Efficiency in Auctions with Private and Common Values: An Experimental Study

By JACOB K. GOEREE AND THEO OFFERMAN*

Auctions are generally not efficient when the object's expected value depends on private and common value information. We report a series of first-price auction experiments to measure the degree of inefficiency that occurs with financially motivated bidders. While some subjects fall prey to the winner's curse, they weigh their private and common value information in roughly the same manner as rational bidders, with observed efficiencies close to predicted levels. Increased competition and reduced uncertainty about the common value positively affect revenues and efficiency. The public release of information about the common value also raises efficiency, although less than predicted. (*JEL* C72, D44)

Auctions are typically classified as either "private value" or "common value." In private value auctions, bidders know their own value for the commodity with certainty but are unsure about others' valuations (e.g., the sale of a painting). In contrast, in common value auctions bidders receive noisy signals about the commodity's value, which is the same for all (e.g., firms competing for the rights to drill for oil). While this dichotomy is convenient from a theoretical viewpoint, most real-world auctions exhibit both private and common value elements. In the recent spectrum auctions conducted by the U.S. Federal Communications Commission (FCC), for example, the different cost structures of the bidding firms constituted a private value element while the uncertain demand for the final consumer product added a common value part. Alternatively, in takeover battles, bidders' valuations are determined by private synergistic gains in addition to the target's common market value.[1]

By focusing on the "extreme" cases, the literature has inadvertently spread the belief that auctions generally lead to efficient allocations. In (symmetric) private value auctions, optimal bids are increasing in bidders' values so the object is awarded to whom it is worth the most, and in common value auctions any allocation is trivially efficient. When both private and common value elements play a role, however, inefficiencies should be expected. The simple intuition for this result is that a bidder with an inferior private value but an overly optimistic conjecture about the common value may outbid a rival with a superior private value. The possibility of inefficiencies in multisignal auctions

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[1] Even the standard examples of pure private or pure common value auctions are not entirely convincing. When a painting is auctioned, for instance, it may be resold in the future and the resale price will be the same for all bidders, which adds a common value element. And in the oil drilling example, a private value element is introduced when the costs of exploiting the tract differ (substantially) between firms.

was first discussed by Eric S. Maskin (1992) and further explored by Partha Dasgupta and Maskin (2000), Wolfgang Pesendorfer and Jeroen M. Swinkels (2000), Philippe Jehiel and Benny Moldovanu (2001), and Goeree and Offerman (2002).

This paper reports a series of first-price auction experiments in which each bidder receives a private and a common value signal. To determine the optimal bid, the two pieces of information have to be combined and the more weight bidders assign to their common value signals the lower is expected efficiency. For instance, if bidders ignore their common value signals, the auction becomes a fully efficient private value auction. In the other extreme, when bidders ignore their private value information, the auction is no more efficient than a random allocation rule. Rational bidders react to both pieces of information, resulting in some intermediate degree of inefficiency.[2]

Observed bids will differ from rational ones when subjects do not (sufficiently) incorporate the negative information conveyed by winning. Indeed, there exists substantial experimental and empirical evidence that in pure common value auctions, bidders often ignore this adverse selection effect and forgo some profits as a result: the *winner's curse*.[3] While naive bidding obviously results in higher revenues for the seller, its effect on efficiency has not yet been explored. One goal of this paper is to measure the degree of inefficiencies that results when both private and common value elements play a role.

A second goal is to evaluate factors that are predicted to raise efficiency and revenues. In particular, we examine the impact of (i) a decrease in the uncertainty about the common value, (ii) an increase in the number of bidders, and (iii) the public release of information about the common value. Intuitively, a decrease in uncertainty about the common value makes the private values more important, enhancing efficiency.[4] An increase in the number of bidders has no effect on efficiency in pure common value auctions, but improves efficiency in auctions with private values since it tends to raise the highest signal. In auctions with both common and private values, there is an additional benefit: with more competition, bidders weigh their common value information less, resulting in more efficient outcomes. Due to this latter effect even a moderate increase in competition may substantially improve efficiency. Finally, the seller often possesses information about the object for sale. Paul R. Milgrom and Robert J. Weber (1982) have shown that the public release of such information is beneficial to the seller because it raises revenues. In addition, public information disclosure raises efficiency because it reduces the uncertainty about the common value.

The paper is organized as follows. The design of the auctions is explained in Section I. Section II gives the necessary theoretical background. The experimental design is described in Section III. Section IV reports the experimental results and provides an analysis of individual and aggregate data. Section V concludes. The deriva-

[2] Robert Forsythe et al. (1989) also consider auctions with common and private value elements. In their setup the seller has the option to truthfully reveal the quality of the item, which resolves all uncertainty about the common value part and transforms the auction into a private value auction. When the seller provides no information, optimal bids are based on bidders' private value information and their common prior about quality. In the sequential Nash equilibrium, the seller reveals the quality of the item and buyers assume the worst when no information is revealed. The fully efficient outcome predicted by the sequential Nash equilibrium is observed in the final periods of the experiment. Oliver Kirchkamp and Moldovanu (2000) report an experiment that compares the efficiency properties of the English auction and the second-price auction when bidders' valuations are interdependent.

[3] For experimental evidence, see Max H. Bazerman and William F. Samuelson (1983), John H. Kagel and Dan Levin (1986), Douglas Dyer et al. (1989), Kagel et al. (1989), Barry Lind and Charles R. Plott (1991), Susan Garvin and Kagel (1994), Levin et al. (1996), Christopher Avery and Kagel (1997), Kagel and Levin (1999), James C. Cox et al. (2001), and Goeree and Offerman (2001). Field studies also suggest that bidders are bothered by a winner's curse, see, e.g., E. C. Capen et al. (1971), Richard Roll (1986), Richard H. Thaler (1988), and Orley Ashenfelter and David Genesove (1992).

[4] For example, when licenses to operate in a market are auctioned, interested firms will have to estimate the uncertain (but common) demand for the consumer product they will sell. There will be more uncertainty associated with licenses for new markets (e.g., wireless local loop frequencies for multimedia applications) than with licenses for well-established markets (e.g., vendor locations at fairs).

tion of the Nash equilibrium bids for one of the treatments is given in the Appendix.

I. Design of the Auctions

We consider a first-price auction in which each bidder receives a private value signal, t_i, and an independent common value signal, v_i. A bidder's valuation is the sum of her private value and the common value, V. The common value is defined as the average of bidders' common value signals:[5]

$$V = \frac{1}{n} \sum_{i=1}^{n} v_i. \tag{1}$$

This formulation for the common value has previously been used in both theoretical and experimental work.[6] An advantage of the average formulation is that it is easier to explain and understand than the "traditional" formulation of the common value, where V has some known prior distribution and bidders' signals are draws conditional on the particular realization of V (Robert Wilson, 1977). While being simpler, the average formulation captures the two main features of the traditional formulation: (i) the value of the object for sale is the same for all bidders, and (ii) in order not to fall prey to a winner's curse, bidders should take into account the information conveyed by winning. The difficulty of overcoming this adverse selection problem may be mitigated, however, since bidders will naturally be inclined to think about others' signals when forming an estimate of the average.[7] This should be kept in mind when comparing our results to previous experimental work using the traditional formulation.

When the seller publicly announces her own estimate, v_0, of the common value, the expression in (1) changes to

$$V = \frac{1}{n+\lambda} \left(\lambda v_0 + \sum_{i=1}^{n} v_i \right) \tag{2}$$

where λ represents the precision or quality of the seller's signal. The seller's estimate is of the same quality (higher quality) as a bidder's estimate when $\lambda = 1$ ($\lambda > 1$).[8]

II. Theoretical Background

While few people would dispute that most real-world auctions exhibit both private and common value features, surprisingly little is known about their equilibrium properties. The difficulty with multiple signals is how to combine the different pieces of information into a single statistic that can be mapped into a bid (Milgrom and Weber, 1982 p. 1097). This is not a problem, however, for the average formulation in (1). It is routine to verify that the summary statistic is given by the "surplus" $s_i = v_i/n + t_i$.[9] The optimal bids then follow from the work of Milgrom and Weber (1982) who characterize the Nash equilibrium for standard auctions when bids depend on a univariate statistic.

First, let us introduce some notation. Due to symmetry we can, without loss of generality, focus on bidder 1 who has surplus $s_1 = v_1/n + t_1$. Let y_1 denote the highest surplus of the $n - 1$ others: $y_1 = \max_{j=2,\dots,n} (v_j/n + t_j)$, and let

[5] Hence, at the time of bidding a bidder knows only part of her valuation.

