Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt*

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Abstract

This paper proposes a dynamic politico-economic theory of fiscal policy whose driving force is the intergenerational conflict over debt, taxes and public goods. Subsequent generations of voters choose fiscal policy through repeated elections. The presence of young voters induce quasi-geometric discounting, with a low short-run discount rate. This translates into a demand for fiscal discipline, i.e., low taxes and low debt accumulation. We characterize the Markov Perfect Equilibrium of the voting game. In spite of low altruism, the political equilibrium may converge to a stationary debt level which is bounded away from the endogenous debt limit. The equilibrium can reproduce a number of qualitative and quantitative features of fiscal policy in modern economies.

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1 Introduction

Lack of commitment is an inherent feature of modern democracies, where elected governments cannot tie the future voters' hands. The impossibility to sustain time-inconsistent optimal policies is often a burden on future generations. For instance, limited commitment induces excessive capital taxation causing low investments and low future capital stock (see, e.g., Klein and Ríos-Rull 2003).

In this paper, we propose a dynamic politico-economic theory of debt and fiscal policy where the inability of governments to commit *benefits* future generations. Agents of different age groups (the "young" and the "old") vote every period over public good provision and its financing. Government expenditure is financed by levying distortionary taxes and by issuing debt that can be sold in an international market at an exogenous interest rate. Voters are only imperfectly altruistic towards future generations, i.e., they discount future felicities accruing to their descendants at a higher rate than they discount their own future felicities. In this environment – which encompasses the standard OLG model as a particular case – young agents dislike debt accumulation more than do the old, since debt crowds out future public-good provision within their lifetime. Moreover, the repeated political influence of young cohorts makes fiscal policy time inconsistent. Formally, the political equilibrium can be represented as the maximization of the discounted utility of a single decision-maker (a *planner*) endowed with preferences inducing quasi-geometric discounting: the planner's discount factor is larger in the first period than in all subsequent periods. Therefore, contrary to earlier theories, the lack of commitment results in higher patience and fiscal discipline.

This time inconsistency has dramatic long-run implications: If voters could commit the path of future fiscal policy, the equilibrium would feature "public poverty" in the long run, i.e., governments would accumulate large debt and eventually become unable to provide public goods. Instead, if fiscal policy is decided sequentially through repeated elections, the long-run equilibrium may feature low debt and positive public good provision.

Our theory identifies the factors strengthening and weakening fiscal discipline. One such factor is distortionary taxation. When voting over the current budget the young contemplate its implications on next-period public good provision. Leaving a large debt forces the next government to make a fiscal adjustment: it must increase taxes, reduce expenditure, or expand debt further. When the lion's share of the future government's response is a cut in expenditure, young voters support a strong fiscal discipline. Conversely, the more the future government responds by increasing taxes and rolling over debt, the more the young accept a lax debt policy. Thus, expectations about future fiscal policy shape current fiscal policy. We embed such expectations into a dynamic voting game, and focus on Markov Perfect Equilibria (MPE) where the strategies of current voters are conditioned only on pay-off-relevant state variables. The response of future governments turns out to depend on tax distortions. Intuitively, the more distortionary future taxation, the less future governments will be tempted to increase taxes, and the more they will instead cut public good provision. Therefore, the fiscal discipline becomes stronger the larger and the more convex tax distortions are.

In spite of its stylized nature, the model contains the key ingredients of a theory of long-run debt level, and can be used as the workhorse of a quantitative analysis. In this paper, we perform a first step in this direction. We show that under a reasonable calibration of parameters the political equilibrium yields a steady-state labor tax rate of 27%, a ratio of public-to-private consumption of 20% and an annualized debt-to-GDP ratio of 120%.[1] The model has interesting predictions for how debt should respond to demographic changes: it should increase when the share of old in the population grows. This is consistent with the observation that government debt has increased in ageing societies since the 1980s.

It is instructive to examine the role of bequests. In the benchmark model we make the plausible assumption that agents cannot leave negative bequests. In an extension, we allow the old to pass on both assets and debts to their children. In this case, the economy converges to both public and private poverty, i.e., in the long run agents inherit so high private and public debt that they have no consumption possibilities.

From a methodological standpoint, we provide a characterization of the MPE of a dynamic game in terms of a system of Generalized Euler Equations (GEEs). The notion of GEE was first introduced by Harris and Laibson (2001) and Krusell *et al.* (2002) in the context of consumption choices under time-inconsistent preferences. Our formulation is close to Klein *et al.* (2008) who characterize, as we do, the GEE for government expenditure. We extend their approach in two important respects. First, we focus on public debt, while they impose balanced budget. Second, in our model the source of time inconsistency is an intergenerational conflict. The GEEs provide an intuitive characterization of the dynamics. Moreover, in some particular cases, we attain analytical solutions.

Our paper contributes to the politico-economic literature studying the determinants of government debt. Three forebears are Persson and Svensson (1989) and Alesina and Tabellini

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[1] These figures correspond to U.S. averages over the last decades, once the implicit Social Security debt is included, see Section 5.1 for details.

(1990a, 1990b), who emphasized political conflict as a driving factor for public debt in models without intergenerational conflict. Although the observation that conflicts of interest within societies can give rise to time-inconsistent fiscal policy is not *per se* new, most existing theories emphasize political failures leading to excess short-termism and deficit bias (e.g., Azzimonti Renzo 2009). In contrast, our theory emphasizes a novel opposite force leading to fiscal moderation even in a world where agents are not altruistic towards future generations. A very different source of fiscal moderation arises in closed infinite-horizon models with endogenous interest rates, *à la* Lucas and Stokey (1983). There, a government with positive debt has the incentive to use fiscal policy to increase future private consumption in order to reduce the interest rate and, hence, reduce outstanding debt. Thus, the optimal debt policy is time inconsistent, and a sequence of governments lacking commitment would reduce the stock of debt until this falls to zero in the long run.[2] An important difference between our theory and these models is that in our open-economy environment a large government debt implies that future resources are committed to interest payments to the rest of the world. In contrast, in closed-economy environments any government debt is mirrored by private savings. Thus the scope for public debt to lead to the immiseration of future generations is stronger in our theory.

More closely related is Cukierman and Meltzer (1989, CM) who construct a model where agents who are heterogenous in altruism and cannot leave negative bequests vote on public debt and taxation. In their benchmark model, highly altruistic agents are indifferent with respect to debt policy, due to Ricardian equivalence. Thus, fiscal policy is chosen according to the preferences of the most bequest-constrained agent. In spite of some common ground, our mechanism is very different and stems from the interaction between two elements that are absent in CM: rational dynamic voting (in CM agents are myopic and take future fiscal policy as given) and public good provision (in CM the government spends its revenue on transfers).[3] CM also extend their analysis to a closed economy where debt crowds out capital and, hence, increases the rate of return to capital and lowers wages. In this environment, a conflict arises between individuals with different endowments of human and non-human wealth. These issues are absent in our theory, due to an exogenous interest rate and no intragenerational heterogeneity.

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[2] See Alesina and Tabellini (1990a) and Bortoli and Nunes (2008). In both of these papers, the economy converges to zero debt in the absence of political uncertainty. However, they also consider scenarios where governments with a taste for spending on different public goods alternate in office stochastically. This introduces a deficit bias. The combination of the two effects leads to an positive debt level.

[3] In CM agents can use private savings and bequests to offset the effects of higher public debt on future government expenditure. In contrast, in our model this is not possible since savings are an imperfect substitute for public goods. Moreover, even if the government spent its revenue on public good provision, fiscal discipline would not arise in CM because their voters would not foresee that increasing debt crowds out future expenditure.

A more recent related paper is Battaglini and Coate (2007) who analyze fiscal policy and government debt in a stochastic infinite-horizon model of legislative bargaining without any intergenerational conflict. In their model agents would like to commit to large government savings when the value of the public good is low, and debt accumulation and public-good provision when the value of the public good is high. However, legislators can also divert resources to pork-barrel transfers to geographically defined districts. Due to this political conflict, legislators opt for inefficient transfers instead of government savings when the debt is low. Consequently, the equilibrium features too much debt, too little public-good provision, and stationary debt dynamics.

Our model shares the prediction of Alesina and Tabellini (1990a) and Battaglini and Coate (2007) that the (average) long-run debt level is determined, contrary to Barro (1979). This prediction is also borne out in Aiyagari et al. (2002) where a planner can commit to future policies but markets are incomplete, and Yared (2009), where voters try to discipline a self-interested government. We view our contribution as complementary to these papers, emphasizing a different mechanism for mean reversion of debt in the absence of commitment.

Our paper is also related to the recent politico-economic literature on dynamic fiscal policy, where agents vote repeatedly on taxation and redistribution. These papers also focus on MPE, but assume balanced government budget (see, e.g., Krusell and Ríos-Rull (1999), Hassler et al. (2003), Bassetto (2008), and Klein *et al.* (2008)). Chen and Song (2005) and Gonzalez Eiras and Niepelt (2009) focus on the political sustainability of a pension system with MPE (see Boldrin and Rustichini (2000) for an analysis with non-Markov strategies).

The paper is organized as follows. In section 2 we describe the model environment. Section 3 characterizes the commitment solution and the political equilibrium. Section 4 provides analytical solutions for two particular cases. Section 5 analyzes the general case with the aid of a calibrated example. Section 6 extends the analysis to the case in which agents can leave negative bequests. Section 7 concludes. The proofs of the Lemmas and Propositions are contained in Appendixes A and B. The latter is available from our webpages.

2 Model Economy

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period. Successive generations are linked by dynastic ties. Each agent has one child implying a constant population size. Agents consume two goods: a private good (c) and a public good (g), provided by the government.

Private goods can be produced via two technologies – market and household production.

Market production is subject to constant returns, and agents earn a pre-tax hourly wage w. Wages are subject to a linear tax rate, $\tau \in [0,1]$. We abstract from taxes on consumption and capital income. Incorporating such taxes would be technically complicated because such analysis would, as we shall see, require adding a state variable (namely savings). We conjecture that the effect of adding such taxes would be the same as if the interest rate were endogenous, where higher debt would tend to increase the taxes (due to the larger cost of servicing the debt) and hence reduce the interest rate.

The household production technology is represented by the production function $y_H = F(h)$, where the total time endowment is unity, $h \in [0,1]$ is the market labor supply, and $1-h$ is the time devoted to household production. The function F has the following properties: $F'(h) < 0$, $F''(h) \leq 0$, $F'''(h) \leq 0$, $F(1) = 0$, and $-F'(1) > w$. Since the government cannot tax household production, taxation distorts the time agents work in the market. Agents choose the allocation of their time so as to maximize total after-tax labor income, denoted by $A(\tau)$:

$$A(\tau) \equiv \max_{h \in [0,1]} \left\{ (1-\tau) wh + F(h) \right\}. \tag{1}$$

This program defines the optimal market labor supply $H(\tau) = -(F')^{-1}((1-\tau)w)$, where $H'(\tau) \leq 0$ and $H''(\tau) \leq 0$, and where the envelope theorem implies $A'(\cdot) = -wH(\tau)$. Let $e(\tau) \equiv -(dH(\tau)/d\tau)(\tau/H(\tau))$ denote the tax elasticity of labor supply. The assumptions on F ensure that $e'(\tau) \geq 0$. Moreover, let $\bar{\tau}$ denote the tax rate corresponding to the top of the Laffer curve: $\bar{\tau} \equiv \arg\max_{\tau} \tau \cdot H(\tau)$. Standard algebra shows that $e(\bar{\tau}) = 1$, hence, $e(\tau) < 1$ for all $\tau < \bar{\tau}$.

Consider the preferences of a young agent in dynasty i, born in period t. In the benchmark case, we assume additively separable preferences over private and public good consumption. The case of non-separable utility is studied as an extension in section 5.5. More formally,

$$U_{Y,i,t} = \tilde{u}(c_{Y,i,t}) + u(g_t) + \beta\left(\tilde{u}(c_{O,i,t+1}) + u(g_{t+1}) + \lambda U_{Y,i,t+1}\right), \tag{2}$$

where the subscripts Y and O stand for "young" and "old", respectively, β is the discount factor, and $\lambda \geq 0$ is the altruistic weight on the utility of the agent's child (denoted by $U_{Y,i,t+1}$). We assume the functions $\tilde{u}$ and u to be strictly increasing and strictly concave, with $\lim_{x \to 0} u'(x) = \infty$. Agents maximize (2) subject to the budget constraint

$$c_{Y,i,t} + \frac{c_{O,i,t+1}}{R} = x_{Y,i,t} + A(\tau) - \frac{x_{O,i,t+1}}{R}. \tag{3}$$

where $x_{Y,t}$ is the wealth inherited by the young agent at time t and $x_{O,t+1}$ are the bequests the old agent leaves to her offspring. We omit dynasty subscripts henceforth.

We maintain throughout that $\lambda\beta R < 1$, emphasizing imperfect altruism. In the benchmark case we abstract from bequests by assuming that (i) agents *cannot* leave negative bequests, and (ii) λ is sufficiently small so that agents *do not want to* leave positive bequests.

Assumption 1 $x_{O,t+1} \geq 0$ *for all* $t \geq 0$ *(negative bequests are not allowed).*

Lemma 1 *Let* $\hat{x} < \infty$ *denote the bequest of the young in the first period. Then,* $\exists\ \bar{\lambda}(\hat{x}) > 0$ *(defined in the proof) such that, for all* $\lambda < \bar{\lambda}(\hat{x})$, *the desired bequest would be negative for all feasible tax sequence* $\{\tau_t\}_{t=0,\ldots,\infty}$ *such that* $\tau_t \in [0, \bar{\tau}]$*. Then, given Assumption 1,* $x_{Y,t} = x_{O,t} = 0$ *for all* $t \geq 1$.

The intuition of the proof of Lemma 1 is simple. Absent constraints, old agents would choose bequests so that $\tilde{u}'(c_{O,t}) = \lambda\tilde{u}'(c_{Y,t})$, so desired bequests increase with λ. However, these could be negative, since young agents also have a labor income. Labor income, in turn, has a natural lower bound, since no rational government would ever tax beyond the top of the Laffer curve. The existence of a Laffer curve guarantees that the consumption of the young be positive and its marginal utility be finite for *any* feasible (and rational) fiscal policy. Thus, one can always find a range of low λ's such that the old would like to grab resources from their children rather than leave bequests. An extreme example is the canonical OLG model without altruism, where old agents would set, if they could, infinitely negative bequests. Motivated by Lemma 1, we focus on low λ and abstract from bequests in our main analysis.[4] However, since the case of unconstrained bequests helps highlight the mechanism behind our results, we relax Assumption 1 in section 6. Absent bequests, the optimal consumption in both periods is only a function of total after-tax labor income: $c_Y = c_Y(A(\tau))$ and $c_O = c_O(A(\tau))$, where $c_Y'(\cdot) > 0$, $c_O'(\cdot) > 0$, and $\tilde{u}'(c_Y)/\tilde{u}'(c_O) = \beta R$.

Fiscal policy is determined every period through repeated elections. Given an inherited debt b, the elected government chooses the tax rate (τ), the public good provision (g) and the debt accumulation (b'), subject to the following dynamic budget constraint:[5]

$$b' = g + Rb - \tau w H(\tau). \tag{4}$$

Both private agents and governments have access to an international capital market providing borrowing and lending at the constant gross interest rate R. The government is committed to

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[4] A number of empirical studies document that only a small fraction of the population leave significant bequests (see, e.g., Hurd, 1989, Leitner and Ohlsson, 2001). Moreover, part of these bequests are involuntary. These observations motivate the focus of our analysis on low altruism.

