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COMPETITIVE BIDDING IN AUCTIONS WITH PRIVATE AND COMMON VALUES*

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The objects for sale in most auctions possess both private and common value elements. This salient feature has not yet been incorporated into a strategic analysis of equilibrium bidding behaviour. This paper reports such an analysis for a stylised model in which bidders receive a private value signal and an independent common value signal. We show that more uncertainty about the common value has a negative effect on efficiency. Information provided by the seller decreases uncertainty, which raises efficiency and seller's revenues. Efficiency and revenues are also higher when more bidders enter the auction.

Auctions have traditionally been divided into two categories. In *private value auctions*, bidders know their own value for the commodity with certainty but are unsure about others' valuations. Vickrey (1961) provides an early strategic analysis of private value auctions for the case when values are independent across bidders. The standard textbook example associated with his model is the sale of a painting; buyers' valuations for the painting may differ but are assumed to be independent. In contrast, *common value auctions* pertain to situations in which the object for sale is worth the same to everyone, but bidders have different private information about its true value. A well known example where the common value set-up applies is the auctioning of oil drilling rights, which, to a first approximation, are worth the same to all competing bidders. Common value models were introduced by Wilson (1969, 1977) who also provided the first equilibrium analysis of the winner's curse. See Klemperer (1999) for a survey of this early literature and more recent advances in auction theory.

Most real-world auctions, however, are not exclusively common value or private value. In his extensive survey of empirical studies on auctions, Laffont (1997) concludes: 'In general, economic theory is not able to tell us in which particular model we are. Most empirical studies clearly involve some private value element as well as some common value element...' For instance, when a painting is auctioned to a private collector, it may be resold in the future and the resale price will be the same for all bidders. This adds a common value component to the auction. In the oil drilling example, private value differences may arise when a superior technology enables some firms to exploit the rights better than others.

This paper develops a model in which bidders' information consists of both private and common value components. Our stylised two-signal model nests the two paradigms of auction theory in the simplest possible way. The main focus of

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this paper is on a strategic analysis: we derive the equilibrium bidding functions for our model and make explicit how they vary with the private and common value signals. Our analysis generates insights about efficiency and revenues in auctions with private and common values that could not have been obtained from a private or common value model alone.

For example, in the absence of reserve prices and entry fees, efficiency is not an issue in (symmetric) private or common value auctions. In the independent private values context, Vickrey (1961) has shown that optimal bids are increasing in bidders' values, and hence, the object is awarded to the bidder that values it the most. In common value auctions, all bidders value the object the same so any allocation rule is (trivially) efficient. However, when bidders possess both private and common value information, inefficiencies should be expected. The intuition for this result is straightforward: a bidder with a moderate private value and an overly optimistic estimate of the common value may outbid a rival with a superior private value but more realistic conjectures about the common value.

Maskin (1992) first pointed out that auctions may result in inefficient allocations when signals are multi-dimensional, a result that has been generalised by Jehiel and Moldovanu (2001) by allowing for allocative externalities. Dasgupta and Maskin (2000) extend Maskin's earlier work and show that a generalised Vickrey auction provides a 'second best' mechanism in the sense that it is 'constrained efficient'. An interesting related paper is that of Pesendorfer and Swinkels (2000). In the context of a two-signal, uniform price auction, they derive conditions under which efficiency is restored when the number of bidders becomes large. None of these papers, however, contains a strategic analysis. Dasgupta and Maskin and Jehiel and Moldovanu prove their results using mechanism design arguments without characterising the optimal bidding functions. Such a characterisation is also lacking in Pesendorfer and Swinkels (2000), who remark, 'As a consequence of the two-dimensional type space we have not been able to prove existence of equilibrium. Our results instead hinge on a partial characterisation of what equilibria must look like in the limit, if they exist.' Existence in their set-up is not guaranteed, however, as Jackson's (1999) counterexample shows.

In contrast, we explicitly derive the equilibrium for our model, which makes it possible to determine the extent of inefficiency that can result from the use of auctions. We show that more uncertainty about the common value implies a higher level of expected inefficiency. In the limit when the uncertainty about the common value is so large as to override the private value information, auctions are no more efficient than a random allocation rule. In the other extreme when there is no uncertainty about the common value at all, the auction reduces to an efficient private value auction.

The observation that more uncertainty about the object's value results in more inefficiency has implications for the auctioning of (government) licences to operate in a market. The cost structure of bidding firms constitutes a private value element in such cases, while the uncertain demand for the consumer products introduces a common value aspect. The efficiency losses that can be expected in these situations may be mitigated when firms repeatedly participate in similar auctions, since this provides them with the opportunity to learn about consumer demand. For instance, vendor locations at fairs are often auctioned on an annual basis, which enables bidders to accumulate information about demand over time. Consequently, the common value of the location can be determined more precisely, thereby reducing possible efficiency losses. In contrast, the infrequent auctioning of landing slots at airports involves more uncertainty about the common value, since bidders face the difficult task of predicting consumer demand for a longer period. The FCC auctions used to allocate the spectrum provide another example where the uncertainty about the common value aspect is likely to be large. First, these auctions are held infrequently. Second, estimating demand is even more complicated when the frequencies are used in emerging markets like the market for mobile phones. As a result, efficiency losses may be more pronounced.

Another benefit of our strategic analysis is that it is possible to evaluate policies to enhance efficiency and revenues. One such measure is the disclosure of the seller's information about the common value. The seller's information reduces uncertainty about the common value, which raises efficiency. On average, the seller's revenue is also higher when information is revealed, which makes it likely that such disclosure will indeed occur. The intuition is that with less uncertainty, bidding will be more aggressive. The positive effects of the seller's information are stronger the more precise the information. Another factor improving efficiency is an increase in competition: expected efficiency and expected revenue increase with each extra bidder. In the limit when the number of bidders goes to infinity, an efficient allocation again materialises. Interestingly, the effect of more competition on efficiency and revenues is stronger than the effect of information provided by the seller.

This paper is organised as follows. Section 1 introduces the model. In Section 2 we derive the summary statistic and discuss when a reduction to a one-dimensional signal is possible. Section 3 characterises the equilibrium for standard winner-pay auctions (first price, second price, and English) and provides general formulas for the seller's revenue, the winner's profit, and total surplus. In Section 4 we determine the effects of increased uncertainty, more competition, and the public disclosure of information on efficiency and revenues and illustrate our results with an example. Section 5 concludes.

1. The Model

Consider an auction where n bidders compete for a single object such as a government contract, drilling rights for an offshore tract, the right to use a certain radio frequency etc. Suppose bidder $i = 1,\ldots,n$, has an unbiased estimate, v_i of the object's true value, V, which is the same for all bidders. Two ways to model the common value have been proposed. In the 'traditional' formulation (Rothkopf, 1969; Wilson, 1969, 1977) the common value has some known prior distribution and bidders' signals are draws conditional on the particular realisation of V. This realisation, unknown at the time of bidding, is revealed when the object is obtained.[1] Alternatively, the common value can be modelled as the average of bidders' signals:[2]

$$V = \frac{1}{n}\sum_{i=1}^{n} v_i \tag{1}$$

It is important to point out that the two formulations have the same qualitative features. First, the commodity for sale is worth the same to all bidders. Second, in both formulations bidders should realise that winning means that their signal is likely to be too optimistic. In order not to fall prey to a 'winner's curse', bidders should shade their bids accordingly.