[6] Theoretical papers include Wulf Albers and Ronald M. Harstad (1991), Sushil Bikhchandani and John G. Riley (1991), Vijay Krishna and John Morgan (1997), Paul Klemperer (1998), and Jeremy I. Bulow et al. (1999). Experimental papers include Avery and Kagel (1997) and Charles A. Holt and Roger Sherman (2000).

[7] Nevertheless, Holt and Sherman (2000) report clear evidence of a winner's curse in a two-person first-price auction experiment using the average formulation in (1). Avery and Kagel (1997) find similar evidence in a second-price auction.

[8] Equation (2) can be motivated as follows: suppose the variance of the bidders' signals is σ^2_b (the same for all bidders) and the variance of the seller's signal is σ^2_a. The best (i.e., unbiased and smallest variance) estimator of the commodity's value is then given by $V = (\lambda v_0 + \sum_{i=1}^{n} v_i)/(n + \lambda)$ where $\lambda = \sigma^2_b/\sigma^2_a$ is a measure of the (relative) quality of the seller's information.

[9] A bidder's expected payoff is: π^e = (expected gain − expected payment) × probability of winning. The expected payment and the probability of winning are independent of a bidder's private and common value signals (but will depend on her bid and others' bidding strategies). Moreover, for the average formulation of the common value, bidder i's expected gain equals her surplus, $s_i = v_i/n + t_i$, plus terms that are independent of her signals. The first-order conditions for profit maximization therefore determine optimal bids in terms of the surplus, s_i.

Y_1 (Y_2) be the maximum (second highest) of all n surplus draws. To keep the notation simple we use only one expectation symbol, e.g., the expected private value of the winner $E(t_{winner}) = E_{Y_1}(E(t|s = Y_1))$ is written as $E(t|s = Y_1)$. Proofs of the propositions in this section can be found in Goeree and Offerman (2002).

PROPOSITION 1: *The n-tuple of strategies* $(B(\cdot), \ldots, B(\cdot))$, *where*

$$(3) \quad B(x) = E(V + t_1|s_1 = x, Y_1 = x) - E(Y_1 - y_1|s_1 = x, Y_1 = x)$$

is an equilibrium of the first-price auction. The winner's expected profit is $\pi_{winner} = E(Y_1) - E(Y_2)$ *and the seller's expected revenue is* $R = E(V) + E(t_{winner}) - \pi_{winner}$.

The intuition behind (3) is straightforward. Since Y_1 is the highest of all surplus draws, the first term on the right side represents what the commodity is worth (on average) to a bidder assuming that her surplus, x, is the highest. The second term shows how much she shades her bid to make a profit.

When the seller publicly reveals information about the common value, the optimal bids in (3) have to be adjusted. The new optimal bids are functions of the summary statistic $s_i' = v_i/(n + \lambda) + t_i$. With this new surplus variable and a corresponding redefinition of the order statistics, the Nash equilibrium bids again follow from Proposition 1. In other words, the functional form in (3) remains valid.[10]

Expected efficiency depends only on the weights bidders place on their common value signals, and any factor that reduces these weights will positively affect efficiency. This intuition underlies the comparative statics results of Proposition 2.

PROPOSITION 2: *Expected efficiency in a Nash equilibrium rises when* (*i*) *more bidders enter the auction,* (*ii*) *information is publicly released, and* (*iii*) *the variance of the common value signals is reduced.*

We end this section by discussing two models of bidding behavior that include rational bidding as a special case. First, assume subjects weigh their private and common value signals differently. When the summary statistic is $s_i^\alpha = \alpha v_i + t_i$, optimal bids become

$$(4) \quad B^\alpha(x) = E(V + t_1|s_1^\alpha = x, Y_1^\alpha = x) - E(Y_1^\alpha - y_1^\alpha|s_1^\alpha = x, Y_1^\alpha = x)$$

which reduce to the Nash bids in (3) if and only if $\alpha = 1/n$. Second, previous experiments based on pure common value auctions have demonstrated that subjects often fail to take into account the information conveyed by winning. Therefore we also consider a model of naive bidding in which bidders replace others' common value signals by their unconditional expected value:

$$(5) \quad B_{curse}^\alpha(x) = E(V + t_1|s_1^\alpha = x) - E(Y_1^\alpha - y_1^\alpha|s_1^\alpha = x, Y_1^\alpha = x).$$

We refer to (5) for $\alpha = 1/n$ as the "Naive" benchmark. Even though $B_{curse}^\alpha(x) > B^\alpha(x)$ for all x and α, the models predict the same winner because (4) and (5) are functions of the same summary statistic, s_1^α. Hence, there is no efficiency loss due to the winner's curse.

III. Design of the Experiments

We conducted seven treatments as shown in the leftmost column of Table 1. The labels "low" and "high" indicate whether the variance of the common value distribution was small or large (column 6), the number indicates group size (column 4), and the "+" (or "++") sign indicates that subjects were once (twice) experienced (column 2). In the experiment, subjects earned points, which were converted into guilders at the end of the experiment at a rate of 4 points = 1 guilder

[10] The Nash bid is: $B(x) = E(V + t_1|v_0, s_1' = x, Y_1' = x) - E(Y_1' - y_1'|s_1' = x, Y_1' = x)$, with the common value, V, given by (2). The winner's profit is $\pi_{winner} = E(Y_1') - E(Y_2')$ and the seller's revenue is $R = E(V) + E(t|s' = Y_1') - \pi_{winner}$.

TABLE 1—SUMMARY OF EXPERIMENTAL DESIGN

Treatment	Subjects' experience	Number of subjects	Number of bidders per group	Private values	Common values	Quality of seller's signal
Low-3	none	30	3	$U[75, 125]$	$U[75, 125]$	$\lambda = 1$
Low-3+	once	18	3	$U[75, 125]$	$U[75, 125]$	$\lambda = 1$
High-3	none	30	3	$U[75, 125]$	$U[0, 200]$	$\lambda = 1$
High-3+	once	18	3	$U[75, 125]$	$U[0, 200]$	$\lambda = 1$
High-6	none	54	6	$U[75, 125]$	$U[0, 200]$	$\lambda = 1$
High-6+	once	18	6	$U[75, 125]$	$U[0, 200]$	$\lambda = 1$
High-3++	twice	21	3	$U[75, 125]$	$U[0, 200]$	$\lambda = 7$

(≈ \$0.50).[11] The experiment was completely computerized and consisted of two parts.[12] Subjects received the instructions for the second part only after all 20 periods of the first part were finished.[13]

A. *Part 1: The Basic Setup*

The first part of the experiment lasted 20 periods. Bidders were given a starting capital of 120 points, which they did not have to pay back. In each period, subjects' private values, t_i, were uniformly distributed between 75 and 125, i.e., $t_i \sim U[75, 125]$, as shown in column 5 of Table 1. Common value signals were $U[0, 200]$ distributed in the high-3 and high-6 treatments, and were $U[75, 125]$ distributed in the low-3 treatments. Both private and common value signals were identically and independently distributed across subjects and periods, and the procedure for generating the signals was common knowledge. Finally, we used a different set of private and common value signals for each treatment.

Bids were restricted to lie between the lowest and highest possible valuation for the commodity. In treatments low-3 and low-3+, subjects had to enter integer bids between 150 and 250 points, while in the other treatments bids had to be between 75 and 325 points. At the end of a period subjects were told the bids in their group ordered from high to low, the common value, and whether or not they won the auction. (In case of a tie, the winner was selected at random from the highest bidders.) Subjects only received information about others' bids, not about others' private or common value signals, and the winner's profit was communicated only to the winner. The reason for disclosing all losing bids is partly motivated by real-world auctions, where all bids are often observable. This procedure, which has previously been used by Kagel and Levin (1986), may alleviate the winner's curse as subjects learn from observing others' decisions.

B. *Part 2: Public Information Disclosure*

The second part lasted from periods 21 to 30, and was designed to evaluate the effects of public information disclosure on efficiency, revenues, and profits. A subject's starting capital in period 21 equaled the total amount earned in part 1 plus a 60-point bonus. While the effects of increased competition and increased uncertainty about the common value were investigated in a *between* subject design, a *within* subject design was used to determine the effects of public information release. In each period, subjects made two decisions. The first decision was the same as in the one described in part 1. After all subjects made their first decision they received an additional signal about the value of the object, the *seller's signal,* and were asked to bid again.[14] The seller's signal was an

[11] For statistical reasons, group composition was kept constant during the whole experiment. Subjects did not have this information to avoid repeated-game effects.

[12] The experiment was programmed using the *RatImage* Toolbox (Klaus Abbink and Abdolkarim Sadrieh, 1995).

[13] A translation of the instructions is available from the authors on request.

