University of Zurich

Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 101

Degreasing the Wheels of Finance

Aleksander Berentsen, Samuel Huber and Alessandro Marchesiani

December 2012

Degreasing the Wheels of Finance

Aleksander Berentsen
University of Basel and Federal Reserve Bank of St. Louis

Samuel Huber
University of Basel

Alessandro Marchesiani
University of Minho

December 17, 2012

Abstract

Can there be too much trading in financial markets? To address this question, we construct a dynamic general equilibrium model, where agents face idiosyncratic preference and technology shocks. A financial market allows agents to adjust their portfolio of liquid and illiquid assets in response to these shocks. The opportunity to do so reduces the demand for the liquid asset and, hence, its value. The optimal policy response is to restrict (but not eliminate) access to the financial market. The reason for this result is that the portfolio choice exhibits a pecuniary externality: An agent does not take into account that by holding more of the liquid asset, he not only acquires additional insurance but also marginally increases the value of the liquid asset which improves insurance for other market participants.

1 Introduction

Policy makers sometimes propose and implement measures that prevent agents readjusting their portfolios frequently. A case in point are holding periods or differential tax treatments, where capital gains taxes depend on the holding period of an asset. This paper addresses a basic question: Can it be optimal to increase frictions in financial markets in order to reduce the frequency of trading? Or, to phrase this question differently: Can the frequency at which agents trade in financial markets be too high from a societal point of view?

The main message of our paper is that restricting access to financial markets can be welfare-improving. At first, this result seems to be counter-intuitive: How can it be possible that agents are better-off in a less flexible environment? The reason for this result is that in our environment the portfolio choices of agents exhibit a pecuniary externality. This externality can be so strong that the optimal policy response is to reduce the frequency at which agents can trade in financial markets; i.e., we provide an example of an environment, where degreasing the wheels of finance is optimal.

We derive this result in a dynamic general equilibrium model with two nominal assets: a liquid asset and an illiquid asset.[1] By liquid (illiquid), we mean that the asset can be used (cannot be used) as a medium of exchange in goods market trades.[2] Agents face idiosyncratic liquidity shocks, which generate an ex-post inefficiency in that some agents have "idle" liquidity holdings, while others are liquidity-constrained in the goods market. This inefficiency generates an endogenous role for a financial market, where agents can trade the liquid for the illiquid asset before trading in the goods market. We show that restricting (but not eliminating) access to this market can be welfare-improving.

The basic mechanism generating this result is as follows. The financial market has two effects. On the one hand, by reallocating the liquid asset to those agents who have an immediate need for it, it provides insurance against the idiosyncratic liquidity shocks. On the other hand, by insuring agents against the idiosyncratic liquidity shocks, it reduces the demand for the liquid asset ex-ante and thus decreases its value. This effect can be so strong that it dominates the benefits provided by the financial market in reallocating liquidity.

In a sense made precise in the paper, the financial market allows market participants to free-ride on the liquidity holdings of other participants. An agent does not take into account that by holding more of the liquid asset he not only acquires additional insurance against his own idiosyncratic liquidity risks, but he also marginally increases the value of the liquid asset which improves insurance for other market participants. This pecuniary externality can be corrected by restricting, but not eliminating, access to this financial market.

2 Literature Review

Our framework is related to the literature that studies the societal benefits of illiquid government bonds, which started with Kocherlakota's (2003) observation that if government money and government bonds are equally liquid, they should trade at par, since the latter constitutes a risk-free nominal claim against future money.[3] In practice, though, government bonds trade at a discount, indicating that they are less liquid than money.[4] Kocher-

[1] Our basic framework is the divisible money model developed in Lagos and Wright (2005). The main departure from this framework is that we add government bonds and a secondary bond market, where agents can trade bonds for money after experiencing an i.i.d. liquidity shock.

[2] We call an asset that can be exchanged for consumption goods *liquid* and one that cannot be exchanged *illiquid.* In our model, the liquid asset is fiat money and the illiquid asset is a one-period government bond. The fact that government bonds cannot be used as a medium of exchange to acquire consumption goods is the consequence of certain assumptions that we impose on our environment as explained in the main discourse of this paper.

[3] Some other papers that study the societal benefits of illiquid bonds are Boel and Camera (2006), Shi (2008), Andolfatto (2011), and Berentsen and Waller (2011). All these papers show, among other things, that Kocherlakota's result holds in a steady state equilibrium as well.

[4] According to Andolfatto (2011, p.133), the illiquidity of bonds "is commonly explained by the fact that bonds possess physical or legal characteristics that render them less liquid than money ... which raises

lakota's surprising answer to this observation is that it is socially beneficial that bonds are illiquid. The intuition for this result is that a bond that is as liquid as money is a perfect substitute for money and hence redundant, or in the words of Kocherlakota (2003, p. 184): "If bonds are as liquid as money, then people will only hold money if nominal interest rates are zero. But then the bonds can just be replaced by money: there is no difference between the two instruments at all."

Kocherlakota (2003) derives this result in a model, where agents receive a one-time i.i.d. liquidity shock after they choose their initial portfolio of money and illiquid bonds. After experiencing the shock, agents trade money for bonds in a secondary bond market. Many aspects of our environment are similar to Kocherlakota (2003) and Berentsen and Waller (2011).[5] However, our key result is different and novel. We show that it is not only optimal to create illiquid nominal bonds, but that one needs to go one step further: It can be efficient to restrict the ability of agents to trade them for money in a secondary bond market. That is, it is optimal to reduce the frequency at which agents trade money for illiquid bonds.

The Shi (2008) framework differs from Kocherlakota and the follow-up papers. In Shi, there is no secondary bond market. Rather, he assumes that agents are allowed to use bonds and money to pay for goods in some trade meetings, while they can only use money to pay for goods in some other trade meetings. He shows that such a legal restriction can be welfare-improving.[6]

It is worthwhile to present more details of the Shi (2008) framework, in order to compare it to our model. There are two types of goods: red and green. The costs of production are the same for the two colors, but the marginal utilities differ. Let θ denote the relative marginal utility for the red goods.[7] Once agents are matched, they receive a *matching shock*: with a 50 percent probability the red good is produced and with a 50 percent probability the green good is produced. In each match, buyers make a take-it-or-leave-it offer.

The key result in Shi is that the legal restriction discussed above is welfare-improving if the relative marginal utility of red goods is less than one, but not too small. The intuition

---

the question of what purpose it might serve to issue two nominally risk-free assets, with one intentionally handicapped (hence discounted) relative to the other."

[5] However, the questions studied in Berentsen and Waller (2011) are unrelated to the questions studied in this paper. The starting point of Berentsen and Waller (2011) is the observation that in monetary economies, when households face binding liquidity constraints, they can either acquire additional liquidity by selling assets or by borrowing. They show that these different methods for relaxing liquidity constraints lead to equivalent allocations under optimal policy.

[6] Since in the Shi framework agents can use money and bonds to pay for goods in some matches, it is more closely related to the literature that studies competing media of exchange (see, for example, Geromichalos, Licari, and Lledó, 2007; Lagos and Rocheteau, 2007, 2008, 2009; Lester, Postlewaite, and Wright, 2012).

[7] Formally, consumption utility is represented by the utility function $\theta^j u\left(c^j\right)$, where c^j is consumption of good j, and j denotes the good type: G (green) and R (red). θ^j is a scaling parameter. It is assumed that $\theta^G = 1$ and $\theta^R = \theta > 0$.

is that in the economy without this legal restriction agents consume the same amount of goods in all matches, because money and bonds are perfect substitutes (see Table 1, where $q_1^R = q_1^G$). This allocation is inefficient, because efficiency requires that consumption of green goods is higher than consumption of red goods (see Table 1, where the efficient quantities satisfy $q_*^R < q_*^G$). The legal restriction, thus, shifts consumption from the red good to the more highly valued green good. This smoothes marginal utilities across green and red matches, which is welfare increasing.

In Shi, the welfare improvement arises because the legal restriction shifts consumption towards the more desired good. In Kocherlakota and in our model, this mechanism is absent, since there is only one good and, hence, only one efficient quantity (see Table 1, where the efficient quantity is denoted q_*). Hence, the welfare benefits of creating illiquid bonds with a secondary bond market arises because it increases consumption. In Table 1, this result is indicated with the inequality $q_1 < q_2 < q_*$, where q_1 (q_2) is the quantity consumed when bonds are liquid (illiquid). In Shi, this effect is absent, since when $\theta = 1$, the legal restriction does not change the allocation in his framework.

Table 1: Consumed quantity in Shi, Kocherlakota and our Model[a]

Case[b]	Description	Shi (IS)[c]	Kocherlakota	Our model[d]
1)	Liquid bonds	$q_1^G = q_1^R < q_*^R < q_*^G$	$q_1 < q_*$	$q_1 < q_*$
2)	Ill. bonds with SBM	-	$q_1 < q_2 < q_*$	$q_1 < q_2 < q_*$
3)	Partially ill. bonds without SBM	$q_3^R < q_1^R = q_1^G < q_3^G$	-	-
4)	Ill. bonds with restricted SBM	-	-	$q_4^{NA} < q_4^A < q_*$

[a]Variables labeled with a * indicate efficient quantities. In Shi, the efficient quantities differ in green matches, q_*^G, and red matches, q_*^R, while in our model there is only one efficient quantity, q_*. [b]The lower-case index of the variables refers to the equilibrium quantities in the following cases: 1) An economy with liquid bonds; 2) An economy with illiquid bonds and with unrestricted access to the secondary bond market (SBM); 3) An economy with partially liquid bonds without a secondary bond market; 4) An economy with illiquid bonds and with restricted access to the secondary bond market. [c]IS refers to the "Imperfect Substitutability" equilibrium in Shi. [d]q_4^A and q_4^{NA} are the consumption quantities for agents that have access and no access to the SBM in our model.

We next consider our key result, which is that it can be welfare-improving to set $\pi < 1$. There are two effects of such a policy. First, it introduces variance in the marginal utilities across matches, since agents who have access trade different quantities than agents who have no access. In Table 1, the former trade the quantity q_4^A, and the latter trade the quantity q_4^{NA}, with $q_4^{NA} < q_4^A$. Introducing consumption variability is clearly inefficient. Nevertheless, since this policy can increase both consumption quantities, it can be welfare-improving. Since our mechanism adds a wedge between marginal utilities across matches, while Shi's mechanism reduces such a wedge, it should be clear that our mechanism of reducing access to the secondary bond market is very different from Shi's legal restriction model.

Our paper is also related to the macroeconomic literature that studies the implications of pecuniary externalities for welfare (e.g., Caballero and Krishnamurthy, 2003; Lorenzoni, 2008; Bianchi and Mendoza, 2011; Jeanne and Korinek, 2012; Korinek, 2012). In this literature, the fundamental friction is limited commitment; i.e., agents have a limited ability to commit to future repayments. Due to this friction, borrowing requires collateral, and a pecuniary externality arises, because agents do not take into account how their borrowing decisions affect collateral prices, and through them the borrowing constraints of other agents. As a consequence, the equilibrium is characterized by overborrowing, which is defined as "the difference between the amount of credit that an agent obtains acting atomistically in an environment with a given set of credit frictions, and the amount obtained by a social planner who faces the same frictions but internalizes the general-equilibrium effects of its borrowing decisions" (see Bianchi and Mendoza, 2011, p.1).[8]

This pecuniary externality effect has been used to study *credit booms* and *busts*. In a model with competitive financial contracts and aggregate shocks, Lorenzoni (2008) identifies excessive borrowing ex-ante and excessive volatility ex-post. In Bianchi and Mendoza (2011), cyclical dynamics lead to a period of credit expansion up to the point where the collateral constraint becomes binding, followed by sharp decreases in credit, asset prices and macroeconomic aggregates (see also Mendoza and Smith, 2006; Mendoza, 2010). Jeanne and Korinek (2012) study the optimal policy involved in credit booms and busts. They find that it is optimal to impose cyclical taxes to prevent agents from excessive borrowing. They emphasize that the level of the tax be adjusted for the vulnerability of each sector in the economy.

In all of these papers, agents do not internalize the effect of fire-sales on the value of other agents' assets, and, therefore, they overborrow ex-ante. Our paper differs from this literature, because it is *not* a model of crisis: there are neither aggregate shocks nor multiple steady-state equilibria. The pecuniary externality is present in "normal" times; i.e., in the unique steady state equilibrium.[9] Second, we propose a novel policy response to internalize

[8] Related to this literature are studies on financial accelerators (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997) or endogenous borrowing constraints (e.g, Kehoe and Levine, 1993; Berentsen, Camera, and Waller, 2007).

