University of Zurich

Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 198

Monetary Policy with Asset-Backed Money

David Andolfatto, Aleksander Berentsen and Christopher Waller

June 2015

Monetary Policy with Asset-Backed Money*

David Andolfatto

Federal Reserve Bank of St. Louis and Simon Fraser University

Aleksander Berentsen

University of Basel and Federal Reserve Bank of St. Louis

Christopher Waller

Federal Reserve Bank of St. Louis and University of Notre Dame

May 2015

*The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St.Louis, the Federal Reserve System, or the Board of Governors. Andolfatto: Federal Reserve Bank of St.Louis, P.O. Box 442, St.Louis, MO 63166-0442 (email David.Andolfatto@stls.frb.org). Berentsen: Economics Department, University of Basel, Peter-Merian-Weg 6, Postfach, CH-4002 Basel, Switzerland (email aleksander.berentsen@unibas.ch). Waller: Federal Reserve Bank of St.Louis, P.O. Box 442, St.Louis, MO 63166-0442 (email cwaller@stls.frb.org).

Abstract

We study the use of asset-backed money in a neoclassical growth model with illiquid capital. A mechanism is delegated control of productive capital and issues claims against the revenue it earns. These claims constitute a form of asset-backed money. The mechanism determines (i) the number of claims outstanding, (ii) the dividends paid to claim holders, and (iii) the structure of redemption fees. We find that for capital-rich economies, the first-best allocation can be implemented and price stability is optimal. However, for sufficiently capital-poor economies, achieving the first-best allocation requires a strictly positive rate of inflation. In general, the minimum inflation necessary to implement the first-best allocation is above the Friedman rule and varies with capital wealth.

Keywords: Limited Commitment, Asset-Backed Money, Optimal Monetary Policy

JEL: D82, D83, E61, G32

1 Introduction

The end of Bretton Woods in 1971 ushered in the era of fiat currencies. This decoupling of currency from a commodity standard raised many issues among economists, such as price-level determinacy, the optimal rate of inflation, and most importantly, who should be in charge of the monetary system – the government or the private sector? Friedman (1969), Klein (1974, 1976), and Hayek (1976) argued strenuously that privately managed monetary arrangements were feasible and would lead to the best economic outcomes. Under this system, a commodity-backed private currency would pave the way to price stability – that is, zero inflation. The main point of contention was whether or not it is essential in such a system for a government to provide a monopoly currency. In short, the debate centered on whether a government fiat currency offers unique advantages.

The inflation of the 1970s rekindled this debate in the 1980s, as reflected in the work by Barro (1979), King (1983), Wallace (1983), Sargent and Wallace (1983), and Friedman and Schwartz (1986). Again, the discussion on private monetary systems focused on commodity money backed by gold or silver. However, Fama (1983) argued that asset-backed claims were sufficient and actually offered advantages over a specie-backed currency. According to this arrangement, the financial intermediary would not issue liabilities redeemable in specie, since the claims would be *equity* claims. The financial intermediary was simply a conduit for transferring the returns on the underlying assets to the claim holders. Nevertheless, Fama argues that due to information and computation costs, fiat currency would still be needed for "hand-to-hand" transactions. While the Great Moderation and the decline in worldwide inflation since the early 1980s caused the profession to lose interest in this topic, the recent financial crisis has led to renewed public debate on the necessity of having a government fiduciary currency, most notably from the "End the Fed" supporters in the United States.

Although the literature on privately managed monetary systems focuses on many dynamic issues such as price stability, surprisingly, none of this work has used choice theoretic, dynamic general equilibrium models.[1] Much of the analysis is static, purely intuitive, or focuses on historical episodes. Another problematic issue is that the underlying frictions giving rise to the need for currency were not well specified. This was an obvious problem recognized early as evidenced by Helpman's (1983, p. 30) discussion of Fama's paper:

> The argument for an uncontrolled banking system is made on efficiency grounds by means of the frictionless neoclassical model of resource al-

[1] Notable exceptions are Sargent and Wallace (1983), who study a commodity money economy in an overlapping-generations framework, and Berentsen (2006), who studies the private provision of fiat currency in a random matching model with divisible money.

> location. But this framework does not provide a basis for arguing the desirability of price level stabilization. If indeed stabilization of the price level is desirable, we need to know precisely what features of the economy lead to it. Then we have to examine whether such features make an uncontrolled banking system desirable. This problem is of major importance, but it is not addressed in the paper.

Modern monetary theory has made clear progress in addressing Helpman's critique of Fama's work by specifying the frictions needed to make a medium of exchange essential for trade.[2] Furthermore, due to advances in macroeconomic and monetary theory, we are now able to use choice theoretic, dynamic general equilibrium models to revisit these issues. These tools allow us to address a variety of questions regarding the impact of an asset-backed money (ABM) on the real economy and the price level. For example, is price-level stability desirable? More generally, how should the monetary instrument be designed and managed to achieve a good allocation? Our objective in this paper is to build a macroeconomic model to provide answers to questions of this nature.

We use the Aruoba and Wright (2003) neoclassical growth model to study the use of intermediated assets as media of exchange. As in their model, a medium of exchange is needed for some transactions. Physical capital is assumed to be illiquid and thus cannot serve directly as a medium of exchange. However, rather than fiat currency, we assume that a mechanism is delegated control of the stock of physical capital and issues equity claims against the revenues that it earns. These claims are used as a medium of exchange and constitute a form of ABM.[3] The mechanism consists of a set of rules that specifies (i) the number of claims outstanding, (ii) the dividends paid out to claim holders, and (iii) the redemption fee charged for disbursing the dividend.

We focus our attention on implementing first-best allocations. We find that for capital-rich economies, the first-best allocation can be implemented and price stability is optimal as Fama suggested. However, for sufficiently poor economies, achieving the first-best allocation requires a strictly positive inflation.

Literature Aruoba and Wright (2003) assume that physical capital is illiquid in the sense that capital cannot be used as a payment instrument in the goods market. To facilitate trade in that market, a new asset is introduced – a fiat money object (more generally, government debt) that is assumed to be liquid. As is standard, they find that the Friedman rule ($\mu = \beta < 1$) is an optimal policy. That is, if lump-sum

[2] These frictions include a lack of record-keeping (public communication of individual trading histories) and a lack of commitment.

[3] William Roberds pointed out to us that the central banks of 300 to 600 years ago operated exactly in this manner. See Fratianni (2006).

taxation is available, then it should be used to contract (deflate) the money supply to generate an efficient real rate of return on money. Lagos and Rocheteau (2008) modify the Aruoba and Wright (2003) model by permitting capital to circulate as a payment instrument. In some cases, capital is over-accumulated and the introduction of a second asset – again, fiat money or government debt – can improve efficiency. The optimal policy is again the Friedman rule ($\mu = \beta < 1$).

Our approach contrasts with the previous two papers in the following ways. First, we assume that capital is illiquid, but that intermediated claims to capital are not. This assumption is innocuous if the stock of ABM is fixed. However, we allow the stock of ABM to change over time. This is one distinction between our paper and Lagos and Rocheteau (2008).[4] Second, we do not (although we could) introduce a second asset (such as fiat money or government debt). Consequently, monetary policy in our model is restricted to managing the supply of ABM. Third, first-best implementation in our model is possible even without lump-sum taxes, at least, in patient economies. Fourth, we find that an optimal mechanism may require *inflation* – which is equivalent to a policy of persistent (and predictable) dilution of the outstanding stock of ABM. This latter result is consistent with Andolfatto (2010) who abstracts from capital. Our results are more general in that the minimum inflation necessary to implement the first-best allocation varies with an economy's capital stock. In particular, first-best implementation can be consistent with moderate deflations away from the Friedman rule.

Although we have only one medium of exchange, our paper is related to a body of research that studies the coexistence of fiat money and other assets as media of exchange. This literature finds that if a real asset can be used as a medium of exchange *and* its fundamental value is sufficiently high, then the first-best allocation can be obtained. However, if the fundamental value is too low, the real asset will carry a liquidity premium and the first-best allocation will be unattainable (see Waller, 2003; Geromichalos et al., 2007; and Lagos and Rocheteau, 2008). It then follows that introducing a second asset (typically fiat money) to reduce this liquidity premium will lead to better allocations. Thus, fiat money is essential even though real assets are available as exchange media. With respect to this literature, we are able to show that even when the fundamental value is too low, mechanisms can be designed to attain the first-best allocation without the need for a second asset.