[14] This procedure to evaluate the effects of a public signal does not affect subjects' choices in an undesired way (Kagel and Levin, 1986; Kagel et al., 1987).

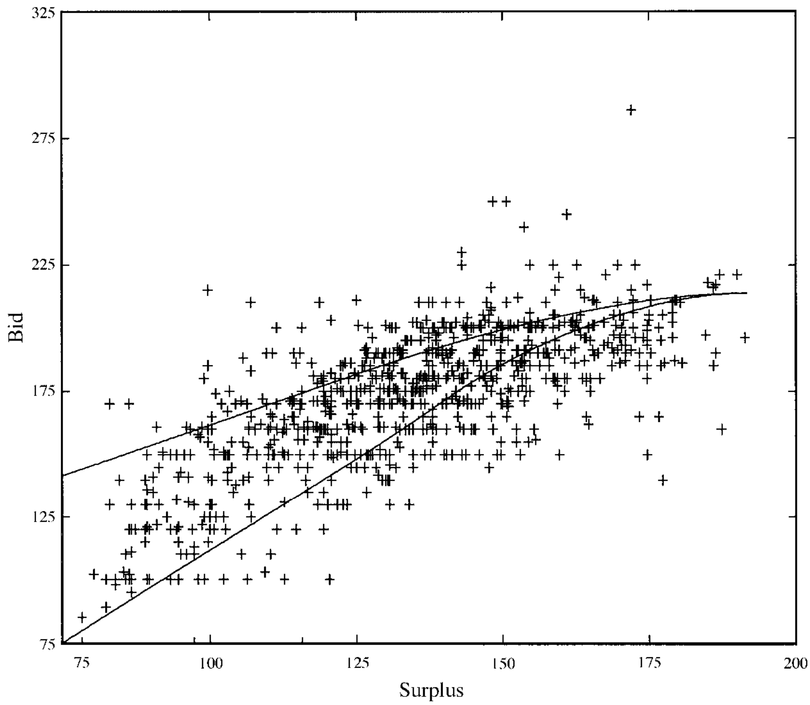

FIGURE 1A. BIDS (+) IN PART 1 OF TREATMENT HIGH-3 TOGETHER WITH NASH BIDS (LOWER LINE) AND NAIVE BIDS (TOP LINE)

independent draw from the same distribution as the common value signals of the bidders. Everyone in the group received the same seller's signal, and subjects' private and common value signals for the second decision were the same as for the first. In all treatments, except high-3++, the common value in part 2 was given by equation (2) with $\lambda = 1$. In treatment high-3++ the quality of the seller's signal was higher, $\lambda = 7$, as indicated in the final column of Table 1. Only one of the two decisions was actually paid out. Decisions with and without a public information signal had an equal chance of being selected for payment, and subjects learned which decision was chosen only after everyone had made both decisions. They only received information pertaining to the decision selected, and the information provided was analogous to that in part 1.

C. *Subjects and Bankruptcy*

Subjects were recruited at the University of Amsterdam. The experiment was finished within two hours and subjects on average earned 61.25 guilders (≈ $30.60). Their starting capital of 120 points provided some buffer against bankruptcy.[15] A subject went bankrupt when her cash balance became negative, in which case she knew she had to leave the experiment without receiving any money. If a subject went bankrupt in a treatment with six bidders per group, the computer bid zero for this subject for the remainder of the experiment. The other bidders in the group then proceeded as before, now with one less opponent.[16] If a subject went bankrupt in a treatment with three bidders per group, the computer submitted Nash bids for this person for the remainder of the experiment. (We did not use zero bids in this case because we feared it would make collusion too easy.) The other two bidders proceeded as

[15] Even when everyone plays according to the Nash strategy there is some chance that the winner loses money.

[16] Of course, if one of six bidders enters a zero bid the theoretical predictions are affected (both for the Nash benchmark and for alternative models). We take this into account when analyzing the data.

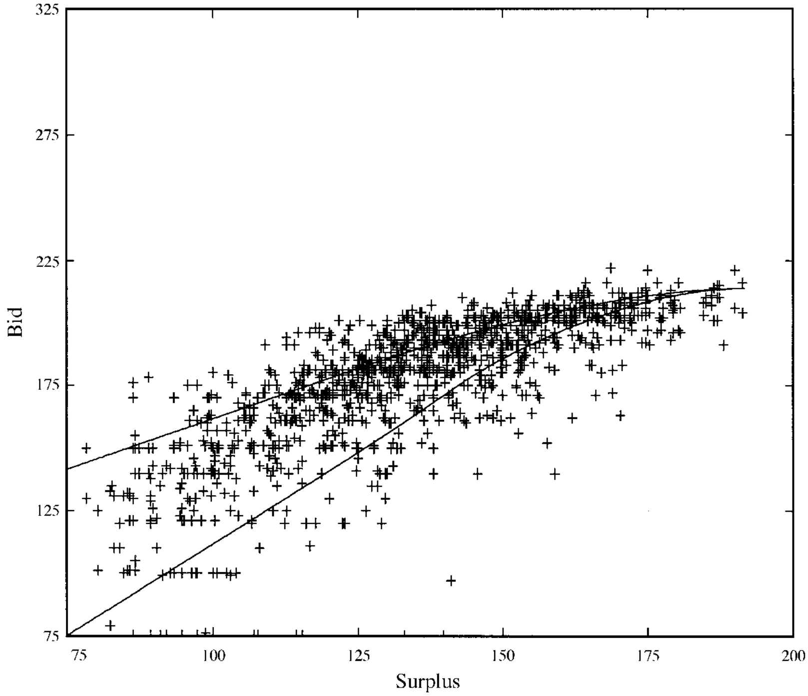

FIGURE 1B. BIDS (+) IN PARTS 1 OF TREATMENTS HIGH-3+ AND HIGH-3++ TOGETHER WITH NASH BIDS (LOWER LINE) AND NAIVE BIDS (TOP LINE)

before, now facing one human and one "computerized Nash" opponent. The exact details of these bankruptcy procedures were communicated to all bidders when a bankruptcy occurred. The periods after a bankruptcy in a group of three were completed only to give the remaining two bidders a chance to earn some money; data from these periods were discarded.

Subjects who did not go bankrupt could voluntarily subscribe for one of the experienced sessions. (Subjects who went bankrupt were not given this opportunity.) The majority of subjects in experienced sessions participated in the same treatment as in their inexperienced session. Treatment high-3++ was conducted two months after the other treatments, and subjects in this treatment had participated in two earlier sessions.

IV. Experimental Results and Analysis

The analysis of the laboratory data is divided into two parts. In Section IV, Subsection A, we report realized efficiency levels and in Section IV, Subsection B, we test the comparative statics predictions. First, we discuss the occurrence of the winner's curse in the different treatments.

Figures 1–3 show actual bids together with predictions of the Nash benchmark (bottom line) and the Naive benchmark (top line). In all treatments, the majority of the data are "sandwiched" between the Nash and Naive bids. Note that actual bids tend to increase in surplus and that the amount of overbidding (relative to Nash) tends to be higher when a subject's surplus is smaller. This is indicative of naive bidding: winning the auction is more informative about others' common value signals when own surplus is small, so neglecting this information leads to a larger bias.

Table 2 gives a more detailed account of the winner's curse. The third row of the table displays the fraction of observed bids that exceed the object's expected value given the empirical distribution of bids and common value signals. A winner's curse is negligible in treatments low-3 and low-3+, which is not surprising since the adverse selection problem is relatively small

