## Online Appendix For "Competing Engines of Growth: Innovation and Standardization"

### Proof of Lemma 5

We take a linear approximation of the dynamical system (40) around $(n^{ss}, \chi^{ss})$ :

$$
\begin{aligned}
\frac{\dot{\chi}}{\chi} &\simeq F_\chi(\chi^{ss}, n^{ss}) \cdot (\chi - \chi^{ss}) + F_n(\chi^{ss}, n^{ss}) \cdot (n - n^{ss}) \\
\frac{\dot{n}}{n} &\simeq G_\chi(\chi^{ss}, n^{ss}) \cdot (\chi - \chi^{ss}) + G_n(\chi^{ss}, n^{ss}) \cdot (n - n^{ss})
\end{aligned}
$$

where subscripts denote partial derivatives and

$$
\begin{aligned}
F(\chi, n) &\equiv r(n) - \rho - \frac{y(n) - \mu_L m(n) n - \chi}{\mu_H}, \\
G(\chi, n) &\equiv \left(\frac{1-n}{n}\right) \frac{y(n) - \chi}{\mu_H} - m(n)\left(1 + (1-n)\frac{\mu_L}{\mu_H}\right).
\end{aligned}
$$

Thus, $F_\chi(\chi, n) = \frac{1}{\mu_H} > 0$ and $G_\chi(\chi, n) = -\left(\frac{1-n^{ss}}{n^{ss}}\right)\frac{1}{\mu_H} < 0$. Solving for the schedules such that, respectively, $\dot{\chi} = 0$ and $\dot{n} = 0$ yields:

$$
\begin{aligned}
\chi(n)|_{\dot{\chi}=0} &= y(n) - \mu_L m(n) n - \mu_H (r(n) - \rho) \\
\chi(n)|_{\dot{n}=0} &= y(n) - \mu_L m(n) n - \mu_H m(n)\left(\frac{n}{1-n}\right),
\end{aligned}
$$

with slopes:

$$
\chi'(n)|_{\dot{\chi}=0} = -\frac{F_n(\chi^{ss}, n^{ss})}{F_\chi(\chi^{ss}, n^{ss})} \quad \text{and} \quad \chi'(n)|_{\dot{n}=0} = -\frac{G_n(\chi^{ss}, n^{ss})}{G_\chi(\chi^{ss}, n^{ss})}.
$$

Suppose there were trajectories featuring both innovation and standardization converging to $(n^{ss}, \chi^{ss})$. Then, either one or both eigenvalues of the linearized system would be negative. We show that this is impossible and that under the sufficient conditions of the Proposition both eigenvalues must be positive. Let the two eigenvalues of the linearized system be denoted by $\lambda_1$ and $\lambda_2$. We know that

$$
\begin{aligned}
\lambda_1 + \lambda_2 &= F_\chi(\chi^{ss}, n^{ss}) + G_n(\chi^{ss}, n^{ss}) \\
\lambda_1 \cdot \lambda_2 &= F_\chi(\chi^{ss}, n^{ss}) \cdot G_n(\chi^{ss}, n^{ss}) - F_n(\chi^{ss}, n^{ss}) \cdot G_\chi(\chi^{ss}, n^{ss}).
\end{aligned}
$$

**Claim 1** *The following inequality holds*

$$
-\frac{F_n(\chi^{ss}, n^{ss})}{F_\chi(\chi^{ss}, n^{ss})} < -\frac{G_n(\chi^{ss}, n^{ss})}{G_\chi(\chi^{ss}, n^{ss})}.
$$