[5] Hereafter, unless specified otherwise, we omit time indexes and switch to a recursive notation with primes denoting next-period variables.

not repudiate the debt. This implies that debt cannot exceed the present discounted value of the maximum tax revenue that can be collected:

$$b \leq \frac{\bar{\tau} w H\left(\bar{\tau}\right)}{R-1} \equiv \bar{b}, \tag{5}$$

where $\bar{b}$ denotes the natural debt limit and, recall, $\bar{\tau}$ is the tax rate attaining the top of the Laffer curve. The constraint (5) rules out government Ponzi schemes. Throughout the paper we restrict attention to the increasing portion of the Laffer curve, i.e., $\tau \leq \bar{\tau}$, since larger taxes would never be chosen under the political mechanism that we consider. We restrict debt to be in a compact set, $b \in \left[\underline{b}, \bar{b}\right]$.[6] This restriction, together with the government budget constraint (4), implies that also g and τ are bounded: $\tau \in [0, \bar{\tau}]$, and $g \in [0, \bar{g}]$.

2.1 Probabilistic Voting

Next, we turn to the political determination of fiscal policy. The indirect utility of young and old voters are, respectively:

$$\begin{aligned} U_Y\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right) &= \phi\left(A\left(\tau\right)\right)+u\left(g\right)+\beta\left(u\left(g'\right)+\lambda U_Y\left(\boldsymbol{b}',\boldsymbol{\tau}',\boldsymbol{g}'\right)\right), & (6)\\ U_O\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right) &= \tilde{u}\left(c_O\left(A\left(\tau_{-1}\right)\right)\right)+u\left(g\right)+\lambda U_Y\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right). & (7)\end{aligned}$$

where τ_{-1} denotes the tax rate in the period when the current old were young, and $\phi\left(A\left(\tau\right)\right) \equiv \tilde{u}\left(c_Y\left(A\left(\tau\right)\right)\right)+\beta\tilde{u}\left(c_O\left(A\left(\tau\right)\right)\right)$ is the discounted lifetime utility of private consumption.[7] The properties of the utility function $\tilde{u}$ guarantee that $d\phi/dA > 0$ and $d^2\phi/dA^2<0$. In (6), primes denote next period's variables and boldface variables are vectors, defined as follows: $\boldsymbol{x} = [x, x', x'', ...] = [x, \boldsymbol{x}']$.

We model the political mechanism as a two-candidate probabilistic voting model *à la* Lindbeck and Weibull (1987), which is extensively discussed in Persson and Tabellini (2000). In this model, agents cast their votes on one of two office-seeking candidates. Voters' preferences may differ not only over fiscal policy, but also over other orthogonal policy dimensions about which the candidates cannot make binding commitments. In a probabilistic voting equilibrium, both candidates propose the same fiscal policy, which turns out to maximize a weighted sum of individual utilities where the weights may differ between young and old agents.[8] Thus, the

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[6] The lower bound on b simplifies the analysis since it avoids uninteresting corner solutions in taxes and market labor supply when the government is very rich. This restriction is innocuous since $\underline{b}$ can be chosen to be so small that it will not bind in the political equilibrium.

[7] Due to additively separable preferences there is no interaction between the wealth of the old $(A\left(\tau_{-1}\right))$ and any political choice variable. Given our focus on Markov equilibria, τ_{-1} can be ignored, as we will discuss later. For this reason, with some abuse of notation, we write $U_O\left(\boldsymbol{b},\boldsymbol{\tau},\boldsymbol{g}\right)$ instead of $U_O\left(\boldsymbol{b},\tau_{-1},\boldsymbol{\tau},\boldsymbol{g}\right)$.

[8] The weights can differ due to differences (between young and old) in their focus on fiscal policy relative to the orthogonal issues. The political clout of each group reflects the relative proportion of "swing voters", or the ability of the group to organize lobbies (see Persson and Tabellini, 2000).

equilibrium policy can be represented as the choice each period of τ, g and b' maximizing a weighted average indirect utility of young and old households, given b. We denote the weights of the old and young as ω and $1-\omega$, respectively. Then, the *political objective function* is given by

$$U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) = (1-\omega) U_Y(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}) + \omega U_O(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g}), \tag{8}$$

subject to (4) and (5).

Probabilistic voting has been widely used in the recent fiscal policy literature (see, e.g., Dixit and Londegran 1998, Strömberg 2004, Hassler et *al.* 2005, Gonzalez Eiras and Niepelt 2008). We chose this political mechanism because it is both tractable and allows us to isolate the lack of commitment from the confounding effect of other sources of political inefficiency. It can be viewed as the microfoundation to a planning problem without commitment, where the planner attaches weights to individuals in different age groups equal to the respective political influence. This links our analysis explicitly to the macroeconomic dynamic fiscal policy literature with and without commitments (see, e.g., Klein and Ríos-Rull 2003, Klein, et *al.* 2008).[9]

Using (6)-(7), pre-multiplying all terms by the constant $(1+\lambda)/(1+\omega\lambda)$, and ignoring the predetermined term $\tilde{u}(c_O(A(\tau_{-1})))$, we can write (with a slight abuse of notation) the sequential representation of the political objective function:

$$U_0 = (1+\lambda) u(g_0) + \delta\lambda\phi(A(\tau_0)) + \delta \sum_{t=1}^{\infty} (\beta\lambda)^t ((1+\lambda) u(g_t) + \lambda\phi(A(\tau_t))), \tag{9}$$

where $\delta \equiv 1 + (1-\omega)/(\lambda(1+\lambda\omega)) \geq 1$ captures the influence of the young on the political objective function. Note that δ is decreasing in ω. Thus, a larger influence of the young (higher δ) means more concern for current private consumption – and, hence, a higher aversion to current taxation – and a higher weight on future felicities. In contrast a larger weight on the old (lower δ) means more intense preferences for current public good provision.

The political objective function, (9), resembles the present discounted utility of an individual decision maker (e.g., a benevolent planner) whose preferences induce quasi-geometric discounting, giving rise to time inconsistent choices. In earlier papers, political conflict led to excess discounting in the first period (see, e.g., Alesina and Tabellini 1990a and Azzimonti Renzo 2009). In contrast, in our model the time inconsistency takes the form of extra patience: since $\delta \geq 1$, the discount factor in the initial period ($\delta\beta\lambda$) exceeds the discount factors

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[9] In a recent prominent contribution, Battaglini and Coate (2007) analyze fiscal policy in a dynamic setting with the aid different political model based on legislative bargaining. Their model is especially useful to capture conflicts of interest across geographical districts, while probabilistic voting seems better suited to study the intergenerational conflict which is the focus of this paper.

of subsequent periods ($\beta\lambda$). The source of time inconsistency is the political representation of the young who have a direct (as opposed to altruistic) concern for next-period public-good consumption. In contrast, the young and the old agree on the intertemporal trade off between the utilities accruing to subsequent future generations. Thus, they discount future utilities geometrically from the second period onwards.

The first-period felicity in the political objective function also differs from that of subsequent periods in that the discounted utility from private consumption of the young, $\phi\left(A\left(\tau\right)\right)$, has a higher intratemporal weight relative to public-good consumption in the first period. This is again due to the direct influence of the young whose disposable income depends on τ. To gain more intuition, consider the sequential representation of the preferences of the young:

$$U_0^Y = u\left(g_0\right) + \left(1+\lambda^{-1}\right)\sum_{t=1}^{\infty}\left(\beta\lambda\right)^t u\left(g_t\right) + \sum_{t=0}^{\infty}\left(\beta\lambda\right)^t \phi\left(A\left(\tau_t\right)\right).$$

The young have quasi-geometric preferences over g because the first cohort of young agents cares directly about g_1. An alternative interpretation is that there is a shortfall of weight on $u\left(g_0\right)$ since young voters do not care about the utility of the initially old. In contrast, the preference of the old feature geometric discounting over both g and τ.[10]

3 Equilibrium

We first characterize the (Ramsey) policy sequence that would be chosen by the first generation of voters if they could commit the entire future path of fiscal policy. Then, we move to the political equilibrium which is the main contribution of this paper.

3.1 The Commitment Solution

Consider, first, the commitment problem. Since $\delta \geq 1$, the maximization of (9) subject to (5) and a sequence of government budget constraints, (4), does not admit a standard recursive representation. However, it admits the following two-stage recursive formulation (proof omitted).

Lemma 2 *The commitment problem is characterized as follows;*

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[10] If altruism were double-sided, the political objective function would still be time inconsistent. Suppose the young cared about the old with an altruism factor λ_Y. Then, the political objective function would have the same form as in (9). However, δ would be replaced by $\hat{\delta} \equiv 1+\left(1/\lambda - \omega/\left(1-\omega+\lambda\omega\right)\right)/\left(1+\lambda_Y+\omega/\left(1-\omega+\lambda\omega\right)\right) \geq 1$, where $\hat{\delta} = 1$ if and only if $\omega = 1$ (see Appendix B for details). This expression is smaller and closer to unity the larger is λ_Y. Intuitively, the only way preferences can be time consistent with some influence of the young is if the young would like to do exactly what pleases the old, i.e. $\lambda_Y \to \infty$.

(i) After the initial period, policies are the solution to the recursive problem

$$V_O^{comm}(b) = \max_{\{\tau, g, b'\}} \left\{ v(\tau, g) + \beta \lambda V_O^{comm}(b') \right\}, \tag{10}$$

subject to (4)-(5), where

$$v(\tau, g) \equiv (1+\lambda) u(g) + \lambda \phi(A(\tau)). \tag{11}$$

(ii) In the initial period, policies solve the following problem, subject to (4)-(5):

$$\{\tau_0, g_0, b_1\} = \arg \max_{\{\tau_0, g_0, b_1\}} \left\{ v(\tau_0, g_0) + (\delta - 1) \lambda \phi(A(\tau_0)) + \delta \beta \lambda V_O^{comm}(b_1) \right\}. \tag{12}$$

Consider, first, the particular case in which $\omega = \delta = 1$, i.e., old agents dictate their preferred policy. Then, the political objective function is time consistent, and the commitment solution coincides with the allocation that would be chosen sequentially by subsequent generations of old voters. Standard arguments establish that the program is a contraction mapping, and, hence, a solution exists and is unique (see Lemma 7 in Appendix B). To solve the program, we combine the FOCs with respect to τ and g and invoke $A'(\tau) = -wH(\tau)$ to obtain the following intratemporal optimality condition:

$$\lambda \phi'(A(\tau)) = (1 - e(\tau))(1+\lambda) u'(g), \tag{13}$$

Since $e'(\tau) > 0$, and $\phi(\cdot)$ and $u(\cdot)$ are concave, higher g is associated with lower τ and, hence, higher private consumption. Intuitively, the planner equates the marginal cost of taxation (foregone utility from private consumption of the young) to its marginal benefit (marginal utility of public good consumption, adjusted by the marginal cost of raising public funds). In addition, standard analysis leads to an Euler equation for public consumption:

$$\frac{u'(g)}{u'(g')} = \beta \lambda R. \tag{14}$$

Consider next the general case when also the young have political influence ($\omega < 1$). After the initial period, the optimality conditions (13)-(14) continue to characterize the commitment solution, irrespective of ω. Since $\beta \lambda R < 1$, (14) implies that public good provision declines asymptotically to zero.[11] Then, (13) implies that taxes converge to the top of the Laffer curve. Finally, (4) implies that debt converges asymptotically to the natural limit, $\bar{b}$.

Proposition 1 *The commitment solution converges to "public poverty":*
$\lim_{t\to\infty} b_t = \bar{b}$, $\lim_{t\to\infty} g_t = 0$ *and* $\lim_{t\to\infty} \tau_t = \bar{\tau}$.

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[11] If $\beta \lambda R = 1$, the solution would be stationary, and debt, taxes, and consumption would remain constant at their initial levels, as in Barro (1979).

However, in the general case the initial-period problem is different from that of the following periods. The young demand less government expenditure and more fiscal discipline (i.e., lower taxation and debt accumulation) than do the old. As a result, the commitment solution is time inconsistent: if at some point in future the living voters were allowed to re-optimize, they would deviate from the policy sequence dictated by the initial generation.

3.2 The Political Equilibrium

In this section we consider the political equilibrium under repeated voting. In equilibrium, the objective function (9) is maximized, but policies are chosen sequentially without commitment. Thus, a political equilibrium is a Nash equilibrium of the dynamic game between successive generations of voters. The set of equilibria is potentially large. We restrict attention to Markov Perfect Equilibria (MPE) where agents condition their voting strategies only on pay-off-relevant state variables. In principle, consecutive periods are linked by two state variables: the government debt, b, and the private wealth of the old. However, since preferences are separable between private and public goods consumption, the wealth of the old does not affect their preference over fiscal policies. Therefore, b is the only pay-off-relevant state variable.[12]

Definition 1 *A Markov Perfect Political Equilibrium (MPPE) is defined as a 3-tuple* $\langle B, G, T\rangle$*, where* $B : [\underline{b}, \bar{b}] \to [\underline{b}, \bar{b}]$ *is a debt rule,* $b' = B(b)$, $G : [\underline{b}, \bar{b}] \to [0, \bar{g}]$ *is a government expenditure rule,* $g = G(b)$*, and* $T : [\underline{b}, \bar{b}] \to [0, \bar{\tau}]$ *is a tax rule,* $\tau = T(b)$*, such that:*

1. $\langle B(b), G(b), T(b)\rangle = \arg\max_{\{b' \in [\underline{b}, \bar{b}], g \in [0, \bar{g}], \tau \in [0, \bar{\tau}]\}} U(\boldsymbol{b}, \boldsymbol{\tau}, \boldsymbol{g})$, *subject to (4), (5), and (8), where* $\boldsymbol{b} = [b, b', B(b'), B(B(b')), ...]$, $\boldsymbol{\tau} = [\tau, T(b'), T(B(b')), T(B(B(b'))), ...]$, *and* $\boldsymbol{g} = [g, G(b'), G(B(b')), G(B(B(b'))), ...]$.

2. *The government budget constrained is satisfied:*

$$B(b) = G(b) + Rb - T(b) \cdot w \cdot H(T(b)) \tag{15}$$

Definition 2 *A MPPE is said to be differentiable (DMPPE) if the equilibrium functions* $\langle B, G, T\rangle$ *are continuously differentiable in the interior of their domain,* $(\underline{b}, \bar{b})$*.*

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[12] Note that we abstract throughout the paper from taxes on the returns to savings or on consumption. If the government could also tax savings, the wealth of the old would become an additional state variable, and the analysis would be substantially more complicated. Note that – given the focus on Markov strategies – young voters would support 100% taxation on sunk savings, since this taxbase is inelastic. In contrast, the old would oppose any such tax, and the strength of their opposition would be inversely related to their wealth, due to probabilistic voting. The equilibrium tax rate would be intermediate.

In words, the government chooses the current fiscal policy (taxation, expenditure and debt accumulation) subject to the budget constraint, under the expectation that future fiscal policies will follow the equilibrium policy rules, $\langle B(b), G(b), T(b)\rangle$. Furthermore, the vector of policy functions must be a fixed point of the system of functional equations in part 1 and 2 of the definition, where part 2 requires the equilibrium policies to be consistent with the resource constraint. The following Lemma is a useful step to characterize the MPPE.

