Redistribution, Distance to Frontier, and Growth

Master Thesis

University of Zurich

Department of Economics

Prof. Dr. Josef Zweimüller

Supervisor: Dr. Christian Kiedaisch
Author: Noe Reidt
Course of studies: Economics
Email: reidtnoe@gmail.com
Student ID: 11-744-596
Hand in date: 23.08.2016
Abstract

This thesis presents a theoretical analysis of the effect of redistribution on growth with respect to the distance to the technological frontier. Far few studies consider that redistribution has a direct effect on growth and an indirect effect through the change in inequality. This analysis however is based on four channels through which redistribution could have a direct effect on growth and five channels through which redistribution could have an indirect effect. Firstly, it is analysed what the general effect of redistribution on growth is through each channel and secondly, how this effect depends on the distance to the technological frontier. The effect of one important and well-known channel, namely the physical and human capital investment channel, is analysed by a theoretical model. The results show that redistribution can have an effect on growth directly and indirectly and that this effect is highly dependent on the distance to the technological frontier. These results need to be considered to get a better understanding of the impact of redistribution on growth.
# Table of content

1. Introduction ................................................................. 1

2. Inequality and redistribution ........................................ 3

3. Distance to the technological frontier .............................. 8

4. Income inequality, redistribution, distance to frontier and growth .......... 10

4.1 Direct effects .................................................................. 11

   4.1.1 Information asymmetries ......................................... 12

   4.1.2 Implementation costs ............................................... 16

   4.1.3 Redistribution and incentives: Equity versus efficiency? .... 17

   4.1.4 Social freeloader ..................................................... 24

4.2 Indirect effects ............................................................... 26

   4.2.1 Physical and human capital investment ..................... 26

   4.2.2 Social instability ..................................................... 46

   4.2.3 Incentives ............................................................... 47

   4.2.4 Earning-related preferences ................................... 48

   4.2.5 Political economy .................................................. 50

4.3 Conclusion table ........................................................... 50

5. Conclusion ....................................................................... 53

References ........................................................................... 55
Figures

Figure 1: Income and life expectancy by Sen (1997, p. 387).......................................................... 3
Figure 2: Direct and indirect effects. Figure based on Thewissen (2013, p. 6).................................11
Figure 3: Dynamic process of bequests without exogenous growth..............................................30
Figure 4: Dynamic process of bequests without exogenous growth and with redistribution. 33
Figure 5: Dynamic process of bequests without exogenous growth and with credit market. 35
Figure 6: Dynamic process of bequests with exogenous growth....................................................38
Figure 7: Dynamic process of bequests for a developing country with exogenous growth....43
Figure 8: Dynamic process of bequests for a developed country with exogenous growth....44
Figure 9: Innovative goods variety by expenditure group by Kiedaisch (2016, p. 2). ...............49

List of tables

Table 1: Conclusion table ..................................................................................................................51
1. Introduction

In 2014 the famous book “Capital in the Twenty-First Century” of Piketty (2014) intensified the discussion about inequality and redistribution. Inequality is increasing in developed countries (e.g. Piketty, 2014), but also in developing countries (e.g. United Nations Development Programme, 2013) and this trend seems to continue over the next decades. By now huge amounts of money are transferred by redistribution and redistribution belongs to one of the most important government actions. Therefore, it’s essential to know what economic consequences a redistribution policy has for a country.

A direct consequence of redistribution is that it could affect economic growth by itself. For a long time there was the general meaning that redistribution by itself has a negative effect on growth. The most frequently mentioned arguments were that the transfer would be costly and that redistribution decreases the incentive to work hardly. (Okun, 1975) However, more recent studies challenge this general result because they detected channels through which redistribution could have a positive impact on growth, like the establishment of a kind of social insurance against risks. Another, but not less important consequence is the change of inequality caused by redistribution, which could affect economic growth as well. The question about how inequality affects growth has been a highly discussed topic in economics for years. In the 1950es and 60es there was the general understanding that inequality is beneficial for growth as it increases the savings which in turn increases the investments. (e.g. Kaldor, 1957) However, in the 1990es several theories and empirical papers challenged this understanding and state that the effect of inequality on growth could be negative as well. It can be seen that the effect of a redistribution mechanism on growth is an enduring and controversial topic, but it remains uncertain if a general statement holds for different countries.

In this regard, the pioneering paper of Galor and Zeira (1993) generates an interesting relation: It states that the effect of inequality on growth depends on the development level of a country. This relation coincides with more recent studies about the effectiveness of growth strategies regarding the distance to the technological frontier.
These studies argue that different distances to the technological frontier require different growth strategies. Consequently, it is important to consider this point in the analysis of the effect of inequality and redistribution on growth.

Therefore, this thesis provides a comprehensive overview over the effects of redistribution and inequality on growth with respect to the distance to the technological frontier. The thesis reviews some of the most important channels from the economic literature, through which inequality and redistribution could affect growth. As a second step, it is analysed theoretically how the effect of these channels depends on the distance to the technological frontier. Furthermore, the probably most famous channel “Physical and human capital investment” is analysed with a new theoretical model based on Galor and Zeira (1993).

The thesis is structured as follows: Chapter 2 gives an understanding about the concept of inequality and redistribution and Chapter 3 about the concept of the distance to the technological frontier. These two Chapters are considered as the basis of this thesis. The main part can be found in Chapter 4. Chapter 4 contains the topic about the effect of inequality and redistribution on growth with respect to the distance to the technological frontier. As a last point, a conclusion can be found in Chapter 5.
2. Inequality and redistribution

This chapter gives a short introduction to the world of inequality and redistribution from the point of view of an economist. The majority of economists use the income inequality as a measure for inequality. This method is a verifiable and commonly accepted approach. However, Sen (1997) suggests that the economists should shift their attention from income inequality to economic inequality. The economic inequality focuses on what a person is able to get out of its income: The income functions help individuals to achieve states which they desire and to do things which they value.

Figure 1 illustrates that the relationship between income, personal achievements and freedom may not be constant if we assume that the quality of life correlates with the life expectancy. Figure 1 shows, for example, that the GDP per capita in Sri Lanka is much less than the GDP in Gabon, but the life expectancy in Sri Lanka exceeds the one in Gabon by almost 20 years. It follows that some factors may impede the conversion of income into life expectancy for this example. Some sources could lead to systematic variations in the conversion of income inequality to economic inequality. For example (1) personal heterogeneities: The needed income to achieve a lifestyle may differ
between different physical characteristics like age, disability, illness or gender. (2) Environmental diversities: flooding, heavy rainfalls and so on could impede the conversion and (3) variation in social climate: the public health care, the public educational arrangements and the extent of crime in the region could also influence the conversion. If the state focuses on the economic inequality instead of the income inequality it may have a better understanding about the real inequality. (Sen, 1997) Nevertheless, this thesis will focus on the income inequality due to it’s easier to measurability and it suits better to the purpose of this work.

Income inequality is measurable as a periodic or as a lifetime inequality. The periodic income inequality considers the income of all individuals over a period, independent of their characteristics. A problem may be that the periodic inequality does not consider that a senior worker earns more than a beginner. The concept of lifetime inequality tries to solve this problem by considering the age-income profile. This approach justifies some periodic inequalities as inevitable and natural. Paglin (1975) argues that the focus on periodic inequality measured by the Gini coefficient may not be meaningful and could distort the reality. One should rather use a tool which accounts for the age-income profile, for example the Paglin-Gini coefficient. However, the consideration of the lifetime inequality would go beyond the scope of this thesis. Therefore, this work focuses on the periodic income inequality and neglect the lifetime income inequality.

The topic around inequality is often accompanied by the delicate subject of poverty in economic literature. A higher inequality does not necessary implicate a higher poverty, but it implicates a better possibility to hamper poverty by the use of redistribution. Poverty could have a lot of negative consequences, as for a poor individual as well as for a society. A highly discussed consequence of poverty is that poor individuals are not able to satisfy basic needs like education and health. The need of an amount of money to get access to the education or health system may exclude some poor individuals from the advantages of these systems. In the case of education this could lead to the famous poverty trap. Poor individuals or even poor dynasties may never be able to overcome the threshold in order to get education and therefore they will be trapped in poverty. The lack of access to the health system could have even more
serious consequences for an individual because literally its life could depend on it.

Deaton (2003) shows empirically that poverty has a negative effect on population health but also that income inequality is no major determinant of population health. However, the use of redistribution in order to lower inequality could reduce the poverty and therefore be beneficial for population health.

According to Piketty (2014), an increase in inequality belongs to the capitalism. He claims that the rate of return on capital is generally higher than the growth rate of income and of output in a capitalistic society. Hence, capital may reproduce faster than income and output increases. Furthermore, he argues that uncontrolled growth of inequality could be dangerous for the democracy and the economy and therefore it is necessary to control this growth of inequality by using redistribution. All in all, the impact of income inequality on welfare leads to an incentive to change the income inequality on purpose by using redistribution. Following are three motives from Boadway and Keen (2000) which may drive the incentive to redistribute.

(i) Redistributed social justice: This motive focuses on the individual sense of fairness. It is acknowledged that people do rather consider the relative income than the absolute income (e.g. Easterlin, 1974, Loewenstein, Thompson and Bazerman, 1989, or Akerlof and Yellen, 1990). The consideration of the relative income displays that individuals are interested in what others get. Furthermore, Cappelen, Sørensen and Tungodden (2010) show that if individuals are not responsible for their output then they consider inequality as unfair and Fehr and Falk (2002) show that individuals are willing to give up some income to remove inequality they sense as unfair. As a consequence, the consideration of ethical aspects leads to an incentive to redistribute in order to achieve social justice.

(ii) Efficient redistribution: The possibility to achieve an efficiency gain through a change in the income inequality drives here the incentive to redistribute. According to Boadway and Keen (2000) there are at least three different meanings of the term efficiency gain in the literature: a Pareto improvement, a potential Pareto improvement and a total output increase. The following will focus on the efficiency gain as a Pareto improvement.
One may think that the term redistribution by itself excludes the possibility of a Pareto improvement due to its nature that it takes away from someone and gives it to others. But this is not necessary the case. The motivation to redistribute could for example be caused by altruistic behaviour. In this case, the redistribution would be voluntary and could increase the utility of the receiver and of the donor. Therefore, if some individuals possess altruistic behaviour then redistribution could lead to a Pareto improvement. (Hochman and Rodgers, 1969)

However, not only the intrinsic motivation to redistribute might lead to a Pareto improvement, but also the possible effect of income inequality and redistribution on growth. If the dynamic gains outweigh the static losses then a long term Pareto improvement would be possible. Furthermore, redistribution could lead to a kind of social insurance as an individual with bad luck would receive a part of the more lucky individuals: If there is uncertainty in the future and some individuals are risk averse then a social insurance could be beneficial for the individuals. The utility from a social insurance may exceed the loss of the donors caused by redistribution and therefore may lead to a Pareto improvement. The topic “social insurance” will be discussed more briefly in Section 4.1.3.2.

An extension of the traditional redistribution which focuses on a pure money transfer to a “modern” redistribution which includes non-monetary components, for example a road link between two cities, could also lead to a Pareto improvement. Such “modern” redistribution might offer a solution for the famous free-rider problem which can occur when public goods are privately provided. As a result, public goods may be underfinanced. This problem could be overcome by “modern” redistribution because the public goods could be provided by means of a redistribution mechanism to overcome the inefficiency of a free-rider problem. Hence, a Pareto improvement might be possible. However, this work focuses on the traditional meaning of redistribution and only considers pure money transfers. The redistribution could increase the efficiency through even more channels like using intergenerational transfers to attain dynamic
efficiency gains or using redistribution to overcome distortions which prevent the economy to achieve the first best outcome like credit market restrictions or distorted incentive constraints. (Boadway and Keen, 2000)

(iii) Redistribution as expropriation: Self-interested individuals with the power to redistribute have the incentive to redistribute in their favour. Individuals could, for example, use redistribution to enrich themselves or to keep the power, respectively to strength it.

