Balanced Growth and Empirical Proxies of the Consumption-Wealth Ratio[1]

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Abstract

Empirical proxies of the aggregate consumption-wealth ratio in terms of a cointegrating relationship between consumption (c), asset wealth (a) and labour income (y), commonly referred to as *cay*-residuals, play an important role in recent empirical research in macroeconomics and finance. This paper shows that the balanced-growth assumption made in deriving *cay* implies a second cointegrating relationship between the three variables; the three great ratios $c - a$, $c - y$ and $a - y$ should all be individually stationary. In U.S. data I find evidence for this second cointegrating relationship once I control for deterministic trends and a structural break. The fact that *cay* is a linear combination of two stationary great ratios has a number of important implications. First, without additional identifying restrictions, the residual from a cointegrating regression can no longer be interpreted as an approximation of the aggregate consumption-wealth ratio. I discuss an identifying assumption that may still allow to do so. Secondly, predictive regressions of asset prices on a combination of two stationary great ratios, must do at least as well as regressions on *cay* alone. Still, *cay* proves remarkably robust as an indicator of aggregate asset price cycles. The findings here also inform a recent debate about the role of look-ahead bias in *cay*: in order to identify transitory components in asset prices, households do not need to identify the parameters of the *cay*-relation.

KEYWORDS: CONSUMPTION-WEALTH RATIO, BALANCED GROWTH, COINTEGRATION, ASSET RETURN PREDICTABILITY.

JEL-CLASSIFICATION: E21, G12

1 Introduction

In two very influential recent papers, Lettau and Ludvigson (2001, 2004) have suggested an empirical characterization of the consumption-wealth ratio in terms of a cointegrating relationship between consumption, asset wealth and labour income known as the *cay*-residual. This residual has proven extremely successful as a predictor of asset prices and plays a central role in recent empirical research in empirical finance and macroeconomics.

It has generally been argued that the *cay*-relationship can be derived from a minimum of theoretical restrictions via a log-linearization of the average household's intertemporal budget constraint. However, one key assumption made in this derivation is that the economy follows a balanced growth path so that the aggregate portfolio shares of physical and human capital are constant in the long-run. In this paper, I show that this implies that the consumption-asset (ca), the consumption income (cy)and the asset income (ay) ratios should all be individually stationary. Hence, there should be two linearly independent long-run (cointegrating) relationships between the three variables – consumption, income and asset wealth should share a single stochastic trend. This important empirical implication of the Lettau-Ludvigson framework has so far not been spelled out in the literature. This paper explores its consequences for the construction of empirical proxies of the consumption-wealth ratio and for the role of *cay* as a predictor of asset prices.

I re-examine the U.S. data set used by Lettau and Ludvigson (2004) in search of evidence for this second cointegrating relationship. In my analysis, I recognize that in all economic models, the long-run means of great ratios are functions of the deep parameters . To the extent that these parameters change gradually or shift abruptly, one should expect to see trends or sudden breaks in the great ratios (see e.g. Attfield and Temple (2006)). I therefore allow for deterministic trend terms in cy and ay and I account for the possibility of a structural break as suggested by the results in some recent papers on the *cay*-relation. This structural break occurs at around the mid-point of the sample, i.e. around 1978, and it not only affects the trend growth rates of consumption and labour income but also the measurement of the long-run relationship between these variables. Once the break is explicitly modelled, extant tests indicate the two cointegrating relationships predicted by the theoretical framework.

One the one hand, these results constitute an important empirical corroboration of the theoretical approach underlying the two Lettau-Ludvigson studies – if the second cointegrating relationship can indeed be found in the data, then consumption, asset wealth and income follow a single stochastic trend, as is implied by the balanced-growth assumption. First, this is a necessary condition for the construction of an empirical proxy of the consumption wealth ratio. Secondly, it also substantially facilitates the interpretation of the joint long-run dynamics of the three variables in the light of standard macroeconomic theory.

One the other hand, the potential presence of a second cointegrating relationship also complicates the interpretation of estimated *cay* residuals along at least two dimensions: if the cointegrating space is two-dimensional, then the *cay*-proxy of the consumption-wealth ratio will not be econometrically identified. Without further identifying restrictions, the researcher will therefore not be able to estimate the share of physical assets and human capital in total wealth. I argue that such an identifying restriction could be to assume that economic agents chose a minimum variance portfolio of physical assets and human capital. Using this assumption, I construct a minimum-variance portfolio and show that – at least in the U.S. data set used here – *cay* proxies the implied consumption-wealth ratio remarkably well. Still, at a general level, the results here should caution against the naively associating *cay* residuals with the consumption-wealth ratio: this association is possible only under rather strong theoretical assumptions. While I suggest an informal way of gauging the plausibility of these assumptions in a given data set, my results suggest that the *cay* approach rests on much stronger *a priori* theoretical presumptions than has commonly been believed.

Secondly, the balanced-growth assumption could potentially also have implications for the predictability of stock returns and for the size of transitory components in asset prices. Under the balanced growth assumption, *cay* is just a particular linear combination in a two-dimensional cointegrating space so that any combination of two of the three great ratios *ca*, *cy* and *ay* must do at least as well as *cay* in predicting stock returns. Interestingly enough, however, my results suggests that alternative linear combinations of *cy* and *ay* do not significantly outperform *cay* in predicting stock markets. Hence, even though the second cointegrating relationship would seem to draw into question the validity of *cay* as a proxy of the consumption-wealth ratio and as predictor of asset prices, the results here suggest that *cay* – though generally estimated from an econometric setup that is misspecified under the maintained null – proves remarkably robust in both respects.

Finally, the considerations here also have immediate implications for the recent debate about look-ahead-bias in the *cay*-residual. If *cay* is just a linear combination of two directly observable stationary great ratios, say *cy* and *ay*, then in order to identify transitory components in asset prices, a forecaster does not have to estimate the parameters of the *cay*-relation first. She can use *cy* and *ay* directly and does therefore not need long spans of data to identify the parameters of the *cay*-relationship.

2 The issue

2.1 The framework

The framework used by Lettau and Ludvigson (2001, 2004)[1] builds on Campbell and Mankiw (1989) and assumes that the aggregate consumption-wealth ratio is a stationary variable. Starting from the aggregate budget constraint

$$W_{t+1} = (1 + r_{t+1})(W_t - C_t) \tag{1}$$

where W_t is aggregate wealth, r_t its net return and C_t consumption, Campbell and Mankiw use this assumption to log-linearize the intertemporal budget constraint around the long-run mean of $c - w = \ln(C_t/W_t)$, so that

$$c_t - w_t = \mathbf{E}_t \sum_{j=1}^{\infty} \rho^j \left(r_{t+j} - \Delta c_{t+j}\right) \tag{2}$$

where ρ is a constant smaller than one. According to (2), the consumption wealth ratio predicts returns to wealth or future declines in consumption. The assumption made in deriving (2) as well as the fact that consumption growth and returns can usually be characterized as non-integrated ($I(0)$) variables suggests that c and w should cointegrate. Aggregate wealth is, however, not directly observable, since it is composed of both physical (asset) wealth as well as human capital:

$$W_t = A_t + H_t \tag{3}$$

where A_t is asset wealth and H_t is human capital. To proxy for W_t in terms of observable variables, L&L as well as virtually the entire literature inspired by their papers assume that the shares of physical (asset) wealth and the share of human wealth in total wealth are if not constant so to the least stationary, so that (3) can be log-linearized to obtain

$$w_t = \gamma a_t + (1 - \gamma) h_t$$

where $\gamma = \mathbf{E}(\exp(a_t - w_t))$ is the long-run mean of the share of asset wealth in total wealth. In a second step, L&L then assume that the stochastic trend in human capital can be captured by its dividend – labour income. Denoting log-labour income with y_t and assuming that $z_t = y_t - h_t$ is $I(0)$, we get

$$w_t = \gamma a_t + (1 - \gamma) y_t + (1 - \gamma) z_t$$

Plugging into (2), one obtains

$$c - \gamma a_t - (1 - \gamma) y_t = \mathbf{E}_t \sum_{j=1}^{\infty} \rho^j \left[r_{t+j} - \Delta c_{t+j}\right] + (1 - \gamma)\, z_t. \tag{4}$$

[1] Fore brevity, we refer to these papers as “L&L“.

Since the RHS of this equation just differs from the RHS of (2) only by the $I(0)$ process z_t, $c - \gamma a_t - (1-\gamma)y_t$ should be $I(0)$ as well. Hence, the logs of consumption, asset wealth and labour income should cointegrate with cointegrating vector $[1, -\gamma, -(1-\gamma)]'$. This is the *cay*-relationship that is the focus of L&L. To the extent that z_t – the transitory part of labour income – is small, *cay* should therefore capture the variation in the consumption wealth ratio $c - w$. Lettau and Ludvigson show that this is indeed the case: in U.S. data, labour income is hardly predictable. Since c also behaves almost like a random walk, *cay* mainly predicts changes in asset wealth.