Using a mechanism design approach, Hu and Rocheteau (2014a, 2014b) gain additional insights into the coexistence issue. Hu and Rocheteau (2014a) investigate the coexistence of fiat money and higher-return assets in an economy with pairwise meetings, where fiat money and risk-free capital compete as means of payment. They

[4] In Lagos and Rocheteau (2008) capital is privately owned and managed. Here, the mechanism manages the capital stock and can use the capital return to back the ABMs. Thus, while the mechanism has no access to lump-sum taxes in our model, it is delegated control of real wealth and controls the dividend stream.

find that in any stationary monetary equilibrium, capital has a higher rate of return than fiat money. In Hu and Rocheteau (2014b), they study the effects of monetary policy on asset prices. Although some research questions are similar to ours, such as which conditions are necessary for first-best allocations to be obtained, our paper differs from this literature in that we do not study coexistence issues, since we restrict attention to cases where a single exchange medium is sufficient.[5]

2 Environment

2.1 Preferences and technologies

Our environment is based on Aruoba and Wright (2003). Time t is discrete and the horizon is infinite. Each period is divided into two subperiods, which we refer to below as the AM and PM (subperiods), respectively. There is a $[0,1]$ mass of *ex ante* identical agents with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sigma \left[u(q_t^c) - q_t \right] + \theta U\left(C_t\right) + \overline{N} - N_t \right\}. \tag{1}$$

Agents are randomly assigned to one of three states in the AM: *consumers*, *producers*, or *idlers*. The probability of becoming a consumer is σ, the probability of becoming a producer is σ, and the probability of becoming an idler is $1-2\sigma$, where $0<\sigma<1/2$. The AM good is nonstorable. A consumer derives flow utility $u(q_t^c)$ from consuming the AM good q_t^c, and a producer derives flow utility $-q_t$ from producing the AM good q_t at time t. The AM flow utility for idlers is normalized to zero. Assume $u''<0<u'$ and $u(0)=0$, $u'(0)=\infty$. As there is an equal mass of producers and consumers, feasibility and efficiency imply $q_t^c = q_t$. The discount factor is restricted to lie in the interval $\sigma<\beta<1$, and we assume $u\left(q^*\right)q^* \geq \left(1-\sigma\right)\sigma$.[6]

All agents have the same preferences and opportunities in the PM. Their PM flow utility payoff is given by $\theta U\left(C_t\right)+\overline{N}-N_t$, where C_t is consumption of the PM good, N_t denotes the aggregate labor input at time t, and $\overline{N}$ is the endowment of time.[7]

[5] Another related strand of this literature assumes there is an asymmetric information problem with the real asset (Rocheteau, 2011; and Lester et al., 2012) or that the real asset has better consumption-hedging properties for some agents than others (Jacquet and Tan, 2012).

[6] These are sufficient conditions for existence of an ABM equilibrium as explained in the proofs. The former condition requires that agents cannot be too impatient. The latter condition requires that the gains from trading are sufficiently large. Either, given the trading probability σ, the match surplus $u\left(q^*\right)-q^*$ has to be sufficiently large, or, given the match surplus, the trading probability has to be sufficiently large.

[7] Since preferences are linear in N_t, individuals are in fact indifferent across lotteries that deliver N_t in expectation. It is useful to keep this in mind when we consider the properties of decentralized allocations below.

Throughout the paper, we assume that $\overline{N}$ is sufficiently large such that the constraint $\overline{N} \geq N_t$ never binds. Assume that $U'' < 0 < U'$ with $U(0) = -\infty$ and $U'(0) = \infty$. The parameter θ indexes the relative weight agents place on consumption vis-à-vis labor in their preferences and will play an important role in our analysis below.

Production of the PM output is standard neoclassical: $Q_t = F(K_t, N_t)$, where F exhibits all the usual properties, and K_t and N_t are the aggregate capital and labor inputs, respectively. Let $f(k) \equiv F(K/N, 1)$, where $k \equiv K/N$. The resource constraint is given by

$$C_t = F(K_t, N_t) + (1 - \delta)K_t - K_{t+1}, \tag{2}$$

for all $t \geq 0$, with $K_0 > 0$ given and where $0 \leq \delta \leq 1$ is the rate of capital depreciation.

2.2 First-best allocation

We define a first-best allocation to be a sequence $\{K_{t+1}, N_t, C_t, q_t\}_{t=0}^{\infty}$ that maximizes (1) subject to (2), with $K_0 > 0$ given. The steady-state first-best allocation constitutes a set of numbers (K^*, k^*, C^*, q^*) satisfying:

$$\begin{aligned} u'(q^*) &= 1 && (3) \\ \beta\left[f'(k^*) + 1 - \delta\right] &= 1 && (4) \\ \left[f(k^*) - f'(k^*)k^*\right]\theta U'(C^*) &= 1 && (5) \\ \left[f(k^*)/k^* - \delta\right]K^* &= C^* && (6) \end{aligned}$$

where $N^* = K^*/k^*$. Lemma 1 follows immediately from (3)-(6).

Lemma 1 *q^* and k^* are determined independently of θ. $K^*(\theta)$, $N^*(\theta)$ and $C^*(\theta)$ are strictly increasing in θ.*

3 Asset-backed money

Aruoba and Wright (2003) examine the properties of equilibria under the assumptions that (i) agents are anonymous; (ii) capital cannot be used as a payment instrument in the AM market; and (iii) there is a fiat money instrument that can be used as a payment instrument in the AM market; and (iv) the government has access to a lump-sum tax instrument. They find that the Friedman rule (i.e., to deflate at the rate of time preference) is an optimal policy.

In this paper, we maintain assumptions (i) and (ii) used in Aruoba and Wright (2003), but we remove assumptions (iii) and (iv) and replace them with two others. First, we assume that the monetary instrument takes the form of an *asset-backed*

money (ABM) instrument. Second, we assume that all trade must be voluntary – that is, there is no lump-sum tax instrument. Our purpose is to examine the properties of an optimal monetary policy when money takes the form of ABM and when all trade must be sequentially rational.

3.1 Capital and labor markets

We assume that all markets are competitive.[8] In particular, capital K_t and labor N_t are traded in a competitive factor market in the PM.[9] Factor market equilibrium implies that capital and labor earn their respective marginal products. Consequently, in the PM, the real wage and rental rates, respectively, satisfy

$$w(k_t) = f(k_t) - f'(k_t)k_t \tag{7}$$

$$r(k_t) = f'(k_t). \tag{8}$$

3.2 Frictions, market structure and mechanisms

The key friction is anonymity, which precludes private credit arrangements and generates the need for an exchange medium. One possibility is to follow Lagos and Rocheteau (2008) and have capital serve as a payment instrument. These authors assume a competitive market structure and demonstrate that for some parameters, capital is overaccumulated. They then introduce a mechanism (interpreted as a monetary authority) that manages a supply of fiat money that competes with capital as an exchange medium. When their mechanism is programmed to destroy fiat money at the rate of time preference (the Friedman rule), the resulting equilibrium implements the first-best allocation.

The Lagos and Rocheteau (2008) mechanism is endowed with a coercive power that is used to extract resources from the population.[10] These resources are necessary to finance an optimal deflation. The coercive aspect of their mechanism invites one to interpret it as a government with the power to exact lump-sum taxes. We want to restrict attention to mechanisms that have no coercive power. To the extent that our mechanism requires an income source resembling tax revenue, the mechanism must induce individuals to pay these taxes voluntarily. Because payments made under

[8] Bargaining frictions play no essential role in our analysis, so we abstract from them for simplicity. Note, however, since producers have linear disutility in the AM, the outcome in the AM is the same when buyers make take-it-or-leave-it offers to producers.

[9] We maintain the assumption of Aruoba and Wright (2003) that individuals cannot produce PM goods themselves. They can only supply labor to firms and earn wage ω.