[9] Rojas-Breu (2013) also identifies a pecuniary externality that is present in the steady state equilibrium. In her model, some agents use credit cards and some fiat money to acquire consumption goods. She shows that restricting the use of credit cards can be welfare improving. The intuition for this result is that marginally increasing the fraction of agents that use credit cards can have a general equilibrium effect on the price level, which makes the agents that have no credit card worse off. This effect can be so strong that overall welfare decreases. In contrast to our model, in her model restricting the use of credit cards is a local optimum only, since it would be optimal to endow all agents with credit cards.

the pecuniary externality by showing that reducing the frequency of trading can be optimal. In contrast, Jeanne and Korinek (2012) propose a Pigouvian tax on borrowing and Bianchi (2011) proposes a tax on debt to internalize the pecuniary externality.

3 The Model

Time is discrete, and in each period there are three markets which open sequentially.[10] In the first market, agents trade money for nominal bonds. We refer to this market as the *secondary bond market*. In the second market, agents produce or consume market-2 goods. We refer to this market as the *goods market*. In the third market, agents consume and produce market-3 goods, receive money for maturing bonds, and acquire newly issued bonds. We refer to this market as the *primary bond market*. All goods are nonstorable, which means that they cannot be carried from one market to the next.

There is a $[0, 1]$ continuum of infinitely lived agents. At the beginning of each period, agents receive two idiosyncratic i.i.d. shocks: a preference shock and an entry shock. The preference shock determines whether an agent can produce or consume market-2 goods. With probability $1 - n$ an agent can consume but not produce, and with probability n he can produce but not consume. Consumers in the goods market are called *buyers*, and producers are called *sellers*. The entry shock determines whether agents can participate in the secondary bond market. With probability π they can, and with probability $1 - \pi$ they cannot. Agents who participate in the secondary bond market are called *active*, while agents who do not are called *passive*. For active agents, trading in the secondary bond market is frictionless.

In the goods market, agents meet at random in bilateral meetings. We represent trading frictions by using a reduced-form matching function, $\zeta\mathcal{M}(n, 1 - n)$, where $\zeta\mathcal{M}$ specifies the number of trade matches in a period and the parameter ζ is a scaling variable, which determines the efficiency of the matching process (see e.g., Rocheteau and Weill, 2011). We assume that the matching function has constant returns to scale, and is continuous and increasing with respect to each of its arguments. Let $\delta(n) = \zeta\mathcal{M}(n, 1 - n)(1 - n)^{-1}$ be the probability that a buyer meets a seller. The probability that a seller meets a buyer is denoted by $\delta^s(n) = \delta(n)(1 - n)n^{-1}$. In what follows, to economize on notation, we suppress the argument n, and refer to these probabilities as δ and δ^s, respectively.

In the goods market, buyers get utility $u(q)$ from consuming q units of market-2 goods, where $u'(q), -u''(q) > 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Sellers incur the utility cost $c(q) = q$

[10] Our basic framework is the divisible money model developed in Lagos and Wright (2005). This model is useful, because it allows us to introduce heterogeneous preferences while still keeping the distribution of asset holdings analytically tractable. The main departure from Lagos and Wright (2005) is that we add a secondary bond market.

from producing q units of market-2 goods.[11]

As in Lagos and Wright (2005), we impose assumptions that yield a degenerate distribution of portfolios at the beginning of the secondary bond market. That is, we assume that trading in the primary bond market is frictionless, that all agents can produce and consume market-3 goods, and that the production technology is linear such that h units of time produce h units of market-3 goods. The utility of consuming x units of goods is $U(x)$, where $U'(x), -U''(x) > 0$, $U'(0) = \infty$, and $U'(\infty) = 0$.

Finally, agents discount between, but not within, periods. The discount factor between two consecutive periods is $\beta = 1/(1+r)$, where $r > 0$ is the real interest rate.

3.1 First-best allocation

For a benchmark, it is useful to derive the planner allocation. The planner treats all agents symmetrically. His optimization problem is

$$\mathcal{W} = \max_{h,x,q} \left[\delta (1-n) u(q) - \delta^s nq\right] + U(x) - h, \tag{1}$$

subject to the feasibility constraint $h \geq x$. The efficient allocation satisfies $U'(x^*) = 1$, $u'(q^*) = 1$, and $h^* = x^*$. These are the quantities chosen by a social planner who dictates consumption and production.[12]

3.2 Pricing mechanism

In what follows, we study the allocations that are attainable in a market economy. To this end, we assume that the primary and secondary bond markets are characterized by perfect competition. In contrast, we will investigate several pricing mechanism for the goods market. The baseline case is random matching and generalized Nash bargaining. However, we will also study random matching with Kalai bargaining, price-taking and competitive search. We are in particular interested in how the different pricing mechanisms affect the portfolio choices of the agents in the primary and the secondary bond markets.

3.3 Money and bonds

The description of the environment in this subsection closely follows Berentsen and Waller (2011).[13] There are two perfectly divisible and storable financial assets: money and one-

[11] We assume a linear utility cost for ease of exposition. It is a simple generalization to allow for a more general convex disutility cost.

[12] Since our planner can dictate quantities, there is no need for either money or bonds to achieve the first-best allocation.

[13] However, the questions investigated in Berentsen and Waller (2011) are different to the questions studied in this paper (see the literature review).

period, nominal discount bonds. Both are intrinsically useless, since they are neither arguments of any utility function nor are they arguments of any production function. Both assets are issued by the central bank in the last market. Bonds are payable to the bearer and default free. One bond pays off one unit of currency in the last market of the following period.

At the beginning of a period, after the idiosyncratic shocks are revealed, agents can trade bonds and money in the perfectly competitive secondary bond market. The central bank acts as the intermediary for all bond trades by recording purchases and sales of bonds. Bonds are book-keeping entries – no physical object exists. This implies that agents are not anonymous to the central bank. Nevertheless, despite having a record-keeping technology over bond trades, the central bank has no record-keeping technology over goods trades. Since agents are anonymous and cannot commit, a buyer's promise in the goods market to deliver bonds to a seller in the primary bond market is not credible.

Since bonds are intangible objects, they cannot be used as a medium of exchange in the goods market: hence they are illiquid. It has been shown in Kocherlakota (2003), Andolfatto (2011), and Berentsen and Waller (2011) that in similar environments to the one studied here, it is optimal that bonds are illiquid. All these papers assume unrestricted access to bond markets. One of our contributions to this literature is to show that it is not only optimal that bonds are illiquid, but that it can be optimal to reduce their liquidity further by restricting access to bond markets.

To motivate a role for fiat money, search models of money typically impose three assumptions on the exchange process (Shi 2008):[14] a double coincidence problem, anonymity, and costly communication. First, our preference structure creates a single-coincidence problem in the goods market, since buyers do not have a good desired by sellers. Second, agents in the goods market are anonymous, which rules out trade credit between individual buyers and sellers. Third, there is no public communication of individual trading outcomes (public memory), which, in turn, eliminates the use of social punishments in support of gift-giving equilibria. The combination of these frictions implies that sellers require immediate compensation from buyers. In short, there must be immediate settlement with some durable asset, and money is the only such durable asset. These are the micro-founded frictions that make money essential for trade in the goods market. In contrast, in the last market all agents can produce for their own consumption or use money balances acquired earlier. In this market, money is not essential for trade.[15]

Denote M_t as the per capita money stock and B_t as the per capita stock of newly issued

[14] See also Araujo (2004), Kocherlakota (1998), Wallace (2001), and Aliprantis, Camera and Puzzello (2007) for discussions of what makes money essential.

[15] One can think of agents as being able to barter perfectly in this market. Obviously in such an environment, money is not needed.

bonds at the end of period t. Then M_{t-1} (B_{t-1}) is the beginning-of-period money (bond) stock in period t. Let ρ_t denote the price of bonds in the primary bond market. Then, the change in the money stock in period t is given by

$$M_t - M_{t-1} = \tau_t M_{t-1} + B_{t-1} - \rho_t B_t. \tag{2}$$

The change in the money supply at time t is given by three components: a lump-sum money transfer ($T = \tau_t M_{t-1}$); the money created to redeem B_{t-1} units of bonds; and the money withdrawal from selling B_t units of bonds at the price ρ_t. We assume there are positive initial stocks of money M_0 and bonds B_0, with $\frac{B_0}{M_0} > \frac{n}{1-n}$. For $\tau_t < 0$, the government must be able to extract money via lump-sum taxes from the economy.

4 Agent's Decisions

For notational simplicity, the time subscript t is omitted when understood. Next-period variables are indexed by $+1$, and previous-period variables are indexed by -1. In what follows, we look at a representative period t and work backwards, from the primary bond market (the last market) to the secondary bond market (the first market).

4.1 Primary bond market

In the primary bond market, agents can consume and produce market-3 goods. Furthermore, they receive money for maturing bonds, buy newly issued bonds, adjust their money balances by trading money for goods, and receive the lump-sum money transfer T. An agent entering the primary bond market with m units of money and b units of bonds has the indirect utility function $V_3(m, b)$. An agent's decision problem in the primary bond market is

$$V_3(m, b) = \max_{x,h,m_{+1},b_{+1}} \left[U(x) - h + \beta V_1(m_{+1}, b_{+1})\right], \tag{3}$$

subject to

$$x + \phi m_{+1} + \phi \rho b_{+1} = h + \phi m + \phi b + \phi T, \tag{4}$$

where ϕ is the price of money in terms of market-3 goods. The first-order conditions with respect to m_{+1}, b_{+1} and x are $U'(x) = 1$, and

$$\frac{\beta \partial V_1}{\partial m_{+1}} = \rho^{-1} \frac{\beta \partial V_1}{\partial b_{+1}} = \phi, \tag{5}$$

where the term $\beta \partial V_1 / \partial m_{+1}$ ($\beta \partial V_1 / \partial b_{+1}$) is the marginal benefit of taking one additional unit of money (bonds) into the next period, and ϕ ($\rho\phi$) is the marginal cost of doing so.

Due to the quasi-linearity of preferences, the choices of b_{+1} and m_{+1} are independent of b and m. It is straightforward to show that all agents exit the primary bond market with the same portfolio of bonds and money. The envelope conditions are

$$\frac{\partial V_3}{\partial m} = \frac{\partial V_3}{\partial b} = \phi. \tag{6}$$

According to (6), the marginal value of money and bonds at the beginning of the primary bond market is equal to the price of money in terms of market-3 goods. Note that equations (6) imply that the value function V_3 is linear in m and b.

4.2 Goods market

For the goods market, we make various assumptions of how the terms of trade are determined. The baseline case is random matching and generalized Nash bargaining. In section 5.5, we also consider competitive pricing, competitive search, and Kalai bargaining.

Generalized Nash Bargaining A matched buyer and seller bargain over the terms of trade (q, d), where q is the quantity of goods and d is the amount of money exchanged in the match. In what follows, we assume that the bargaining outcome satisfies the generalized Nash bargaining solution.

The seller's net payoff in a meeting in the goods market is given by $-c(q) + V_3(m + d, b) - V_3(m, b)$ and the buyer's net payoff is given by $u(q) + V_3(m - d, b) - V_3(m, b)$. Using the linearity of V_3 with respect to m and b, the bargaining problem can be formulated as follows:

$$\begin{gathered}(q, d) = \arg\max \left[u(q) - \phi d\right]^{\theta} \left[-c(q) + \phi d\right]^{1-\theta} \\ s.t.\ d \leq m.\end{gathered} \tag{7}$$

where θ is the buyer's bargaining weight, and m is the buyer's money holding. The constraint states that the buyer cannot spend more money than the amount he brought into the match. If the buyer's constraint is nonbinding, the solution is given by $d < m$ and $q = q^*$, where q^* satisfies $u'(q^*) = 1$. If the buyer's constraint is binding, the solution is given by $d = m$ and

$$\theta u'(q)\left[-c(q) + \phi d\right] = (1 - \theta)\, c'(q)\left[u(q) - \phi d\right]. \tag{8}$$

This latter condition can be written as follows:

$$\phi m = z\left(q\right) \equiv \frac{\theta c\left(q\right) u'(q) + \left(1-\theta\right) u(q) c'\left(q\right)}{\theta u'(q) + \left(1-\theta\right) c'\left(q\right)}. \tag{9}$$

This is by now a routine derivation of the Nash bargaining solution in a Lagos and Wright-type model. More details can be found in Lagos and Wright (2005) or Nosal and Rocheteau (2011).