To change the income inequality one has a lot of different possibilities to redistribute. Much noticed redistribution tools are the relative capital income taxation and the relative labor income taxation. This work mainly uses a relative labor income tax with a lump-sum-redistribution.
3. Distance to the technological frontier

The distance to the technological frontier represents country specific technological development levels compared with technological development levels of other countries at the same point in time. It shows how far a country is away from the best performance achieved by any other country, in which the technological level of the technological leader acts as the frontier. Therefore, this distance illustrates the relative position of a country regarding the technological level compared with the other countries.

The technological performance is an important input factor in many growth models (e.g. Romer, 1990). Hence, different technological levels are implementable and therefore it is possible to analyse the theoretical consequences of the distance to the technological frontier.

An interesting aspect is the relation between the possibility to imitate technologies and the distance to the technological frontier. The farther away a country is from the frontier, the more imitable technologies does it have. A country near to the frontier might reach technological progress mainly through innovation due to the lack of imitable technologies. This induces that the strategy of a country in order to reach technological progress may depend on the distance to the technological frontier: A country near to the technological frontier may follow an innovation-based strategy while a country which is farther away may rather follow an imitation-based strategy.

Different economic papers support the claim that growth maximizing strategies should depend on the distance to the technological frontier, in particular: Acemoglu, Aghion and Zilibotti (2002) analyse the topic about the importance of manager selection (low and high skilled managers) in different development levels, whereas Vandenbussche, Aghion and Meghir (2006) refer more generally to the optimal composition of human capital for a certain development level. Amable, Demmou and Ledezma (2008) analyse the effects of competition on countries with different technological levels and Acemoglu, Zilibotti and Aghion (2003) deal with the relation between vertical integration and the distance to the technological frontier. The relation between
growth maximizing policies and the distance to the technological frontier combined with the topic about inequality and redistribution is used as the basis for the next chapter.
4. Income inequality, redistribution, distance to frontier and growth

The relation of income inequality and growth is a highly discussed topic in economics. It is generally acknowledged that income inequality has an impact on growth, but how and in which direction this effect acts on growth is controversial. The presence of an impact on growth leads to certain relevance of income inequality regarding to growth enhancing policies and strategies. Growth maximizing policies might require adjustments in income inequality. The common way to do this is the use of redistribution. A redistribution mechanism could induce the level of the desirable income inequality in order to maximize growth. However, the use of redistribution by itself could also have a separate impact on growth, for example by changing incentives. This means that one, who wants to adjust the income inequality to maximize growth, has to consider the effect on growth caused by redistribution as well. This leads to an even more complex problem.

The theoretical effect of income inequality and redistribution on growth is ambiguous and depends on at least three topics: (1) Individual thinking and behaviour: This topic is about the relation of individual nature and the income inequality and redistribution. It contains for example the sense of fairness, the attitude to risk, incentive based motivation to work and the social culture. (2) Political institutions: The political institutions provide the general framework for the society. They lay down rules and are responsible for their observation. These rules act as the boundaries for individual behaviour and determine, for example, the degree of protection of property, the market rules or the public health system, but also things like the degree of equality of opportunities for different individuals. (3) Distance to the technological frontier: The distance to the technological frontier shows the relative position of a country compared with the other countries. As shown in Chapter 3, different distances to the technological frontier implicate different needs and strategies to maximize growth. For example, a change of incentives caused by inequality and/or redistribution could have another impact on a country with a long distance to the technological frontier than on a country which is closer to the technological frontier. This third topic is considered as
basis for the way forward: The following will analyse different channels by which income inequality or redistribution could affect growth with respect to the distance to the technological frontier. The considered channels and the functioning of the cycle are illustrated in Figure 2.

4.1 Direct effects

The channels in which redistribution affects growth are classified as direct effects. As the name implies these are the effects with direct consequences on growth.

The literature around Okun (1975) and Mirrlees (1971) presented the idea of the famous equity-efficiency trade-off. They argued that the use of a redistributive taxation in order to decrease the income inequality decreases the efficiency for example due to changes in incentives. Furthermore, Okun (1975) argued that the use of redistribution by itself is inefficient due to the so called leaky bucket problem: The money taken from individual A has to be carried in a leaky bucket in order to give it to individual B. Some money is going to be lost due to the leaky bucket and individual B is not getting everything. However, the empirical works of Thewissen (2013) and Ostry,
Berg and Tsangarides (2014) do not support the existence of an equity-efficiency trade-off. They both find that redistribution, as long as it is not too extensive, is roughly growth neutral. The following discusses theoretically how redistribution could affect growth with respect to the distance to the technological frontier.

4.1.1 Information asymmetries
This channel focuses on the effect which an upcoming redistribution may have. Assuming firstly the individuals anticipate the redistribution and secondly there are information asymmetries between the government and the individuals regarding to their true income. The inclusion of these two assumptions leads for an individual to an incentive to reveal a lower income than it actually has in order to profit more or lose less from redistribution. In other words, the “incomes before redistribution” will be distorted. This could lead to two main problems:

1. The redistribution and therefore the estimated effect from the redistribution may be distorted. This could impede the effort to maximize growth.
2. The ability to reveal a lower taxable income than it actually is, may correlate positively with the relative income and that could lead to socio political problems. An individual might use, for example, black markets or the possibility to hide income at a foreign bank to reveal a lower taxable income than it has. In the following, it is argued that the upper statement could be reasonable under some assumptions.

Mincer (1974) shows that the wage function over time is often concave and mainly increasing, except at the end where it decreases in some cases. The following does not take this possible decrease into account and assumes that the wage function is concave and increasing over time. An individual is able to learn more about the financial system and to get better financial connections with every transaction it does. Furthermore, it is assumed that a high education correlates positively with high skills (e.g. Spence, 1973) and therefore the high educated individuals are able to learn faster from every transaction than the low educated individuals. If we consider that the high educated individuals
rather get a high income (e.g. Moretti, 2004, or Ashenfelter and Rouse, 1999) then we get the following framework:

The old and highly educated individuals tend to have a huge income. They did a lot of transactions in their lifetime (are therefore very experienced) and they have learned quickly from every transaction. Therefore, they have an excellent knowledge about the financial system and a huge financial network. The high educated middle-aged individuals and the low educated old individuals tend to have a high income. The high educated middle-aged individuals do not have that much experience but learn fast and the old low educated individuals have a lot of experience but do not learn that fast. This leads to a good knowledge about the financial system and a large financial network for these two groups. This mechanism continues until the youngest individuals with no education, which have the poorest knowledge about the financial system.

If the ability to cheat on the financial system correlates positively with the knowledge about it and with the integration in the financial system, then the upper framework leads to the conclusion that a higher income correlates with the ability to reveal a lower taxable income than one actually has. Another interesting and reasonable assumption is that an individual needs at least some knowledge and some connections to the financial network in order to have a realistic chance to cheat successfully. Without an advanced ability to cheat, the risk to be caught may be too excessive. This increases the importance of knowledge and leads to a bigger difference in the opportunities to cheat between individuals with high knowledge and individuals with less of it. Another argument for the correlation between the ability to cheat and the relative income is that the individuals with a relative high income are more likely able to pay for a financial expert or to use power to enforce their interests. However, the detection of a cheating individual may lead to punishment and/or to reputational damage. This may harm an individual’s connection to the financial network and decrease the actual
and future incomes. Consequently, cheating is risky and uncertain, but could be worthwhile for some individuals.

The consequences of cheating individuals could be an increase of corruption, mutual distrust, criminality and so on, which in turn could lead to social political problems. Furthermore, some individuals might see the ability to reveal a lower income than it actually is as unfair, which could increase the social political problems as well.

When the ability to reveal a lower taxable income than it actually is correlates positively with the relative amount of income and this actually leads to social political problems, then a country has the incentive to do something against it. One way to prevent or at least hamper these problems is the use of monitoring. Monitoring is a tool with which one can reduce information asymmetries. Therefore, if the government induces a monitoring system combined with a potential punishment for detected cheaters, then it could reduce the incentive to cheat. Generally, the use of monitoring is costly. This leads to the following problem regarding to the effect of redistribution on growth with respect to information asymmetries: Either the government uses redistribution without monitoring and faces the two problems outlined above which may impede the effect on growth or it uses monitoring to decrease the information asymmetry and accounts for the monitoring costs which may also impede the effect on growth. For the following, it is assumed that the country prefers to use monitoring.

Monitoring without a punishment will not be effective because then it is often worthwhile to give a try and monitoring loses its deterrent effect. Theoretically, the government could induce an enormous punishment and a little monitoring effort to lower the incentive to cheat without paying high monitoring costs. However, assuming an independent court decides about the punishment, then this will not be possible on purpose. The following simple model should illustrate the relation between the amount of redistribution and of the monitoring costs:

A country has \( n \) individuals and an unequal income distribution. Assuming a two period model where an individual \( i \) has an increasing and concave utility function \( U(c_{1i}, c_{2i}) \)
and receives only in the first period a personal work income, which may be affected through a redistribution mechanism. Furthermore, an individual decides about its saving plan and its optimal consumption path at the end of the first period. An individual $i$ chooses its optimal consumptions $c_{1i}(R)$ and $c_{2i}(R)$ which maximize its utility dependent on the amount of redistribution. We assume that only the individuals which would be negative affected by redistribution \( \frac{\partial c_{ji}^*(R)}{\partial R} < 0 \) have the ability to cheat. The government of the country announces the implication of a monitoring mechanism at the beginning of the first period. This monitoring mechanism will randomly observe the work of \( n_m \) individuals and their resulting incomes without that the individuals know who is monitored and who is not. A cheating individual gets a punishment if it is detected. This punishment lowers an individual`s utility by \( v \). Therefore, an individual which would be negative affected by redistribution cheats as long as \( \left( 1 - \frac{n_m}{n} \right) * U(c_{1i}(0), c_{2i}(0)) + \frac{n_m}{n} * \left( U(c_{1i}(R), c_{2i}(R)) - v \right) > U(c_{1i}(R), c_{2i}(R)) \) holds. This implicates that the government has to monitor \( \frac{n_m}{n} \geq \frac{U(0)-U(R)}{U(0)+v-U(R)} \), where $U(0) = U(c_{1i}(0), c_{2i}(0))$ and $U(R) = U(c_{1i}(R), c_{2i}(R))$ holds, to hinder an individual $i$ to cheat. And because $\frac{\partial c_{ji}^*(R)}{\partial R} < 0$ holds, it is straightforward that $n_m$ increases with $R$ and that the higher $n_m$ the higher are the monitoring cost and therefore the higher is the negative effect on growth.

With a view to the distance to the technological frontier, the informational infrastructure becomes relevant. It is reasonable that the amount of accruing costs for a government in order to monitor the income of an individual depends directly on the level of the country specific information infrastructure (e.g. Acemoglu and Zilibotti, 1999). According to Monteiro and Hanseth (1996) one of the important aspects of the informational infrastructure is the connection between the technological and the nontechnological elements. Therefore, the country specific informational infrastructure depends on the technological level. Consequently, the farther a country is away from the technological frontier the higher might be the monitoring costs and therefore the more negative may be the effect on growth. This could be a problem, especially for technological poor countries: Even if redistribution by itself would enhance growth, the relative high monitoring costs caused by inefficiencies could negatively outweigh
the effect. Therefore, information asymmetries may impede especially technological poor countries to tap the full potential for growth through redistribution.