Lemma 3 *The MPPE (part 1 of Definition 1) admits the following two-stage formulation:*

$$\langle B(b), G(b), T(b)\rangle = \arg \max_{\left\{b' \in \left[\underline{b}, \bar{b}\right], g \in [0, \bar{g}], \tau \in [0, \bar{\tau}]\right\}} \left\{v(\tau, g) + (\delta - 1)\lambda\phi(A(\tau)) + \delta\beta\lambda V_O(b')\right\}, \tag{16}$$

where $v(.)$ *is defined as in (11), subject to (4) and (5), and* V_O *satisfies the functional equation;*

$$V_O(b') = v(T(b'), G(b')) + \beta\lambda V_O(B(b')). \tag{17}$$

The difference between the commitment solution and the political equilibrium can be seen by comparing (10) with (17). In the political equilibrium, the first generation of voters cannot choose the entire future policy sequence, but take the mapping from the state variable into the (future) policy choices as given. For this reason, there is no maximization operator in the definition of V_O. The two programs are identical if and only if $\omega = \delta = 1$ (only the old vote), i.e., when the commitment solution is time consistent.

Why does the commitment solution differ from the MPPE? Recall that the children are more fiscally disciplined than their parents. In the commitment solution, the intergenerational conflict plays out after one period, as discussed above. In contrast, the conflict is persistent in the MPPE, as a new generation of young voters enters the stage in each election. As a result, the political equilibrium delivers, as we will see, more fiscal discipline. If a differentiable political equilibrium exists, it can be characterized by applying standard recursive methods to the FOCs of (16)-(17). The results are summarized by the following Proposition.

Proposition 2 *A DMPPE is fully characterized by a system of two functional equations:*

1. *A trade-off between private and public good consumption*

$$\delta\lambda\phi'(A(\tau)) = (1 - e(\tau))(1 + \lambda)u'(g). \tag{18}$$

where $g = G(b)$ *and* $\tau = T(b)$.

2. A Generalized Euler Equation (GEE) for public good consumption:

$$\frac{u'(g)}{u'(g')} = \beta\lambda R - \underbrace{(\delta - 1)\beta\lambda G'(b')}_{\textit{the disciplining effect}}, \tag{19}$$

where $g = G(b), g' = G(b'), \tau = T(b)$ *and* $b' = g + Rb - \tau w H(\tau) \equiv B(b)$.

Consider, first, equation (18). The only difference between (18) and (13) lies in the δ term appearing in the left-hand side of (18): increasing the weight of the young (i.e., increasing δ) increases the marginal disutility of taxation (LHS) in the political objective function. Thus, the MPPE features lower taxes than does the commitment solution.

The GEE, (19), is the key equilibrium condition. The ratio between the marginal utilities of public good consumption in two consecutive periods consists of two terms. The first, $\beta\lambda R$, is the standard Euler-equation term appearing in the commitment solution, (14). The second, which we label as the *disciplining effect*, arises from the dynamic game between successive generations of voters, and was absent from the commitment solution. Young voters anticipate that an increase in the future debt will prompt a fiscal adjustment. This effect hinges on the forward-looking voting of the young, and vanishes when the old have full political power. The derivative $G'(b')$ describes the effect of the future fiscal adjustment on next-period government expenditure. Although a global characterization of G' is not available – except in particular cases discussed below –, we can establish that in a neighborhood of any steady state $G' < 0$, i.e., higher debt is associated with lower public spending.[13] Consequently, the disciplining effect increases the growth rate of public expenditure. As in a standard Euler equation, high growth of g is attained by reducing expenditure and increasing public savings today.

Consider the comparative statics of the influence of the young: Increasing δ magnifies the disciplining effect thereby restraining debt accumulation. Moreover, conditional on b, it reduces taxes and government expenditure – see equation (18). The case of majority voting with a majority of young is a limit case of Proposition 2 where $\delta = 1 + \lambda^{-1}$, maximizing fiscal discipline.

The commitment solution coincides with the MPPE when only the old have political influence. In this case, a standard contraction mapping argument ensures the existence and uniqueness of the MPPE (proof omitted).

Lemma 4 *Assume that* $\omega = \delta = 1$. *Then, the MPPE induces the same allocation as the commitment solution. Consequently, the MPPE exists and is unique.*

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[13] In steady state, $u'(g) = u'(g')$. Thus, the GEE (19) reduces to $G'(b^*) = -(1 - \beta\lambda R) / ((\delta - 1)\beta\lambda) < 0$, where $G'(b^*)$ is independent of b^*. It can also be established that G is concave in the neighborhood of a steady state, as long as b converges monotonically to the steady state.

In Appendix B, we provide sufficient conditions for the equilibrium policy functions to be continuous and differentiable (namely, for the equilibrium to be a DMPPE) in the $\omega = 1$ case (see Lemma 8). The crux is to impose restrictions on preferences and on the household technology that guarantee the concavity of the return function in the contraction mapping.

Extending the proof of existence and uniqueness of the MPPE to the general case of $\omega < 1$ is not straightforward. This is a common problem, as dynamic games generally do not admit a contraction-mapping formulation. However, Judd (2004) provides a strategy for proving local existence and uniqueness in such environments. He proposes to perturb the GEE in the neighborhood of a particular parameter configuration for which the problem is a contraction mapping. Here, we exploit the same strategy, by perturbing the equilibrium around the $\omega = 1$ case. The following proposition establishes local existence and uniqueness of the DMPPE.

Proposition 3 *Let* $\left\langle \bar{B}(b), \bar{G}(b), \bar{T}(b) \right\rangle$ *denote equilibrium policies when* $\omega = 1$. *Assume that* $\bar{B}(b), \bar{G}(b), \bar{T}(b)$ *are continuously differentiable. Suppose that*

$$\left| \frac{u''(g)}{\beta\lambda R u''(g')} - \bar{G}'(b') \left(1 - \frac{\phi'(A(\tau))\, u''(g)\, wH(\tau)(1 - e(\tau))/u'(g)}{\phi''(A(\tau))\, A'(\tau) + \phi'(A(\tau))\, e'(\tau)/(1 - e(\tau))} \right) \right| > 1, \tag{20}$$

where $g = \bar{G}(b)$, $g' = \bar{G}(b')$, $\tau = \bar{T}(b)$ *and* $b' = \bar{B}(b)$. *Then, for* δ *close to unity, there exists a unique DMPPE.*

The proof, which follows Judd (2004), is in Appendix B. Note that condition (20) is imposed on the equilibrium functions of the case with $\omega = \delta = 1$, for which existence and uniqueness are guaranteed (see Lemma 4). Thus, condition (20) can be verified either analytically in special cases admitting closed-form solutions (as we do in section 4.1), or numerically.

3.3 No Altruism

The analysis of the previous sections nests the standard OLG model without altruism. When $\lambda = 0$, equations (18) and (19) simplify to

$$(1 - \omega)\, \phi'(A(\tau)) = (1 - e(\tau))\, u'(g), \tag{21}$$

$$\frac{u'(g)}{u'(g')} = -\underbrace{(1 - \omega)\, \beta G'(b')}_{\text{the disciplining effect}} \tag{22}$$

where $g = G(b), g' = G(b')$, $b' = B(b)$, and $\tau = T(b)$. In this case, the social planner problem becomes trivial: the planner only cares about the first two periods and, hence, chooses $g = 0$, $b = \bar{b}$ and $\tau = \bar{\tau}$ thereafter. In contrast, the dynamics of the political equilibrium are isomorphic to the case of positive altruism. The old would like to set $b' = \bar{b}$ and $\tau = \bar{\tau}$ in order to maximize current public good consumption. However, the young care about next-period public good provision and each period exercise fiscal discipline. The case of no altruism is appealing for its simplicity, and we will dwell on it in the next section.

4 Two Analytical Examples

In this section we highlight some salient features of the political equilibrium by studying two special cases that admit analytical solutions – one with lump-sum taxation and one with a linear household-production technology. We parameterize the utility to be logarithmic, $\tilde{u}(c) = \log(c)$ and $u(g) = \theta \log(g)$, where $\theta > 0$ is a parameter describing the intensity of preferences for public good consumption. The household production technology is $F(h) = X \cdot \left(1 - h^{1+\xi}\right) / (1+\xi)$, where $\xi > 0$ is the inverse of the Frisch elasticity. For simplicity, we set $\lambda = 0$.

4.1 Example I: Inelastic Labor Supply

In this example we set $X = 0$ and focus on inelastic market labor supply ($H = 1$).[14] Hence, $A(\tau) = (1-\tau) w$, $e(\tau) = 0$, $\bar{\tau} = 1$, and $\bar{b} = w/(R-1)$. Equation (18) then simplifies to

$$g = \frac{\theta w (1-\tau)}{(1-\omega)(1+\beta)}. \tag{23}$$

Substituting (23) into the government budget constraint, (4), yields;

$$b' = \left(1 + \frac{(1+\beta)(1-\omega)}{\theta}\right) g + Rb - w. \tag{24}$$

To obtain a solution, we guess that G is linear; $G(b) = \gamma\left(\bar{b} - b\right)$. Then, the GEE, (19), yields $\left(\bar{b} - B(b)\right) / \left(\bar{b} - b\right) = \gamma (1-\omega) \beta$. This equation, together with the budget constraint, (24), the equilibrium condition $b' = B(b)$, and the expression for $\bar{b}$ given above, lead to $\gamma = \theta R / ((1-\omega)(1+\beta)(1+\theta) + \omega\theta)$. Substituting g by its equilibrium expression in (23)-(24) yields a complete characterization, summarized in Proposition 4 (proof in the text).

Proposition 4 *Assume inelastic labor supply, implying that* $\bar{b} = w/(R-1)$. *Then, there exists a DMPPE characterized by the following policy functions:*

$$\tau = T(b) = 1 - \frac{1}{w} \frac{(1-\omega)(1+\beta)R}{(1-\omega)(1+\beta)(1+\theta) + \theta\omega} \left(\bar{b} - b\right) \tag{25}$$

$$g = G(b) = \frac{\theta R}{(1-\omega)(1+\beta)(1+\theta) + \theta\omega} \left(\bar{b} - b\right), \tag{26}$$

$$b' = B(b) = \bar{b} - \frac{(1-\omega)\beta\theta R}{(1-\omega)(1+\beta)(1+\theta) + \theta\omega} \left(\bar{b} - b\right). \tag{27}$$

---

[14] Since the Laffer curve has no interior maximum, the sufficient condition of Lemma 1 is not satisfied in this example. However, since $\lambda = 0$ agents would like to leave negative bequests if unconstrained.

Note that $G'(.) = -\gamma < 0$. Hence, the disciplining effect in equation (19) increases the growth rate of public spending and decreases the growth rate of debt. This is a global property in this analytical example. In fact, due to the linear dynamics, the disciplining effect does not change with the debt level. For simplicity we restrict attention to the case in which the interest rate is not too large (i.e., $R < (1+\beta)(1+\theta)/(\beta\theta)$). Then the debt converges gradually to its maximum level $\bar{b}$.

FIGURE 1 HERE

Figure 1 illustrates the political equilibrium with the aid of a numerical example. τ increases linearly with debt (panel *a*), while g declines linearly with debt (panel *b*). Panel *c* shows the law of motion of debt converging to $\bar{b}$. Panel *d* shows the time path of b starting out with $b_0 = 0$. As the figure shows, generation after generation, private and public consumption are progressively crowded out by debt repayment to the lenders. The debt builds up gradually: the old would like to set $b' = \bar{b}$, but they meet opposition from the young who are concerned about public expenditure in their old age. The concern for public good consumption is crucial to prevent an immediate resource depletion: if $\theta = 0$, all voters would agree to set $b' = \bar{b}$, and the young would secure their private consumption in old age through savings. In contrast, under commitment debt would attain the maximum level after two periods. Thus, future generations benefit from their political empowerment, although the long-run outcome is the same as under commitment.[15]

4.2 Example II: Linear Household Technology

In the previous section we showed that if the labor supply were inelastic, debt would converge asymptotically to the natural limit when R is sufficiently low. In this section, we construct a tractable example where debt remains bounded away of its natural limit. The qualitative debt dynamics of this example carry over to the general case of section 5.

We assume that taxation does not distort labor supply as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$. However, if $\tau > \bar{\tau}$, agents stop working in the market, and the tax revenue falls to zero. When the interest rate is sufficiently high (though not so high to induce asset accumulation), an economy starting from low initial debt converges in finite time to a steady state where $\tau = \bar{\tau}$,

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[15] In an earlier version of this paper, we considered $\lambda \geq 0$ (see Song et al. 2007). The results are qualitatively similar. The growth rate of debt is higher under commitment than in the political equilibrium. Interestingly, if λ is sufficiently large the political equilibrium implies a *falling* government debt, whereas debt would still converge to $\bar{b}$ under commitment.

$b < \bar{b}$ and $g > 0$.[16] In Appendix B, we provide a full characterization of the equilibrium which involves discontinuous policy functions and multiple steady states.[17] In a neighborhood of the lowest steady state, the equilibrium policy rules are:

$$b' = B(b) = b^* \equiv \bar{b}\left(1 - \frac{\theta(1-\bar{\tau})}{(1-\omega)\bar{\tau}(1+\beta)}\right) \tag{28}$$

$$\tau = T(b) = \begin{cases} \bar{\tau} - \frac{(1-\omega)(1+\beta)R}{w((1-\omega)(1+\beta)+\theta)}(b^* - b) & \text{if } b \in [\underline{b}, b^*) \\ \bar{\tau} & \text{otherwise} \end{cases} \tag{29}$$

$$g = G(b) = \begin{cases} \frac{w\theta(1-\bar{\tau})}{(1-\omega)(1+\beta)} + \frac{\theta R}{(1-\omega)(1+\beta)+\theta}(b^* - b) & \text{if } b \in [\underline{b}, b^*) \\ b^* + \bar{\tau}w - Rb & \text{otherwise} \end{cases} \tag{30}$$

FIGURE 2 HERE

Figure 2 plots the equilibrium in the neighborhood of the lowest steady state ($b = b^*$). Panel *a* shows the equilibrium tax policy: taxes increase linearly with the debt level as long as $b < b^*$. Thereafter, T is flat at $\tau = \bar{\tau}$. Panel *b* shows the equilibrium expenditure: public good provision declines linearly with the debt level as long as $b < b^*$. To the right of b^*, the government cannot increases taxes further, and must adjust entirely on the expenditure side. Thus, the government expenditure function becomes steeper. Panel *c* shows that the debt policy is flat around b^*. Therefore, if the initial debt level is sufficiently close to b^*, debt converges to b^* in one period and remains constant thereafter. Finally, panel *d* shows an example of the time path of b where convergence occurs in the first period.

We now discuss the intuition for these dynamics. In the linear equilibrium of example I, the concern of young voters for next period's public good provision was insufficient to prevent ever-growing debt accumulation (unless R was very large). The reason was that along the linear equilibrium path, the next government would respond to a larger debt not only by cutting expenditure, but also by increasing taxes and debt proportionally. As a result, each generation of voters passed the bill on to the next one at the cost of only a small sacrifice of public good consumption. However, passing the bill on becomes harder when tax distortions are convex. In example II, this effect is particularly stark. As debt approaches b^* and taxes approach $\bar{\tau}$, voters anticipate that future generations will not be able to set taxes above $\bar{\tau}$. The adjustment can only be made on government expenditure, and therefore the disciplining effect of the young becomes very large at the steady state. The point that in the presence of

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[16] When the interest rate is low, the equilibrium is qualitatively similar to that of of section 4.1: $\lim_{t\to\infty} b_t = \bar{b}$ and $\lim_{t\to\infty} g_t = 0$. However, since taxes cannot go above by $\bar{\tau}$, private consumption does not fall to zero.

