Designing All-pay Auctions

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Abstract: This paper shows that wide classes of effort distributions can be implemented as mixed-strategy equilibria of symmetric two-player all-pay auctions with endogenous prizes. Implementation becomes harder with more players and with asymmetric players. We also ask how all-pay auctions have to be designed to induce high expected winning efforts without generating wasteful efforts of losers. All-pay auctions with endogenous prizes do better than all-pay auctions with fixed prizes; almost-linear prize functions approximate the optimum. We apply the results to innovation policy and to the design of incentive systems within organizations. Experimental evidence provides weak support for some of the theoretical results.

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1 Introduction

An all-pay auction with endogenous prize (AEP) is a contest in which players simultaneously exert costly efforts. As in a standard all-pay auction, only the player with the highest effort wins a prize. However, in an AEP the prize is not fixed exogenously; instead it is determined by a *prize function* that maps efforts into prizes. AEPs capture the essence of more complicated strategic interactions. Applications of the framework include innovation contests, promotion tournaments and competitive procurement.

Previous work (Kaplan et al. (2003) and Siegel (2009, 2010)) has characterized the mixed-strategy equilibria (MSE) of AEPs with complete information. We contribute to the literature by analyzing the design of AEPs. We think of the designer as maximizing some statistic of the effort distribution. We therefore first ask which effort distributions she can choose from, that is, which distributions are compatible with equilibrium behavior in a suitable AEP. Our baseline model with two players suggests that "anything goes": Any effort distribution with bounded and positive density on a compact interval from 0 to any positive value can be implemented as the equilibrium of some AEP. With more than two players, this result is overturned: Implementation is impossible for distributions with bounded density; and for unbounded densities implementation becomes more difficult as the number of players increases.

For the design problem, we focus on situations where the efforts of losers only serve to put competitive pressure on the winner – otherwise these efforts are pure waste.[2] As efforts are social costs, maximizing expected *total* effort is not a reasonable objective. Instead, we focus on maximizing the expected *winning* effort for fixed expected total effort. We use a constructive approach to this design problem. We consider a class of all-pay auctions with prize function αx^γ for efforts x, a scale parameter $\alpha > 0$ and a concavity parameter $\gamma \in [0, 1)$. For any desired level of expected *total* efforts, there exists a continuum of parameter vectors (γ, α) so that the AEP with prize function αx^γ implements this expected effort level. Within such a class of AEPs with fixed expected total efforts, the expected *highest* effort is strictly increasing in γ. Moreover, as γ approaches 1, the expected highest effort approaches twice the expected average effort, which is the theoretical maximum that can be achieved with *any* prize function. Thus, the general design problem can be solved approximately by focusing on the specific class of prize functions. Alternatively, one can approach the optimum

[2] We will argue below that this is a good approximation for procurement situations or for innovations without much uncertainty where only the best product will be produced.

by using effort hurdles, that is, pay positive prizes only above a certain effort level. Fixing expected efforts, the upper bound for the expected winning effort can be approximated for any fixed $\gamma \in [0, 1)$ by choosing the hurdle as high as possible within an equivalence class.

We apply our approach to the analysis of incentives for product innovation by assuming that higher effort corresponds one-to-one to higher product quality. Our setting fits best for the polar case of innovations for which effort duplication has no intrinsic value.[3] If a buyer of an innovation who cares about the expected winning quality (expected highest effort) finds a way to commit to a quality-dependent (effort-dependent) prize function, she can benefit from using it, and the gains are largest if the prize function is close to linear. Commitment to such a prize function may be difficult if quality is not perfectly verifiable, however. We therefore show that it is also possible to approximate the optimum within a very simple class of prize functions, namely with step functions, that is, with a suitable combination of a fixed prize and an effort hurdle. Verifying that the effort is above the hurdle is easier than verifying the exact quality level.

More broadly, the analysis contributes to the understanding of innovation policy. Suppose there are two instruments to influence product innovations, *patent policy* and *redistribution policy*. Patent policy rescales the expected product market profits of a patent owner by a constant factor.[4] Redistribution policy makes the reward function more concave, thus benefiting low quality innovators relative to high quality innovators. Different policy choices correspond to different prize functions in an AEP. Whereas more generous patent policy and less redistribution both increase expected total innovation effort, for fixed expected total innovation effort expected product quality (corresponding to the *highest* innovation effort) is usually higher when patents have shorter duration. The analysis also suggests that relying on rigid novelty standards might be a more appropriate way to incentivize innovation than relying on a high patent duration.