2.2 A second cointegrating relationship

One key assumption made in obtaining the prediction that the *cay*-residual is a cointegrating relationship and that it proxies for the aggregate consumption wealth ratio is that the shares of physical and financial assets and human capital in total wealth , γ and $(1-\gamma)$, are constant in the long run. This is tantamount to saying that A_t/W_t is stationary and hence the logarithm of A/W, given by $a_t - w_t$, must also be a non-integrated ($I(0)$) process with a finite and constant unconditional mean. The same must be true for $h - w$ and – since $z_t = y_t - h_t$ is assumed stationary as well – also for $y - w$. Hence, all three variables c, a and y will cointegrate pairwise with w_t :

$$c_t - w_t \sim I(0) \tag{5a}$$
$$a_t - w_t \sim I(0) \tag{5b}$$
$$y_t - w_t \sim I(0) \tag{5c}$$

The first is just a restatement of (2) above. The second and third follow from the fact that the portfolio shares of human and physical capital in total wealth are assumed constant in the long-run. Clearly, any linear combination of $c - w$, $a - w$ and $y - w$ must therefore also be stationary. Since w_t is unobservable, it its sufficient to concentrate on those three linear combinations of (5) that eliminate w_t:

$$c - a \sim I(0) \tag{6a}$$
$$c - y \sim I(0) \tag{6b}$$
$$a - y \sim I(0) \tag{6c}$$

Only two of these linear combinations are, however, linearly independent. The first and the second of these great ratios are the (log) consumption-asset and the consumption-income ratios to which, in analogy to L&L, we refer as *ca* and *cy* respectively. The third one is the asset-income ratio (*ay*). Clearly, any linear combination of ca, cy, ay will also constitute a valid representation of one of the two cointegrating relationships, so that in particular cay can be written as

$$cay = \gamma ca + (1-\gamma)cy = cy - \gamma ay = ca - (1-\gamma)ay$$

Hence, the assumptions made in obtaining the representation (4) of cay actually imply the presence of two cointegrating relationships between consumption, asset wealth and income. This point has so far been overlooked in the literature. It poses a big challenge to the validity of the entire framework presented in the previous subsection, since neither L&L nor any of the studies applying this framework to other countries and data sets have actually detected this second cointegrating relationship in the data. Therefore, the second cointegrating relationship raises two questions: first, why has the second cointegrating relationship been so elusive in the data? And secondly, how does its presence affect the interpretation of cay as a proxy of the consumption-wealth ratio and as a predictor of asset price changes?

3 Another look at the data

3.1 Stationarity of the great ratios?

The data set used here is the one used in Lettau's and Ludvigson's (2004) paper and ranges from 1952Q4 to 2003Q1.

Figure (1) presents the graphs of the three potentially stationary relations, $ca = c - a$, $cy = c - y$ and $ya = y - a = -ay$. I formally examine the stationarity of ca, cy and ay in table 1. Based on Johansen's (1991) test the null of no cointegration cannot be rejected in any of the three pairs of variables. Once one cointegrating relationship is imposed, however, the cointegrating vector estimated using Johansen's procedure come relatively close to their theoretical value $\left[\begin{array}{cc} 1 & -1 \end{array}\right]'$ and for the system consisting of c and y and the pair y,a we cannot reject the hypothesis that the cointegrating vector is actually $\left[\begin{array}{cc} 1 & -1 \end{array}\right]$.

Ocular inspection of figure (1) suggests that at least two of the three great ratios are individually trending; cy seems to trend more strongly in the first half of the sample period, whereas the downward drift in ca is more evenly spread.

There are at least two possible economic explanations for trends in the great ratios.

Rudd and Whelan (2006) have recently argued that the intertemporal budget constraint (1) applies to all funds spent on consumption and that, therefore, total consumption rather than non-durables consumption (as is the case in the Lettau-Ludvigson data set) should be used in the empirical implementation of the cay relationship. In a related paper, Palumbo, Rudd and Whelan (2006) also show that real non-durables consumption expenditure has been trending downwards as a share of total consumption. In principle this feature of the data could help explain the downward trend in the great ratios.[2]

Another explanation has recently been emphasized by Hahn and Lee (2006): the intertemporal budget constraint (1) does have to hold for each individual household, but if households are heterogeneous and if the structure of household heterogeneity drifts slowly over time, then this may induce a trend in the aggregate consumption-wealth ratio. Indeed, Hahn and Lee (2006) find that a deterministic trend cannot actually be excluded from the trivariate cointegrating relationship between c, a and y and they conclude that the coefficients of the *cay*-relationship as estimated by Lettau and Ludvigson are likely to be biased by the omission of this linear trend.

While it is beyond the scope of this paper to identify the economic forces that may induce such a trend in *cay*, it is important to recongnize that in each economic model, the long-run mean of the great ratios is ultimately a function of deep parameters (see the discussion in Attfield and Temple (2006)). If these parameters change gradually or abruptly it appears plausible that the great ratios of which *cay* is a linear combination could be individually trending and subject to breaks. Clearly, the omission of such trends from an econometric model may have severe consequences for the number of cointegrating restrictions that are picked up by extant tests.[3]

Table 2, panel I, reports regressions of the three great ratios on a deterministic trend and a constant. In all three cases this trend is found to be highly significant. But even the inclusion of deterministic trend does not generally allow me to reject the non-stationarity of the great ratios: as is apparent from the last lines of table (2), standard unit-root tests on the ensuing regression residual do not generally reject the null and if so, they are just marginally significant. Based on Johansen's test for cointegration, I cannot reject non-stationarity in a single one of the three cases.

What I wish to argue here is that the trend in the three great ratios may have been subject to at least one major break over the sample period and that this break may have contributed to our failure to detect the second cointegrating relationship in the data. In fact, the recent literature on the consumption-wealth ratio reports considerable evidence for a structural break in the *cay* relationship. Hahn and Lee (2001, 2006) show that the

[2] Interestingly, we find our cointegration results to be reported below to be quite independent of whether we use total or non-durables and services consumption. We still stick to the use of the original Lettau_Ludvigson data set, mainly because our findings turn out to have important consequences for the debate about look-ahead bias in *cay* which has mainly been conducted based on this data set.

[3] Note that the great ratios might have deterministic components that are not contained in *cay*. In this case, *cay* would define that linear combination of the great ratios that eliminates these deterministic terms. We discuss this possibility below.

coefficients of the *cay*-relation are unstable between the first and the second half of their sample period. Brennan and Xia (2005) report that the forecasting power of *cay* for stock markets is considerably weaker in the second half of the sample period. The ocular evidence from figure (1) supports the view that the first half and the second half of the sample period are different: in particular the drift in *cy* seems much steeper during the first half. Formal stability tests for the bivariate VECMs also support the notion that there is a structural break in the second half of the 1970s. I therefore allow the trend in the three great ratios to break in 1978Q1. Panel II of Table (2) reports regressions of *cy*, *ca* and *ya* on a deterministic trend and a trend break variable that takes the values $t - t_0$ for $t < t_0$ and $t = 0$ for $t > t_0$ where t_0 is the date of the break which – following Hahn and Lee – I locate in 1977Q4. As is apparent, both trend terms are found to be significant in all three great ratios. Now, I clearly reject the unit root when I run ADF-tests on the regression residuals and the Johansen tests are significant at least at the 90 percent level.[4] This suggests that all three great ratios can indeed be characterized as trend stationary, once a break in the deterministic trend is explicitly modelled. Figure (2, a-c) conveys an optical impression of the great ratios, once with only a deterministic trend , once with both the trend cum break removed. As is apparent, the inclusion of a trend break makes a major difference, in particular for *cy*: while *cy* would only cross its mean twice during the sample period if only a linear trend is removed, it looks much more clearly mean reverting if the trend break is explicitly modelled. A similar, though somewhat less pronounced pattern emerges for the other two great ratios. This is only additional informal evidence but it seems to speak very strongly; a merely trend stationary process would seem unlikely to display the pattern observed here.

Allowing for the presence of deterministic trends in the great ratios and in particular, for a break in these trends seems to go a long way in explaining why extant tests have failed to detect the second cointegrating relationship that is implied by the balanced growth assumption on which the entire L&L approach is based. As discussed here and elsewhere, such trends are not *a priori* incompatible with the intertemporal budget constraint. It should therefore now appear natural to impose the second cointegrating relationship in a trivariate characterization of the dynamics of the three variables in a vector error correction framework. I explore the implications of this approach next.

[4] Strictly speaking the critical values for the Johansen test are not valid if a trend break is included in the cointegrating space. In the trivariate system studied below, I also report tests based on simulated critical vaues, with very similar conclusions.

