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Asymmetric Information in Frictional Markets for Liquidity: Collateralized Credit vs Asset Sale

Florian Madison

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Asymmetric Information in Frictional Markets for Liquidity∗
Collateralized Credit vs Asset Sale

Florian Madison†
University of Basel, Switzerland

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Abstract
The aim of this paper is to identify an optimal contract design for institutions exchanging liquidity on a decentralized asset market with Lucas-trees subject to asymmetric information. Within a search-theoretic dynamic general equilibrium model, I establish a non-equivalence between collateralized credit and an outright sale of assets, and show through a signaling game with Perfect Bayesian Nash Equilibria, why depositing assets as collateral is a dominant strategy. In addition, I provide a compelling reason why financial institutions apply haircuts to securities offered as collateral, which justifies the observed over-collateralization on the over-the-counter market for liquidity since the eruption of the global financial crisis.

JEL Classification: D82, E44, G12, G21
Keywords: Liquidity, Asymmetric Information, Collateral, Undefeated Equilibrium

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†Madison (florian.madison@unibas.ch): Economic Theory Division, Department of Economics, University of Basel, Switzerland
1 Introduction

Whenever a financial institution is in need of liquidity, it has three options to acquire money in the short run – unsecured borrowing, secured borrowing, or selling assets. With the eruption of the global financial crisis in 2007, however, confidence in financial institutions was shaken and concerns about their solvency were raised. This loss of trust literally brought the market for unsecured credit to a halt and the demand for collateral as an insurance against potential default increased substantially. As a result of these developments, collateralized credit, and in particular repurchase agreements, became the common source of funding.\(^1\) Amongst the first to document this run on repo were Gorton and Metrick (2012\(^\text{a,b}\)), while Krishnamurthy et al. (2014) and Copeland, Martin and Walker (2014) later confirmed this observation, pointing out that this run specifically affected over-the-counter markets, characterized by bilateral trades. Although difficult to measure, conservative estimates state that the total daily trading volume on the US repo market amounts 3.8 trillion US\$, whereas overall approximately 84\% of all transactions on over-the-counter derivatives are collateralized.\(^2\) In doing so, these secured loan agreements use a large array of financial assets, including sovereign bonds, corporate bonds, and asset-backed securities.

Around the same time, as outlined by Gorton (2008) and Gorton and Ordoñez (2014), particular asset classes started to suffer from impaired recognizability, resulting in imperfect information regarding the quality of the assets traded on the financial markets. This information friction directly reduced the liquidity of these assets, which manifested itself in increased haircuts and overcollateralization.\(^3\) As shown by Gorton and Metrick (2012\(^\text{b}\)), average haircuts for nine asset-backed security and corporate debt classes rose from zero to approx. 50\% in the time period between 2007 and 2009.\(^4\) As a consequence, financial institutions were forced to provide higher amounts of collateral to secure the same loan size, or reallocate their portfolio towards less risky assets, such as government bonds, in order to avoid being subject to the applied haircut.

Being aware of these developments, the question arises as to why financial institutions in need of liquidity prefer to use their assets as collateral to secure a bilateral credit, instead of selling them to acquire the needed money. Animated by this question, the aim of this paper is to identify an optimal contract design for institutions exchanging liquidity on a decentralized asset market subject to asymmetric information.

The basic model used for this analysis is the unified infinite-time dynamic general equilibrium model established by Lagos and Wright (2005) and the extension introduced by Rocheteau (2011) to account for the information sensitivity of the exchanged asset. After receiving an idiosyncratic consumption opportunity, agents bilaterally exchange assets against consumption goods, using asset sales, collateralized credit, or a mix of both, to settle their trade.

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\(^1\)A collateralized credit is defined as a bilateral credit agreement secured by an asset with a value of at least the loan size to insure the lender against default of the borrower, as characterized in Kiyotaki and Moore (1997). In a repurchase agreement, an agent sells an asset today, but at the same time commits to buy it back at a later point in time at a price agreed on today. Going forward, I will treat these two means of payment equivalent, since I abstain from legal concerns and re-hypothecation.

\(^2\)Estimates are provided by Copeland, Davis, LeSueur and Martin (2014). Further estimates including reverse repurchase agreements are available in Copeland et al. (2012) and Gorton and Metrick (2012b).

\(^3\)A haircut is defined as the difference between the market value of an asset used as collateral and the effective size of the granted loan.

\(^4\)For some asset classes, haircuts even rose to a 100\% and were thus not accepted as collateral anymore.
Given this setup, I consider a signaling game with Perfect Bayesian Equilibria, in which the informed agent proposes an offer to the uniformed counterparty. However, in contrast to Rocheteau (2011), the equilibrium outcome is not solely restricted to the Riley (1979) least-inefficient separating equilibrium, but also allows for a Hellwig (1987) efficient pooling equilibrium by introducing the refinement of the undefeated equilibrium established by Mailath et al. (1993). To rule out Pareto inefficiencies, the decision on which terms-of-trade to choose is endogenous and solely taken by the high-quality asset holder in the economy.

The provided theory shows that information frictions regarding the quality of the particular assets traded trigger the non-equivalence between collateralized credit and asset sales, when allowing for pooling equilibria to exist. Agents holding sufficiently many high-quality assets prefer a pooling collateralized credit subject to a haircut over any other sale or collateralized credit agreement. This complements the equivalence result established by Lagos (2011) under full information, as well as Rocheteau (2011) when only considering separating equilibria, by providing a non-equivalence when allowing for pooling equilibria. The intuition for this result emerges from the properties of the pooling equilibrium, as elaborated below.

Whenever an agent decides to sell his assets in a pooling equilibrium, the equilibrium price corresponds to the expected value, since any other offer involving a higher price will not be accepted by the uninformed counterparty purchasing the asset. Given these terms of trade, high-quality asset holders incur a loss on every unit of the asset sold, while agents holding low-quality assets are able to exploit the information friction and benefit on the cost of the high-quality asset holders. The resulting equilibrium allocation is inefficient. Thus, in order for a high-quality asset holder to circumvent this loss, an alternative settlement option, allowing for a more beneficial allocation, is desirable. A collateralized credit subject to a haircut allows for this, since on one side, repayment of the outstanding credit obligation enables the borrower to consume the high return of the pledged asset in the following period. On the other side, the borrower can obtain the desired consumption, if in possession of enough assets to compensate for the applied haircut. This amount, and thus the degree of overcollateralization by the high-quality asset holder, depends on the severity of asymmetric information prevailing in the economy, and is determined through the haircut imposed by the counterparty granting the loan. As a result, from the perspective of the agent holding high-quality assets, such a collateralized credit dominates any other sale or credit agreement. Low-quality asset holders imitating the offer in turn are indifferent between repayment and default, and the contract is desirable to lenders, since the applied haircut eliminates credit risk.

If assets are relatively scarce, however, the optimal terms of trade change, since the concomitant overcollateralization associated to a collateralized credit offsets its benefits. Whether high-quality asset holders then sell at a pooling price, or choose to separate themselves by asset retention, depends on the distribution of asset-qualities in the economy. Thus, in an economy with a particularly high share of low-quality assets in the market, all but the Riley (1979) least-inefficient separating equilibrium can be dismissed. As a result, high-quality asset holders effectively separate themselves from low-quality asset holders through asset-retention. However, the better terms of trade come at the cost of inefficiently low consumption due the performed signaling. On the contrary, if the share of low-quality as-
set holders is relatively small, allocations endogenously change to a Hellwig (1987) efficient pooling equilibrium, since the high-quality asset holder chooses to avoid the cost of separation. This result was promoted by Li and Rocheteau (2008) and Bajaj (2016), applying the \textit{undefeated equilibrium} to the second-generation frameworks established by Trejos and Wright (1995) and Shi (1995).

Although this paper focuses on bilateral over-the-counter contracts with assets subject to private information, a more general analysis involving other, more liquid, asset classes is not precluded. This allows, first, to generate a run on repo, induced by an overall decline in the average quality of assets in the economy, and second, to identify an increase in haircuts concomitant with a scarcity of safe assets, capturing the developments during the recent global financial crisis and supporting the observations of Gorton and Metrick (2012\textsuperscript{a,b}).

1.1 Related Literature

There is a broad range of research literature that analyzes collateralized credit contracts. Bester (1985) shows that collateral allows agents on the interbank market to overcome asymmetric information about the solvency of the respective counterparty willing to borrow. Flannery (1996), however, argues that while other market participants may know about the overall solvency of the respective counterparty, they are not aware of the individual assets in the portfolio. Given this lack of information, private credit will tend to collapse, providing a role for an alternative payment system, such as a government discount window.

The monetary theory literature, however, mostly considered these two means of settlement to be equivalent so far. This has been shown by Lagos (2011) under information symmetry, as well as by Rocheteau (2011) under information asymmetry, when only considering separating equilibria. A first distinction under complete information has been provided by Berentsen and Waller (2011). In their paper, they analyze the trade-off that liquidity-constrained agents face in their decision to either sell assets outright (inside bonds) or borrow against collateral (outside bonds). They conclude that any allocation in an economy with inside bonds can be replicated by an economy with outside bonds, but not vice versa. Monnet and Narajabad (2012), however, show that if agents face uncertainty about the value of holding the security in the future, they prefer to conduct a repurchase agreement, since they might end up needing the asset for an upcoming consumption opportunity. Parlatore (2017) analyzes this trade-off in the presence of uncertainty regarding the outcome of the financed risky projects. In her model, assets entail a liquidity and a collateral premium, since they enable external financing. The main results show that with an increase in liquidity, the liquidity premium on an asset increases, while the collateral premium declines. Tomura (2016) explains the use of repurchase agreements by a hold up problem introduced through a deadline on retrieving cash. An urgent need to liquidate bonds on an over-the-counter market weakens the bargaining power of the selling party and de-incentivizes the acquisition of that bond in the first place. A repurchase agreement subject to a haircut, however, allows an agent to circumvent this inefficiency. Gottardi et al. (2016) in their paper focus on the aspect of re-hypothecation to explain the rise of repurchase agreements in the last years. However, none of these papers have considered information frictions regarding the quality of the traded asset.
The paper builds on the extensive literature analyzing the relationship between asset prices and asymmetric information, pioneered by Akerlof (1970) and followed by numerous contributions by Rothschild and Stiglitz (1976), Riley (1979), Wilson (1980), Burdett and Judd (1983), Hellwig (1987), and Eisfeldt (2004), among others. When applying games with information frictions to the extended monetary theory literature, models are usually either restricted to separating or pooling equilibria. A prominent example for the former are Velde et al. (1999), who were amongst the first to study the effect of asymmetric information on asset prices in a search-and-matching environment, by considering a signaling game with pooling equilibria. Papers using the Cho and Kreps (1987) Intuitive Criterion to restrict the set of equilibria to separating ones involve Nosal and Wallace (2007), Guerrieri et al. (2010), and Rocheteau (2011). While Nosal and Wallace (2007) focus on the threat of counterfeited money in an economy, Rocheteau (2011) shows that in the case of assets subject to private information, high-types can effectively separate themselves from low-types through asset retention. By applying the undefeated equilibrium by Mailath et al. (1993) in this paper, the choice among pooling and separating equilibria is endogenized, allowing for transitions between the two types of equilibria to be analyzed.