Next, we consider a firm that can incentivize effort of its employees using a combination of a fixed bonus and a minimum performance level (a hurdle) below which no bonus is given. Our analysis implies that, to maximize expected highest efforts, firms should pay relatively high bonuses, but be restrictive in the

[3]Intuitively, this requires that (i) there is not much uncertainty about the right approach to innovation (so that duplication does not increase the total probability of innovation) and (ii) the successful innovator is shielded from ex-post competition, as in a patent system, so that duplication does not increase ex-post competition.

[4]For instance, as the patent length increases in a static market environment, the innovator will enjoy the rent from innovation for a longer time.

conditions for awarding them. Conversely, to maximize expected lowest effort, firms should use low bonuses without effort hurdles.

We extend the analysis of the baseline model in three directions. First, we provide implementation results for more complex equilibria. Implementation is possible for MSE with support consisting of unions of compact intervals. Second, we consider asymmetric AEP where one player has lower effort costs than the other. We show that asymmetry reduces the scope for implementation: Once the effort distribution for one player is fixed, so is the effort distribution of the other one. Also, the strong (low-cost) player obtains rents. We find that, for given expected rents, the expected effort is particularly high for extreme cases (fixed-prize tournaments and AEP with almost-linear prize functions). Third, we allow for all-pay auctions where the effort of the runner-up reduces the prize of the winner ("negative prize externalities").[5] The MSE of such contests are often identical to AEPs where the payoff depends only on the winner's prize. Thus, allowing for negative prize externalities does not necessarily increase the scope for implementation.

Finally, we provide evidence from a laboratory experiment. The observations provide some support for the result that a suitably designed AEP can yield high expected winning efforts for given expected average effort. Contrary to our theoretical results, however, the evidence suggests that an AEP and an all-pay auction with negative prize externalities may induce different behavior even when they have the same symmetric MSE: In particular, with negative externalities, average efforts are higher than without.

Our analysis of implementation and design of AEPs builds from several papers that have characterized the MSE of AEP for symmetric and asymmetric contestants.[6] Kaplan et al. (2003) have treated a symmetric two-player game with symmetric information where firms commit to the timing of an innovation. After a simple transformation of variables, the game is almost identical to our AEP with symmetric players.[7] Like us, depending on the prize function, the authors obtain (i) equilibria with randomization on an interval containing zero efforts, (ii) equilibria with an atom at zero and randomization on an interval starting with a positive effort, (iii) equilibria with non-connected support. The asymmetric case has been treated by Kaplan et al. (2003) and Siegel (2009,

[5]For discussions of such all-pay auctions with negative prize externalities see Skaperdas 1992, Chung 1996, Baye et al. 2010). Darai et al. (2010) includes an experimental analysis of the Bertrand investment game, a special example. Sacco and Schmutzler (2008) describe a solution procedure for the discrete case.

[6]Konrad (2009) provides a broader survey of the contest literature.

[7]Essentially, one can define the effort as the inverse of the time needed for innovation.

2010) under mutually exclusive assumptions.[8] In some important dimensions, Siegel's equilibrium characterization is more general than ours and the one in Kaplan et al. (2003). He considers more than two players and multiple prizes and he allows for investments that are conditional on winning or losing as well as for very general types of prize and cost asymmetries.[9]

Contrary to the existing literature, we address implementation and design issues. To this end, we independently derive the MSE for prize functions that are compatible with the assumptions of Siegel and Kaplan et al., but more general in some dimensions. Accordingly, in the asymmetric case we obtain equilibria without small positive efforts of the strong player (as in Kaplan et al. 2003) as well as equilibria with small positive efforts (as in Siegel 2009, 2010).[10]

A more broadly related literature allows for asymmetric information in AEP. Kaplan et al. (2002) derive comparative statics results when the prize function differs across player types, which are private information. In a similar setting, Cohen et al. (2008) consider the optimal prize function for a principal who maximizes total effort or expected highest effort (net of expected prizes).[11] We do not deal with asymmetric information, because we want to focus on which effort distributions can be generated endogenously rather than as a reflection of type distributions.