[17] Multiple steady states is a fragile property. Numerical simulations suggest that they vanish when the labor supply distortion becomes smooth (i.e., $\xi < 1$). However, the robust features of this equilibrium are captured by the local dynamics around the lowest steady state, plotted in Figure 2.

commitment problems governments can sustain a higher expenditure when taxation is more distortionary echoes an argument made in a different context by Krusell *et al.* (1997).

5 A Calibrated Economy

The intuition behind the result of example II carries over to the general case with smooth labor supply distortions ($\xi \in (0, \infty)$). In this case, an analytical characterization of the equilibrium is not available. Instead, we perform numerical analysis, using a standard projection method with Chebyshev collocation (Judd, 1992) to approximate T and G, exploiting the equilibrium conditions (18) and (19). As a robustness check on the numerical results we also use the algorithm proposed by Krusell, Kuruscu and Smith (2002), which is based on the calculation of higher-order derivatives of (15), (18) and (19). Using fourth order derivatives identifies the same (internal) steady state as the one we analyze below, up to the fourth decimal point for debt. Even outside of the steady state the two solutions are quantitatively similar (see Figure A3 in Appendix B).

5.1 Benchmark Calibration

We calibrate the parameters as follows. Since agents live for two periods, we let a period correspond to thirty years. Accordingly, we set $\beta = 0.985^{30}$ and $R = 1.025^{30}$, implying a 1.5% annual discount rate and a 2.5% annual interest rate, consistent with the average real long-term on U.S. government bonds between 1960 and 1990. As we have no strong prior on ω, we simply assume equal political weights on the young and old ($\omega = 0.5$).[18] We normalize the wage to unity and set X to target a ratio of market consumption to total consumption (including the value of leisure and home production) to 0.3, following Apps and Rees (1996, Table 2). Since $\bar{\tau} = \xi / (1 + \xi)$, we set $\xi = 1.5$ to target a labor tax associated with the top of the Laffer curve of 60%. This implies a Frisch elasticity of 2/3.[19] In the sensitivity analysis we show that our results are robust to a wide range of values for this elasticity.

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[18] Proposition 3 establishes existence and uniqueness in the neighborhood of $\omega = 1$. Standard caveats apply as we extend a local result to lower ω's. However, we have solved for a range of economies holding constant the parameters of Table 1 and varying ω. The numerical routine always converges to a set of policy functions satisfying (up to numerical approximations) the equilibrium conditions. Moreover, the equilibrium policy functions change with continuity, namely, small changes in ω leads to small changes in the equilibrium functions (see Figure A3 in Appendix B). In none of the simulations we have found more than one equilibrium for each parameter configuration.

[19] Empirical estimates of the aggregate Frisch elasticity have a wide range. Micro estimates of the Frisch elasticity along the intensive margin – based on people who remain employed – indicate a low elasticity, at least for men. Macro estimates tend to be higher, as they include adjustments along the extensive margin. The real business cycle literature often assumes an elasticity of unity (Cooley and Prescott, 1995).

The two remaining parameters θ and λ are set so that in steady state the model matches the empirical debt-to-output ratio and the ratio of labor tax revenue to labor income. The average labor income tax has been about 27% in the US in the last thirty-five years, so we set $\tau^* = 0.27$.[20] The ratio of explicit US federal debt to GDP has been around 40% over the last decades. However, the government has also significant pension liabilities. Van den Noord and Herd, (1993) estimate these to be 60-90% of GDP. Thus, the total US debt is 100-130% of annual GDP. One period in our model is 30 years. Our notion of aggregate production abstracts from capital. With one period being 30 years and an empirical labor's share of output of 0.67, we target a steady-state level of debt to labor earnings of $b/wH = 120\% \times 0.67/30 = 6\%$. Table 1 summarizes the parameters.

Table 1: Calibration

Target observation		*Parameter*	
Annual discount rate	1.5%	β	0.985^{30}
Annual interest rate	2.5%	R	1.025^{30}
Average tax on labor	27%	θ	0.092
Tax rate corresponding to the top of the Laffer Curve	60%	ξ	1.5
Debt-GDP ratio (including Social Security liabilities)	120%	λ	0.674
Ratio of market-to-total consumption	30%	X	2.722
Relative political weight young-old	equal	ω	0.5

Figure 3 plots the equilibrium functions of our calibrated economy. As in example I of section 4.1, taxes are increasing in b (panel a) and public expenditure is decreasing in b (panel b). However, the debt policy is now a strictly convex function of b which crosses the 45-degree line twice: at an interior steady-state level ($b = 0.025$) and at the natural debt limit. Only the interior steady-state is stable. Thus, for any initial $b < \bar{b}$, the economy converges to the internal steady state (see panel d). The steady-state government expenditure is $g^* = 0.086$, implying a 19.9% ratio of public expenditure to private market consumption.[21] Panel d shows the transition of debt towards the steady-state.

FIGURE 3 HERE

Let us compare the calibrated economy with the analytical examples. In all cases, the tax function is non-decreasing and concave, while the expenditure function is decreasing and

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[20] In the period 1970-2005, the aggregate tax revenue minus corporate taxes minus social security contributions in the US was on average 18% (source: OECD). With a labor share of 0.67, this implies an average tax rate on labor of 27%. Klein and Ríos-Rull (2003) report an average income tax rate of 24% for the period 1947-90.

[21] The corresponding number for the U.S. over the period 1970-2005 happens to be precisely 19.9% when public goods are measured as expenditures on defense, highways, and a number of public goods provided on the state and local level (see Appendix B for a comprehensive list of items).

concave. In example I, where taxation is not distortionary, an increase in debt causes a proportional increase in taxation and cut in expenditure, so as to keep g/c constant. In example II, the policy functions are piece-wise linear with a kink at the steady state. This is because taxation is non-distortionary to the left of $\bar{\tau}$ and infinitely distortionary to the right of it. Accordingly, the g/c ratio is constant for $b \leq b^*$, and decreasing thereafter. In the general case of $\xi \in (0, \infty)$, as b increases, the tax function, $T(b)$ becomes less steep, whereas the expenditure function, $G(b)$, becomes steeper. Namely, at high debt levels, the government responds to debt accumulation by cutting expenditure more than by increasing taxes. Hence, the ratio of public-to-private consumption falls as b increases. This fall in relative government expenditure is what deters voters from demanding more debt in steady state.

The qualitative findings of an internal steady state are robust to a large range of all parameter values (in all experiments of this section we checked that the sufficient condition of Lemma 1 continues to be satisfied). We know from Example I that an internal steady state hinges on tax distortions. However, an internal steady state exists for even very low Frisch elasticities. For instance, consider changing the values of the Frisch elasticity while keeping the other parameters as in Table 1. If $\xi = 3$ (low Frisch elasticity), the steady state features a tax rate of 46% and debt-earnings ratio $b/wH = 23\%$ (an internal steady state actually exists for as low a Frisch elasticity as 0.01!).[22] If $\xi = 1$ (high Frisch elasticity), the corresponding figures are 17% and -2% (i.e., a government surplus). Thus, the more distortionary are taxes, the lower is the debt and the lower are taxes in steady-state.

As far as altruism is concerned, an interior steady state may cease to exist if λ is too low. For instance, if we fix all other parameters as in Table 1 and vary λ, a steady-state with $b < \bar{b}$ is sustained only if $\lambda > 0.55$. For lower λ's the economy converges to the maximum debt. However, an equilibrium with an interior steady state can be sustained even for $\lambda = 0$, as long as we increase either the interest rate or the Frisch elasticity. For instance, it is possible to match the same tax rate and debt-GDP ratio as in Table 1 if we set $\theta = 0.05$, $R = 1.055^{30}$, and $\xi = 0.5$ (β and ω being as in Table 1). Thus, on the one hand altruism is not crucial for our qualitative results. On the other hand, it may be quantitatively important: if parents were totally selfish, only a relatively high Frisch elasticity can sustain an equilibrium with realistic features.

The results are robust to changes in ω. If we fix all other parameters as in Table 1 and vary ω, a steady-state with $b < \bar{b}$ is sustained only if $\omega < 0.77$. If we set $\omega = 0.7$ while keeping

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[22] The case of $\xi = 3$ is consistent with the calibration of Trabandt and Uhlig (2006) who have a specification of the labor supply distortion similar to ours.

the other parameters as in Table 1, the steady state tax rate is 47% and the debt-earnings ratio is 63%. If $\omega = 0.3$, the corresponding figures are 13% and -12%. The results confirm that increasing the influence of the young strengthens fiscal discipline.

Finally, we investigated the robustness of the results to changes in the utility functions by considering u being CRRA; $u(g) = \left(g^{1-\sigma} - 1\right)/(1-\sigma)$, while maintaining $\tilde{u}(c) = \log(c)$ (the specification of $\tilde{u}$ has hardly any effect on the sustainability of an internal steady state). Note that preferences are still separable between c and g, an assumption we relax in section 5.5. The results are qualitatively similar to the log-log case discussed above. The steady-state debt level is decreasing with σ, and an internal steady state can be sustained as long as $\sigma \geq 0.48$. Intuitively, a larger σ makes agents more concerned about reduced future public good provision, thereby strengthening fiscal discipline. In Appendix B we report the simulated policy functions for $\sigma = 1.5$ and $\sigma = 0.75$.

5.2 Commitment vs. Markov Equilibrium

It is interesting to compare the DMPPE in the benchmark calibration of Figure 3 with the corresponding commitment (Ramsey) solution. We already know that under commitment debt converges to $\bar{b}$ and g converges to zero (Proposition 1). In Figure 4, we compare the transitional dynamics for economies starting with zero debt.

FIGURE 4 HERE

In the first period, the commitment solution features slightly lower taxes (τ_0) and higher government spending (g_0) than the DMPPE. Consequently, b_1 is higher under commitment. Government expenditure is significantly larger in the second period (g_1) than in the first period (g_0). This comes at the expense of a larger increase in the debt inherited by agents born in period two. Thereafter, debt accumulates at a higher rate in the commitment solution where taxes and spending converge, respectively, to the top of the Laffer curve (panel b) and to zero (panel c) and debt converges to $\bar{b}$ (panel a). All generations born in period two or later are strictly worse off in the commitment solution, while the agents who are alive at the time of the initial vote are better off.

5.3 A Fiscal Shock

Consider the effect of fiscal shocks in the MPPE. Suppose that the economy is hit by a one-period surprise war requiring an exogenous spending of Z units. During the war, the government's budget constraint changes to $b' = g + Rb - \tau w H(\tau) + Z$, and then, as peace returns, it reverts to (4). The shock occurs at the beginning of the period, before the government sets g, τ and b'. Suppose the economy is initially in the steady state b^* of Figure 3. The shock is equivalent to an exogenous increase in debt from b^* to $b^* + Z/R$. Thus, the government reacts by increasing τ and decreasing g in wartime. The time path of the fiscal adjustment is shown in panel A of Figure 5: the fiscal shock is absorbed by a combination of cuts in non-war expenditure and increases in debt and taxation. After the war debt, taxes and expenditure revert slowly to their original steady state.[23] These results stand in contrast with the normative model of Barro (1979), where the war should be financed by debt and, following the principle of tax smoothing, taxes and non-war expenditure should be adjusted permanently so as to guarantee a smooth repayment of the excess debt.

FIGURE 5 HERE

The prediction of our positive theory is consistent with the empirical evidence of Bohn (1998) who found the debt-to-output ratio to be highly persistent but mean reverting in the US. Namely, a fiscal shock induces an increase in the debt-to-output ratio on impact and a subsequent reversion towards its initial level – precisely as in Figure 5. In Song et al. (2007) we document that the same pattern holds in a panel of 21 OECD countries.

5.4 A Demographic Transition

Our theory can be extended to encompass a growing population. This allows us to study the effect of demographic changes such as the ageing population in OECD countries during the last decades. We assume that the resource cost of public-good provision is proportional to the population size. This avoids that the model features scale effects. For our purposes, this is a conservative assumption, as our results would be strengthened if economies with larger populations had a technological advantage in the provision of public goods.

We consider an economy undergoing a fully anticipated demographic transition such that in $t = 1$ it has an annualized population growth rate of 1%, and for all $t \geq 2$ it has a constant population and is identical to the benchmark calibration of Table 1. A falling population growth has two effects in the model. First, by increasing the relative size of the old cohort, it increases their political influence. More formally, let N_t denote the measure of the cohort

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[23] We have also studied recurrent wars, assuming that the state of the economy (war or peace) evolves following a first-order stationary Markov process. The results are similar to those of a surprise war. However, the positive probability of future wars induces an additional precautionary motive for public savings during peacetime, which is also present in the commitment solution, and it turns out that with recurrent wars, even the commitment solution features mean-reverting debt dynamics. Details are available upon request.

born at t. Then, the political weight of the young and old become, respectively, $(1-\omega)\, N_t$ and ωN_{t-1}. As population growth N_t/N_{t-1} becomes smaller, δ falls during the transition, causing a reduction in fiscal discipline. Second, the fall in population growth reduces the share of working population and the size of the taxbase.[24]

Panel B of Figure 5 shows the impulse response of debt per capita, public good per capita and taxes along the demographic transition. For illustrative purposes, we assume initial debt per capita to be equal to the final steady-state debt per capita. Clearly, this choice is arbitrary, but the main qualitative results do not hinge on it. At $t = 1$, when the share of young agents is large, debt falls, as anticipated above. Taxes are low and public good provision is high due to the large tax base. From $t = 2$ onwards, taxes grow and public good provision fall. Most interestingly, debt starts growing and eventually converges to the steady state. Thus, our theory predicts that ageing societies increase debt accumulation, in line with the empirical observation that since the 1980s an increasing share of old voters has been accompanied by a rising government debt, especially in quickly ageing societies like Japan. The U-shaped behavior of debt in the example also resembles the post-war pattern for debt in the US and most Western European countries.[25]

5.5 Non-Separable Utility

In this section, we generalize the analysis to non-separable preferences between private and public consumption. We assume $u(c,g) = \log\left(\left(\left(1/\left(1+\theta\right)\right)c^{\rho} + \left(\left(\theta/\left(1+\theta\right)\right)\right)g^{\rho}\right)^{1/\rho}\right)$ where $\rho < 1$. This specification encompasses the benchmark separable utility as $\rho \to 0$. Private and public good consumption are substitutes if $\rho > 0$ and complements if $\rho < 0$. This generalization has interesting implications: for instance, if agents can substitute private for public health services they may be less concerned for future public good provision and, hence, be less averse to public debt.

The analysis must take into account two new features. First, private savings now depend not only on current taxes but also on the current and next-period public good provision. Second, the private wealth of the old is a pay-off relevant state variable, since it affects the marginal utility of public expenditure of the old and, hence, the probabilistic voting equilibrium outcome. Formally, the equilibrium policy functions depend now on a two-dimensional state

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[24] Formally, the government budget constraint must be rewritten as $b_{t+1}\left(N_t + N_{t+1}\right)/\left(N_{t-1} + N_t\right) = g_t + Rb_t - \left(N_t/\left(N_{t-1} + N_t\right)\right)\tau_t w H\left(\tau_t\right)$ where b and g are now debt and public good per capita, respectively.

