Optimal Design and ρ-Concavity

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- Objective: Find a tight criterion for regularity

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$$F_\xi(x) = \int_{X(x,\xi)} f(\tilde{x})d\tilde{x}$$

for any $x \in X_\xi$, where $X(x,\xi) = X_\xi \cap (-\infty; x]$.

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- Write $f > 0$ if $f(x) > 0$ for all $x \in X_\xi$.
- If $f > 0$ define *virtual valuation*: $J_{\xi,f}(x) = x - \overline{F}_\xi(x)/f(x)$ on X_ξ (reference to ξ will be dropped).

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  - **Condition (Z):** At any $x \in X$, the limit $\lim_{\varepsilon \to 0+} f(y+\varepsilon)$ exists and $\limsup_{\varepsilon \to 0+} f(y-\varepsilon) \leq g(y) = \lim_{\varepsilon \to 0+} f(y+\varepsilon)$.

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    - Intuition: (Right-hand upper Dini) derivative of the density needs to be sufficiently informative.

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- **Proposition 1.** *Consider an arbitrary random variable* ξ *that takes on values in an open interval* $X \subseteq \mathbb{R}$ *with probability one, and let* $f > 0$ *be a density function for* ξ*. Then* $\overline{F}(x) > 0$ *on* X*. Moreover, provided that both (Z) and (CL) are satisfied,* J_f *is nondecreasing if and only if* $1/\overline{F}$ *is convex on* X*.*

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- Proof technical due to minimalistic assumptions.
- Two useful implications.

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- The rate by which this density increases over time is the "zoom rate" $f(x)/(1 - F(x))^2$.
- Increasing zoom rate = new criterion.

Condition on density

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- Tightest possible within ρ-concavity

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- Density functions with jumps

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- **Also in the paper:** Discussion of regularity checks in cases where criteria have no bite

Table I. Distributions *with* $(-\frac{1}{2})$-concave density functions.†

Name of distribution	Interval X	P.d.f. $f(x)$	C.d.f. $F(x)$	Concavity ρ of p.d.f.
Any with log-concave density	– see Bagnoli and Bergström (2006, Table 1) –			≥ 0
Pareto ($\beta \geq 1$)	$(1;\infty)$	$\beta x^{-\beta-1}$	$1 - x^{-\beta}$	$-\frac{1}{\beta+1}$
Log-normal ($\sigma_L^2 \leq 2$)	$(0;\infty)$	$\propto \frac{1}{x}\exp(-\frac{(\ln x - \mu_L)^2}{2\sigma_L^2})$	*	$-\frac{\sigma_L^2}{4}$
Student's t ($n \geq 1$)	$\mathbb{R}$	$\propto (1+\frac{x^2}{n})^{-\frac{n+1}{2}}$	*	$-\frac{1}{n+1}$
Cauchy	$\mathbb{R}$	$\frac{1}{\pi(1+x^2)}$	$\frac{1}{2}+\frac{\arctan x}{\pi}$	$-\frac{1}{2}$
F distr. ($m_1, m_2 \geq 2$)	$(0;\infty)$	$\propto \frac{x^{\frac{m_1}{2}-1}}{(m_1 x + m_2)^{\frac{m_1+m_2}{2}}}$	*	$-\frac{2}{m_2+2}$
Mirror-im. of Pareto ($\beta \geq 1$)	$(-\infty;-1)$	$\beta(-x)^{-\beta-1}$	$(-x)^{-\beta}$	$-\frac{1}{\beta+1}$

†) The symbol $\propto$ indicates that the density function, for fixed parameters, is proportional to the term given in the table; for cumulative distribution functions marked with *, there is no closed-form representation.

Table I. (continued) Distributions *with* $(-\frac{1}{2})$-concave density functions.