Few people would dispute that most auctions have both private and common value features. For instance, when licences to operate in a market are auctioned, a common value aspect results from the uncertain demand for the product to be provided, while the different cost structures of the bidding firms adds a private value part. This example motivates the information structure of our model:[3]

ASSUMPTION 1. *There are $n \geq 2$ risk neutral bidders. Bidder $i = 1,\ldots,n$ receives a private cost signal, c_i, and an independent common value signal, v_i. Cost and value signals are assumed to be independently and identically distributed across bidders.*

The density of cost signals is denoted by $f_c(c)$ with support $[c_L, c_H]$ where $c_L \geq 0$. Similarly, $f_v(v)$ is the density of value signals, with support $[v_L, v_H]$. We consider the case where $v_L \geq c_H$, so that bidders will always wish to participate. The assumption of risk neutrality implies that when bidder i receives the object and pays an amount b, her net utility is $V - c_i - b$.

2. From Two to One Dimensions: A Problem?

Before we give a formal derivation of the equilibrium strategies, it is useful to provide some intuition. When bidders receive multiple signals they must somehow combine the different pieces of information into a summary statistic since their bid is just a single number. Milgrom and Weber (1982, p. 1097) point out that '... the derivation

[1] The traditional formulation can be illustrated with the auctioning of oil drilling rights. A (common) prior is formed as bidders repeatedly compete for different tracts. For each new tract to be auctioned, bidders receive an estimate of the true amount of oil.

[2] Previous papers that have used the average formulation include: Bikhchandani and Riley (1991), Albers and Harstad (1991), Krishna and Morgan (1997), Klemperer (1998), Bulow *et al.* (1999), and Bulow and Klemperer (2002). One possible motivation for this formulation is the auctioning of licences to operate in emerging markets. In such cases, no prior information exists about the distribution of the common value and residual uncertainty about the object's true value is likely to remain after the auction is closed. The best approximation for the value of the licence is then its market resale price. For instance, consider a bidder who enters a spectrum auction with the intention of reselling the rights to use the frequency. Prospective buyers will infer others' value signals from their bids and use this information to re-evaluate the licence's value. Assuming that the 'quality' or precision of signals is uniform across bidders, the best estimate of the resale price of the licence is the average of all signals.

[3] Obviously, alternative motivations are possible. For instance, the common value signal could pertain to the resale price of a painting, while the private value signal represents a bidder's own taste for the painting. (We will return to this example in Section 4.) Alternatively, in 'take-over' battles, the estimated 'market value' of the firm could be represented by the common value signal while the private value indicates the expected 'value added' of the bidder.

of a summary statistic from several pieces of information is in general a difficult task…' For our model, however, the summary statistic follows from the following observation. A bidder's expected payoff in any winner-pay auction is: $\pi^e =$ (expected gain − expected payment) × probability of winning. The expected payment and the probability of winning are independent of a bidder's private and common value signals (but will depend on her bid and others' bidding strategies). Moreover, bidder i's expected gain equals her 'surplus', $s_i = v_i/n - c_i$ plus terms that are independent of her signals.[4] The first-order conditions for profit maximisation therefore determine optimal bids in terms of the surplus, s_i It is important to point out that the aggregation of private and common value information into a single statistic is possible because of the independence assumption. In Wilson's traditional formulation where bidders' common value signals are affiliated it is generally much more difficult (if not impossible) to find the summary statistic.

In our model, the information conveyed by the cost and common value signals is thus adequately summarised by a one-dimensional surplus signal. Before applying the standard apparatus developed for univariate signals, however, we have to deal with the following caveat: in general, a higher surplus does not imply a higher value signal, and hence, optimal bids may not be increasing in surplus. This problem is illustrated by the following two-bidder example.

Example 1. Let costs be uniformly distributed on the intervals [0, 0.5] and [100, 100.5], and let values be uniformly distributed on [201, 202] and [399, 400]. With two bidders, surplus is given by $s = v/2 - c$, which is (non-uniformly) distributed on [0, 1], [99, 100], [100, 101] and [199, 200]. Note that $99 < s < 100$ implies $399 < v < 400$, while $100 < s < 101$ implies $201 < v < 202$, so $E(v|s)$ is not monotonically increasing in s. Now, suppose, in contradiction, that optimal bids are increasing in surplus and bidder 1, say, has the higher surplus. The expected value of the object to bidder 1 is given by $\frac{1}{2}v_1 + \frac{1}{2}E(v|s \leq s_1) - c_1 = s_1 + \frac{1}{2}E(v|s \leq s_1)$. It is straightforward to show that $E(v|s \leq 100) = 300.5$ and $E(v|s \leq 101) = 266.5$, so the expected value is $250\frac{1}{4}$ for $s_1 = 100$ and $234\frac{1}{4}$ for $s_1 = 101$. Hence, optimal bids cannot be increasing in surplus.

The underlying problem is that there is no natural way to order signals in two dimensions. As a result, a one-dimensional statistic may not adequately rank bidders. This problem would not occur if a higher surplus signalled 'good news' about the value, i.e. when a higher surplus implies, on average, a higher value. Example 1 shows this property cannot be expected when the densities of cost and value signals are not unimodal. To avoid problems of non-monotonicity and non-existence we restrict ourselves to the following class of unimodal densities.[5]

Assumption 2. *The densities $f_v(v)$ and $f_c(c)$ are logconcave.*

[4] The use of the word surplus for $s_i = v_i/n - c_i$ is somewhat misleading since a bidder's true surplus from the auction (in case she wins) is $V - c_i$. A bidder's 'type' would be a more correct, though less informative, name for s_i

[5] Athey (2001) discusses more general conditions (the 'single-crossing' property) that guarantee existence of a pure-strategy equilibrium in auctions where bidders receive a univariate signal.

Logconcavity means that the (natural) log of the density is concave. This mild restriction is met by many commonly used densities such as the uniform, normal, chi-square, exponential densities etc.[6] The next lemma shows that the assumption of logconcavity ensures that surplus and values are positively correlated (see the Appendix on www.res.org for a proof).

LEMMA 1. *Under Assumption 2, the conditional expectations* $E(v|s = x)$ *and* $E(v|s \leq x)$ *are non-decreasing in x. Furthermore,* $E(c|s = x)$ *and* $E(c|s \leq x)$ *are non-increasing in x.*

In the next Section we derive the equilibrium bidding functions for our model. Lemma 1 shows that logconcavity makes a reduction from two to one dimension possible, and Milgrom and Weber (1982) characterise the equilibrium for standard auctions when bids are based on a univariate statistic. Since we know how to aggregate the private and common value information into a single statistic, the derivation of the optimal bids is straightforward. The explicit characterisation of the equilibrium allows us to study interesting economic phenomena such as the effects of uncertainty, public release of information, and competition on efficiency and revenues.