All in all, the inclusion of information asymmetries and a potential upcoming redistribution could lead to inefficiencies in the redistribution process. The government is able to use monitoring in order to overcome the information asymmetries, but this causes monitoring costs which may impede growth. The little model above showed that the amount of monitoring costs is positively correlated with the amount of redistribution. Moreover, the consideration of country specific informational infrastructures leads to the conclusion that the farther away a country is from the technological frontier the higher might be the monitoring costs of individual’s incomes and therefore the more negative might be the effect on growth.

4.1.2 Implementation costs
The implementation costs are the accruing costs for the use of redistribution by itself and one of the main reasons for the leaky bucket problem stated by Okun (1975). These costs emerge because the implementation of a redistributive mechanism needs resources in form of labor and capital to work out. So the implementation costs take away resources, which could be invested in growth enhancing projects, and therefore redistribution indirectly harms growth through the implementation costs channel. As a consequence, redistribution in order to enhance growth is accompanied by implementation costs which in turn may impede growth.

In order to analyse the effect of the distance to the technological frontier on the implementation costs we assume that a government establishes an office for redistribution. This office has to do everything to execute a possible redistribution: It has to collect information, induce and execute the redistributive mechanism and analyse the results. To execute these tasks the office works with the technology parameter $A$ and needs labor $L$ and capital $K$ which cause the implementation costs through wages and interests respectively. Assuming that the office works according to the neoclassical production function $Y = A \times F(K, L)$, where $\frac{\partial Y}{\partial K}, \frac{\partial Y}{\partial L} > 0$ and $\frac{\partial^2 Y}{\partial K^2}, \frac{\partial^2 Y}{\partial L^2} < 0$ holds, the output $Y$ is the demanded redistribution by the government. Consider that the higher the level of the technology parameter $A$, the more efficient
the office uses its resources. This may be reasonable because the performance of the office depends directly on the quality of its equipment and of its workers. If the government demands a redistribution of \( Y_R \), then we get \( \frac{Y_R}{A} = F(K, L) \). This shows that the higher \( A \), the less resources are needed by the Ofr in order to produce \( Y_R \). Hence, the amount of implementation costs caused by redistribution is lower and therefore less harmful for growth the closer a country is to the technological frontier.

4.1.3 Redistribution and incentives: Equity versus efficiency?
As mentioned in Section 4.1, Okun (1975) and Mirrlees (1971) argued that there is a trade-off between equity and efficiency. The inclusion of a progressive redistribution mechanism in order to achieve more equality could reduce the incentive for individuals to increase their personal working income (Adam and Browne, 2010). This may lead to growth affecting changes in the behaviour of individuals. The rationale behind this is the fact that the inclusion of a redistribution mechanism leads to less performance-based individual earnings that in turn could affect an individual’s willingness to get out of the comfort zone to achieve a better performance. However, it is not universally acknowledged that this trade-off exists or has a relevant effect. According to Polanyi (1957), there is rather an equity-efficiency trade-in than a trade-off. The primal argument in his construction is that he sees the human nature as more social and not only as self-interested. Therefore, economic incentives are not that important and redistribution could be a natural tool to enhance the efficiency.

Nevertheless, the economic incentives for self-interested individuals have priority in the following analysis. Smith (1776) argued already that economic incentives are a key point for economic progress and a big topic in the economic literature is the discussion about the best kind of redistribution mechanism with respect to the preservation of economic incentives (e.g. S. Allegrezza, Heinrich and Jesuit, 2004, Adam, Brewer and Shephard, 2006, or Adam and Browne, 2010). However, the following focuses on the effect of a given redistribution mechanism on the economic incentives with respect to growth and the distance to the technological frontier. It will be shown that one can not necessarily assume that redistribution has a negative impact on the economic incentives.
Assuming the government induces an income taxation $\tau \ast y_j$, where $\tau \in (0,1)$ and $y_j$ is the income of an individual $j$, which finances a subsidy $R$ given to every individual in the country. In view of the equity-efficiency relation the implementation of a redistribution mechanism could lead to changes in two growth affecting arguments: In “optimal working efforts” and in “the incentive to take risks”.

4.1.3.1 Optimal working effort

The inclusion of a redistribution mechanism as above has two different effects on an individual’s decision about the working effort:

1. The non-labor income will be increased. An individual receives the subsidy $R$ irrespective of its working decision. In a static model of labor supply with individual decisions about consumption and leisure an increase in non labor income has naturally a negative effect on the amount of optimal individual working hours (if an interior solution exists). As a result, the implication of a subsidy $R$ would lead to a lower labor supply than before.

2. The labor income will be decreased. The inclusion of an income taxation $\tau \ast y_j$, directly leads to a lower labor income for every individual. In a static model of labor supply with individual decisions about consumption and leisure, a decrease in labor income has naturally an ambiguous effect on the amount of optimal individual working hours (if an interior solution exists). The ambiguous effect is caused by the possible reverse impact of the substitution effect and the income effect. Therefore, a lower work income may theoretically not necessary lead to lower individual working hours. However, Chetty et al. (2012) showed empirically, by analysing the Hicksian elasticity and the Frisch elasticity, that an individual generally decreases its working hours as a response to lower work income. This means that the lower work income caused by the redistribution mechanism seems to lower the labor supply.

Additionally, an overall effect of the implementation of the redistribution mechanism could be a higher rate of unemployment because individuals might have less incentive to seek jobs (Kenworthy, 2003). Hence, we assume that an individual will actually work less if a redistribution mechanism is induced. If the output is produced according to a
neoclassical production $Y = A \ast F(K, L)$ as in Section 4.1.2, then it is comprehensible that less labor input decreases economic growth. The question arises how this effect on growth is related to the distance to the technological frontier.

One point to consider is the ability of a country to use a new technology efficiently as fast as possible so that it has an effect on growth. Assuming an individual is using new technologies during the worktime and is able to learn by doing; this means that the more a new technology is used the more efficient is its use. The time needed to adopt the new technology in order to have a positive effect on growth is therefore extended by the reduction of work time caused by redistribution due to less time to learn by doing. This could lead to a delayed and probably less positive overall effect of the new technology on growth. Furthermore, if we consider vertical innovations and the concept of creative destruction from Schumpeter (1942) then growth could be negatively affected not only from the delay of growth enhancing effects realized through the new technology but also from the extended time without the growth enhancing effects of the replaced technology until the new technology works.

A country is able to imitate technologies from other countries and/or to develop innovations. It is reasonable to assume that it is easier and cheaper to imitate existing technologies than to develop innovations. Furthermore, as argued in Chapter 3, the farther a country is away from the technological frontier the more imitable technologies does it have. Assuming two countries differ only in their distance to the technological frontier: The poor country is far away from the frontier whereas the rich is at the frontier. Due to the easier possibility to imitate, the poor country will adopt more technologies than the rich one and catch up. This means that a country which is farther away from the technological frontier would face more new technologies. Therefore, the higher time exposure to adopt new technologies caused by redistribution would have a more negative effect on growth for the poor country than for the rich country.

Another point comes from the possible relation between education costs and the distance to the technological frontier considering the relation of the amount of possibilities to imitate technologies and the distance to the technological frontier.
Under the assumption that innovation is more complicated and therefore more skill based than imitation, Vandenbussche, Aghion and Meghir (2006) argue that skilled labor may have a more positive effect on growth the closer a country is to the technological frontier. A worker has to be educated in order to be high skilled and this causes education costs. The effect of the human capital composition on growth leads to a strategic choice for a growth maximizing country regarding to the amount of education expenditures. As a consequence, a country closer to the frontier invests more in education than a country which is farther away. This implicates that there might be more costly educated individuals at developed countries. Consequently, a decrease of every workers work time harms growth more, the closer a country is to the technological frontier due to the higher amount of educated workers, who work less as well.

4.1.3.2 The incentive to take risks

If an individual has some options with uncertain outcome, the incentives to take risks become important. The following focuses on the change of the incentives to take risks caused by a redistribution mechanism with respect to growth and the distance to the technological frontier. The effects can be separated in “the effect of a decrease in the performance based compensation” and in “the effect of a basis income”. The reason behind this: a separate analysis of these effects gives a more comprehensible framework about the relation to growth and to the distance to the technological frontier.

The effect of a decrease in the performance based compensation:

The implementation of an income taxation $\tau \cdot y_j$, where $\tau \in (0,1)$ and $y_j$ is the income of an individual $j$, as a part of a redistribution mechanism leads to a decrease in the performance based compensation. The problem regarding the incentives to take risks arises when uncertain outcomes are included. The implementation of income taxation could then hinder an individual to invest or to put effort in a risky project because the outcome is to some extent socialized in case of success. With regard to the distance to the technological frontier and growth, the topic around imitation and innovation is interesting. An innovation describes the invention and the successful
implementation of a total new technology. An innovator is able to patent the innovation and therefore to act as a monopolist with a high output, but the invention and the implementation of a new technology may be complex, effortful and its success is uncertain. As a consequence, the development of innovations causes high costs and has an uncertain outcome. An imitation describes the transfer and implementation of an existing technology. This technology is not patentable and therefore everyone is able to use it. The imitation of a technology is not that complex and its output is relative certain because it is already a standing technology in other countries.

The implementation of an income taxation lowers the incentive to put effort in the development of innovations because the upside risk (the possibility to get rich) is to some extent socialized, whereas the high costs, the high effort and the risk to develop the innovation are here still privatized. The effect of an income taxation on the incentive to imitate may be relatively low due to the low amount of costs and its relative certain outcome. The possible decline in the incentives to innovate caused by the income taxation may impede technological progress and therefore growth.

To analyse the relation between the effect on growth and the distance to the technological frontier consider again that the individuals of a country have more opportunities to imitate existing technologies the farther away their country is from the technological frontier.

For the following, a poor country which is far away from the technological frontier and a rich country which is close to the technological frontier are considered. If only one individual puts effort in imitating a particular technology then it would have an outcome in the amount of a successful development of an innovation but with lower costs. However, a possible competition could lower the outcome of an imitated technology for an individual because it is not patentable. Therefore, the majority of the individuals might focus primarily on imitating until the competition is too big and only then they will consider the possibility of innovating. Consequently, the individuals of the poor country will spend a high amount of their resources in order to imitate existing technologies with a certain output. They will not spend that much on the expensive and complex development of innovations due to enough opportunities to
imitate. However, the individuals in the rich country will spend a high amount of their resources in the development of innovations due to the lack of opportunities to imitate existing technologies. Consequently, the poor country reaches technological progress mainly through imitation and a rich country mainly through innovation. As described above, the implementation of income taxation may primarily impede the incentive to innovate and therefore could lower the amount of innovations. Consequently, the innovation-based growth of the rich country will be more negatively affected by income taxation than the imitation-based growth of a poor country. Therefore, the effect of the implementation of income taxation as a part of the redistribution mechanism on growth may be less negative the farther a country is away from the technological frontier.