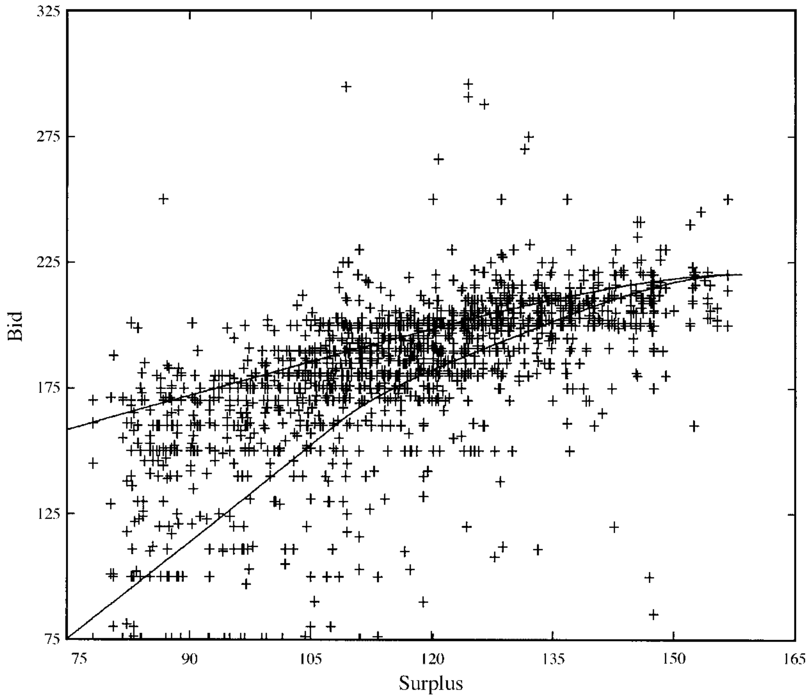

FIGURE 2A. BIDS (+) IN PART 1 OF TREATMENT HIGH-6 TOGETHER WITH NASH BIDS (LOWER LINE) AND NAIVE BIDS (TOP LINE)

in these treatments. In the other treatments, however, the winner's curse is clearly present. The fourth row indicates that subjects rarely bid above the object's expected value predicted by the Naive model. The latter result is mirrored by the second row of the middle panel: average bids predicted by the Naive model are significantly higher than actual averages.[17] The bottom panel of Table 2 shows how costly overbidding is in the inexperienced treatments high-3 and high-6. In high-3, winners realize only about half the available Nash profits and in high-6, winners even lose money on average. In low-3, winners make less profit than predicted by either the Naive or Nash benchmark. One reason for these lower profits is that the auction is not always won by the bidder with the highest surplus (see row 2), as predicted by Nash/Naive bidding.

The performance of experienced subjects is somewhat better. First, no bankruptcies occurred in the experienced treatments, while seven inexperienced subjects went bankrupt (6 percent). Second, earnings are higher in the experienced treatments.[18] Nevertheless, experienced subjects still fall prey to the winner's curse and in treatments high-3+, high-3++, and high-6+ they systematically overbid at a

[17] Bids show no systematic time trend within a treatment. Nevertheless, some aspects in the data are consistent with *learning direction* theory (Reinhard Selten and Joachim Buchta, 1998). Most notably, when winning the auction results in a loss, subjects increase their bid factor ($s + (n - 1)/nE(v) -$ bid) on average by 22.9 (10.3) in the inexperienced (experienced) treatment.

[18] This improved performance may be either the result of learning, selection, or both. Subjects that subscribed for an experienced session earned, on average, 1.62 points per period in the inexperienced sessions, while those that did not subscribe earned 1.03 points. This supports the idea that selection plays a role, although the difference between the earnings is far from significant (a Mann-Whitney test with subjects as the unit of observation yields $p = 0.77$). The 45 subjects that participated twice in the same treatment earned somewhat higher profits and deviated slightly less from Nash (in an absolute sense) in the experienced session. Thus, there are also some (weak) indications of learning, although learning mainly occurs within the inexperienced session, and not between the inexperienced and experienced sessions.

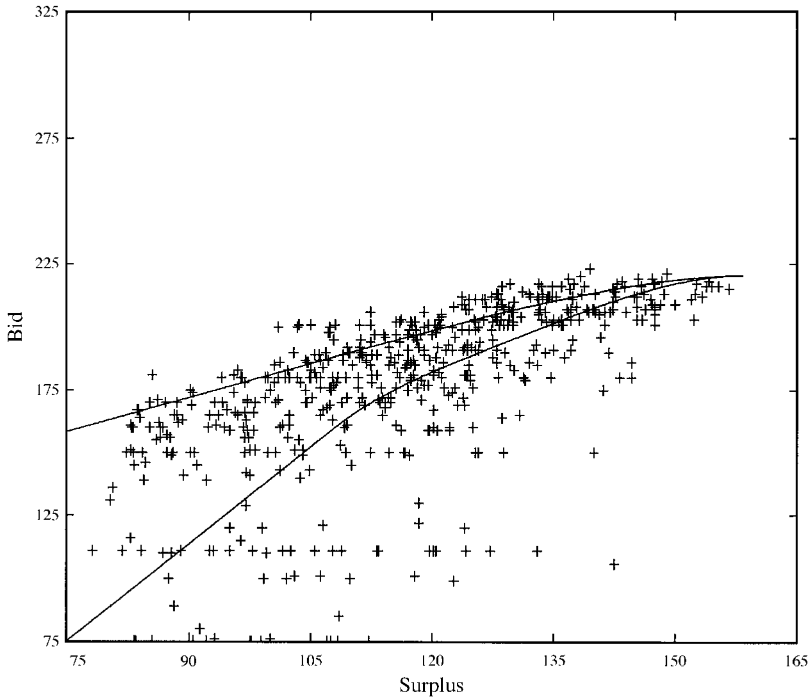

FIGURE 2B. BIDS (+) IN PART 1 OF TREATMENT HIGH-6+ TOGETHER WITH NASH BIDS (LOWER LINE) AND NAIVE BIDS (TOP LINE)

considerable cost. In high-6+, bids are now cautious enough to result in a small profit, but in high-3+, bids are even somewhat higher than in the inexperienced treatment high-3.[19] To conclude, even though the average formulation for the common value naturally stimulates bidders to think about others' signals, the data clearly show evidence of a winner's curse.[20]

A. *Efficiency Levels*

The efficiency levels realized in the experiment are determined as follows. Let t_{winner} denote the private value of the winner and let $t_{\min}(t_{\max})$ be the minimal (maximal) private value in the group. Then:

$$(6) \quad realized\ efficiency = \frac{t_{winner} - t_{\min}}{t_{\max} - t_{\min}} \times 100 \text{ percent.}$$

The efficiency level predicted by a benchmark is obtained by replacing t_{winner} with the private

[19] One possible explanation for why bids fall in treatment high-6 while they rise in treatment high-3 is that in high-3 losers experience regret more frequently when they learn their value is higher than the winning bid (46.3 percent in high-3 versus 36.4 percent in high-6). Furthermore, when they experience regret, it is stronger: in high-3 an average of 39 points is left on the table versus 21 points in high-6. Thus, regret may have caused an upward pressure on bids in high-3.

[20] It is often argued that economic institutions correct individual biases. In market settings, for instance, "biased" traders can learn from "unbiased" traders via signals provided by market prices. There is some experimental evidence that markets may alleviate the effects of judgmental biases (e.g., Colin F. Camerer, 1987; Ananda R. Ganguly et al., 2000). Interestingly, the selection process in auctions may aggravate individual biases, since the bidder with the strongest curse tends to win the auction. The data confirm this intuition. A logistic regression with the probability to win the auction as the dependent variable and "surplus" and "curse" (= actual bid − Nash bid) as independent variables, shows that the estimated parameter for surplus is 0.10 (s.d. 0.003), the estimated parameter for curse is 0.06 (s.d. 0.003) and the estimated parameter for the constant is −14.45 (s.d. 0.443). Hence, subjects with a stronger curse have a higher probability of determining the price for the commodity. This selection force may weaken in the long run, however, as bidders with more severe curses go bankrupt.

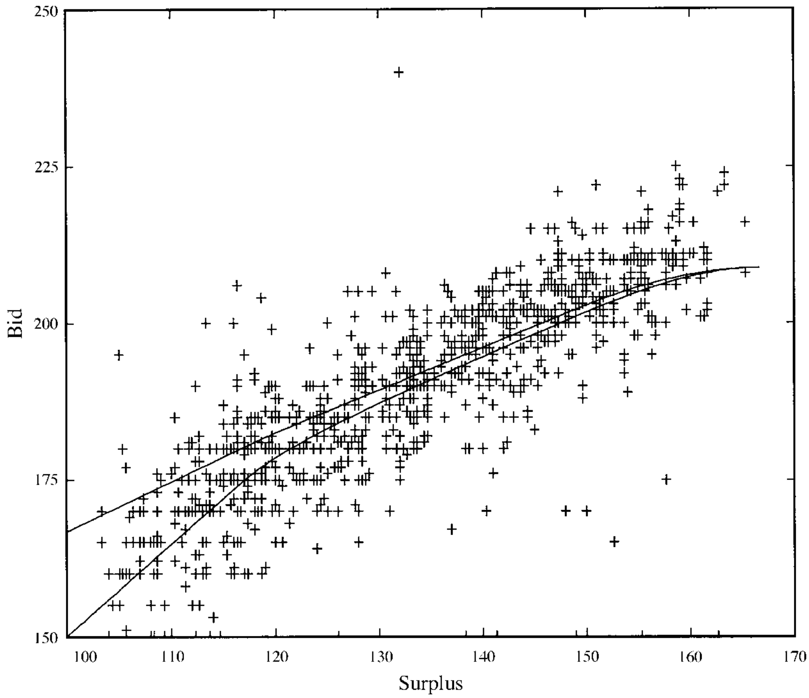

FIGURE 3A. BIDS (+) IN PART 1 OF TREATMENT LOW-3 TOGETHER WITH NASH BIDS (LOWER LINE) AND NAIVE BIDS (TOP LINE)

value of the bidder predicted to win. Since the Nash and Naive model select the same winner the predicted efficiencies are identical.