The promoted non-equivalence also borrows elements from the literature on security design under asymmetric information. Boot and Thakor (1993) define an optimal security design from an issuer’s perspective facing different investors. The results show that for a high-type issuer it is beneficial to split the information sensitive assets, rather than selling a composite security. Since they only consider pooling equilibria, low-types have no other option than to imitate the strategy of the high-type, as their true type would be revealed otherwise. Demarzo and Duffie (1999) analyze a similar question, but by considering separating equilibria. Their results show that high-quality asset holders separate themselves from low-quality asset holders by asset retention, whereas the security design problem involves a trade-off between the utility costs of asset retention and the impaired liquidity through less retention.

Concerning the organization of the paper, Section 2 presents the environment of the discussed theoretical framework. Section 3 studies the bargaining game. Using these insights, Section 4 then analyzes the agents’ optimal choice of asset holdings in the centralized market. Section 5 discusses the resulting equilibrium, followed by the conclusion in Section 6.

2 Environment

The model is based on the unified search-theoretic framework established by Lagos and Wright (2005) and the extension introduced by Rocheteau (2011). Time is discrete, starts at $t = 0$, and continues forever. Each period is divided into two subperiods: a centralized market (CM), and a decentralized market (DM), as visualized in Figure 1. The discount factor across periods is $\beta \in (0, 1)$, where $\beta = (1 + r)^{-1}$ and $r$ is the time rate of discount. There is a continuum $[0, 1]$ of infinitely lived agents with an expected lifetime utility from date $t = 0$ onwards defined as:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ u(q_t) - c(q_t) + U(x_t) - h_t \right], \quad (1)$$
with $q$ being the consumption good in the decentralized market, $x$ the consumption good in the centralized market, and $h$ hours worked in the centralized market. For tractability, the period utility is separable across subperiods. The utility function in the decentralized market, $u(q)$, is twice continuously differentiable, strictly increasing in $q$, and concave, $u'(q) > 0 > u''(q)$. The value of the function at $q = 0$ is zero, the limit of the derivative towards zero is positive infinity, $u'(0) = \infty$, and the limit of the derivative towards infinity is zero, $u'(\infty) = 0$, fulfilling the Inada (1963) conditions. $c(q) = q$ is a linear cost function in the decentralized market. The production technology in the centralized market is linear with labor being the only input, $h = x$, and the utility of consuming the general good is linear with $U(x) = x$. In the following, the timing of events and the two markets are discussed in detail, starting with the centralized market.

![Figure 1: Environment](image)

**Centralized Market:** In the first subperiod, all agents produce and consume a general good, $x$, settle claims of the previous period, and adjust their asset holdings. Contrary to Lagos and Wright (2005) and Rocheteau (2011), however, there is no fiat money and the production of real assets is endogenous. Stored general goods serve as perfectly divisible one-period lived real assets, $z$, in the subsequent decentralized market, where they are used either as a direct medium of exchange or as collateral to secure credit.

**Decentralized Market:** At the beginning of the decentralized market, each agent is privately informed about the stochastic real return, $0 \leq \kappa_L < \kappa_H < \infty$, of his asset in the subsequent settlement market, $t + 1$, similar to the fruits of a Lucas (1978) tree. The realization of this idiosyncratic information shock is common to all assets held by an agent, but independent across agents. With probability $\pi \equiv \text{Prob}[\kappa = \kappa_H]$, the investment succeeds and yields a terminal return $\kappa = \kappa_H$, while with the complementary probability, $1 - \pi$, it fails and yields $\kappa = \kappa_L$. The expected return is $R = \pi \kappa_H + (1 - \pi) \kappa_L$, where the condition $\beta R \leq 1$ needs to be imposed, since otherwise, if $\beta R > 1$, agents would store infinite amounts of assets, which is inconsistent with equilibrium.

In addition to the idiosyncratic information shock, agents learn their role in the decentralized market. With probability $n \in (0, 1)$, agents want to produce, but cannot consume the search good $q$, while with complementary probability, $1 - n$, agents want to consume, but cannot produce. From here onwards, I refer to consumers in the decentralized market as buyers and to producers in the decentralized market as sellers, with the corresponding subscripts $b$ and $s$.

Based on the idiosyncratic preference and information shocks, agents meet bilaterally and

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5 Assets are perfectly divisible and can be interpreted as not publicly traded assets such as corporate bonds, asset-backed securities, or private equity, among others. In order to interpret the information asymmetry in this economy, I follow the logic of Plantin (2009) stating that agents acquire some private information about the return of their asset simply by holding it. This is especially relevant for the set of asset classes described.
trade perfectly divisible shares of assets, \( y \in [0, z] \), against search goods, \( q \). All market participants and trades are anonymous and trading histories are private information, which precludes unsecured bilateral credit. Given there is a double coincidence of wants problem, a medium of exchange is essential for trade to occur, as shown in Kocherlakota (1998), Wallace (2001), and Shi (2006). Payment takes place either through immediate settlement, bilateral collateralized credit, or a combination of both. When engaging in a collateralized credit, agents face a borrowing constraint, as defined by Kiyotaki and Moore (1997) in a non-monetary economy, where buyers pledge their real asset as collateral and agree to fulfill their credit obligation in the subsequent centralized market at a price defined today. Default is an endogenous decision for the buyer, whereas the only cost of defaulting is the loss of collateral. If the buyer fulfills his credit obligation, however, repayment of the collateral by the seller is enforceable. Before moving on to the bargaining game and deriving the steady-state equilibria of the model, I consider the planner’s problem.

**Social Optimum:** Without loss of generality, it is assumed that the social planner treats all agents symmetrically and maximizes their expected lifetime utility for a stationary allocation, \((q, z)\), according to:

\[
(1 - \beta)W = [(1 - n)u(q_b) - nq_s] + (\beta R - 1) z.
\]  

While the first term on the right-hand side of (2) denotes the expected utility from consuming and producing in the decentralized market, the second term represents the utility of producing assets and receiving the discounted expected return, \(\beta R\), in the centralized market of the subsequent period. Since all agents enter the period with the same amount of assets, \(q_b\) and \(q_s\) are the same for all of them. Therefore, market clearing implies that \(q_s = \frac{1 - n}{n} q_b\).

Solving the planner’s problem, the first-best allocation, \((q^*, z^*)\), satisfies \(q = q^*\), where \(u'(q^*) = 1\). Hence, at the optimum, the marginal utility of consumption in the decentralized market equals the marginal cost of production. Concerning the asset, from a planner’s perspective, \(z^* = 0\) if \(\beta R < 1\), and \(z^* \in [0, \infty]\) is indeterminate for \(\beta R = 1\). Hence, if assets are costly to carry along the period, they have no use in this economy from a planner’s point of view.

### 3 Bargaining Game

**Centralized Market:** The expected utility of an agent entering the centralized market with \(z \in \mathbb{R}_+\) units of real assets, their corresponding return, \(\kappa \in \{\kappa^L, \kappa^H\}\), and \(l \in \mathbb{R}_+\) units of credit obligations from the previous period, is:

\[
W(z, l; \kappa) = \max_{x, h, z'} x - h + \mathbb{E}V(z'; \kappa')
\]

\[
\text{s.t. } x + z' = h + l + z\kappa,
\]

where an agent finances his end-of-period asset balances, \(z'\), his centralized market consumption, \(x\), and his outstanding credit obligations, \(l\), through inputs of labor, \(h\), and the asset returns, \(z\kappa\), realized in the centralized market. \(V(z'; \kappa')\) is the value of entering the
decentralized market with \( z' \) units of the asset, where the expected value, \( \mathbb{E} \), captures the uncertainty about the future role in the decentralized market, and the terminal return, \( \kappa' \), of the asset, \( z' \). For simplicity, and without loss of generality, it is assumed that the buyer’s asset holdings, \( z \), are common knowledge in the match, but not the return, \( \kappa \). Using the agent’s budget constraint to eliminate \( x - h \) in the objective function, the following first-order and envelope conditions emerge:

\[
\mathbb{E} V'_{z'} = 1; \quad W_z = \kappa; \quad W_l = -1, \tag{5}
\]

where \( \mathbb{E} V'_{z'} \) is the marginal value of taking another unit of the real asset into the decentralized market, and \( W_z \) and \( W_l \) are the partial derivatives of \( W(z, l; \kappa) \) with respect to \( z \) and \( l \). The usual properties like linearity and independence apply. Hence, \( W(z, l; \kappa) \) is linear in wealth with the intercept, \( W(0, 0; \kappa) \), and the slope, \( \kappa \). Moreover, the amount of assets an agent chooses to produce and carry into the decentralized market, \( z' \), is independent of his current asset holdings when entering the centralized market, \( z \).