Value functions The value function of a buyer entering the goods market with m units of money and b units of bonds is

$$V_2^b(m,b) = \delta\left[u\left(q\right) + V_3\left(m-d,b\right)\right] + \left(1-\delta\right) V_3(m,b). \tag{10}$$

With probability δ, the buyer has a match and the terms of trade are (q,d). Under these terms, he receives consumption utility $u\left(q\right)$ and expected continuation utility $V_3\left(m-d,b\right)$. With probability $1-\delta$ he has no match and receives expected continuation utility $V_3(m,b)$. To derive the marginal indirect utility of money and bonds, we take the total derivatives of (10) with respect to m and b, respectively, and use (6) to replace $\frac{\partial V_3}{\partial m}$ and $\frac{\partial V_3}{\partial b}$ to get

$$\frac{\partial V_2^b}{\partial m} = \delta\left[u'(q)\frac{\partial q}{\partial m} + \phi\left(1-\frac{\partial d}{\partial m}\right)\right] + \left(1-\delta\right)\phi \qquad \text{and} \qquad \frac{\partial V_2^b}{\partial b} = \phi. \tag{11}$$

If the buyer's constraint (7) is nonbinding, then $\frac{\partial q}{\partial m} = 0$ and $\frac{\partial d}{\partial m} = 0$. In this case, the buyer's envelope conditions (11) satisfy $\frac{\partial V_2^b}{\partial m} = \frac{\partial V_3}{\partial m} = \phi$. If the constraint is binding, then $\frac{\partial q}{\partial m} = \frac{\phi}{z'(q)}$ and $\frac{\partial d}{\partial m} = 1$. In this case, the buyer's envelope conditions (11) can be rewritten as follows:

$$\frac{\partial V_2^b}{\partial m} = \delta\phi\frac{u'(q)}{z'\left(q\right)} + \left(1-\delta\right)\phi \qquad \text{and} \qquad \frac{\partial V_2^b}{\partial b} = \phi. \tag{12}$$

The first equality simply states that a buyer's marginal utility of money has two components: With probability δ he has a match, and by spending the marginal unit he receives utility $\phi u'(q) z'\left(q\right)^{-1}$, and with probability $1-\delta$ he has no match, in which case by spending the marginal unit of money in the last market he receives utility ϕ. The second equality states that a buyer's marginal utility of bonds at the beginning of the goods market is equal to the price of money in the last market, since bonds are illiquid in the goods market.

The value function of a seller entering the goods market with m units of money and b units of bonds is

$$V_2^s(m,b) = \delta^s\left[-c\left(q\right) + V_3\left(m+d,b\right)\right] + \left(1-\delta^s\right) V_3(m,b). \tag{13}$$

The interpretation of (13) is similar to the interpretation of (10) and is omitted. Taking the total derivative of (13) with respect to m and b, respectively, and using (6) to replace $\frac{\partial V_3}{\partial m}$ and $\frac{\partial V_3}{\partial b}$, yields the seller's envelope conditions:

$$\frac{\partial V_2^s}{\partial m} = \frac{\partial V_2^s}{\partial b} = \phi. \tag{14}$$

These conditions simply state that a seller's marginal utility of money and bonds at the beginning of the goods market are equal to the price of money in the last market. The reason is that a seller has no use for these two assets in the goods market.

4.3 Secondary bond market

Let $(\hat{m}, \hat{b})$ denote the portfolio of an active agent after trading in the secondary bond market, and let φ denote the price of bonds in terms of money in the secondary bond market. Then, an active agent's budget constraint satisfies

$$\phi m + \varphi \phi b \geq \phi \hat{m} + \varphi \phi \hat{b}. \tag{15}$$

The left-hand side of (15) is the sum of the real values of money and bonds with which the agent enters the secondary bond market, and the right-hand side is the real value of the portfolio with which the agent leaves the secondary bond market.

Trading is further constrained by two short-selling constraints: Active agents cannot sell more bonds, and they cannot spend more money, than the amount they carry from the previous period: that is

$$\hat{m} \geq 0, \;\; \hat{b} \geq 0. \tag{16}$$

Let $V_1^j(m, b)$ denote the value functions of an active buyer ($j = b$) or an active seller ($j = s$). Then, an active agent's decision problem is

$$V_1^j(m, b) = \max_{\hat{m}, \hat{b}} V_2^j(\hat{m}, \hat{b}) \quad s.t. \quad \text{(15) and (16).}$$

The secondary bond market's first-order conditions for active agents are

$$\frac{\partial V_2^j}{\partial \hat{m}} = \phi \lambda^j - \lambda_m^j, \quad \text{and} \quad \frac{\partial V_2^j}{\partial \hat{b}} = \varphi \phi \lambda^j - \lambda_b^j, \tag{17}$$

where, for $j = b, s$, λ^j are the Lagrange multipliers on (15), and λ_m^j and λ_b^j are the Lagrange multipliers on (16).

Finally, let $V_1(m, b)$ denote the expected value for an agent who enters the secondary bond market with m units of money and b units of bonds before the idiosyncratic shocks are realized. Then, $V_1(m,b)$ satisfies

$$V_1(m,b) = \pi\,(1-n)\,V_1^b(m,b) + \pi n V_1^s(m,b) + (1-\pi)\,(1-n)\,V_2^b(m,b) + (1-\pi) n V_2^s(m,b).$$

Note that passive buyers and passive sellers cannot change their portfolios and so their value functions at the beginning of the secondary bond market are $V_2^b(m,b)$ and $V_2^s(m,b)$, respectively.

The envelope conditions in the secondary bond market are

$$\frac{\partial V_1}{\partial m} = \pi\phi\left[(1-n)\,\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\,\frac{\partial V_2^b}{\partial m} + n\frac{\partial V_2^s}{\partial m}\right], \tag{18}$$

$$\frac{\partial V_1}{\partial b} = \pi\phi\varphi\left[(1-n)\,\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\,\frac{\partial V_2^b}{\partial b} + n\frac{\partial V_2^s}{\partial b}\right]. \tag{19}$$

According to (18), the marginal value of money at the beginning of the period consists of four components: With probability $(1-n)\,\pi$ an agent is an active buyer, in which case he receives the shadow value of money λ^b; with probability $n\pi$ he is an active seller, in which case he receives the shadow value of money λ^s; with probability $(1-n)\,(1-\pi)$ he is a passive buyer, in which case he receives the marginal value of money at the beginning of the goods market; with probability $n\,(1-\pi)$ he is a passive seller, in which case he receives the marginal value of money at the beginning of the goods market.

5 Monetary Equilibrium

We focus on symmetric, stationary monetary equilibria, where all agents follow identical strategies and where real variables are constant over time. Let $\eta \equiv B/B_{-1}$ denote the gross growth rate of bonds, and let $\gamma \equiv M/M_{-1}$ denote the gross growth rate of the money supply. These definitions allow us to write (2) as follows:

$$\gamma - 1 - \tau = \frac{B_{-1}}{M_{-1}}\,(1-\rho\eta)\,. \tag{20}$$

In a stationary monetary equilibrium, the real stock of money must be constant; i.e., $\phi M = \phi_{+1} M_{+1}$, implying that $\gamma = \phi/\phi_{+1}$. Furthermore, the real amount of bonds must be constant; i.e., $\phi B = \phi_{-1} B_{-1}$. This implies $\eta = \gamma$, which we can use to rewrite (20) as

$$\gamma - 1 - \tau = \frac{B_0}{M_0}\,(1-\rho\gamma)\,. \tag{21}$$

The model has three types of stationary monetary equilibria. In what follows, we characterize these types of equilibria. To simplify notation, let

$$\Psi(q) \equiv \delta \frac{u'(q)}{z'(q)} + 1 - \delta. \tag{22}$$

Furthermore, in what follows we assume that $\Psi(q)$ is decreasing in q. This assumption guarantees that our stationary monetary equilibrium derived below is unique.[16]

5.1 Type I equilibrium

In a type I equilibrium, an active buyer's bond constraint in the secondary bond market does not bind, and a seller's cash constraint in the secondary bond market does not bind. In the Appendix, we show that a type I equilibrium can be characterized by the four equations stated in Lemma 1.

Lemma 1 *A type I equilibrium is a time-independent list* $\{q, \hat{q}, \rho, \varphi\}$ *satisfying*

$$\begin{aligned} \varphi &= 1, & (23)\\ \frac{\gamma}{\beta} &= \pi + (1-\pi)\left[(1-n)\Psi(q) + n\right], & (24)\\ \rho &= \frac{\beta}{\gamma}, & (25)\\ u'(\hat{q}) &= z'(\hat{q}). & (26) \end{aligned}$$

In a type I equilibrium, the seller's cash constraint in the secondary bond market does not bind. This can only be the case if he is indifferent between holding money or bonds, which requires $\varphi = 1$; i.e., that equation (23) holds. According to (25), the price of bonds in the primary bond market is equal to its fundamental value β/γ. The reason for this result is that bonds in the primary market attain no liquidity premium (see our discussion below), since an active buyer's constraint on bond holdings in the secondary bond market does not bind.

According to (26), active buyers consume the quantity $\hat{q}$ that satisfies $u'(\hat{q}) = z'(\hat{q})$. If $\theta < 1$, then $\hat{q} < q^*$, so they consume the inefficient quantity even as $\beta \to \gamma$. If $\theta = 1$, then $\hat{q} = q^*$, and they consume the efficient quantity. From (24), the consumed quantity for passive buyers, q, is inefficient for all θ.

[16] Lagos and Wright (2005, p. 472) investigate under which conditions $\frac{u'(q)}{z'(q)}$ is decreasing in q. They argue that $\frac{u'(q)}{z'(q)}$ "is monotone if $\theta \approx 1$, or if $c(q)$ is linear and $u'(q)$ log-concave." For a comprehensive study of existence and uniqueness of equilibrium in the Lagos and Wright framework see Wright (2010).

5.2 Type II equilibrium

In a type II equilibrium, an active buyer's bond constraint in the secondary bond market does not bind, and a seller's cash constraint in the secondary bond market binds. In the Appendix, we show that a type II equilibrium can be characterized by the four equations stated in Lemma 2.

Lemma 2 *A type II equilibrium is a time-independent list* $\{q, \hat{q}, \rho, \varphi\}$ *satisfying*

$$\frac{1}{\varphi} = \Psi(\hat{q}), \tag{27}$$

$$\frac{\gamma}{\beta} = \frac{\pi}{\varphi} + (1-\pi)\left[(1-n)\Psi(q) + n\right], \tag{28}$$

$$\rho = \frac{\beta}{\gamma}, \tag{29}$$

$$z(q) = z(\hat{q})(1-n). \tag{30}$$

The interpretations of the equilibrium equations in Lemma 2 are similar to their respective equations in Lemma 1. The key difference is that the price of bonds in the secondary bond market satisfies $\varphi < 1$. The reason is that now an active seller's constraint on money holdings is binding. Consequently, money is scarce and so buyers are willing to sell a fraction of their bonds at a discount; i.e., $\varphi < 1$. Note that a buyer's constraint on bond holdings is still nonbinding, since he is only selling a fraction of his bonds. Accordingly, the price of bonds in the primary bond market, ρ, continues to be equal to its fundamental value, β/γ, as in the type I equilibrium.

Finally, (30) reflects the fact that the cash constraints of the active and passive buyers in the goods market are binding. Consequently, consumption of market-2 goods is inefficiently low for both active and passive buyers.

5.3 Type III equilibrium

In a type III equilibrium, both the active buyer's bond constraint and the active seller's cash constraint in the secondary bond market bind. In the Appendix, we show that a type III equilibrium can be characterized by the four equations stated in Lemma 3.

Lemma 3 *A type III equilibrium is a time-independent list* $\{q, \hat{q}, \rho, \varphi\}$ *satisfying*

$$\frac{1}{\varphi} = \frac{B_0}{M_0}\frac{1-n}{n}, \tag{31}$$

$$\frac{\gamma}{\beta} = \pi\left[(1-n)\Psi(\hat{q}) + n/\varphi\right] + (1-\pi)\left[(1-n)\Psi(q) + n\right], \tag{32}$$

$$\rho = \frac{\beta}{\gamma}\left\{1 + \pi(1-n)\left[\varphi\Psi(\hat{q}) - 1\right]\right\}, \tag{33}$$

$$z(q) = z(\hat{q})(1-n). \tag{34}$$

According to (33), the price of bonds in the primary bond market ρ includes two components: the *fundamental value* of bonds, β/γ, and the *liquidity premium*, $\frac{\beta}{\gamma}\pi(1-n)\left[\varphi\Psi(\hat{q}) - 1\right]$. The liquidity premium is increasing in π and equal to zero at $\pi = 0$. In contrast, there is no liquidity premium in the type I and type II equilibria, since an active buyer's constraint on bond holdings is not binding.

Furthermore, from (31), note that the price of bonds in the secondary bond market, φ, is constant (in contrast to the type II equilibrium). The reason is that in Lemma 3, φ is obtained from the secondary bond market budget constraint, (15). In contrast, in Lemmas 1 and 2 it is obtained from the secondary bond market first-order conditions (17). Finally, (34) has the same interpretation as (30).

5.4 Regions of equilibria

In the following Lemma, we characterize three non-overlapping regions in which these different types of equilibria exist.