Table 3 shows the realized efficiency levels by blocks of ten periods. Note that actual efficiency levels of the experienced treatments are roughly constant and in the same range as those observed in the last 20 periods of the inexperienced treatments. The low efficiency levels in the initial ten periods of the inexperienced treatments could be due to "noisy" bidding behavior, which causes the ranking of bids to differ from the ranking of surpluses. An alternative explanation is that bidders initially place too much weight on their common-value signal. In the remainder of this section, we use the individual bid data to discriminate between these two explanations.

The Nash and Naive benchmarks make point predictions. To evaluate these models an assumption has to be made about how players err. We invoke a common assumption: for each of the benchmarks a random error term is added to the predicted bid. The error terms are drawn from a truncated normal distribution with mean 0 and variance σ^2, and are identically and independently distributed across subjects and periods.[21] This method of transforming deterministic models into stochastic models may be criticized on theoretical grounds but there is no a priori reason why one model is favored over another. So this procedure seems adequate to compare the "goodness-of-fit" of different benchmark models.

Table 4 reports the estimation results for the first ten periods and the final 20 periods separately. The top panel pertains to the Nash and Naive benchmarks. There is no obvious ranking of the two models: the likelihoods of the Nash benchmark are higher in treatments low-3 and low-3+, but the reverse is true in, for instance, high-3+ and high-3++. Glancing at Figures 1–3 it seems plausible that there is some heterogeneity among subjects, with some

[21] The distribution is truncated to ensure that bids stay between the lower and upper limit on bids.

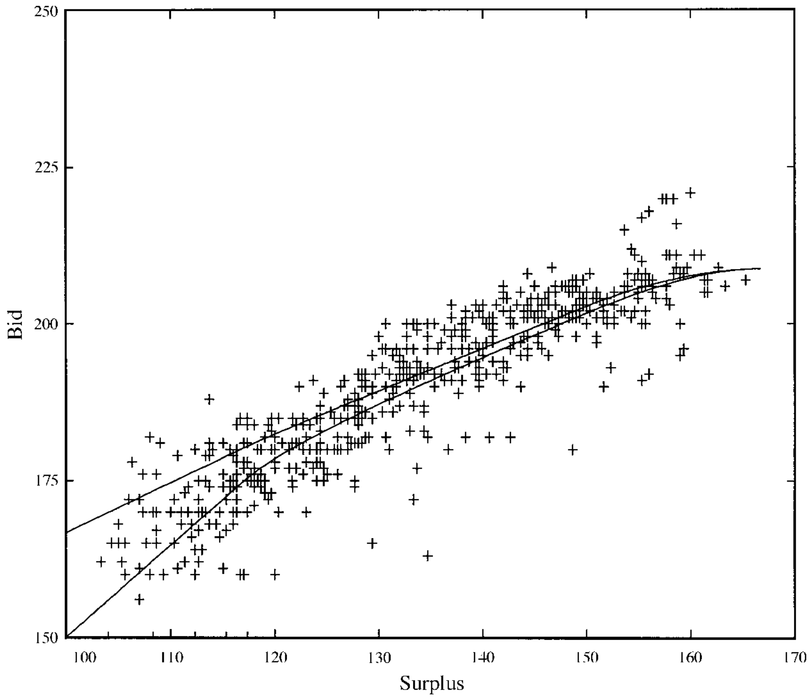

FIGURE 3B. BIDS (+) IN PART 1 OF TREATMENT LOW-3+ TOGETHER WITH NASH BIDS (LOWER LINE) AND NAIVE BIDS (TOP LINE)

bidders suffering from the winner's curse while others do not. This is tested in the Nash-Naive combined model, which allows subjects to bid according to either the Nash or Naive benchmark.[22] This combined model yields a much higher likelihood than either of the two individual models.

Finally, the $B^{\alpha} - B^{\alpha}_{curse}$ model generalizes the Nash-Naive model by allowing bidders to give their common value signals weight α, which is estimated from the data. (Recall that the Nash-Naive model assumes that $\alpha = 1/n$.) The bottom panel of Table 4 shows that the inclusion of the weight α results in a small, albeit significant, increase in likelihood in the high-3 treatments, but adds nothing in the other treatments. These results suggest that, while a substantial fraction of the subjects fall prey to the winner's curse, subjects roughly weigh their common value information in the same manner as rational bidders. The main reason for the lower realized efficiencies in the first ten periods of the inexperienced sessions is that behavior is initially more noisy as indicated by the higher σ-estimates.[23]

[22] To be precise, the unconditional likelihood $L(x_{i,1}, \ldots, x_{i,10})$ of player i's choices $x_{i,t}$ in periods 1–10 is:

$$L(x_{i,1}, \ldots, x_{i,10}) = p \prod_{t=1}^{10} L(x_{i,t}|Nash) + (1-p) \prod_{t=1}^{10} L(x_{i,t}|Naive),$$

where $L(x_{i,t}|Nash)$ represents the conditional probability of $x_{i,t}$ predicted by the Nash model, $L(x_{i,t}|Naive)$ represents the conditional probability of $x_{i,t}$ given the Naive model, and p is the probability that a subject plays according to the Nash benchmark. The Nash and Naive benchmarks are nested as special cases (i.e., $p = 1$ or $p = 0$).

[23] We also estimated a model in which bidders make a "logit" best response to the empirical distribution of bids (with or without a winner's curse). This model resulted in a worse fit of the data (i.e., a 10–20 percent reduction in the log-likelihood per observation). Finally, we estimated a "discount" model in which bids are determined as a fraction

TABLE 2—WINNER'S CURSE STATISTICS

Statistic	Low-3	High-3	High-6	Low-3+	High-3+	High-6+	High-3++
Percentage of auctions with positive profits	75	62	48	81	61	52	64
Percentage of auctions won by highest surplus	79	65	51	76	73	64	72
Percentage of bids (winning bids) > E_{emp}[a]	19	45	57	12	54	38	37
	(14)	(21)	(49)	(8)	(33)	(30)	(24)
Percentage of bids (winning bids) > E_{Naive}[b]	7	6	17	3	7	9	3
	(10)	(10)	(33)	(2)	(9)	(9)	(4)
	Observed and predicted average bids and profits[c]						
Actual bids	189.6	172.6	182.4	189.5	179.5	176.7	175.6
Naive bids	191.5	186.7	194.3	191.5	187.0	194.6	187.0
	0.09	*0.01*	*0.01*	*0.12*	*0.05*	*0.11*	*0.02*
Nash bids	188.3	159.9	171.1	188.3	160.4	169.3	160.7
	0.28	*0.01*	*0.01*	*0.17*	*0.03*	*0.29*	*0.02*
Actual profits	7.27	11.88	−2.75	9.62	10.55	5.34	12.44
Naive profits	12.02	8.54	2.67	11.72	8.83	2.23	9.31
	0.01	*0.33*	*0.02*	*0.05*	*0.17*	*0.29*	*0.05*
Nash profits	13.34	21.77	10.01	13.05	21.96	9.83	22.45
	0.01	*0.01*	*0.01*	*0.03*	*0.03*	*0.11*	*0.02*

[a] E_{emp} is the object's expected value given the empirical distribution of bids and common-value signals.
[b] E_{Naive} is the expected value assuming a winner's curse: $E_{Naive} = v_i/n + t_i + (n - 1)/nE(v)$.
[c] The p values of a Wilcoxon rank test comparing predictions of the benchmark models with actual data are displayed in italics. Groups are the unit of observation. Test results are based on only three pairwise observations in high-6+.

B. *Comparative Statics Predictions*

In this section we consider the impact of changes in the bidding environment on efficiency, winner's profits, and revenues. Table 5 displays the effects of a reduction in the uncertainty about the common value. According to both benchmark models, efficiency should increase when uncertainty decreases, as shown in the second row. This prediction is borne out by the data. The realized efficiency level is significantly higher in treatment low-3 than in high-3, both for inexperienced and experienced subjects (first row).