*Hence,* $\lambda_1 \cdot \lambda_2 > 0$

**Proof.** We need to show that $\chi'(n^{ss})|_{\dot{\chi}=0} < \chi'(n^{ss})|_{\dot{n}=0}$. Define $\Delta(n) = \chi(n)|_{\dot{\chi}=0} - \chi(n)|_{\dot{n}=0} = \mu_H m(n)\left(\frac{n}{1-n}\right) - \mu_H(r(n) - \rho)$. We know that $m(n) = 0$ for $n \geq n^{\max} > n^{ss}$. Thus, at $n = n^{\max}$ we have:

$$\Delta(n^{\max}) = -\mu_H(r(n) - \rho) = -\mu_H\left(\frac{\pi_H(n)}{\mu_H} - \rho\right) < 0$$

by Assumption 1. Next, recall that at $n^{\min} = \frac{H}{H+L}$ we have $y(n) = H + L$, $m(n) = \frac{H+L}{\epsilon}\left(\frac{1}{\mu_H} - \frac{1}{\mu_L}\right)$ and $r(n) = \frac{H+L}{\epsilon\mu_L}$. Thus:

$$\Delta\left(n^{\min}\right) = \mu_H\frac{H+L}{\epsilon}\left(\frac{1}{\mu_H} - \frac{1}{\mu_L}\right)h - \mu_H\left(\frac{H+L}{\epsilon\mu_L} - \rho\right) > 0$$

by Assumption 2. Moreover, we know that $n^{ss}$ is unique. Thus, $\chi(n^{ss})|_{\dot{\chi}=0} = \chi(n^{ss})|_{\dot{n}=0}$ for a unique value of $n^{ss}$. Then, by the intermediate value theorem, $\chi(n)|_{\dot{\chi}=0} > \chi(n)|_{\dot{n}=0}$ for all $n < n^{ss}$ and $\chi(n)|_{\dot{\chi}=0} < \chi(n)|_{\dot{n}=0}$ for all $n > n^{ss}$. ■

Since we know that $\lambda_1 \cdot \lambda_2 > 0$, showing that $\lambda_1 + \lambda_2 > 0$ establishes that the system has two positive eigenvalues and is therefore unstable. The condition $\lambda_1 + \lambda_2 > 0$ can be written as:

$$\chi'(n)|_{\dot{n}=0} = -\frac{G_n(\chi^{ss}, n^{ss})}{G_\chi(\chi^{ss}, n^{ss})} > \frac{F_\chi(\chi^{ss}, n^{ss})}{G_\chi(\chi^{ss}, n^{ss})} = -\left(\frac{n}{1-n}\right)\frac{1}{\mu_H}. \tag{43}$$

A sufficient condition for (43) to be satisfied is that the locus $\dot{n} = 0$ be upward sloping in a neighborhood of the BGP. Let us first consider the case where $\rho \to 0$. Using $y(n) = \epsilon[n\pi_H(n) + (1-n)\pi_L(n)]$ and $m(n) = \frac{\pi_H(n)}{\mu_H} - \frac{\pi_L(n)}{\mu_L}$ into $\chi(n)_{|\dot{n}=0}$:

$$\chi(n)|_{\dot{n}=0} = y(n) - m(n)\left(\frac{n}{1-n}\mu_H + n\mu_L\right) = [\pi_H(n)\,n] \cdot A(n)$$

where $A(n) \equiv \left[\frac{\mu_L}{\mu_H} + \frac{\pi_L(n)}{\pi_H(n)}\left(1 + \frac{\epsilon}{n}\right) + \frac{1}{1-n}\left(\frac{\pi_L(n)}{\pi_H(n)}\frac{\mu_H}{\mu_L} - 1\right)\right]$. We know that the factor $\pi_H(n)\,n$ is inverted U-shaped, with a maximum at $n^* > n_{\min}$ (it corresponds to the maximum $g$ characterized in Section 2.7). Thus, for $n \in [n_{\min}, n^*]$ a sufficient condition for $\chi'(n)_{|\dot{n}=0} > 0$ is $\frac{\partial A(n)}{\partial n} > 0$:

$$\frac{\partial A(n)}{\partial n} = \frac{\partial}{\partial n}\left(\frac{\pi_L(n)}{\pi_H(n)}\right)\left(1 + \frac{\epsilon}{n}\right) - \frac{\pi_L(n)}{\pi_H(n)}\frac{\epsilon}{n^2} + \frac{1}{(1-n)^2}\left(\frac{\pi_L(n)}{\pi_H(n)}\frac{\mu_H}{\mu_L} - 1\right) + \frac{\partial}{\partial n}\left(\frac{\pi_L(n)}{\pi_H(n)}\right)\frac{1}{1-n}$$

