.

Proposition E. ***(Pooling offer without commitment)*** *If producers cannot commit to future actions, then* $\bar{q}_{\mathcal{C}} = \bar{q}_{\mathcal{S}} < q^*$ *and* $\bar{y}_{\mathcal{C}} = \bar{y}_{\mathcal{S}}$ *in* (21)-(22). *Proof in Appendix H.*

Proposition E shows that if producers cannot commit to future actions, the allocations for a pooling collateralized credit are equivalent to the pooling asset sale characterized in (21)-(22). The reason is intuitive. If a high-type consumer were to offer $(\bar{q}, \bar{y}_{\mathcal{C}}, \bar{l})$ in (23)-(25), a producer understands that the expected return of the pledged asset is $R > \kappa^L$, and thus has an incentive to default on the consumer. Hence, for a high-type consumer, such an offer would be equivalent to selling high-quality assets at the low price, κ^L. The resulting surplus is strictly smaller than pledging a lower quantity of assets priced at the expected value, R, rendering allocations equivalent to an asset sale.

The results in proposition D and E show that in the absence of producer's commitment, collateralized credit and asset sales are equivalent, yielding inefficient allocations for both separating and pooling offers. Hence, for collateralized credit to dominate asset sales, and thus efficiency to occur in the presence of information frictions, it is vital to have commitment by lenders. This motivates technologies like central counterparties, public record-keeping, or long-term relationships (reputation). While the work of Diamond (1989), Kehoe and Levine (1993), Alvarez and Jermann (2000), and Gu et al. (2013) discusses these technologies in

[9] Note that the inability of consumers to commit cannot be altered, since relaxing this friction would result in perfect credit markets, eliminating the need for a medium of exchange, and hence the demand for assets.

the context of perfect credit markets, this paper sheds new light on collateralized credit, highlighting the importance of commitment on the lender's side.

Proposition F. ***(Undefeated equilibrium without commitment)*** *If producers cannot commit to future actions and* $\kappa^L > 0$*, there exists a threshold value* $\hat{\pi} \in (0, 1)$ *such that:*

(i) If $\pi < \hat{\pi}$*, then* $\bar{S}_c(a; \kappa^H) < S_c(a; \kappa^H)$ *and the equilibrium is separating.*

(ii) If $\pi > \hat{\pi}$*, then* $\bar{S}_c(a; \kappa^H) > S_c(a; \kappa^H)$ *and the equilibrium is pooling.*

where $S_c(a; \kappa^H)$ *denotes the high-type consumer's surplus offering a separating offer defined in* (18)-(20)*, and* $\bar{S}_c(a; \kappa^H)$ *the high-type consumer's surplus offering a pooling offer defined in* (21)-(22)*. If* $\kappa^L = 0$*, a pooling offer is the only sustainable offer. Proof in Appendix I.*

Relying on Definition A, Proposition F determines the offer selection in the absence of producers' commitment. The results are in line with Li and Rocheteau (2008) and Bajaj (2018), studying the undefeated equilibrium in a Shi (1995) and Trejos and Wright (1995) second-generation monetary search model with indivisible assets. If the fraction of low-type consumers in the economy is large (i), the expected return, R, is low, and thus separation is worthwhile for the high-type consumer. On the other hand, if the fraction of low-type consumers is relatively low (ii), the reduced consumption incurred through asset retention is not justified, and thus selling assets at the pooling price dominates a separating offer. If $\kappa^L = 0$, analog to Proposition A and B, a pooling sale remains as the only sustainable offer.

5 General Equilibrium

This section incorporates the solutions of the bargaining game into the general equilibrium structure of the model and determines the agents' choice of asset holdings in the CM. Since said decision takes place before the idiosyncratic information shock is realized, the agents' decision does not impart any private information about the assets' future return. The following market clearing condition holds with N being the total number of consumers and producers in the economy:

$$\int_{j \in N} a(j) dj = A. \tag{29}$$

Using the linearity of the CM value function (5) yields:

$$a \in \arg\max_{a} -(\psi - \beta)a + \mathbb{E}S_i(a;\kappa) \geq 0, \tag{30}$$

where $\mathbb{E}S_i(a;\kappa)$ is the consumer and producer's expected DM surplus with $i = \{c, p\}$. Given (30), an agent chooses a maximizing her expected surplus in the DM net of the cost of holding assets, $-(\psi - \beta)a$. If $\psi > \beta$, assets are costly to hold and hence there is a uniqure solution to (30). For $\psi < \beta$, however, demand would be inifinite, and hence there is no solution. Therefore, in equilibrium, the latter case is ruled out, assuring $\psi \geq \beta$. Given the signaling game, suggesting zero surplus for the producer, consumers will be the only ones accumulating assets in the CM. Furthermore, since consumers are homogeneous prior to the realization of the information shock at the end of the CM, each consumer enters the DM with the same amount of assets, a.

Definition B. An equilibrium is a list of asset holdings in the CM, terms of trade in the DM, and aggregate asset supply, $\{a(\cdot), [q(\cdot), y_{\mathcal{S}}(\cdot), y_{\mathcal{C}}(\cdot), l(\cdot), d(\cdot)], A\}$, such that:

(i) $a(\cdot)$ is a solutut
ion to (30);

(ii) $[q(\cdot), y_{\mathcal{S}}(\cdot), y_{\mathcal{C}}(\cdot), l(\cdot), d(\cdot)]$ is a solution to (10)-(14);

(iii) $A \in \mathbb{R}_+$ is the total supply of assets in the economy; and

(iv) The market clearing condition for A, (29), holds.