The effect of increased uncertainty on winner's profits and revenues depends on the benchmark. Nash predicts that bids are lower because of the increased probability of a winner's curse and, as a result, profits are higher. In contrast, naive bidders neglect the fact that winning is informative and hence are insensitive to the increased risk of a winner's curse. In fact, they bid *higher* when there is more uncertainty, because the maximum surplus, $v/n + t$, is higher when the common value signals are drawn from $U[0,200]$ than when they are drawn from $U[75,125]$. The third row of Table 5 shows that actual profits are lower with less uncertainty, although less so than predicted by Nash (both for inexperienced and experienced bidders). Revenue results are the opposite. Nash predicts that revenues will decrease when the uncertainty about the common value increases, while the Naive benchmark predicts that revenues will increase.[24] Again, Nash correctly predicts the downward shift in observed revenues and the actual change is close to the predicted change for inexperienced bidders, but it is too small for experienced bidders.

of the (rational or naive) expected value of the object. This model yielded similar log-likelihoods as the ones in Table 4. We prefer the benchmarks of Section II, however, as they have a more sound theoretical foundation.

[24] Note that the predicted change in revenues is not equal to the change in profits because the total surplus is higher in low-3 than in high-3.

TABLE 3—OBSERVED EFFICIENCIES (IN PERCENTAGES) BY BLOCKS OF TEN PERIODS

	Inexperienced			Once experienced			Twice experienced		
Efficiency	1–10	11–20	21–30	1–10	11–20	21–30	1–10	11–20	21–30
	High-3			High-3+			High-3++		
Actual	54	68	68	73	62	72	71	69	71
Nash/naive	71	73	75	72	72	75	75	73	76
	0.01	*0.07*	*0.07*	*0.92*	*0.05*	*0.12*	*0.46*	*0.31*	*0.18*
	High-6			High-6+					
Actual	72	81	85	93	89	90			
Nash/naive	94	86	91	92	88	91			
	0.01	*0.05*	*0.07*	*1.00*	*1.00*	*1.00*			
	Low-3			Low-3+					
Actual	79	87	89	90	91	86			
Nash/naive	96	93	98	97	92	97			
	0.01	*0.11*	*0.01*	*0.03*	*0.92*	*0.04*			

Notes: The p value of a Wilcoxon rank test comparing a model's efficiency with realized efficiency is displayed in italics. Groups are the unit of observation.

Another important determinant for efficiency of market outcomes is the degree of competition. Since Nash and Naive bids depend on the surplus $v/n + t$, both predict that efficiency levels increase with the number of bidders, as shown in the second row of Table 6. The data accord with this prediction. For both inexperienced and experienced bidders there is a sharp increase in the actual efficiency level when the number of bidders is increased from three to six (first row).[25] The middle and bottom panel of Table 6 show that for both inexperienced and experienced bidders, revenues increase with the number of bidders while winner's profits fall.

Finally, consider the case where the seller publicly reveals an independent estimate of the common value. This improves bidders' estimates of the object's value, resulting in a more efficient allocation. According to the Nash benchmark, the disclosure of the seller's information also raises revenue. These predicted effects are stronger the higher the quality of the seller's information. However, one of the central findings in Kagel and Levin (1986) is that the positive effect of a public information signal on revenue is mitigated when bidders fall prey to the winner's curse. The intuition is that the seller's information helps naive bidders with high signals realize their signal is too optimistic.

Table 7 shows that, by and large, bidders change their bid in the direction predicted by Nash, both when the quality of the seller's signal is similar to that of the bidders ($\lambda = 1$) and when it is higher ($\lambda = 7$). The most common deviation is that actual bids do not increase when Nash predicts they should. Bidders do not seem to realize that higher bids are warranted since the extra information mitigates the winner's curse.

Table 8 shows that the public disclosure of a seller's signal when it is of the same quality as bidders' signals has no effects on efficiency, revenues, and profits (i.e., the effects are economically small, not systematic, and most often statistically insignificant). Note, however, that this lack of effect is consistent with the predictions of the different benchmark models.[26] In

[25] In contrast, if bidders would have weighed their private value and common value signal equally, say, an increase in group size from three to six would have resulted in a substantially smaller increase in efficiency (from 63 percent to 65 percent in high-6 and from 62 percent to 66 percent in high-6+).

[26] These results are also consistent with the results reported in Kagel and Levin (1986). They investigate the effect of a public signal in a pure common value auction and find that revenues increase less than predicted by Nash. They report that a public signal raises revenue when bidders do not fall prey to a winner's curse, while revenue decreases when they do. Note that in our treatment high-3+, where the winner's curse is most prevalent, the decrease in revenue is largest.

TABLE 4—MAXIMUM LIKELIHOOD RESULTS FOR PERIODS 1–10 AND PERIODS 11–30 (ITALICS)[a]

Statistic	Low-3 (n = 300, *n = 600*)	High-3 (n = 300, *n = 519*)	High-6 (n = 527, *n = 981*)	Low-3+ (n = 180, *n = 360*)	High-3+ (n = 180, *n = 360*)	High-6+ (n = 180, *n = 360*)	High-3++ (n = 210, *n = 420*)
				Nash ($\alpha = 1/n$)			
σ_{Nash}	11.6 *8.4*	29.7 *25.4*	33.6 *27.9*	6.2 *6.4*	29.4 *28.1*	29.5 *31.0*	27.6 *26.2*
$-\log L$	3.82 *3.53*	4.74 *4.60*	4.87 *4.71*	3.22 *3.26*	4.74 *4.70*	4.76 *4.79*	4.68 *4.64*
				Naive ($\alpha = 1/n$)			
σ_{Naive}	11.7 *8.8*	27.9 *21.9*	29.5 *26.2*	6.6 *7.0*	16.9 *18.5*	28.0 *34.3*	18.3 *23.2*
$-\log L$	3.86 *3.59*	4.75 *4.50*	4.80 *4.68*	3.31 *3.37*	4.24 *4.34*	4.75 *4.95*	4.33 *4.56*
				Nash-Naive Combined ($\alpha = 1/n$)			
σ_{Nash}	8.2 *6.0*	26.4 *21.4*	36.0 *27.6*	3.9 *4.9*	20.2 *27.3*	28.1 *30.6*	22.5 *23.8*
σ_{Naive}	15.5 *15.5*	20.0 *15.8*	13.9 *12.5*	7.5 *10.4*	13.9 *12.9*	7.5 *8.8*	10.5 *12.7*
p[b]	0.63 *0.83*	0.59 *0.45*	0.53 *0.44*	0.60 *0.77*	0.12 *0.22*	0.62 *0.72*	0.34 *0.37*
$-\log L$	3.77 *3.38*	4.60 *4.32*	4.57 *4.31*	3.10 *3.20*	4.08 *4.15*	4.30 *4.49*	4.08 *4.21*
				$B^{\alpha} - B^{\alpha}_{curse}$ Combined			
σ	8.1 *5.6*	27.4 *21.9*	36.6 *27.6*	3.8 *4.8*	20.0 *20.3*	26.8 *30.7*	23.5 *25.3*
σ_{curse}	14.8 *13.4*	20.0 *15.9*	14.0 *12.6*	7.5 *10.6*	12.9 *13.8*	6.5 *8.8*	11.5 *12.2*
p	0.61 *0.75*	0.52 *0.32*	0.51 *0.44*	0.59 *0.78*	0.11 *0.11*	0.67 *0.72*	0.24 *0.33*
α[c]	0.46 *0.40*	0.42 *0.47*	0.25 *0.17*	0.37 *0.28*	0.39 *0.43*	0.13 *0.17*	0.42 *0.42*
$-\log L$	3.76 *3.37*	4.60 *4.30*	4.56 *4.31*	3.09 *3.19*	4.06 *4.10*	4.28 *4.49*	4.06 *4.18*
				Random			
$-\log L$	4.62	5.53	5.53	4.62	5.53	5.53	5.53

[a] The negative of the mean log-likelihood per choice is displayed. The top number in each cell gives the estimate for periods 1–10, and the bottom number (in italics) gives the estimate for periods 11–30.
[b] The parameter p denotes the probability that a subject correctly incorporates the information conveyed in winning.
[c] The parameter α is the relative weight that bidders assign to their common value signals.

contrast, the predicted effects of a high-quality seller's signal (treatment high-3++) are substantial. There is a significant effect on realized efficiency, although smaller than expected. The effect on actual profits and revenues is rather small and not significant. The observed changes are roughly consistent with naive bidding, and sharply contrast the large drop in profits and large rise in revenues predicted by the Nash benchmark.