3.2 A trivariate VECM specification

The deterministic terms in the great ratios imply a VECM-specification of the form

$$\mathbf{\Gamma}(\mathbf{L})\mathbf{\Delta x}_t = \boldsymbol{\alpha} \begin{bmatrix} \boldsymbol{\beta}' & \boldsymbol{\delta}_{trend} & \boldsymbol{\delta}_{break} \end{bmatrix}] \begin{bmatrix} \mathbf{x}_{t-1} \\ t \\ \min(0, t-t_0) \end{bmatrix} + \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 stepdum + \boldsymbol{\varepsilon}_t \quad (7)$$

where $\mathbf{x}_t = \begin{bmatrix} c_t & a_t & y_t \end{bmatrix}$, $\mathbf{\Gamma}(\mathbf{L})$ is a 3×3 matrix polynomial in the lag operator, $\boldsymbol{\beta}$ is the 3×2-matrix of cointegrating vectors and $\boldsymbol{\alpha}$ the 3×2 matrix of error correction adjustment loadings. The vectors δ_{trend} and δ_{break} give the coefficients on the two deterministic trend terms. The trend shift in the great ratios modelled by the deterministic terms must be caused by a shift in the trend growth rate of at least one of the three endogenous variables. Therefore, an additional step dummy has to be included in the short-run dynamics which is loaded with the vector of coefficients $\boldsymbol{\mu}_1$. Finally, $\boldsymbol{\mu}_0$ is a vector of intercept terms.

It is well known that the inclusion of trend breaks and similar deterministic terms invalidates the standard critical values used in cointegration testing. I therefore simulate critical values for the particular configuration of deterministic terms considered here using the program Disco available from Bent Nielsen's web page. Table (3), panel I, provides the cointegration tests. As is apparent, these strongly signal the presence of two cointegrating relationships, as implied by the theoretical framework. Once again, the importance of the trend break is highlighted by these tests. The table also reports the Johansen tests for the case when only a deterministic but non-breaking trend is included. In this case, very much as in the Lettau and Ludvigson-paper, I can only detect a single cointegrating relation. This suggests that the presence of structural breaks could indeed be responsible for the fact that extant tests fail to signal the presence of a second cointegrating relationship in the *cay* framework.

Panel II of Table (3) also gives the estimated cointegrating vectors. These, indeed, come close to the ones that we have estimated from the bivariate relationships. One is virtually the consumption income ratio, the other one can be interpreted as proxying the portfolio share, $a - y$. Though the point estimates of the coefficients on y seem to deviate somewhat from their theoretical value unity, I can accept the hypothesis that $\beta_y = 1$ at high probability levels. The table therefore also gives the cointegrating vectors estimated under this restriction. In this case, the coefficients on the two trend terms turn out to be virtually identical to the coefficients obtained from the regression of *cy* and $-ya$ respectively on the deterministic trends.[5]

[5]Note also that restricting the trend on the system without structural break to zero

This further supports the notion that it is the great ratios that form the stationary directions in the VECM, even though their stochastic stationarity may have been affected by gradual shifts and exogenous breaks.

As memorandum items for my discussion below, the second column of the same table also gives the results obtained from the model that only includes the non-breaking trend. In accordance with the above tests results and by way of comparison with earlier studies, this model is estimated with only one cointegrating relation. If the trend term is left unrestricted, the cointegrating vector aligns very well with the results reported in Hahn and Lee, I estimate $[1, -0.16, -0.58]$ and I find the deterministic trend coefficient highly significant. The lower part of the column also reports the cointegrating vector obtained when the trend coefficient is restricted to zero; this yields $\boldsymbol{\beta}' = \left[\begin{array}{ccc} 1 & -0.26 & -0.63 \end{array}\right]$.

I now proceed to estimating the dynamic adjustment parameters in the VECM by imposing detrended and break-adjusted great ratios cy and ay as the stationary directions in the system, so that

$$\begin{aligned} \left[\begin{array}{c} \widetilde{ca}_t \\ \widetilde{ay}_t \end{array}\right] &= \left[\begin{array}{ccc} \boldsymbol{\beta}' & \boldsymbol{\delta}_{trend} & \boldsymbol{\delta}_{break} \end{array}\right] \left[\begin{array}{c} \mathbf{x}_{t-1} \\ t \\ \min(0, t - t_0) \end{array}\right] \\ &= \left[\begin{array}{c} c - a - 0.00t - 0.002tb_t \\ a - y - 0.002t - 0.005tb_t \end{array}\right] \end{aligned} \tag{8}$$

where the tilde denotes the purely stochastic component of the respective great ratio and $tb_t = \min(0, t - t_0)$ is the trend break variable.

Table (4) presents the coefficient estimates of the VECM, $\boldsymbol{\Gamma}(\mathbf{L})$, and in particular of the two vectors of adjustment loadings $\boldsymbol{\alpha}$ in (7) above. A first key feature of the results is that the adjustment coefficients in the asset wealth equation is large and significant on both cointegrating relationships. This is in line with the findings reported in Lettau and Ludvigson who also find a big role for asset wealth in the error correction dynamics of their system. Note also that, quite in line with most economic theories, consumption does not seem to react significantly to past cointegration errors. But unlike in the L&L model, here I also find the coefficient on cy in the income equation to be highly significant.[6] Furthermore, it is also worth noting that the step dummy is highly significant in the labour income equation, reflecting the significance of the structural break.

I now turn to exploring the implications of the second cointegrating relationship for the dynamics of the three variables.

generates the cointegrating vector estimated by Lettau and Ludvigson, the very point highlighted by Hahn and Lee (2006).

[6]Even though our income concept comprises only labour income, this finding seems reminiscent of Cochrane's observation that the consumption-GNP ratio mainly predicts fluctuations.in aggregate income.

a) The size of transitory components: I start by examining how the size and variability of the transitory component of the three variables is affected. First, I identify permanent and transitory shocks in the VECM and conduct variance decompositions. Secondly, I also perform a decomposition of consumption, income and asset wealth into a trend and a cycle component.

I identify transitory shocks following the (equivalent) procedures outlined in Johansen (1995) and Gonzalo and Ng (2001). This identification starts from the insight that the error correction term in the VECM can be annihilated by premultipliyng (7) with the orthogonal complement of the matrix of adjustment loadings $\boldsymbol{\alpha}$, that I denote with $\boldsymbol{\alpha}'_{\perp}$. Hence, permanent shocks must be identified as

$$\boldsymbol{\pi} = \boldsymbol{\alpha}'_{\perp}\boldsymbol{\varepsilon}_t$$

Then, imposing that the vector of transitory shocks τ, be orthogonal to π, one can obtain the following representation of τ as

$$\boldsymbol{\tau} = \boldsymbol{\alpha}'\boldsymbol{\Omega}^{-1}\boldsymbol{\varepsilon}_t$$

where Ω is the variance-covariance matrix of the residuals $\boldsymbol{\varepsilon}_t$.

As shown e.g. in Becker and Hoffmann (2006), this identification is sufficient to conduct variance decompositions. Note that it will not be sufficient to uncover impulse responses to all three shocks. For example, in the model studied here, there is one permanent and two transitory shocks and any nonsingular transformation $\mathbf{S}\boldsymbol{\tau}$ of $\boldsymbol{\tau}$ will also qualify as a vector of transitory shocks that is orthogonal to $\boldsymbol{\pi}$. Hence, in the present model, impulse responses to the permanent shock are readily obtained – I return to this issue below – but identification of transitory shocks would require an additional identifying assumption.

Table (4) presents the contribution of permanent and transitory shocks to the variability of the three variables. The variance decompositions confirm Lettau's and Ludvigson's findings of a sizeable transitory component in asset wealth. It is also the case that consumption variability is largely driven by permanent shocks, even though at short horizons transitory shocks seem to play some role. But in contrast to Lettau and Ludvigson, my results here reveal a considerable role of transitory shocks for the dynamics of income.

I further investigate the size of the transitory components by conducting a permanent-transitory (P-T) decomposition of the cointegrated system along the lines of Gonzalo and Granger (1995) who suggest to decompose $\mathbf{x}_t$ as

$$\begin{aligned}\mathbf{x}_t &= \left[\boldsymbol{\beta}_{\perp}(\boldsymbol{\alpha}_{\perp}\boldsymbol{\beta}'_{\perp})^{-1}\boldsymbol{\alpha}_{\perp} + \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\right]\mathbf{x}_t \\ &= \mathbf{x}_t^P + \mathbf{x}_t^T\end{aligned} \tag{9}$$

where '$\perp$ ' again denotes the orthogonal complement of a matrix and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the adjustment loadings and cointegrating relations from (7) above.

Two points are worth noting from (9): in systems with more than one cointegrating relationship, the transitory component of the three variables will no longer be a scalar multiple of the cointegrating relationship but rather a particular linear combination of all the cointegrating relations in the system. This also implies that the transitory components of the three variables here are not necessarily perfectly correlated as they would be in Lettau's and Ludvigson's VECM.

Figure (3) plots the transitory components obtained from the above decomposition. As is clearly apparent, the presence of a second cointegrating relationship generates substantial transitory components not only in asset wealth but also in labour income and – to a lesser extent in consumption. Still, the transitory component in asset wealth appears to be the most volatile and most sizeable in terms of its average absolute deviation from the mean.