[10] We use the word *mechanism* as short hand for a policy rule, or *protocol*. While the mechanism is exogenous as far as individual agents are concerned, one could think of agents as collectively determining the protocols under which they wish to be governed.

these conditions look more like voluntary "fee for service" exchanges, our mechanism could be interpreted as a non-government agency programmed to serve private-sector stakeholders in a non-coercive manner. Our preference, however, is to remain agnostic with regard to interpretation and focus on the nature of optimal policy. However, if it helps to fix ideas, the reader is free to interpret our mechanism as a government agency with monopoly rights over money-issuance, but with no power to tax.[11]

Our mechanism is delegated control rights over the capital stock, which we assume to be illiquid.[12] Because transition dynamics are peripheral to the points we wish to emphasize below, we simplify by restricting attention to steady states in what follows. Thus, we assume that the mechanism is programmed to maintain an initial capital stock K_0, with K_0 being arbitrary (as long as it is feasible).[13] In particular, when we set $K = K^*$ we implicitly assume $K_0 = K^*$.[14] All other quantities, such as consumption and labor effort, are chosen by individuals in the usual way.

In exchange for the control rights over capital K, the mechanism issues paper notes that constitute legal claims against the net income generated by the mechanism. We refer to these paper notes as *asset-backed money* (ABM). Let M_t denote the supply of ABM at date t. The mechanism expands the supply of ABM at the constant (gross) rate $\mu = M_{t+1}/M_t$. Newly issued ABM is injected into (or withdrawn from) the economy in the PM subperiod only. We assume that ABM constitutes the only medium of exchange. Let $(\phi_{1,t}, \phi_{2,t})$ denote the competitive price of ABM measured in units of AM and PM output, respectively. Let $S_t = (\mu - 1)\phi_{2,t}M_t$ denote the seigniorage revenue collected by the mechanism measured in units of PM goods. Of course, in a steady state $\phi_{2,t}M_t = \phi_{2,t+1}M_{t+1}$, so that $S_t = S$ and $\phi_{2,t}/\phi_{2,t+1} = \mu$.

Apart from capital rental income and seigniorage revenue, there is a third potential source of earnings for the mechanism. In particular, it may collect revenue in the form of redemption fees T, again, measured in units of PM output. These combined earnings are used to finance investment $I = \delta K$ and an aggregate dividend D. Thus, the mechanism is subject to a budget constraint:

$$\delta K + D = r(k)K + (\mu - 1)\phi_2 M + T. \tag{9}$$

[11] We restrict our analysis to a single money-issuing agency to avoid the delicate (and interesting) issues that arise when there are potentially multiple competing currencies.

[12] In general, we could assume that control rights are delegated over only some fraction of the capital stock.

[13] For the readers interested in transition dynamics, the mechanism could be programmed to follow a path for the capital stock $\{K_{t+1}\}_{t=0}^{\infty}$, with $K_0 > 0$ given. Any such path is permitted as long as the implied net investment at each date can be financed from retained earnings. One such path is the first-best solution.

[14] No agent (or agency) in our economy chooses the capital stock. The point we want to investigate here is the optimal design of policy for a given capital stock. This issue is of interest, since even if the capital is efficient, the level of economic activity in the AM market need not be. It is in these circumstances that we ask how a well-designed monetary policy and ABM instrument might implement the first-best allocation (3)-(6).

We now describe specifically how the ABM is designed. First, one unit of ABM in the PM entitles the holder to a dividend d_t. In most setups, it would be sufficient to define d_t as the dividend per unit of outstanding ABM. Things are a little different here, because we assume that in order to collect this dividend, an individual must pay a redemption fee τ (individuals are allowed to keep their ABM post redemption). Let $Z_t \leq M_t$ denote the units of ABM that are presented for redemption. We define $d_t = D/Z_t$, where Z_t depends on individual behavior.

Let $m_t(i)$ denote the ABM holdings of an individual i in the PM at date t. As the redemption choice is voluntary, an agent will collect the dividend if, and only if, $d_t m_t(i) \geq \tau$. Define the indicator function for individual $i \in [0,1]$ by $\chi_t(i) \in \{0,1\}$, where $\chi_t(i) = 1$ means that the redemption option is exercised at date t. Then $Z_t = \int m_t(i)\chi_t(i)di$ and $T = \tau \int \chi_t(i)di$.[15]

Matters are greatly simplified if at this stage we invoke a result that is known to hold in this class of quasilinear models. In particular, the equilibrium distribution of ABM at the beginning of the PM will be massed over three points $\{0, M, 2M\}$. That is, the mass σ of AM consumers enter the PM with zero units of ABM, the mass $(1-2\sigma)$ of AM idlers enter with M units of ABM, and the mass σ of AM producers enter with $2M$ units of ABM. Below, we design the ABM in a manner such that the mass $(1-2\sigma)$ of idlers and the mass σ of producers will voluntarily exercise the redemption option. In this case, $Z_t = M_t$ and $\int \chi_t(i)di = 1-\sigma$. Moreover, we have

$$D = d_t M_t \geq \tau \text{ and } T = (1-\sigma)\tau. \tag{10}$$

Formally then, given some initial capital stock K_0 and initial stock of ABM M_0, a *mechanism* can be defined here as three objects (D, μ, τ) satisfying (9) and (10), together with the redemption rule that is built into the design of ABM. Note that, since D is constant, a growing stock of ABM means that the dividend d_t is decreasing over time.

We define a *passive mechanism* as a mechanism with the property $(\mu, \tau) = (1, 0)$. In this case, dividends consist solely of net capital income, $D = [r(k) - \delta]\, K$. Any mechanism that is not a passive mechanism is referred to below as an *active mechanism*.

[15] Note that it is not without loss in generality that we assume that d and τ are independent of m. For example, one could imagine the mechanism could do better by letting τ be a function of m (possibly decreasing) and by specifying that dm is a general function of m (not necessarily linear as we assume). This is not relevant for the parametrizations for which our mechanism delivers the allocation (1)-(4), but may be for those cases where it does not.

4 Individual decision-making

We now drop time subscripts from our notation. Any variable without a time subscript is to be understood as a contemporaneous variable. A "one-period ahead" variable x_{t+1} will have the notation x^+. Likewise, we denote x_{t-1} as x^-.

4.1 The PM market

An individual who enters the PM with m units of ABM faces the budget constraint

$$c + \phi_2 m^+ = (\phi_2 + \chi d)m + wn - \chi\tau, \tag{11}$$

where $m^+ \geq 0$ denotes the ABM carried into the next-period AM, and n represents the individual's labor supply.[16] Note that $n < 0$ is permitted here and is interpreted as a household buying leisure at market price w. The act of redemption is sequentially rational if, and only if,

$$dm \geq \tau. \tag{12}$$

This, in turn, implies that only agents entering the PM market with large enough money balances ($m \geq \tau/d$) will exercise the redemption option. In what follows, we conjecture (and later verify) that the PM redemption option is exercised by those who were producers and idle agents in the previous AM market. Consumers in the previous AM will have depleted their ABM balances to a point that makes redemption suboptimal.

Associated with the asset positions (m, m^+) are the value functions $W(m)$ and $V(m^+)$, which must satisfy the following recursive relationship:

$$W(m) \equiv \max\left\{\theta U(c) - n + \beta V(m^+) : c = (\phi_2 + \chi d)m + wn - \phi_2 m^+ - \chi\tau\right\}. \tag{13}$$

Assuming that V is strictly concave (a condition that can be shown to hold in the relevant range of parameters considered below), for a given redemption choice χ, the following first-order conditions describe optimal behavior:

$$1 = w\theta U'(c) \tag{14}$$

$$\phi_2 = w\beta V'(m^+). \tag{15}$$

By the envelope theorem:

$$W'(m) = (1/w)(\phi_2 + \chi d). \tag{16}$$

[16] Note that our dividend/fee structure implies a nonlinear mechanism and is therefore potentially exploitable by a coalition of agents. The assumed lack of commitment, however, rules out the formation of a cooperative coalition.