[25] Debt was initially high due to the war shock: (in 1946, the US Federal debt-GDP ratio was 122%), and fell gradually until the end of 1970s, reaching a trough of 33% in 1981. During this period, the population share over 40 went from 35% in 1948 to 36% in 1981. Thereafter, debt increased reaching 68% in 2008, while the population share over 40 went up to 46%. In the same period, taxation grew and the share of government purchase of goods and services fell. Both facts are consistent with the impulse response of Figure 5.

vector: $\tau = T(s_{-1}, b)$, $g = G(s_{-1}, b)$, and $b' = B(s_{-1}, b)$.

A formal characterization of the DMPPE in terms a system of functional equations analogue to those in Proposition 2 is deferred to Appendix B, where we also present some numerical results for different levels of ρ, using otherwise the same parameter as in Table 1. Here, we summarize the findings. When c and g are substitutes ($\rho > 0$), the demand of public good falls and fiscal discipline is weaker. Taking the steady-state debt level of Figure 3 as the initial condition, voters support an initial tax cut, lower public good provision and an increase in government debt. Debt converges to a higher steady state level with higher taxes and lower public good provision. An internal steady state is sustained as long as $\rho < 0.45$. The dynamics have the opposite sign (tax increase, higher public good provision and lower government debt) when c and g are complements ($\rho < 0$). Interestingly, when c and g are substitutes (complements) both private and public consumption are lower (higher) in the long-run than in the benchmark case.

6 Unconstrained Bequests

In the analysis thus far, we have ruled out negative bequests. In this section, we relax this constraint.[26] Although we do not view negative bequests as realistic, solving the model without constraints sheds light on the mechanism of the theory and allows us to derive additional implications about the dynamics of debt.

We assume that the old bequeath before the young decide their savings. This rules out the possibility for the young to choose their savings strategically in order to attract more bequests.[27] Since the old care about their children's utilities, our timing guarantees that there is no within-household disagreement about savings after the old have set bequests. For simplicity, we restrict attention to log utility. In this environment, by controlling bequests the old *de facto* dictate the intrahousehold allocation of private consumption. In Appendix B we show that under log utility the consumption of the young and old agents are, respectively, $c_Y = \lambda (1+\lambda)^{-1} c$, and $c_O = (1+\lambda)^{-1} c$, where c denotes total household consumption. Substituting these conditions into the expressions of U_Y and U_O, respectively, yields the following sequential representation of the political objective function:

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[26] The analysis of this section is general and does not require any restriction on altruism. However, for coherence with the focus of our main section, we continue to emphasize the case in which $\beta\lambda R < 1$. In this case, agents will eventually leave negative inheritances, although positive bequests are possible during finite transitions.

[27] Another potential source of strategic behavior could be labor supply. However, in our model there is no pure leisure, so the allocation of time between market and household production is affected by neither wealth nor bequests.

Lemma 5 *The political objective function can be written as:*

$$U_0 = \log(c_0) + \theta \log(g_0) + \delta \sum_{t=1}^{\infty} (\beta\lambda)^t (\log(c_t) + \theta \log(g_t)), \tag{31}$$

where, recall, $\delta \equiv 1 + (1-\omega)/(\lambda(1+\lambda\omega))$.

Since the political objective function under probabilistic voting coincides with that of a benevolent planner without commitment, we will henceforth refer to the *planner's objective function*. The planner's objective function (31) can be compared with its analogue in the case of no bequests, (9). Now, the weight of the young (captured by δ) has no effect on the intratemporal trade off between c and g in the first period.[28] However, as before, δ affects the intertemporal trade off. Consequently, the planner's preferences are time-inconsistent and feature quasi-geometric discounting. Thus, the time-consistent allocation is the solution of a dynamic game between the planner and her future selves as in Krusell and Smith (2003). We restrict attention to MPE as in the rest of the paper. Private savings is now an additional state variable because savings affect the marginal utility of consumption and hence taxation. The planner maximizes utility taking as given the policy rules governing her future selves' choices from the second period onward. The planning problem admits the following two-stage recursive representation:

$$V(s_{-1}, b) = \max_{\{c,\tau,g,b'\}} \left\{ \log(c) + \theta \log(g) + \delta\beta\lambda V^b(s, b') \right\}, \tag{32}$$

where

$$V^b(s_{-1}, b) = \log(C(s_{-1}, b)) + \theta \log(G(s_{-1}, b)) + \beta\lambda V^b(s, b'), \tag{33}$$

$C(s_{-1}, b)$ and $G(s_{-1}, b)$ are equilibrium private consumption and public expenditure functions. The maximization is subject to the government budget constraint, (4), and a budget constraint for private wealth, $s = Rs_{-1} + A(\tau) - c$, plus the respective no-Ponzi game conditions.

It is useful to distinguish between two planners endowed with different instruments. An N-planner (*non-empowered planner*) chooses fiscal policy sequentially and without commitment, being subject to the *implementability* constraint that private consumption is chosen optimally by the households. The N-planner allocation is equivalent to the MPPE of Section 3, except that now agents can leave any bequests. In contrast, an E-planner (*empowered planner*) has an additional instrument: she controls the intertemporal allocation of private consumption. The E-planner is meant to be an instructive normative (second-best) benchmark and helps build the intuition for the main results.

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[28] Note that this is a non-robust feature. With general utility, the weight of the young would also affect the planner's relative intratemporal weights of c and g. However, the main equilibrium features are similar to those obtained in the log case.

6.1 The E-planner Allocation

We start by analyzing the E-planner's problem. The following Proposition follows from the FOCs and envelope conditions of the program (32)-(33).[29]

Proposition 5 *Assume that $s_{-1} \geq \underline{s}$ and $b \leq \bar{b}$. A differentiable time-consistent E-planner allocation satisfies the following system of functional equations:*

1. *A trade-off between private and public good consumption*

$$\Delta^{\tau}\left(s_{-1}, b\right) \equiv \left(1 - e\left(\tau\right)\right)\frac{\theta}{g} - \frac{1}{c} = 0, \tag{34}$$

2. *A GEE for public good consumption:*

$$\Delta^{g}\left(s_{-1}, b\right) \equiv -\frac{\theta}{g} + \lambda\beta R\frac{\theta}{g'} - \left(\delta - 1\right)\lambda\beta\left(\frac{\theta}{g'}G_2\left(s, b'\right) + \frac{1}{c'}C_2\left(s, b'\right)\right) = 0, \tag{35}$$

3. *A GEE for private consumption:*

$$\Delta^{c}\left(s_{-1}, b\right) \equiv -\frac{1}{c} + \lambda\beta R\frac{1}{c'} + \left(\delta - 1\right)\lambda\beta\left(\frac{\theta}{g'}G_1\left(s, b'\right) + \frac{1}{c'}C_1\left(s, b'\right)\right) = 0, \tag{36}$$

where, in the three equations above, $g = G\left(s_{-1}, b\right)$, $c = C\left(s_{-1}, b\right)$, $\tau = T\left(s_{-1}, b\right)$, $g' = G\left(s, b'\right)$, $c' = C\left(s, b'\right)$, $s = Rs_{-1} + A\left(\tau\right) - c$, and $b' = g + Rb - \tau wH\left(\tau\right)$.

Equations (34)-(35) are the analogues of equations (18)-(19). As in the benchmark case, time inconsistency vanishes when $\omega = 1$, in which case the GEE is a standard Euler equation. The third term in (35) is the strategic (*disciplining*) effect which now incorporates the additional term $C_2\left(s, b'\right)/c'$ capturing the fact that a debt-financed increase in g affects future private consumption by reducing the total wealth of the planner. In the no-bequest case, $C_2 = 0$ since there the future consumption of the current young was independent of b'. The GEE for private consumption, (36), did not feature in the no-bequest case, since there every agent was born with zero private wealth. The first two terms yield a standard Euler equation for private consumption. The third term is a strategic effect: by saving, the planner increases her future self's wealth which in turn affects the future provision of c and g. Perhaps surprisingly, a full analytical characterization of the E-planner allocation can be found.

Proposition 6 *The E-planner allocation is characterized as follows:*

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[29] In the rest of this section we ignore, for simplicity, issues associated with negative taxation and corner solutions in labor supply. Internal solutions could be guaranteed by joint restrictions on the parameters and state space, but these are in general fairly complicated. Without imposing such restrictions, the E-planner solution may feature asset accumulation, as we will see, even though $\beta\lambda R < 1$. However, we continue to impose that neither governments nor dynasties can play Ponzi schemes and are subject to a natural debt limit.

1. *The tax policy function,* $T(s_{-1},b)$, *is the unique solution to the following equation:*
$$\theta\left(1-e\left(T\left(s_{-1},b\right)\right)\right)\left(\frac{A\left(T\left(s_{-1},b\right)\right)}{R-1}+s_{-1}\right)=\frac{w\tau H\left(T\left(s_{-1},b\right)\right)}{R-1}-b. \tag{37}$$
The other equilibrium policy functions are given by:
$$\begin{array}{rr} G\left(s_{-1},b\right)=\left(R-\varphi\right)\left(\frac{w\tau H(\tau)}{R-1}-b\right), & C\left(s_{-1},b\right)=\left(R-\varphi\right)\left(\frac{A(\tau)}{R-1}+s_{-1}\right), \\ B\left(s_{-1},b\right)=\varphi b+\left(1-\varphi\right)\frac{w\tau H(\tau)}{R-1}, & S\left(s_{-1},b\right)=\varphi s_{-1}-\left(1-\varphi\right)\frac{A(\tau)}{R-1}, \end{array}$$
where $\tau=T(s_{-1},b)$ *and* $\varphi\equiv\left(1+\frac{(\delta-1)(1-\beta\lambda)}{1+\beta\lambda(\delta-1)}\right)\beta\lambda R>\beta\lambda R$.

2. *Along the equilibrium path, the tax rate is constant, and equal to* $\tau=T\left(\hat{s}_{-1},\hat{b}_0\right)$, *where* $\hat{s}_{-1}$ *and* $\hat{b}_0$ *denote initial conditions,* $g'/g=c'/c=\varphi$ *and* $g/c=\theta\left(1-e\left(\tau\right)\right)$.

The E-planner chooses a constant tax rate, and uses public debt to ensure that along transition the ratio between private and public consumption is kept constant, as required by condition 1 in Proposition 5. This can be viewed as an extension of the tax-smoothing result of Barro (1979). Note that the growth rate of c and g is larger than $\beta\lambda R$, so that a positive wealth accumulation can be sustained even though $\beta\lambda R<1$. Note that (s_{-1},b) evolve over time albeit in a way such that $T(s_{-1},b)$ remain constant.

6.2 The N-planner Allocation (DMPPE)

With the aid of the E-planner allocation, we can now analyze the N-planner allocation, which is equivalent to the MPPE, i.e., the allocation we are ultimately interested in. We start from the households' saving decision which provides the implementability constraint for the N-planner (proof omitted).

Lemma 6 *The growth rate of total consumption follows a standard Euler equation,*
$$\frac{c'}{c}=\beta\lambda R. \tag{38}$$
Moreover, consumption and savings satisfy:
$$c = C\left(s_{-1},b\right)=\left(1-\beta\lambda\right)\left(Rs_{-1}+W\left(s_{-1},b\right)\right) \tag{39}$$
$$s = Rs_{-1}+A\left(\tau\right)-C\left(s_{-1},b\right), \tag{40}$$
where W *denotes the discounted future after-tax income satisfying the recursion:* $W\left(s_{-1},b\right)=A\left(T\left(s_{-1},b\right)\right)+R^{-1}W\left(s,b'\right)$, s *is given by (40), and* $b'=B\left(s_{-1},b\right)$. *The function* $W\left(s_{-1},b\right)$ *exists and is unique.*

Note that private consumption growth in (38) is lower than in the E-planner allocation. Intuitively, the old – who dictate saving decisions – have a lower discount factor than the E-planner. $C(s_{-1}, b)$ is the consumption function along the equilibrium path. However, the planner needs to know how current consumption responds to fiscal policy deviations in the current period. To this aim, let $\tilde{C}(g, \tau, b', s_{-1})$ denote current consumption as a function of the current fiscal policy under the assumption that equilibrium policies apply from next period onward. If g, τ and b' are evaluated in equilibrium, we have $\tilde{C}(G(s_{-1}, b), T(s_{-1}, b), B(s_{-1}, b), s_{-1}) = C(s_{-1}, b)$. Then, (39) implies that $\tilde{C}$ must satisfy the following recursive expression:[30]

$$\tilde{C}\left(g, \tau, b', s_{-1}\right) = (1-\beta\lambda)\left(Rs_{-1} + A(\tau) + R^{-1}W\left(Rs_{-1} + A(\tau) - \tilde{C}\left(g, \tau, b', s_{-1}\right), b'\right)\right). \tag{41}$$

The N-planner allocation is the solution to program (32)-(33), subject to the same constraints as in the E-planner problem, plus the implementability constraints

$$c = \tilde{C}\left(g, \tau, b', s_{-1}\right), \ \ s = Rs_{-1} + A(\tau) - \tilde{C}\left(g, \tau, b', s_{-1}\right). \tag{42}$$

The next proposition characterizes the solution to this program.[31]

Proposition 7 *Assume that $s_{-1} \geq \underline{s}$ and $b \leq \bar{b}$. A differentiable time-consistent N-planner allocation satisfies the following system of functional equations:*

$$0 = \Delta^g(s_{-1}, b) + \Delta^\tau(s_{-1}, b) \cdot \frac{C_2(s, b')}{\lambda\beta R} \tag{43}$$

$$0 = \Delta^c(s_{-1}, b) - \Delta^\tau(s_{-1}, b) + \beta\lambda R \cdot \Delta^\tau\left(s, b'\right) - \Delta^\tau(s_{-1}, b) \cdot \frac{C_1(s, b')}{\lambda\beta R} \tag{44}$$

where Δ^g, Δ^c and Δ^τ are defined in Proposition 5, $C(s, b') = \lambda\beta R \cdot \tilde{C}(g, \tau, b', s_{-1})$, and b' and s satisfy the government and household intertemporal budget constraints, respectively.

Proposition 5 defined three wedges, Δ^g, Δ^τ and Δ^c, that the E-planner would set to zero. The N-planner does not control private consumption and, therefore, has only two independent fiscal policy instruments. As she cannot set all three wedges to zero, she must trade off one wedge against another.

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[30] From Lemma 6 off-equilibrium consumption and discounted future after-tax income must satisfy

$$\tilde{C}\left(g, \tau, b', s_{-1}\right) = (1-\beta\lambda)\left(Rs_{-1} + \tilde{W}\left(g, \tau, b', s_{-1}\right)\right),$$

$$\tilde{W}\left(g, \tau, b', s_{-1}\right) = A(\tau) + R^{-1}W\left(Rs_{-1} + A(\tau) - \tilde{C}\left(g, \tau, b', s_{-1}\right), b'\right).$$

Substituting away $\tilde{W}$ yields expression (41).

[31] The Euler equation allows us to eliminate $\tilde{C}$ and its derivative and express all GEEs as functions of C. See the proof of Proposition 7 for details.