**Decentralized Market:** The bargaining game between a buyer and a seller in the decentralized market has the structure of a signaling game, i.e., the informed agent moves first and makes the offer. A strategy for the buyer is to specify an offer, \( (q_b, y_{S,b}, y_{C,b}, l_b) \), where \( q_b(z; \kappa_b) \) is the amount of goods received, \( y_{S,b}(z; \kappa_b) \) the share of assets sold, \( y_{C,b}(z; \kappa_b) \) the share of assets deposited as collateral, and \( l_b(z; \kappa_b) \) the credit obligation in period \( t + 1 \), all as a function of the buyer’s type, \( \kappa_b \), and his asset holdings, \( z \). In doing so, the transfer of assets is constrained by the agent’s current asset holdings, i.e., \( y_{S,b} + y_{C,b} \leq z \). Given the offer placed by the buyer, the seller updates his beliefs about the terminal return of the buyer’s asset, \( \kappa_b \), and defines an acceptance rule, \( A_b \), that specifies the set of acceptable offers. Due to bilateral matching and thus \( q_b = q_s, y_{S,b} = y_{S,s}, y_{C,b} = y_{C,s}, \) and \( l_b = l_s \), I refrain from the subscripts, \( b \) and \( s \), for these variables going forward. Hence, the buyer’s payoff in the state \( \kappa_b \) is:

\[
V_b(z; \kappa_b) = \begin{cases} 
  u(q) + \beta [(1 - d)W(z - y_{S,b}, l; \kappa_b) + dW(z - y_{S,b} - y_{C,b}, 0; \kappa_b)] & \mathbb{I}_{A_b} \\
  \beta W(z, 0; \kappa_b) (1 - \mathbb{I}_{A_b}), & \end{cases} \tag{6}
\]

where \( \mathbb{I}_{A_b} \) is an indicator function equal to one if the buyer’s offer is in the seller’s set of acceptable offers, i.e., \( (q, y_{S,b}, y_{C,b}, l) \in A_b \), and zero otherwise. If engaging in a collateralized credit, default is a binary choice variable, \( d \in \{0; 1\} \), allowing buyers to default on their credit obligation, where \( d = 0 \) denotes repayment, and \( d = 1 \) default. For convenience, there is no matching friction and hence, buyers and sellers meet with probability one. Using the linearity of the agent’s value function in the centralized market, the buyer’s payoff in case of trade can be reduced to his surplus, \( S_b(z; \kappa_b) = \{ u(q) - \beta [y_{S,b} \kappa_b + (1 - d)l + dy_{C,b}] \} \mathbb{I}_{A_b} \).

Similarly, the seller’s payoff is given by:

\[
V_s(z; \kappa_s) = \begin{cases} 
  -q + \beta [(1 - d)W(z + y_{S,b}, l; \kappa_s) + dW(z + y_{S,b} + y_{C,b}, 0; \kappa_s)] & \mathbb{I}_{A_s} \\
  \beta W(z, 0; \kappa_s) (1 - \mathbb{I}_{A_s}), & \end{cases} \tag{7}
\]

---

\(^6\)This assumption allows to ignore how beliefs about the composition of the agents’ portfolio are being formed.

\(^7\)This is in line with the lotteries introduced by Berentsen et al. (2002) under indivisibility, where agents in a bilateral trade are able to offer their asset probabilistically.
which can be reduced to $S_s(z; \kappa_s) = \{-q + \beta[y_S\kappa_b + (1 - d)l + dy_C\kappa_b]\}I_{A_s}$. In order for a seller to accept the offer made by the buyer, he has to form expectations about the terminal return, $\kappa_b$, of the buyer’s asset. Let $\lambda = \text{Prob}[\kappa_b = \kappa_b^H \mid (q, y_S, y_C, l)] \in \{0, 1\}$ represent the seller’s posterior belief that the buyer’s asset is of high quality, $\kappa_b = \kappa_b^H$, conditional on the offer, $(q, y_S, y_C, l)$, made. The posterior expected return can therefore be formulated as:

$$E_{\lambda}[\kappa_b] = \lambda(q, y_S, y_C, l)\kappa_b^H + [1 - \lambda(q, y_S, y_C, l)]\kappa_b^L,$$

(8)

determining the seller’s acceptance rule, $A_s(\lambda)$:

$$A_s(\lambda) = \{(q, y_S, y_C, l) \in F : -q + \beta[y_S + dy_C]\left[\lambda\kappa_b^H + [1 - \lambda]\kappa_b^L\right] + \beta(1 - d)l \geq 0\}.$$  

(9)

Hence, for a given belief system, $\lambda(q, y_S, y_C, l)$, in order for a seller to accept an offer made by the buyer, it has to yield a non-negative expected surplus. Assuming a tie-breaking rule according to which a seller agrees to any offer that makes him indifferent between accepting and rejecting, the buyer chooses an offer that maximizes his surplus, $S_b(z; \kappa_b)$, taking as given the acceptance rule of the seller, $A_s(\lambda)$, and the feasibility constraints. Therefore, the buyer’s problem reduces to:

$$S_b(z; \kappa_b) = \max_{d \in \{0; 1\}} \left[ (1 - d) \max_{q,l,y_S,y_C} \left[ u(q) - \beta y_S\kappa_b - \beta l \right] + d \max_{q,l,y_S,y_C} \left[ u(q) - \beta y_S\kappa_b - \beta y_C\kappa_b \right] \right]I_{A_s},$$

(10)

s.t. $(q, y_S, y_C, l) \in F = \mathbb{R}_+ \times [0, z]$

s.t. $A_s(\lambda)$.

I consider Perfect Bayesian Equilibria, where an equilibrium for this bargaining game is a profile of strategies for the buyer and the seller, and a system of beliefs. If $(q, y_S, y_C, l)$ is an offer made by the buyer, then $\lambda(q, y_S, y_C, l)$ is derived from the seller’s prior belief according to Bayes’ rule. Without restriction, every allocation in this bargaining game can be a Perfect Bayesian Equilibrium. Therefore, in order to narrow the set and exclude unreasonable out-of-equilibrium beliefs, the equilibrium has to be refined. I apply the undefeated equilibrium established by Mailath et al. (1993), endogenously selecting among the least-inefficient separating equilibrium by Riley (1979) and the most-efficient pooling equilibrium by Hellwig (1987). Among the sustainable separating and pooling offers, the refinement chooses the one maximizing the lifetime utility of the high-type buyer, $V_b(z; \kappa_b^H)$, thereby eliminating potential Pareto inefficiencies. This includes the choice between collateralized credit and asset sale. In the following, separating and pooling equilibria are derived separately, starting with the least-inefficient separating equilibrium.\footnote{I decided to forgo a benchmark scenario under full-information at this stage of the paper, since the resulting allocations are equivalent to the ones corresponding to the low-type buyer’s offer, (14), in the separating equilibrium.}
3.1 Separating Equilibrium

When considering separating equilibria, the undefeated equilibrium by Mailath et al. (1993) relies on the Cho and Kreps (1987) Intuitive Criterion. The proposed offer, \((q, y_S, y_C, l)\), fails the Intuitive Criterion if an out-of-equilibrium offer, \((\tilde{q}, \tilde{y}_S, \tilde{y}_C, \tilde{l})\), exists, such that the following is true:

\[
u(\tilde{q}) - \beta [\tilde{y}_S \kappa_b^H + (1-d)\tilde{l} + d\tilde{y}_C \kappa_b^H] > u(q^H) - \beta [y_S H \kappa_b^H + (1-d)l^H + dy_C \kappa_b^H]
\] (11)

\[
u(\tilde{q}) - \beta [\tilde{y}_S \kappa_b^L + (1-d)\tilde{l} + d\tilde{y}_C \kappa_b^L] < u(q^L) - \beta [y_S L \kappa_b^L + (1-d)l^L + dy_C \kappa_b^L]
\] (12)

\[-\tilde{q} + \beta [\tilde{y}_S \kappa_b^H + (1-d)\tilde{l} + \tilde{y}_C \kappa_b^H] \geq 0
\] (13)

where the first inequality corresponds to the surplus of a high-type buyer, the second to the low-type buyer, and the third to the seller’s beliefs. According to (11), the offer, \((\tilde{q}, \tilde{y}_S, \tilde{y}_C, \tilde{l})\), would make the high-type strictly better off. The low-type, however, would be strictly worse off, and according to (13), the offer would be accepted by the seller, since he believes that it comes from a high-type. Such offers are ruled out by the Intuitive Criterion, since a low-type buyer would always have an incentive to deviate to his original offer, \((q^L, y_S^L, y_C^L, l^L)\).

In order to solve for the separating equilibrium of this bargaining game, three steps need to be fulfilled. In a first step (Lemma A), it needs to be shown that under the Cho and Kreps (1987) Intuitive Criterion, no pooling equilibrium with \(y > 0\) can exist. Second, it needs to be shown that among all Perfect Bayesian Equilibria, only the least-inefficient separating equilibrium survives the Intuitive Criterion (Lemma B). And third, a system of beliefs is constructed that supports the least-inefficient separating equilibrium, hence explicitly defining the sellers’ acceptance rule, \(A_s(\lambda)\).

Lemma A. In a separating equilibrium, there exists no pooling offer with \(y > 0\).\(^9\)

Sketch of Proof: Assume that there exists a pooling equilibrium with \(y > 0\), in which all assets are traded at the same pooling price. The price a high-type buyer would demand for one additional unit of his asset in that equilibrium is higher than the ask price of the low-type buyer. Hence, the high-type would be willing to sell a lower amount of his asset in order to receive better terms of trade compared to the pooling price. Due to the concavity of \(u(q)\), this would increase the high-type buyer’s surplus, while the low-type buyer would be worse off, since his decentralized market consumption would be lower than in the initially proposed pooling equilibrium. Given that reasoning, the seller would attribute such a separating offer to a high-type buyer and accept it, since he was willing to accept the pooling offer in the first place.

Lemma A states that any equilibrium, in which agents exchange assets against goods, is separating under the Cho and Kreps (1987) Intuitive Criterion. Thus, we determine the optimal separating offer for a low- and a high-type buyer, (14) and (18), individually.

---

\(^{9}\)A sketch of proof has been provided below. The full proof can be found in Appendix A, corresponding to Lemma B.
Buyer’s Problem - Low-Type: In equilibrium, the low-type buyer can do no worse than to reveal his type and make the same offer he would make under complete information, since this offer is always acceptable to the seller, independent of his beliefs. However, at the same time, he cannot do any better, since otherwise the offer would have to be pooled with the offer of a high-type buyer. Such out-of-equilibrium offers are ruled out by the Intuitive Criterion. Hence, the problem of a low-type buyer maximizing his surplus, $S_b^L(z; \kappa_b^L)$, takes the following form:

$$\begin{align*}
(q^L, y_S^L, y_C^L, l^L, d) & \in \arg \max_{q, y_S, y_C, l, d} \, u(q) - \beta \left[ y_S \kappa_b^L + (1 - d) l + d y_C \kappa_b^L \right] \tag{14} \\
\text{s.t.} & - q_S + \beta y_S \kappa_b^L \geq 0 \tag{15} \\
\text{s.t.} & - q_C + \beta \left[ (1 - d) l + d y_C \kappa_b^L \right] \geq 0 \tag{16} \\
\text{s.t.} & 0 \leq y_S + y_C \leq z, \tag{17}
\end{align*}$$

where (15)-(16) correspond to the seller’s participation constraints, and (17) to the buyer’s feasibility constraint. From (16), one can see that $l^L \leq y_C^L \kappa_b^L$ needs to hold in equilibrium, since otherwise an incentive for strategic default would exist, irrespective of the chosen offer. This corresponds to the Kiyotaki and Moore (1997) borrowing constraint in non-monetary economies.