The effect of a basic income:

The implementation of a subsidy $R$ as a part of a redistribution mechanism leads to a basic income for every individual independent of their performance. If uncertain outcomes are included, then this basic income acts as a kind of social insurance: An individual with bad luck still receives this basic income independent of its outcome and because the basic income comes from a redistributive mechanism, some of the individual risk is going to be spread on the shoulders of the society. It is well known that shifting risks on the basis of a mandatory social insurance is accompanied by the famous moral hazard problematic. To analyse the effect of a social insurance on growth the ex-ante moral hazard becomes interesting. With regard to the incentives of an individual to take risks it is clear that the ex-ante moral hazard caused by the social insurance tempt an individual to have a higher incentive to take risks. And considering again the topic about imitating and innovating, then this would lead to a higher incentive to put effort into the risky development of an innovation while the incentive to imitate stays the same. Consequently, a basic income could trigger a higher technological progress and higher economic growth by more innovations through the “incentive to take risks” channel.

As described in Chapter 3, the importance of innovation for growth might increase the closer a country is to the technological frontier. Therefore, the positive effect of a
social insurance on growth respecting the incentives to take risks could be higher the closer a country is to the technological frontier.

In summary, it can be stated that the effect of a redistribution mechanism on the incentives could have theoretically ambiguous consequences for growth. The implementation of a redistribution mechanism might lower an individual’s incentive to work and therefore impede growth. The relation between the distance to the technological frontier and the incentive to work with respect to growth seems to be ambiguous. Furthermore, the effect of redistribution on the incentive to take risks seems to be ambiguous as well. On one hand a decrease of the performance-based compensation caused by redistribution lowers the incentive to take risks, but on the other hand an increase in the non-labor income caused by redistribution acts as a social insurance, which may increase the incentive to take risks.

4.1.3.3 Empirical reflection
According to Ostry, Berg and Tsangarides (2014) the equity-efficiency trade-off is rejected by the data based on averages and the effect of redistribution on growth is insignificant. The empirical studies of Thewissen (2013) and Lindert (2004) do as well deny the existence of an equity-efficiency trade-off. However, Ostry, Berg and Tsangarides (2014) give evidence that further redistribution for a country which already redistributes a lot could impede growth and therefore such a country might face an equity-efficiency trade-off. In the following, some possible reasons for that based on the theory presented in Section 4.1.3.1 and Section 4.1.3.2 are discussed.

If it is assumed that these results are driven by the theory presented in Section 4.1.3.1 and Section 4.1.3.2 then this would lead to the following conclusion: As long as redistribution is not too big, the growth enhancing effect of a social insurance cancel out the growth decreasing effect of a decrease in the performance based compensation on the incentives to take risks as well as the effect of a decrease of the optimal working effort. But if redistribution overcomes a threshold value then the positive effect on growth of a social insurance may be exceeded and the equity-efficiency trade-off appears. However, the assumption that the result is only driven by the presented theory may not be reasonable. A more reasonable explanation for the
upper result could be that an individual’s behaviour is rigid and that it does not adjust its behaviour on incentive changes caused by the redistribution mechanism as long as the redistribution is not too big. This might be reasonable because individuals may have a plan for the future from which they dislike to deviate from. If the redistribution overcomes an individual’s threshold then the incentive to change its behaviour is bigger than the rigidity of it. This would lead to behavioural responses to the change of the incentives as discussed in Section 4.1.3.1 and Section 4.1.3.2 with a possible equity-efficiency trade-off as a consequence. However, these empirical results could have plenty of reasons. In summary, it can be said that a general equity-efficiency trade-off is rejected by several empirical works.

4.1.4 Social freeloader

This topic refers to the problem of adverse selection caused by a progressive redistribution mechanism. If there are many countries with unequal income distributions and with different redistributive systems, then factor mobility could lead to an adverse selection problem. The contributors (high earning individuals) would have the incentive to move to the country with the lowest redistribution while the net beneficiaries (low earning individuals) would prefer to move to a country with a high redistribution (Sandmo, 2002). If income correlates positively with the ability to enhance growth then the social freeloader problem matters for the growth maximizing choice about the optimal redistribution mechanism. It would lead to a competition between the countries regarding to a progressive redistribution, where the countries would have the incentive to lower their progressive redistribution in order to enhance their growth at the expenses of the growth of other countries. The consequence would be that the social freeloader problem impede the use of a redistribution mechanism which could potentially increase growth.

An interesting framework comes from the paper of Meltzer and Richard (1981) which claims that at least in democratic countries higher inequality may lead to higher pressure for redistribution (see Section 4.2.5). If there are two countries with different inequalities at the beginning, then the country with the higher initial inequality may not be able to prevent the contributors from going and the net beneficiaries from coming. This leads unavoidably to a two-tier system, where one country has the
growth enhancing high earning individuals while the other has just the low earning individuals.

In general, the presence of a social freeloader problem may prevent a country from using redistribution as a tool to enhance growth as the following example shows. According to Caminada, Goudswaard and Vliet (2010) this problem was one reason for the establishment of the European Union. They argue that the European Union should prevent the participating countries from a social race to the bottom and lead to convergence of the different social protection systems. Moreover, they show empirically that the social protection systems of the participating countries actually converged since the establishment of the European Union. However, the future trend is doubtful. Globally there are currently 60 million refugees, which is the highest amount since the second world war, due to wars, political oppression, economic issues and so on (Kämper, 18.06.2015). One of the main targets of the refugees, which are often not prepared for the foreign economy or not allowed to work in it, is the European Union. This influx of refugees increases the incentive for the European countries to deviate from a general social protection system in order to prevent refugees from coming to their country.

The relation between the social freeloader problem caused by the implementation of a redistribution mechanism and growth with respect to the technological frontier is straightforward: Countries which only differ in the distance to the technological frontier have different output and income levels (under normal conditions). The closer a country to the technological frontier the higher is its income level. If these countries induce the same redistribution mechanism, then the countries closer to the technological frontier are more attractive for net beneficiaries than those farther away because they will benefit more from rich countries. Consequently, the negative effect of a social freeloader problem caused by redistribution on growth might increase the closer a country is to the technological frontier.
4.2 Indirect effects

The channels in which inequality after redistribution affects growth are classified as indirect effects. A redistribution mechanism affects the net inequality and therefore it has an indirect effect on growth through those channels.

Keynes (1920) and Kaldor (1957) provide a long-lasting and general accepted understanding that income inequality is beneficial for growth. But more recent empirical literature states with few exceptions (e.g. Li and Zou, 1998) that income inequality tends to have in general a negative impact on growth. For example, Alesina and Perotti (1996) show that investment and income inequality is reversely related and therefore more income inequality may impede growth through the investment channel, Ostry, Berg and Tsangarides (2014) that net inequality is negatively correlated with faster and longer lasting growth and Berg et al. (2012) that income inequality is one of two main risk factors for sustained growth. Barro (2000) and Barro (2008) agree that net inequality has in general a negative effect on growth, but they show that this effect is even more negative for developing countries and might be positive for developed countries. The following discusses theoretically how the income inequality could affect growth with respect to the distance to the technological frontier.

4.2.1 Physical and human capital investment

This channel focuses on the effect of inequality on growth through physical and human capital investment under some restrictions. The following is segmented in two subchapters: The first is about decreasing investment returns and the second about increasing investment returns with setup costs.

4.2.1.1 Decreasing investment returns

This chapter focuses on the effect of inequality after redistribution on growth regarding to physical and human capital investments with respect to two restrictions. The first restriction is that the credit market is imperfect and the second that every individual is able to invest in physical and human capital, but with decreasing investment productivity. Whether the second restriction holds in reality is questionable: If, for example, investments in health are considered then diminishing marginal productivity is reasonable (e.g. Grossmann, 1972), but whether this holds for
example for investments in education is doubtful. When there is decreasing investment productivity, then a higher inequality combined with an imperfect credit market could lower the average productivity of investments because it is not possible to allocate the resources to the most productive investments. As a consequence, a higher inequality would be detrimental for growth. To analyse the relation of inequality on growth with respect to the distance to the technological frontier, consider that the efficiency of the credit market depends on the technology level of a country. This implicates that a country far away from the technological frontier might have a less efficient credit market, with the consequence that it is more difficult to allocate the resources to the most productive investments. Therefore, a higher inequality might have a higher negative effect the farther a country is away from the technological frontier in the case of decreasing investment productivity.

4.2.1.2 Increasing Investment returns and setup costs

In reality it is often the case that an individual has to pay a certain amount to be able to invest in human or physical capital. Take, for example, a tuition fee, which has to be paid in order to be allowed to participate in the education process. It’s not possible to pay just half and get half of the education because then one will not get the necessary certificate. Or take a look at the establishment of a firm: It is often required to do some costly basic investments to establish a successful firm at the market. If an individual is able to buy a production machine but has no money left to run it, then the firm will not work and no returns are generated. Therefore, an individual needs a certain amount of money to be able to do these basic investments. If such setup costs are combined with an imperfect credit market restriction, then the effect of inequality on growth might look different than above as analysed in the following OLB model. This model is related to the model of Galor and Zeira (1993).

Individuals and restrictions

Assuming a small economy produces one final good \( Y \), which can be used for consumption or investment. The final good is produced with basic labor \( B \), human capital \( X_H \) and physical capital \( X_K \). An individual \( i \) born in period \( t \) is endowed with one unit of basic labor and with bequest \( w_{t,i} \) in terms of final goods, which it gets from its
parent, and lives for two periods. In the first period an individual is able to invest his wealth and in the second period he gets his investment returns and decides about his optimal consumption and his optimal bequest for his child before he dies. It is assumed that an individual chooses its consumption \( c_{t+1} \) and its bequest \( w_{t+1} \) in period \( t + 1 \) in order to maximise its utility function

\[
u_{t+1} = (1 - \beta)ln c_{t+1} + \beta ln w_{t+1}
\]

where \( \beta \in (0,1) \) describes the rate at which an individual allocates its resources to the bequest for its child \( w_{t+1} \). Notice that the individuals only differ in their inherited wealth from other individuals. All individuals have the same preferences and the same basic labor. However, the individuals face two restrictions: The first restriction is about setup costs in investment opportunities. At the beginning of period \( t \) every individual has three investment opportunities:

1. The traditional investment opportunity. An individual could lend any amount \( f_T \) at the market and gets \( (1 + r)f_T \) in terms of basic labor, where \( r \) is the interest rate.
2. Investment in human capital: An individual \( i \) is able to go to university for an indivisible tuition fee of \( f_H \). As a return it gets \( x_{i,H} \) human capital.
3. Investment in physical capital: An individual \( i \) could establish a firm. An amount of \( f_K \) is needed to be successful at the market. In return the individual gets \( x_{i,K} \) physical capital. For the following it is assumed that \( f_K > f_H \) holds. This might be reasonable because the establishment of a new firm is usually way more expensive than the tuition fee for a university.

The second restriction is that the credit market is imperfect. The following identification of a credit market imperfection is from Galor and Zeira (1993): Assuming that the world interest rate \( r > 0 \) is constant over time. An individual is able to lend any amount of money at this rate. An individual can also borrow from the credit market. It is assumed that a borrower is able to evade debt payments, for example by moving to another country, but this is costly. However, the lender can monitor the borrower in order to decrease the incentive to default, but this causes monitoring costs \( m \) depending on the implemented monitoring effort. Furthermore, it is assumed
that a borrower is still able to evade the debt payments even if the lender monitors him, but then the default costs would be \( \gamma * m \), where \( \gamma > 1 \) holds. \( \gamma \) can be seen as a parameter which describes the consequences of monitoring on the borrower. This means that a higher monitoring effort increases the default costs and decreases the incentive to cheat. Therefore, if competitive financial intermediations operate on zero profits, then an individual, who borrows an amount \( d \), has to pay an interest rate \( i_d \) which covers lender’s interest rate \( r \) and lender’s monitoring costs \( m \):

\[
d * i_d = d * r + m \tag{2}
\]

The lenders choose \( m \) to be high enough in order to avoid cheating:

\[
d(1 + i_d) = \gamma * m \tag{3}
\]

Equation (2) and (3) leads to an incentive compatibility constraint:

\[
i_d = \frac{\gamma * r + 1}{\gamma - 1} > r \tag{4}
\]

This induces that the credit market is imperfect, since \( i_d > r \) holds.