3. Equilibrium Strategies, Revenues, Winner's Profit, and Total Surplus

We start with the first-price auction, then we consider the second-price auction and finally the English auction.[7] Due to symmetry we can, without loss of generality, focus on bidder 1 with surplus $s_1 = v_1/n - c_1$. Lower case letters are used to denote the highest surplus of the $n - 1$ other bidders, e.g. $y_1 = \max_{j=2,\ldots,n}(v_j/n - c_j)$, and capital letters indicate order statistics when they pertain to all n bidders, e.g. Y_1 is the maximum of n surplus draws. To keep the notation simple we only use one expectation symbol, e.g. the winner's expected cost $E(c_{\text{winner}}) = E_{Y1}[E(c|s = Y_1)]$ is written as $E(c|s = Y_1)$. Proofs of the propositions can be found in the Appendix on www.res.org.

PROPOSITION 1. *The n-tuple of strategies* $(B(\ ),\ldots,B(\ ))$, *where*

$$B(x) = E(V - c_1|s_1 = x, Y_1 = x) - E(Y_1 - y_1|s_1 = x, Y_1 = x), \tag{2}$$

is an equilibrium of the first-price auction.

The intuition behind (2) is as follows: the first term on the right side represents what the commodity is worth (on average) to a bidder assuming that her surplus, $s_1 = x$, is the highest and the second term shows how much she 'shades' her bid.

Next we turn to a second-price auction in which the highest bidder wins at the second highest price. The optimal bid function for this case is the same as for the first-price auction except that a bidder now assumes that she has the highest

[6] See, for instance, Caplin and Nalebuff (1991) for a more extensive list and for applications of the logconcavity assumption in models of oligopolistic competition.

[7] We will not discuss the Dutch auction separately, since it is strategically equivalent to the first-price auction.

surplus and one other bidder has that same surplus. The shade term on the right side of (2) therefore disappears.

PROPOSITION 2. *The n-tuple of strategies $B(\ ),\ldots,B(\ )$), where*

$$B(x) = \mathrm{E}(V - c_1 | s_1 = x, y_1 = x), \tag{3}$$

is an equilibrium of the second-price auction.

Note the resemblance between (3) and Vickrey's dominant strategy solution for the private value second-price auction, i.e. bidding one's value. When the expected value of the commodity depends on others' information, a dominant strategy no longer exists. Instead, a bidder's optimal strategy is to bid the *expected* value assuming she has the highest surplus and this surplus is equal to the second highest surplus. To see this, note that if she bids less than (3) while others bid according to (3), she could lose from bidders with a slightly lower surplus than hers and forego some profit. Alternatively, if she bids higher than (3), she could win from bidders with a slightly higher surplus than hers and pay too much, resulting in a negative profit.

Since our model contains the standard private value auction and common value auction as special cases, it is interesting to see how bidding behaviour in these familiar settings compares with the results of Propositions 1 and 2. For instance, in the independent private values model, increased competition leads to more aggressive bidding because an increase in the number of bidders reduces the chance of winning. The opposite effect of the number of bidders on optimal bids is predicted for the common value model, at least for some examples. The intuition is that with more bidders, winning is more informative, i.e. an even stronger indication that the winner's signal was 'too optimistic'.

Sometimes these contradicting comparative statics results are used in empirical work to determine which model applies: private or common value (Paarsch, 1992). In our model, in which bidders possess both private value and common value information, there is no clear-cut effect of increasing the number of bidders: optimal bids may increase, decrease, or may be non-monotonic in the number of bidders. The procedure to classify the auction on the basis of this numbers effect may therefore be less useful.

To get some insight into this result, consider a bidder with a low value and low cost signal. With only one competitor, winning signals bad information about the other's value and, hence, about the value of the object for sale. In order not to fall prey to a winner's curse, the optimal bid has to take this adverse selection effect into account. In contrast, with an infinite number of bidders, a bidder's value signal becomes irrelevant and optimal bids are based on costs only. In this case, the expected value of the object is simply its unconditional expected value. So, with a low cost and value signal, the optimal bid is higher with an infinite number of bidders than with two bidders. However, an analogous argument shows that bidders with high cost and high value signals bid *less* with more rivals.

Figure 1 illustrates the dependence of optimal bids on the number of bidders in a second-price auction when common value signals are uniformly distributed on [2, 3] and costs are uniformly distributed on [0, 1]. The optimal bid decreases with n when $(v, c) = (3,0.1)$, is non-monotonic when $(v, c) = (2.4, 0.05)$, and increases with n when $(v, c) = (2.3, 0.01)$. Unlike in the pure private or common value case, the ranking of bidders may change when others enter the auction. For example, the bidder with $(v, c) = (3, 0.1)$ outbids the other two when $n \leq 7$ but has the lowest bid when $n \geq 12$. Hence, the introduction of new bidders may change the winner of the auction and make it more efficient (see Section 4), *even when these bidders themselves have no chance of winning*. The intuition behind this result is that when more bidders enter the auction, the common value signal becomes less important, which exemplifies the cost differences between firms and makes an efficient outcome more likely.

Our derivation of the optimal bidding functions for the English auction follows Milgrom and Weber (1982). In the variant we consider, the auctioneer continuously raises the price and bidders publicly reveal when they withdraw from the auction. Bidders who have dropped out are not allowed to re-enter. With only two bidders, a bidder's strategy is described by a single number that indicates how high the bidder is willing to go before conceding: the English auction with two bidders is equivalent to a second-price auction. However, with more than two bidders the prices at which some bidders drop out convey information to those who remain active. Suppose k bidders have dropped out at bid levels $b_1 \leq \ldots \leq b_k$ Bidder 1's strategy can be described by functions $B_k(s_1;b_1,\ldots,b_k)$, which specify how high bidder 1 is willing to bid given that k bidders dropped out at levels $b_1,\ldots,b_k$ (and given her surplus s_1). The equilibrium strategy $B(\bullet) = [B_0(\bullet),\ldots,B_{n-2}(\bullet)]$ is then defined recursively.