Buyer’s Problem - High-Type: Among all incentive compatible offers, in equilibrium, the least-costly offer a high-type buyer can place is the one maximizing his surplus, while fulfilling the seller’s participation constraints and the Cho and Kreps (1987) Intuitive Criterion. For a reasonable system of beliefs, any other offer would give the high-type buyer an incentive to deviate, as shown in Lemma A. Thus, the high-type buyer’s problem, maximizing his surplus, $S_b^H(z; \kappa_b^H)$, is:

$$\begin{align*}
(q^H, y_S^H, y_C^H, l^H, d) & \in \arg \max_{q, y_S, y_C, l, d} \, u(q) - \beta \left[ y_S \kappa_b^H + (1 - d) l + d y_C \kappa_b^H \right] \tag{18} \\
\text{s.t.} & \quad u(q) - \beta \left[ y_S \kappa_b^H + (1 - d) l + d y_C \kappa_b^H \right] \leq S_b^L(z; \kappa_b^L) \tag{19} \\
\text{s.t.} & \quad - q_S + \beta y_S \kappa_b^H \geq 0 \tag{20} \\
\text{s.t.} & \quad - q_C + \beta \left[ (1 - d) l + d y_C \kappa_b^H \right] \geq 0 \tag{21} \\
\text{s.t.} & \quad 0 \leq y_S + y_C \leq z, \tag{22}
\end{align*}$$

where (19) is an incentive compatibility constraint ensuring that the low-type buyer has no incentive to mimic the high-type’s offer, (20)-(21) are the seller’s participation constraints, and (22) is a feasibility constraint, ruling out short-sales. Again, following Kiyotaki and Moore (1997), to rule out strategic default, $l^H \leq y_C^H \kappa_b^H$, needs to hold in equilibrium.

In order for the high-type’s offer, $(q^H, y_S^H, y_C^H, l^H)$, to fulfill the acceptance rule of the seller, a belief system, $\lambda(q, y_S, y_C, l)$, has to be generated that is consistent with the offer in (18) and satisfies the Intuitive Criterion. Beliefs are determined from Bayes’ rule and take the following form:

(i) $\lambda(\tilde{q}, \tilde{y}_S, \tilde{y}_C, \tilde{l}) = 0 \quad \forall \ (\tilde{q}, \tilde{y}_S, \tilde{y}_C, \tilde{l}) \notin \mathcal{O}$, and $u(\tilde{q}) - \beta \left[ \tilde{y}_S \kappa_b^L + (1 - d) \tilde{l} + d \tilde{y}_C \kappa_b^L \right] > u(q^L) -$
\[
\beta \left[ y_S^L \kappa_b^L + (1 - d) l^L + dy_C^L \kappa_b^L \right]
\]

(ii) \( \lambda(q, y_S, y_C, l) = 1 \quad \forall \quad (q, y_S, y_C, l) \notin \mathcal{O}, \) and \( u(q) - \beta \left[ y_S^L \kappa_b^L + (1 - d) l^L + dy_C^L \kappa_b^L \right] \leq u(q^L) - \beta \left[ y_S^L \kappa_b^L + (1 - d) l^L + dy_C^L \kappa_b^L \right], \)

where \( \mathcal{O} \) is the set of equilibrium offers. Hence, any out-of-equilibrium offer, \((q, y_S, y_C, l) \notin \mathcal{O},\) that increases the payoff of the low-type buyer compared to the complete-information offer in (14), is attributed to the low-type buyer, while any other out-of-equilibrium offer is attributed to the high-type. This results in the following acceptance rule:

\[
\mathcal{A}_s(\lambda) = \left\{ (q, y_S, y_C, l) \in \mathcal{F} : u(q^H) - \beta \left[ y_S^H \kappa_b^H + (1 - d) l^H + dy_C^H \kappa_b^H \right] \leq S_b^L(z; \kappa_b^L), \right. \\
\left. \quad \text{and} \quad -q^H + \beta \left[ y_S^H \kappa_b^H + (1 - d) l^H + dy_C^H \kappa_b^H \right] \geq 0 \right\}. \tag{23}
\]

**Definition A.** A separating equilibrium of this bargaining game is a pair of strategies and a belief system, \( \{[q(z; \kappa_b), y_S(z; \kappa_b), y_C(z; \kappa_b), l(z; \kappa_b), d(z; \kappa_b)], A_s(\lambda), \lambda, \} \), such that:

(i) The terms of trade, \([q(z; \kappa_b), y_S(z; \kappa_b), y_C(z; \kappa_b), l(z; \kappa_b), d(z; \kappa_b)], \) are a solution to the buyer’s bargaining problem, (14) and (18), with \( \kappa \in \{\kappa^L, \kappa^H\} \) respectively.

(ii) The seller’s acceptance rule, \( A_s(\lambda), \) is given by (23).

(iii) The belief system, \( \lambda(q, y_S, y_C, l), \) satisfies Bayes’ rule and the Intuitive Criterion (11)-(13).

**Lemma B.** In a separating equilibrium of this bargaining game, all but the least-inefficient separating equilibrium can be dismissed. The corresponding decentralized market allocations, \((q^L, y_S^L, y_C^L, l^L, d)\) and \((q^H, y_S^H, y_C^H, l^H, d),\) solving (14) and (18), are:

\[
q_S^L = q_C^L = \min\{q^*, \beta z \kappa_b^L\} \tag{24}
\]
\[
y_S^L = y_C^L = \min\{q_C^L / \beta \kappa_b^L - y_C^L, z - y_C^L\} \tag{25}
\]
\[
q_S^H = q_C^H = \frac{\kappa_b^H}{\kappa_b^L} \left[ u(q_S^H) - S_b^L(z; \kappa_b^L) \right] \tag{26}
\]
\[
y_S^H = y_C^H = q_C^H / \beta \kappa_b^H - y_C^H. \tag{27}
\]

with \( i \in \{S, C\}, \) \( l^L = y_C^L \kappa_b^L, \) \( l^H = y_C^H \kappa_b^H, \) \( d \in \{0, 1\}, \) and \( q^* \) solves \( u'(q^*) = 1. \) Proof in Appendix A.

Considering the low-type buyer first, (24) shows that trade is efficient if the discounted real balances are large enough to compensate the seller for his disutility of production, i.e., if \( \beta z \kappa_b^L \geq q^* \). If the low-type buyer is constrained in his asset holdings, however, then \( q^L < q^* \). Turning to the equilibrium allocations of the high-type buyer, (26) shows that \( q^H \) is uniquely determined and has to fulfill: \( q^H \in [0, q^L) \), where the offer, \((q^H, y_S^H, y_C^H, l^H),\) corresponds to
the lowest possible $q^H$ that solves equation (26).\footnote{This can be verified as follows: If $q^H = 0$, the left-hand side of equation (26) is lower than the right-hand side, and if $q^H = q^L$, the opposite holds. In addition to this, for all $q^H \leq q^*$, the left-hand side is increasing in $q^H$.} Using the fact that $q^H < q^L$, which implies $y^H = \beta y^H k^H_0 < \beta y^L k^L_0 = q^L$, it immediately follows that $y^H < y^L \leq z$. Hence, low-type buyers cannot do better than to reveal their type by placing a complete-information offer, whereas high-type buyers, using the low-type’s offer as a benchmark, separate themselves to secure better terms of trade. This separation takes place through asset retention, whereas a high-type buyer signalizes his high-future returns, $\kappa^H$, through his willingness to carry parts of the asset into the subsequent centralized market himself. The consequent better terms of trade, however, come at the cost of lower consumption in the decentralized market. The amount of assets retained is thereby exactly at the threshold, such that the low-type buyer has no incentive to mimic the high-type’s offer anymore, which is in line with the results in Demarzo and Duffie (1999) and Guerrieri and Shimer (2014).\footnote{If we had complete information regarding the asset quality, the high-type buyer would consume a weakly higher quantity than the low-type buyer, $q^H \geq q^L$, and at the same time spend a lower fraction of his assets, $y^H \leq y^L$.}

Hence, low-type buyers cannot do better than to reveal their type by placing a complete-information offer, whereas high-type buyers, using the low-type’s offer as a benchmark, separate themselves to secure better terms of trade. This separation takes place through asset retention, whereas a high-type buyer signalizes his high-future returns, $\kappa^H$, through his willingness to carry parts of the asset into the subsequent centralized market himself. The consequent better terms of trade, however, come at the cost of lower consumption in the decentralized market. The amount of assets retained is thereby exactly at the threshold, such that the low-type buyer has no incentive to mimic the high-type’s offer anymore, which is in line with the results in Demarzo and Duffie (1999) and Guerrieri and Shimer (2014).

This result corresponds to Gresham’s Law, which states that the existence of lemons in the market crowds out good assets.

**Proposition A.** In a separating equilibrium, a buyer is indifferent between asset sale, $S$, collateralized credit, $C$, and strategic default, $D$. Proof in Appendix B.

The decentralized market allocations provided in Lemma B show that in a separating equilibrium, low- and high-type buyers are indifferent between immediate settlement and collateralized credit, supporting the equivalence result established by Rocheteau (2011). Additional to that, buyers are indifferent between repayment and default, due to the seller’s acceptance rule, $A_s(\lambda)$, given by equation (23).

**Lemma C.** In a separating equilibrium of this bargaining game, separation takes place through asset retention. For $z > 0$ and $\beta z \kappa^L < q^*$:

$$\frac{\partial y^H}{\partial z} = \frac{u'(q^L) - 1}{\frac{\kappa^H}{\kappa^L} u'(q^H) - 1} \in (0, 1). \tag{28}$$

For $\beta z \kappa^L \geq q^*$, however, $\frac{\partial y^H}{\partial z} = 0$. Proof in Appendix C.