1. Production function

Assume for now that a country produces the final good according to

\[
Y_t = B + \sigma X_H + \Phi X_K \tag{5}
\]

where \( B \) is accumulated basic labor, \( X_H \) accumulated human capital and \( X_K \) accumulated physical capital. In the second period an individual’s input is rewarded according to rates of return and the profitability constraints

\[
\sigma > (1 + r)f_H \tag{PC1}
\]

\[
\Phi > \sigma + (1 + r)(f_K - f_H) \tag{PC2}
\]

are assumed to be satisfied. This implicates that it would be worthwhile to scale up an investment level. When there is no credit market at all, then an individual \( i \) needs at least \( w_{t,i} \geq f_H \) to be able to invest in human capital, \( w_{t,i} \geq f_K \) to invest in physical capital and \( w_{t,i} \geq f_H + f_K \) to invest in human and physical capital in the first period. This means that \( w^1_{c,H} = f_H, w^1_{c,K} = f_K \) and \( w^1_{c,H,K} = f_H + f_K \) are the wealth thresholds: An individual with less initial wealth will not be able to invest. Considering these
thresholds and the rates of return $\frac{\partial Y_t}{\partial B} = 1$, $\frac{\partial Y_t}{\partial X_H} = \sigma$ and $\frac{\partial Y_t}{\partial X_K} = \Phi$ it is possible to set up the budget constraints for an individual $i$ with $w_{t,i}$:

$$c_{t+1} + w_{t+1} = \begin{cases} 
(1 + r)w_{t,i} + 1, & \text{if } 0 \leq w_{t,i} < w_{c,H}^1 \\
\sigma + (1 + r)(w_{t,i} - f_H) + 1, & \text{if } w_{c,H}^1 \leq w_{t,i} < w_{c,K}^1 \\
\Phi + (1 + r)(w_{t,i} - f_K) + 1, & \text{if } w_{c,K}^1 \leq w_{t,i} < w_{c,H,K}^1 \\
\Phi + \sigma + (1 + r)(w_{t,i} - f_K - f_H) + 1, & \text{if } w_{c,H,K}^1 \leq w_{t,i} 
\end{cases}$$

(6)

Note that the +1 in the budget constrain is the income from the inherent basic labor of an individual. Based on the budget constraints the individuals are able to solve their maximisation problem. Considering the utility function in Equation (1) every individual will choose its optimal future bequest according to $w_{t+1}^* = \beta \ast (\text{end of period total wealth})$. This means that the initial wealth distribution in period $t$ determines the wealth distribution in period $t + 1$. A child who inherits $w_{t+1}$ (and gets one unit of basic labor) is able to invest its wealth in period $t + 1$. In period $t + 2$ it gets its investment return and chooses its consumption $c_{t+2}$ and bequest $w_{t+2}$ for its child. As a consequence, the wealth distribution in period $t$ is the start of a dynamic process and the wealth of dynasties is determined by the initial wealth distribution. An interesting case is illustrated in Figure 3.

![Figure 3: Dynamic process of bequests without exogenous growth.](image)

The mechanism is as follows: An individual receives a bequest of $w_t$ in period $t$ and leaves bequest $w_{t+1}$ for its child. When the curve at the point $w_t$ is above the 45
degree line, then the individual will leave a bigger bequest for its child than it received from its parent. As a consequence, the child starts its life with a higher value on the x-axis than its parent. However, when the curve at the point $w_i$ is below the 45 degree line, then a parent leaves fewer bequests for its child than it received from its parent and its child starts with a lower value on the x-axis. This process continues until a parent leaves the same bequest for its child than it receives from its parent. In the case of Figure 3, it can be seen that the bequest of a dynasty could converge to one of four steady states over time.

The bequest of dynasties, which initial get $0 \leq w_{t,i} < w_{c,H}^1$, converges in the long term to steady state $A$. This means that no individual of these dynasties will ever be able to get an education or to build up a firm. They are trapped in poverty and will only use the traditional investment. Point $B$ marks the steady state at which the bequest of dynasties, which initial get $w_{c,H}^1 \leq w_{t,i} < w_{c,K}^1$, converges in the long term. Therefore, those individuals will never invest in physical capital but in traditional investment and in human capital due to the profitability constraint 1. Furthermore, the bequest of dynasties, which initial get $w_{c,H}^1 \leq w_{t,i} < w_{c,K}^1$, will converge to the steady state $C$. They will always invest in physical capital and in the traditional investment, but not in human capital due to the profitability constraint 2. Finally, the bequest of dynasties, which initially get $w_{c,H,K}^1 \leq w_{t,i}$, will converge to the steady state $D$. Those individuals will use all three investment opportunities. The results above are quite interesting; they implicate that there is no equality of opportunities at all. An individual stays in the same group of investment as its parent. However, when the individuals differ in their abilities and the output of human and physical capital investments depends on the ability, then this might be very inefficient. Consider, for example, a very intelligent individual who is not able to go to university even if it would be worthwhile for the economy because its father was poor. Or consider an individual who has a great idea for a start-up company but does not have the capital available to establish a firm. The reason is that the individuals are not able to borrow money to exploit one`s full potential. As a consequence, there would be a waste of human potential. This result highlights the importance of an efficient credit market, scholarships and supports for start-up companies.
Reconsider that the initial wealth distribution describes the entire dynamic process. The performance of a dynasty depends on the initial bequest. As a consequence, the initial wealth distribution matters for output as illustrated in the following example: If two individuals have $w_t = w_{c,H}^1 - \varepsilon$, where $\varepsilon$ is a very small number, then final good would be produced only by basic labor in the short and in the long term. However, a wealth distribution, where individual $j$ has $w_{t,j} = w_{c,H}^1 - 2 * \varepsilon$ and individual $i$ has $w_{t,i} = w_{c,H}^1$, would enable one individual to invest in human capital without changing the aggregate wealth. As the wealth distribution matters it is not far to seek that inequality matters as well. If it is assumed that $f_H$ is below and $f_K$ above average wealth, then, taking into account that physical capital has the higher return than human capital, a strong upper class might be desirable to get a high output. When more inequality strengthens this upper class, then a higher initial inequality might be beneficial for output as long as it does not weaken the underclass (which is able to invest in human capital) too much.

In the following, a country can somehow redistribute wealth just before an individual chooses its consumption and bequest in order to maximize the long term $Y_t$. The country is able to take away money from individuals (the same amount per individual), who belong to a certain investment group like human capital investors, and give it to individuals, who belong to another investment group like physical capital investors. With this kind of redistribution, a country becomes able to exploit the differences in the rates of return of the investments and the dynamic long term development for different initial bequests. The redistribution mechanism illustrated in Figure 4 takes advantage of the higher rate of return for physical capital and of the dynamic development in order to maximize $Y_t$ in the long term by maximizing the amount of physical capital investors in the long run.
Additionally, Figure 4 considers a case where the income distribution is skewed on the right. Therefore, a decrease of the end of period wealth for every traditional investor by some amount will release relatively more resources than a decrease of the end of period wealth for every physical capital investor by the same amount. This is because there are more individuals who invest traditionally than individuals who invest in physical capital. It can be seen that there are two long term equilibria: The dynasties which have an initial wealth (in the period after the redistribution) of $0 \leq w_{t,i} < w_{c,H}^1$ fall in absolute poverty and converge to point $E$ and the dynasties which have an initial wealth of $w_{t,i} > w_{c,H}^1$ converge to point $F$. Those individuals will invest everything in physical capital in the long run. As a consequence, there will be two different groups of wealth in the long term: One contains very poor individuals and the other physical capital investors. However, whether this long term wealth distribution is desirable from other perspectives is questionable.

When there is a credit market, even an imperfect one as described above, then the result becomes more complex. An individual is now able to borrow some money at certain conditions in order to overcome the setup costs. As a consequence, the wealth thresholds go down (under the assumption that $i_d$ is not too big). Assuming that PC1 and PC2 are still satisfied, then an individual $i$ with $w_{t,i} < f_H$ needs
$$\sigma + (1 + i_d)(w_{t,i} - f_H) \geq (1 + r)w_{t,i}$$  \hspace{1cm} (7)$$
to be willing to use the imperfect credit market and to invest in human capital. The critical wealth threshold for human capital $w_{c,h}^2$ is the smallest $w_{t,i}$ for which Equation (7) is still satisfied:

$$w_{c,H}^2 = \frac{(1 + i_d)f_H - \sigma}{(i_d - r)}$$  \hspace{1cm} (8)$$

With the same procedure we get the wealth threshold for physical capital $w_{c,k}^2$ (under the assumption that $w_{c,k}^2 > f_H$ holds) and the wealth threshold for human and physical capital $w_{c,h,k}^2$ (under the assumption that $w_{c,h,k}^2 > f_k$ holds):

$$w_{c,k}^2 = \frac{\sigma - \Phi + (1 + i_d)f_k - (1 + r)f_H}{(i_d - r)}$$  \hspace{1cm} (9)$$

$$w_{c,h,k}^2 = \frac{(1 + i_d)f_H + (i_d - r)f_k - \sigma}{(i_d - r)}$$  \hspace{1cm} (10)$$

It can be seen that the wealth threshold actually goes down comparing to the case without credit market. As a consequence, the budget constraint for different initial wealth levels changes. An individual $i$ with initial bequest of $w_{t,i}$ maximize its utility function in Equation (1) subject to

$$c_{t+1} + w_{t+1} = \begin{cases} 
(1 + r)w_{t,i} + 1, & \text{if } 0 \leq w_{t,i} < w_{c,H}^2 \\
\sigma + (1 + i_d)(w_{t,i} - f_H) + 1, & \text{if } w_{c,H}^2 \leq w_{t,i} < f_H \\
\sigma + (1 + r)(w_{t,i} - f_H) + 1, & \text{if } f_H \leq w_{t,i} < w_{c,k}^2 \\
\Phi + (1 + i_d)(w_{t,i} - f_k) + 1, & \text{if } w_{c,k}^2 \leq w_{t,i} < f_k \\
\Phi + (1 + r)(w_{t,i} - f_k) + 1, & \text{if } f_k \leq w_{t,i} < w_{c,h,k}^2 \\
\Phi + \sigma + (1 + i_d)(w_{t,i} - f_h - f_k) + 1, & \text{if } w_{c,h,k}^2 \leq w_{t,i} < f_H + f_K \\
\Phi + \sigma + (1 + r)(w_{t,i} - f_H - f_K) + 1, & \text{if } f_H + f_K \leq w_{t,i} 
\end{cases}$$  \hspace{1cm} (11)$$

Here again, every individual will choose its optimal future bequest according to $w_{t+1} = \beta \ast$ (end of period total wealth), which is the starting shot for a dynamic development of future bequests for a dynasty. One possible and interesting case is illustrated in Figure 5.
It can be seen that there are seven possible long term equilibria. $G$, $I$, $K$ and $M$ are stable equilibria, whereas $H$, $J$ and $L$ are unstable equilibria. A dynasty will only be in an unstable equilibrium when the initial bequest is $w_H$ for equilibrium $H$, $w_J$ for $J$ or $w_L$ for $L$. If a dynasty is not in an unstable equilibrium, then it will converge to a stable one. The dynasties which initially get $0 \leq w_{t,i} < w_H$ converge to equilibrium $G$ and therefore they will be trapped in poverty in the long run. However, not every dynasty will be in poverty in the short run. Dynasties which initially get $w_{t,i}^2 \leq w_{t,i} < w_H$ are able to invest in human capital at the beginning, but fall in poverty after a while. As a consequence, it is possible that a dynasty drops an investment level back over time. A reason for this is that the use of the credit market takes away some of the profit from an investment. Dynasties which initially get $w_H < w_{t,i} < w_J$ converge to equilibrium $I$. Some of them will invest in physical capital at the beginning, but at the end they will all be human capital investors. Furthermore, dynasties which initially get $w_J < w_{t,i} < w_L$ converge to equilibrium $K$ and are therefore physical capital investors in the long run and those dynasties which initially get $w_L < w_{t,i}$ converge to equilibrium $M$. They will invest in human and physical capital from the beginning.