Consider, first, the GEE for public consumption, (43). On the one hand, setting $\Delta^g = 0$ would require a low public consumption today and a high growth rate of g. On the other hand, setting $\Delta^\tau = 0$ while holding τ constant (as the E-planner would do) would require that she keeps constant the g/c ratio, letting c and g grow at the same rate. However, it is impossible for the N-planner to achieve both objectives because households are more impatient than she is, and choose a low private consumption growth. When trading off these objectives, the N-planner chooses a lower growth rate of g than would the E-planner, but one that exceeds the growth rate of private consumption. As a result, the MPPE features positive wedges Δ^g and Δ^τ as well as an increasing g/c ratio.[32]

Consider, next, the GEE for taxes, (44). The E-planner would set $\Delta^c = \Delta^\tau = 0$. However, the N-planner cannot achieve $\Delta^c = 0$ as private consumption is controlled by the old. Thus, the N-planner uses the tax sequence to trade off the two wedges. In general, taxes will not be constant since the wedges change over time.

Contrary to the E-planner allocation, a full analytical characterization of the equilibrium is not available. Nevertheless we can establish a key long-run property of the model.

Corollary 1 *Assume* $\beta\lambda R < 1$. *Then, the DMPPE of Proposition 7 features* $\lim_{t\to\infty} c_t = 0$ *and* $\lim_{t\to\infty} g_t = 0$.

In sharp contrast with the no-bequest economy of section 3 the DMPPE features both private and public poverty in the long run. The intuition is simple: On the one hand, old agents with a low altruism control private consumption and choose a consumption sequence converging to zero. On the other hand, although the fiscal policy is subject to the disciplining influence of the young, a falling c would open an arbitrarily large gap between the marginal utility of private and public good consumption unless it were accompanied by a fall in g. More formally, as the marginal utility of private consumption tends to infinity, that of public good consumption must also tend to infinity, or, else, the intratemporal wedge would grow without bound. This tension was absent in the model of section 3. There, private consumption was protected by the existence of a Laffer curve and by the inability of the old to leave negative bequests, implying that the marginal utility of private consumption was bounded in any equilibrium. This made it possible to sustain equilibria where g does not fall to zero.

Figure 6 illustrates the equilibrium dynamics of the E-planner and N-planner allocations for the parameter values of Table 1. It displays the time path of private (*panel a*) and public-good (*panel b*) consumption, tax rate (*panel c*), g/c (*panel d*), private savings (*panel e*) and public

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[32] Note that $C_2(s, b') < 0$, since a higher debt decreases total wealth. Thus, the sign of the two wedges must be the same. Since the current c is too high for the taste of the planner, Δ^τ must be positive.

debt (*panel f*) in the E-planner and N-planner (DMPPE) allocations. The initial condition is in all cases the steady state of the benchmark economy of Table 1, which is plotted in the graphs for comparison. As shown in Proposition 6, the E-planner attains a permanently higher private consumption growth than the N-planner. Moreover, she lets c and g grow at a common rate. Under this calibration, the discipline effect of the E-planner is so strong that she accumulates both private and public wealth, inducing an ever growing sequence of c and g.[33] In contrast, the N-planner allocation converges to private and public poverty, consistent with Corollary 1. While the E-planner chooses a constant tax rate that is higher than the steady-state in the no-bequest economy, the tax rate falls over time in the N-planner allocation. Finally, the government accumulates assets in both the E-planner and the N-planner allocation (whereas b remains constant in the benchmark case). However, while in the E-planner allocation agents also accumulate private wealth, this is depleted in the N-planner allocation.

FIGURE 6 HERE

This extension allows us to draw some interesting lessons. It shows that in a world of imperfect altruism, if a small open economy decided to start enforcing debt passed on from parents to their heirs, this economy would accumulate both private and public debt, and experience an increasingly negative foreign asset position.

7 Conclusion

In this paper, we have constructed a theory of intergenerational private and public wealth transmission under the maintained assumptions that fiscal policy is determined by an elected government, agents have finite lives and low altruism, and the government cannot default on its debt. We have studied the equilibrium dynamics in different environments. In the benchmark model, we have maintained that agents cannot die with a negative private wealth, and considered alternative assumptions about the governments' ability to tie future governments' hands (commitment). In this environment, if the fiscal policy path were set on behalf of the first generation of voters with full commitment, the economy would fall into public poverty: future generations would end up being taxed on the top of the Laffer curve to repay the public debt accumulated by their ancestors. In contrast, if fiscal policy is decided sequentially by elected governments, the long-run equilibrium may feature positive private and public consumption.

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[33] In the E-planner problem, we do not have to impose a lower bound on b since we have analytical solution. To avoid uninteresting complications, we have allowed negative taxes in these simulations.

The model can be used as a workhorse for a positive analysis of debt policy. As a first step in this direction, we have shown with the aid of a calibrated example that the political equilibrium can sustain a long-run debt level that is in the range of the empirical observations for contemporary Western economies. Moreover, the model predicts changes in debt policy during demographic transitions that are consistent with the empirical observation that ageing societies have engaged in significant debt accumulation since the 1980s.

The mechanism through which the repeated influence of young voters affects policy over time can be important beyond the fiscal-policy debate. For instance, the discount factor is a key parameter for both positive and normative analyses of environmental sustainability. Our theory suggests that even though individuals may have a low intergenerational altruism, governments may be under pressure to intervene in defense of the environment, due to the repeated political influence of young agents who fear a depletion of environmental resources during their lifetime.

Tractability has been achieved at the cost of important assumptions. We plan to relax some of these in future research. For instance, in some work in progress we enrich the asset structure, introducing physical capital. In another paper in progress, we introduce intragenerational conflict by assuming that agents have heterogenous productivity, and derive testable implications about the propensity to debt of governments of different political color. Finally, we have maintained throughout that governments are committed to repay their debt and ruled out government Ponzi schemes. Integrating our analysis with the insights of the sovereign-debt literature may give rise to novel insights.

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Appendix A: Proofs of Lemmas and Propositions

Proof of Lemma 1. The proof strategy is based on constructing a tax sequence such that – conditional on λ – the incentive to bequeath is maximum, and then finding the range of low λ's such that agents do not wish to leave positive bequests even in this case. Since the incentive to bequeath is maximum when the next generation has a low private consumption relative to the current generation, we construct a worst-case scenario in which agents born at time zero face zero taxes, whereas agents born at period one and later are taxed at the top of the Laffer curve, $\tau_t = \bar{\tau}$. Then, we show that for sufficiently low lambda, agents do not leave positive bequests even in this scenario. Suppose that $\tau_t = \bar{\tau}$ for all $t \geq 1$, and that $x_{Y,t} = 0$. Given Assumption 1 and the constant disposable income, $\lambda \leq (\beta R)^{-1} \Rightarrow x_{O,t} = 0$, for $t > 1$. Moreover, for $t > 1$, $c_{Y,t} = \underline{c_Y}$ and $c_{O,t+1} = \underline{c_O}$, where $\{\underline{c_Y}, \underline{c_O}\} \in (R^+)^2$ is the solution of the individual optimization, characterized by $\tilde{u}'\left(\underline{c_Y}\right)/\tilde{u}'\left(\underline{c_O}\right) = \beta R$ and $\underline{c_Y} + \underline{c_O}/R = A(\bar{\tau})$ .Suppose, next, that $\tau_0 = 0$ and $x_{Y,0} = \hat{x}$. Let $\{\overline{c_Y}(\hat{x}), \overline{c_O}(\hat{x})\} \in (R^+)^2$ be the the solution of the individual optimization $(\tilde{u}'(\overline{c_Y}(\hat{x}))/\tilde{u}'(\overline{c_O}(\hat{x})) = \beta R$ and $\overline{c_Y}(\hat{x}) + \overline{c_O}(\hat{x})/R = A(0) + \hat{x})$. Let $\bar{\lambda}(\hat{x})$ be such that $\bar{\lambda}(\hat{x}) = \tilde{u}'(\overline{c_O}(\hat{x}))/\tilde{u}'\left(\underline{c_Y}\right)$. Since $\tilde{u}'(\overline{c_O}(\hat{x})) > 0$ and $0 < \tilde{u}'\left(\underline{c_Y}\right) < \infty$, then $\bar{\lambda}(\hat{x}) > 0$. An agent endowed with $\lambda = \bar{\lambda}(\hat{x})$ find it optimal to leave zero bequests, whereas any agent with $\lambda < \bar{\lambda}(\hat{x})$ would strictly prefer to leave negative bequests. Since this is forbidden by Assumption 1, then $\lambda < \bar{\lambda}(\hat{x}) \Rightarrow x_{O,1} = 0$. Moreover, since $\overline{c_O}(\hat{x})$ is the upper bound to the consumption of an old agent who started with an inherited wealth $\hat{x}$, while $\underline{c_Y}$ is the lower bound to the consumption of a young agent, no agent with $\lambda \leq \bar{\lambda}(\hat{x})$ will choose positive bequest for any feasible tax sequences not exceeding the top of the Laffer curve. QED

Proof of Proposition 1. Since $\beta\lambda R < 1$, then equation (14) implies that $\lim_{t\to\infty} u'(g_t) = \infty$, and hence $\lim_{t\to\infty} g_t = 0$. Since $\lim_{t\to\infty} u'(g_t) = \infty$, then (13) implies that $\lim_{t\to\infty} e(\tau_t) = 1$, which in turn implies that $\lim_{t\to\infty} \tau_t = \bar{\tau}$. These two facts, together with (4), establish that $\lim_{t\to\infty} b_t = \bar{b}$. QED

Proof of Lemma 3. Recall the sequential formulation of the political objective function, (9):

$$U_t = v(\tau_0, g_0) + (\delta - 1)\lambda\phi(A(\tau_0)) + \delta\sum_{t=1}^{\infty}(\lambda\beta)^t v(g_t, \tau_t).$$

Along the equilibrium path $\langle g_t, \tau_t\rangle = \left\langle G\left(B^t(b)\right), T\left(B^t(b)\right)\right\rangle$. Given the policy rules $T(b)$, $G(b)$ and $B(b)$, the discounted utility of the old can then be written as $V_O(b) \equiv \sum_{t=0}^{\infty}(\lambda\beta)^t v\left(G\left(B^t(b)\right), T\left(B^t(b)\right)\right)$, where $V_O(b)$ satisfies the functional equation (17). Therefore, part 1 of the Definition 1 of MPPE can be rewritten as equation (16) subject to (4) and the function V_O solving (17). QED

Proof of Proposition 2. The FOCs of the program (16) with respect to τ and g (after substituting away b' using (4)) yield:

$$\begin{aligned} -\delta\lambda\phi'(A(\tau)) - \delta\beta\lambda V_O'(b')(1 - e(\tau)) &= 0, & (45)\\ (1+\lambda)u'(g) + \delta\beta\lambda V_O'(b') &= 0, & (46)\end{aligned}$$

where we have used the definition of $e(\tau)$ and the envelope condition, $A'(\tau) = -wH(\tau)$. Combining the two FOCs yields equation (18).

Next, consider (17). Differentiating $V_O(b)$ using (18) yields

$$V_O'(b) = u'(G(b))\begin{pmatrix} (1+\lambda)G'(b) - \\ \frac{1+\lambda}{\delta}(1 - e(T(b)))wH(T(b))T'(b)\end{pmatrix} + \beta\lambda V_O'(B(b))B'(b). \quad (47)$$

Leading by one period equation (47) yields an expression for $V_O'(b')$ which can be used, together with (46), to eliminate $V_O'(b')$ and $V_O'(B(b'))$.[34] The resulting expression is:

$$\frac{1+\lambda}{\delta\beta\lambda}u'(g) = (1+\lambda)u'(G(b'))\left(\frac{R}{\delta} - \left(1 - \frac{1}{\delta}\right)G'(b')\right).$$

Rearranging terms leads to the GEE, (19). QED

Proof of Lemma 5. The sequential representation of the utility of the young yields

$$\begin{aligned} U_{Y,t} &= \log(c_{Y,t}) + \theta\log(g_t) + \frac{1}{\lambda}\sum_{j=1}^{\infty}(\beta\lambda)^j\left(\log(c_{O,t+j}) + (1+\lambda)\theta\log(g_{t+j}) + \lambda\log(c_{Y,t+j})\right)\\ &= Q_Y + \log(c_t) + \theta\log(g_t) + \frac{1}{\lambda}\sum_{j=1}^{\infty}(\beta\lambda)^j\left((1+\lambda)\left(\log(c_{t+j}) + \theta\log(g_{t+j})\right)\right),\end{aligned}$$

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[34] $B'(b)$ is obtained by differentiating $B(b)$ from equation (15).

where the second line follows from the fact that $c_Y = \lambda\left(1+\lambda\right)^{-1} c$, and $c_O = \left(1+\lambda\right)^{-1} c$, and Q_Y is a collection of constants. Following the same procedure for the old, we obtain:

$$U_{O,t} = Q_O + \left(1+\lambda\right)\left(\log\left(c_t\right) + \theta \log\left(g_t\right)\right) + \sum_{j=1}^{\infty} \left(\beta\lambda\right)^j \left(\left(1+\lambda\right)\left(\log\left(c_{t+j}\right) + \theta \log\left(g_{t+j}\right)\right)\right).$$

Finally, setting $U_t = \left(1+\lambda\omega\right)^{-1}\left(\left(1-\omega\right) U_{Y,t} + \omega U_{O,t}\right)$ yields the planner's objective function, (31). QED

Proof of Proposition 5. The FOCs of the program (32)-(33) w.r.t. τ, g and c yield:

$$\delta\beta\lambda\left(V_1^b\left(s,b'\right)A'\left(\tau\right) - V_2^b\left(s,b'\right)\left(1-e\left(\tau\right)\right)wH\left(\tau\right)\right) = 0, \tag{48}$$

$$\frac{\theta}{g} + \delta\beta\lambda V_2^b\left(s,b'\right) = 0, \tag{49}$$

$$\frac{1}{c} - \delta\beta\lambda V_1^b\left(s,b'\right) = 0, \tag{50}$$

where subscripts denote partial derivatives. First, using (49)-(50) to eliminate $V_2^b\left(s,b'\right)$ and $V_1^b\left(s,b'\right)$ from (48), and recalling that $A'\left(\tau\right) = -wH\left(\tau\right)$, yields (34). Next, differentiating $V^b\left(s_{-1},b\right)$ w.r.t. its arguments, and applying the Envelope theorem, yields:

$$V_1^b\left(s_{-1},b\right) = \left(1-\frac{1}{\delta}\right)\frac{C_1\left(s_{-1},b\right)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_1\left(s_{-1},b\right)}{g} + \beta\lambda R V_1^b\left(s,b'\right), \tag{51}$$

$$V_2^b\left(s_{-1},b\right) = \left(1-\frac{1}{\delta}\right)\frac{C_2\left(s_{-1},b\right)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2\left(s_{-1},b\right)}{g} + \beta\lambda R V_2^b\left(s,b'\right). \tag{52}$$

We now use (49)-(50) to eliminate $V_1^b\left(s_{-1},b\right)$, $V_1^b\left(s_{-1},b\right)$, $V_2^b\left(s,b'\right)$ and $V_1^b\left(s,b'\right)$ from (51)-(52), respectively, and lead the expressions by one period. This yields

$$-\frac{\theta}{\delta\beta\lambda g} = \left(1-\frac{1}{\delta}\right)\frac{C_2\left(s_{-1},b\right)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2\left(s_{-1},b\right)}{g} - \frac{R}{\delta}\frac{\theta}{g'},$$

$$\frac{1}{\delta\beta\lambda c} = \left(1-\frac{1}{\delta}\right)\frac{C_2\left(s_{-1},b\right)}{c} + \left(1-\frac{1}{\delta}\right)\frac{\theta G_2\left(s_{-1},b\right)}{g} + \frac{R}{\delta}\frac{1}{c'}.$$