Lemma C characterizes the relationship between the amount of assets used by a high-type buyer in the decentralized market, $y^H$, and the total amount of assets carried along the period, $z$. This identifies the degree of asset retention and thus, the high-type’s marginal willingness to use another unit of his asset in equilibrium. Two cases need to be distinguished. If the low-type’s liquidity needs are satiated, hence if $\beta z \kappa^L_0 \geq q^*$, an additional unit of the asset does not relax the high-type’s incentive compatibility constraint and thus, his willingness to use another unit of the asset is zero, i.e., $\partial y^H / \partial z = 0$. If $\beta z \kappa^L_0 < q^*$, however, an additional unit of the asset raises the low-type’s surplus. As a result, the high-type’s marginal willingness to use another unit of his assets is positive. However, since he can only use a fraction of each additional asset without violating the incentive compatibility
constraint, the marginal willingness to sell is less than one, i.e., \( \partial y^H / \partial z \in (0, 1) \), as shown in equation (28).

**Lemma D.** For \( z > 0 \), it holds that \( \frac{\partial (y^H / z)}{\partial \kappa^H} < 0 \) and \( \frac{\partial (y^H / z)}{\partial \kappa^L} > 0 \). If \( \kappa^L \rightarrow 0 \), then \( \lim_{\kappa^L \rightarrow 0} \frac{y^H}{z} = 0 \). Proof in Appendix D.

Given Lemma D, the relative amount of assets a high-type buyer chooses to retain, \( y^H / z \), depends on the degree of asymmetric information in the economy. The higher the distance between \( \kappa^L \) and \( \kappa^H \), the more binding is the incentive compatibility constraint, and thus the lower the relative amount of assets, \( y^H / z \), used. As soon as the lemon’s value approaches zero, i.e., \( \kappa^L \rightarrow 0 \), separation breaks down. This limiting case corresponds to the result established in Nosal and Wallace (2007) in a counterfeiting equilibrium.

### 3.2 Pooling Equilibrium

Having determined the equilibrium allocations in the least-costly separating equilibrium, I now consider the Hellwig (1987) most-efficient pooling equilibrium, maximizing the surplus of the high-type buyer.\(^{12}\) The problem of a high-type buyer placing a pooling offer, \( (\bar{q}, \bar{y}_S, \bar{y}_C, \bar{I}) \), is:

\[
(q, y_S, y_C, I, d) \in \arg \max_{q, y_S, y_C, I, d \in \{0, 1\}} u(q) - \beta \left[ y_S \kappa^H_b + (1 - d) l + d y_C \kappa^H_b \right]
\]

\( \text{s.t.} - q_S + \beta y_S \left[ \pi \kappa^H_b + (1 - \pi) \kappa^L_b \right] \geq 0 \) (30)

\( \text{s.t.} - q_C + \beta \left[ (1 - d) l + d y_C \kappa^L_b \right] \geq 0 \) (31)

\( \bar{S}_b(z; \kappa^L_b) - S^L_b(z; \kappa^L_b) \geq 0 \) (32)

\( 0 \leq y_S + y_C \leq z \), (33)

where (30)-(31) are the seller’s participation constraints, (32) the low-type buyer’s individual rationality constraint, and (33) a feasibility constraint. Note that in order to prevent strategic default, the Kiyotaki and Moore (1997) collateral constraint, implicitly provided in (31), needs to hold explicitly for the low-type buyer, i.e., \( \bar{I} \leq \bar{y}_C \kappa^L_b \). According to (32), in order for the proposed offer to be part of the Hellwig (1987) most-efficient pooling equilibrium, the low-type buyer’s surplus needs to be at least as high as his surplus under full-information, as determined in (14).

**Definition B.** A pooling equilibrium of this bargaining game is a pair of strategies and a belief system, \( \{\bar{q}(z; \kappa_b), y_S(z; \kappa_b), y_C(z; \kappa_b), I(z; \kappa_b), d(z; \kappa_b)\}, A_s(\lambda), \lambda \} \), such that:

(i) The terms of trade, \( [\bar{q}(z; \kappa_b), \bar{y}_S(z; \kappa_b), \bar{y}_C(z; \kappa_b), \bar{I}(z; \kappa_b), d(z; \kappa_b)] \), are a solution to the high-type buyer’s maximization problem, (29).

(ii) The seller’s acceptance rule, \( A_s(\lambda) \), is given by (9).

\(^{12}\) I restrict attention to this kind of pooling equilibrium and abstain from all other possible equilibria, as defined in Mailath et al. (1993).
The belief system, \( \lambda(q, y_S, y_C, l) \), satisfies Bayes’ rule.

**Lemma E.** In a pooling equilibrium of this bargaining game, all but the most-efficient pooling equilibrium can be dismissed. The corresponding decentralized market allocations, \((\bar{q}, \bar{y}_S, \bar{y}_C, \bar{l}, d)\), solving (29), are:

\[
\bar{q}_S = \min\{\bar{q}_S, \beta z R\} \quad (34)
\]

\[
\bar{q}_C = \min\{q^*, \beta z \kappa_b^L\} \quad (35)
\]

\[
\bar{y}_S = \min\{\bar{q}_S/\beta R, z - \bar{y}_C\} \quad (36)
\]

\[
\bar{y}_C = \min\{\bar{q}_C/\beta \kappa_b^L, z - \bar{y}_S\} \quad (37)
\]

\[
\bar{l} = \bar{y}_C \kappa_b^L \quad (38)
\]

where \( d \in \{0, 1\} \) and \( q^* \) solves \( u'(q^*) = 1 \). Proof in Appendix E.

Let us first consider the equilibrium allocations (34) and (36) corresponding to a pure sale offer, \((\bar{q}_S, \bar{y}_S)\). Since the seller is unaware of the buyer’s asset quality, \( \kappa_b \), he is only willing to accept such terms of trade if his production costs are covered in expectation, i.e., if \( \bar{q}_S \leq \beta \bar{y}_S R \). Since the high-type buyer knows that his future return is higher than the price he can get under these terms of trade, he only consumes a low quantity, \( \bar{q}_S < q^* \), and carries the remaining assets into the centralized market to consume their return, \((z - \bar{y}_S)\kappa_b^H\). Hence, using the asset as a direct medium of exchange in a pooling equilibrium does not allow a high-type buyer to consume the efficient quantity, \( q^* \), in the decentralized market.

Consider now the case where a high-type buyer chooses to offer a collateralized credit, \((\bar{q}_C, \bar{y}_C, \bar{l}, d)\), resulting in the equilibrium allocations (35), (37), and (38). Due to rational expectations, a seller knows that every low-type buyer will default on his credit obligation if the deposited collateral is evaluated at the expected value, i.e., \( d = 1 \) for \( \kappa_b = \kappa_b^F \) if \( \bar{l} = \beta \bar{y}_C R \). Given that reasoning, a seller is only willing to engage in a collateralized credit, if the high-type buyer is accepting a haircut that eliminates the credit risk associated to the low-type buyer. As a consequence, to fully insure the seller against strategic default, the size of the applied haircut corresponds to the discounted difference between the market price, \( R \), and the return of a low-quality asset, \( \kappa^L \), as determined in Lemma F.

**Lemma F.** In a pooling collateralized credit, the seller imposes a haircut of the size:

\[
\mathcal{H} = \beta (R - \kappa^L), \quad (39)
\]

on each asset used as collateral. Proof in Appendix F.

Given that rationale, a collateralized credit allows for socially-efficient consumption, \( \bar{q}_C = q^* \), for all \( \pi \in [0, 1] \), if the high-type buyer possesses enough assets to purchase the desired quantity, i.e., if the feasibility constraint, (33), is non-binding. Proposition B elaborates on the correspondence between the amount of assets brought into the period, \( z \), and the resulting
allocations in a pooling equilibrium.

Proposition B. In a pooling equilibrium, for $\kappa^L > 0$, there exists a threshold value, $\bar{z} \in (0, \infty)$, such that:

(i) For $z > \bar{z}$, a high-type buyer strictly prefers a collateralized credit over an asset sale;
(ii) For $z < \bar{z}$, asset sale dominates collateralized credit; and
(iii) For $z = \bar{z}$, the high-type buyer is indifferent between collateralized credit and asset sale.

If $\kappa^L = 0$, a pooling sale is the only sustainable offer. Proof in Appendix G.

The results of the bargaining game show that the equilibrium allocations depend on the amount of assets carried along the period, $z$. If assets are relatively plentiful, i.e., if $z > \bar{z}$, the high-type buyer prefers a collateralized credit subject to a haircut, $H = \beta(R - \kappa^L)$, over selling his asset below fundamental value, since it allows him to consume $\bar{q}_C$ without sacrificing the high return, $\kappa^H$. As a consequence, in equilibrium, there is over-collateralization, eliminating the incentive for strategic default of the low-type buyer. Since the high-type buyer is aware of his assets’ high return, he fulfills his credit obligation, $l$, while the low-type buyer is indifferent between repayment and default, since the pooling offer corresponds to his complete-information offer determined in (14). Hence, the only beneficial deviation for a low-type buyer would be the equilibrium allocations (34) and (36) corresponding to a pure pooling asset sale, since given the pooling price, $\beta R$, they allow low-types to exploit the information friction and benefit at the expense of the high-types. However, given Proposition B, the seller is aware of the fact that a high-type buyer will never offer a pooling sale contract for $z > \bar{z}$, and therefore attributes such an offer to a low-type buyer, revealing his true asset return, $\kappa^L$. Hence, despite the information friction, for $z > \bar{z}$, high-type buyers can consume the desired quantity in the decentralized market without sacrificing the high returns of the pledged asset by engaging in a collateralized credit.

Consider now the case in which asset holdings, $z$, are particularly low. If $z < \bar{z}$, a fire-sale equilibrium installs itself, and high-type buyers prefer selling their assets at a pooling price, rather than engaging in a secured credit contract. Thus, being aware of required over-collateralization induced by the haircut defined in (39), a high-type buyer is better off selling all his assets at the pooling price, $\beta R$, rather than purchasing the relatively low quantity corresponding to the low-type’s complete information offer, $q_C = q^L$. A similar logic applies in the limiting case in which the return of the low-quality asset is equal to zero, i.e., $\kappa^L = 0$. Given the fact that sellers use the return of the low-quality asset as a benchmark to determine the terms of trade for a collateralized credit, such a contract is not sustainable anymore. As a result, asset sale, i.e., $(\bar{q}_S, \bar{y}_S)$, remains as the only feasible pooling offer.