For $\rho \to 0$, in the BGP we have $\frac{\pi_L(n)}{\pi_H(n)} = \frac{\mu_L}{\mu_H} n$ and $\frac{\partial}{\partial n}\left(\frac{\pi_L(n)}{\pi_H(n)}\right) = \frac{\epsilon-1}{\epsilon}\frac{\pi_L(n)}{\pi_H(n)}\frac{1}{(1-n)n} = \frac{\epsilon-1}{\epsilon(1-n)}\frac{\mu_L}{\mu_H}$. Thus:

$$\frac{\partial A(n)}{\partial n} = \frac{\mu_L}{(1-n)\mu_H}\left[\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + \frac{\mu_H}{\mu_L}\right]. \tag{44}$$

A sufficient condition for $\chi'(n)|_{\dot{n}=0} > 0$ is then: $\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + \frac{\mu_H}{\mu_L} > 0$. Noting that in the BGP $\frac{\pi_L(n)}{\pi_H(n)} = \frac{\mu_L}{\mu_H} n \to \frac{\mu_H}{\mu_L} = \left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}} n$, the sufficient condition become:

$$\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + \left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}} n > 0$$

Since this expression is increasing in $n$, we only need to verify that it is positive at $n_{\min}$. At $n_{\min}$ we have $\left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}} = 1$ and the condition becomes:

$$\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + n > 0. \tag{45}$$

Substituting $n_{\min} = h/(1+h)$ yields (28) in the text.

Finally, for $n > n^*$, rewrite the necessary condition as:

$$\frac{\chi'(n)|_{\dot{n}=0}}{\chi(n)|_{\dot{n}=0}} = \frac{\pi'_H(n)}{\pi_H(n)} + \frac{1}{n} + \frac{\partial A}{\partial n}\frac{1}{A} > -\left(\frac{n}{1-n}\right)\frac{1}{\mu_H \chi(n)|_{\dot{n}=0}}$$

From equations (16) and (17):

$$\frac{\pi'_H(n)}{\pi_H(n)} + \frac{1}{n} = \frac{1}{\epsilon}\frac{[y(n)]^{\frac{1-\epsilon}{\epsilon}}}{\epsilon-1}\left(\frac{H}{n}\right)^{\frac{\epsilon-1}{\epsilon}} - \frac{1}{\epsilon}\frac{[y(n)]^{\frac{1-\epsilon}{\epsilon}}}{\epsilon-1}\left(\frac{L}{1-n}\right)^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{n\epsilon}.$$

Substituting $[y(n)]^{\frac{\epsilon-1}{\epsilon}} = (1-n)^{\frac{1}{\epsilon}} L^{\frac{\epsilon-1}{\epsilon}} + n^{\frac{1}{\epsilon}} H^{\frac{\epsilon-1}{\epsilon}}$ into this expression, we have

$$-\frac{1}{\epsilon(\epsilon-1)}\frac{(1-n)^{-1}}{1+\left(\frac{n}{1-n}\right)^{\frac{1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}}} < \frac{\pi'_H(n)}{\pi_H(n)} + \frac{1}{n}$$

This implies that $\frac{\partial A(n)}{\partial n}\frac{1}{A(n)} > \frac{1}{\epsilon(\epsilon-1)}\frac{\frac{1}{1-n}}{1+\left(\frac{n}{1-n}\right)^{\frac{1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}}}$ is a sufficient condition for $\chi'(n)|_{\dot{n}=0} > 0$. Using (44) and the fact that in BGP $\frac{\pi_L(n)}{\pi_H(n)} = \frac{\mu_L}{\mu_H} n$, it can be verified that this condition is satisfied when (45) holds, for any $n \in [n^*, 1]$. In sum, (45) is sufficient to prove that the dynamical system with both innovation and standardization is locally unstable in the limit where $\rho \to 0$. By continuity, the same result applies for $\rho < \bar{\rho}$ for some $\bar{\rho} > 0$ sufficiently small.