In fact, the transitory components of asset prices is almost unaffected by the modelling of the second cointegrating relationship and the structural breaks. For comparison, figure (4) plots the the Lettau-Ludvigson transitory component obtained from a model with one cointegrating relationship and no trends in the cointegrating space. It also reproduces the transitory component of asset prices from figure (3). The two transitory components virtually have the same size and their correlation almost reaches 0.9. Note that the L&L transitory component is nothing else than a negative multiple of *cay*. Hence, the interpretation of *cay* as a transitory component of asset wealth is not affected by the second cointegrating relationship nor by the drift terms or breaks! Only the size of transitory components in consumption and in particular in income is!

Another feature worth noting is that the transitory components of consumption and income in figure 3 are almost perfectly correlated. One way to interpret the close comovement between consumption and labour income is as evidence for the presence of credit market constraints or rule-of-thumb consumers along the lines of Campbell and Mankiw (1989). Regressing the unrestricted transitory component in consumption on that of income yields a coefficient of 0.38 and an R^2 of virtually unity. In the metric of Mankiw and Campbell this suggests that around 40 percent of labour income accrues to rule-of-thumb or credit-constrained consumers.

The distinction between rule-of-thumb consumers and forward looking consumers may be useful in interpreting the time trend in the great ratios. As suggested by Hahn and Lee, some household heterogeneity may be required to reconcile the trends in the *cay*-relationship with the aggregate budget constraint (1). If efficiency in the U.S. financial system has increased over time as more households have obtained access to credit and stock markets, this could potentially account for such a trend.

b) A single stochastic trend The second cointegrating relationship substantially facilitates the interpretation of the joint long-run dynamics of consumption, income and asset wealth: consumption, income and asset wealth must share a single common stochastic trend. It is therefore straightforward to study the response of the three variables to the common permanent shock π_t along the lines of King et al. (1991). Figure (4) plots the impulse responses of the three variables. First it is noteworthy that the response of consumption is broadly consistent with macroeconomic theory – after a trend shock, consumption quickly reaches its new long-run level, while income and asset wealth adjust somewhat more sluggishly. Still, the adjustment in consumption is not immediate. This is in line with the existence of a transitory component in consumption as discussed above. Fully forward-looking consumers should adjust their consumption level immediately. It is also worth noting that asset wealth shows some interesting non-monotonic adjustment. Assets seem to overshoot their long-run level, a result that is consistent with the observed short-run volatility of asset prices. As both income and consumption settle onto their long-run levels, however, so does asset wealth. The shape of the asset wealth response is consistent with a substantial temporary component in asset prices that could be triggered by what is ultimately a permanent shock to consumption, income and asset wealth. This feature of the asset response may therefore also help explain why I find a relatively lower role of transitory shocks for asset wealth even though the size of the transitory component in a is unchanged vis-a-vis Lettau and Ludvigson's paper.

4 Reinterpreting *cay*

The Lettau-Ludvigson approach suggests two different, though intimately related interpretations of *cay*: first, at a theoretical level, *cay* is an approximation of the unobservable aggregate consumption-wealth ratio $c - w$. Secondly, it is also an empirically successful indicator of transitory fluctuations in financial assets and in particular, in asset prices. It is the coincidence of these two interpretations that accounts for much of the theoretical appeal of the *cay* approach: a variable that is so central in many macroeconomic models – the consumption wealth ratio – uncovers temporary variation in asset prices. I now address in turn, how the presence of a second cointegrating relationship affects both of these interpretations.

4.1 *cay* as a proxy of the consumption-wealth ratio

If the cointegrating space is two-dimensional, it would appear that a proxy of the consumption-wealth ratio cannot be consistently estimated from a simple cointegrating regression. However, if only one cointegrating relationship between c, a and y is specified in an econometric model, the estimated relation will generally not only be stationary, it will also reflect a linear combination of minimum-variance in the cointegrating space. My argument here is that this minimum-variance property allows us to interpret the *cay*-residual as a factor that mimics the consumption-wealth ratio that is associated with the wealth portfolio with the smallest variance. Hence, if we introduce the additional assumption that the average household holds a portfolio of human capital and assets that minimizes the (short-term) variability of total wealth, then the estimated *cay* will also be a proxy of $c - w$.[7]

I now construct a minimum-variance portfolio from the transitory components obtained from the VECM with two stationary relations. This can be done by backing out γ from the minimization problem

$$\min_{\gamma} \left\{ var \left[w^T \right] \right\} = \min_{\gamma} \left\{ var \left[\gamma a^T + (1-\gamma) y^T \right] \right\}$$

where $a^T = a - a^p$ and $y^T = y - y^p$ are the cyclical or transitory components of assets and income obtained from the VECM with two cointegrating relations respectively, w^T is the transitory component of total wealth and the superscript p denotes the permanent component. The solution for γ is

$$\gamma = \frac{var(y^T) - cov(a^T, y^T)}{var(a^T - y^T)}$$

Based on my estimates of a^T and y^T identified from the Granger-Gonzalo decomposition, I calculate $\gamma = 0.26$. I then obtain w^T and construct a measure of the consumption-wealth ratio as

$$c - w = c^T - w^T = \boldsymbol{\gamma}' \mathbf{x}_t^T$$

where I have defined $\boldsymbol{\gamma}' = [1, -\gamma, \; -1-\gamma]'$.[8] Figure (5) plots my estimates of $c^T - w^T$ and w^T against the sample estimate of the *cay*-relation.[9] The correlation of $c^T - w^T$ with *cay* is 0.99 , that of w^T with *cay* is 0.83!

[7] If consumption, income and physical and financial asssets follow a single stochastic trend, the composition of the wealth portfolio is irrelevant for the long-run variance of wealth and therefore for the long-run variance of consumption. The choice of the portfolio weights γ and $(1-\gamma)$ will then only affect the variance of wealth around the trend and it is only this variability that optimising agents will be able to minimize through their choice of γ.

[8] Here, I have used that $c - w$ cointegrates so that $c^P - w^P$ is constant. For convenience, I assume this constant to be zero.

[9] To take account of the deterministic terms, the vector of transitory components is constructed as

$$\mathbf{x}_t^T = \alpha(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1} \left[\begin{array}{ccc} \boldsymbol{\beta}' & \boldsymbol{\delta}_{trend} & \boldsymbol{\delta}_{break} \end{array} \right] \left[\begin{array}{c} \mathbf{x}_{t-1} \\ t \\ \min(0, t-t_0) \end{array} \right]$$

These findings strongly suggest that the *cay*-residual – even though it may have been estimated from what is a misspecified model under the maintained assumptions – is likely to be an excellent approximation of the consumption-wealth ratio– provided the average household holds a portfolio that minimizes the cyclical variability of wealth. Interestingly, this holds true even though such residuals will generally have been estimated without allowance for structural breaks and deterministic trend terms. One potential explanation for this finding is that the minimum-variance property of the *cay*-residual will also seek to minimize the in-sample variability induced by deterministic terms.

Note that my estimate of $\gamma = 0.26$ also seems highly plausible as measure of the portfolio share of assets in total wealth. Lettau and Ludvigson argue that along a balanced growth path, the portfolio weights γ and $1-\gamma$ should correspond to the long-run capital and labour shares of the economy. While this may not necessarily be true if capital is, e.g. more risky than human capital, my estimate of $\gamma = 0.26$ is still close to the values for the capital share typically used in the RBC literature. Furthermore, $\gamma = 0.26$ exactly corresponds to the coefficient on a in the *cay*-relation. Indeed, under the maintained assumption that the great ratios are stationary, the cointegrating space is spanned by

$$\boldsymbol{\beta}' = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

and the portfolio vector $\boldsymbol{\gamma}$ is an exact linear combination of $\boldsymbol{\beta}'$ so that $\boldsymbol{\gamma} = \boldsymbol{\beta}\mathbf{R}$ for some non-singular matrix $\mathbf{R}$. I can then write

$$c^T - w^T = \boldsymbol{\gamma}'\boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\mathbf{x}_t = \mathbf{R}'\boldsymbol{\beta}'\boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\mathbf{x}_t = \boldsymbol{\gamma}'\mathbf{x}_t = c_t - \gamma a_t - (1-\gamma)y_t$$

Hence, if the great ratios define the stationary relations, the vector $\boldsymbol{\gamma}$ of portfolio weights must also define the very linear combination of the *levels* of the process that approximates the consumption-wealth ratio. To the extent that the transitory part in consumption is not too volatile, so that minimizing the variability in w^T comes close to also minimizing the variability in $c^T - w^T$, this linear combination can then be estimated by means of a cointegrating regression.

While the findings reported here rehabilitate *cay*-like residuals as empirical proxies of the transitory component in aggregate wealth, this rehabilitation comes at some cost: it has often been claimed that the derivation of the *cay* residual rests on minimal theoretical assumptions because it is based on the log-linearization of the intertemporal budget constraint (1) alone. The results put forward in this paper suggest that things are not that simple. The balanced-growth assumption made in deriving *cay* from (1) implies a second cointegrating relationship which makes it econometrically impossible to identify the consumption-wealth ratio without a further identifying assumption. Such an assumption will almost inevitably be based on economic theory, e.g. on optimizing behaviour by economic agents. In this respect, the *cay*-residual is much more than just the log-linearized version of a budget constraint.