Proposition G. ***(Asset pricing)*** *There exists a unique solution to* (30)*, corresponding to:*

$$\psi = \beta[1 + \mathcal{L}], \tag{31}$$

with:

$$\mathcal{L}_j = (1-\pi)\kappa^L\big[u'(q^L) - 1\big] + \pi\kappa^H\big[u'(q^H) - 1\big]\Omega \tag{32}$$

$$\bar{\mathcal{L}}_{\mathcal{S}} = R\big[u'(\bar{q}_{\mathcal{S}}) - 1\big] \tag{33}$$

$$\bar{\mathcal{L}}_{\mathcal{C}} = \kappa^L\big[u'(\bar{q}_{\mathcal{C}}) - 1\big], \tag{34}$$

where $j \in \{\mathcal{S}, \mathcal{C}\} = \{$*sale; credit*$\}$*, and* $\Omega = [u'(q^L) - 1]/[(\kappa^H/\kappa^L)u'(q^H) - 1]$*. Proof in Appendix J.*

Equation (32) denotes the marginal surplus (liquidity premium) of an additional asset in the DM in a separating equilibrium. The first part on the right-hand side represents the marginal surplus of a low-type consumer, whereas the second part corresponds to the one of a high-type consumer, defined as the liquidity value of a high-return asset under full-information, multiplied by the high type's marginal willingness to spend under private information, $\Omega = \partial y^H/\partial a \in [0, 1)$, where $\Omega = 0$ for $q^L = q^*$. Due to the binding incentive compatibility constraint, (12), to successfully separate from a low-type consumer, a high-type consumer faces a binding resaleability constraint. As a consequence, only a fraction of each additional unit of an asset can be used as a medium of exchange in the DM. Equation (33) characterizes the liquidity premium in the event of a pooling sale (and thus a pooling credit if a producer lacks commitment). Lastly, equation (34) corresponds to the consumer's liquidity premium in a pooling collateralized credit if a producer can commit, whereas the difference to (33) stems from the applied haircut, $\mathcal{H} = R - \kappa^L$.

6 Optimal Policy

Having determined the general equilibrium results of the model, let us now discuss optimal policy.

Proposition H. ***(Optimal asset supply)*** *First-best consumption, $\beta a^* \kappa^L = q^*$, occurs for $A = A^*$ with $\mathcal{L}_j = \bar{\mathcal{L}}_{\mathcal{C}} = 0$ and $\bar{\mathcal{L}}_{\mathcal{S}} > 0$. If $A < A^*$, and hence $\psi > \beta$, then $\partial\mathcal{L}/\partial a < 0$. Proof in Appendix K.*

Proposition H revisits the relationship between the liquidity premium, $\mathcal{L}$, and a change in the asset supply, A. From Proposition A and B we know that $q^L = q^*$ (and thus $\bar{q}_{\mathcal{C}} = q^*$ if a producer can commit) if the consumer's feasibility constraint is slack, and hence $\psi = \beta$, suggesting $\mathcal{L}_j = \bar{\mathcal{L}}_{\mathcal{C}} = 0$. For $\psi > \beta$, however, $q^L < q^*$, implying $\mathcal{L} > 0$ for all settlement strategies, where the liquidity premium decreases with an increase in the asset supply, $\partial\mathcal{L}/\partial a < 0$. Thus, by increasing the asset supply in the economy, the opportunity cost of holding assets decreases, resulting in increased consumption.

Should the asset supply be rigid, consumption can be increased by exchanging information-sensitive assets for risk-free bonds. Proposition I summarizes the implications of an open market operation in an environment with $\psi > \beta$, analog to the large scale asset purchases conducted in response to the Great Recession.

Proposition I. ***(Open market operations)*** *Assume $\psi > \beta$. An open market operation, exchanging A units of information-sensitive assets for $B \in \mathbb{R}_+$ units of one-period lived risk free bonds in the CM, improves allocations. Efficiency occurs for $B = B^*$ with $\beta[a\kappa^L + b^*] = q^*$ if a producer can commit, and $B = B^{**} > B^*$ with $\beta b^{**} = q^*$ if there is zero commitment. Proof in Appendix L.*

To mitigate the inefficiencies associated with trade under private information, interventions need to be conducted before the information friction materialized, following Mankiw (1986), Tirole (2012), and Chiu and Koeppl (2016) among others.[10] More precisely, exchanging A units of information-sensitive assets for B units of liquid one-period risk-free bonds in the CM. To do so, the following government budget constraint needs to hold:

$$\psi A + B = \omega B + A, \tag{35}$$

where ω denotes the CM price for one unit of the risk-free bond paying one unit of the numéraire good in the subsequent CM. Hence, by selling B bonds at price ω in the CM, the government raises enough money to purchase information sensitive assets A, whereas the average return, $R = 1$, covers the return on bonds. The size of the open market operation depends on commitment frictions. If a producer cannot commit to future actions, for first-best consumption to occur for both low- and high-types, all information-sensitive assets need to be replaced, suggesting $A = B^{**}$ such that $\beta b^{**} = q^*$. If a producer can commit to future actions, however, first-best consumption occurs for $\beta[a\kappa^L + b^*] = q^*$, and thus $B^* < B^{**}$. Interestingly, in the latter case, different to Rocheteau (2011), even if information-sensitive assets and risk-free bonds coexist, there is no pecking order, i.e., there is no preference to spend risk-free bonds first. This novel result stems from the pooling collateralized credit determined in Proposition B. High-type consumers do not forgo their high returns in the CM by paying with the information-sensitive asset and hence have no incentive to retain their high-return assets and pay with bonds first.