4.2 The AM market

A consumer who enters the AM with m units of money faces the following problem:

$$V_c(m) \equiv \max\left\{u(q) + W(m') : m' = m - \phi_1^{-1}q \geq 0\right\}.$$

Let λ denote the Lagrange multiplier associated with the constraint $m' \geq 0$. Then the optimality condition is given by

$$\phi_1 u'(q) = (1/w)(\phi_2 + \chi d) + \lambda, \tag{17}$$

where, here, we use (16). If the constraint is slack, then $\lambda = 0$ and $\phi_1 m \geq q$. Otherwise, the constraint binds so that $\phi_1 m = q$. By the envelope theorem:

$$V_c'(m) = \phi_1 u'(q). \tag{18}$$

A producer who enters the AM with m units of money faces the following problem:

$$V_p(m) \equiv \max\left\{-q + W(m') : m' = m + \phi_1^{-1}q\right\}.$$

In equilibrium, the following must hold:

$$\phi_1 = (1/w)\left(\phi_2 + \chi d\right), \tag{19}$$

where, here, we use (16). Finally, for the idle agents who enter with m units of money, $V_i(m) = W(m)$. For them and for the producers, the envelope theorem yields

$$V_p'(m) = V_i'(m) = \phi_1, \tag{20}$$

where we once again use (16).

5 Stationary ABM equilibrium

For a given mechanism, we focus on symmetric stationary equilibria. Such equilibria meet the following requirements: (i) Households' decisions are optimal; (ii) the decisions are symmetric across all sellers and symmetric across all buyers; (iii) markets clear at every date, and (iv) all real quantities are constant across time. In particular, the aggregate real value of outstanding ABM is constant: $\phi_2^- M^- = \phi_2 M$ such that $\phi_2^- = \mu\phi_2$.

We now gather the restrictions implied by individual behavior. We use (15), (18), and (20) to form $\phi_2 = w\beta\left\{\sigma\phi_1^+ u'(q^+) + \sigma\phi_1^+ + (1-2\sigma)\phi_1^+\right\}$. Backdating this expression by one period and collecting terms, yields

$$\phi_2^- = w(k)\beta L(q)\phi_1, \tag{21}$$

where

$$L(q) \equiv \sigma \left[u'(q) - 1\right] + 1. \tag{22}$$

The function $L(q)$ is related below to the notion of a liquidity premium. Note that $L'(q) < 0$ and $L(q^*) = 1$ (zero liquidity premium at the first-best allocation).

Multiply both sides of (19) and (21) by M and use $\phi_2^- = \mu\phi_2$ to derive

$$\phi_1 M = \left[1/w(k)\right](\phi_2 M + D) \tag{23}$$

$$\mu\phi_2 M = w(k)\beta L(q)\phi_1 M, \tag{24}$$

where, recalling (10), $D = dM$. Use (9) to replace D in (23) and then solve (23) and (24) for $\phi_1 M$ and $\phi_2 M$ to derive

$$\phi_1 M = \frac{1}{w(k)\left[1 - \beta L(q)\right]}\left\{\left[r(k) - \delta\right]K + T\right\} \tag{25}$$

$$\phi_2 M = \frac{\beta L(q)}{\mu\left[1 - \beta L(q)\right]}\left\{\left[r(k) - \delta\right]K + T\right\}. \tag{26}$$

From condition (14), we have

$$w(k)\theta U'\left(\left[f(k)/k - \delta\right]K\right) = 1, \tag{27}$$

since $c = \left[f(k)/k - \delta\right]K$.[17]

Next, from the consumer's choice problem, the constraint $\phi_1 m \geq q$ either binds or is slack. If the value function $V(m)$ is strictly concave, then quasilinear preferences imply that all agents enter the AM with identical money holdings. If $V(m)$ is linear, then our assumption of symmetry implies (without loss) the same thing. Market clearing implies $m = M$ at every date, so that $\phi_1 m \geq q$ implies

$$\phi_1 M \geq q^* \text{ or } \phi_1 M = q < q^*. \tag{28}$$

When the economy is away from the first-best allocation, the equilibrium distribution of money at the beginning of the PM is as follows: Consumers hold zero money, idlers hold M units of money, and producers hold $2M$ units of money. This distribution of money continues to be an equilibrium as we approach the first-best allocation. Recall from the discussion surrounding (10) that we restrict attention to incentive schemes that satisfy $dM \geq \tau$, so that producers strictly prefer to exercise the redemption option, while idle agents weakly prefer to do so. In this case, using (9) and (10), the redemption constraint $dM \geq \tau$ can be written as follows:

$$\left[r(k) - \delta\right]K + (\mu - 1)\phi_2 M \geq \sigma\tau. \tag{29}$$

[17] The functional relationship $c = \left[f(k)/k - \delta\right]K$ can be derived from the ressource constraint $C = F(K, N) - \delta K$: aggregate consumption C must be equal to PM output $F(K, N)$ minus the PM goods needed to maintain the steady-state capital stock. This equation can be written as $C = \left[f(k)/k - \delta\right]K$. From (14) all households consume the same quantity c, and since the measure of households is 1, we obtain $c = C$.

Finally, since agents lack commitment, the allocation must be sequentially rational. This requirement is automatically satisfied for a passive (linear) mechanism. There is a question as to whether the first-best allocation can be implemented with a linear mechanism. In general, the answer is no (i.e., the implementation fails for low θ economies). In this case, an active (nonlinear) mechanism is necessary and sequential participation constraints need to be checked explicitly.

The key sequential participation constraint to check is that of a consumer who enters the PM with zero ABM balances. This agent must work hard (sacrifice much transferable utility) to accumulate ABM. On the equilibrium path, his payoff for rebalancing his wealth in the PM is given by $W(0)$. If the sacrifice of rebalancing is too high, an individual would forgo the opportunity to do so and enter the next AM market with zero ABM. The individual will still consume in the PM market, and is free to work in the AM market. If he does work in the AM, he takes his ABM and spends it in the PM. Along this path, the individual never consumes in the AM. Let $\tilde{W}(0)$ denote the value of the alternate strategy.[18] Then, sequential rationality requires that

$$W(0) \geq \tilde{W}(0). \tag{30}$$

There is a second sequential rationality constraint: The sellers in the AM market must be willing to produce q for m units of money. It is straightforward to show that in equilibrium this constraint is satisfied.

Definition 1 *For a given mechanism (D, μ, τ) satisfying (9) and (10), a stationary ABM equilibrium is characterized by a set of quantities and prices $(q, k, \phi_1, \phi_2, w, r)$ satisfying (7), (8), and (25) through (30).*

6 Optimal mechanisms

We now study the properties of mechanisms that are consistent with a first-best implementation (i.e., the implementation of the first-best allocation as a competitive equilibrium). In what follows, we restrict attention to mechanisms that are programmed to start with a capital stock $K = K^*(\theta)$. For arbitrary mechanisms (in our class of mechanism) with these properties, the level of economic activity in the AM, q, may or may not be efficient. The goal is to identify the additional properties needed for first-best implementation.

[18] We provide a formal derivation of $\widetilde{W}(0)$ in the Appendix as a part of the proof to Lemma 4.

6.1 Passive mechanism

We first determine conditions under which a passive mechanism ($\mu = 1$ and $\tau = 0$) is optimal. In this case, $\chi = 1$ trivially for all agents, so the redemption condition (29) can be ignored. From (9), this implies that the aggregate real dividend is equal to capital income net of depreciation expense; that is, $D = [f'(k) - \delta] K$. It also implies that ABM prices (ϕ_1, ϕ_2) are constant over time. The question here is whether the constraint (28) is binding or not. To begin, we first assume that the constraint is slack and then verify this will indeed be the case in a certain region of the parameter space.

From (28), the debt constraint is slack if $\phi_1 M \geq q^*$. Under a passive mechanism and a slack debt constraint, (25) and (26) reduce to

$$\phi_1 M = \frac{1}{w(k)(1-\beta)} [r(k) - \delta] K \geq q^* \tag{31}$$

$$\phi_2 M = \frac{\beta}{(1-\beta)} [r(k) - \delta] K. \tag{32}$$

Using (4) and (8), we have $(1/\beta) = [r(k^*) - \delta]/(1-\beta)$. Combine this latter result with (31) and (32), assuming that $K = K^*(\theta)$, to obtain:[19]

$$\phi_1 M = K^*(\theta)/[w(k^*)\beta] \geq q^* \tag{33}$$

$$\phi_2 M = K^*(\theta). \tag{34}$$

Thus, under a passive mechanism that implements the first-best allocation, the value of the outstanding stock of ABM is equal to the value of the capital stock ($\phi_2 M = K^*(\theta)$); that is, the Tobin's Q relation holds.[20] Furthermore, if the debt constraint is slack, from (28) we have $\phi_1 M \geq q^*$, which implies

$$K^*(\theta) \geq q^* w(k^*)\beta. \tag{35}$$

Note that $K^*(\theta)$ is increasing in θ. In the Proposition below, we will use this property to derive a critical value θ_0.