PROPOSITION 3. *The n-tuple of strategies* $(B(\bullet),\ldots,B(\bullet))$, *with* $B(\bullet)$ *defined in (4), constitutes an equilibrium of the English auction.*

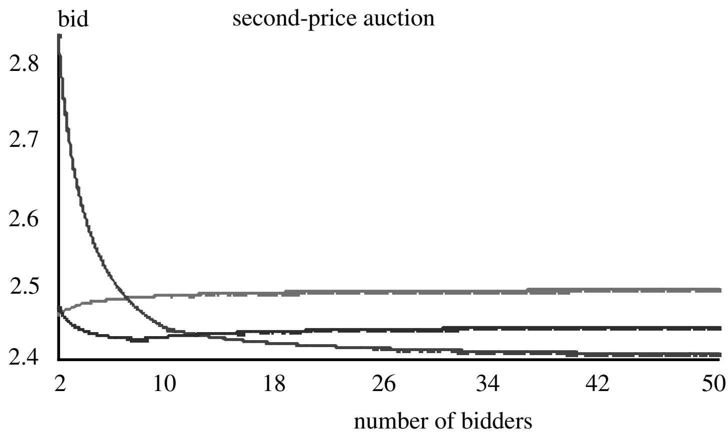

Fig. 1. *Optimal Bidsin a Second-Price Auction when Private Costs are Uniform on* [0,1] *and Common Value Signals are Uniform on* [2,3]

$$\begin{aligned} B_0(x) &= \mathrm{E}(v_1 - c_1 | s_1 = x), \\ B_k(x; b_1, \ldots, b_k) &= \frac{n-k}{n} \mathrm{E}(v_1 | s_1 = x) - \mathrm{E}(c_1 | s_1 = x) \\ &\quad + \frac{1}{n} \sum_{j=0}^{k-1} \mathrm{E}\left[v_1 \middle| B_j\left(s_1; b_1, \ldots, b_j\right) = b_{j+1}\right]. \end{aligned} \tag{4}$$

The intuition behind (4) is straightforward: given her surplus and the information conveyed in others' drop-out levels, the highest a bidder is willing to go is given by the expected value of the commodity assuming that all other active bidders have the same surplus. For instance, consider the bid function $B_0(s_1)$ which pertains to the case when no bidder has dropped out yet. If all other bidders were to drop out at level $B_0(s_0)$, then bidder 1's expected payoff is: $s_1 + (n-1)/n\ \mathrm{E}(v|s = s_0) - B_0(s_0) = s_1 + (n-1)/n\ \mathrm{E}(v|s = s_0) - \mathrm{E}(v - c|s = s_0) = s_1 - s_0$. Using the bidding strategy B_0 bidder 1 remains active until she is indifferent between winning and quitting. Similar interpretations can be given for the B_k with $k \geq 1$.

We end this Section with a 'revenue and efficiency equivalence' result. Our model predicts that the seller's profit is the same for all standard auctions thereby providing a more general setting for which Vickrey's celebrated revenue equivalence result holds. While revenue equivalence is sensitive to particular assumptions of a model (e.g. independence of signals and risk neutrality),[8] it produces useful benchmarks from which the effects of varying specific assumptions of the model can often be deduced; see Klemperer (1999, 2000).

Interestingly, our model predicts that efficiency is the same for all standard auctions. One might have expected that an English auction would be more efficient than a sealed-bid auction. In an English auction, part of the uncertainty about the common value is resolved as the remaining bidders observe the price levels at which low-surplus bidders drop out. These prices reveal their surpluses and thus provide information about their common value signals. In the final stage of the English auction with only two active bidders left, all but one of the surpluses have been revealed, allowing for more precise beliefs about the common value. Note, however, that also in the English auction optimal bids are increasing in surplus, and so the highest-surplus bidder will remain active the longest. Thus, the firstprice, secondprice, and English auctions are efficiency equivalent because in all three auctions the highest-surplus bidder wins.

PROPOSITION 4. *In all three auctions, the total surplus is* $W = E(V) - E(c|s = Y_1)$, *the winner's profit is* $\pi_{winner} = E(Y_1) - E(Y_2)$, *and the seller's revenue is* $R = W - [E(Y_1) - E(Y_2)]$.

[8] Milgrom and Weber (1982) show that revenue equivalence no longer holds when bidders' signals are affiliated. Holt (1980) studies the effects of risk aversion on revenues of different auction formats.

4. Comparative Statics

Full efficiency requires that bidders ignore their common value information and bid according to their private cost only. In a Nash equilibrium, however, optimal bids depend on the summary statistic $s = v/n - c$ and both private and common value signals are taken into account. In this Section, we study how efficiency changes as more weight is given to the common value signals. We consider mean-preserving transformations of the form $v \to \alpha v + (1 - \alpha)\mathrm{E}(v)$, which leave the expected value of the common value unchanged but increase its variance.[9] Under such a transformation, the summary statistic simply changes to $s(\alpha) = \alpha v/n - c$ (as additive constants merely shift the optimal bids).

Intuitively, the probability $\mathrm{P}(\alpha)$ that the low-cost bidder wins the auction is a decreasing function of α. In one extreme where surplus contains no information about costs $\mathrm{P}(\alpha) = 1/n$ while it is 1 when surplus and costs are perfectly correlated.[10] However, the probability of winning is not necessarily a good measure of inefficiency. Even when the lowest-cost bidder does not receive the commodity, the winner's expected cost could still be very low so that the efficiency loss is small. A better measure of efficiency is the total expected surplus generated by the auction $W = \mathrm{E}(V) - \mathrm{E}(c_{\text{winner}})$, i.e. the pie to be divided between the bidders and the seller.

Proposition 5. *An increase in lowers total expected surplus and expected revenues.*

The revenue result is intuitive since bidders' information rents, and hence their profits, rise with. Proposition 5 shows that the seller benefits if bidders attach less weight to their common value information. For instance, suppose the seller also possesses private information, represented by v_0 about the common value V. Under the assumption that the precision of the signal v_0 is the same as those of the bidders, the common value changes as follows:[11]

[9] This mean-preserving transformation enables us to study the effects of increased competition and public information release. In practice, it is, of course, not possible to change the common value signals in this way directly, except in an experimental setting; see Goeree and Offerman (2002). Nevertheless, the extent to which there is uncertainty about the common value may vary across auctions (see the Introduction for some examples). By studying the effects of an increase in α we can compare these situations of different uncertainty.

[10] Note that $\mathrm{P}(\alpha)$ is given by $\mathrm{Prob}[s_1\alpha = \max_{j=1,\cdots,n} s_j\alpha | c_1 = \min_{j=1,\cdots,n} c_j]$, or

$$\begin{aligned}\mathrm{P}(\alpha) &= \mathrm{Prob}[c_1 - c_2 < \alpha(v_1 - v_2)/n, \ldots, c_1 - c_n < \alpha(v_1 - v_n)/n | c_1 - c_2 < 0, \cdots, c_1 - c_n < 0] \\ &= \mathrm{Prob}[v_1 - v_2 > n(c_1 - c_2)/\alpha, \ldots, v_1 - v_n > n(c_1 - c_n)/\alpha | c_1 - c_2 < 0, \cdots, c_1 - c_n < 0].\end{aligned}$$

The top line implies $P(0) = 1$ and the bottom line implies that $P(\alpha)$ is decreasing in α with $\lim_{\alpha \to \infty} P(\alpha) = 1/n$.