Proposition 1 *There exist critical values* γ_L *and* γ_H*, with* $\beta \leq \gamma_L \leq \gamma_H < \infty$*, where* γ_L *is the value of* γ *that solves* $u'(\hat{q}) = z'(\hat{q})$*, and* γ_H *is the value of* γ *that solves* $\Psi(\hat{q}) = \frac{B_0}{M_0}\frac{1-n}{n}$*. If* $\beta \leq \gamma < \gamma_L$*, equilibrium prices and quantities are characterized by Lemma 1; if* $\gamma_L \leq \gamma < \gamma_H$*, they are characterized by Lemma 2; and if* $\gamma_H \leq \gamma$*, they are characterized by Lemma 3.*

The following table summarizes the bond prices φ and ρ and the relevant multipliers in the three equilibria:

Table 2: Bond prices, equilibrium regions and multipliers

Value of φ	Value of ρ	Inflation range	Multipliers
$\varphi = [\Psi(\hat{q})]^{-1} = 1$	$\rho = \beta/\gamma$	$\beta \leq \gamma < \gamma_L$	$\lambda_m^s = \lambda_b^b = 0$
$\varphi = [\Psi(\hat{q})]^{-1} < 1$	$\rho = \beta/\gamma$	$\gamma_L \leq \gamma < \gamma_H$	$\lambda_m^s > \lambda_b^b = 0$
$\varphi = \frac{M_0}{B_0}\frac{n}{1-n}$	$\rho = \frac{\beta}{\gamma}\left\{1 + \pi(1-n)\left[\varphi\Psi(\hat{q}) - 1\right]\right\}$	$\gamma_H \leq \gamma$	$\lambda_m^s, \lambda_b^b > 0$

In the type I and II equilibria ($\beta \leq \gamma < \gamma_H$), the constraint on bond holdings of active buyers does not bind ($\lambda_b^b = 0$) in the secondary bond market. This implies that the return on bonds in the secondary bond market, $1/\varphi$, has to be equal to the expected return on money, $\Psi(\hat{q})$. It also implies that the price of bonds in the primary bond market, ρ, must equal the *fundamental value* of bonds, β/γ. The economics underlying this result are straightforward. Since active buyers do not sell all their bonds for money in the secondary bond market, bonds in the primary bond market have no *liquidity premium,* and so the Fisher equation holds; i.e., $1/\rho = \gamma/\beta$.[17]

In contrast, in the type III equilibrium, the constraint on bond holdings of active buyers binds in the secondary bond market. Consequently, bonds attain a liquidity premium, and the Fisher equation does not hold; i.e., $1/\rho < \gamma/\beta$.[18]

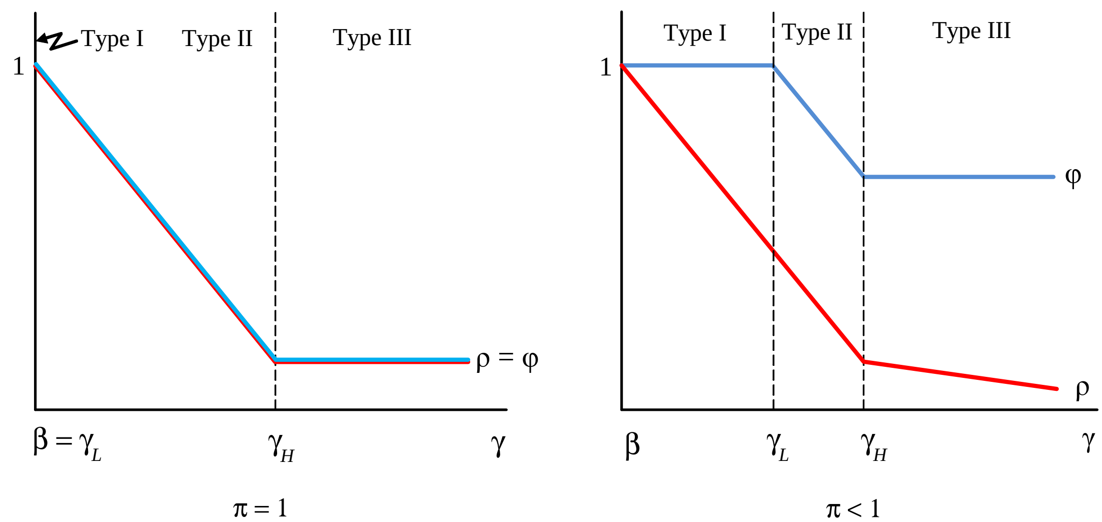

Figure 1: Bond prices when $\pi = 1$ and $\pi < 1$.

Figure 1 graphically characterizes the bond prices, φ and ρ, as a function of γ in the three types of equilibria. An interesting aspect of the model is that when $\pi = 1$, the two bond prices are equal for any value of γ.[19] Furthermore, the type I equilibrium only exists at $\gamma = \beta$. In contrast, there is a strictly positive spread $\varphi - \rho$, when $\pi < 1$ and $\gamma > \beta$.

Why is there a positive spread $\varphi - \rho$ if $\pi < 1$? If $\pi < 1$, the price ρ reflects the fact that bonds can only be traded with probability π in the secondary bond market. In contrast,

[17] The Fisher equation requires that $1/\rho = \gamma/\beta$.

[18] In a similar environment, Geromichalos and Herrenbrueck (2012) also analyze under what conditions a liquidity premium exists in the primary financial market.

[19] To see this, consider, first, equations (32) and (33). Setting $\pi = 1$ and rearranging yields $\rho = \varphi = \frac{M_0}{B_0}\frac{n}{1-n}$. Consider, next, equations (28) and (29). Again, setting $\pi = 1$ and rearranging yields $\rho = \varphi = \frac{\beta}{\gamma}$. Finally, at $\pi = 1$, the type I equilibrium only exists under the Friedman rule $\gamma = \beta$.

the price φ reflects the fact that active agents can trade bonds with probability 1 in the secondary bond market. Thus, the positive spread is because the bonds in the secondary bond market have a higher liquidity premium than the bonds in the primary bond market.

As can be seen in Figure 1, when, $\pi < 1$, the price of bonds in the secondary bond market, φ, is constant and equal to 1 in the type I equilibrium; it is decreasing in the type II equilibrium; and it is constant in the type III equilibrium. The price of bonds in the primary bond market, ρ, follows a different pattern. In the type I and type II equilibrium, it is equal to the fundamental value of bonds, β/γ, whereas in the type III equilibrium, it contains a liquidity premium. The lower π in the type III equilibrium is, the larger is the difference between φ and ρ.

5.5 Other pricing mechanisms

Here we discuss how the key equations change when we assume one of the other pricing mechanisms mentioned above. Using the Kalai bargaining solution is straightforward. Competitive pricing and competitive search are a bit more involved, and we present the derivations in the Appendix.

Kalai bargaining The Nash bargaining solution is non-monotonic (see Aruoba, Rocheteau, and Waller; 2007). In contrast, the Kalai bargaining solution, also referred to as *proportional* bargaining (Kalai; 1977), is monotonic and because of this property it is increasingly used in monetary economics.[20] It can be formalized as follows:

$$(q,d) = \arg\max u(q) - \phi d$$
$$s.t. \qquad u(q) - \phi d = \theta \left[u(q) - c\left(q\right)\right] \qquad \text{and} \qquad d \leq m.$$

When the buyer's cash constraint is binding, the solution is $d = m$ and

$$\phi m = z^{K}\left(q\right) \equiv \theta c\left(q\right) + \left(1-\theta\right) u(q), \tag{35}$$

where the superscript K refers to Kalai bargaining. When the buyer's constraint (7) is binding, the Kalai bargaining solution differs from the Nash bargaining solution unless $\theta = 0$ or $\theta = 1$. When the constraint is nonbinding, Nash bargaining and Kalai bargaining yield the same solution.

[20] The Kalai bargaining solution is discussed in Aruoba, Rocheteau, and Waller (2007) and is used, for example, in Rocheteau and Wright (2010), Lester, Postlewaite, and Wright (2012), He, Wright, and Zhu (2012), and Trejos and Wright (2012). For a textbook treatment of the Kalai bargaining solution see Nosal and Rocheteau (2011).

It is straightforward to study the model under Kalai bargaining. One only needs to replace $z(q)$ with $z^K(q)$ in Lemmas 1-3.

Competitive search Let $\eta(n)$ be the seller's contribution to the matching process; i.e., the elasticity of the matching function with respect to the measures of sellers. In the Appendix, we show that under competitive search, the terms of trade satisfy

$$\phi m = z^P(q)\text{, where} \tag{36}$$

$$z^P(q) \equiv \frac{[1-\eta(n)]\, c(q)\, u'(q) + \eta(n)\, u(q) c'(q)}{[1-\eta(n)]\, u'(q) + \eta(n)\, c'(q)}.$$

It is straightforward to study the model under competitive search. One only needs to replace $z(q)$ with $z^P(q)$ in Lemmas 1-3.

Competitive pricing Competitive pricing differs from random matching and bargaining along two dimensions. Obviously, there is no random matching, meaning that agents trade with certainty, since in competitive equilibrium buyers and sellers trade against the market. To make the results comparable, however, we assume that buyers and sellers can enter the goods market only probabilistically with probability δ and δ^s, respectively. The benefit of this assumption is that all differences in results are due to the pricing mechanism since the number of trades is equal under all pricing protocols.

The second difference is that there is no bargaining; instead, the competitive price adjusts to equate aggregate demand and aggregate supply. The market clearing condition for the goods market is

$$\delta(1-n)\left[\pi\hat{q} + (1-\pi)q\right] = \delta^s n q_s, \tag{37}$$

where $\hat{q}$ (q) is the quantity consumed by a buyer who has (no) access to the secondary bond market.

In the Appendix, we show that competitive pricing yields the same allocation as random matching and bargaining if the buyers have all the bargaining power; i.e., $\theta = 1$. In particular, the terms of trade satisfy[21]

$$z^C(q) \equiv q. \tag{38}$$

It is then straightforward to study the model under competitive pricing. One only needs to replace $z(q)$ with $z^C(q)$ in Lemmas 1-3.

[21] In general, the condition is $z^C(q_s, q) \equiv c'(q_s)\, q$, where q_s is a seller's production. With a linear cost function, $c(q_s) = q_s$, the condition reduces to $z^C(q) \equiv q$.

6 Optimal Participation

In this section, we discuss the following. First, we show that if agents have a choice to participate in the secondary bond market, they strictly prefer to do so. Second, we show that such a participation choice involves a negative pecuniary externality. Third, we discuss optimal secondary market participation.

6.1 Endogenous participation

So far, we have assumed that participation in the secondary bond market is determined by the exogenous idiosyncratic participation shock π. Suppose instead that each agent has a choice. Recall that $V_1^b(m,b)$ is the expected lifetime utility of a buyer at the beginning of the secondary bond market, and $V_2^b(m,b)$ is the expected lifetime utility of a buyer at the beginning of the goods market who had no access to the secondary bond market. Then, for a buyer, it is optimal to participate if

$$V_1^b(m,b) \geq V_2^b(m,b).$$

Note that the exact experiment here is to keep all prices at their equilibrium values for a given participation rate π, and, then, to ask the question, whether a single buyer would prefer to enter the secondary bond market. The move of a single buyer from passive to active does not change equilibrium prices.

Lemma 4 *In any equilibrium, $V_1^b(m,b) - V_2^b(m,b) \geq 0$.*

According to Lemma 4, a buyer is always better off when participating in the secondary bond market. To develop an intuition for this result, note that, as shown in the proof of Lemma 4,

$$V_1^b(m,b) - V_2^b(m,b) = u(\hat{q}) - \hat{q} - [u(q) - q] - i(\hat{q} - q), \tag{39}$$

where $i = (1-\varphi)/\varphi$ is the nominal interest rate. A passive buyer's period surplus is $u(q) - q$, while an active buyer's surplus is $u(\hat{q}) - \hat{q} - i(\hat{q} - q)$, where the term $i(\hat{q} - q)$ measures the utility cost of selling bonds to finance the difference $\hat{q} - q \geq 0$. The difference $u(\hat{q}) - \hat{q} - [u(q) - q]$ is strictly positive, while the term $-i(\hat{q} - q)$ is negative. The reason is that in any equilibrium $q \leq \hat{q} \leq q^*$. Thus, the equilibrium interest rate cannot be too large in order for (39) to be positive. In the proof of Lemma 4, we replace i in (39) for all three types of equilibria and find that $V_1^b(m,b) - V_2^b(m,b) > 0$.

We now turn to the sellers. For them, we also find that they are better off when participating in the secondary bond market.

Lemma 5 *In any equilibrium,* $V_1^s(m,b) - V_2^s(m,b) \geq 0$.