After rearranging terms, this yields (35)-(36). QED

Proof of Proposition 6. The proof consists of guessing-and-verifying that the policy functions $T\left(s_{-1},b\right)$, $G\left(s_{-1},b\right)$, $C\left(s_{-1},b\right)$ and $B\left(s_{-1},b\right)$ given in the Proposition, and the ensuing equilibrium law of motion of c and g ($c'/c = g'/g = \varphi$) satisfy the equilibrium conditions of Proposition 5. First, using (37) and the guesses of G and C yields

$$\theta\left(1-e\left(\tau\right)\right) = \frac{\frac{w\tau H(\tau)}{R-1} - b}{\frac{A(\tau)}{R-1} + s_{-1}} = \frac{G\left(s_{-1},b\right)}{C\left(s_{-1},b\right)}$$

which verifies the intratemporal trade-off, (34), while proving that $T\left(s_{-1},b\right)$ must be constant along the equilibrium path (suppose not, then g/c would change over time, contradicting that $c'/c = g'/g$). To show that $T\left(s_{-1},b\right)$ is the unique solution to (37) note that the right-hand side of (37) is non-negative for $\tau = \bar{\tau}$ (since $b \leq \bar{b}$) and is continuous and monotone increasing in τ for $\tau \leq \bar{\tau}$ (since $A' \leq 0$ and $e' \geq 0$). Moreover, the left-hand side of (37) is zero for $\tau = \bar{\tau}$ (since $e(\bar{\tau}) = 1$) and is continuous and monotone decreasing in τ for $\tau \leq \bar{\tau}$ (since $e'(\tau) \geq 0$ and $A'(\tau) \geq 0$). A standard fixed-point argument establishes uniqueness. Next, differentiating the equilibrium policy function (recalling that $A' = -wH(\tau)$ and $e(\tau) \equiv -\tau H'(\tau)/H(\tau)$), and leading the expressions one period, yields the following partial derivatives: $G_1(s,b') = (R-\varphi)\frac{wH(\tau)}{R-1}(1-e(\tau))T_1(s,b')$, $G_2(s,b') = (R-\varphi)\left(\frac{wH(\tau)}{R-1}(1-e(\tau))T_2(s,b') - 1\right)$, $C_1(s,b') = -(R-\varphi)\left(\frac{wH(\tau)}{R-1}T_1(s,b') - 1\right)$, and $C_2(s,b') = -(R-\varphi)\frac{wH(\tau)}{R-1}T_2(s,b')$. Note that in all expressions $\tau = T(s_{-1},b)$. Next, rewrite the GEE for g, (35) as

$$\begin{aligned} 0 &= -\frac{g'}{g} + \lambda\beta R - (\delta-1)\lambda\beta\left(G_2(s,b') + \frac{g'}{\theta c'}C_2(s,b')\right) \\ &= -\varphi + \lambda\beta R - (\delta-1)\lambda\beta\left(G_2(s,b') + (1-e(\tau))C_2(s,b')\right) \end{aligned}$$

Plugging in the expressions of φ, G_2, and C_2, and simplifying terms, verifies the GEE, (35). Similarly, rewrite the GEE for c, (36) as

$$\begin{aligned} 0 &= -\frac{c'}{c} + \lambda\beta R + (\delta-1)\lambda\beta\left(\frac{c'\theta}{g'}G_1(s,b') + C_1(s,b')\right) \\ &= -\varphi + \lambda\beta R + (\delta-1)\lambda\beta\left(\frac{G_1(s,b')}{1-e(\tau)} + C_1(s,b')\right) \end{aligned}$$

Plugging in the expressions of φ, G_1, and C_1, and simplifying terms, verifies the GEE, (36). Finally, we must verify that the government and private budget constraints hold, i.e., $B(s_{-1},b) = G(s_{-1},b) + Rb - \tau wH(\tau)$ and $S(s_{-1},b) = Rs_{-1} + A(\tau) - C(s_{-1},b)$. Given the expressions of B, G, S and C, it is straightforward to verify that both conditions hold. QED

Proof of Proposition 7. First note that the household Euler equation (38) implies that

$$C(Rs_{-1} + A(\tau) - c, g + Rb - \tau wH(\tau)) = \lambda\beta Rc, \tag{53}$$

where $c = \tilde{C}(g,\tau,b',s_{-1})$. Differentiating (53) yields

$$\tilde{C}_{s_{-1}}(g,\tau,b',s_{-1}) = \frac{RC_1(s,b')}{\lambda\beta R + C_1(s,b')}, \tag{54}$$

$$\tilde{C}_{\tau}(g,\tau,b',s_{-1}) = \frac{C_1(s,b')A'(\tau) - C_2(s,b')(1-e(\tau))wH(\tau)}{\lambda\beta R + C_1(s,b')}, \tag{55}$$

$$\tilde{C}_{g}(g,\tau,b',s_{-1}) = \frac{C_2(s,b')}{\lambda\beta R + C_1(s,b')}. \tag{56}$$

Now, consider the program (32), subject to the implementability constraint (42). The FOCs w.r.t. τ and g are, respectively:

$$\left(\frac{1}{c} - \delta\beta\lambda V_1^b(s,b')\right)\tilde{C}_\tau + \delta\beta\lambda\left(\begin{array}{c} V_1^b(s,b')A'(\tau) \\ -V_2^b(s,b')(1-e(\tau))wH(\tau) \end{array}\right) = 0, \tag{57}$$

$$\left(\frac{1}{c} - \delta\beta\lambda V_1^b(s,b')\right)\tilde{C}_g + \frac{\theta}{g} + \delta\beta\lambda V_2^b(s,b') = 0. \tag{58}$$

Differentiating $V^b(s_{-1},b)$ w.r.t. s_{-1} yields

$$\begin{aligned} V_1^b(s_{-1},b) &= \left(\frac{1}{c}-\beta\lambda\delta V_1^b(s,b')\right)\frac{\tilde{C}_{s_{-1}}}{\delta}+\left(1-\frac{1}{\delta}\right)\frac{1}{c}C_1(s_{-1},b) \\ &\quad +\left(1-\frac{1}{\delta}\right)\frac{\theta}{g}G_1(s_{-1},b)+R\beta\lambda V_1^b(s,b'), \end{aligned} \tag{59}$$

where we use (57)-(58), and the fact that $C_1(s_{-1},b)=\tilde{C}_{s_{-1}}+\tilde{C}_\tau T_1(s_{-1},b)+\tilde{C}_g G_1(s_{-1},b)$. Substituting out $V_2^b(s,b')$ in (57) and (58), using (55)-(56) to eliminate $\tilde{C}_g$ and $\tilde{C}_\tau$, recalling that $A'(\tau)=-wH(\tau)$, and rearranging terms establish:

$$\delta\beta\lambda V_1^b(s,b')=\theta\frac{e(1-\tau)}{g}+\frac{C_1(s,b')}{\lambda\beta R}\left(-\frac{1}{c}+\frac{\theta e(1-\tau)}{g}\right). \tag{60}$$

Using (60) to substitute out $V_1^b(s,b')$ from the RHS of (59), then leading the resulting expression one period and applying (60) again to eliminate $V_1^b(s,b')$ from the LHS establishes (44).

Similarly, differentiating $V^b(s_{-1},b)$ w.r.t. b yields

$$V_2^b(s_{-1},b)=\left(1-\frac{1}{\delta}\right)\frac{C_2(s_{-1},b)}{c}+\left(1-\frac{1}{\delta}\right)\frac{\theta G_2(s_{-1},b)}{g}-\frac{1}{\delta}\frac{\theta R}{g}, \tag{61}$$

where we use (57) and (58), and the fact that $C_2(s_{-1},b)=\tilde{C}_\tau T_2(s_{-1},b)+\tilde{C}_g(G_2(s_{-1},b)+R)$. Substituting (60) back into (58) yields:

$$\delta\beta\lambda V_2^b(s,b')=-\left(\left(1+\frac{C_1(s,b')}{\lambda\beta R}\right)\left(\frac{1}{c}-\frac{\theta e(1-\tau)}{g}\right)\right)\frac{C_2(s,b')}{\lambda\beta R+C_1(s,b')}-\frac{\theta}{g}, \tag{62}$$

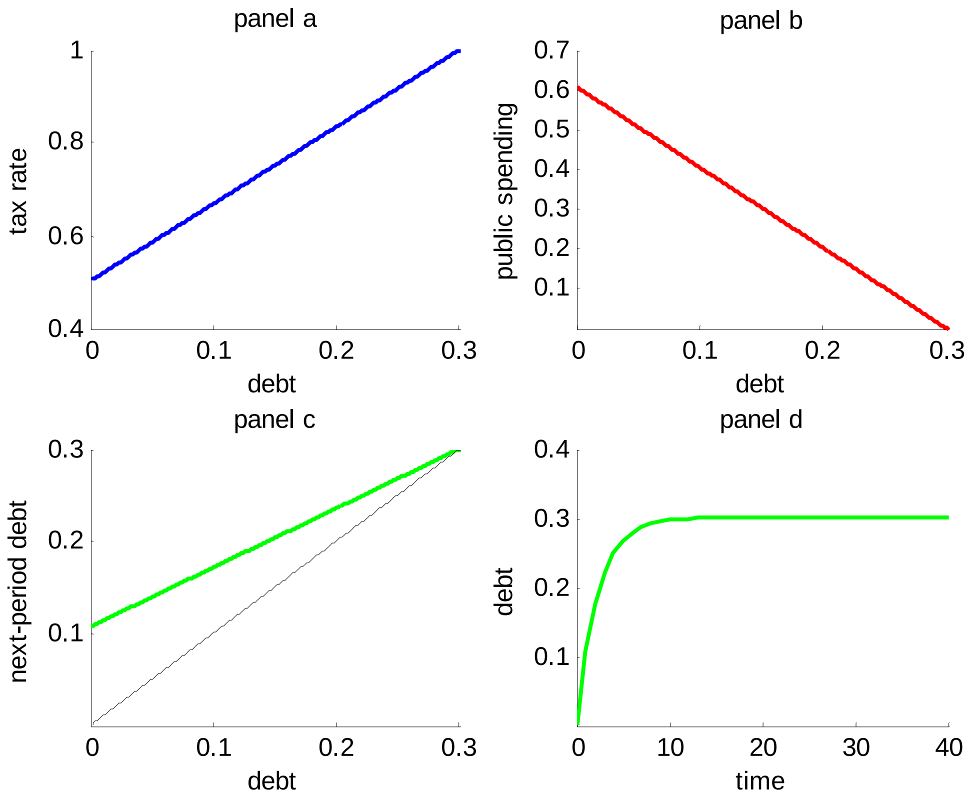
where we use (56) to substitute out $\tilde{C}_g$. A combination of (61) and (62) establishes (43). QED

Proof of Corollary 1. That $\lim_{t\to\infty}c_t=0$ follows immediately from (38). To prove that $\lim_{t\to\infty}g_t=0$, rearrange (44):

$$\begin{aligned} &\frac{\theta\lambda\beta R(1-e(\tau'))}{g'}-\frac{\theta(1-e(\tau))}{g}+\beta\lambda(\delta-1)\frac{\theta}{g'}G_1(s,b') \\ =&\left(\frac{\theta(1-e(\tau))}{g}-\frac{1}{c}(1+\beta\lambda(\delta-1))\right)\frac{C_1(s,b')}{\lambda\beta R}. \end{aligned} \tag{63}$$

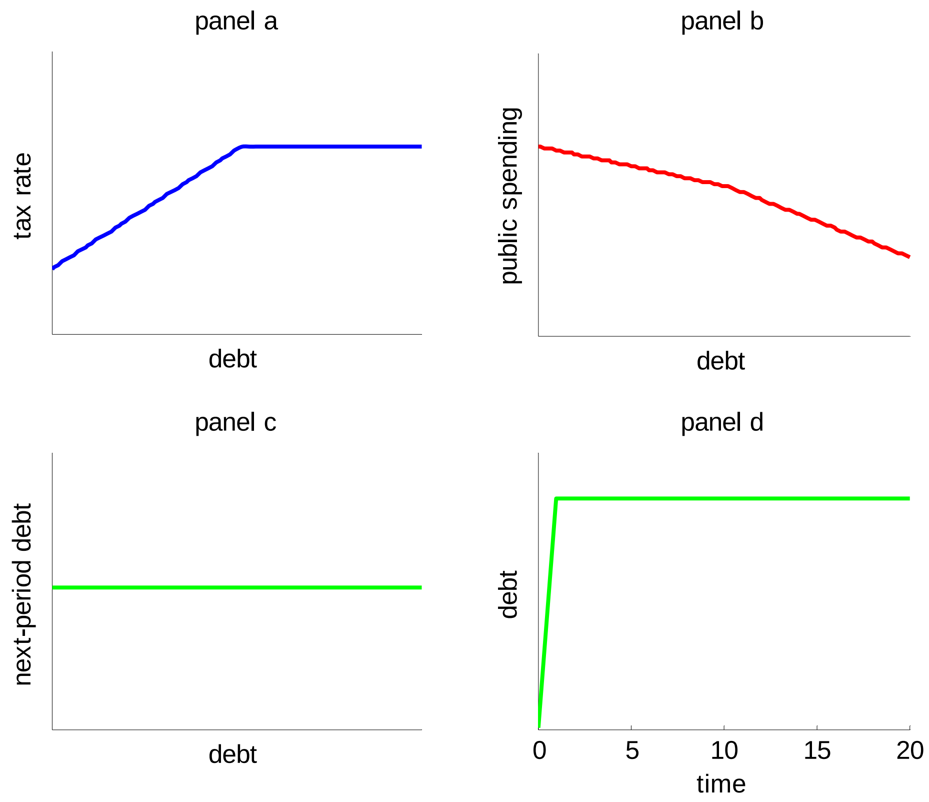
First, we claim that $\lim_{t\to\infty}G_1(s_t,b_{t+1})>-\infty$ and $\lim_{t\to\infty}C_1(s_t,b_{t+1})>0$. The former follows from the assumptions that G is continuous and that $g\geq 0$. The latter follows from the fact that as $\lim_{t\to\infty}C(s_t,b_{t+1})=0$, then $\lim_{t\to\infty}C_1(s_t,b_{t+1})>0$. Next, suppose for contradiction that $\lim_{t\to\infty}g_t>0$. Then, the LHS of equation (63) would be finite, while the RHS would go to $-\infty$. This yields a contradiction, and establishes then that $\lim_{t\to\infty}g_t=0$. QED

Figure 1: Example I (Inelastic Labor Supply)



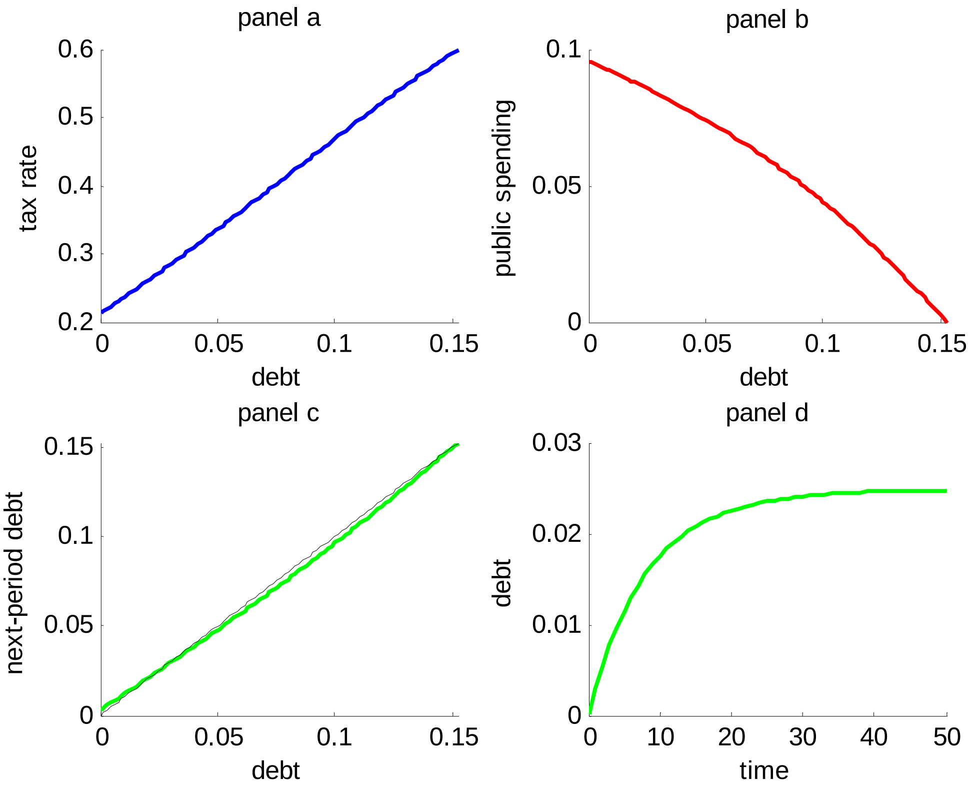
The figure plots the equilibrium policy functions T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium time path of b (panel d). The parameter values are: $\beta = 0.985^{30}$, $\lambda = 0$, $R = 1.05^{30}$, $\omega = 0.50$, $\theta = 1.00$ and $w = 1$. The natural debt limit is $\bar{b} = 0.30$.