Proposition 1 *Under a passive mechanism, there exists a unique $0 < \theta_0 < \infty$ that satisfies $K^*(\theta_0) = q^* w(k^*)\beta$, such that the following is true:*
For economies with $\theta \geq \theta_0$, the competitive monetary equilibrium is efficient and Tobin's Q holds. That is, (33) and (34) hold.

[19] In the Appendix, we show that any mechanism such that $K = K^*(\theta)$ implies $k = k^*$ (see Lemma 5).

[20] Tobin's Q states that the market value of a firm must be equal to its replacement cost. In the context of our model, $\phi_2 M$ is the value of the firm, and $K^*(\theta)$ is the replacement cost, since it costs $K^*(\theta)$ to acquire $K^*(\theta)$ capital.

For economies with $0 < \theta < \theta_0$, the competitive monetary equilibrium is inefficient, and Tobin's Q does not hold – the value of outstanding ABM exceeds the value of the capital stock. That is,

$$\phi_1 M = q < q^* \tag{36}$$
$$\phi_2 M > K^*(\theta). \tag{37}$$

According to Proposition 1, if $\theta < \theta_0$, the household's preference for the PM-good is low, and so the first-best capital stock is small. Consequently, the value of ABM in the AM $\phi_1 M$ is too low for sellers to be willing to produce the first-best amount of AM goods. Furthermore, the value of the outstanding stock of money in the PM, $\phi_2 M$, is larger than the value of the capital stock, $K^*(\theta)$, and so Tobin's Q does not hold.

Proposition 1 is related to the nonmonetary equilibrium of Lagos and Rocheteau (2008), where capital serves as a medium of exchange. They find that if the efficient capital stock is small, then the equilibrium capital stock is too high and consumption in the AM is too small. Here, since the mechanism is endowed with $K^*(\theta)$, the capital stock is efficient, but the value of money that is backed by the capital stock is too high in equilibrium, and consumption in the AM is too small.

To develop some further intuition for the inefficiency of the allocation when $\theta < \theta_0$, note that conditions (36) and (37) imply the following relations:

$$\frac{\phi_2}{\phi_1} = \frac{p_1}{p_2} > \frac{K^*(\theta_0)}{q} > \frac{K^*(\theta_0)}{q^*} > \frac{K^*(\theta)}{q^*}.$$

Thus for $\theta < \theta_0$, AM goods are too expensive relative to PM goods (and relative to the first-best). The reason is that a low θ means a low demand for PM consumption and hence a low demand for the capital that would produce such consumption. But aside from its use in production, capital (ABM) is also used as a medium of exchange (for a constant supply of ABM, there is a one-for-one correspondence between capital and a claim to capital in the form of ABM). Hence a low demand for capital also implies a low real stock of ABM. The resulting shortage of ABM implies that not enough AM consumption can be financed. To restore efficiency, we want a mechanism that somehow reduces ϕ_2 and increases ϕ_1.

6.2 Active mechanisms

Proposition 1 shows that first-best implementation under a passive mechanism is not possible for low θ economies. We now investigate whether active mechanisms can implement the first-best allocation in regions of the parameter space where a passive mechanism fails; i.e., we now restrict attention to economies with $\theta < \theta_0$. The essential problem with ABM in this region of the parameter space is that its real

rate of return is too low to motivate AM producers to supply the first-best level of output. An active mechanism can potentially mitigate this problem by subsidizing the ABM dividend (hence, the rate of return on ABM). There are two instruments available to finance this subsidy: seigniorage and redemption fee revenue.

Case 1: $\mu > 1$ **and** $\tau = 0$. In what follows, we ask whether seigniorage helps to implement the first-best allocation in the absence of redemption fees. Since $\tau = T = 0$, we can write (25) and (26) as follows

$$\phi_1 M = \frac{1}{w(k)\left[1-\beta L(q)\right]}\left[r(k)-\delta\right]K \tag{38}$$

$$\phi_2 M = \frac{\beta L(q)}{\mu\left[1-\beta L(q)\right]}\left[r(k)-\delta\right]K. \tag{39}$$

Under a slack debt constraint, $q^* \leq \phi_1 M$, and for an efficient capital stock, $K = K^*(\theta)$, (38) and (39) reduce to

$$\phi_1 M = K^*(\theta)/\left[w(k^*)\beta\right] \geq q^* \tag{40}$$

$$\phi_2 M = \mu^{-1}K^*(\theta) \tag{41}$$

Note that (40) and (41) are identical to (33) and (34) apart from the μ term in (41). Since (33) and (40) are identical, they both define the same critical value θ_0 independently of μ. Consequently, seigniorage alone does not expand the set of economies for which the mechanism can attain the first-best allocation. The intuition for this result follows from a well-known proposition in monetary theory: namely, that newly created money, injected as proportional transfers, is superneutral.[21] From (40), $\phi_1 M$ is independent of μ. Note that while $\phi_2 M$ is decreasing in μ, this does not affect the PM allocation, because the ABM that agents have available in the PM is given by $\phi_2 M\mu$, which includes the newly printed money and which is by (41) a constant.

Case 2: $\tau > 0$. We have just shown that using seigniorage alone to finance the dividend cannot improve the allocation when $\tau = 0$ and $\theta < \theta_0$. The following lemma characterizes the redemption-fee income necessary to implement the first-best allocation.

Lemma 2 *For economies with $\theta < \theta_0$, the minimum fee income T $[= (1-\sigma)\tau]$ required to implement the first-best allocation is*

$$T^*(\theta) = \left[K^*(\theta_0) - K^*(\theta)\right](1-\beta)\beta^{-1}. \tag{42}$$

$T^*(\theta)$ is independent of the rate of ABM creation μ.

[21] Note that super-neutrality also holds when $q = \phi_1 M < q^*$. This can be seen from (38) which in this case determines q independent of μ for a given K.

The minimum fee income $T^*(\theta)$ is decreasing in θ, since an increase in θ increases the optimal capital stock, and a larger capital stock generates more capital income. For $\theta \geq \theta_0$, no fee income is needed to implement the first-best allocation.

If agents could be forced to pay the minimum fee $\tau^* = T^*(1-\sigma)^{-1}$, this would be the end of the story. But, of course, coercion is ruled out by our assumption that all trade must be voluntary. Consequently, we need to find conditions under which the fee income T^* can be collected from voluntary contributions. In particular, the implied redemption fee τ^* needs to satisfy the redemption condition (29).

Lemma 2 also states that the minimum fee income is independent of inflation μ (ABM dilution). This suggests that money creation continues to be super neutral. However, we will see below that this is not true, because μ affects the redemption constraint.

Lemma 3 *Under $T^*(\theta)$, the redemption constraint (29) satisfies*

$$(1-\beta)\beta^{-1}K^*(\theta) + (\mu-1)\mu^{-1}K^*(\theta_0) \geq \sigma(1-\sigma)^{-1}T^*(\theta). \tag{43}$$

An increase in μ relaxes the redemption constraint (43).

Lemma 3 clearly shows that inflation (ABM dilution) relaxes the redemption constraint. The intuition is that a higher inflation rate dilutes the real value of existing ABM and therefore increases the opportunity cost of failing to collect the dividend.

In the proof of Lemma 3, we also show that under the minimum fee income the value of money in the PM satisfies $\phi_2 M = \mu^{-1}K^*(\theta_0)$. The value of money is clearly higher than the one under a passive policy, where it is $\phi_2 M = \mu^{-1}K^*(\theta)$ (see 41). The reason is that the fee income is used to increase the dividend which induces sellers to produce more in the AM.

Proposition 2 *There exists a unique critical value $0 < \theta_1 < \theta_0$, where θ_1 solves $K^*(\theta_1) = \sigma K^*(\theta_0)$, such that the following is true:*
If $\theta \geq \theta_1$, the first-best allocation can be implemented with $\mu = 1$.
If $\theta < \theta_1$, the first-best implementation requires that $\mu > 1$.