[10] An intervention of that sort in the DM would have no such effect, since the information friction already materialized, and hence terms of trade are equivalent to the ones in a bilateral match between a consumer and a producer.

7 Conclusion

This paper identifies an optimal settlement strategy for institutions bilaterally exchanging information-sensitive assets in OTC markets. The terms of trade are determined using a signaling game refined by the *undefeated equilibrium*, endogenously selecting between a separating and a pooling offer. The results show that (non-)equivalence of collateralized credit and asset sale depends on information and commitment frictions. Allocations are non-equivalent if lenders can commit to future action. If assets are relatively plentiful, agents prefer a pooling collateralized credit subject to a haircut over any other sale or collateralized credit agreement. If assets are scarce, however, the concomitant over-collateralization associated with a collateralized credit offsets the beneficial terms of trade, shifting settlement to asset sales at a pooling price. Policy recommendations include increasing the total supply of information-sensitive assets or conducting open market operations, replacing information-sensitive assets with risk-free bonds. Once commitment frictions increase, the equivalence of collateralized credit and asset sale, albeit inefficient, is restored.

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Appendix - Proofs

A. Proof of Proposition A

When studying separating equilibria, the *undefeated equilibrium* relies on the Cho and Kreps (1987) *Intuitive Criterion.* The proposed offer, $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)$, satisfies the intuitive criterion of there exists no out-of-equilibrium offer, $(\tilde{q}, \tilde{y_{\mathcal{S}}}, \tilde{y_{\mathcal{C}}}, \tilde{l})$, that satisfies:

$$u(\tilde{q}) - \beta[\tilde{y}_{\mathcal{S}}\kappa^H + (1-d)\tilde{l} + d\tilde{y}_{\mathcal{C}}\kappa^H] > u(q^H) - \beta[y_{\mathcal{S}}^H\kappa^H + (1-d)l^H + dy_{\mathcal{C}}^H\kappa^H], \tag{A.1}$$

$$u(\tilde{q}) - \beta[\tilde{y}_{\mathcal{S}}\kappa^L + (1-d)\tilde{l} + d\tilde{y}_{\mathcal{C}}\kappa^L] < u(q^L) - \beta[y_{\mathcal{S}}^L\kappa^L + (1-d)l^L + dy_{\mathcal{C}}^L\kappa^L], \tag{A.2}$$

$$-\tilde{q} + \beta[\tilde{y}_{\mathcal{S}}\kappa^H + (1-d)\tilde{l} + d\tilde{y}_{\mathcal{C}}\kappa^H] \geq 0. \tag{A.3}$$

According to (A.1)-(A.3), the out-of-equilibrium offer, $(\tilde{q}, \tilde{y_{\mathcal{S}}}, \tilde{y_{\mathcal{C}}}, \tilde{l})$, would make the high-type consumer strictly better off and the low-type consumer strictly worse off, as it would be accepted by the producer believing it comes from a high-type consumer. Such offers are ruled out by the intuitive criterion. Being aware of that, the proof proceeds in two steps.

First, it needs to be shown that no pooling equilibrium with $y_i > 0$ can exist, and that among all Perfect Bayesian Equilibria, only the least-inefficient separating equilibrium survives:

First proof: By contradiction. Consider first the left panel of Figure 4, displaying the surplus of a low-type consumer, a high-type consumer, and a producer in a pooling equilibrium. Suppose high- and low-types make the same pooling offer, $(q, y; \kappa^L) = (q, y; \kappa^H) \neq (0, 0)$, leading to the respective surpluses:

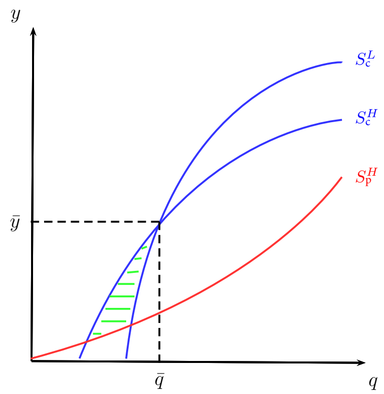
$$S_c(a; \kappa^H) = u(\bar{q}) - \beta\bar{y}\kappa^H, \tag{A.4}$$

$$S_c(a; \kappa^L) = u(\bar{q}) - \beta\bar{y}\kappa^L, \tag{A.5}$$

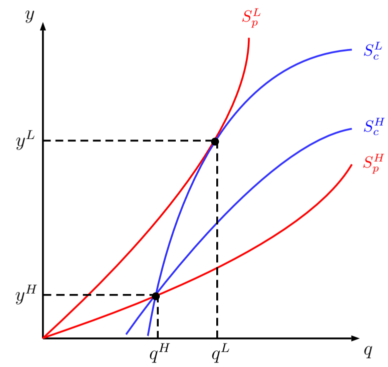
and to the offer being accepted by the producer, since she believes she is facing a high-type consumer, given her participation constraint:

$$S_p \equiv \{(\bar{q}, \bar{y}) : -c(\bar{q}) + \beta\bar{y}\kappa^H \geq 0\}, \tag{A.6}$$

with $c'(q) > 0$ and $c''(q) > 0$ for generality. One can immediately see that $S_c(a; \kappa^L)$ is steeper than $S_c(a; \kappa^H)$, given the Spence-Mirrlees single crossing property. The proposed equilibrium offer, $(\bar{q}, \bar{y})$, is located above S_p, since, by Bayes' rule, it is only accepted if $\pi < 1$, i.e., there