Figure 2: Example II (Linear Household Technology)



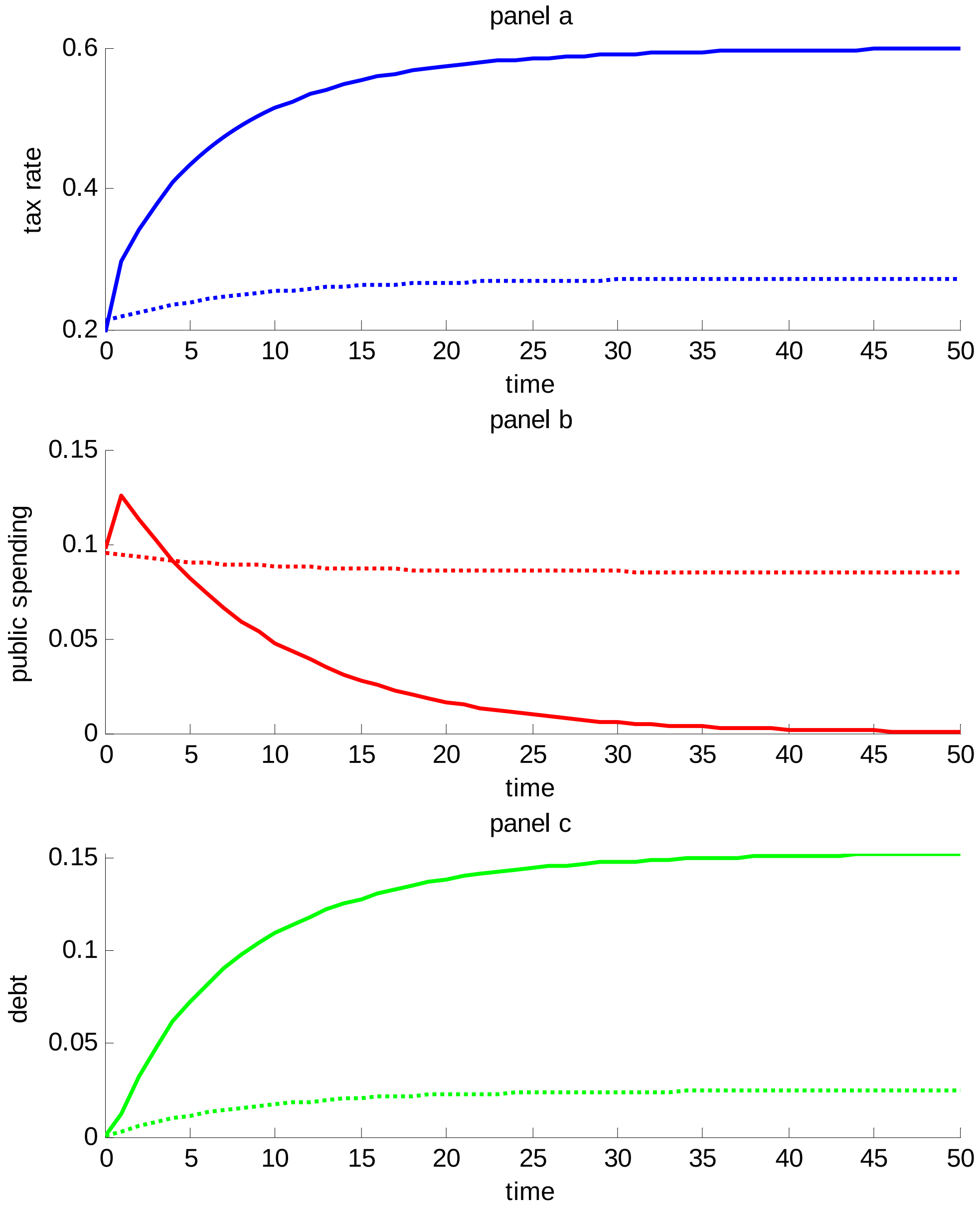
The figure plots the equilibrium policy functions T(b) (panel a), G(b) (panel b), B(b) (panel c) and the equilibrium time path of b (panel d). The parameter values are: $\beta = 0.985^{30}$, $\lambda = 0$, $R = 1.05^{30}$, $\omega = 0.50$, $\theta = 1.00$, $w = 1$ and $\bar{\tau} = 0.60$. The natural debt limit is $\bar{b} = 0.18$.

Figure 3: Benchmark Calibration



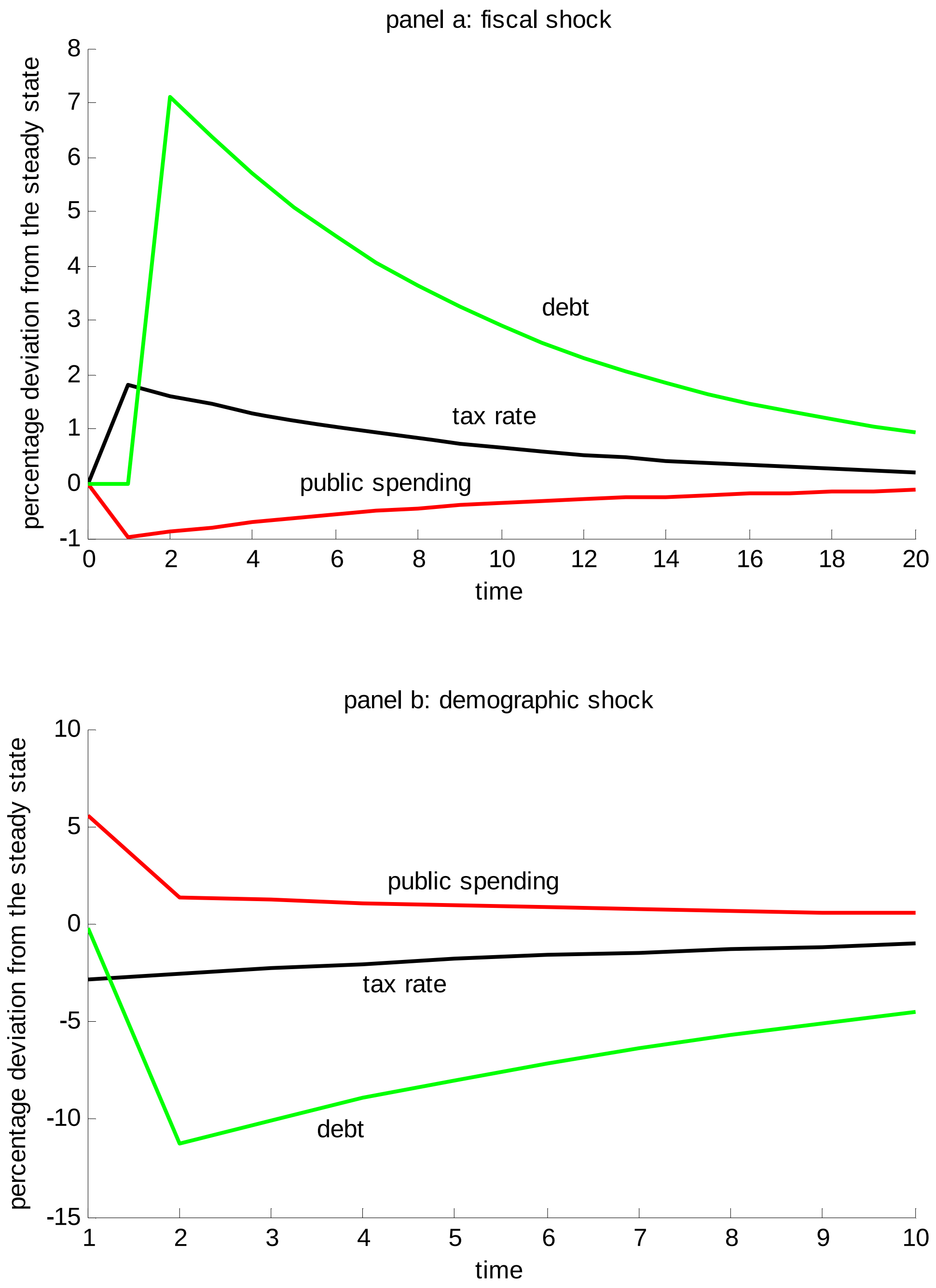
The figure plots the equilibrium policy functions for the calibrated economy of Table 1: T(b) (panel a), G(b) (panel b), B(b) (panel c) and the corresponding equilibrium time path of b (panel d). Parameter values are those of the benchmark calibration (see Table 1).

Figure 4: Ramsey versus Markov



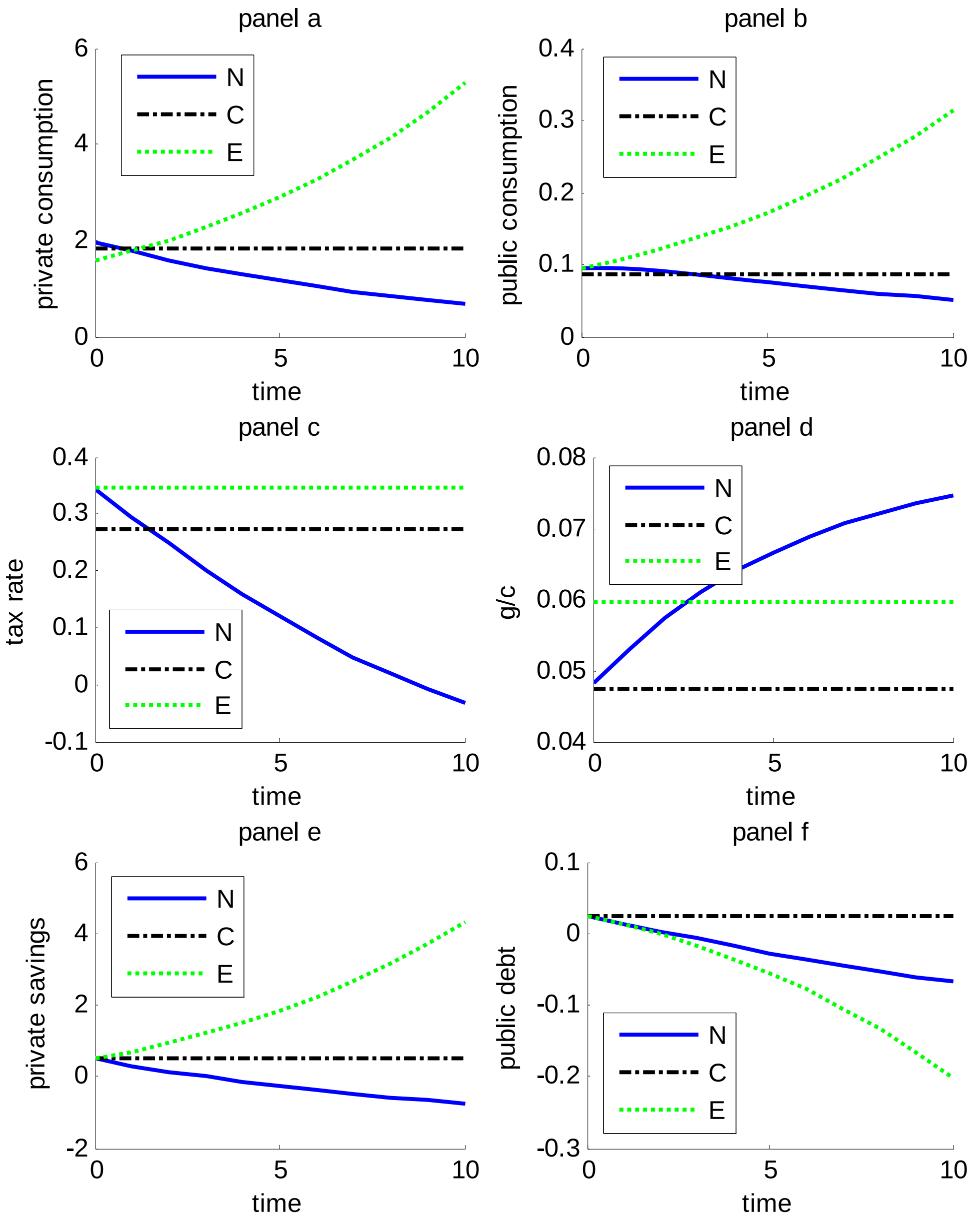
The figure shows the Ramsey (solid lines) and Markov equilibrium (dotted lines) time paths of taxes (panel a), public spending (panel b) and debt (panel c). The parameter values are those of the benchmark calibration (see Table 1).

Figure 5: Impulse Response Functions



Panel a shows impulse-response functions of a fiscal (war) shock of 1% of GDP. Panel b shows impulse-response functions of a demographic shock (panel b) where annualized population growth falls from 1% to zero. The parameter values in Table 1. The initial debt at period 1 in panel b is that of the final steady state.

Figure 6: Equilibrium with no Bequest Constraints



The figure shows the equilibrium time paths of total private consumption (panel a), public consumption (b), tax rate and g/c (panel c and d), private wealth and public debt (panel e and f). The solid, dotted and dashed lines are for the E- and N-planner allocations and equilibrium with non-negative bequests (benchmark model), respectively. Parameter values are as in the benchmark calibration (see Table 1). The initial conditions (b and s) are set equal to the steady state levels in the benchmark model with non-negative bequests.