[11] This way of modelling the effect of public release of information is appropriate in the following setting. Consider the sale of a painting where bidders are uncertain about its authenticity. The potential future resale price forms the common value component in this auction. In the absence of other meaningful sources of information about the painting's authenticity, an average of the experts' judgments will determine the resale price. If the seller hires her own expert to investigate the painting, and publicly reveals this expert's estimate, the common value changes as proposed in (5).

$$V' = \frac{1}{n+1}\sum_{i=0}^{n} v_i. \tag{5}$$

The effect of this extra piece of information on bidders' strategies is captured by the new surplus variable $s' = v/(n+1) - c$.[12] Proposition 5 implies that the seller's information increases total surplus and revenues. The intuition is that the seller's information reduces uncertainty about the common value. Reduced uncertainty makes the private cost component more important, which raises the chance that a low-cost bidder wins. At the same time, reduced uncertainty leads to more aggressive bidding and thus to higher revenues.[13,14]

The effect of an increase in competition is twofold: the surplus variable is redefined and the number of draws is increased. With $n + 1$ bidders, the common value is given by (1) with $1/n$ replaced by $1/(n + 1)$ and with one signal added.[15] To isolate the latter effect, we scale the common value signals of all $n + 1$ bidders by $\alpha_0 = (n + 1)/n$, so that their surplus is the same as when there are n bidders. Let $Y_1'(\alpha_0)$ denote the highest of the $n + 1$ scaled signals.

LEMMA 2. $E_{n+1}[c|s = Y_1'(\alpha_0)] \leq E_n(c|s = Y_1)$.

This Lemma (see the Appendix on www.res.org for a proof) shows that with one more bidder the winner's expected cost decreases even when uncertainty about

[12] Note that the seller's signal, v_0 does not enter the new surplus. Hence, for the way in which optimal bids depend on bidders' signals it does not matter whether or not the seller's information is disclosed. However, disclosure of this information will affect the *magnitudes* of bids as bidders will compete more (less) aggressively when the seller's news is good (bad).

[13] One may wonder whether a truthful report is in the seller's best interest. Milgrom and Weber (1982) present theoretical arguments why no reporting policy leads to a higher expected revenue than the policy of always reporting honestly. Ashenfelter (1989) provides empirical evidence that sellers' price estimates of impressionist paintings are 'very highly correlated with the actual prices fetched' and that the estimates are 'very close to unbiased'.

[14] There are also situations where the seller is better informed about the commodity's true value than the bidders. A natural way to model this is to weigh the seller's signal by a factor $\lambda > 1$. The expression for the common value then becomes $V = (\lambda v_0 + \Sigma_{i=1}^{n} v_i)/(n + \lambda)$. This formulation can be motivated as follows: consider the case in which the variance of the bidders' signals is σ_b^2 (the same for all bidders) and the variance of the seller's signal is σ_s^2. The 'best' (i.e. unbiased and smallest variance) estimator of the commodity's value is then given by $V = (\lambda v_0 + \sum_{i=1}^{n} v_i)/(n + \lambda)$ where $\lambda = \sigma_b^2/\sigma_s^2$ is a measure of the (relative) quality of the seller's information. The summary statistic now becomes $s = v/(n + \lambda) - c$ and Proposition 5 thus implies that an increase in the quality of the seller's information leads to higher expected total surplus, higher expected revenue, and lowers the winner's expected profit.

[15] In our model it becomes easier for each bidder to predict the common value when more bidders enter the auction. This is not the case in the traditional formulation of the common value, where each bidder's signal is an unbiased draw conditional on the particular realisation of the common value. However, also in that case the expected value *conditional on winning* is easier to predict when bidders are added. To see this, consider the case where bidders' signals are uniformly distributed around the true common value with support $[-\in, \in]$. When the number of rivals grows large, a bidder with signal v knows for sure that the common value is $v - \in$. Also for general distributions and finite numbers of bidders, the variance of the common value conditional on winning decreases as more bidders enter the auction. A real-world example where the introduction of extra bidders reduces the variance of the object's value is the following. Suppose the winning bidder in an auction for a house is unsure whether she will resell in a year. The house's resale price will be determined by the bidder with the second-highest signal, and the expected value of the second-highest signal is higher and less variable when more bidders compete for the house.

the common value increases by a factor $\alpha_0 = (n + 1)/n$. Together with Proposition 5, this proves that more competition raises total expected surplus. Bidders' information rents fall with an increase in competition and since the total pie to be divided is bigger, seller's revenues rise.[16,17]

Interestingly, the effect of one more bidder on efficiency and revenues is greater than the effect of information disclosure by the seller, at least when the seller's signal is not more precise than a bidder's signal. Both lead to the same redefinition of the surplus, which has positive effects on revenues and efficiency since it reduces the uncertainty of the common value. However, in the case of one more bidder there is also one more value and cost draw, which further lowers the winner's expected cost. When faced with the choice to gather more information or to attract more bidders, a seller may want to choose for the latter. This is similar to Bulow and Klemperer's (1996) finding that an optimally structured bargaining process following an auction with n bidders yields less revenue than an auction with $n + 1$ bidders.

We end this Section with a brief illustration of our results. Suppose the private value, $t_i \geq 0$, represents a bidder's idiosyncratic taste for a painting while the estimate, $v_i \geq 0$, of the resale price adds a common value part. Since the common value is the same for all bidders, efficiency only depends on private value differences. Consider the following efficiency measure that takes on values between 0% and 100%:

$$\Omega = \frac{t_{\text{winner}} - t_{\text{min}}}{t_{\text{max}} - t_{\text{min}}} \times 100\%,$$

where t_{max}, t_{min} and t_{winner} are the maximum, minimum, and winner's taste draw respectively. Figure 2 shows attained efficiency as a function of the number of bidders (bottom line), both with the public announcement of the seller's signal (middle line), and one extra bidder (top line). The efficiency loss is substantial with only a few bidders and disappears as more bidders enter the auction. As argued above, the effect of increased competition is greater than the public disclosure of the seller's information. The effect of uncertainty about the common value is illustrated in Figure 3. Note that efficiency decreases when the uncertainty increases, even though the mean of the common value distribution remains unchanged.

[16] The winner's profit can be written in terms of the (inverse of the) hazard rate $h(x) = [1 - F_s(x)]/f_s(x)$:

$$\pi_{\text{winner}} = \int_{s_L}^{s_H} h(x)^{-1} dF_s^n(x),$$

see McAfee and McMillan (1987). First, consider the transition from n to $n + 1$ bidders while scaling the value signals by $\alpha_0 = (n + 1)/n$ so that bidders' surpluses remain the same. The winner's expected profits are then given by the above equation with F^n replaced by F^{n+1} Since logconcavity ensures that the inverse hazard rate, $1/h(x)$, is everywhere non-decreasing (see Lemma A2 in the Appendix) and F^{n+1} first-order stochastically dominates F^n, the resulting profit is less. Hence bidders' profits decrease with the number of bidders even when values are scaled by α_0, and will be even lower without this scaling.

[17] Compte and Jehiel (2002) consider a setting with asymmetric bidders and identify cases where expected welfare is lower when one more bidder participates in the auction.