(a) Pooling Equilibrium



(b) Separating Equilibrium

Figure 4: Pooling versus Separating Equilibria

are some low-type consumers in the economy. The shaded-area indicates the set of offers that increase the utility of the high-type, decrease the utility of the low-type, and are acceptable to the producer, i.e., fulfill (11) assuming that $\lambda = 1$. Thus, there exists an offer involving lower consumption, q, and a lower transfer, y, that would make the high-type consumer better off. Being aware that such an offer would only be proposed by a high-type since it makes the low-type worse off, the producer accepts such a separating offer, given she was willing to accept the initial pooling offer, $(\bar{q}, \bar{y})$, in the first place. Hence, under the Cho and Kreps (1987) intuitive criterion, pooling offers are not compatible, and therefore, if an equilibrium exists, it has to be characterized by separating offers, i.e., different offers for the high- and the low-type buyers. The respective offers are described in (15)-(17) and (18)-(20), and illustrated in the right-hand panel of Figure 4.

Low-type offer: The low-type consumer's offer is at the tangency point of the producer's participation constraint, $-c(q^L)+\beta y^L \kappa^L \geq 0$, and the low-type consumer's surplus, $S_c(a; \kappa^L)$. Thus, the low-type makes a full-information offer. To solve for it, set the producer's participation constraint equal to zero since we face take-it-or-leave-it offers by the consumers. This determines the corresponding amount of assets used, $y_{\mathcal{S}}^L$ and $y_{\mathcal{C}}^L$. Then, plug the participation constraint, (11), holding with strict equality, into the low-type's maximization problem, (10), and maximize with respect to $q_{\mathcal{S}}^L$ and $q_{\mathcal{C}}^L$.

High-type offer: To satisfy the producer's participation constraint and the incentive compatibility constraint, a high-type consumer has to make an offer to the left of (and including) $S_c(a; \kappa^L)$ and above (and including) $S_c(a; \kappa^H)$. The corresponding utility-maximizing offer is at the intersection of these two curves, implying $q^H < q^L \leq q^*$. Hence, high-type consumers always consume less than the low-types and retain a fraction of their high-return assets, i.e., $y^H < y^L \leq a$. To obtain $q_{\mathcal{S}}^H$, $q_{\mathcal{C}}^H$, $y_{\mathcal{S}}^H$, and $y_{\mathcal{C}}^H$, set the incentive compatibility constraint, (12), to equality and plug in the producer's participation constraint, (11).

Second Proof: A system of beliefs is constructed that supports the least-inefficient separating equilibrium, hence defining the producer's acceptance rule, (11):

In order for the high-type's offer, $(q^H, y_{\mathcal{S}}^H, y_{\mathcal{C}}^H, l^H)$, to fulfill the acceptance rule of the producer, a belief system, $\lambda(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)$, has to be generated that is consistent with the offers in (15)-(17) and (18)-(20) and satisfies the Intuitive Criterion. Beliefs are determined from Bayes' rule and take the following form:

(i) $\lambda(\tilde{q}, \tilde{y}_{\mathcal{S}}, \tilde{y}_{\mathcal{C}}, \tilde{l}) = 0 \;\forall\; (\tilde{q}, \tilde{y}_{\mathcal{S}}, \tilde{y}_{\mathcal{C}}, \tilde{l}) \notin \mathcal{O}$, and $u(\tilde{q}) - \beta[\tilde{y}_{\mathcal{S}}\kappa^L + (1-d)\tilde{l} + d\tilde{y}_{\mathcal{C}}\kappa^L] > u(q^L) - \beta[y_{\mathcal{S}}^L\kappa^L + (1-d)l^L + dy_{\mathcal{C}}^L\kappa^L]$

(ii) $\lambda(\tilde{q}, \tilde{y}_{\mathcal{S}}, \tilde{y}_{\mathcal{C}}, \tilde{l}) = 1 \;\forall\; (\tilde{q}, \tilde{y}_{\mathcal{S}}, \tilde{y}_{\mathcal{C}}, \tilde{l}) \notin \mathcal{O}$, and $u(\tilde{q}) - \beta[\tilde{y}_{\mathcal{S}}\kappa^L + (1-d)\tilde{l} + d\tilde{y}_{\mathcal{C}}\kappa^L] \leq u(q^L) - \beta[y_{\mathcal{S}}^L\kappa^L + (1-d)l^L + dy_{\mathcal{C}}^L\kappa^L]$,

where $\mathcal{O}$ is the set of equilibrium offers. Hence, any out-of-equilibrium offer, $(\tilde{q}, \tilde{y}_{\mathcal{S}}, \tilde{y}_{\mathcal{C}}, \tilde{l}) \notin \mathcal{O}$, that increases the payoff of the low-type consumer compared to the full-information offer in (15)-(17), is attributed to the low-type consumer, while any other out-of-equilibrium offer is attributed to the high-type consumer. ■

B. Proof of Lemma A

From Proposition A, if $a > 0$, then q^H is the unique solution in $[0, q^L)$ to:

$$u(q^L) - q^L = u(q^H) - q^H \frac{\kappa^L}{\kappa^H}. \tag{A.7}$$

To determine the first part of Lemma A, differentiate (A.7) to obtain:

$$\frac{\partial q^H}{\partial \kappa^L} = \frac{[u'(q^L) - 1]\beta a + \beta y^H}{u'(q^H) - \frac{\kappa^L}{\kappa^H}} > 0, \tag{A.8}$$

$$\frac{\partial q^H}{\partial \kappa^H} = -\frac{\frac{\kappa^L}{\kappa^H}\beta y^H}{u'(q^H) - \frac{\kappa^L}{\kappa^H}} < 0, \tag{A.9}$$

where $q^L = \beta y^L \kappa^L$ with $y^L = a$ for $q^L < q^*$, $q^H = \beta y^H \kappa^H$, and $0 < y^H < y^L$. Using