4.2 *cay* and asset prices

Under the maintained hypothesis that *cy* and *ay* are individually stationary, *cay* is just a particular linear combination of these two great ratios. This also implies that predictive regressions of equity premia or asset prices on *cy* and *ay* will perform at least as well as a regression on *cay* alone. This should affect the measurement of transitory components in asset prices. The second cointegrating relationship also informs the recent debate about the role of look-ahead bias for the predictive power of *cay*: since the great ratios *ca*, *cy* and *ay* are – in principle – directly observable, their joint predictive power cannot be subject to look-ahead bias; it is not necessary to first estimate the parameters of *cay* from a long sample in order to do at least as well as *cay* in forecasting excess returns.

Under the theoretical assumptions made in section 2, it is trivially true that *cay* is a linear combination of the great ratios. But it is not clear *a priori* to what extent it is true if the great ratios and are subject to deterministic drifts and breaks. Table (6) therefore reports regressions of the *cay* residual on $\widetilde{cy}$ and $\widetilde{ya}$ and the deterministic trend terms t and $\min(t - t_{0,}0)$. I consider two different measures of *cay*. The first is the original *cay* used by Lettau and Ludvigson (2004) which is constructed with the cointegrating vector $\beta = [1, -0.30, -0.60]$ estimated from a dynamic OLS regression. I refer to this residual as cay_{LL}.The second one is constructed based on the cointegrating vector $\beta = [1, -0.26, -0.63]$ which is estimated by Johansen's FIML procedure. This is the cointegrating vector also reported in table (3) above and the associated *cay*-residual is the one that has been used in the paper so far.

In both regressions, the two detrended great ratios are highly significant and also have very similar coefficients. Furthermore, the fit of both regressions is overwhelming with an R^2 of 0.97 and 0.98. There is, however, an interesting difference in as far as the deterministic terms are concerned. In the original Lettau-Ludvigson cay_{LL},both the trend and the break term are highly significant. In the *cay*-residual based on $\beta = [1, -0.26, -0.63]$ only the coefficient on the trend break term remains marginally significant but it is much smaller than in the Lettau-Ludvigson *cay* so that this version of the *cay* residual can essentially be written as[10]

[10] Since this measure of the *cay*-residual appears immune against the Hahn-Lee caveat, I continue to use it as my preferreed measure in the remainder of this paper. Whenever a distinction is necessary and may matter for the results, I abbreviate the Lettau-Ludvigson residual with cay_{LL}.

$$cay = 0.80\widetilde{cy} + 0.26\widetilde{ya} + 0.28$$

Hahn and Lee (2006) argue that the (unmodelled) deterministic components in the Lettau-Ludvigson *cay* are a main driver behind the predictive power of the residual for asset prices. This point is examined in table (7), where I report regressions of asset returns on the two *cay* measures. My analysis is based on two different measures of asset prices. The first are excess returns on the CRSP index. The second is broad measure of asset returns that I construct from the asset data used in Lettau and Ludvigson.[11] This broad measure has the advantage that it does not only capture fluctuations in stock markets but also fluctuations in other financial and, in particular, in physical assets such as housing.

Based on the broad measure of returns, I find that the Lettau-Ludvigson cay_{LL} seems to outperform *cay* by a wide margin. This result highlights the potential importance of the Hahn-Lee caveat. But it should be noted that *cay* remains an important predictor of aggregate asset prices with R^2 peaking at 0.19 at the 2 year horizon. The lower two panels report similar regressions for equity (excess) returns. The predictive power of both *cay* measures is now very similar. Interestingly, the R^2on excess returns are generally higher than those obtained on the broad asset return measures , which supports Lettau's and Ludvigson's claim that *cay* is, in particular, a good indicator of the equity risk premium.

Table (8) presents long-horizon regressions of the broad return measure on the observable great ratios *cy* and *ay*. For each forecasting horizon, line *I* reports regressions on the detrended versions of *cy* and *ay*, whereas line *II* gives the regressions on $\widetilde{cy}$ and $\widetilde{ay}$, i.e. taking account of both a linear trend and the break in 1978. While controlling for a linear trend alone would suggest that the joint predictive power of the observable great ratios still by far exceeds that of *cay*(or even ca_{LL}), this is not so clearly the case for $\widetilde{cy}$ and $\widetilde{ay}$. Certainly, in the regressions in the second line, the adjusted R^2 measure at all forecasting horizons exceeds the R^2 from a regression with *cay* (as reported in the previous table). This fact per se should not be surprising, since *cay* is almost an exact linear combination of $\widetilde{cy}$ and $\widetilde{ay}$. It is however, doubtful that $\widetilde{cy}$ and $\widetilde{ya}$ really explain a significantly larger

[11]he law of motion for asset wealth can be written as $A_{t+1} = (1 + r_{t+1})(A_t + Y_t - C_t)$. Dividing through with A_t, taking logarithms and solving backwards it is straightforward to show that $a_{t+1} = \sum_{l=1}^{t+1} r_{t+l} + a_0 + \sum_{l=0}^{t} \log(1 + (Y_l - C_l)/A_l)$. The aggregate asset price measure I construct is $p_t = a_t - \sum_{l=0}^{t} \log(1 + (Y_l - C_l)/A_l)$. Under the null that asset returns are unpredictable, $r_{t+k} = r + v_{t+k}$, where r is a constant and v_{t+k} is *i.i.d.*. Then $\mathbf{E}_t(p_{t+k} - p_t) = kr$, i.e. p_t follows a random walk with drift and should therefore not be predictable from *cay* or other variables.

portion of the variation in asset returns than does *cay*: at short horizons, the combination of $\widetilde{cy}$ and $\widetilde{ay}$ generates an adjusted R^2 that only exceeds that of the *cay*-regression by a factor of $1.2 - 1.4$. Though this factor increases to 2 at the five year horizon, the regressions are only significant up to an horizon of up to $3 - 4$ years,very much as the *cay*-only based regressions. Indeed, as I show in lines III and IV, none of the two great ratios makes an independent contribution to predicting asset prices if it is included along with *cay* as a regressor.

Table (9) provides long-horizon regressions of excess returns on the observable great ratios. Based on the linear trend alone (line I) , *cy* and *ay* outperform *cay*by a wide margin, but the results are even more pronounced once the trend break is also controlled for (line II respectively). In this case, R^2 reaches 0.6 at the 6-year horizon. As lines III and IV show, both great ratios also make a significant independent contribution to predicting excess returns if they are included along with *cay* as a regressor.

While *cy* and *ay* seem able to uncover predictable dynamics in stock prices and in particular in equity risk premia to a degree that by far exceeds the predictive power of *cay*, this is not generally true for a broader concept of asset prices. For the broader concept, the great ratios together are just as good as *cay*. One possible interpretation for this finding could be based on Cochrane et al.'s (2005) recent argument that return predictability may arise from portfolio adjustment alone in a model with several Lucas trees. Under the balanced growth assumption asset wealth and human capital share a single common trend and their long-run portfolio shares are fixed. So, an asymmetric shock to, say, asset wealth, must either forebode a similar adjustment in human capital (if the shock is permanent), or a re-adjustment to the permanent value of a (if the shock is transitory). Hence, we would expect that relative returns – such as that of equity versus bonds – are even more highly predictable from the interaction of the error-correction terms than are are aggregate returns on a broad measure of asset wealth. The aggregate consumption-wealth ratio predicts aggregate asset price fluctuations and, in particular, risk premia; the observable great ratios also predict the price effects of relative portfolio adjustment, so that they outperform *cay* on this account. Still, *cay* remains remarkably robust as an indicator of transitory components in aggregate asset prices.

Out-of-sample prediction and look-ahead bias Brennan and Xia (2005) have pointed out that the predictive power of the *cay* residual could possibly be spurious: even if only past values of c, a and y are used in out-of-sample predictive regressions for asset prices, the *cay*-cointegrating coefficients will have been estimated from the full sample. They argue that if these coefficients are re-estimated from past data every period, *cay* does not beat the random walk in forecasting asset prices. Brennan and Xia conclude that the

Lettau-Ludvigson results are subject to look-ahead bias.

The results in this paper have immediate implications for this debate: as long as *cay* is an exact linear combination of stationary and directly observable great ratios, a household (or a researcher seeking to identify household expectations) does not have to estimate the coefficients of the *cay*-relation from a long sample in order to identify the transitory component in asset prices. Hence, if the balanced-growth assumption holds, look-ahead bias in *cay* cannot actually be a problem. Certainly, as my earlier analysis has shown, the great ratios *ca* and *ay* may only be stationary, after the removal of some deterministic terms. This will make the comparison of out-of-sample predictive power of the great ratios with *cay* somewhat less straightforward.