TABLE 5—EFFECTS OF UNCERTAINTY ABOUT THE COMMON VALUE

	Inexperienced			Experienced		
Measure	High-3	Low-3	p value	High-3+	Low-3+	p value
Actual efficiency (percent)	62	85	0.00	69	89	0.00
Efficiency predicted by Nash/Naive (percent)	73	96	0.00	73	95	0.00
Actual winner's profit	11.88	7.27	0.27	10.55	9.62	0.38
Winner's profit predicted by Nash	21.77	13.34	0.00	21.96	13.05	0.00
Winner's profit predicted by Naive	8.54	12.02	0.10	8.83	11.72	0.52
Actual revenues	194.1	202.4	0.03	198.2	200.8	0.20
Revenues predicted by Nash	187.0	198.7	0.00	187.2	198.5	0.00
Revenues predicted by Naive	200.2	200.1	0.94	200.4	199.9	0.69

Notes: The third (sixth) column reports p values for Mann-Whitney rank tests, which compares treatments high-3 and low-3 (high-3+ and low-3+). Groups are the unit of observation.

TABLE 6—EFFECTS OF INCREASED COMPETITION

	Inexperienced			Experienced		
Measure	High-3	High-6	p value	High-3+	High-6+	p value
Actual efficiency (percent)	62	79	0.00	69	91	0.02
Efficiency predicted by Nash/Naive (percent)	73	90	0.00	73	90	0.02
Actual winner's profit	11.88	−2.75	0.01	10.55	5.34	0.04
Winner's profit predicted by Nash	21.77	10.01	0.00	21.96	9.83	0.02
Winner's profit predicted by Naive	8.54	2.67	0.00	8.83	2.23	0.07
Actual revenues	194.1	211.8	0.00	198.2	208.3	0.02
Revenues predicted by Nash	187.0	202.8	0.00	187.2	203.8	0.02
Revenues predicted by Naive	200.2	210.1	0.00	200.4	211.4	0.02

Notes: The third (sixth) column reports p values for Mann-Whitney rank tests, which compares treatments high-3 and high-6 (high-3+ and high-6+). Groups are the unit of observation.

The data suggest the following explanation. Without the seller's information there is a substantial winner's curse in periods 21–30. The seller's signal helps subjects to form a better estimate of the common value, thereby alleviating the winner's curse. After the seller's signal has been revealed subjects bid on average somewhat higher (178.2 versus 171.9), but now the winner's curse has disappeared. In fact, they bid somewhat less than the Nash benchmark predicts (184.6).

V. Conclusions

The majority of the theoretical and empirical literature on auctions pertains to either private or common value auctions. A remarkable feature of these polar cases is that both yield fully efficient allocations (in a Nash equilibrium). Most real-world auctions, however, exhibit both private and common value elements and inefficiencies should be expected, even in a Nash equilibrium. This paper reports a series of first-price auction experiments in which bidders receive a private value signal and an independent common value signal. We investigate the extent of inefficiency that occurs with financially motivated bidders. In addition, we test several policies aimed at reducing inefficiencies.

As expected, a fraction of the bidders fall prey to the winner's curse and this curse is more severe when winning is more informative. While there is systematical overbidding in most treatments, bidders aggregate their private and common value information in roughly the same

TABLE 7—QUALITATIVE EFFECT OF AUCTIONEER'S SIGNAL ON BIDS

Signal quality	Nash bid	$b_1 > b_2$	$b_1 = b_2$	$b_1 < b_2$	Percentage
Seller's signal weight $\lambda = 1$ ($n = 1{,}570$)	$N_1 > N_2$	334	71	39	28.3
	$N_1 = N_2$	38	11	19	4.3
	$N_1 < N_2$	133	196	729	67.4
	Percentage	32.3	17.7	50.1	100
Seller's signal weight $\lambda = 7$ ($n = 210$)	$N_1 > N_2$	58	0	5	30.0
	$N_1 = N_2$	1	0	0	0.5
	$N_1 < N_2$	22	8	116	69.5
	Percentage	38.6	3.8	57.6	100

Notes: N_1 (N_2) denotes the Nash bid without (with) seller's signal and b_1 (b_2) is the actual bid without (with) seller's signal. When $\lambda = 1$, $\chi^2_{Pearson} = 637.83$ ($p = 0.00$), and when $\lambda = 7$, $\chi^2_{Pearson} = 111.77$ ($p = 0.00$).

TABLE 8—EFFECTS OF THE PUBLIC DISCLOSURE OF THE AUCTIONEER'S SIGNAL

	Efficiency (percent)		Winner's profit			Revenue		
Treatment	Actual	Nash/Naive	Actual	Nash	Naive	Actual	Nash	Naive
Low-3								
Without	89	98	6.58	11.82	10.50	201.7	197.7	199.0
With	90	98	6.65	11.90	10.84	202.0	198.2	199.2
	0.62	*0.18*	*0.76*	*0.58*	*0.12*	*0.51*	*0.58*	*0.88*
Low-3+								
Without	86	97	6.92	11.55	10.20	200.8	197.6	199.0
With	90	98	8.00	6.80	10.58	200.8	198.3	199.4
	0.27	*0.32*	*0.17*	*0.59*	*0.14*	*0.83*	*0.35*	*0.60*
High-3								
Without	68	75	15.71	20.36	8.44	192.8	189.5	201.4
With	66	82	11.33	18.07	9.90	194.6	191.3	199.5
	0.67	*0.04*	*0.21*	*0.09*	*0.09*	*0.58*	*0.40*	*0.16*
High-3+								
Without	72	75	10.12	20.60	7.82	198.2	187.8	200.6
With	76	82	12.42	18.25	9.58	194.7	190.8	199.5
	0.35	*0.07*	*0.25*	*0.12*	*0.12*	*0.06*	*0.17*	*0.60*
High-6								
Without	85	91	0.01	9.18	0.84	207.7	200.0	208.3
With	82	93	1.13	8.46	2.06	207.9	204.1	210.5
	0.21	*0.11*	*0.95*	*0.14*	*0.01*	*0.21*	*0.01*	*0.01*
High-6+								
Without	90	91	3.60	7.67	−0.70	205.3	202.1	210.5
With	90	93	6.13	7.13	0.70	205.3	205.6	212.1
	1.00	*0.32*	*0.11*	*0.59*	*0.11*	*1.00*	*0.11*	*0.11*
High-3++								
Without	73	76	11.89	21.76	8.59	196.2	187.1	200.3
With	82	94	13.14	14.01	12.30	194.4	196.2	197.9
	0.09	*0.02*	*0.31*	*0.02*	*0.20*	*0.50*	*0.06*	*0.87*

Notes: Each third row reports p values for a Wilcoxon rank tests comparing the entry with and without an auctioneer's signal. Groups are the unit of observation.

manner as rational bidders would. As a result, realized efficiencies are of the same magnitude as predicted by Nash. Large differences occur only in the first ten periods of the inexperienced sessions, and seem mostly due to initially more erratic behavior.

When the bidding environment is changed, the Nash benchmark often correctly predicts the direction of adjustments in bids but overstates their magnitudes. The residual discrepancy between observed and predicted changes is best explained by assuming that a fraction of the subjects bid according to the Naive model. For example, an increase in uncertainty about the common value leads to a substantial decrease in efficiency, accompanied by a slight increase in winner's profits and a slight decrease in the seller's revenues. These results are in line with Nash predictions, but the effects on profits and revenues are smaller than predicted because the increase in uncertainty aggravates the winner's curse. The public release of high-quality information about the common value positively affects efficiency, although again less so than predicted by Nash.

Finally, our results indicate that more competition is a robust way to enhance efficiency, reduce winner's profits, and raise seller's revenues. The reasons for these positive effects are partly "statistical": with more bidders, the winner will on average have better information (i.e., higher signals). More importantly, however, an increase in competition induces bidders to weigh their own common value signal significantly less, which makes their private value information more important and an efficient outcome more likely.

Appendix: Derivation of the Nash Equilibrium Bids for Treatment High-3

Recall from Section II that the Nash equilibrium bids are given by

$$\text{(A1)} \quad B(x) = E(V + t_1 | s_1 = x,\ Y_1 = x) - E(Y_1 - y_1 | s_1 = x,\ Y_1 = x),$$

where the surplus variable is defined as $s = v/n + t$, and Y_1 is the maximum of the n surplus draws. For treatment high-3, the surplus variable is the sum of two uniformly distributed random variables: $t \sim U[75, 125]$ and $v \sim U[0, 200]$. It is useful to decompose the support of s into three regions: $R_I = [75, 125] \cup R_{II} = [125, 75 + 200/3] \cup R_{III} = [75 + 200/3,\ 125 + 200/3]$. The density of the surplus variable can then be worked out as: $f_I(s) = 3(s - 75)/10{,}000$, $f_{II}(s) = 3/200$, and $f_{III}(s) = 3(575/3 - s)/10{,}000$, i.e., the density has the shape of a "trapezoid."