$q^H = \beta y^H \kappa^H$, differentiating, and combining with $\frac{\partial q^H}{\partial \kappa^H}$ and $\frac{\partial q^H}{\partial \kappa^L}$, yields:

$$\frac{\partial y^H}{\partial \kappa^L} = \frac{[u'(q^L) - 1]a + y^H}{\kappa^H u'(q^H) - \kappa^L} > 0, \tag{A.10}$$

$$\frac{\partial y^H}{\partial \kappa^H} = -\frac{u'(q^H) y^H}{\kappa^H u'(q^H) - \kappa^L} < 0. \tag{A.11}$$

The fact that $y^H = 0$ if $\kappa^L = 0$ is straighforward from (A.7). To determine the second part of Lemma A, differentiate (A.7) and use $\frac{\partial q^L}{\partial a} = \beta \kappa^L$ to obtain:

$$\frac{\partial q^H}{\partial a} = \frac{\beta \kappa^L [u'(q^L) - 1]}{u'(q^H) - \frac{\kappa^L}{\kappa^H}}. \tag{A.12}$$

Use $q^H = \beta y^H \kappa^H$ and differentiate to obtain:

$$\frac{\partial q^H}{\partial a} = \beta \kappa^H \frac{\partial y^H}{\partial a}. \tag{A.13}$$

Combine (A.12) and (A.13) to obtain:

$$\frac{\partial y^H}{\partial a} = \frac{u'\left(q^L\right) - 1}{\frac{\kappa^H}{\kappa^L} u'\left(q^H\right) - 1} \in (0, 1), \tag{A.14}$$

for $\beta a \kappa^L < q^*$, since $u'(q^L) < u'(q^H)$ and $\kappa^L < \kappa^H$. For $\beta a \kappa^L = q^*$, and hence $u'(q^L) = 1$, however, $\frac{\partial y^H}{\partial a} = 0$. ■

C. Proof of Proposition B

When studying pooling equilibria, the *undefeated equilibrium* by Mailath et al. (1993) relies on the *most-efficient pooling equilibrium* proposed by Hellwig (1987), selecting the equilibrium maximizing the high-type consumer's surplus. I proceed in two steps: asset sales, $\mathcal{S}$, followed by collateralized credit, $\mathcal{C}$.

Asset sale: Upon receiving a pooling sale offer, $(\bar{q}_{\mathcal{S}}, \bar{y}_{\mathcal{S}})$, a producer's updated beliefs correspond to her initial beliefs, i.e., $\pi = \lambda$, since the proposed offer does not reveal any information with respect to the asset quality of the consumer. Consequentially, in order for a producer to accept the proposed offer, her expected profit needs to be non-negative, and hence $-\bar{q}_{\mathcal{S}} + \beta \bar{y}_{\mathcal{S}} R \geq 0$. Solving (11) for $\bar{y}_{\mathcal{S}}$ using $\lambda = \pi$, plugging into the high-type consumer's maximization problem, (10), and taking the derivative w.r.t. $\bar{q}_{\mathcal{S}}$ yields:

$$u'(\bar{q}_{\mathcal{S}}) = \kappa^H/R > 1, \tag{A.15}$$

and hence $\bar{q}_{\mathcal{S}} < q^*$.

Collateralized credit: Proof by contradiction. Suppose a producer would accept the posted collateral to be prized at the expected value, i.e. $\bar{l} = \bar{y}R$, and hence $\bar{q}_{\mathcal{C}} = \beta\bar{y}_{\mathcal{C}}R$, rendering collateralized credit and asset sales in a pooling equilibrium equivalent. In that scenario, each low-type consumer would default on her credit obligation, i.e. $d = 1$, resulting in a loss for the producer, since the posted collateral ($\bar{y}\kappa^L$) is worth less than the disutility of production ($-\bar{y}R$) exerted in the DM. High-type consumers, on the other hand, would repay their credit, i.e., $d = 0$, since their posted collateral ($\bar{y}\kappa^H$) is worth more than the credit obligation ($\bar{y}R$), leaving the producer with zero surplus. Given the producer's commitment to repay collateral in CM at the price defined in the DM, a strictly negative surplus is the consequence:

$$\begin{aligned} S_p &= -\bar{q}_{\mathcal{C}} + \beta[(1-\pi)\bar{y}_{\mathcal{C}}\kappa^L + \pi\bar{y}_{\mathcal{C}}R] \\ &= \beta(1-\pi)\bar{y}_{\mathcal{C}}[\kappa^L - R] < 0, \end{aligned} \tag{A.16}$$

for $\bar{q}_{\mathcal{C}} = \beta\bar{y}_{\mathcal{C}}R$ and $\pi < 1$, violating the producer's acceptance rule, (11). Hence, to guarantee a strictly non-negative surplus for the producer, i.e., $S_p \geq 0$, she needs to make sure that a low-type consumer has no incentive to default on her credit obligation. To ensure that, a producer only accepts the proposed contract if the asset is priced at the low-value, i.e., $\bar{l} = \bar{y}_{\mathcal{C}}\kappa^L$, and hence $\bar{q}_{\mathcal{C}} = \beta\bar{y}_{\mathcal{C}}\kappa^L$, yielding $S_p = 0$. Since consumer's take-it-or-leave-it offers eliminate offers that yield $S_p > 0$, the contract is unique and characterized by (23)-(25) in Proposition B. ■