4 General Equilibrium

Before proceeding to the selection between a pooling and a separating equilibrium, we first determine the agents’ optimal asset accumulation in the centralized market. By doing so, we
incorporate the solutions of the bargaining game in Section 3 into the general equilibrium structure of the model. The timing of events implies that the agents’ asset choice does not impart any private information about the assets’ future return, since agents choose their asset holdings before being privately informed about the quality of the asset and their role in the goods market. Using the linearity of the centralized market value function, (3), the expected lifetime utility of an agent entering the decentralized market is:

\[
\mathbb{E}V(z; \kappa) = \pi \left[ (1 - n)V_b(z; \kappa^H_b) + nV_s(z; \kappa^L_s) \right] + (1 - \pi) \left[ (1 - n)V_b(z; \kappa^L_b) + nV_s(z; \kappa^L_s) \right],
\]

with

\[
V_b(z; \kappa_b) = u[q(z; \kappa_b)] - \beta[y_S(z; \kappa_b)\kappa_b + (1 - d)l(z; \kappa_b) + dy_C(z; \kappa_b)\kappa_b] + \beta z \kappa_b
\]

\[
V_s(z; \kappa_s) = -q(z; \kappa_b) + \beta[y_S(z; \kappa_b)\kappa_b + (1 - d)l(z; \kappa_b) + dy_C(z; \kappa_b)\kappa_b] + \beta z \kappa_s
\]

where \( q \in \{q^L, q^H, \bar{q} \} \), \( y_S \in \{y^L_S, y^H_S, \bar{y}_S \} \), \( y_C \in \{y^L_C, y^H_C, \bar{y}_C \} \) and \( l \in \{l^L, l^H, \bar{l} \} \) solve (14), (18), and (29), all as a function of the buyer’s type, \( \kappa_b \), and his asset holdings, \( z \).\(^{13}\) Substituting (40) into the centralized market value function, (3), the agent chooses his real asset holdings, \( z \), to maximize his expected surplus in the goods market, net of the cost of holding real assets, \( 1 - \beta R \), according to:\(^{14}\)

\[
\max_z -\left( 1 - \beta R \right) z + (1 - n) \left[ \pi \left[ u(q) - \beta[y_Sk^H_b + (1 - d)l + dy_Ck^H_b] \right] \right] + (1 - \pi) \left[ u(q) - \beta[y_Sk^L_b + (1 - d)l + dy_Ck^L_b] \right].
\]

**Definition C.** An equilibrium is a list of asset holdings and terms of trade in the decentralized market, \{\( z(\cdot), [q(\cdot), y_S(\cdot), y_C(\cdot), l(\cdot), d(\cdot)] \}\}, such that:

(i) \( z(\cdot) \in \mathbb{R}_+ \) is a solution to (43) for all agents; and (ii) \( \{q(z; \kappa_b), y_S(z; \kappa_b), y_C(z; \kappa_b), l(z; \kappa_b), d(z; \kappa_b)\} \) is a solution to (14) if \( \kappa_b = \kappa^L_b \) and (18) if \( \kappa_b = \kappa^H_b \) in a separating equilibrium, and (29) in a pooling equilibrium.

By inserting the solutions to the bargaining games, (14) and (18), in a separating equilibrium,
and (29) in a pooling equilibrium, the following first order conditions emerge:

\[(1 - \beta R) \leq (1 - n) \left[ (1 - \pi) \beta k_b^L \left[ u'(q^L) - 1 \right] + \pi \beta k_b^H \left[ u'(q^H) - 1 \right] \right] \quad (44)\]

\[(1 - \beta R) \leq (1 - n) \beta \kappa_b^L \left[ u'(\bar{q}_S) - 1 \right] \quad (45)\]

\[(1 - \beta R) \leq (1 - n) \beta k_b^C \left[ u'(\bar{q}_C) - 1 \right], \quad (46)\]

where (44) corresponds to a separating equilibrium, (45) to a pooling-sale equilibrium, and (46) to a pooling-credit equilibrium. If \(\beta R < 1\), real assets are costly to hold and hence there is a unique solution to (43), satisfying (44), (45), and (46). If \(\beta R = 1\), real assets are costless to hold and \(z \in [0, \infty)\), and if \(\beta R > 1\), real assets yield a positive return and hence there is no solution to (43), since everyone would store infinite amounts of assets. Lemma G summarizes.

**Lemma G.** The exists a unique solution to (43), where an agent’s optimal choice of asset holdings in the centralized market, \(z\), is a solution to:

(i) (44) in a separating equilibrium;

(ii) (45) in a pooling-sale equilibrium; and

(iii) (46) in a pooling-credit equilibrium.

**Proof in Appendix H.**

Equation (44) denotes the marginal surplus of an additional unit of the asset in the decentralized market in a separating equilibrium. The first part on the right-hand side corresponds to the marginal surplus of a low-type buyer, and the second part to the one of a high-type buyer, which is defined as the liquidity value of an asset with \(\kappa = \kappa^H\) under full-information, multiplied by the high type buyer’s marginal willingness to spend under private information, \(\partial y^H / \partial z \in (0, 1)\), defined in (28). Since the high-type buyer needs to fulfill the incentive compatibility constraint, (19), to successfully separate himself from the low-type buyers by asset retention, he faces a binding resaleability constraint. Thus, only a fraction of each additional unit of an asset can be used as medium of exchange in the decentralized market. Equations (45) and (46) in turn correspond to the buyer’s marginal benefit in a pooling sale and a pooling collateralized credit, respectively, whereas the difference stems from the applied haircut defined in (39).

5 Discussion

Having determined the bargaining solutions to (14), (18), and (29) in the decentralized market, and (43) in the centralized market, we now revisit the original research question: Under the presence of information frictions, do individuals subject to a liquidity shock prefer engaging in a collateralized credit rather than selling assets? Definition D defines the eligible equilibrium terms of trade, \([q(z; \kappa_b), y_S(z; \kappa_b), y_C(z; \kappa_b), l(z; \kappa_b), d(z; \kappa_b)]\).
Definition D. Under the undefeated equilibrium, the full set of Perfect Bayesian Equilibria for the buyer’s problem, \( (10) \), consists of a set of offers, \((q, y_S, y_C, l)\), defined by:

(i) \( (14) \) and \( (18) \) in a separating equilibrium; and
(ii) \( (29) \) in a pooling equilibrium.

All out-of-equilibrium offers, \((\tilde{q}, \tilde{y}_S, \tilde{y}_C, \tilde{l})\), are associated to a low-type buyer holding \( \kappa_b = \kappa^L_b \).

The choice, whether to select a pooling or a separating offer, is solely triggered by the lifetime utility of the high-type buyer, \( V_b(z; \kappa^H_b) \). Depending on the degree of adverse selection prevailing in the economy, one defeats the other, thus eliminating Pareto inefficiencies and maximizing gains from trade.\(^{15}\) Lemma H summarizes this line of reasoning by characterizing the properties of the undefeated equilibrium by Mailath et al. (1993).

Lemma H. The buyer’s offer, \((q, y_S, y_C, l)\), and the resulting equilibrium in the bargaining game with private information is:

(i) \( (q^H, y^H_S, y^H_C, l^H) \) in \( (18) \), and thus \( (q^L, y^L_S, y^L_C, l^L) \) in \( (14) \), if \( V_b(z; \kappa^H_b) > \bar{V}_b(z; \kappa^H_b) \).
(ii) \( (\tilde{q}, \tilde{y}_S, \tilde{y}_C, \bar{l}) \) in \( (29) \) if \( \bar{V}_b(z; \kappa^H_b) > V_b(z; \kappa^H_b) \).

with \( V_b(z; \kappa^H_b) \) being the lifetime utility of a high-type buyer in a separating equilibrium, and \( \bar{V}_b(z; \kappa^H_b) \) in a pooling equilibrium. The seller’s corresponding beliefs, \( \lambda(q, y_S, y_C, l) \), supporting this equilibrium are such that:

(i) \( \lambda = 0 \) for all offers that make the high-type buyer strictly worse off, and are preferred to \( (\tilde{q}, \tilde{y}_S, \tilde{y}_C, \bar{l}) \) by the low-type buyer;
(ii) \( \lambda = 1 \) for all offers that make the low-type buyer strictly worse off than \((q^L, y^L_S, y^L_C, l^L)\); and
(iii) \( \lambda = \pi \) for all other offers. Proof in Appendix J.

Being aware of the mechanism selecting among separating and pooling offers, we now focus on the buyer’s choice between asset sale and collateralized credit. Consider first the case where \( z > \tilde{z} \), i.e., when assets are relatively cheap to carry along the period. From Proposition A we know that in a separating equilibrium, for any \( z \), high-type buyers are indifferent between asset sale and collateralized credit, since the respective terms of trade are equivalent for both settlement methods. This result is in line with the findings in Rocheteau (2011). In a pooling equilibrium, however, collateralized credit is strictly preferred, since it allows a high-type buyer to consume the desired quantity in the decentralized market, without sacrificing the high return of the possessed asset, as elaborated in Proposition B. Thus, for \( z > \tilde{z} \), a pooling collateralized credit subject to the haircut defined in \( (39) \) strictly dominates any other pooling or separating offer. Proposition C summarizes.

Proposition C. For \( z > \tilde{z} \), high-type buyers prefer the pooling collateralized credit defined in Lemma E over any other sale or collateralized credit agreement. High-type buyers honor

\(^{15}\) Figure 3 in Appendix I demonstrates this line of reasoning.
their credit obligation, while low-type buyers are indifferent between repayment and default. Proof in Appendix K.

Consider now an economy in which assets are relatively scarce, i.e., \( z < \tilde{z} \). From Proposition B we know that in a pooling equilibrium high-type buyers prefer to sell their asset at a pooling price rather than engaging in a collateralized credit subject to a haircut, \( \mathcal{H} \). Thus, the concomitant over-collateralization offsets the beneficial terms of trade when assets are relatively scarce. However, what remains to be identified is whether high-type buyers prefer to separate instead. Lemma I summarizes.

Lemma I. Considering asset sales only, for given asset values, \( 0 < \kappa_L < \kappa_H < \infty \), there exists a threshold value, \( \tilde{\pi} \in (0, 1) \), such that:

(i) For \( \tilde{\pi} < \pi \), the equilibrium is pooling;
(ii) For \( \tilde{\pi} > \pi \), the equilibrium is separating; and
(iii) For \( \tilde{\pi} = \pi \), the high-type buyer is indifferent.