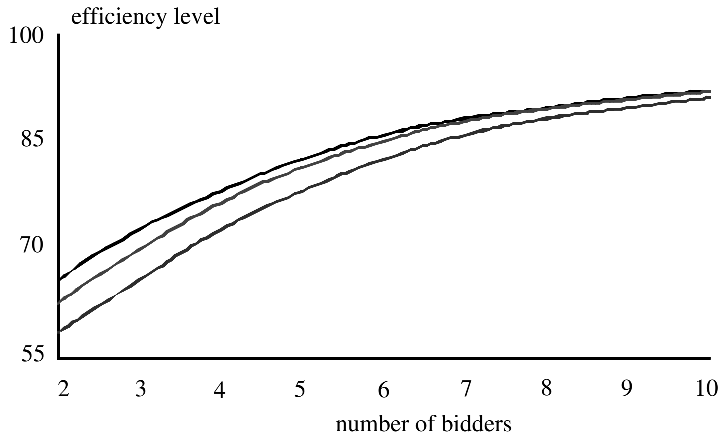

Fig. 2. *Efficiencyas a Function of the Number of Bidders when Private Values are Uniform on* [7,8] *and Common Value Signals are Uniform on* [5,10]

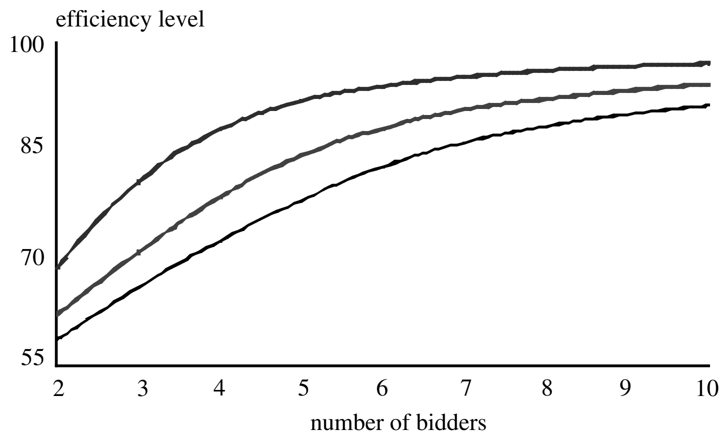

Fig. 3. *Effectsof Uncertainty when Private Values are Uniform on* [7,8] *and Common Value Signals are Uniform on* [7,8] *(Top),* [6,9] *(Middle),* [5,10] *(Bottom)*

5. Conclusion

Currently many governments use auctions to allocate licences for industries with technically limited entry. The results reported in this paper temper an overly enthusiastic attitude towards auctions: when the object for sale has both private and common value elements, inefficiencies should be expected; see also Dasgupta and Maskin (2000); Jehiel and Moldovanu (2001). In all standard auction formats, bidders with a superior private value but a pessimistic signal about the common value can lose against bidders with an inferior private value but an optimistic common value signal. The magnitude of the efficiency losses depends on the degree of uncertainty about the common value: more uncertainty implies a higher level of expected inefficiency. It is thus easy to construct (theoretical) examples that generate large inefficiencies.

We do not suggest, however, abandoning the use of auctions when allocating scarce rights of entry. Only in the extreme case when the common value information completely overrides the private value information, do auctions approach the more or less random allocation of traditional methods like administrative procedures, lotteries, and 'first come first served'. In this sense, our analysis still supports McMillan's (1995) argument that auctioning licences to operate in a market should be favoured over such traditional methods. However, the seller can and should actively try to reduce expected inefficiencies. Our analysis provides two tools that raise efficiency. First, any information gathered by the seller about the common value increases expected efficiency. The higher the quality of the seller's information, the higher the expected efficiency generated by the auction. Second, the auction should be designed such that all potentially interested bidders indeed participate, since more competition results in a more efficient allocation. These results are illustrated in Figures 2 and 3: efficiency losses can be substantial with only two or three bidders, but are negligible with seven bidders or more.

Our results suggest a number of directions for future research. In situations where the same set of bidders interacts repeatedly, it is possible that social ties develop between bidders, which might lead them to work together. Our static model does not address the possibility of collusion among the bidders.[18] It would be interesting, however, to determine the effects of collusion in our model: suppose bidders can find a mechanism to communicate their information (private and common value) truthfully to the other members of their bidding ring. Paradoxically, this may *improve* efficiency for two reasons. First, each bidding ring selects the bidder with the best private value among its members, thus lowering the chance that a bidder with an inferior value wins the auction. Second, when members of a bidding ring pool their information about the common value, the uncertainty about the common value diminishes, which further mitigates the possibility of inefficiencies. On the other hand, collusion also implies that there are fewer bidders, which has a negative effect on efficiency.

Our results have implications for structural econometric models of auctions, e.g. Laffont *et al.* (1995). Such models start with the equilibrium bidding functions for the private value case and use data on winning bids to determine the distribution of bidders' values. This information can be used by the seller to determine the optimal reservation price. As we have shown, the introduction of some common value elements may have a large effect on the optimal bidding functions and their comparative statics properties (see Section 3).

[18] See, for instance, Graham and Marshall (1987) for an analysis of collusion in second-price and English auctions with private values. McAfee and McMillan (1992) consider collusion in first-price
1 auctions. Robinson (1995) investigates the stability of bidding rings for various auction forms. Krishna and Morgan (1997) study the effects of joint bidding in a common value auction using the average formulation for the common value employed in this paper.

Besides auctions there are many other situations of interest where both private and common values play a role. In elections, for instance, voters' preferences over candidates' positions constitute a private value component, while the 'quality' of the candidate can be interpreted as a common value element (Feddersen and Pesendorfer, 1997). Likewise, in Cournot market games, firms' costs are private while the consumer demand forms a common value element; a firm that is overly optimistic about demand may produce more than a more efficient rival. In the special case when the common value equals the average of bidders' signals, the approach of this paper suggests how equilibrium behaviour can be analysed in the presence of both private and common values.

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Appendix: Proofs

Below we shall need the following properties implied by logconcavity.

LEMMA A1. *If $f(x, y)$ is logconcave then so are its marginals $f(x)$ and $f(y)$.*

Proof. See An (1998).

COROLLARY A1. *The density $f_s(s)$ is logconcave in s.*

Proof. The joint density of v and s is given by $f(v, s) = f_v(v)\, f_c(v/n-s)$. It is straightforward to verify that under Assumption 2 the Hessian of $\log [f(v, s)]$ is negative semi-definite, i.e. $f(v, s)$ is logconcave. Lemma A1 thus implies that $f_s(s)$ is logconcave. Q.E.D.