D. Proof of Lemma B

To determine the effect of an increae in the distance between the high and the low asset return, $\kappa^H - \kappa^L$, on $\bar{y}_{\mathcal{S}}$, in (i), differentiate the first order condition:

$$u'(\bar{q}_{\mathcal{S}}) - \frac{\kappa^H}{\pi\kappa^H + (1-\pi)\kappa^L} = 0, \tag{A.17}$$

with respect to κ^H using $\bar{q}_{\mathcal{S}} = \beta \bar{y}_{\mathcal{S}} R$ to get:

$$\frac{\partial \bar{y}_{\mathcal{S}}}{\partial \kappa^H} = \frac{(1-\pi)\kappa^K - \pi u''(\bar{q}_{\mathcal{S}})\bar{q}_{\mathcal{S}}}{\beta u''(\bar{q}_{\mathcal{S}})} < 0, \tag{A.18}$$

given $u''(\bar{q}_{\mathcal{S}}) < 0$. Since $R = \pi\kappa^H + (1-\pi)\kappa^L = 1$, an increase in κ^H implies a decrease in κ^L, and therefore:

$$\frac{\partial \bar{y}_{\mathcal{S}}}{\partial (\kappa^H - \kappa^L)} < 0. \tag{A.19}$$

Parts (ii) and (iii) are, given $\bar{q}_{\mathcal{S}} = \min\{q^*, \beta a \kappa^L\}$, straightforward. ■

E. Proof of Proposition C

As defined in Definition A, an equilibrium of the bargaining game in the DM is a pair of strategies and a belief system, $\{[q(a;\kappa), y_{\mathcal{S}}(a;\kappa), y_{\mathcal{C}}(a;\kappa), l(a;\kappa), d(a;\kappa)], \lambda(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)\}$, where $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^H)$ is the equilibrium offer of a high-type consumer, $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^L)$ the offer of a low-type consumer, and $\lambda(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l)$ the system of beliefs. The offer can be either separating or pooling. Assuming that we only consider equilibria in which the consumer's offers are accepted, an alternative equilibrium, $\{(q', y'_{\mathcal{S}}, y'_{\mathcal{C}}, l'; a, \kappa^H), (q', y'_{\mathcal{S}}, y'_{\mathcal{C}}, l'; a, \kappa^L), \lambda(q', y'_{\mathcal{S}}, y'_{\mathcal{C}}, l')\}$, defeats the original equilibrium, if the following holds:

(i) For $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^H) \neq (q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^L)$, if:

$$u\left(q^H\right) - \beta(y^H_{\mathcal{S}} + d y^H_{\mathcal{C}})\kappa^H - \beta(1-d)l^H < u\left(q'^H\right) - \beta(y'^H_{\mathcal{S}} + d y'^H_{\mathcal{C}})\kappa^H - \beta(1-d)l'^H.$$

(ii) For $(q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^H) = (q, y_{\mathcal{S}}, y_{\mathcal{C}}, l; a, \kappa^L) = (\bar{q}, \bar{y}_{\mathcal{S}}, \bar{y}_{\mathcal{C}}, \bar{l})$, if:

$$\begin{aligned} u\left(\bar{q}\right) - \beta(\bar{y}_{\mathcal{S}} + d\bar{y}_{\mathcal{C}})\kappa^H - \beta(1-d)\bar{l} &< u\left(\bar{q}'\right) - \beta(\bar{y}'_{\mathcal{S}} + d\bar{y}'_{\mathcal{C}})\kappa^H - \beta(1-d)\bar{l}' \\ u\left(\bar{q}\right) - \beta(\bar{y}_{\mathcal{S}} + d\bar{y}_{\mathcal{C}})\kappa^L - \beta(1-d)\bar{l} &< u\left(\bar{q}'\right) - \beta(\bar{y}'_{\mathcal{S}} + d\bar{y}'_{\mathcal{C}})\kappa^L - \beta(1-d)\bar{l}', \end{aligned}$$

where (i) corresponds to a separating equilibrium, and (ii) to a pooling. The alternative equilibrium defeats the original equilibrium in two cases. First, as summarized in (i), if there exists a profitable deviation for the high-type in the separating equilibrium, or second, in a pooling equilibrium (ii) if both types have an incentive to deviate. If there is no such alternative equilibrium offer, the original equilibrium is undefeated. Proposition A and B characterize the offers.

To determine the equilibrium offer, and hence the threshold $\tilde{a} \in (0, a^*)$, we compare the surplus' of a high-type consumer for each settlement strategy, defined as:

$$S_c(a;\kappa^H) = u(q^H) - q^H, \tag{A.20}$$

$$\bar{S}_{c,\mathcal{C}}(a;\kappa^H) = u(\bar{q}_{\mathcal{C}}) - \bar{q}_{\mathcal{C}}, \tag{A.21}$$

$$\bar{S}_{c,\mathcal{S}}(a;\kappa^H) = u(\bar{q}_{\mathcal{S}}) - \bar{q}_{\mathcal{S}}\kappa^H/R, \tag{A.22}$$

with $q^L = \bar{q}_{\mathcal{C}} \leq q^*$, $q^H \in [0, q^L)$, and $\bar{q}_{\mathcal{S}} < q^*$ defined in (18), (21), and (23). Assume $\kappa^L > 0$, and hence $q^L, \bar{q}_{\mathcal{S}}, q^H \neq 0$. If $a = a^*$, where a^* solves $\bar{q}_{\mathcal{C}} = \beta a^* \kappa^L = q^*$, then:

$$\bar{S}_{c,\mathcal{C}}(a;\kappa^H) > S_c(a;\kappa^H), \tag{A.23}$$

since $q^H \in [0, q^L)$ given (15) and (18), and

$$\bar{S}_{c,\mathcal{C}}(a;\kappa^H) > \bar{S}_{c,\mathcal{S}}(a;\kappa^H), \tag{A.24}$$

since $\bar{q}_{\mathcal{S}} < q^*$ given (21). If $a \to 0$, however, then $\bar{q}_{\mathcal{C}} = \beta a \kappa^L \to 0$, and thus:

$$\bar{S}_{c,\mathcal{S}}(a;\kappa^H) > \bar{S}_{c,\mathcal{C}}(a;\kappa^H), \tag{A.25}$$

given $\bar{q}_{\mathcal{C}} = \beta a \kappa^L < \bar{q}_{\mathcal{S}} = \beta \bar{y}_{\mathcal{S}} R$. However,

$$\bar{S}_{c,\mathcal{C}}(a;\kappa^H) > S_c(a;\kappa^H), \tag{A.26}$$

still holds given the binding incentive compatibility constraint, (18), and thus $u(q^H) - q^H \frac{\kappa^L}{\kappa^H} = u(q^L) - q^L$, suggesting $u(q^H) - q^H < u(q^L) - q^L = u(\bar{q}_{\mathcal{C}}) - \bar{q}_{\mathcal{C}}$, and hence $S_c(a;\kappa^H) < \bar{S}_{c,\mathcal{C}}(a;\kappa^H)$. Consequentially, there exists a threshold, $\tilde{a} \in (0, a^*)$, for which the high-type consumer's surplus in a pooling sale and a pooling credit are equivalent, i.e.,:

$$\bar{S}_{c,\mathcal{C}}(\tilde{a};\kappa^H) = \bar{S}_{c,\mathcal{S}}(\tilde{a};\kappa^H), \tag{A.27}$$

defining $\tilde{a}$. Given that rationale, the following conditions emerge. If $\kappa^L > 0$, for $a > \tilde{a}$, a pooling collateralized credit dominates any other settlement strategy, while for $a < \tilde{a}$, a pooling asset sale is the dominant strategy.

Last but not least, if $\kappa^L = 0$, then $q^L = \bar{q}_{\mathcal{C}} = q^H = 0$ and $\bar{q}_{\mathcal{S}} > 0$, and hence a pooling sale is the unique solution. ■

F. Proof of Lemma C

Following the proof of Proposition C, the threshold value $\tilde{a} \in (0, a^*)$ satisfies (A.27) suggesting:

$$u(\beta \bar{y}_{\mathcal{S}} R) - \beta \bar{y}_{\mathcal{S}} \kappa^H = u(\beta \tilde{a} \kappa^L) - \beta \tilde{a} \kappa^L \tag{A.28}$$

given (21)-(25). Given $R = 1$ and $\partial \bar{y}_{\mathcal{S}} / \partial(\kappa^H - \kappa^L) < 0$, for the above identity to hold, $\tilde{a}$ needs to decrease following an increase in κ^L, and thus $\partial \tilde{a} / \partial \kappa^L < 0$ implying $\partial \tilde{a} / \partial \kappa^H > 0$. ■

G. Proof of Proposition D

As shown in Proposition A, the offers (15)-(17) and (18)-(20) price assets at their fundamental values, κ^L and κ^H, rendering the consumer's default decision irrelevant. Consequentially, there is also no strict incentive for the producer to default, suggesting allocation equivalence for both commitment and no-commitment of producers. ■

H. Proof of Proposition E

Proof by contradiction. If offered an asset as collateral at the low price, i.e., $\bar{q}_{\mathcal{C}} = \beta a \kappa^L$ and $\bar{l} = \bar{y}_{\mathcal{C}} \kappa^L$, given the producer's participation constraint:

$$\mathbb{E} S_p(a; \kappa) = -\bar{q} + \beta(\bar{y}_{\mathcal{S}} + d\bar{y}_{\mathcal{C}})[\lambda \kappa^H + (1-\lambda)\kappa^L] + \beta(1-d)\bar{l} \geq 0, \tag{A.29}$$

with $\lambda = \pi$, a producer has a strict incentive to default on the consumer, since the expected return of the pledged asset is higher than the credit obligation. For a high-type consumer, such an offer would be equivalent to selling the high-return assets at the low price, κ^L, resulting in the consumer's surplus:

$$S_c(a; \kappa^H) = u(q^L) - \beta a \kappa^H. \tag{A.30}$$

Since such an offer yields a positive surplus for the producer, it is strictly dominated by selling assets at the pooling price, R, as proven in Proposition B. Consequently, high-type consumers only pledge their asset as collateral, if priced at the expected value, R, rendering sale and collateralized credit equivalent. ■

I. Proof of Proposition F

From (A.20) and (A.22) we know that a high-type consumer's surplus in a separating equilibrium is independent of π, and monotonically increasing in π in a pooling sale equilibrium. Consider two limiting cases:

If $\pi \to 1$, since a high-type consumer's surplus is independent of π in a separating equilibrium, consumption $q^H \in [0, q^L)$ is equivalent to (18) in Proposition A. In a pooling equilibrium, since $R \to \kappa^H$ with $\pi \to 1$, it follows that $\bar{q}_{\mathcal{S}} \to q^*$ given $\max_{\bar{q}_{\mathcal{S}}} u(\bar{q}_{\mathcal{S}}) - \bar{q}_{\mathcal{S}} \frac{\kappa^H}{R}$. Hence, for $\pi \to 1$, it follows that $\bar{S}_c(a; \kappa^H) > S_c(a; \kappa^H)$.