These results show that the implication of a credit market, even an imperfect one, enables more individuals to invest. Therefore, policies which enhance the efficiency of the credit market could be beneficial for growth. Reconsider that the efficiency of the
credit market is caused by the opportunity of a borrower to avoid debt payments by moving to another country. In this regard, a policy which increases the international cooperation to chase these debt avoiders could increase the efficiency of the credit market. Furthermore, Figure 5 illustrates that the poverty trap is avoidable: When the interest rate for poor borrowers is small enough (see mf in Figure 5), then every poor dynasty would be able to overcome poverty over time. This result supports the use of microfinance as a solution to avoid the poverty trap.

As from now the analysis considers the case without a credit market because then the analysis becomes clearer and more understandable without changing the general result.

2. Production function with exogenous growth

For the next section the production function slightly changes to

$$Y_t = A_t \cdot (B + \sigma X_H + \Phi X_K)$$ (12)$$

, where the new factor $A_t$ describes the technological level of a country at time $t$. It is assumed that $A_t$ grows exogenously according to

$$A_{t+1} = A_t \cdot (1 + \alpha) = A_0 \cdot (1 + \alpha)^{t+1}$$ (13)$$

Furthermore, the investment opportunities adjust to the economic growth. It becomes more difficult to get basic labor in technological advanced countries. Hence, the traditional investment leads to $\frac{(1+r)}{A_t}$ basic labor instead of $(1+r)$. This might be reasonable because a higher technological level might increase the requirements for basic labor. As the economy increases, the price level rises as well, this in turn increases the setup costs for human and physical capital investment. An individual could invest $f_{t,H} = (1 + \alpha)^t f_H$ in period $t$ and gets $x_H$ human capital. Furthermore, it could invest $f_{t,K} = (1 + \alpha)^t f_K$ and get $x_K$ physical capital.

Reconsider that an individual invests in period $t$ and gets the returns in period $t + 1$. This implicates that the individuals have to invest under the conditions in period $t$ but gets the rates of return for their investment under the conditions in $t + 1$. One possible explanation for this is that the technological progress is induced by the investments. Therefore, the productivity of basic labor, human capital and physical
capital is based on the next technological level. The rates of return in period \( t + 1 \) are 
\[
\frac{\partial y_{t+1}}{\partial B} = A_{t+1}, \quad \frac{\partial y_{t+1}}{\partial X_H} = A_{t+1} \sigma \quad \text{and} \quad \frac{\partial y_{t+1}}{\partial X_K} = A_{t+1} \Phi.
\]
It is assumed that the profitability constraints
\[
\sigma > \frac{(1 + r)}{A_0} f_H \quad \text{(PC3)}
\]
\[
\Phi > \sigma + \frac{(1 + r)}{A_0} (f_K - f_H) \quad \text{(PC4)}
\]
are satisfied. If there is no credit market, then the threshold for investing in human capital is \( w_{t,c,H}^3 = f_{t,H} \), for investing in physical capital \( w_{t,c,K}^3 = f_{t,K} \), and for investing in both \( w_{t,c,H,K}^3 = f_{t,H} + f_{t,K} \). Reconsider that every individual is endowed with one basic labor. Hence, an individual \( i \) with the initial bequest \( w_{t,i} \) faces the following budget constraint:
\[
c_{t+1} + w_{t+1} = \begin{cases} 
A_{t+1} (1 + r) \frac{w_{t,i} + 1}{A_t} & \text{if } 0 \leq w_{t,i} < w_{t,c,H}^3 \\
A_{t+1} \left( \sigma + \frac{(1 + r)}{A_t} (w_{t,i} - f_{t,H}) + 1 \right) & \text{if } w_{t,c,H}^3 \leq w_{t,i} < w_{t,c,K}^3 \\
A_{t+1} \Phi + \frac{(1 + r)}{A_t} (w_{t,i} - f_{t,K}) + 1 & \text{if } w_{t,c,K}^3 \leq w_{t,i} < w_{t,c,H,K}^3 \\
A_{t+1} (1 + r) \frac{w_{t,i} + 1}{A_t} & \text{if } w_{t,c,H,K}^3 \leq w_{t,i} 
\end{cases}
\]
(14)
Consider that the thresholds can be rearranged so that \( \frac{w_{t,c,H}^3}{(1 + a)^t} = f_H, \frac{w_{t,c,K}^3}{(1 + a)^t} = f_K \) and \( \frac{w_{t,c,H,K}^3}{(1 + a)^t} = f_H + f_K \) holds. An individual maximizes its utility function in Equation (1) subject to the budget constraint.
It maximizes its future bequest \( w_{t+1} \) such that
\[
\frac{w_{t+1}}{(1 + a)^{t+1}} = \begin{cases} 
\beta \left( (1 + r) \frac{w_{t,i}}{(1 + a)^t} + A_0 \right) & \text{if } 0 \leq \frac{w_{t,i}}{(1 + a)^t} < f_H \\
\beta \left( A_0 (\sigma + 1) + (1 + r) \left( \frac{w_{t,i}}{(1 + a)^t} - f_H \right) \right) & \text{if } f_H \leq \frac{w_{t,i}}{(1 + a)^t} < f_K \\
\beta \left( A_0 (\Phi + 1) + (1 + r) \left( \frac{w_{t,i}}{(1 + a)^t} - f_K \right) \right) & \text{if } f_K \leq \frac{w_{t,i}}{(1 + a)^t} < f_H + f_K \\
\beta \left( A_0 (\sigma + \Phi + 1) + (1 + r) \left( \frac{w_{t,i}}{(1 + a)^t} - f_K - f_H \right) \right) & \text{if } f_H + f_K \leq \frac{w_{t,i}}{(1 + a)^t} 
\end{cases}
\]
(15)
holds. Here again, the initial wealth distribution initiates a dynamic process which could look like Figure 6.
It can be seen that our model with exogenous growth can be analysed with the same procedure as the model without exogenous growth. Note that here the axes take the exogenous growth into account. However, the interpretation of this figure is very similar to the one without exogenous growth. Therefore, it is referred to page 31.

3. Production function with exogenous growth and distance to the technological frontier

For the next analysis the production function again slightly changes to

$$Y_t = A_t \times (B + f(\bar{A} - A_t)X_H + g(\bar{A} - A_t)X_K)$$

(16)

, where $\bar{A}$ is the technological frontier. It is assumed that $\bar{A}$ is constant. A possible explanation could be that it is easier for countries away from the frontier to increase their technological level. Therefore, an increasing $A_t$ and constant $\bar{A}$ could display a catch-up process.

We assume that a country is able to use human and physical capital for innovating and for imitating technologies. Reconsider that a country away from the frontier has the possibility to imitate technologies from more advanced countries. The imitation of technologies increases the productivity and therefore it could be beneficial for growth.

An advantage of imitating is that one is able to adopt an already existing and successful technology. Therefore, it becomes possible to enhance the technological level without
high risk and without a high mental effort. However, in order to be able to imitate an existing technology successfully, there is a need of an appropriate infrastructure. It is assumed that there has to be a firm which provides such an infrastructure in order to use an imitated technology. As a consequence, the amount of physical capital affects the capacity of a country to imitate technologies. Consider that copying an existing technology does not require a huge mental effort as already argued above. This might be reasonable because the huge mental effort to invent this technology is already made by others. Hence, imitating existing technologies requires a high physical capital effort, but no or just a small human capital effort. As a consequence, imitating is a driving factor for the productivity of physical capital, but has no or just a small effect on the productivity of human capital.

However, instead of imitation, every country has the possibility to put effort into innovation. Unlike imitation, an innovation develops a world’s first technology. The process of innovation carries some risks that no new technology is developed or that the technology is not successful at the market. Furthermore, the process of innovation might be a knowledge intensive activity because one has to develop a technology based on the advanced technologies at the technological frontier. The use of experts and specialists could lower the risks of innovating and provide the required knowledge. It is assumed that the development of a world’s first technology is made by a university. The higher the amount of individuals in the university the higher is the ability to develop a new technology. As a consequence, the ability to develop a new technology depends directly on human capital in our model. This implication is supported by several empirical data (e.g. Dakhli and Clercq, 2004). Finally, a firm is needed to convert the new technology in a successful product and therefore to finish the process of innovation. However, in contrast to the process of imitating, where the quantity of firms is crucial, the more complex process of innovating leads to just a few innovations, hence, there are only few firms required. As a consequence, innovating is a main factor for the productivity of human capital, but has just a small effect on the productivity of physical capital.

Considering the discussion above, it might be reasonable to make the simplifying assumption that the factor $f$ in Equation (16) is driven by the effect of innovating,
where the small effect of imitating is neglected, and that the factor $g$ in Equation (16) is driven by the effect of imitating, where the small effect of innovating is neglected. As argued in Chapter 3, the farther away a country is from the frontier the more imitable technologies does it have. Furthermore, assuming that the technologies become more advanced the closer they are to the frontier, the imitable technologies are easier to get the farther away a country is from the frontier. Therefore, it is reasonable that $rac{\partial g(\bar{A}-A_t)}{\partial (\bar{A}-A_t)} > 0$ holds. This implicates that the closer a country is to the technological frontier the less positive is the effect of imitating. However, the lack of imitable technologies for developed countries increases the incentive to innovate. Furthermore, it might become easier to innovate the closer a country is to the frontier because innovations are mostly based on advanced technologies at the frontier. Hence, a country which is more used to those advanced technologies might be more innovative than others. Therefore, countries close to the frontier might have a competitive advantage over countries farther away in the run for innovations. As a consequence, it is reasonable to assume that $rac{\partial f(\bar{A}-A_t)}{\partial (\bar{A}-A_t)} < 0$ holds.

$rac{\partial g(\bar{A}-A_t)}{\partial (\bar{A}-A_t)} > 0$ and $rac{\partial f(\bar{A}-A_t)}{\partial (\bar{A}-A_t)} < 0$ displays the following: A country which is initially far away from the technological frontier has a relative high productivity of physical capital. The more this country catches up to the frontier, the less important is physical capital and the more important is human capital. This mechanism is related to the one of Galor and Moav (2004). They show that the physical capital was very important after the Industrial Revolution and that human capital became more important over time. However, this is not exactly the same as here because this analysis is focused on the consequences of technological differences between countries at one point in time.