In the type I equilibrium, the nominal interest rate is $i = 0$. In this case, $V_1^s(m,b) = V_2^s(m,b)$. In the type II and type III equilibria, the nominal interest rate is $i > 0$. In this case, the seller strictly prefers to enter, since $V_1^s(m,b) > V_2^s(m,b)$.

6.2 Optimal secondary bond market participation

Lemmas 4 and 5 show that if agents have a choice, they will participate in the secondary bond market. In this section, we explain why restricting participation to the secondary bond market can be welfare-improving. The reason is straightforward. The secondary bond market provides insurance against the idiosyncratic liquidity shocks. At the end of a period in the primary bond market, agents choose a portfolio of bonds and money. At this point, they do not know yet whether they will be buyers or sellers in the following period. At the beginning of the following period, this information is revealed, and they can use the secondary bond market to readjust their portfolio of money and illiquid bonds.

From a welfare point of view, the benefit of the secondary bond market is that it allocates liquidity to the buyers and allows sellers to earn interest on their idle money holdings. The drawback of this opportunity is that the secondary bond market reduces the incentive to self-insure against the liquidity shocks. This lowers the demand for money in the primary bond market, which depresses its value. This effect can be so strong that it can be optimal to restrict access to the secondary bond market. The basic mechanism can be seen from the following welfare calculations.

The welfare function can be written as follows:

$$(1-\beta)\,\mathcal{W} = (1-n)\,\delta\left\{\pi\left[u(\hat{q}) - \hat{q}\right] + (1-\pi)\left[u(q) - q\right]\right\} + U(x^*) - x^*, \tag{40}$$

where the term in the curly brackets is an agent's expected period utility in the goods market, and $U(x^*) - x^*$ is the agent's period utility in the primary bond market.

Differentiating (40) with respect to π yields

$$\begin{aligned} \frac{1-\beta}{(1-n)\,\delta}\frac{d\mathcal{W}}{d\pi} &= \left[u(\hat{q}) - \hat{q}\right] - \left[u(q) - q\right] \\ &\quad + \pi\left[u'(\hat{q}) - 1\right]\frac{d\hat{q}}{d\pi} + (1-\pi)\left[u'(q) - 1\right]\frac{dq}{d\pi}. \end{aligned} \tag{41}$$

The contribution of the first two terms to the change in welfare is always positive, since in any equilibrium $\hat{q} \geq q$ (with strict inequality for $\gamma > \beta$). However, the derivatives $\frac{d\hat{q}}{d\pi}$ and $\frac{dq}{d\pi}$ can be negative, reflecting the fact that increasing participation reduces the incentive to self-insure against idiosyncratic liquidity risk.[22] Reducing the incentive to self-insure reduces the demand for money and hence its value, which then reduces the consumption quantities q and $\hat{q}$.

Whether restricting participation is welfare-improving depends on which of the two effects dominates. One can show that in the type I and in the type II equilibria it is always optimal to set $\pi = 1$. In contrast, restricting participation in the type III equilibrium can be welfare-improving. Whether it is depends on preferences and technology. In the following, we calibrate the model to investigate whether restricting access to the secondary bond market is optimal under reasonable parameter values.[23]

7 Quantitative Analysis

We choose a model period as one quarter. The functions $u(q)$ and $c(q)$ have the forms

$$u(q) = Aq^{1-\alpha}/(1-\alpha) \qquad \text{and} \qquad c(q) = q.$$

For the matching function, we follow Kiyotaki and Wright (1993) and choose

$$\mathcal{M}(B,S) = BS/(B+S),$$

where $B = 1 - n$ is the measure of buyers and $S = n$ the measure of sellers in the goods market. Therefore, the matching probability of a buyer in the goods market is simply given by $\delta = \zeta\mathcal{M}(B,S)/B = \zeta n$.

The parameters to be identified are as follows: (i) preference parameters: (β, A, α); (ii) technology parameters: (n, π); (iii) bargaining power: θ; (iv) policy parameters: the money growth rate γ and the bonds-to-money ratio $\mathcal{B}$. Finally, we set $\zeta = 1$ for all but one calibration, where as a robustness check we choose $\zeta = 0.5$.

To identify these parameters, we use US-data from the first quarter of 1960 to the fourth quarter of 2010. All data sources are provided in the Appendix. Table 3 lists the identification restrictions and the identified values of the parameters.

[22] A sufficient condition for these derivatives to be negative is that $\Psi(q) = \delta\frac{u'(q)}{z'(q)} + 1 - \delta$ is decreasing in q, which is an assumption throughout the paper.

[23] In an earlier version of the paper, we provided an analytical proof that if inflation is sufficiently high, it is optimal to restrict access to the secondary bond market for $u(q) = \ln(q)$ and perfect competition in the goods market.

Table 3: Calibration targets

Parameter	Target description	Target value
β	average real interest rate r	0.991
γ	average change in the consumer price index	1.01
$\mathcal{B}$	average bonds-to-money ratio	3.52
ζ	set equal to 1	1.00
π	average price of gov. bonds with a maturity of 3 months	0.987
A	average velocity of money (annual)	6.72
n	average price of gov. bonds with a remaining maturity of 7 days	0.999
θ	retail sector markup	.300

The gross growth rate of the money supply $\gamma = 1.01$ matches the average quarterly change in the consumer price index. We set $\beta = 0.991$ so that the real interest rate in the model matches the data, measured as the difference between the rate on AAA corporate bonds and the change in the consumer price index. The bonds-to-money ratio $\mathcal{B} = 3.52$ matches the average empirical bonds-to-money ratio which we calculate as the ratio of the total public debt to the M1 money stock.[24]

The parameters θ, π, n, and A are obtained by matching the following targets simultaneously. First, we set θ such that the markup in the goods market matches the retail data summarized by Faig and Jerez (2005). They provide a target markup of $\mu = 0.3$ (30 percent).[25] Second, we set π to match the average price of government bonds with a maturity of 3 months, which is $\rho = 0.987$. Note, from Proposition 1, that $\rho = 0.987 > \beta/\gamma = 0.982$ implies that we are in the type III equilibrium. Third, we interpret the price φ as the price of a government bond with a remaining maturity of 7 days; i.e., $\varphi = \rho^{4/52} = 0.999$, and we use it to calibrate n.[26] Fourth, we set A to match the average velocity of money. The model's velocity of money is[27]

$$v = \frac{Y}{\phi M_{-1}} = \frac{1 + (1-n) * \delta * [\pi z(\hat{q}) + (1-\pi)\, z(q)]}{z(q)},$$

which depends on i via q and $\hat{q}$, and on A and α via the function $z(q)$. Although there are

[24] This definition is in line with Martin (2012).

[25] See Aruoba, Waller and Wright (2011) or Berentsen, Menzio and Wright (2011) on calibrating LW-type models, including matching the markup data.

[26] We show in the robustness analysis that our results are not very sensitive to the choice of φ.

[27] The real output in the goods market is $Y_{GM} = (1-n) * \delta * [\pi \phi \hat{m} + (1-\pi)\, \phi m]$, where $\phi \hat{m} = z(\hat{q})$ and $\phi M_{-1} = \phi m = z(q)$, and the real output in the primary bond market is $Y_{PBM} = 1$. Accordingly, total real output of the economy adds up to $Y = Y_{GM} + Y_{PBM}$, and the model-implied velocity of money is $v = Y/\phi M_{-1}$.

alternative ways to fit this relationship, we set A to match the average $Y/\phi M_{-1}$, using $M1$ as our measure of money.

Our targets discussed above and summarized in Table 3 are sufficient to calibrate all but one parameter, the elasticity of the utility function α. Most calibrations of variants of the Lagos and Wright (2005) framework find α to be around 0.2. We, therefore, first present the calibration results for the value of $\alpha = 0.2$ and, then, show the effects of different values of α later on.[28]

7.1 Baseline results and robustness checks under Nash bargaining

Table 4 presents the results for the baseline calibration and four robustness checks under generalized Nash bargaining. The robustness checks are defined as follows: In the calibration labeled "markup", we target a markup in the goods market of 40 percent instead of 30 percent; in the calibration labeled "high $\mathcal{B}$", we target $\mathcal{B} = 4.5$ instead of $\mathcal{B} = 3.5$; in the calibration labeled "high φ", we target a remaining maturity of government bonds of 1 day instead of 7 days; and in the calibration labeled "low δ", we set $\zeta = 0.5$ instead of $\zeta = 1$.

TABLE 4: NASH BARGAINING[a]

	Description	baseline	markup	high $\mathcal{B}$	high φ	low δ
A	goods market utility weight	1.48	1.52	1.56	1.48	1.54
n	number of sellers	.778	.778	.818	.779	.778
θ	buyer's bargaining share	.446	.362	.470	.449	.539
π	calibrated π	.689	.650	.692	.676	.593
π^*	optimal π[b]	.698	.632	.672	.676	.587
s_{GM}	goods market size	.351	.337	.347	.347	.158

[a] Table 4 displays the calibrated values for the key parameters A, n and θ for the value of $\alpha = 0.2$. It also displays the calibrated value of π, the optimal value of π (π^*) and the size of the goods market (s_{GM}). [b] π^* is calculated numerically by searching for the welfare maximizing value of π, holding all other parameters at their calibrated values.

[28] Most monetary models that calibrate variants of the Lagos and Wright (2005) framework, set α to match the elasticity of money demand with respect to the nominal interest rate. We cannot do this, because in our framework the interest rate on ρ represents the yield on 3-month government bonds, while related studies work with the AAA Moody's corporate bond yield to calculate the elasticity of money demand. Using United States data from 1960 to 2010, we obtain an empirical elasticity of money demand with respect to the yield on 3-month government bonds of $\xi_{gov} = 0.05$. The elasticity of money demand in our model is negative by construction, which precludes the use of this target.

Table 4 presents the key parameter values for the baseline calibration and the robustness checks when $\alpha = 0.2$. To address the question of whether there is too much trading in the secondary bond market, we also calculate the optimal entry probability π^* for each case. It is calculated as follows. For each set of calibrated parameter values, we numerically search for the value of π that maximizes ex-ante welfare, defined by (40).

We find two key results. First, our calibrations always yield an entry probability π, which is strictly below 1. Second, the optimal entry probability π^* is below the calibrated entry probability for a sufficiently high markup, a high bonds-to-money ratio, a high value of φ, and a low matching probability δ. In contrast, under the baseline calibration, we find that the access to the secondary bond market should be slightly increased in order to maximize ex-ante welfare.

In Table 4, we also provide the estimates of the model-implied goods market share on total output, $s_{GM} = Y_{GM}/Y$. Under Nash bargaining, the goods market share on total output is in the area of 35% for $\zeta = 1$, and for a lower matching probability ($\zeta = 0.5$) it is in the area of 16%, which is in line with the estimates of Berentsen, Menzio and Wright (2011) and related studies.

7.2 The effect of the elasticity of the utility function

How sensitive are our results to the choice of α? In order to answer this question, we recalibrate each model presented in Table 4 for different values of $\alpha \in (0, 1)$, and draw the difference $\pi^* - \pi$ in Figure 2.

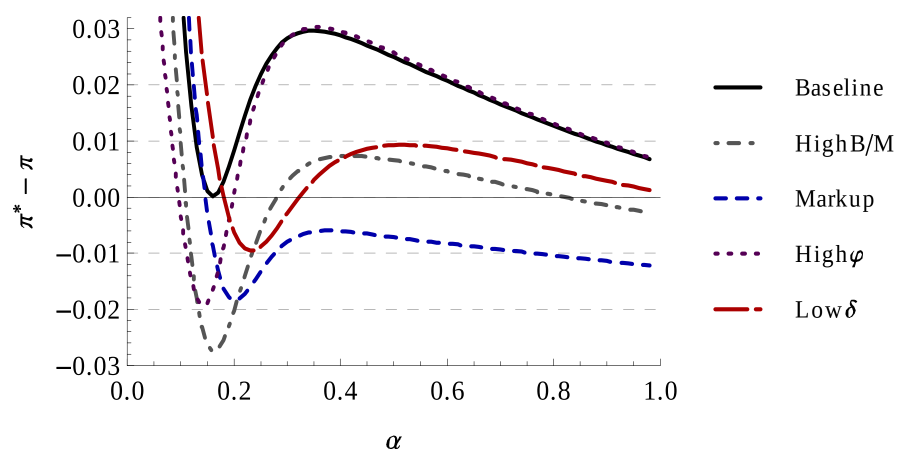

FIGURE 2: $\pi^* - \pi$ FOR DIFFERENT VALUES OF α

For the baseline calibration under Nash bargaining, we find for any $\alpha \in (0, 1)$, that $\pi < \pi^*$. In contrast, for the four robustness checks "markup", "high $\mathcal{B}$", "high φ" and "low

δ" there is a strictly positive range for which there is too much entry; i.e., $\pi > \pi^*$. A higher markup in the goods market appears to have the largest effect, since $\pi^* - \pi < 0$ for any value of $\alpha > 0.14$.