In table 10, I therefore report the results of an empirical exercise, in which I compare the out-of-sample forecast performance of the two great ratios (with and without trend – model I and II respectively) to that of a range of rival models. The rivals are different versions of *cay* (models III, IV and V) and a random walk model (VII). In addition, I also consider a model in which c, a and y figure individually as predictors (VI). As pointed out by Lettau and Ludvigson (2005) in their reply to Brennan and Xia, this model does not presume knowledge of the *cay*-relation. To account for Hahn's and Lee's point that the predictive power of the consumption-wealth ratio might largely stem from an unmodelled linear trend, model VI also includes a deterministic trend term. There are three versions of the *cay*-model: in model III, I use the *cay*-residual estimated from the whole sample, in model IV, a trend is included along with *cay* and in model V, to examine the role of look-ahead bias, I follow Brennan and Xia and reestimate the *cay* residual each period.

The results are quite clear: none of the models can outperform the *cay*-residual estimated from the whole sample. This is true, quite irrespective of whether a trend is included along with *cay* or not. But once only past information is used in the estimation of *cay* , i.e. *cay* is reestimated every period, its predictive power is hardly better than that of a random walk or of the levels-specification with trend. This is just an instance of the Brennan-Xia result. What is new here is that the great ratio specifications now really come into their own: in particular the specification without a trend does a lot better than either the re-estimated *cay*, the levels specification or the random walk setup – and by construction, *ca* and *cy* cannot suffer from look-ahead bias.

In the second and third panels of table 10, I examine to what extent these results are sensitive to the structural break that others and myself have identified in the middle of the sample period. The pattern stays largely the same across subperiods: the great ratios generally outperform the random-walk and in particular in the first period, they outperform the re-estimated *cay* model and the level specification by a remarkable margin. Though in the second sub-period, a trend is needed along with the two great ratios –

which may hint at the importance of the structural break – this does not change the overall conclusion emerging from the exercise presented here: a linear combination of *ca* and *cy* always outperforms the random walk and does at least as well as the *cay*-residual that is estimated from the same information set.[12] Thus, Lettau and Ludvigson's main conclusion remains immune against look-ahead bias: it is possible to outperform a random walk out of sample with a appropriate linear combination of consumption, income and asset wealth.

5 Discussion and Conclusion

Lettau and Ludvigson have suggested a by now very popular approach to approximating the consumption-wealth ratio as a cointegrating relationship between consumption, asset wealth and labour income commonly called the *cay* residual. One key assumption that underlies the interpretation of *cay* as an approximation of the consumption-wealth ratio is that the shares of human capital and asset wealth in total wealth are constant in the long run. In this note I have demonstrated that this actually implies that the consumption asset (*ca*) , the consumption-labour income (*cy*) and the asset-income (*ay*) ratios should all be individually stationary. Hence, there should be two linearly independent cointegrating relations between the three variables – consumption, income and asset wealth should share a single stochastic trend.

I have explored the reasons why earlier studies have not generally detected this second cointegrating relationship. While cointegration tests could generally have very low power, a structural break in the trend growth rates of consumption and income in the particular sample used by Lettau and Ludvigson can provide an explanation for this low power; once this break is explicitly modelled, the second cointegrating relationship predicted by the theoretical framework is picked up by the extant tests.

While these results simplify the interpretation of the joint long-run dynamics of consumption, income and asset wealth in the light of standard economic theory, they strongly affect the interpretation of *cay*-like residuals as approximations of the consumption wealth ratio: in the presence of a second cointegrating relationship, *cay* becomes a particular linear combina-

[12] When estimated from the whole sample, *cay* seems to outperform a combination of *ca* and *cy*. In view of the results in this paper, this could well be an artefact of the unmodelled structural break in the *cay* relationship: as shown earlier, the empirical *cay* residual is not an exact linear combination of *ca* and *ay*, possibly because the drifts and the structural break in *ca* and *ay* affect the estimate of the *cay*-relation and bias the sum of the coefficients away from unity. Clearly, without an economic underpinning for the structural breaks identified here, one should be very cautious in interpreting this result as demonstrating that the predictive ability of *cay* is superior to that of the combined great ratios.

tion in a two-dimensional cointegrating space and we cannot generally hope to obtain an approximation of the consumption -wealth ratio from a simple cointegrating regression alone.

However, I have shown that *cay* remains a good indicator of transitory components in asset prices and that – under the additional assumption that the average household holds a minimum-variance portfolio of physical assets and human wealth – one may still be able to interpret it as a proxy of the consumption-wealth ratio. This assumption however, shows that the interpretation of *cay* as a proxy of the consumption-wealth ratio does not rest on the rather innocuous log-linearization of an intertemporal budget constraint alone.

Finally, the results provided here should be informative with respect to the recent debate about look-ahead bias in *cay*: if *cy* and *ay* are individually stationary, then in order to identify transitory components in asset prices, households and researchers do not need to identify the parameters of the *cay*-relation first – which may only be estimable *ex post* from very long samples of data. Rather, the great ratios carry at least the same amount of information.

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Table 1: Cointegration results for great ratios

Panel I: Johansen's tests for cointegration

	$[c, y]$	$[c, a]$	$[y, a]$
Trace Test	3.84	5.53	4.55
p-val	[0.9091]	[0.7512]	[0.8503]

Panel II: Estimates of CI-vector $\beta = \begin{bmatrix} 1 & \beta_2 \end{bmatrix}'$

	$[c, y]$	$[c, a]$	$[y, a]$
β_2	-0.938	-0.832	-0.842
std. deviation	(0.047)	(0.073)	(0.117)
p-Value of $\beta_2 = -1$	[0.18]	[0.031]	[0.2405]

NOTES: Cointegration tests based on bivariate VECM with 2 lags in the variable pair indicated at the head of each column. p-values in panel I are baed on the response surface method by Doornik (1998) and obtained using the procedure in the package JMulti (Lütkepohl and Krätzig (2004)).

Table 2: Trend stationarity of great ratios

	Panel I: deterministic trend			Panel II: deterministic trend and break		
	$c-a$	$c-y$	$y-a$	$c-a$	$c-y$	$y-a$
t	-0.0009	-0.0005	-0.0004	-0.0025	0.0004	-0.0029
	(-10.8604)	(-14.2166)	(-3.3694)	(-21.4555)	(9.3622)	(21.8682)
$\min(t-t_0)$	—	—	—	0.0034	-0.0017	0.0051
				(15.8234)	(-25.0342)	(21.4090)
const	-1.5735	0.2515	-1.8250	-1.3195	0.1203	-1.4399
	(-168.4468)	(60.8328)	(-144.1113)	(-76.6371)	(21.4005)	(-74.6037)
ADF $t-$test	-2.0611**	-1.4260	-1.6974*	-3.1317***	-3.5337***	-3.2189***
Johansen test	11.97	14.21	14.32	26.81*	26.81 **	25.31*

NOTES: Regressions of the great ratios on deterministic components, t-values in parentheses. The last two lines give unit root tests on the regression residuals (t-stat of an augmented Dicke-Fuller test with two lags) and of Johansen's system cointegration test with a trend restricted to the cointegrating space. 1,2 or 3 Stars denote significance at the 90, 95% and the 99% levels respectively. The corresponding critical values for the ADF test are -1.67, -1.99 and -2.65 respectively. Those for the Johansen test are 23.32, 25.73 and 30.67.

Table 3: Cointegrating results in the trivariate VECM

Panel I: Cointegration tests

	trend cum break	90%	95%	CV	trend only	90%	95%
$r \geq 1$	43.83	33.17	35.95		31.49	39.73	42.77
$r \geq 2$	24.16	14.63	16.62		11.60	23.32	25.73

Panel II: Estimated cointegrating vectors

2 CI relations imposed, cum break						1 CI-relation imposed, no break			
Coefficients (unrestricted)					p-values	Coefficients			
β_c	β_a	β_y	δ_{trend}	δ_{break}	$H_o : \beta_y = -1$	β_c	β_a	β_y	δ_{trend}
1	–	–0.67	-0.002	0.001	[0.18]	1	-0.16	-0.58	-0.001
–	1	-0.77	-0.003	0.005	[0.15]				
Coefficients (restricted)						Coeffs (restricted)			
1	—	-1	-0.000	0.002		1	-0.26	-0.63	—
	1	-1	-0.002	0.005					

NOTES: The critical values are based on 10000 simulations in DisCo (Johansen and Nielsen (1993)), assuming $T = 202$ observations.

Table 4: VECM with 2 stationary relations

	Equation		
	Δc_t	Δa_t	Δy_t
Δc_{t-1}	**0.1770**	0.1023	**0.3377**
	(2.2373)	(0.2803)	(2.1662)
Δa_{t-1}	**0.0416**	0.0863	**0.0782**
	(2.5821)	(1.1613)	(2.4646)
Δy_{t-1}	0.0729	-0.0336	-0.1055
	(1.7728)	(-0.1770)	(-1.3021)
$\widetilde{cy}_{t-1}$	0.0058	**0.3635**	**0.1074**
	(0.2259)	(3.0483)	(2.1083)
$-\widetilde{ya}_{t-1}$	-0.0107	**0.1014**	-0.0208
	(-1.3992)	(2.8742)	(-1.3769)
$step_t$	0.0010	0.0009	**0.0026**
	(0.5559)	(0.3103)	(2.1358)
$const$	**0.0030**	0.0044	**0.0026**
	(5.6764)	(1.8366)	(2.5340)
$\overline{R}^2$	0.17	0.04	0.15

NOTES: coefficient estimates of the VECM

$$\mathbf{\Gamma}(\mathbf{L})\mathbf{\Delta x}_t = \boldsymbol{\alpha} \begin{bmatrix} \widetilde{ca}_{t-1} & \widetilde{ay}_{t-1} \end{bmatrix}' + \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 step_t + \boldsymbol{\varepsilon}_t$$

where $\begin{bmatrix} \widetilde{ca}_t & \widetilde{ay}_t \end{bmatrix}'$ is the vector of cointegrating relations as defined in equation (8) and $step_t$ is a step dummy that is 0 until 1977:Q4 and one afterwards.