LEMMA A2. *The hazard rate, $h(x) = f(x)/[1-F(x)]$, of a logconcave density, $f(x)$, is everywhere non-decreasing.*

Proof. Let $f(x)$ be logconcave then

$$f'(x)[1 - F(x)] = \int_x^{x_H} f'(x)f(t)\mathrm{d}t \geq \int_x^{x_H} f'(t)f(x)\mathrm{d}t \geq -f(x)^2,$$

where the first inequality follows since logconcavity of f implies that $f'(x)/f(x)$ is non-increasing in x, so $f'(x)/f(x) \geq f'(t)/f(t)$ for all $t \geq x$. It is straightforward to rewrite the above equation as: $\{f(x)/[1-F(x)]\}' \geq 0$. Q.E.D.

LEMMA A3. *Let X and Y be random variables. The conditional expectation $\mathrm{E}(Y|X \leq x)$ satisfies*

$$\frac{\partial}{\partial x}\mathrm{E}(Y|X \leq x) = [\mathrm{E}(Y|X = x) - \mathrm{E}(Y|X \leq x)]\frac{f_X(x)}{F_X(x)}.$$

Proof. Write the conditional expectation $\mathrm{E}(Y|X \leq x)$ as

$$\mathrm{E}(Y|X \leq x) = \int_{x_L}^{x} \mathrm{E}(Y|X = t)\frac{f_X(t)}{F_X(x)}\mathrm{d}t.$$

Differentiating the right side with respect to x yields the desired result. Q.E.D.

Proof of Lemma 1. Since $f(v|s) = f_v(v)f_c(v/n-s)/f_s(s)$, logconcavity of $f_c(\cdot)$ is equivalent to

$$\partial_v \partial_s \log[f(v|s)] \geq 0.$$

This means that $\partial_s \log[f(v|s)] = \partial_s f(v|s)/f(v|s)$ is increasing in v, or, $\partial_s f(v_1|s)/f(v_1|s) \geq \partial_s f(v_2|s)/f(v_2|s)$ for all $v_1 \geq v_2$. This inequality can be rewritten as $\partial_s[f(v_1|s)/f(v_2|s)] \geq 0$. Hence, for all $s_1 \geq s_2$ we have $f(v_1|s_1)/f(v_2|s_1) \geq f(v_1|s_2)/f(v_2|s_2)$.[19] Since this is true for all $v_1 \geq v_2$, it remains true if we integrate v_1 from v to v_H and integrate v_2 from v_L to v, which yields $[1-F(v|s_1)]/F(v|s_1) \geq [1-F(v|s_2)]/F(v|s_2)$, or, $F(v|s_1) \leq F(v|s_2)$. In other words, $F(v|s_1)$ first-order stochastically dominates $F(v|s_2)$, which implies $\mathrm{E}(v|s_1) \geq \mathrm{E}(v|s_2)$ for all $s_1 \geq s_2$. So $\mathrm{E}(v|s = x)$ is non-decreasing in x. Lemma A3 applied to $X = s$, $Y = v$ yields: $\partial_x \mathrm{E}(v|s \leq x) = [\mathrm{E}(v|s = x) - \mathrm{E}(v|s \leq x)]f_s(x)/F_s(x)$, which is non-negative since $\mathrm{E}(v|s = x)$ is non-decreasing in x. The proofs for the conditional expectations of the cost are similar. Q.E.D.

Below we use the following notation. The conditional density of y_1 given $y_1 \leq s$ is: $f_{y_1}(x|y_1 \leq s) = f_{y_1}(x)/F_{y_1}(s) = (n-1)f_s(x)F_s(x)^{n-2}/F_s(s)^{n-1}$ for $x \leq s$, the corresponding distribution function is $F_{y_1}(x|y_1 \leq s) = F_s(x)^{n-1}/F_s(s)^{n-1}$.

Proof of Proposition 1. An equivalent way to write (2) is: $B(x) = (n-1)/n\mathrm{E}(v|s \leq x) + \mathrm{E}(y_1|y_1 \leq x)$ and it derivative is easily derived from Lemma A3: $B'(x) = (n-1)/n\,[\mathrm{E}(v|s = x) - \mathrm{E}(v|s \leq x)]\,f_s(x)/F_s(x) + [x - \mathrm{E}(y_1|y_1 \leq x)]f_{y_1}(x)/F_{y_1}(x)$. The last term is positive (the event $y_1 = x$ occurs with probability zero) and the first term is non-negative by Lemma 1. Hence, $B(\cdot)$ is increasing. Suppose that bidders 2,…,n bid according to (2). Bidder 1's expected profit is:

$$\pi_1^e(b) = \left\{s_1 + \frac{n-1}{n}\mathrm{E}[v|s \leq B^{-1}(b)] - b\right\}F_s^{n-1}[B^{-1}(b)].$$

From Lemma A3, the derivative of the expected profit with respect to b evaluated at $b = B(x)$ is:

$$\pi_1^e[B'(x)] = \frac{f_{y_1}(x)}{B'(x)}\left[s_1 + \frac{1}{n}\mathrm{E}(v|s = x) + \frac{n-2}{n}\mathrm{E}(v|s \leq x) - B(x) - B'(x)\frac{F_{y_1}(x)}{f_{y_1}(x)}\right].$$

Using the expression for B' derived above it is straightforward to verify that the term in the brackets equals $s_1 - x$. Together with monotonicity of $B(\cdot)$ this shows $B(s_1)$ is bidder 1's unique optimal bid. Q.E.D.

Proof of Proposition 2. Note that (3) is equivalent to: $B(x) = x + 1/n\,\mathrm{E}(v|s = x) + (n-2)/n\,\mathrm{E}(v|s \leq x)$. The last two terms are non-decreasing in x (Lemma 1), so $B(\cdot)$ is increasing. Assume that bidders 2,…,n bid according to (3). Bidder 1's expected payoff is:

[19] In other words, $f(v|s)$ satisfies the monotone likelihood property, see Milgrom (1981).

$$\pi_1^e(b) = \left\{ s_1 + \frac{n-1}{n} \mathrm{E}[v|s \leq B^{-1}(b)] - \int_{s_L}^{B^{-1}(b)} B(x) f_{y_1}[x|s \leq B^{-1}(b)] \mathrm{d}x \right\} F_s^{n-1}[B^{-1}(b)].$$

The second term in the brackets on the right side represents the sum of the values of bidders 2 to n given that their surpluses are less than $B^{-1}(b)$. An equivalent way of writing this term is by choosing one of them to have the highest surplus, $y_1 \leq B^{-1}(b)$, while the $n-2$ other have surpluses less than or equal to y_1. The expected profit then becomes

$$\begin{aligned} \pi_1^e(b) &= \int_{s_L}^{B^{-1}(b)} \left[s_1 + \frac{1}{n} \mathrm{E}(v|s = x) + \frac{n-2}{n} \mathrm{E}(v|s \leq x) - B(x) \right] f_{y_1}(x) \mathrm{d}x \\ &= \int_{s_L}^{B^{-1}(b)} (s_1 - x) \mathrm{d}F_{y_1}(x). \end{aligned}$$

Together with monotonicity of $B^{-1}(\cdot)$ this proves that the unique optimal bid is $B(s_1)$. Q.E.D.