The result that a strictly positive inflation can be necessary for optimality is, at first glance, not entirely obvious. In a large class of monetary models, the Friedman rule is an optimal policy. In models where inflation is desirable, there is usually a redistributive motive at work (e.g., Levine, 1991; Berentsen, Camera, and Waller, 2005; or Molico, 2006), or inflation relaxes borrowing constraints (Berentsen, Camera, and Waller, 2007). These motives are absent here. The rationale for the optimality of inflation is that it relaxes the redemption constraint. A higher inflation rate dilutes

the real value of existing ABM and therefore increases the opportunity cost of failing to collect the dividend. Inflation therefore encourages redemption for any given fee and hence permits a higher fee to be charged, which is then used to finance the higher dividend. The higher dividend in turn increases the real rate of return on ABM, incentivizing production.

The intuition for the key results of the paper can be summarized as follows. There are two critical values: $0 < \theta_1 < \theta_0$. For $\theta \geq \theta_0$, a passive mechanism ($\mu = 1$ and $\tau = 0$) can implement the first-best allocation. The stock of capital is large, and so capital earnings are sufficient to pay a dividend that induces producers to produce q^*. For $\theta_1 \leq \theta < \theta_0$, the efficient capital stock is intermediate, and capital earnings are insufficient to attain q^*. In this case, the mechanism charges a fee ($\mu = 1$ and $\tau > 0$). The fee income that is necessary to induce producers to produce q^* is so small that the redemption constraint does not bind. For that reason, there is no need for inflation (ABM dilution), and, in fact, we show that the ABM is super neutral for $\theta_1 \leq \theta$. For $\theta < \theta_1$, the economy is capital poor, and capital earnings are small. In this case, the mechanism requires a large fee income in order to induce producers to produce q^*. However, agents are not willing to pay this, but are instead willing to use their ABM without exercising the redemption option. To restore efficiency, the mechanism must resort to inflation (ABM dilution); i.e., ($\mu > 1$ and $\tau > 0$). A strictly positive inflation dilutes the real value of the existing ABMs in the PM and increases the opportunity cost of failing to collect the dividend. A strictly positive inflation, therefore, encourages redemption for any given fee and hence permits a higher fee to be charged, which is then used to finance the higher dividend.

Minimum inflation rate. We end this section by deriving the range of μ for which raising the minimum fee income $T^*(\theta)$ through voluntary contributions is incentive-feasible. For this purpose, it is useful to define the following expressions:

$$\Omega(\mu) \equiv (1-\sigma)\left[\beta(1-\beta)^{-1}(\mu-1)\mu^{-1}+1\right] \text{ and } \Psi(\theta) \equiv \frac{K^*(\theta_0)-K^*(\theta)}{K^*(\theta_0)}.$$

Proposition 3 *The minimum fee income $T^*(\theta)$ can be raised through voluntary contributions for any μ satisfying*

$$\Omega(\mu) \geq \Psi(\theta). \tag{44}$$

Furthermore, there exists a minimum inflation rate $\mu^*(\theta)$ such that if $\mu \geq \mu^*(\theta)$,the first-best allocation is implementable with a voluntary fee.

Proposition 3 states that the efficient allocation can be attained with a voluntary fee $\tau^* = (1-\sigma)^{-1}T^*(\theta)$ if (44) holds. The term $\Psi(\theta)$ is decreasing in $\theta \leq \theta_0$ with $\Psi(\theta_0) = 0$ and $\Psi(0) = K^*(\theta_0)$. This term reflects the scarcity of ABM. If the first-best capital stock is low because θ is low, then $\Psi(\theta)$ is large. In this case, the fee

income must be high, which makes it difficult to satisfy the redemption constraint. Consequently, an increase in θ relaxes (44).

The term $\Omega(\mu)$ is increasing in μ with $\Omega(\beta) = 0$ and $\lim_{\mu\to\infty}\Omega(\mu) = (1-\sigma)\left[\beta(1-\beta)^{-1}+1\right]$. Since $\Omega(\mu)$ is increasing in μ, inflation (ABM dilution) relaxes this constraint. Since $\Omega(\mu)$ is increasing in μ and since $\beta > \sigma$ (this is shown in the proof), there exists a critical value μ^* such that $\Omega(\mu^*) = \Psi(\theta)$. $\mu^*(\theta)$ is the minimum inflation rate such that the first-best allocation is implementable with a voluntary fee. That is, for any if $\mu \geq \mu^*(\theta)$ the first-best allocation is implementable with a voluntary fee.

Note that $\mu^*(\theta)$ can be greater or less than 1. If it is less than one, the first-best allocation can be implemented with a deflation. This is possible if preferences and technology are such that the economy approximates the first-best allocation under a passive mechanism, such that the difference $K^*(\theta_0) - K^*(\theta)$ is small. In this case, the fee income required to attain the first-best q^* is small – the dividend being sufficiently high to induce voluntary redemptions for a small enough fee. Having said this, note that higher inflation rates are also consistent with the first-best implementation so that deflation, while possible, is not necessary.

7 Conclusion

When commitment and record-keeping are limited, media of exchange are necessary to facilitate trade. Exactly what form these exchange media should take and how their supply should be managed over time remain open questions. In theory, a properly managed supply of fiat money may be sufficient, but as shown in this paper, fiat money is generally not necessary. The concept of *monetary policy* should be extended to include the management of intermediated exchange media such as asset-backed money. Our model does not address the question of whether the responsibility for the money supply should reside in the public or private sector. Because our results do not involve lump-sum taxation, our paper suggests that private intermediaries, like the central banks of ancient times, could, in principle, be left to manage the money supply.

We also show that with asset-backed money, optimal monetary policy can be very different from the type usually obtained with fiat money. With fiat money and lump-sum taxation, the Friedman rule is typically optimal. In contrast, with ABM and in the absence of lump-sum taxation, we find that for capital-poor economies, achieving the first-best allocation may sometimes require a strictly positive inflation rate. In general, the minimum inflation necessary to implement the first-best allocation is above the Friedman rule and varies with capital wealth.

8 Appendix

Proof of Proposition 1. Consider inequality (35). The existence of a unique value θ_0 such that $K^*(\theta_0) = q^* w(k^*)\beta$ follows from the fact that $K^*(\theta)$ is strictly increasing in θ and that (q^*, k^*) are independent of θ (see Lemma 1). Combining (33) and $K^*(\theta_0) = q^* w(k^*)\beta$ yields

$$\frac{\phi_1 M}{q^*} = \frac{K^*(\theta)}{K^*(\theta_0)}.$$

From (28), if the debt constraint is binding, $q = \phi_1 M$. For $\theta < \theta_0$, Lemma 1 implies that $K^*(\theta) < K^*(\theta_0)$, and so $q = \phi_1 M < q^*$. Furthermore, if the debt constraint binds, Tobin's Q does not hold. To see this, rewrite (26) as follows:

$$\frac{\phi_2 M}{K\left[r(k) - \delta\right]} = \frac{\beta L(q)}{1 - \beta L(q)}.$$

Since $q < q^*$, we have $L(q) > 1$ (a positive liquidity premium) so that

$$\frac{\phi_2 M}{K\left[r(k) - \delta\right]} = \frac{\beta L(q)}{1 - \beta L(q)} > \frac{\beta}{1 - \beta}.$$

Use (4) and (8) to replace $\beta\left(1 - \beta\right)^{-1}$ to obtain

$$\frac{\phi_2 M}{K} > \frac{r(k) - \delta}{r(k^*) - \delta}.$$

If the capital stock is efficient; that is, if $K = K^*(\theta)$, then from Lemma 5, $k = k^*$ (see Lemma 5). Accordingly, we have $\phi_2 M > K^*(\theta)$. Thus, if the economy is characterized by an efficient but small capital stock, the value of the outstanding ABM exceeds the value of the capital stock. In particular, Tobin's Q does not hold. ■

Proof of Lemma 2. To derive $T^*\left(\theta\right)$, set $q = q^* = \phi_1 M$, $K = K^*\left(\theta\right)$, and $k = k^*$ in equation (25) to obtain

$$q^* = \frac{1}{w(k^*)\left(1 - \beta\right)}\left\{\left[r(k^*) - \delta\right] K^*\left(\theta\right) + T\right\}.$$

Use (4) and (8) to replace $r(k^*) - \delta = \beta\left(1 - \beta\right)^{-1}$ to obtain

$$q^* = \frac{K^*\left(\theta\right)}{w(k^*)\beta} + \frac{T}{w(k^*)\left(1 - \beta\right)}.$$