Table 5: Variance decompositions based on 2 stationary relations

Variance share of transitory component

	Horizon k in quarters						
	1	2	4	8	12	16	24
$c_{t+k} - \mathbf{E}_t(c_{t+k})$	0.1330	0.1245	0.0953	0.0617	0.0448	0.0353	0.0258
	[0.04-0.25]	[0.04-0.23]	[0.03-0.17]	[0.02-0.12]	[0.01-0.08]	[0.01-0.07]	[0.01-0.05]
$a_{t+k} - \mathbf{E}_t(a_{t+k})$	0.6540	0.6222	0.5732	0.5043	0.4576	0.4228	0.3718
	[0.47-0.80]	[0.45-0.77]	[0.42-0.71]	[0.36-0.63]	[0.33-0.58]	[0.30-0.54]	[0.2-0.48]
$y_{t+k} - \mathbf{E}_t(y_{t+k})$	0.8721	0.8167	0.7233	0.5478	0.4197	0.3334	0.2331
	[0.76-0.96]	[0.69-0.82]	[0.59-0.83]	[0.43-0.65]	[0.33-0.51]	[0.26-0.41]	[0.18-0.29]

Notes: The variance decomposition is the mean across 250 bootstrap replications of the model, the numbers in rectangular brackets give the 95% confidence intervals obtained from the bootstrap.

Table 6: cay as linear combination of great ratios

	cay_{LL}	cay
$\widetilde{cy}_t$	0.82	0.80
	(88.53)	(78.41)
$\widetilde{ay}_t$	0.29	0.26
	(106.45)	(86.53)
t	-0.0001	-0.0000
	(-29.30)	(-1.08)
$\min(t - t_0, 0)$	0.0002	0.0000
	(19.27)	(4.60)
$const$	0.908	0.28
	(254.21)	(70.78)
$\overline{R}^2$	0.99	0.98

NOTES: OLS regression of cay_{LL} and cay on the deterministic terms and on the great ratios. $cay_{LL} = c - 0.3a - 0.6y$ is the Lettau-Ludvigson-cay, whereas $cay = c - 0.26a - 0.63y$ is based on the cointegrating vector estimated by FIML.

Table 7: Long-horizon regressions of asset returns on *cay* **and** cay_{LL}

$$\sum_{l=1}^{k} r_{t+l} - r_{t+l}^{f} = \mathbf{z}_t'\boldsymbol{\delta}_k + v_{kt}$$

$\mathbf{z}_t$	Horizon k in quarters						
	1	2	4	8	12	16	20
Panel I: Aggregate asset returns on cay_{LL}							
cay_{LL}	0.5106	1.0009	1.8167	3.3771	4.3417	4.3623	4.5481
	(3.7715)	(3.6621)	(3.3145)	(4.1877)	(3.8874)	(2.8581)	(2.7926)
$\overline{R^2}$	0.1077	0.1772	0.2353	0.3106	0.2601	0.1549	0.1033
Panel II: Aggregate asset returns on *cay*							
cay	3.7715	3.6621	3.3145	4.1877	3.8874	2.8581	2.7926
	(3.2536)	(2.9835)	(2.4702)	(2.5356)	(2.0190)	(1.4046)	(1.0997)
$\overline{R^2}$	0.0702	0.1135	0.1430	0.1903	0.1338	0.0551	0.0230
Panel III: Excess stock market returns on cay_{LL}							
cay_{LL}	1.8160	3.4859	6.2120	10.3546	12.8615	13.8875	16.9512
	(3.9625)	(3.9511)	(3.8185)	(5.2619)	(7.3338)	(6.8522)	(5.4022)
$\overline{R^2}$	0.0855	0.1526	0.2580	0.4052	0.4255	0.3704	0.3529
Panel IV: Excess stock market returns on *cay*							
cay	2.0324	3.8553	6.7739	11.2227	13.3658	13.7944	15.6080
	(4.6595)	(4.6116)	(4.4898)	(6.0746)	(5.9340)	(5.2692)	(4.9552)
$\overline{R^2}$	0.0858	0.1491	0.2475	0.3914	0.4134	0.3666	0.3325

NOTES: OLS regressions. $t-$statistics are based on heteroskedasticity and autocorrelation consistent standard errors based on Newey and West (1987), using a window width of $k+1$.

Table 8: Predictive regresssions of aggregate asset returns on great ratios

$$\sum_{l=1}^{k} r_{t+l} - r_{t+l}^{f} = \mathbf{z}_t' \boldsymbol{\delta}_k + v_{kt}$$

Horizon k in quarters	Regression number	trend only cy	trend only ay	trend and break cy	trend and break ay	cay	R^2
$k=1$	I	0.4875	0.1273				0.1086
		(4.2695)	(3.1564)				
	II			0.5126	0.1217		0.0942
				(3.5712)	(3.0610)		
	III				-0.0282	0.5278	0.0682
					(-0.6143)	(2.8388)	
	IV			0.1704		0.3814	0.0759
				(1.2655)		(2.5691)	
$k=4$	I	1.7445	0.4609				0.2478
		(3.5704)	(2.6248)				
	II			1.5032	0.5084		0.2012
				(2.5269)	(2.9458)		
	III				0.0990	1.3685	0.1439
					(0.5513)	(1.7611)	
	IV			0.1085		1.5330	0.1392
				(0.1924)		(2.2596)	
$k=8$	I	3.3120	0.8518				0.3546
		(4.0414)	(2.5665)				
	II			2.5186	1.0168		0.2842
				(2.3365)	(3.3375)		
	III				0.3561	2.1391	0.2124
					(0.8754)	(1.4743)	
	IV			-0.2671		3.0599	0.1876
				(-0.2040)		(2.3397)	
$k=16$	I	3.7753	0.5779				0.3409
		(3.3751)	(1.0346)				
	II			3.0232	1.2342		0.1399
				(1.4402)	(2.5496)		
	III				0.4993	1.7838	0.0728
					(0.5853)	(0.6522)	
	IV			0.0659		2.6370	0.0500
				(0.0241)		(1.1984)	

NOTES: see table 7

Table 9: Predictive regresssions of excess returns on great ratios

$$\sum_{l=1}^{k} r_{t+l} - r_{t+l}^{f} = \mathbf{z}_t' \boldsymbol{\delta}_k + v_{kt}$$

Horizon k in quarters	Regression number	trend only cy	trend only ay	trend and break cy	trend and break ay	cay	R^2
$k=1$	I	1.8035	0.5559				0.0943
		(5.2850)	(4.5282)				
	II			1.7548	0.5649		0.0914
				(3.9625)	(4.2529)		
	III				0.0270	1.9730	0.0813
					(0.1737)	(3.5025)	
	IV			0.1945		1.9564	0.0857
				(0.9377)		(4.4070)	
$k=4$	I	6.0814	1.9264				0.2929
		(5.1683)	(4.7750)				
	II			4.6921	2.2045		0.3064
				(3.3196)	(4.6042)		
	III				0.8163	4.9746	0.2776
					(1.8903)	(2.7870)	
	IV			0.5197		6.5664	0.2517
				(0.8493)		(4.4351)	
$k=8$	I	10.1177	3.1932				0.4690
		(7.4920)	(6.9389)				
	II			6.9072	3.8350		0.5214
				(4.1951)	(6.2500)		
	III				1.8112	7.2038	0.4826
					(2.6845)	(3.4218)	
	IV			0.9186		10.8365	0.4023
				(1.2117)		(5.6751)	
$k=16$	I	12.7711	3.9858				0.4788
		(7.7646)	(5.8669)				
	II			9.1543	5.2963		0.5650
				(4.8109)	(10.8392)		
	III				2.5911	9.1572	0.5056
					(3.0523)	(3.5522)	
	IV			1.5518		(12.4482)	0.3908
				(1.4603)		(4.1925)	

NOTES: see table 7

Table 10: Out of sample forecast performance of different models

	great ratios		*cay*-based			Levels	RW
	I	II	III	IV	V	VI	VII
	with trend	w/o trend	whole sample	whole sample +trend	re-estimated	with trend	with trend
				Sample period 1952:4-2002:4			
$RMSPE$	0.0897	0.0938	0.0872	0.0881	0.0914	0.0915	0.0915
Pseudo R^2	0.0191	-0.0253	0.0463	0.0364	0.0003	-0.0008	0
				Sample period 1952:4-1977:4			
RMSPE	0.0944	0.0939	0.0899	0.0917	0.0954	0.0967	0.0966
Pseudo R^2	0.0232	0.0279	0.0700	0.0505	0.0128	-0.0006	0
				Sample period 1978:1-2002:4			
RMSPE	0.0899	0.0859	0.0857	0.0875	0.0859	0.0898	0.0875
Pseudo R^2	-0.0279	0.0180	0.0206	-0.0008	0.0173	-0.0267	0