Proof of Proposition 3. Note that each B_k is strictly increasing in x. Suppose bidders $2,\ldots,n$ bid according to (4). When bidder 1 wins the auction her expected profit is: $s_1 + 1/n \sum_{j=1}^{n-1} \mathrm{E}(v|s = s_{j+1}) - B_{n-2}(s_2)$, where the s_j are the realisations of the others' surpluses arranged in increasing order, i.e. $s_2 \geq \cdots \geq s_n$. Using the definition of B_{n-2} the expected payoff can be written as: $s_1 + 1/n\, \mathrm{E}(v|s = s_2) - 2/n\, \mathrm{E}(v|s = s_2) + \mathrm{E}(c|s = s_2) = s_1 - s_2$. So bidder 1's expected profit is positive only when she has the highest surplus, and using $B(\cdot)$ she wins iff $s_1 = Y_1$. Hence, $B(\cdot)$ is the optimal bidding strategy for player 1. Q.E.D.

Proof of Proposition 4. Expected total surplus equals the winner's valuation of the commodity:

$$W = \int_{s_L}^{s_H} \left[x + \frac{n-1}{n} \mathrm{E}(v|s \leq x) \right] f_{Y_1}(x) \mathrm{d}x,$$

so $W = \mathrm{E}(s|s = Y_1) + (n-1)/n\, \mathrm{E}(v|s \leq Y_1) = 1/n\, \mathrm{E}(v|s = Y_1) + (n-1)/n\, \mathrm{E}(v|s \leq Y_1) - \mathrm{E}(c|s = Y_1)$. The first two terms represent the expected value of the commodity given that one bidder has the highest surplus and the others have lower surpluses: this is just $\mathrm{E}(V)$. Hence $W = \mathrm{E}(V) - \mathrm{E}(c|s = Y_1)$.

The Envelope Theorem implies that the derivative of the equilibrium profit $\pi^*(s) = \pi^e[B(s)]$ with respect to a bidder's surplus s equals the equilibrium probability of winning. When surpluses are i.i.d. across bidders the equilibrium probability of winning is simply $F_s(s)^{n-1}$ and the equilibrium profit is therefore $\pi^*(s) = \int_{s_L}^{s} F_s(x)^{n-1} \mathrm{d}x$. This is the expected equilibrium profit of a bidder with surplus s in any of the three auctions discussed above. The *ex ante* expected equilibrium *payoffs* accrue to the winning bidder, so

$$\pi_{winner} = n \int_{s_L}^{s_H} \int_{s_L}^{s} F_s^{n-1}(x) \mathrm{d}x\, \mathrm{d}F_s(s) = n \int_{s_L}^{s_H} [1 - F_s(x)] F_s^{n-1}(x) \mathrm{d}x,$$

where the last expression is obtained by changing the order of integration. Partially integrating this expression yields: $\pi_{\text{winner}} = \mathrm{E}(Y_1) - \mathrm{E}(Y_2)$. Finally, revenue is the difference between total surplus and the winner's profit: $R = W - \pi_{\text{winner}} = W - [\mathrm{E}(Y_1) - \mathrm{E}(Y_2)]$. Q.E.D.

Proof of Proposition 5. Let $\alpha_1 > \alpha_2 > 0$. Suppose, without loss of generality, that $s_1(\alpha_1) = \max_{j=1,\ldots,n} s_j(\alpha_1)$ and $s_k(\alpha_2) = \max_{j=1,\ldots,n} s_j(\alpha_2)$. There are two cases to be considered. If $k = 1$, the costs are the same for both α s. If $k \neq 1$, we have: $\alpha_1 v_1/n - c_1 > \alpha_1 v_k/n - c_k$ and $\alpha_2 v_k/n - c_k > \alpha_2 v_1/n - c_1$ (ties occur with probability zero). Dividing the first inequality by α_1 and the second by α_2 and adding the result yields: $c_1/\alpha_1 + c_k/\alpha_2 < c_1/\alpha_2 + c_k/\alpha_1$, or, equivalently, $c_k\,(1/\alpha_2 - 1/\alpha_1) < c_1\,(1/\alpha_2 - 1/\alpha_1)$. Hence $c_1 > c_k$ when $k \neq 1$. In other words, for some realisations of the value and cost signals, the winner's cost is less for $\alpha = \alpha_2$ than for $\alpha = \alpha_1$, while it is the same for other realisations. *A fortiori*, the expected winner's cost (i.e. the winner's cost averaged over all possible realisations of the value and cost signals) is less for $\alpha = \alpha_2$. So an increase in α lowers total expected surplus. Next we show that an increase in α does not lower bidders' expected profits. Note that the distribution function, $F_{s(\alpha)}(x)$, of $s(\alpha) = \alpha v/n - c$ is given by

$$F_{s(\alpha)}(x) = \int_{v_L}^{v_H} f_v(v)[1 - F_c(\alpha v/n - x)]\mathrm{d}v,$$

from which it follows that $\mathrm{d}F_{s(\alpha)}(x)/\mathrm{d}\alpha = -1/n\,\mathrm{E}[v|s(\alpha) = x] f_{s(\alpha)}(x) \leq 0$, i.e. the distribution $F_{s(\alpha)}(\cdot)$ is stochastically increasing in α. The difference between the first and second order statistic is:

$$\mathrm{E}[Y_1(\alpha)] - \mathrm{E}[Y_2(\alpha)] = n \int_{s_L(\alpha)}^{s_H(\alpha)} F_{s(\alpha)}^{n-1}(x)[1 - F_{s(\alpha)}(x)]\mathrm{d}x.$$

Differentiating this expression with respect to α yields

$$\frac{\mathrm{d}\{\mathrm{E}[Y_1(\alpha)] - \mathrm{E}[Y_2(\alpha)]\}}{\mathrm{d}\alpha} = \frac{1}{n}\{\mathrm{E}[v|s(\alpha) = Y_1(\alpha)] - \mathrm{E}[v|s(\alpha) = Y_2(\alpha)]\}.$$

Lemma 1 together with $Y_1(\alpha) > Y_2(\alpha)$ ensures that the right side is non-negative. Since bidders are no worse off and total expected surplus is lower, the seller's revenue has to be less. Q.E.D.

Proof of Lemma 2.

$$\mathrm{E}_n(c|s = Y_1) = \int_{s_L}^{s_H} \mathrm{E}(c|s = x)\mathrm{d}F_s^n(x),$$

and

$$\mathrm{E}_{n+1}[c|s = Y_1'(\alpha_0)] = \int_{s_L}^{s_H} \mathrm{E}(c|s = x)\mathrm{d}F_s^{n+1}(x).$$

Since $F^{n+1}(\cdot)$ first-order dominates $F^n(\cdot)$ and $\mathrm{E}(c|s = x)$ is non-increasing in x, we have $\mathrm{E}_{n+1}[c|s = Y_1'(\alpha_0)] \leq \mathrm{E}_n(c|s = Y_1)$. Q.E.D.

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