Use $K^*(\theta_0) = q^* w(k^*)\beta$ to replace $w(k^*)$ and solve for T to obtain (42). Finally, since $K^*\left(\theta\right)$ and $K^*\left(\theta_0\right)$ are independent of μ, $T^*\left(\theta\right)$ is independent of μ. ■

Proof of Lemma 3. To derive the value of money in the PM under an active policy, set $K = K^*(\theta_0)$, $q = q^*$, and $T = T^*(\theta)$ in (26) to obtain

$$\phi_2 M = \mu^{-1} K^*(\theta_0) \tag{45}$$

To derive (43), use (4) and (45) to rewrite the redemption constraint (29). Since $K^*(\theta)$, $K^*(\theta_0)$, and $T^*(\theta)$ are independent of μ, an increase in ABM creation μ clearly relaxes the redemption constraint. ■

Proof of Proposition 2. In (43), set $\mu = 1$ to obtain $(1-\beta) K^*(\theta)/\beta \geq \sigma(1-\sigma)^{-1} T^*(\theta)$. Use (42) to substitute $T^*(\theta)$ and simplify to obtain $K^*(\theta) \geq \sigma K^*(\theta_0)$. Since $K^*(\theta)$ is strictly increasing in θ, there exists an unique critical value θ_1 such that $K^*(\theta_1) = \sigma K^*(\theta_0)$. Thus, if $\theta \geq \theta_1$, the first-best allocation can be implemented with $\mu = 1$.
To prove that for $\theta < \theta_1$, first-best implementation requires $\mu > 1$, use (42) to write (43) as follows:

$$(1-\beta)\beta^{-1} K^*(\theta) + (\mu - 1)\mu^{-1} K^*(\theta_0) \geq \sigma(1-\sigma)^{-1} \left[K^*(\theta_0) - K^*(\theta)\right](1-\beta)\beta^{-1}.$$

Rearrange this expression to obtain

$$(1-\sigma)\beta(1-\beta)^{-1}(\mu-1)\mu^{-1} K^*(\theta_0) \geq \sigma K^*(\theta_0) - K^*(\theta). \tag{46}$$

For $\theta < \theta_1$, we have $\sigma K^*(\theta_0) - K^*(\theta) > 0$, which implies that the redemption constraint (46) requires $\mu > 1$. ■

Proof of Lemma 3. In the text, we discuss the properties of the functions $\Omega(\mu)$ and $\Psi(\theta)$. Existence of of a unique value μ such that $\Omega(\mu) = \Psi(\theta)$ requires that $\Omega(\mu) > \Psi(\theta)$ for some μ. Here, we show that a sufficient condition is $\beta > \sigma$ (as assumed throughout the paper). To see this, set $\theta = 0$ to obtain $\Psi(0) = 1$. In this case, $\Omega(\mu) > \Psi(\theta) = 1$ reduces to

$$(1-\sigma)\left[\beta(1-\beta)^{-1}(\mu-1)\mu^{-1} + 1\right] > 1.$$

This inequality can be written as follows:

$$\mu\left(1 - \frac{\sigma}{1-\sigma}\frac{1-\beta}{\beta}\right) > 1.$$

This shows two things. First, for $\theta = 0$ (this is the fiat money case), we need money creation $\mu > 1$. Second, we also need $\beta > \sigma$.

Finally, we need to show that the equilibrium also satisfies the sequential rationality constraint (30). Further below, we show (see Lemma 4) that for an active mechanism a sufficient condition for (30) to hold is

$$\frac{u(q^*)}{q^*} \geq \frac{1-\sigma}{\sigma},$$

in order to implement the first-best allocation. This condition requires that the gains from trading are sufficiently large. Either, given the trading probability σ, the match surplus $u(q^*) - q^*$ has to be sufficiently large, or, given the match surplus, the trading probability has to be sufficiently large. ■

Lemma 4 (Sequential rationality) *Under an active policy, in order to implement the first-best allocation, a sufficient condition for sequential rationality (30) is*

$$\frac{u(q^*)}{q^*} \geq \frac{1-\sigma}{\sigma}. \tag{47}$$

Proof of Lemma 4. In order to derive a sufficient condition for voluntary participation in the AM, we need to calculate the equilibrium payoff for an agent who holds zero ABM at the beginning of the PM and compare his expected lifetime utility with the lifetime utility of a deviator who enters the AM with zero ABM. Since the one-time deviation principle holds, we can focus on a strategy that deviates for one period after which it reverts to the equilibrium strategy.

The equilibrium payoff for an agent who enters the PM with zero ABM is

$$W(0) = \theta U(c^*) - \left[c^* + \phi_2 m^+\right]/w(k^*) + \beta V(m^+),$$

where $V(m^+)$ is calculated as follows. In the AM, with probability σ the agent produces, with probability σ he consumes, and, with probability $(1-2\sigma)$ he is idle. Accordingly, on the equilibrium path, the value function satisfies

$$V(m^+) = \sigma\left[-q^* + W(2m^+)\right] + \sigma\left[u(q^*) + W(0)\right] + (1-2\sigma)W(m^+).$$

The payoff for a one-time deviation is calculated as follows. A deviator chooses $m^+ = 0$ and $c = c^*$, so we have

$$\tilde{W}(0) = \theta U(c^*) - c^*/w(k^*) + \beta\tilde{V}(0).$$

$\tilde{V}(0)$ is calculated as follows. In the AM, with probability σ the deviator produces, and with probability $(1-\sigma)$ he is idle. Accordingly, the value function of a deviator satisfies

$$\tilde{V}(0) = \sigma\left[-q^* + W(m^+)\right] + (1-\sigma)W(0).$$

Sequential rationality requires that $W(0) \geq \tilde{W}(0)$; that is,

$$-\phi_2 m^+/w(k^*) + \beta V(m^+) \geq \beta\tilde{V}(0).$$

Substituting $\beta V(m^+)$ and $\beta\tilde{V}(0)$ yields

$$\begin{aligned}-\phi_2 m^+/w(k^*) + \beta\left\{\sigma\left[-q^* + W(2m^+)\right] + \sigma\left[u(q^*) + W(0)\right] + (1-2\sigma)W(m^+)\right\} \\ \geq \beta\left\{\sigma\left[-q^* + W(m^+)\right] + (1-\sigma)W(0)\right\}.\end{aligned}$$

Simplifying yields

$$-\phi_2 m^+/w(k^*) + \beta\left\{\sigma W(2m^+) + \sigma\left[u(q^*) + W(0)\right] + (1-2\sigma)W(m^+)\right\}$$
$$\geq \beta\left\{\sigma W(m^+) + (1-\sigma)W(0)\right\}.$$

From (13), on the equilibrium path, we have

$$W(m) = W(0) + \left[(\phi_2 + \chi d)m - \chi\tau\right]/w(k^*),$$

where $W(0) = \theta U(c^*) - \left[c^* + \phi_2 m^+\right]/w(k^*) + \beta V(m^+)$. Thus, we have the following:

$$\begin{aligned} W(m^+) &= W(0) + \left[m^+\left(\phi_2^+ + d^+\right) - \tau\right]/w(k^*) \\ W(2m^+) &= W(0) + \left[2m^+\left(\phi_2^+ + d^+\right) - \tau\right]/w(k^*). \end{aligned}$$

Use these expressions to write the inequality as follows:

$$-\phi_2 m^+ + \beta\left\{\sigma\left[2m^+\left(\phi_2^+ + d^+\right) - \tau\right] + \sigma u(q^*)w(k^*) + (1-2\sigma)\left[m^+\left(\phi_2^+ + d^+\right) - \tau\right]\right\}$$
$$\geq \beta\left\{\sigma\left[m^+\left(\phi_2^+ + d^+\right) - \tau\right]\right\}.$$

Simplifying yields

$$-\phi_2 m^+ + \beta\left\{\sigma u(q^*)w(k^*) + m^+\left(\phi_2^+ + d^+\right) - \tau + \sigma\tau\right\} \geq \beta\left\{\sigma\left[m^+\left(\phi_2^+ + d^+\right) - \tau\right]\right\}.$$