Proof in Appendix L.

Given Lemma I, if the fraction of high-types is smaller than the threshold value, i.e., \( \pi < \tilde{\pi} \), high-type buyers are better off separating themselves from the low-types, since \( V_b(z; \kappa_H^b) > \check{V}_{b,S}(z; \kappa_H^b) \) for \( \pi < \tilde{\pi} \). However, if the fraction of low-type buyers is particularly small, offering a pooling contract is beneficial, since it allows the high-type to avoid the cost of separation, yielding \( \check{V}_{b,S}(z; \kappa_H^b) > V_b(z; \kappa_H^b) \) for \( \pi > \tilde{\pi} \). In sum, one can conclude that only if the economy is relatively sound, high-type buyers are willing to offer a pooling sale, which is in line with the properties of the undefeated equilibrium of Mailath et al. (1993), and the results established by Li and Rocheteau (2008) and Bajaj (2016). In the limiting case with \( \kappa_L = 0 \), no separating equilibrium is sustainable anymore, and a pooling sale remains as the only feasible offer. Proposition D summarizes.

Proposition D. For \( z \leq \tilde{z} \), the equilibrium allocations depend on the distribution of types in the economy, \( \pi \). For \( \pi < \tilde{\pi} \), the equilibrium is separating, while for \( \pi > \tilde{\pi} \), the equilibrium is pooling. Proof in Appendix M.

6 Conclusion

The aim of this paper is to identify an optimal contract design for institutions bilaterally exchanging liquidity on an asset market subject to asymmetric information. Using a signaling game, which endogenously selects between a separating and a pooling equilibrium, the results show that in the presence of hidden information regarding the assets’ future return, there exists a non-equivalence between collateralized credit and immediate settlement. If assets

\footnote{Since the terms of trade for asset sale and collateralized credit are equivalent in a separating equilibrium, a comparison of pooling-sale and separating-sale in Lemma I is sufficient.}
are relatively plentiful, agents prefer a pooling collateralized credit subject to a haircut over any other sale or collateralized credit agreement. As a result, there is overcollateralization in equilibrium. If assets are scarce, however, the concomitant overcollateralization associated to a collateralized credit offsets the beneficial terms of trade. The choice between a separating offer and a pooling sale then depends on the distribution of types in the economy. If the relative amount of lemons in the economy is low, high-value asset holders prefer to sell their assets at a pooling price, whereas if the amount of lemons is high, a separating equilibrium installs itself. This result overturns the so far assumed equivalence of immediate settlement and collateralized credit and provides a compelling reason for the increased use of collateralized credit on the over-the-counter markets since the eruption of the global financial crisis in 2007.
References


Appendix

A. Proof of Lemma B

As shown in Lemma A in Section 3.1, the Cho and Kreps (1987) Intuitive Criterion rules out all sorts of pooling equilibria with $y > 0$. In order to prove this, I proceed with a proof by contradiction.

Consider first the left panel of Figure 2, displaying the surplus of a low-type buyer, a high-type buyer, and a seller. Assume that both agents, high- and low-types, make the same pooling offer, $(\bar{q}, \bar{y}) \neq (0, 0)$, leading to the respective surpluses:

$$S^H_L(z; \kappa_b^H) = u(\bar{q}) - \beta \bar{y} \kappa^H$$
$$S^L(z; \kappa_b^L) = u(\bar{q}) - \beta \bar{y} \kappa^L,$$

and to the offer being accepted by the seller, since he believes he is facing a high-type buyer, given his participation constraint:

$$S^H_s = \{(\bar{q}, \bar{y}) : -\bar{q} + \beta \bar{y} \kappa^H \geq 0\}.$$  

One can immediately see that $S^L_b$ is steeper than $S^H_b$, which is due to the Spence-Mirrlees single crossing property and key for obtaining a separating equilibrium. The proposed equilibrium offer, $(\bar{q}, \bar{y})$, is located above $S^H_s$, since, by Bayes’ rule, it is only accepted if $\lambda < 1$, and the seller assumes that there are some low-types in the economy. The shaded-area indicates the set of offers that increase the utility of the high-type, decrease the utility of the low-type and are acceptable to the seller, i.e., fulfill $A_s(\lambda)$ assuming that $\lambda = 1$. Thus, there exists an offer involving lower consumption, $q$, and a lower transfer, $y$, that would make the high-type better off. Being aware that such an offer would only be proposed by a high-type, since it makes the low-type worse off, the seller accepts such a separating offer, given he was willing to accept the initial pooling offer, $(\bar{q}, \bar{y})$, in the first place.

Given that reasoning, under the Cho and Kreps (1987) Intuitive Criterion, pooling offers
are not compatible with equilibrium, and therefore, if an equilibrium exists, it has to be characterized by separating offers, i.e., different offers for the high- and the low-type buyers. The respective offers are described in (14) and (18), and illustrated in the right-hand panel of Figure 2.

The offer of a low-type is at the tangency point of the seller’s participation constraint, \( S^L_s \equiv -q + \beta y \kappa^L \geq 0 \), and the surplus of the buyer holding a low-value asset, \( S^L_b \). Thus, the low-type makes a complete information offer.

The high-type buyer, however, has to make an offer to the left of (and including) \( S^L_b \) and above (and including) \( S^H_s \) in order to satisfy the seller’s participation constraint and the incentive compatibility constraint. The corresponding utility-maximizing offer is at the intersection of these two curves, implying \( q^H < q^L \leq q^* \). Given that, high-type buyers always consume less than the low-types and retain a fraction of their high-quality assets, i.e., \( y^H < y^L \leq z \).

In order to solve for the equilibrium allocations of the low-type buyer, in a first step, set the seller’s participation constraints equal to zero, since we face take-it-or-leave-it offers by the buyers. This determines the corresponding amount of assets used, \( y^L_s \) and \( y^L_c \). Then, plug the participation constraints, (15) and (16), holding with strict equality, into the low-type’s maximization problem, (14), and maximize with respect to \( q^L_s \) and \( q^L_c \).

To determine the equilibrium allocations of the high-type buyer, consider the incentive compatibility constraint, given by equation (19). In order to maximize the high-type buyer’s surplus, this constraint needs to hold with strict equality. Plugging in the seller’s participation constraints, (20) and (21), the equilibrium allocations, \( q^H_s, q^H_c, y^H_s \), and \( y^H_c \) emerge.

\section*{B. Proof of Proposition A}

The equivalence result provided in Proposition A emerges directly from the solutions of the bargaining problem. I proceed in two steps. First, in order to rule out the incentive for strategic default, given Kiyotaki and Moore (1997), the collateral constraints for a low- and a high-type buyer have to take the following form:

\begin{align}
I^L & \leq y^L_c \kappa^L_b \\
I^H & \leq y^H_c \kappa^H_b .
\end{align}  

(50)  
(51)

Thus, the borrower needs to be indifferent between repayment and default. Plugging into the participation constraints, (16) and (21), respectively, and rearranging, the following constraints emerge:

\begin{align}
-q^L_c + \beta y^L_c \kappa^L_b & \geq 0 \\
-q^H_c + \beta y^H_c \kappa^H_b & \geq 0 .
\end{align}  

(52)  
(53)

Comparing these constraints to the participation constraints corresponding to a sale of assets, (15) and (20), equivalence of collateralized credit and asset sale is guaranteed. Combining these two steps, in a separating equilibrium, a buyer is indifferent between asset sale, collat-
eralized credit, and strategic default. ■

C. Proof of Lemma C

In order to prove the asset retention result in Lemma C, consider the equilibrium consumption of a high-type buyer in a separating equilibrium, \( q^H \), given by equation (26):

\[
q^H_S = q^H_C = \frac{\kappa^H_b}{\kappa^L_b} \left[ u(q^H) - S^L_b(z; \kappa^L_b) \right].
\] (54)

Using the fact that \( S^L_b = u(q^L) - q^L \), plugging in \( q^L = \beta y^L \kappa^L_b \) and \( q^H = \beta y^H \kappa^H_b \), and taking the derivative of \( y^H \) with respect to \( z \), yields:

\[
\frac{\partial y^H}{\partial z} = \frac{u'(q^L) - 1}{\kappa^H_b u'(q^H) - 1} \in (0, 1),
\] (55)

for \( \beta z \kappa^L_b < q^* \), since \( u'(q^H) > u'(q^L) \) and \( \kappa^H_b > \kappa^L_b \). For \( \beta z \kappa^L_b \geq q^* \), however, \( \frac{\partial y^H}{\partial z} = 0 \), since \( u'(q^L) = 1 \) for \( q^L = q^* \).

D. Proof of Lemma D

The proof of Lemma D proceeds in a similar fashion than the proof of Lemma C. Consider first the equilibrium consumption of a high-type buyer in a separating equilibrium, \( q^H \), given in equation (26):

\[
q^H_S = q^H_C = \frac{\kappa^H_b}{\kappa^L_b} \left[ u(q^H) - S^L_b(z; \kappa^L_b) \right].
\] (56)

In order to explicitly determine \( y^H \), and thus \( q^H \), a closed form solution is indispensable. Assuming \( u(q) = 2\sqrt{q} \) and solving with respect to \( y^H/z \) gives:

\[
\frac{y^H}{z} = \left( \frac{\kappa^H_b}{\kappa^L_b} \right)^2 \left[ 1 - \sqrt{1 - \frac{\kappa^L_b}{\kappa^H_b} \left( 2\sqrt{q^L - q^L} \right)} \right]^2.\] (57)

Taking the derivative with respect to \( \kappa^L_b \) and \( \kappa^H_b \) yields \( \frac{\partial (y^H/z)}{\partial \kappa^H_b} < 0 \) and \( \frac{\partial (y^H/z)}{\partial \kappa^L_b} > 0 \). Additional to that, it is straightforward to see that \( \frac{y^H}{z} = 0 \) if \( \kappa^L_b = 0 \).