NOTES: the table gives one-quarter ahead root mean squared prediction errors (RMSPE) and pseudo R^2 measures for seven different models. GR is the great ratio model with predictors $\left[\begin{array}{ccc} ca & cy & 1 \end{array}\right]$ and $\left[\begin{array}{cccc} ca & cy & 1 & t \end{array}\right]$ respectively, *cay* whole sample is the Lettau-Ludvigson *cay* for the sample period 1952:4-2002:4, For the "restimated" specification, the *cay* residual is reestimated each period before prediction. For the level specification, following Lettau and Ludvigson (2005), the set of regressors is $\left[\begin{array}{ccccc} c & a & y & t & 1 \end{array}\right]$. RW is the random-walk with drift-specification for asset prices, i.e. constant returns. For each model X, the pseudo R^2-measure is defined as $1 - RMSPE_X/RMSPE_{RW}$. Predictive regressions based on a minimum of 80 observations (50 in the sub-samples).

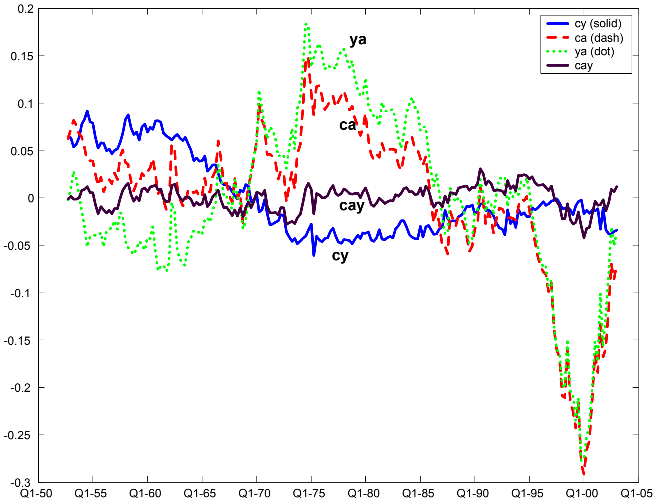

Figure 1: The Great Ratios $cy = c - y$, $ca = c - a$ and $ya = y - a$ along with the cay-residual

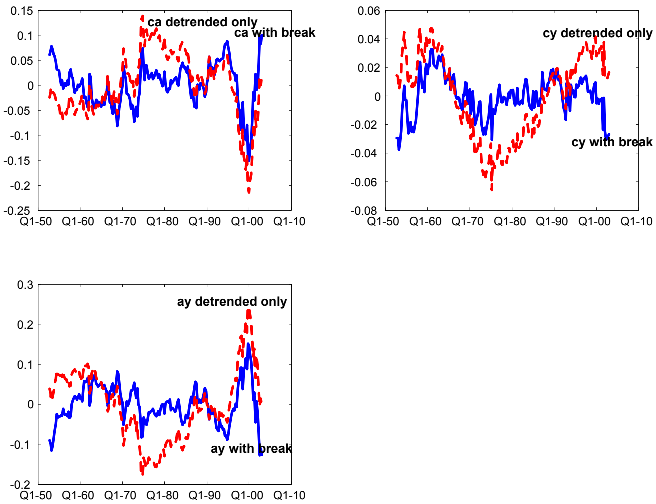

Figure 2: Great Ratios purged of deterministic terms. Trend and break removed (solid /blue line) and trend only removed (dashed/red line).

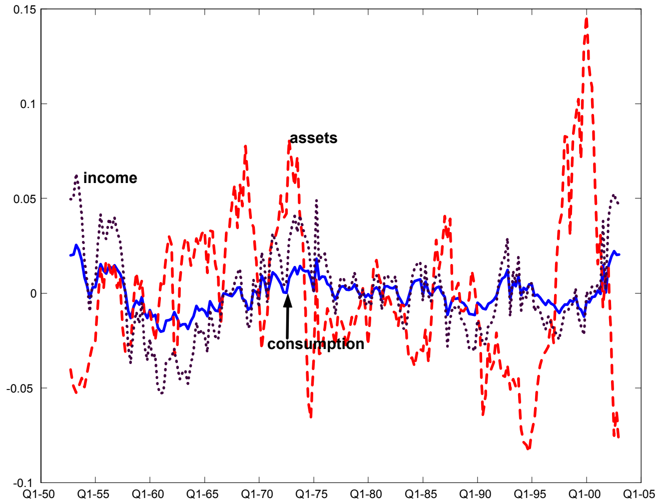

Figure 3: Transitory components from the VECM with 2 stationary relations

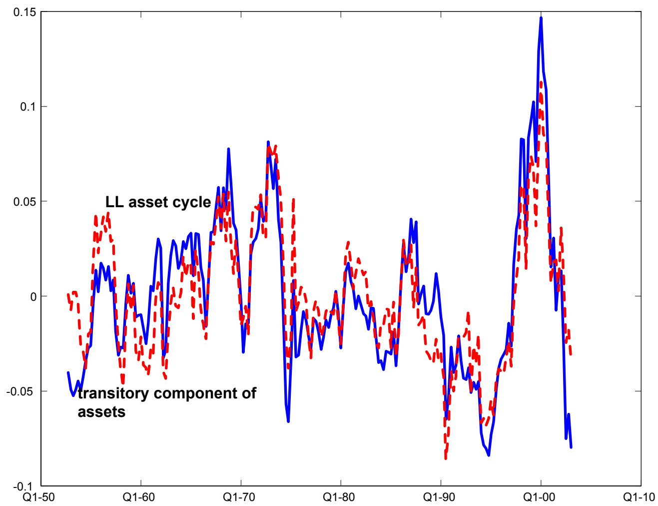

Figure 4: Asset Cycles from Lettau Ludvigson model and from the VECM with 2 stationary relations

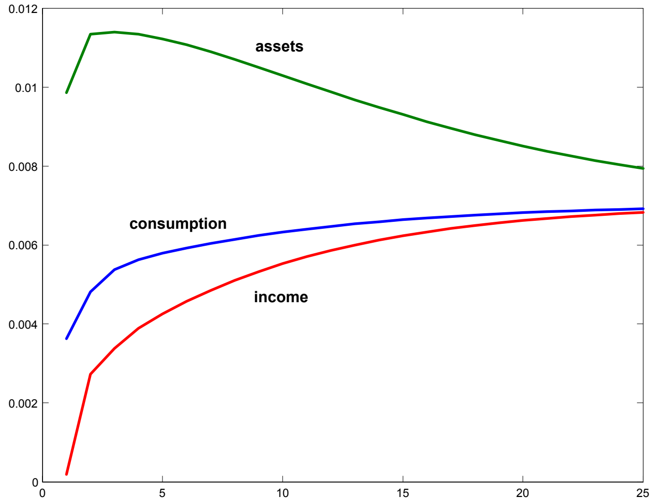

Figure 5: Impulse responses to a permanent shock

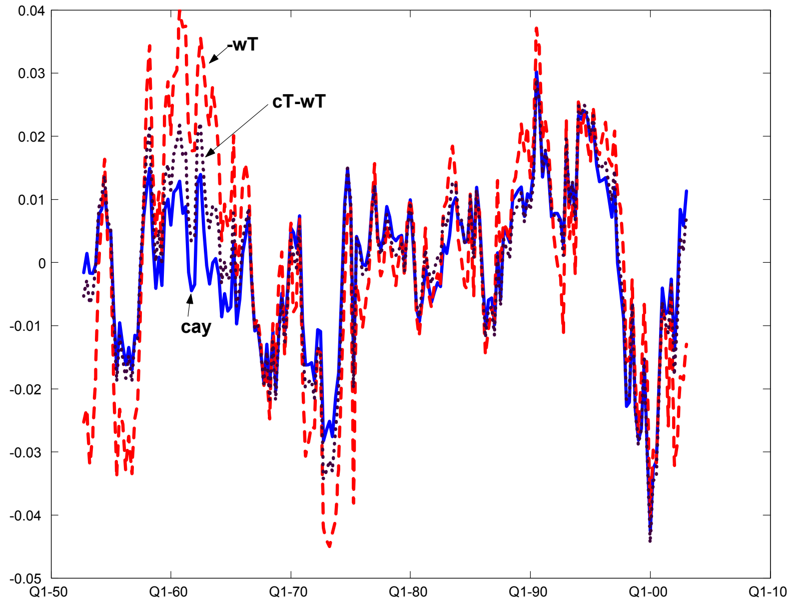

Figure 6: The *cay*-residual, the (negative) minimum-variance wealth portfolio, $-w^T$, and the implied consumption-wealth ratio $c^T - w^T$.