Under those assumptions, a catch-up process over time can be modelled. We assume that a country produces according to the production function in Equation (16). Furthermore, the technological frontier is $\bar{A} = A_0 \times (1 + a)^z$. In the following, we analyse a country which starts a catch-up process at $t = 0$ and grows until $t = z$ holds. The framework is the same as in 2. production function with exogenous growth on page 36. It is assumed that there is no credit market. Furthermore, the economy grows exogenously as in Equation (13). In period $t$, an individual could invest in three
different investment opportunities described on page 36. In period $t + 1$ the investments are rewarded according to the rate of return of the production function in Equation (16) in period $t + 1$ and the individual chooses its optimal consumption and future bequest according to the utility function in Equation (1). It is assumed that the profitability constraints

$$f(\bar{A} - A_t) > \frac{(1 + r)}{A_0} f_H$$  \hspace{1cm} (PC5)

$$g(\bar{A} - A_t) > \frac{(1 + r)}{A_0} f_K$$  \hspace{1cm} (PC6)

$$PC 7:\left\{\begin{array}{l}
g(\bar{A} - A_t) > f(\bar{A} - A_t) + \frac{(1 + r)}{A_0} (f_K - f_H), \; if \; A_t < A_c \\
g(\bar{A} - A_t) = f(\bar{A} - A_t) + \frac{(1 + r)}{A_0} (f_K - f_H), \; if \; A_t = A_c \\
g(\bar{A} - A_t) < f(\bar{A} - A_t) + \frac{(1 + r)}{A_0} (f_K - f_H), \; if \; A_t > A_c
\end{array}\right.$$  \hspace{1cm} (PC7)

holds. Note that the profitability constraint 5 and 6 induces that an individual who is able to invest in human and in physical capital will actually invest in both. If there are individuals who are able to invest in both, then this could account for the small amount of human capital, which might be necessary for an imitation-based strategy or for the small amount of physical capital, which might be necessary for an innovation-based strategy. Profitability constraint 7 shows that there is a critical technological level $A_c$. It is assumed that $A_0 < A_c < A_z$ holds. Note that there are three different developing phases:

(i) A country is a developing country when $A_t < A_c$ holds.

(ii) A country is an emerging country when $A_t = A_c$ holds.

(iii) A country is a developed country when $A_t > A_c$ holds.

We consider a country which is a developing country with $A_t < A_c$ from $t = 0$ to $t = c - 1$, and which emerges at time $t = c$ to an emerging country and finally which is a developed country with $A_t > A_c$ from $t = c + 1$ to $t = z$. Considering the case where $t \in [0, c)$, then every individual would prefer to invest in physical capital instead of human capital due to profitability constraint 7. Therefore, an individual $i$ with bequest $w_{t,i}$, who invests in period $t$, faces the following budget constraint:
\begin{equation}
\begin{aligned}
c_{t+1} + w_{t+1} &= \begin{cases} 
A_{t+1} \left( \frac{1+r}{A_t} w_{t,i} + 1 \right), & \text{if } 0 \leq w_{t,i} < w_{t,c,H}^3 \\
A_{t+1} \left( f(\bar{A} - A_t) + \frac{(1+r)}{A_t} (w_{t,i} - f_{t,H}) + 1 \right), & \text{if } w_{t,c,H}^3 \leq w_{t,i} < w_{t,c,K}^3 \\
A_{t+1} \left( g(\bar{A} - A_t) + \frac{(1+r)}{A_t} (w_{t,i} - f_{t,K}) + 1 \right), & \text{if } w_{t,c,K}^3 \leq w_{t,i} \leq w_{t,i}
\end{cases}
\end{aligned}
\end{equation}

An individual maximizes its utility function in Equation (1) subject to the budget constraint above. It maximizes its future bequest \( w_{t+1} \) such that

\begin{equation}
\begin{aligned}
\frac{w_{t+1}}{(1+a)^{t+1}} &= \begin{cases} 
\beta \left( (1+r) \frac{w_{t,i}}{(1+a)^{t+1}} + A_0 \right), & \text{if } 0 \leq \frac{w_{t,i}}{(1+a)^t} < f_u \\
\beta \left( A_0 (f(\bar{A} - A_t) + 1) + (1+r) \left( \frac{w_{t,i}}{(1+a)^{t+1}} - f_h \right) \right), & \text{if } f_h \leq \frac{w_{t,i}}{(1+a)^t} < f_k \\
\beta \left( A_0 (g(\bar{A} - A_t) + 1) + (1+r) \left( \frac{w_{t,i}}{(1+a)^{t+1}} - f_h \right) \right), & \text{if } f_h \leq \frac{w_{t,i}}{(1+a)^t} < f_k
\end{cases}
\end{aligned}
\end{equation}

holds. An analysis of this result will be provided later on. When the developing country becomes an emerging country at \( t = c \) and \( A_t = A_c \) holds, then an individual is indifferent between investing in physical or human capital. If it is assumed that an individual prefers human capital in case of indifference, then it becomes possible to set up one equation for the budget constraint for case (ii) and (iii). If \( t \in [c,z] \), then every individual prefers to invest in human capital instead of physical capital. Therefore, an individual \( i \) with bequest \( w_{t,i} \), who invests in period \( t \), faces the following budget constraint:

\begin{equation}
\begin{aligned}
c_{t+1} + w_{t+1} &= \begin{cases} 
A_{t+1} \left( \frac{1+r}{A_t} w_{t,i} + 1 \right), & \text{if } 0 \leq w_{t,i} < w_{t,c,H}^3 \\
A_{t+1} \left( f(\bar{A} - A_t) + \frac{(1+r)}{A_t} (w_{t,i} - f_{t,H}) + 1 \right), & \text{if } w_{t,c,H}^3 \leq w_{t,i} < w_{t,c,K}^3 \\
A_{t+1} \left( g(\bar{A} - A_t) + f(\bar{A} - A_t) + \frac{(1+r)}{A_t} (w_{t,i} - f_{t,K} - f_{t,H}) + 1 \right), & \text{if } w_{t,c,K}^3 \leq w_{t,i} \leq w_{t,i}
\end{cases}
\end{aligned}
\end{equation}

An individual maximizes its utility function in Equation (1) subject to the budget constraint above. It maximizes its future bequest \( w_{t+1} \) such that

\begin{equation}
\begin{aligned}
\frac{w_{t+1}}{(1+a)^{t+1}} &= \begin{cases} 
\beta \left( (1+r) \frac{w_{t,i}}{(1+a)^{t+1}} + A_0 \right), & \text{if } 0 \leq \frac{w_{t,i}}{(1+a)^t} < f_u \\
\beta \left( A_0 (f(\bar{A} - A_t) + 1) + (1+r) \left( \frac{w_{t,i}}{(1+a)^{t+1}} - f_h \right) \right), & \text{if } f_h \leq \frac{w_{t,i}}{(1+a)^t} < f_k + f_h \\
\beta \left( A_0 (g(\bar{A} - A_t) + f(\bar{A} - A_t) + 1) + (1+r) \left( \frac{w_{t,i}}{(1+a)^{t+1}} - f_h - f_h \right) \right), & \text{if } f_h + f_k \leq \frac{w_{t,i}}{(1+a)^t}
\end{cases}
\end{aligned}
\end{equation}
holds. In the following we analyse the upper results graphically. The most interesting case of the first period, where a country starts at $t = 0$ as a developing country and becomes an evolving country at $t = c$, is illustrated in Figure 7.

Consider that in the case of Figure 7 it is assumed that the increase of the rate of return of human capital neutralize the decrease of the rate of return of physical capital over time for investors who invest in both. A country starts as a developing country with a long distance to the technological frontier. Hence, the country’s innovative power is low, but its ability to imitate technologies is high. As a consequence, the developing country will follow an imitating-based strategy, which needs a high amount of physical capital but no or just a small amount of human capital. Therefore, the productivity for physical capital is bigger than for human capital. As the technological level of the country increases, the innovative power rises and the ability to imitate technologies falls. This implicates that the rate of return for physical capital decreases while the rate of return for human capital increases. However, as long as $A_t < A_c$ holds, the individuals still prefer to invest in physical capital instead of human capital. To use the ability to imitate technologies efficiently, the wealth distribution should enable individuals to invest in physical capital in order to increase output. If it is assumed that only the upper class is able to invest in physical capital, then a higher inequality might be beneficial for a developing economy.
However, Figure 7 illustrates only the medium-term process because the dynamic process is not finished yet. The process continues at $t = c$ and does not end until $t = z$ as illustrated in Figure 8. Once the country overcomes critical technological level $A_c$ and emerges to a developed country, it shifts its imitation-based strategy to an innovation-based strategy. As a consequence, the productivity of human capital is higher than the productivity of physical capital.

![Figure 8: Dynamic process of bequests for a developed country with exogenous growth](image)

Therefore, the individuals invest in human capital even though they would be able to invest in physical capital. As the technology level increases, the returns for human capital increases even more while the returns for physical capital continue to decrease. It is assumed that the green line is reached at $t = z$. Then there are some long term equilibria: The dynasties with initial bequest of $0 \leq w_t < f_{t,H}$ will never be able to invest in human capital and their bequests converge to steady state $R$. Therefore, they will be in the poverty trap. The dynasties with initial bequest of $f_{t,H} \leq w_t < f_{t,H,K}$ invest in human capital and converge to steady state $S$. However, the dynasties with initial bequest of $f_{t,K} \leq w_t < f_{t,H,K}$ invest in physical capital as long as the country is a developing country and change to human capital when the country becomes developed. The dynasties with $w_t \geq f_{t,H,K}$ will always invest in both and converge to steady state $T$. 
Reconsider that from $t = 0$ to $t = c - 1$ it was important to have a strong upper class because the return for physical capital was higher than for human capital. However, from $t = c + 1$ to $t = z$ the situation is reversed: Here it is important to have a wealth distribution which enables the highest amount of individuals to invest in human capital. As it is assumed that $f_{t,H}$ is below the average, the wealth distribution should be beneficial for the lower class so that the majority of individuals are able to invest in human capital. Furthermore, it becomes more important to prevent individuals from poverty, so that they are able to overcome the human capital threshold. As a consequence, lower inequality might be beneficial for a developed country.

These results illustrate that policies supporting start-up companies are especially important for developing countries. An important point to consider for a developing country nowadays is the social stability in order to decrease the future uncertainty. A social stable developing country is attractive for physical capital investments, but when there is something like war or high corruption, then, independent from the rate of return for physical capital, there is less incentive to invest. Furthermore, the results show that policies implementing scholarships are especially important for developed countries in order to use the full potential of the population.

In summary, it can be stated that if there are decreasing investment return with setup costs and an imperfect credit market, then the distance to the technological frontier has an impact on the effect of inequality on economic output. The reason is that the distance to the technological frontier affects the returns for human and physical capital, which in turn has an effect on the optimal inequality. Assuming that the exogenous growth is caused by a given initial inequality, we can analyse which inequality would be beneficial for growth, considering the distance to the technological frontier. A country far away from the frontier has a high ability to imitate technologies but low innovation power. Therefore, this country might have the incentive to follow an imitating-based strategy. As imitating is physical capital intensive, the returns for physical capital are higher than the return for human capital. Under the assumption that only the upper class is able to invest in physical capital, a high inequality which strengthens this upper class might be beneficial for growth. However, as the country approaches to the technology frontier, the ability to imitate
decreases while the innovative power increases. Therefore, a country will, at a certain technology level, switch from an imitating-based strategy to an innovation-based strategy. To follow an innovation-based strategy, human capital becomes more important. As a consequence, the returns for human capital exceed the returns for physical capital. Under the assumption that the investment threshold for human capital is below the average wealth, a low inequality which strengths the lower class and prevent individuals from the poverty would be beneficial for growth.