E. Proof of Lemma E

To determine the equilibrium allocations in a pooling equilibrium, \((\bar{q}, \bar{y}_S, \bar{y}_C, \bar{l}, \bar{d})\), plug in the sellers participation constraint, (30) and (31), holding with strict equality, into the high-type buyer’s maximization problem, (29), incorporating the Kiyotaki and Moore (1997) collateral constraint, \( \bar{l} \leq \bar{y}_C \kappa^L_b \), and take the derivative with respect to \( \bar{q}_S \) and \( \bar{q}_C \). The following first
order conditions emerge:

\[
u'(\bar{q}_S) = \kappa_H^H / R \tag{58}
\]

\[
u'(\bar{q}_C) = 1, \tag{59}
\]

resulting in the equilibrium allocations provided in Lemma E. ■

F. Proof of Lemma F

Proof by contradiction. Assume the seller’s participation constraint corresponding to a pooling collateralized credit was:

\[-\bar{q}_C + \beta \left[ (1 - d)\bar{l} + d\bar{y}_C R \right] \geq 0, \tag{60}\]

implying a collateral constraint corresponding to:

\[\bar{l} \leq \bar{y}_C R. \tag{61}\]

Every low-type buyer engaging in this collateralized credit contract would choose to default, since the future credit obligation is higher than the actual value of the pledged collateral, i.e.,:

\[\beta \bar{y}_C R > \beta \bar{y}_C \kappa_L^L. \tag{62}\]

Being aware of that circumstance, in order to eliminate that incentive, the seller adjusts his participation constraint such that it explicitly holds for the low-type buyer, and thus takes the following form:

\[-\bar{q}_C + \beta \left[ (1 - d)\bar{l} + d\bar{y}_C \kappa_L^L \right] \geq 0, \tag{63}\]

which is identical to the complete-information offer proposed by the low-type buyer in a separating equilibrium, and thus the worst case scenario. Given that reasoning, the haircut \(H\), defined as the difference between the market value of the asset, \(\beta R\), and the size of the granted loan, \(\beta \kappa_L^L\), is:

\[H = \beta (R - \kappa_L^L). \tag{64}\]

G. Proof of Proposition B

From the high-type buyer’s maximization problem in a pooling equilibrium, (29), we know that for a sufficiently high amount of assets, \(z\), collateralized credit dominates asset sale, since:

\[
\bar{V}_{b,C}(z; \kappa_H^H) > \bar{V}_{b,S}(z; \kappa_H^H). \tag{65}
\]

For \(z \to 0\), however, the opposite holds, due to the concavity of \(u(q)\). Hence, there exists a threshold value, \(\tilde{z} \in (0, \infty)\), such that the high-type buyer’s payoff in a pooling sale and a pooling credit contract are equivalent, and thus:

\[
\bar{V}_{b,C}(z; \kappa_H^H) = \bar{V}_{b,S}(z; \kappa_H^H). \tag{66}
\]

Given that rationale, the following two conditions emerge: For \(z > \tilde{z}\), collateralized credit
dominates asset sale, i.e., $V_{b,C}(z; \kappa_b^H) > V_{b,S}(z; \kappa_b^H)$; and for $z < \bar{z}$, asset sale dominates collateralized credit, i.e., $V_{b,C}(z; \kappa_b^H) < V_{b,S}(z; \kappa_b^H)$. ■

H. Proof of Lemma G

To determine the frist-order conditions in Lemma G, use (43), plug-in the equilibrium allocations in a separating equilibrium (Lemma B) and in a pooling equilibrium (Lemma E), and take the derivative with respect to $z$. Equations (44), (45), and (46) emerge. ■

I. Undefeated equilibrium

Considering Figure 3, the graph on the left-hand side shows an undominated separating equilibrium, and the graph on the right-hand side a dominated one (i.e., a pooling equilibrium), whereas the lines $S^H_b$ and $S^L_b$ reflect indifference curves fulfilling the Spence-Mirrlees single-crossing property. Utility is maximized towards the upper left corner. The decisive factor in determining which of the two offers to choose is the high-type’s surplus, and this again depends on the distribution of types in the economy, and thus the expected return, $R = \pi \kappa^H + (1 - \pi) \kappa^L$.

![Figure 3: Undominated and Dominated Separating Equilibria](image)

J. Proof of Lemma H

As defined in Definition A and B, an equilibrium of the bargaining game in the decentralized market is a 4-tuple $\{(q, y_S, y_C; \kappa_b^H), (q, y_S, y_C; \kappa_b^L), \mathcal{A}_s(\lambda), \lambda\}$, where $(q, y_S, y_C; \kappa_b^H)$ is the equilibrium offer of a high-type buyer, $(q, y_S, y_C; \kappa_b^L)$ the offer of a low-type buyer, $\mathcal{A}_s(\lambda)$ the seller’s acceptance rule, and $\lambda$ the system of beliefs. The offer can either be a separating or a pooling one. Assuming that we only consider equilibria in which the buyer’s offers are accepted, an alternative equilibrium, $\{(q', y'_S, y'_C, l'; \kappa_b^H), (q', y'_S, y'_C, l'; \kappa_b^L), \mathcal{A}_s(\lambda)\}$, defeats the original equilibrium, if the following holds:

(i) For $(q, y_S, y_C; \kappa_b^H) \neq (q, y_S, y_C; \kappa_b^L)$:

$$u(q^H) - \beta [y^H_S \kappa_b^H + (1 - d)l^H + dy^H_C \kappa_b^H] < u(q'^H) - \beta [y'^H_S \kappa_b^H + (1 - d)l'^H + dy'^H_C \kappa_b^H].$$
(ii) For \((q, y_S, y_C, l; \kappa^H_b) = (q, y_S, y_C, l; \kappa^L_b) = (\bar{q}, \bar{y}_S, \bar{y}_C, \bar{l})\):

\[
\begin{align*}
&u(\bar{q}) - \beta [\bar{y}_S \kappa^H_b + (1 - d)\bar{l} + d\bar{y}_C \kappa^H_b] < u(q) - \beta [y_S \kappa^H_b + (1 - d)\bar{l} + d\bar{y}_C \kappa^H_b] \\
&u(\bar{q}) - \beta [\bar{y}_S \kappa^H_b + (1 - d)\bar{l} + d\bar{y}_C \kappa^H_b] < u(q) - \beta [y_S \kappa^H_b + (1 - d)\bar{l} + d\bar{y}_C \kappa^L_b],
\end{align*}
\]

where (i) corresponds to a separating equilibrium, and (ii) to a pooling. The alternative equilibrium defeats the original equilibrium in two cases. First, as summarized in (i), if there exists a profitable deviation for the high-type in the separating equilibrium, or second, in a pooling equilibrium (ii), if both types have an incentive to deviate. If there is no such alternative equilibrium offer, the original equilibrium, determined in (14), (18) and (29), is undefeated.

K. Proof of Proposition C

The proof of Proposition C involves two steps. First, from Proposition B we know that high-type buyers prefer a pooling collateralized credit over a pooling asset sale for \(z > \bar{z}\), since then \(\bar{V}_{b,C}(z; \kappa^H_b) > \bar{V}_{b,S}(z; \kappa^H_b)\) for all \(\pi \in [0, 1]\). However, what remains to be shown in a second step is that for \(z > \bar{z}\), a high-type buyer also prefers a pooling collateralized credit to any separating offer, i.e., \(\bar{V}_{b,C}(z; \kappa^H_b) > V_b(z; \kappa^H_b)\). In order for this result to hold, the following term needs to hold with strict inequality:

\[
\bar{V}_{b,C}(z; \kappa^H_b) = u(\bar{q}_C) + \beta z \kappa^H - \beta \bar{y}_C \kappa^L > u(q^H) + \beta (z - y^H) \kappa^H = V_b(z; \kappa^H_b),
\]

(67)

where the left hand side corresponds to the surplus of a high-type buyer in a pooling collateralized credit, and the right hand side to the surplus in a separating equilibrium. Plugging in the solutions of the bargaining game and rearranging terms yields:

\[
u(q^L) - q^L > u(q^H) - q^H,
\]

(68)

which holds with strict inequality for all \(q^L > q^H\), due to the properties of the utility function, \(u(q)\), fulfilling the Inada (1963) conditions. Thus, for \(z > \bar{z}\), \(\bar{V}_{b,C}(z; \kappa^H_b) > V_b(z; \kappa^H_b)\).

L. Proof of Lemma I

The maximization problem of the high-type buyer, offering a separating contract, (18), has shown that his payoff is independent of the distribution of types in the economy, \(\pi\). In contrast, from (29), the payoff of the high-type buyer offering a pooling sale is strictly decreasing in the amount of low-types in the economy. Assuming the population only consists of high-types, i.e., \(\pi = 1\), then (18) corresponds to (29) with the difference that in the separating equilibrium, the incentive compatibility constraint is binding. As a result, if \(\pi = 1\), a pooling equilibrium dominates a separating equilibrium, i.e.,:

\[
\bar{V}_{b,S}(z; \kappa^H_b) > V_b(z; \kappa^H_b),
\]

(69)

since for \(\pi \rightarrow 1\), \(\bar{y}_S > q^H\) and \(\bar{y}_S > y^H\). Considering now the case where there are only low-types in the economy, i.e., \(\pi = 0\), due to the single-crossing property, the opposite holds:
\[ \bar{V}_{b,S}(z; \kappa_b^H) < V_b(z; \kappa_b^H). \]  

(70)

Hence, there exists a threshold value, \( \tilde{\pi} \in (0, 1) \), such that the payoff of the high-types in the separating and in the pooling equilibrium are equivalent, i.e., \( \bar{V}_{b,S}(z; \kappa_b^H) = V_b(z; \kappa_b^H) \) for \( \pi = \tilde{\pi} \). As a result, the following condition emerges: For all \( \tilde{\pi} < \pi \), it holds that \( \bar{V}_{b,S}(z; \kappa_b^H) > V_b(z; \kappa_b^H) \) and the equilibrium is pooling; and for all \( \tilde{\pi} > \pi \), \( \bar{V}_{b,S}(z; \kappa_b^H) < V_b(z; \kappa_b^H) \), and thus the equilibrium is separating. ■

M. Proof of Proposition D

From Proposition B, we know that there exists a threshold value, \( \tilde{z} \in (0, \infty) \), below which a pooling collateralized credit is dominated. What remains to be shown is which settlement option yields the highest lifetime utility for the high-type buyer under these circumstances. Using the insights from Lemma I, this choice depends on the overall distribution of types in the economy, \( \pi \). The following rule emerges: Given \( z < \tilde{z} \), for \( \pi > \tilde{\pi} \) the equilibrium is pooling, and for \( \pi < \tilde{\pi} \), the equilibrium is separating. ■