Note also that increasing the bonds-to-money ratio from 3.5 to 4.5 results in $\pi^* - \pi < 0$ for $0.1 < \alpha < 0.28$ and $0.82 < \alpha < 1$. This is insofar interesting, since in the US data the bonds-to-money ratio is steadily increasing over time in our sample, and since 1996 it is above the value of 4.5. Furthermore, in 2010 it reached 7.7.

7.3 Other pricing mechanisms

Hereafter, we compare the calibration results for different trading protocols to our baseline calibration under Nash bargaining. The calibration labeled "CS" refers to competitive search, where we set $\theta = 1 - \eta(n) = 1 - \delta'(1-n)n\delta^{-1} = n$ and where δ' is the derivative of δ with respect to the number of sellers. The calibration labeled "Kalai" refers to Kalai bargaining, where we use $z^K(q)$ instead of $z(q)$. Finally, the calibration labeled "CP" refers to competitive pricing.[29] Table 5 presents the parameter values obtained for the different pricing mechanisms for $\alpha = 0.2$.

TABLE 5: OTHER PRICING MECHANISMS[a]

	Description	baseline	CS	Kalai	CP
A	goods market utility weight	1.48	1.42	1.42	1.41
n	number of sellers	.778	.778	.778	.778
θ	buyer's bargaining power	.446	.778	.485	1
π	calibrated π	.689	.789	.668	.828
π^*	optimal π[b]	.698	.868	.735	.911
s_{GM}	goods market size	.351	.388	.344	.402

[a] Table 5 displays the calibrated values for the key parameters A, n and θ for the value of $\alpha = 0.2$. It also displays the calibrated value of π, the optimal value of π (π^*) and the size of the goods market (s_{GM}). [b] π^* is calculated numerically by searching for the welfare maximizing value of π, holding all other parameters at their calibrated values.

In contrast to Nash bargaining, for the three other trading protocols presented in Table 5, the difference of $\pi^* - \pi$ is clearly positive. This indicates that the access to the secondary bond market is too low. Note though, that the result that $\pi^* < 1$ continues to hold. That is,

[29] By setting $\theta = 1$, the model equations reduce to the ones that one obtains from assuming competitive pricing in the goods market (see Appendix II). Notice that the markup-target is only used for the baseline calibration and Kalai bargaining.

it is not optimal to grant unrestricted access to the secondary bond market. Our reading of these results is that our calibration measures the frictions in these markets, and that under certain calibrations the optimal frictions π^* are too high and in others they are too low. However, we always find that eliminating all frictions by setting $\pi = 1$ is suboptimal.

Compared to Nash bargaining, the above-presented pricing mechanisms result on average in a higher goods market share on total output. For Kalai bargaining, we obtain at the lower end $s_{GM} = 34\%$, while at the upper end we obtain $s_{GM} = 40\%$ under competitive pricing.

8 Conclusion

We construct a general equilibrium model with a liquid asset (fiat money) and an illiquid asset (a one-period government bond). Agents experience idiosyncratic liquidity shocks after which they can trade these assets in a secondary bond market. We find that an agent's portfolio choice of liquid and illiquid assets involves a pecuniary externality. An agent does not take into account that by holding more of the liquid asset he not only acquires additional insurance against his own idiosyncratic liquidity risks, but he also marginally increases the value of the liquid asset which improves insurance for other market participants. This pecuniary externality can be corrected by restricting, but not eliminating, access to the secondary bond market.

The key result is that it can be optimal to reduce the frequency of trading in financial markets. This result is consistent with current attempts by the European Parliament to introduce a financial-transactions tax, in order to generate revenue and to dampen excessive trading.

9 Appendix I: Proofs

Proof of Lemma 1. We first note that in *any* equilibrium (i.e., type I, II, and III), a buyer will never spend all his money for bonds in the secondary bond market, implying that $\lambda_m^b = 0$. Furthermore, a seller will never spend all his bonds for money in the secondary bond market, implying that $\lambda_b^s = 0$.

Furthermore, in a type I equilibrium, an active buyer's bond constraint in the secondary bond market does not bind ($\lambda_b^b = 0$), and a seller's cash constraint in the secondary bond market does not bind ($\lambda_m^s = 0$). Using these values for the multipliers, we can rewrite the secondary bond market first-order conditions (17) as follows:

$$\frac{\partial V_2^b}{\partial \hat{m}} = \phi\lambda^b \quad \text{and} \quad \frac{\partial V_2^b}{\partial \hat{b}} = \varphi\phi\lambda^b, \tag{42}$$

$$\frac{\partial V_2^s}{\partial \hat{m}} = \phi\lambda^s \quad \text{and} \quad \frac{\partial V_2^s}{\partial \hat{b}} = \varphi\phi\lambda^s. \tag{43}$$

Furthermore, combining the previous expressions with (12) and (14), we have

$$\lambda^b = \delta\frac{u'(\hat{q})}{z'(\hat{q})} + 1 - \delta \quad \text{and} \quad \varphi\lambda^b = 1, \tag{44}$$

$$\lambda^s = 1 \quad \text{and} \quad \varphi\lambda^s = 1. \tag{45}$$

Then, (45) implies that $\varphi = 1$; i.e., that (23) holds.

Then, from (44), the fact that $\varphi = 1$ immediately implies that $\lambda^b = 1$, which then implies that $u'(\hat{q}) = z'(\hat{q})$; i.e., that (26) holds.

Use (12) and (14) to write (18) and (19) as follows:

$$\frac{\partial V_1}{\partial m} = \pi\phi\left[(1-n)\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\phi\Psi(q) + n\phi\right], \tag{46}$$

$$\frac{\partial V_1}{\partial b} = \pi\phi\left[(1-n)\varphi\lambda^b + n\varphi\lambda^s\right] + (1-\pi)\left[(1-n)\phi + n\phi\right]. \tag{47}$$

Use the primary bond market first order conditions (5) to write the previous equations as follows

$$\frac{\gamma}{\beta} = \pi\left[(1-n)\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\Psi(q) + n\right], \tag{48}$$

$$\frac{\rho\gamma}{\beta} = \pi\varphi\left[(1-n)\lambda^b + n\lambda^s\right] + 1 - \pi. \tag{49}$$

We have already established that in the type I equilibrium $\lambda^b = \lambda^s = \varphi = 1$. This implies, from (49), that $\rho = \beta/\gamma$; i.e., that equation (25) holds. Finally, (24) immediately follows from (48).

Note that if $\theta < 1$, active buyers consume the inefficient quantity, since $\hat{q} < q^*$ even as $\beta \to \gamma$. If $\theta = 1$, $u'(\hat{q}) = 1$, so they consume the efficient quantity $\hat{q} = q^*$. ■

Proof of Lemma 2. We first show that equation (30) holds. In the type II equilibrium, all buyers spend all their money in the goods market. Consequently, $z(q) = m\phi$ and $z(\hat{q}) = \hat{m}\phi$ hold. The last two equations imply

$$z(q) = z(\hat{q})m/\hat{m}. \tag{50}$$

Each active buyer exits the secondary bond market with $\hat{m}$ units of money, while an active seller exits with zero units of money. A passive agent (a seller or a buyer) exits the secondary bond market with m units of money, therefore $M_{-1} = (1-n)\pi\hat{m} + n\pi * 0 + (1-\pi)\,m$. Replacing $m = M_{-1}$, we get $\hat{m} = M_{-1}/(1-n)$. Use $\hat{m} = M_{-1}/(1-n)$ and $m = M_{-1}$ to replace $\hat{m}$ and m in (50), respectively, to get $z(q) = z(\hat{q})(1-n)$; i.e., equation (30) holds.

We now show that (27)-(29) hold. As argued in the proof of Lemma 1, $\lambda_m^b = 0$ and $\lambda_b^s = 0$ in any equilibrium. In a type II equilibrium, an active buyer's bond constraint in the secondary bond market does not bind; i.e., $\lambda_b^b = 0$, and a seller's cash constraint in the secondary bond market binds; i.e., $\lambda_m^s > 0$. Using these values for the multipliers, the secondary bond market first-order conditions (17) can be rewritten as follows:

$$\frac{\partial V_2^b}{\partial \hat{m}} = \phi\lambda^b \qquad \text{and} \qquad \frac{\partial V_2^b}{\partial \hat{b}} = \varphi\phi\lambda^b, \tag{51}$$

$$\frac{\partial V_2^s}{\partial \hat{m}} = \phi\lambda^s - \lambda_m^s \qquad \text{and} \qquad \frac{\partial V_2^s}{\partial \hat{b}} = \varphi\phi\lambda^s. \tag{52}$$

Using the previous expressions in (12) and (14), we obtain

$$\lambda^b = \delta\frac{u'(\hat{q})}{z'(\hat{q})} + 1 - \delta \qquad \text{and} \qquad \varphi\lambda^b = 1, \tag{53}$$

$$\lambda_m^s = \phi\left(\lambda^s - 1\right) \qquad \text{and} \qquad \varphi\lambda^s = 1. \tag{54}$$

From (54), $\lambda_m^s = \phi\left(\lambda^s - 1\right) = \phi\left(\frac{1}{\varphi} - 1\right)$. Note that $\lambda_m^s > 0$ implies $\varphi < 1$.

Expression (27) follows directly from (53). As in Lemma 1, use (12) and (14) to write (18) and (19) as follows:

$$\frac{\partial V_1}{\partial m} = \pi\phi\left[(1-n)\,\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\,\phi\Psi\left(q\right) + n\phi\right], \tag{55}$$

$$\frac{\partial V_1}{\partial b} = \pi\phi\left[(1-n)\,\varphi\lambda^b + n\varphi\lambda^s\right] + (1-\pi)\left[(1-n)\,\phi + n\phi\right]. \tag{56}$$

Use the primary bond market first order conditions (5) to write the previous equations as follows

$$\frac{\gamma}{\beta} = \pi\left[(1-n)\,\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\,\Psi\left(q\right) + n\right], \tag{57}$$

$$\frac{\rho\gamma}{\beta} = \pi\varphi\left[(1-n)\,\lambda^b + n\lambda^s\right] + 1 - \pi. \tag{58}$$

Substituting λ^b and λ^s in (58) yields $\rho = \beta/\gamma$; i.e., equation (29) holds. Finally, (28) immediately follows from (57). ■

Proof of Lemma 3. The proof that equation (34) holds in a type III equilibrium follows the proof that equation (30) holds in Lemma 2, and is not repeated here.

We next show that equation (31) holds. An active agent enters the secondary bond market with a real portfolio $\phi m + \varphi\phi b$ of money and bonds. As a buyer, he sells all his bonds in a the type III equilibrium, and thus he exits the secondary bond market with a portfolio $\phi\hat{m}$. As a seller, he sells all his money and thus exits this market with a portfolio $\varphi\phi\hat{b}$. Therefore $\phi m + \varphi\phi b = \phi\hat{m}$ holds for an active buyer, and $\phi m + \varphi\phi b = \varphi\phi\hat{b}$ holds for an active seller. Combining the two equations yields

$$\hat{m} = \varphi\hat{b}. \tag{59}$$

Immediately after the secondary bond market closes, but before the goods market opens, the stock of money in circulation is in the hands of active buyers and passive agents (sellers and buyers). Active sellers hold no money at the end of the secondary bond market. Consequently, $M_{-1} = \pi(1-n)\hat{m} + \pi n * 0 + (1-\pi)m$. Eliminate m, using $m = M_{-1}$, and rearrange to get

$$\hat{m} = \frac{M_{-1}}{1-n}. \tag{60}$$

The stock of bonds in circulation is in the hands of active sellers and passive agents (sellers and buyers), while active buyers hold no bonds at the end of the secondary bond market. Thus, the stock of bonds is equal to $B_{-1} = \pi\,(1-n) * 0 + \pi n\hat{b} + (1-\pi)b$. Since passive agents do not trade in the secondary bond market, they enter the goods market with the same amount of bonds they had at the beginning of the period, $b = B_{-1}$. Use this equation to eliminate b in the bond stock expression above and get

$$\hat{b} = \frac{B_{-1}}{n}. \tag{61}$$

Replace $\hat{m}$ and $\hat{b}$ in (59) by using (60) and (61), respectively. Since the bonds-to-money ratio is constant over time, we can replace the time $t-1$ stock of money and bonds with their respective initial values. Equation (31) then follows.