An alternative way to write (A1) is

$$\text{(A2)} \quad B(x) = \frac{n-1}{n} E(v | s \le x) + E(y_1 | y_1 \le x).$$

The first term on the right side of (A2) can be written as

$$\text{(A3)} \quad E(v | s \le x) = \int_{75}^{x} E(v | s = y) f_s(y)\, dy$$

with f_s the density of the surplus variable. The second term on the right side of (A2) equals

$$\text{(A4)} \quad E(y_1 | y_1 \le x) = \int_{75}^{x} y\, dF^{n-1}(y_1 | y_1 \le x) = x - \int_{75}^{x} \frac{F_s^{n-1}(y)}{F_s^{n-1}(x)}\, dy$$

with F_s the cumulative distribution corresponding to f_s. The bidding functions on each of the three regions can now be computed from the conditional expectations $E_I(v | s = y) = 3(y - 75)/2$, $E_{II}(v | s = y) = 3(y - 100)$, $E_{III}(v | s = y) = 3(y - 175/3)/2$, and the expressions for the density, f_s, given above. The explicit formulas are:

$$\text{(A5)} \quad B_I(x) = \frac{22}{15} x - 35$$

for $75 \le x \le 125$

$$\text{(A6)} \quad B_{II}(x) = \frac{5(x^3 - 240x^2 + 18{,}125x - 413{,}125)}{3(x - 100)^2}$$

for $125 \le x \le 425/3$, and

(A7) $$B_{III}(x) = \frac{5{,}346x^5 - 3{,}938{,}625x^4 + 1{,}007{,}100{,}000x^3 - 101{,}535{,}468{,}750x^2 + 2{,}786{,}683{,}593{,}750x + 71{,}985{,}771{,}484{,}375}{45(9x^2 - 3{,}450x + 270{,}625)^2},$$

for $425/3 \leq x \leq 575/3$. The optimal bids in (A5)–(A7) are shown as the lower lines in Figures 1A and 1B.

REFERENCES

Abbink, Klaus and Sadrieh, Abdolkarim. "Rat-Image-Research Assistance Toolbox for Computer-Aided Human Behavior Experiments." Working paper, University of Bonn, 1995.

Albers, Wulf and Harstad, Ronald M. "A Framing Effect Observed in a Market Game," in Reinhard Selten, ed., *Game equilibrium models II*. Berlin: Springer-Verlag, 1991, pp. 308–36.

Ashenfelter, Orley and Genesove, David. "Testing for Price Anomalies in Real-Estate Auctions." *American Economic Review*, May 1992 (*Papers and Proceedings*), *82*(2), pp. 501–5.

Avery, Christopher and Kagel, John H. "Second-Price Auctions with Asymmetric Payoffs: An Experimental Investigation." *Journal of Economics and Management Strategy*, Fall 1997, *6*(3), pp. 573–603.

Bazerman, Max H. and Samuelson, William F. "I Won the Auction But Don't Want the Prize." *Journal of Conflict Resolution*, December 1983, *27*(4), pp. 618–34.

Bikhchandani, Sushil and Riley, John G. "Equilibria in Open Common Value Auctions." *Journal of Economic Theory*, February 1991, *53*(1), pp. 101–30.

Bulow, Jeremy I.; Huang, Ming and Klemperer, Paul D. "Toeholds and Takeovers." *Journal of Political Economy*, June 1999, *107*(3), pp. 427–54.

Camerer, Colin F. "Do Biases in Probability Judgment Matter in Markets? Experimental Evidence." *American Economic Review*, December 1987, *77*(5), pp. 981–97.

Capen, E. C.; Clapp, R. V. and Campbell, W. M. "Competitive Bidding in High-Risk Situations." *Journal of Petroleum Technology*, June 1971, *23*(6), pp. 641–53.

Cox, James C.; Dinkin, Samuel H. and Swarthout, James T. "Endogenous Entry and Exit in Common Value Auctions." *Experimental Economics*, October 2001, *4*(2), pp. 163–81.

Dasgupta, Partha and Maskin, Eric S. "Efficient Auctions." *Quarterly Journal of Economics*, May 2000, *115*(2), pp. 341–88.

Dyer, Douglas; Kagel, John H. and Levin, Dan. "A Comparison of Naive and Experienced Bidders in Common Value Offer Auctions: A Laboratory Analysis." *Economic Journal*, March 1989, *99*(394), pp. 108–15.

Forsythe, Robert; Isaac, R. Mark and Palfrey, Thomas R. "Theories and Tests of 'Blind Bidding' in Sealed-Bid Auctions." *RAND Journal of Economics*, Summer 1989, *20*(2), pp. 214–38.

Ganguly, Ananda R.; Kagel, John H. and Moser, Donald V. "Do Asset Market Prices Reflect Traders' Judgment Biases?" *Journal of Risk and Uncertainty*, May 2000, *20*(3), pp. 219–45.

Garvin, Susan and Kagel, John H. "Learning in Common Value Auctions: Some Initial Observations." *Journal of Economic Behavior and Organization*, December 1994, *25*(3), pp. 351–72.

Goeree, Jacob K. and Offerman, Theo. "Winner's Curse without Overbidding." Working paper, University of Virginia, 2001.

______. "Competitive Bidding in Auctions with Private and Common Values." *Economic Journal*, 2002 (forthcoming).

Holt, Charles A. and Sherman, Roger. "Risk Aversion and the Winner's Curse." Working paper, University of Virginia, 2000.

Jehiel, Philippe and Moldovanu, Benny. "Efficient Design with Interdependent Valuations." *Econometrica*, September 2001, *69*(5), pp. 1237–59.

Kagel, John H.; Harstad, Ronald M. and Levin, Dan. "Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study." *Econometrica*, November 1987, *55*(6), pp. 1275–304.

Kagel, John H. and Levin, Dan. "The Winner's

Curse and Public Information in Common Value Auctions." *American Economic Review*, December 1986, *76*(5), pp. 894–920.

______. "Common Value Auctions with Insider Information." *Econometrica*, September 1999, *67*(5), pp. 1219–38.

Kagel, John H.; Levin, Dan; Battalio, Raymond C. and Meyer, Donald J. "First-Price Common Value Auctions: Bidder Behavior and the 'Winner's Curse'." *Economic Inquiry*, April 1989, *27*(2), pp. 241–58.

Kirchkamp, Oliver and Moldovanu, Benny. "An Experimental Analysis of Auctions with Interdependent Valuations." Working paper, University of Mannheim, 2000.

Klemperer, Paul. "Auctions with Almost Common Values: The 'Wallet Game' and Its Applications." *European Economic Review*, May 1998, *42*(3–5), pp. 757–69.

Krishna, Vijay and Morgan, John. "(Anti-) Competitive Effects of Joint Bidding and Bidder Restrictions." Working paper, Princeton University, 1997.

Levin, Dan; Kagel, John H. and Richard, Jean Francois. "Revenue Effects and Information Processing in English Common Value Auctions." *American Economic Review*, June 1996, *86*(3), pp. 442–60.

Lind, Barry and Plott, Charles R. "The Winner's Curse: Experiments with Buyers and with Sellers." *American Economic Review*, March 1991, *81*(1), pp. 335–46.

Maskin, Eric S. "Auctions and Privatization," in Horst Siebert, ed., *Privatization.* Institut fur Weltwirtschaften der Universitat Kiel, Tubingen: Mohr, 1992, pp. 115–36.

Milgrom, Paul R. and Weber, Robert J. "A Theory of Auctions and Competitive Bidding." *Econometrica*, September 1982, *50*(5), pp. 1089–1112.

Pesendorfer, Wolfgang and Swinkels, Jeroen M. "Efficiency and Information Aggregation in Auctions." *American Economic Review*, June 2000, *90*(3), pp. 499–525.

Roll, Richard. "The Hubris Hypothesis of Corporate Takeovers." *Journal of Business*, April 1986, *59*(2), pp. 197–216.

Selten, Reinhard and Buchta, Joachim. "Experimental Sealed Bid First-Price Auctions with Directly Observed Bid Functions," in D. Budescu, I. Erev, and R. Zwick, eds., *Games and human behavior: Essays in honor of Amnon Rapoport.* Mahwah, NJ: Lawrence Erlbaum Associates, 1998, pp. 105–42.

Thaler, Richard H. "Anomalies: The Winner's Curse." *Journal of Economic Perspectives*, Winter 1988, *2*(1), pp. 191–202.

Wilson, Robert. "A Bidding Model of Perfect Competition." *Review of Economic Studies*, October 1977, *44*(3), pp. 511–18.