If $\pi \to 0$, on the other hand, then $R \to \kappa^L$, and hence the high-type consumer's problem offering a pooling sale reduces to $\max_{\bar{q}_{\mathcal{S}}} u(\bar{q}_{\mathcal{S}}) - \bar{q}_{\mathcal{S}} \frac{\kappa^H}{\kappa^L}$, suggesting $\bar{q}_{\mathcal{S}} < q^L$. Consequentially, the incentive compatibility constraint, (12), holds at $(\bar{q}_{\mathcal{S}}, \bar{y}_{\mathcal{S}})$, i.e., $u(\bar{q}_{\mathcal{S}}) - \beta \bar{y}_{\mathcal{S}} \kappa^L \leq u(q^L) - \beta y^L \kappa^L$. Since the producer's participation constraint holds at $-\bar{q}_{\mathcal{S}} + \beta \bar{y}_{\mathcal{S}} \kappa^L = 0$, the producer realizes a positive surplus when trading with a high-type consumer, since $-\bar{q}_{\mathcal{S}} + \beta \bar{y}_{\mathcal{S}} \kappa^H > 0$. A separating offer, on the other hand, guarantees zero surplus for the producer, as established in Proposition A. Hence, for $\pi \to 0$ it follows that $S_c(a; \kappa^H) > \bar{S}_c(a; \kappa^H)$.

Consequentially, there exists a $\hat{\pi} \in (0, 1)$, where for $\pi > \hat{\pi}$, the equilibrium is pooling, and for $\pi < \hat{\pi}$, the equilibrium is separating. ■

J. Proof of Proposition G

To determine the equilibrium asset price, ψ, in Proposition G, take the derivative of (30) with respect to a, taking the market clearing condition, $\int_{j \in N} a(j) dj = A$ with N being the total number of agents in the economy, into account, where q solves (15) and (18) in a separating equilibrium, and (21) and (23) in a pooling equilibrium. (33) and (34) are straightforward. To determine (32), take the derivative of (30) w.r.t a, resulting in:

$$-(\psi - \beta) + \beta(1 - \pi)\kappa^L [u'(q^L) - 1] + \beta\pi \left[\kappa^H u'(q^H) \frac{\partial y^H}{\partial a} - \frac{\partial y^H}{\partial a} \kappa^H \right]. \tag{A.31}$$

To determine $\partial y^H / \partial a$, use the incentive compatibility constraint, (12):

$$u(q^L) - q^L = u(q^H) - q^H \frac{\kappa^L}{\kappa^H}. \tag{A.32}$$

and differentiate w.r.t to a, yielding:

$$\frac{\partial q^H}{\partial a} = \frac{\beta \kappa^L [u'(q^L) - 1]}{u'(q^H) - \frac{\kappa^L}{\kappa^H}}. \tag{A.33}$$

Then, use the fact that $q^H = \beta y^H \kappa^H$ and differentiate w.r.t. a:

$$\frac{\partial q^H}{\partial a} = \beta \kappa^H \frac{\partial y^H}{\partial a}. \tag{A.34}$$

Last but not least, combine (A.33) with (A.34), and plug into (A.31) to get (32) . ■

K. Proof of Proposition H

From Proposition A we know $q^L = q^*$ if $\beta a \kappa^L \geq q^*$, and hence $\mathcal{L}_j = \bar{\mathcal{L}}_{\mathcal{C}} = 0$. To show that $\partial \mathcal{L}/\partial a < 0$ for $a < a^*$ with $\beta a^* \kappa^L = q^*$, the proof proceeds in three steps, distinguishing between pooling credit, pooling sale, and a separating offer.

First, from $\bar{\mathcal{L}}_{\mathcal{C}}$ in (34) we know that $u'(\bar{q}_{\mathcal{C}}) > 1$ if $a < a^*$, and thus $\partial \bar{\mathcal{L}}_{\mathcal{C}}/\partial a < 0$, since an additional unit of the asset relaxes the consumer's feasibility constraint. The same logic applies for $\bar{\mathcal{L}}_{\mathcal{S}}$ in (33). Last but not least, consider the liquidity premium in a separating equilibrium, $\mathcal{L}_j$ in (32). Revisiting Lemma A we know that $\partial y^H/\partial a = 0$ if $a = a^*$, and hence $\partial \mathcal{L}_j/\partial a = 0$, since an additional unit of the asset does not relax the high-type consumer's incentive compatibility constraint. If $a < a^*$, and thus $q^L < q^*$, however, the high-type consumer's marginal willingness to sell or pledge another asset is positive, i.e., $\partial y^H/\partial a \in (0, 1)$, and hence $\partial \mathcal{L}_j/\partial a < 0$. ■

L. Proof of Proposition I

Consider an environment without private information and $\kappa^L = \kappa^H = 1$. From Proposition A it follows that $\beta a = q^*$ if $a \geq a^*$, reflecting the insights from the low-type's full-information offer in (15)-(17). Thus, by replacing information-sensitive assets by risk-free bonds, yielding one unit of the numéraire good in the CM, efficiency occurs for $\beta b^{**} = q^*$. Reintroducing heterogenous returns, feasibility is guaranteed for $AR = B$, where $\psi = \omega = \beta$ holds for both, $\beta(a\kappa^L + b^*) = q^*$ and $\beta b^{**} = q^*$. An intervention of that sort in the DM, in turn, would have no such effect, since the equilibrium notion remains the same, independent of whether a consumer bilaterally trades with a producer or the government. ■