Finally, we show that (32) and (33) hold. In any equilibrium, $\lambda_m^b = 0$ and $\lambda_b^s = 0$. In a type III equilibrium, a seller's cash constraint in the secondary bond market binds; i.e., $\lambda_m^s > 0$, and a buyer's bond constraint in the secondary bond market binds; i.e., $\lambda_b^b > 0$. Using these multipliers, the secondary bond market first-order conditions (17) become

$$\frac{\partial V_2^b}{\partial \hat{m}} = \phi\lambda^b \qquad \text{and} \qquad \frac{\partial V_2^b}{\partial \hat{b}} = \varphi\phi\lambda^b - \lambda_b^b, \tag{62}$$

$$\frac{\partial V_2^s}{\partial \hat{m}} = \phi\lambda^s - \lambda_m^s \qquad \text{and} \qquad \frac{\partial V_2^s}{\partial \hat{b}} = \varphi\phi\lambda^s. \tag{63}$$

Using the previous expressions in (12) and (14), we obtain

$$\lambda^b = \delta\frac{u'(\hat{q})}{z'(\hat{q})} + (1-\delta) \qquad \text{and} \qquad \lambda_b^b = \phi\left(\varphi\lambda^b - 1\right), \tag{64}$$

$$\lambda_m^s = \phi(\lambda^s - 1) \qquad \text{and} \qquad \varphi\lambda^s = 1. \tag{65}$$

Like in a type II equilibrium, $\lambda_m^s = \phi(\lambda^s - 1) = \phi\left(\frac{1}{\varphi} - 1\right)$, and since $\lambda_m^s > 0$, then $\varphi < 1$. Unlike in a type II equilibrium, from (62), we find $\lambda_b^b = \phi\left(\varphi\lambda^b - 1\right) = \phi\left[\varphi\Psi(\hat{q}) - 1\right]$. Since $\lambda_b^b > 0$, $\Psi(\hat{q}) > 1/\varphi$, and so (27) does not hold in a type III equilibrium.

Use (12) and (14) to write (18) and (19) as follows:

$$\frac{\partial V_1}{\partial m} = \pi\phi\left[(1-n)\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\phi\Psi(q) + n\phi\right], \tag{66}$$

$$\frac{\partial V_1}{\partial b} = \pi\phi\left[(1-n)\varphi\lambda^b + n\varphi\lambda^s\right] + (1-\pi)\left[(1-n)\phi + n\phi\right]. \tag{67}$$

Using the primary bond market first-order conditions (5), the previous equations can be rewritten as follows:

$$\frac{\gamma}{\beta} = \pi\left[(1-n)\lambda^b + n\lambda^s\right] + (1-\pi)\left[(1-n)\Psi(q) + n\right], \tag{68}$$

$$\frac{\rho\gamma}{\beta} = \pi\varphi\left[(1-n)\lambda^b + n\lambda^s\right] + 1 - \pi. \tag{69}$$

Substituting λ^b and λ^s in (68) and (69) yields (32) and (33), respectively. ■

Proof of Proposition 1. Derivation of γ_L. The critical value γ_L is the value of γ such that expressions (24) and (28) hold simultaneously; i.e., such that $\Psi(\hat{q}) = 1$. Such a value exists and is unique, since we assume that $\Psi(q)$ is decreasing in q.

Derivation of γ_H. The critical value γ_H is the value of γ such that equations (28) and (32) hold simultaneously; i.e., such that $\Psi(\hat{q}) = \frac{B_0}{M_0}\frac{1-n}{n} > 1$. Again, such a value exists and is unique, since we assume that $\Psi(q)$ is decreasing in q. ■

Proof of Lemma 4. From the buyer's problem in the secondary bond market, $V_1^b(m,b) = V_2^b(\hat{m},\hat{b})$, where $\hat{m}$ and $\hat{b}$ are the quantities of money and bonds that maximize V_2^b. In any equilibrium, the buyer's budget constraint (15) holds with equality. Thus, we can use (15) to eliminate $\hat{b}$ from $V_2^b(\hat{m},\hat{b})$ and get

$$V_1^b(m,b) = V_2^b\left(\hat{m}, \frac{\phi m + \varphi\phi b - \phi\hat{m}}{\varphi\phi}\right). \tag{70}$$

Next, use (4), (10), and (3), to get

$$\begin{aligned} V_1^b(m,b) &= \delta\{u[q(\hat{m})]-\phi d(\hat{m})\}+U(x^*)-x^*+\phi\hat{m}+\phi T-\phi m_{+1} \\ &\quad +\frac{\phi m+\varphi\phi b-\phi\hat{m}}{\varphi}-\phi\rho b_{+1}+\beta V_1(m_{+1},b_{+1}). \end{aligned} \tag{71}$$

Note that the buyer's cash constraint in the goods market binds; i.e., $d(\hat{m})=\hat{m}$. From (2), $T=M-M_{-1}+\rho B-B_{-1}$, and the budget constraint in the goods market satisfies $\hat{m}\phi=z(\hat{q})$ and $m\phi=z(q)$. Furthermore, all agents exit the period with the same amount of money and bonds, hence $m_{+1}=M$ and $b_{+1}=B$. Using these equalities we can rewrite (71) as follows:

$$\begin{aligned} V_1^b(m,b) &= \delta[u(\hat{q})-z(\hat{q})]+U(x^*)-x^* \\ &\quad -\left(\frac{1}{\varphi}-1\right)[z(\hat{q})-z(q)]+\beta V_1(m_{+1},b_{+1}), \end{aligned}$$

where we have used $b=B_{-1}$ and $m=M_{-1}$. Another way to write this is

$$V_1^b(m,b)=\delta[u(\hat{q})-z(\hat{q})]+U(x^*)-x^*-i[z(\hat{q})-z(q)]+\beta V_1(m_{+1},b_{+1}). \tag{72}$$

The active buyer's period surplus is $\delta[u(\hat{q})-z(\hat{q})]$, but he has to pay interest $i=\frac{1}{\varphi}-1$ on the difference $z(\hat{q})-z(q)$.

Along the same lines, for a passive agent one can show that

$$V_2^b(m,b)=\delta[u(q)-z(q)]+U(x^*)-x^*+\beta V_1(m_{+1},b_{+1}). \tag{73}$$

The difference between (72) and (73) is

$$V_1^b(m,b)-V_2^b(m,b)=\delta[u(\hat{q})-z(\hat{q})]-\delta[u(q)-z(q)]-i[z(\hat{q})-z(q)]. \tag{74}$$

We now need to study (74) for the different types of equilibria. For the type I equilibrium, φ comes from (23), thus

$$V_1^b(m,b)-V_2^b(m,b)=\Psi_1\equiv\delta[u(\hat{q})-z(\hat{q})]-\delta[u(q)-z(q)]>0,$$

which is clearly strictly positive, since $q<\hat{q}$. For the type II equilibrium, φ comes from (27), thus

$$\Psi_2\equiv\delta[u(\hat{q})-z(\hat{q})]-\delta[u(q)-z(q)]-\delta\left[\frac{u'(\hat{q})}{z'(\hat{q})}-1\right][z(\hat{q})-z(q)]. \tag{75}$$

For the type III equilibrium, φ comes from (31), thus

$$\Psi_3 \equiv \delta\left[u\left(\hat{q}\right)-z\left(\hat{q}\right)\right]-\delta\left[u\left(q\right)-z\left(q\right)\right]-\left(\frac{B_0}{M_0}\frac{1-n}{n}-1\right)\left[z\left(\hat{q}\right)-z\left(q\right)\right].$$

Note that in the type III equilibrium we have $\delta\frac{u'(\hat{q})}{z'(\hat{q})}+1-\delta \geq \frac{1}{\varphi}=\frac{B_0}{M_0}\frac{1-n}{n}$. Accordingly, $\Psi_3 \geq \Psi_2$. Hence, it is sufficient to show that $\Psi_2 > 0$. To do so, rewrite (75) as follows:

$$u\left(\hat{q}\right)-u\left(q\right)-z\left(\hat{q}\right)+z\left(q\right) > \left[\frac{u'\left(\hat{q}\right)}{z'\left(\hat{q}\right)}-1\right]\left[z\left(\hat{q}\right)-z\left(q\right)\right].$$

Divide both sides of the above inequality by $\hat{q}-q$ and rearrange it to get

$$\frac{\frac{u(\hat{q})-u(q)}{\hat{q}-q}}{\frac{z(\hat{q})-z(q)}{\hat{q}-q}} > \frac{u'\left(\hat{q}\right)}{z'\left(\hat{q}\right)}.$$

The left-hand side is larger than the right-hand side, since we have assumed that $\frac{u'(q)}{z'(q)}$ is a strictly decreasing function of q. Hence, $\Psi_3 \geq \Psi_2 > 0$. ■

Proof of Lemma 5. From an active seller's decision problem in the secondary bond market, $V_1^s(m,b)=V_2^s(\hat{m},\hat{b})$. In any equilibrium, the seller's budget constraint (15) holds with equality. Thus, we can use (15) to eliminate $\hat{b}$ from $V_2^s(\hat{m},\hat{b})$ and get

$$V_1^s\left(m,b\right)=V_2^s\left(\phi\hat{m},\frac{\phi m+\varphi\phi b-\phi\hat{m}}{\varphi\phi}\right). \tag{76}$$

Using (13), the following holds

$$V_1^s\left(m,b\right)=\delta^s\left[-c\left(q\right)+\phi d\right]+V_3\left(\hat{m},\frac{\phi m+\varphi\phi b-\phi\hat{m}}{\varphi\phi}\right),$$

which can be rewritten as follows:

$$\begin{aligned}V_1^s\left(m,b\right) &= \delta^s\left[-c\left(q\right)+\phi d\right]+U(x^*)-x^*+\phi\hat{m}+\frac{\phi m+\varphi\phi b-\phi\hat{m}}{\varphi}\\ &\quad +\phi T-\phi m_{+1}-\phi\rho b_{+1}+\beta V_1(m_{+1},b_{+1}),\end{aligned}$$

by virtue of (3) and (4).

For a passive seller, one can show that

$$\begin{aligned}V_2^s\left(m,b\right) &= \delta^s\left[-c\left(q\right)+\phi d\right]+U(x^*)-x^*+\phi m+\phi b\\ &\quad +\phi T-\phi m_{+1}-\phi\rho b_{+1}+\beta V_1(m_{+1},b_{+1}).\end{aligned}$$

Hence the difference $V_1^s(m,b) - V_2^s(m,b)$ is equal to

$$V_1^s(m,b) - V_2^s(m,b) = \phi\hat{m} - \phi m + \frac{\phi m - \phi\hat{m}}{\varphi} = i(\phi m - \phi\hat{m}).$$

Note that active sellers do not carry any money into the goods market, thus $\phi\hat{m} = 0$. Also note that $\phi m = z(q) > 0$. It turns out that the above difference is positive if $i > 0$. ■

10 Appendix II: Competitive Markets

Under competitive pricing, buyers and sellers take the price of goods as given in the goods market. Under competitive pricing, it is natural to interpret δ and δ^s as participation probabilities in the goods market. In particular, let δ (δ^s) be the probability that a buyer (seller) participates in the goods market. Then the buyer's value function in the goods market is

$$V_2^b(m,b) = \delta \max_q \left[\begin{array}{c} u(q) + V_3(m - pq, b) \\ s.t.\ m \geq pq. \end{array} \right] + (1-\delta) V_3(m,b), \tag{77}$$

where p is the price, and q the quantity of market-2 goods consumed by the buyer. The first-order condition to this problem is

$$u'(q) = p(\phi + \lambda_q), \tag{78}$$

where λ_q is the Lagrange multiplier on $m \geq pq$.

The seller's value function in the goods market is

$$V_2^s(m,b) = \delta^s \max_{q_s} [-c(q_s) + V_3(m + pq_s, b)] + (1-\delta^s) V_3(m,b). \tag{79}$$

The first-order condition to this problem is

$$p\phi = c'(q_s). \tag{80}$$

If the buyer's cash constraint is not binding, the buyer consumes the efficient quantity q^*, where q^* solves $u'(q) = c'(q_s)$. If the cash constraint is binding, then he spends all his money in goods purchases, and consumption is inefficiently low. Note that, in equilibrium, an active buyer will hold more money than a passive buyer. This means that $\lambda_q > \hat{\lambda}_q$. It then follows that $\hat{q} > q$.

The buyer's envelope conditions are

$$\frac{\partial V_2^b}{\partial m} = \phi\delta \frac{u'(q)}{c'(q)} + (1-\delta)\phi \qquad \text{and} \qquad \frac{\partial V_2^b}{\partial b} = \phi, \tag{81}$$

where we have used the envelope conditions in the primary bond market, and the first-order conditions in the goods market. Notice the similarity between (81) and (12). The two expressions are the same if $\theta = 1$. As a consequence of this, active buyers consume the efficient quantity in a type I equilibrium under competitive pricing, while they do not under bilateral matching unless $\theta = 1$.