Name of distribution	Interval X	P.d.f. $f(x)$	C.d.f. $F(x)$	Concavity ρ of p.d.f.
Log-logistic ($\beta \geq 1$)	$(0;\infty)$	$\frac{\beta x^{\beta-1}}{(1+x^\beta)^2}$	$\frac{1}{1+x^{-\beta}}$	$-\frac{1}{\beta+1}$
Inverse gamma ($\alpha \geq 1$)	$(0;\infty)$	$\frac{\exp(-1/x)}{\Gamma(\alpha)x^{\alpha+1}}$	*	$-\frac{1}{\alpha+1}$
Inverse chi-squared ($\nu \geq 2$)	$(0;\infty)$	$\frac{x^{-(\nu/2)-1}}{\Gamma(\nu/2)}\exp(-\frac{1}{2x})$	*	$-\frac{2}{\nu+2}$
Beta prime ($\alpha, \beta \geq 1$)	$(0;\infty)$	$\frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{B(\alpha,\beta)}$	*	$-\frac{1}{\beta+1}$
Pearson ($b_2 \leq \frac{1}{2}$)	– see discussion in the text –			$-b_2$

Table II. Distributions *without* $(-\frac{1}{2})$-concave density functions.

Name of distribution	Interval X	P.d.f. $f(x)$	C.d.f. $F(x)$	Values J'_f	Costs $\overline{J}'_f$
Power ($c<1$)	$(0;1)$	cx^{c-1}	x^c	$\ngtrless 0$	>0
Weibull ($c<1$)	$(0;\infty)$	$cx^{c-1}\exp(-x^c)$	$1-\exp(-x^c)$	$\ngtrless 0$	>0
Gamma ($c<1$)	$(0;\infty)$	$\frac{x^{c-1}\exp(-x)}{\Gamma(c)}$	$*$	$\ngtrless 0$	>0
Chi-Squared ($c<2$)	$(0;\infty)$	$\frac{x^{(c-2)/2}\exp(-x/2)}{2^{c/2}\Gamma(c/2)}$	$*$	$\ngtrless 0$	>0
Chi ($c<1$)	$(0;\infty)$	$\frac{x^{c-1}\exp(-x^2/2)}{2^{(c-2)/2}\Gamma(c/2)}$	$*$	$\ngtrless 0$	>0
Beta ($\nu<1$ or $\omega<1$)	$(0;1)$	$\propto x^{\nu-1}(1-x)^{\omega-1}$	$*$	$\ngtrless 0$	$\ngtrless 0$
Arc-Sine	$(0;1)$	$\frac{1}{\sqrt{\pi x(1-x)}}$	$\frac{2}{\pi}\arcsin(x)$	$\ngtrless 0$	$\ngtrless 0$

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Name of distribution	Interval X	P.d.f. $f(x)$	C.d.f. $F(x)$	Values J'_f	Costs $\overline{J}'_f$
Pareto ($\beta<1$)	$(1;\infty)$	$\beta x^{-\beta-1}$	$1-x^{-\beta}$	<0	>0
Log-normal ($\sigma_L^2>2$)	$(0;\infty)$	$\propto \frac{1}{x}\exp(-\frac{(\ln x-\mu_L)^2}{2\sigma_L^2})$	$*$	mixed (num.)	>0
Student's t ($n<1$)	$\mathbb{R}$	$\propto (1+\frac{x^2}{n})^{-\frac{n+1}{2}}$	$*$	$\ngtrless 0$ (num.)	$\ngtrless 0$ (num.)
F distr. ($m_1<2$ or $m_2<2$)	$(0;\infty)$	$\propto \frac{x^{\frac{m_1}{2}-1}}{(m_1x+m_2)^{\frac{m_1+m_2}{2}}}$	$*$	mixed (num.)	>0
Mirror-im. of Pareto ($\beta<1$)	$(-\infty;-1)$	$\beta(-x)^{-\beta-1}$	$(-x)^{-\beta}$	>0	<0
Log-logistic ($\beta<1$)	$(0;\infty)$	$\frac{\beta x^{\beta-1}}{(1+x^\beta)^2}$	$\frac{1}{1+x^{-\beta}}$	<0	>0
Inverse gamma ($\alpha<1$)	$(0;\infty)$	$\frac{\exp(-1/x)}{\Gamma(\alpha)x^{\alpha+1}}$	$*$	$\ngtrless 0$ (num.)	>0

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Pearson ($b_2 > \frac{1}{2}$)	– see discussion in the text –			mixed (num.)	mixed (num.)

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- Extensions (e.g., for multidimensional types)

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- Implications for both empirical work and theory