Simplifying yields

$$-\phi_2 m^+ + \beta\left\{\sigma u(q^*)w(k^*) + (1-\sigma)m^+\left(\phi_2^+ + d^+\right) - (1-2\sigma)\tau\right\} \geq 0.$$

Note that increasing the fee τ makes it harder to fulfill this condition, since $\sigma < 1/2$. To obtain a sufficient condition, we therefore set $\tau = m^+ d^+$, since in any equilibrium, $m^+ d^+ \geq \tau$. For $\tau = m^+ d^+$, we obtain

$$-\mu\phi_2^+ m^+ + \beta\left\{\sigma u(q^*)w(k^*) + (1-\sigma)m^+\phi_2^+ + \sigma m^+ d^+\right\} \geq 0,$$

since $\phi_2 = \mu\phi_2^+$. We can use (19); that is, $\phi_1 = (1/w)(\phi_2 + d)$ to replace d to obtain

$$-\mu\phi_2^+ m^+ + \beta\left\{\sigma u(q^*)w(k^*) + (1-\sigma)m^+\phi_2^+ + \sigma\left[\phi_1^+ m^+ w(k^*) - m^+\phi_2^+\right]\right\} \geq 0.$$

Since $\mu\phi_2 m = \beta q^* w(k^*)$ and $\phi_1^+ m^+ = q^*$, we can simplify this expression as follows:

$$\sigma\frac{u(q^*)}{q^*} + (1-2\sigma)\beta/\mu \geq 1-\sigma.$$

For $\mu = \beta$, this inequality is always satisfied. It is most difficult to satisfy as μ approaches infinity. In this case, we are left with

$$\frac{u(q^*)}{q^*} \geq \frac{1-\sigma}{\sigma}.$$

■

Lemma 5 *Consider a mechanism that targets $K = K^*(\theta)$. For such a mechanism, $k = k^*$.*

Proof of Lemma 5. We want to show that if $K = K^*(\theta)$, then $N = N^*$, implying that $k = k^*$. On the equilibrium path, the aggregate labor supply is

$$N = \sigma n(0) + \sigma n(2M) + (1 - 2\sigma) n(M),$$

where $n(m)$ is the labor supply of an agent that holds m units of money when entering the PM. From (13), on the equilibrium path, individual labor supply satisfies

$$n = \left[c^* - (\phi_2 + \chi d)m + \phi_2 m^+ + \chi\tau\right] / w(k),$$

implying that

$$\begin{aligned} n(0) &= [c^* + \phi_2 \mu M] / w(k) \\ n(M) &= [c^* - (\phi_2 + d)M + \phi_2 \mu M + \tau] / w(k) \\ n(2M) &= [c^* - (\phi_2 + d)2M + \phi_2 \mu M + \tau] / w(k). \end{aligned}$$

Using the expressions, we can write the aggregate labor supply as follows:

$$\begin{aligned} w(k) N &= \sigma [c^* + \phi_2 \mu M] + \sigma [c^* - (\phi_2 + d)2M + \phi_2 \mu M + \tau] \\ &\quad + (1 - 2\sigma) [c^* - (\phi_2 + d)M + \phi_2 \mu M + \tau]. \end{aligned}$$

Simplifying yields

$$w(k) N = c^* + (1 - \sigma)\tau - dM + (\mu - 1)\phi_2 M.$$

Using the budget constraint of the mechanism $D = dM = [r(k) - \delta] K^* + (\mu - 1)\phi_2 M + \tau(1 - \sigma)$, we obtain

$$w(K^*/N) N = c^* - [r(K^*/N) - \delta] K^*.$$

From (8), we have $r(k) = f'(k)$, and so

$$w(K^*/N) N + f'(K^*/N) K^* = c^* + \delta K^*.$$

The left-hand side is aggregate output and the right-hand side is aggregate demand. Since aggregate demand is at the efficient level, we must have $N = N^*$ and, thus, $k = k^*$.

■

References

[1] Andolfatto, D. (2010). "Essential Interest-Bearing Money," *Journal of Economic Theory* 145, 1319-1602.

[2] Aruoba, S.B. and R. Wright (2003). "Search, Money and Capital: A Neoclassical Dichotomy," *Journal of Money, Credit, and Banking* 35, 1086-1105.

[3] Berentsen, A. (2006). "On the Private Provision of Fiat Currency," *European Economic Review* 50, 1683-1698.

[4] Berentsen, A., G. Camera and C. Waller (2005). "The Distribution of Money Balances and the Non-Neutrality of Money," *International Economic Review* 46, 465-487.

[5] Berentsen, A., G. Camera and C. Waller (2007). "Money, Credit, and Banking," *Journal of Economic Theory* , 135(1), 171-195.

[6] Barro, R. (1979). "Money and the Price Level Under the Gold Standard," *Economic Journal* 89, 13-33.

[7] Fama, E. (1983). "Financial Intermediation and Price Level Control," *Journal of Monetary Economics* 12, 7-28.

[8] Fratianni, M. (2006). "Government Debt, Reputation, and Creditors' Protection: The Tale of San Giorgio," *Review of Finance*, 10:(4), 487-506.

[9] Friedman, M. (1969). "The Optimum Quantity of Money." In the Optimum Quantity of Money and Other Essays (Aldine. Chicago. IL) 1-50.

[10] Friedman, M. and A. Schwartz (1986). "Has Government Any Role in Money?" *Journal of Monetary Economics* 17, 37-62.

[11] Geromichalos, A., J. Licari and J. Suarez Lledo (2007). "Asset Prices and Monetary Policy," *Review of Economic Dynamics* 10, 761-779.

[12] Hayek, F. (1976). Denationalization of Money, Institute of Economic Affairs, London (1978, 2nd extended ed.).

[13] Helpman, E. (1983). "Comments on 'Financial Intermediation and Price Level Control'," *Journal of Monetary Economics* 12, 29-31.

[14] Hu T.-W. and G. Rocheteau (2014). "Monetary Policy and Asset Prices: A Mechanism Design Approach," *Journal of Money Credit and Banking*, forthcoming.

[15] Hu, T.-W. and G. Rocheteau (2014). "On the Coexistence of Money and Higher-Return Assets and Its Social Role," *Journal of Economic Theory* 148, 2520-2560.

[16] Jacquet, N. and S. Tan (2012). "Money and Asset Prices with Uninsurable Risks," *Journal of Monetary Economics* 59, 784-797.

[17] King, R. (1983). "On the Economics of Private Money," *Journal of Monetary Economics* 12, 127-158.

[18] Klein, B. (1974). "The Competitive Supply of Money," *Journal of Money, Credit, and Banking* 6, 423-453.

[19] Klein, B. (1976). "Competing Monies," *Journal of Money, Credit, and Banking* 8, 513-519.

[20] Lagos, R. and G. Rocheteau (2008). "Money and Capital as Competing Media of Exchange," *Journal of Economic Theory* 142, 247-258.

[21] Lagos, R. and R. Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* 113, 463-484.

[22] Lester B., A. Postlewaite, and R. Wright (2012). "Information, Liquidity, Asset Prices and Monetary Policy," *Review of Economic Studies* 79, 1209-1238.

[23] Levine, D. (1991). "Asset Trading Mechanisms and Expansionary Policy," *Journal of Economic Theory*, 54(1), 148-164.

[24] Molico, M. (2006). "The Distribution of Money and Prices in Search Equilibrium," *International Economic Review* 47(3), 701-722.

[25] Rocheteau, G. (2011). "Payments and Liquidity under Adverse Selection," *Journal of Monetary Economics* 58, 191-205.

[26] Rocheteau, G. and R. Wright (2005). "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica* 73, 175-202.

[27] Sargent, T. and N. Wallace (1983). "A Model of Commodity Money," *Journal of Monetary Economics* 2, 163-187.

[28] Wallace, N. (1983). "Legal Restrictions Theory of the Demand for 'Money' and the Role of Monetary Policy," Federal Reserve Bank of Minneapolis Quarterly Review.

[29] Waller, C. (2003). "Comment on 'Search, Money and Capital: A Neoclassical Dichotomy'," *Journal of Money, Credit and Banking*, 35(6, Part 2), 1111-1117.

[30] Waller, C. (2011). "Random Matching and Money in the Neoclassical Growth Model: Some Analytical Results," *Macroeconomic Dynamics* 15(S2), 293-312.