The seller's envelope conditions are exactly the same as (14); i.e., $\frac{\partial V_2^s}{\partial m} = \frac{\partial V_2^s}{\partial b} = \phi$.

Finally, by using the budget constraint of the buyer at equality $pq = m$ and (80) we get

$$\phi m = z^C(q) \equiv c'(q_s)\, q,$$

which is equal to (38) for a linear cost function.

11 Appendix III: Competitive Search

As an alternative trading mechanism we consider the *competitive search* framework proposed by Moen (1997) and Mortensen and Wright (2002), and used in Rocheteau and Wright (2005, 2009). Here, we follow the derivation of the competitive search equilibrium presented in Rocheteau and Wright (2005). We refer to their paper for additional details.

There are agents called submarket makers, who set up submarkets in the goods market and charge an entry fee to participants. This fee is zero in equilibrium, since submarkets can be opened costlessly. Any given submarket is characterized by the terms of trade (q, d) and the number of buyers and sellers $(1 - n, n)$. The timing of the events is as follows. At the beginning of each period, submarket makers announce the terms of trade for each trading post. Given (q, d), agents decide where to go. In each submarket, the terms of trade are predetermined, so agents do not bargain once in a match; however, they still have to search for their partner and may end up without having a match.

Submarket makers design submarkets by maximizing the buyers' payoffs subject to the constraint that a positive number of sellers enter the markets. Note that active buyers and passive buyers may want to consume different quantities, since the former can adjust their balance after the production/consumption shock, while the latter cannot. It turns out that market makers may find it profitable to design different types of submarkets for different types of buyers. In equilibrium, two types of submarkets are open: submarkets for active buyers and submarkets for passive buyers. Nevertheless, in what follows, we can derive the terms of trade in a submarket for agents with money holdings m.

The market maker problem for such a submarket is

$$\max_{n,q,d} \delta \left[u(q) + V_3(m-d,b)\right] + (1-\delta) V_3(m,b) \qquad \text{s.t.}$$
$$\delta^s \left[-c(q) + \phi d\right] = J \qquad \text{and} \qquad d \leq m,$$

where J is the equilibrium expected utility of a seller in the goods market (for more details see Rocheteau and Wright, 2005).

This submarket is designed for buyers with money holdings m. Once in this market, they can consume with probability δ, and they cannot with probability $1-\delta$. The first constraint ensures that a positive number of sellers enter the submarket. Given q and d, this constraint yields n. The second constraint follows from the fact that buyers cannot spend more money than the amount they bring into the goods market.

We can use the linearity of the value function V_3 to rewrite the market maker problem as follows:

$$\max_{n,q,d} \delta \left[u(q) - \phi d\right] + V_3(m,b) \qquad \text{s.t.}$$
$$\delta^s \left[-c(q) + \phi d\right] = J \qquad \text{and} \qquad d \leq m.$$

If the second constraint binds, i.e., $d = m$, the problem above reduces to

$$\max_{n,q} \delta \left[u(q) - \phi m\right] + V_3(m,b) \qquad \text{s.t.}$$
$$\delta^s \left[-c(q) + \phi m\right] = J.$$

If we denote ν the Lagrangian multiplier of the constraint, the first-order conditions are

$$\delta' \left[u(q) - \phi m\right] - \nu \left[-c(q) + \phi m\right] \frac{\delta'(1-n)n - \delta}{n^2} = 0,$$
$$\delta u'(q) + \nu c'(q) \frac{\delta(1-n)}{n} = 0.$$

where δ' is the derivative of δ with respect to the number of sellers. Substituting ν from the first into the second equations yields

$$\delta' \left[u(q) - \phi m\right] + \frac{u'(q)\left[-c(q) + \phi m\right]}{(1-n)c'(q)} \frac{\delta'(1-n)n - \delta}{n} = 0.$$

Solving for ϕm yields $\phi m = z^P(q)$, where

$$z^P(q) \equiv \frac{\left[1-\eta(n)\right] c(q) u'(q) + \eta(n) u(q) c'(q)}{\left[1-\eta(n)\right] u'(q) + \eta(n) c'(q)}, \tag{82}$$

which is the same as (36). Note that $\eta(n) \equiv \frac{\delta'(1-n)n}{\delta}$ is the seller's contribution to the matching process; i.e., the elasticity of the matching function with respect to the measures of sellers. Notice that (82) is the solution of the bargaining problem when $\theta = 1-\eta(n)$. Hence, with competitive search, the Hosios condition is automatically satisfied (for a discussion of efficiency in matching models see Hosios, 1990).

12 Appendix IV: Data Sources

The data we use for the calibration is provided by the U.S. Department of Commerce: Bureau of Economic Analysis (BEA), the Board of Governors of the Federal Reserve System (BGFRS), the Federal Reserve Bank of St. Louis (FRBSL), the U.S. Department of the Treasury: Financial Management Service (FMS), the U.S. Department of Labor: Bureau of Labor Statistics (BLS) and Bloomberg.

Table A.1: Data source

Description	Identifier	Source	Period	Frequency
AAA Moody's corporate bond	AAA	BGFRS	60:Q1-10:Q4	quarterly
Consumer price index	CPIAUCSL	BLS	60:Q1-10:Q4	quarterly
Government bond - 3 months	USGG3M[30]	Bloomberg	60:Q1-10:Q4	quarterly
US total public debt	GFDEBTN	FMS	66:Q1-10:Q4	quarterly
M1 money stock	M1NS	BGFRS	60:Q1-10:Q4	quarterly
Nominal GDP	GDP	BEA	60:Q1-10:Q4	quarterly
Velocity of money	M1V	FRBSL	60:Q1-10:Q4	quarterly

As the total public debt series from the U.S. Department of the Treasury: Financial Management Service is only available from 1966:Q1, we construct the quarterly data in the period from 1960:Q1 to 1965:Q4 with the data provided by http://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm. The definition of quarterly data that we apply is in line with the Federal Reserve Bank of St. Louis FRED® database and defined as the average of the monthly data.

References

[1] Aliprantis, C., Camera, G., and Puzzello, D., 2007, "Anonymous Markets and Monetary Trading," *Journal of Monetary Economics*, 54, 1905-28.

[30] The yields of this index are annualized yields to maturity and pre-tax. The rates are comprised of Generic United States on-the-run government bill/note/bond indices with a maturity of 3 months.

[2] Andolfatto, D., 2011, "A Note on the Societal Benefits of Illiquid Bonds," *Canadian Journal of Economics*, 44, 133-147.

[3] Araujo, L., 2004, "Social Norms and Money," *Journal of Monetary Economics,* 51, 241-256.

[4] Aruoba, S. B., Waller, C., and Wright, R., 2011, "Money and Capital," *Journal of Monetary Economics*, 58, 98-116.

[5] Aruoba, B., Rocheteau, G., and Waller, C., 2007, "Bargaining and the Value of Money," *Journal of Monetary Economics*, 54, 2636-2655.

[6] Berentsen, A., Camera, G., and Waller, C., 2007, "Money, Credit and Banking," *Journal of Economic Theory,* 135, 171-195.

[7] Berentsen, A., and Waller, C., 2011, "Outside Versus Inside Bonds: A Modigliani-Miller Type Result for Liquidity Constrained Economies," *Journal of Economic Theory,* 146, 1852-1887.

[8] Berentsen, A., Menzio, G., and Wright, R., 2011, "Inflation and Unemployment in the Long Run," *American Economic Review*, 101, 371-98.

[9] Bernanke, B., and Gertler, M., 1989, "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14-31.

[10] Bianchi, J., 2011, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101, 3400-3426.

[11] Bianchi, J., and Mendoza, E. G., 2011, "Overborrowing, Financial Crises and 'Macro-prudential' Policy," *International Monetary Found,* working paper # 11/24.

[12] Boel, P., and Camera, G., 2006 "Efficient Monetary Allocations and the Illiquidity of Bonds," *Journal of Monetary Economics*, 53, 1693-715.

[13] Caballero, R. J., and Krishnamurthy, A., 2003, "Excessive Dollar Debt: Financial Development and Underinsurance," *Journal of Finance*, 58, 867-894.

[14] Faig, M., and Jerez, B., 2005, "A Theory of Commerce," *Journal of Economic Theory*, 122, 60-99.

[15] Geromichalos, A., and Herrenbrueck, L., 2012, "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *UC Davis, working paper # 12-20.*

[16] Geromichalos, A., Licari, J. M., and Lledo, J. S., 2007, "Monetary Policy and Asset Prices," *Review of Economic Dynamics*, 10, 761-779.

[17] He, C., Wright, R., and Zhu, Y., 2012, "Housing and Liquidity," *working paper, University of Wisconsin-Madison.*

[18] Hosios, A., 1990, "On the Efficieny of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279-298.

[19] Jeanne, O., and Korinek, A., 2012, "Managing Credit Booms and Busts: A Pigouvian Taxation Approach", *working paper, University of Maryland.*

[20] Kalai, E., 1977, "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica*, 45, 1623-1630.

[21] Kehoe, T., and Levine, D., 1993, "Debt-constrained Asset Markets," *Review of Economic Studies*, 60, 865-888.

[22] Kiyotaki, N., and Moore, J., 1997, "Credit Cycles," *Journal of Political Economy*, 105, 211-248.

[23] Kiyotaki, N., and Wright, R., 1993 "A Search-theoretic Approach to Monetary Economics," *American Economic Review*, 83, 63-77.

[24] Kocherlakota, N., 1998, "Money is Memory," *Journal of Economic Theory,* 81, 232-251.

[25] Kocherlakota, N., 2003, "Social Benefits of Illiquid Bonds," *Journal of Economic Theory,* 108, 179-193.

[26] Korinek, A., 2012, "Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses," *working paper, University of Maryland.*

[27] Lagos, R., and Rocheteau, G., 2007, "Search in Asset Markets: Market Structure, Liquidity, and Welfare," *American Economic Review*, 97, 198-202.

[28] Lagos, R., and Rocheteau, G., 2008, "Money and Capital as Competing Media of Exchange," *Journal of Economic Theory*, 142, 247-258.

[29] Lagos, R., and Rocheteau, G., 2009, "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 77, 403-426.

[30] Lagos, R., and Wright, R., 2005, "A Unified Framework for Monetary Theory and Policy Evaluation," *Journal of Political Economy,* 113, 463-484.

[31] Lester, B., Postelwaite, A., and Wright, R., 2012, "Information, Liquidity, Asset Prices and Monetary Policy," *Review of Economic Studies*, 79, 1209-1238.

[32] Lorenzoni, G., 2008. "Inefficient Credit Booms," *Review of Economic Studies*, 75, 809-833.

[33] Martin, F., 2012, "Debt, Inflation and Central Bank Independence," *working paper, FRB of St. Louis.*

[34] Mendoza, E. G., 2010, "Sudden Stops, Financial Crises, and Leverage," *American Economic Review*, 100, 1941-1966.

[35] Mendoza, E. G., and Smith, K., 2006, "Quantitative Implications of a Debt-deflation Theory of Sudden Stops and Asset Prices," *Journal of International Economics*, 70, 82-114.

[36] Moen, E. R., 1997, "Competitive Search Equilibrium," *Journal of Political Economy,* 105, 385-411.

[37] Mortensen, D., and Wright, R., 2002, "Competitive Pricing and Efficiency in Search Equilibrium," *International Economic Review*, 43, 1-20.

[38] Nosal, E., and Rocheteau, G., 2011, "Money, Payments, and Liquidity," The MIT Press.

[39] Rocheteau, G., and Weill, P. O., 2011, "Liquidity in Frictional Asset Markets," *Journal of Money, Credit and Banking*, 43, 261-282.

[40] Rocheteau, G., and Wright, R., 2005, "Money in Competitive Equilibrium, in Search Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73, 175-202.

[41] Rocheteau, G., and Wright, R., 2009, "Inflation and Welfare in Models with Trading Frictions," in *Monetary Policy in Low Inflation Economies*, by Ed Nosal and Dave Altig, Cambridge University Press.

[42] Rocheteau, G., and Wright, R., 2010, "Liquidity and Asset Market Dynamics," *working paper, University of California, Irvine.*

[43] Rojas-Breu, M., 2013, "The Welfare Effect of Access to Credit," *Economic Inquiry*, 51, 235-247.

[44] Shi, S., 2008, "Efficiency Improvement from Restricting the Liquidity of Nominal Bonds," *Journal of Monetary Economics,* 55, 1025-1037.

[45] Trejos, A., and Wright, R., 2012, "Shi-Trejos-Wright Meets Duffie-Garleanu-Pedersen," *working paper, University of Wisconsin-Madison.*

[46] Wallace, N., 2001, "Whither Monetary Economics?" *International Economic Review,* 42, 847-869.

[47] Wright, R., 2010, "A Uniqueness Proof for Monetary Steady State," *Journal of Economic Theory*, 145, 382-391.