4.2.2 Social instability

This channel considers that inequality might lead to unhappiness which in turn could lead to social instability. The reason could be that individuals are not just interested in what they get, but also in what the others get. It is generally acknowledged that individuals do social comparisons and rather consider the relative income than the absolute income. (e.g. McBride, 2001, or Clark, Frijters and Shields, 2008) This statement is supported by the famous Easterlin paradox stated by Easterlin (1974), which claims for several developed countries (and according to Easterlin et al. (2010) also some developing countries) that increasing incomes have no noticeable effect on the average happiness in long term. When the happiness of individuals depends on the relative income, then a higher inequality would lead to a higher difference in relative incomes and therefore, it would lead to more unhappy individuals. As unhappy individuals tend to have a higher incentive to use disruptive activities, like crime, to change the unsatisfactory state, inequality might affect the social stability negative.

The effect of income inequality on social instability could affect growth in two ways: The first way refers to the potential redundant waste of resources like time and effort for disruptive activities by the relatively poor individuals. A higher inequality increases these resources and therefore impedes growth because the resources spent in disruptive activities could be used for productive activities. (Barro, 2000) The second way refers to the effect of social instability on the amount of investments. A higher social instability increases the risks and uncertainty for a country’s future development, which in turn decreases the incentive to invest and therefore is detrimental for growth. Alesina and Perotti (1996) performed a cross sectional study with a sample of 70 countries for the period of 1960-1985 and found that income
inequality increases the social instability and that the social instability decreases investments which in turn decreases growth.

The inclusion of the distance to the technological frontier seems to have no relevant impact on the effect of inequality on growth through the channel social instability. Muller (1988) showed empirically for 33 democracies in 1960-1980 that the income inequality was negative correlated with social stability and that this effect was independent of the economic development of the considered countries. This might be reasonable because countries that only differ in the distance to the technological frontier just have different levels in absolute incomes, but not in relative incomes.

4.2.3 Incentives

Assume that an income distribution mirrors a ranking of all individuals of a country regarding to their individual output. The individual who achieves the highest output receives the highest income and vice versa. This induces a competition between the individuals regarding to their relative position in the ranking. It is assumed that everyone differs in his inherent talent and that every individual can choose its individual costly effort. Furthermore, the talent and the effort correlate positively with the output. If a higher effort has a high enough impact on the output, then a higher income inequality would intensify the competition for incomes and increase the incentive to use more effort in order to be higher-ranked or to avoid a down-ranking because a change in the rank position then has a bigger effect. The effort increasing effect of a higher inequality would clearly have a positive impact on growth. Furthermore, it is reasonable to assume that innovations lead to a very high output and that its success rate is dependent on individual’s talent and its effort. On this view a higher inequality could lead to a higher incentive to innovate.

To identify the impact of the distance to the technological frontier on the effect of inequality on growth through the incentive channel consider that an individual usually has a concave utility function. When an individual actually has a concave utility function and monotonic preferences, then it has a decreasing marginal utility. We assume that two countries differ only in their distance to the technological frontier. This implicates, under normal conditions, that the country which is closer to the
technological frontier has a higher income level than the country which is farther away. As argued before, the impact of an increase in inequality on growth is caused by an increase in the effort due to an intensified competition. With the inclusion of decreasing marginal utility, this impact on growth depends on the income level: The higher the income level the less the individuals care about the intensified competition for incomes because a slightly higher or slightly lower income then has a smaller impact on utility. As a consequence, the closer a country is to the technological frontier the less are the individuals willing to increase their effort as an answer to an increase in inequality and hence, the less positive is the effect of a higher inequality on growth through the incentive channel.

4.2.4 Earning-related preferences

This channel focuses on the possibility of earning-related preferences. The consumption pattern might change with income. Hence, a rich individual might prefer another consumption bundle than a poor individual. This implicates that income inequality could have an effect on aggregate consumption. One highly discussed topic regarding the earning-related preferences is about earning-related saving preferences. Keynes (1936) argued that the consumption function is concave, where the marginal propensity to consume is higher for poor individuals and lower for rich individuals. This would, under normal conditions, lead to a higher saving rate for rich individuals. (e.g. Kaldor, 1957, or Bourguignon, 1981) When the saving function is convex and monotone increasing in income, then higher inequality would implicate higher aggregate savings and therefore higher inequality could increase investments which might be beneficial for growth. Assume two countries only differ in the distance to the technological frontier. This implicates that those two countries have different average income levels. Based on the assumption of a convex and monotone increasing saving function, a higher inequality leads to a more positive effect on aggregate savings the higher the average income is. Therefore, the closer a country is to the technological frontier the more positive is the effect of income inequality on growth through earning-related saving preferences.

Another topic is about earning-related consume preferences. Wealthy individuals might not face the same consumption pattern as poor individuals. Engel (1857) already
observed in the 19th century that income correlates negatively with the proportion of
income spent on food and named this observation the Engels’s law. If it is assumed
that individuals are risk averse and have monotonic preferences then wealthy
individuals have the higher incentive to consume products with uncertain outcome
than poor individuals. These products with uncertain outcome can be seen as new
products. Additionally, the wealthy individuals are able to consume a lot. This
implies that the higher the amount of wealthy individuals the higher is the demand
for new goods. As a consequence, the incentive to innovate might increase because
the profit of an innovation increases and the risk to put effort in innovation decreases
due to the bigger initial market.

Falkinger and Zweimüller (1996) show empirically that a higher income correlates
positively with the diversity of consumed goods. Furthermore, Kiedaisch (2016)
demonstrates that the wealthier an individual (under the reasonable assumption that
expenditures correlates positively with wealth) the more it consumes innovative goods
as visible in Figure 9.

Therefore, the demand for innovative goods depends on inequality and as a
consequence also on the profit of an innovator. Foellmi and Zweimüller (2006)
developed a model with non-homothetic preferences and an innovation-based growth
model with the reasonable assumption, as visible above, that rich consumers are more
likely to consume innovative goods (at least at the beginning). Assuming that the price
effect dominates the market-size effect, higher inequality would increase the incentive to innovate and therefore enhance growth in this model.

To analyse the relation between the effect on growth and the distance to the technological frontier, reconsider that the individuals of a country have more opportunities to imitate existing technologies the farther away their country is from the technological frontier as discussed in Chapter 3. Under the assumptions made before, inequality would increase the profit of imitating and innovating. Furthermore, a higher inequality might decrease risks for new products as described above. As products which have never been successful at any market contain a higher risk to succeed, the risk decreasing effect of a higher inequality might favour especially those world`s first products - the innovations. Therefore, the positive effect of a higher inequality might be bigger the closer a country is to the technological frontier because the closer the more important becomes innovation.

4.2.5 Political economy
An income distribution is often right skewed, which induces that the median is lower than the mean. As a consequence, the majority of individuals would profit from a redistribution mechanism. It is assumed that the individuals have some power to enforce their interests. This induces pressure for a progressive redistribution mechanism as long as there is inequality. Furthermore, this pressure increases the higher the inequality is because then a poor individual could profit more from a progressive redistribution mechanism. If the pressure is high enough, then the country has to implement a redistribution mechanism with the consequences discussed in Section 4.1. However, the rich individuals might have more power to enforce their interests than the poor individuals. This could mitigate the pressure for redistribution.

4.3 Conclusion table
This Section provides an overview about the results. A + (−) indicates that the effect on growth is positive (negative), a ++ (−−) that the effect is more positive (negative) than the effect with a + (−). A “?” indicates that the effect is uncertain. The column “explanation” points out the subject with which I explained the effect on growth for different distances to the technological frontiers.
### Table 1: Conclusion table

<table>
<thead>
<tr>
<th>Channel</th>
<th>Effect of redistribution on growth</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Far away from tech. frontier</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Close to tech. frontier</td>
<td></td>
</tr>
<tr>
<td><strong>Informational asymmetries</strong></td>
<td>(− −)</td>
<td>Informational infrastructure</td>
</tr>
<tr>
<td><strong>Implementation costs</strong></td>
<td>(− −)</td>
<td>Efficiency</td>
</tr>
<tr>
<td><strong>Redistribution and incentives</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal working effort</td>
<td>(−)</td>
<td>Education costs and amount of new tech.</td>
</tr>
<tr>
<td>Effect of a decrease in performance-based compensation on the incentive to take risks</td>
<td>(−)</td>
<td>(− −)</td>
</tr>
<tr>
<td>Effect of a social insurance on incentive to take risks</td>
<td>(+)</td>
<td>(+ +)</td>
</tr>
<tr>
<td><strong>Social Freeloader</strong></td>
<td>(−)</td>
<td>Income level</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>Effect of inequality on growth</th>
<th>Explanation with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Far away from tech. frontier</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Close to tech. frontier</td>
<td></td>
</tr>
<tr>
<td><strong>Physical and human capital investment</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decreasing investment returns</td>
<td>(− −)</td>
<td>Credit market imperfection</td>
</tr>
<tr>
<td>Increasing investment return</td>
<td>(+)</td>
<td>Rate of return for human and physical capital</td>
</tr>
</tbody>
</table>

51
<table>
<thead>
<tr>
<th></th>
<th>(+)</th>
<th>(+ +)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earning-related preferences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earning-related saving preferences</td>
<td>(+)</td>
<td>(+ +)</td>
<td>Convex and monotone increasing saving function</td>
</tr>
<tr>
<td>Earning-related consumption preferences</td>
<td>(+)</td>
<td>(+ +)</td>
<td>Amount of new goods consumed</td>
</tr>
<tr>
<td><strong>Political economy</strong></td>
<td>(?)</td>
<td>(?)</td>
<td></td>
</tr>
</tbody>
</table>
5. Conclusion

This study has analysed several channels through which redistribution could have an effect on growth respecting the distance to the technological frontier. The channels have been separated in direct effects, which contain information asymmetries, implementation costs, redistribution and incentives and social freeloader, through which redistribution has a direct effect on growth, and in indirect effects, which contain physical and human capital investment, social instability, Incentives, earning-related preferences and political economy, through which the change of inequality affects growth.

The main results are that redistribution has a direct negative effect on growth through the channels information asymmetries, implementation costs and social freeloader. The farther away a country is from the frontier the more negative is the direct effect through the channels information asymmetries and implementation costs and the less negative is the direct effect through the channel social freeloader. The effect through the channel redistribution and incentives is ambiguous. Furthermore, redistribution has a positive indirect effect on growth through the channels incentives and earning-related preferences. The farther away a country is from the frontier the more positive is the indirect effect through the channel incentive and the less positive is the indirect effect through the channel earning-related preferences. A negative indirect effect on growth has redistribution through the channel social instability and, under the assumption of decreasing investment returns, the channel physical and human capital investment. The indirect effect of the latter becomes more negative the farther the country is away from the frontier. However, under the assumption of increasing investment returns, the channel physical and human capital investment leads to a positive indirect effect of redistribution on growth for a country far away from the frontier and to a negative indirect effect on growth for a country close to the frontier.

This study provides a comprehensive overview over several channels through which inequality and redistribution have an effect on growth. It was shown whether the effect through a channel is negative or positive and how this effect changes with the
distance to the technological frontier. In view of the discussion about customized
growth strategies for certain development levels, especially the results regarding the
distance to the technological frontier might be interesting nowadays. However, the
separate analysis of these channels leaves the question open what the overall effect of
redistribution is on growth. Therefore, for future research it would be interesting to
combine the effects of those channels in order to get an overall effect of redistribution
on growth.
References


Statutory Declaration

I declare with these words that the thesis with the title "Redistribution, Distance to Frontier, and Growth" has been prepared and written by myself. Every used source in this thesis is stated with a quotation. This thesis has not been published before in the same or a similar way.

Malans,