In Section 2, we introduce the innovation policy example. Section 3 presents this model with symmetric players and simple prize functions. Section 4 contains the main results for the baseline model. Section 5 discusses applications. Section 6 discusses extensions. Section 7 provides experimental evidence. Section 8 concludes.

[8] Kaplan et al. (2003) deal only with the two-player case and assume that the prize becomes zero for zero efforts. Siegel assumes that prizes are positive for minimal efforts (and that net prizes are declining in efforts).

[9] Chowdhury (2010) analyzes a modified all-pay auction where prizes are fixed, but not guaranteed even for a player who has exerted effort. If the probability that a prize is actually distributed is increasing in the effort of the high-effort player, the game is isomorphic to an AEP with endogenous prizes.

[10] We also provide a recursive formula for MSE with arbitrarily many connected components in the symmetric case. Though such equilibria can also arise in Kaplan et al. (2003) and Siegel (2012), there is no analogous result there.

[11] Interestingly, the optimal prize function can be decreasing in efforts for suitable prize functions when there are many participants.

2 A Motivating Example

We first introduce a stylized reduced-form model of innovation policy as an example of an AEP. In Period 0, two symmetric firms exert costly R&D efforts $x_i \in \mathbb{R}^+$ to obtain a product innovation. Higher efforts correspond to higher product quality. In each period $1, 2, ...,$ market demand is $D(p; x_i)$, which is decreasing in p and increasing in x_i. Marginal production costs are zero. The firm that exerted the highest effort obtains a patent; for the firm that does not obtain the patent its effort has no value. *Patent policy* specifies a number T of monopoly periods (patent length) for the successful innovator. Let $p^m(x_i)$ be the monopoly price corresponding to quality x_i. The successful firm i obtains the per-period monopoly profit $\Pi^m(x_i) = p^m(x_i)D(p^m(x_i); x_i)$ for T periods and zero profits thereafter.[12] We capture *redistribution policy* by a concave function g of profits Π^m. We think of this policy in a broad sense, capturing tax policy, competition policy, R&D subsidies, regulation, etc. Because g is concave, redistribution implies higher incentives for increasing efforts from low to medium rather than from medium to high efforts. With a discount factor $\delta > 0$, a system with patent and redistribution policy corresponds to an AEP where the players simultaneously choose efforts x_i, and the winner obtains a prize

$$a(x_i) = \left(\frac{1 - \delta^{T+1}}{1 - \delta} \right) g\left(\Pi^m(x_i)\right). \tag{1}$$

In this stylized setting, the design of patent and redistribution policy thus corresponds to the choice of a prize function.

In Sections 3 and 4, we analyze a reduced-form model which contains the innovation game as a special case: This game is an all-pay auction, as all firms exert effort and only one obtains a prize. The prize is endogenous, a function of the effort of the winning player. In Section 5.1, we apply the general results to the innovation policy example. We will ask how a designer who cares about expected product quality would choose redistribution policy, patent length and the novelty standards under which a patent is awarded.

3 Baseline Model

We consider a game with complete information. In the baseline case, we assume that the game is symmetric.[13] Risk-neutral players $i \in \mathcal{I} \equiv \{1, 2, ..., n\}$

[12] This (rather stark) assumption corresponds to perfect competition or entry of a stronger firm at the end of the monopoly period.

[13] In Section 6.2, we will consider the case of asymmetric players.

simultaneously choose efforts x_i from $X_i = \mathbb{R}^+$ at costs $K(x_i)$.

Assumption A1: $K(x_i) = x_i$.

Let $\mathbf{x}_{-i} = (x_1, ..., x_{i-1}, x_{i+1}, ..., x_n)$. Player i wins a prize $a(x_i)$ with probability $p(x_i, \mathbf{x}_{-i}) = 1$ if $x_i > \max_{j \neq i} x_j$.[14] His expected payoff is

$$\pi_i(x_i, \mathbf{x}_{-i}) = p(x_i, \mathbf{x}_{-i}) a(x_i) - x_i. \tag{2}$$

Except where otherwise mentioned, we maintain the following assumptions on the *prize function* $a(x_i)$, a: $\mathbb{R}^+ \to \mathbb{R}^+$.

Assumption A2: *$a(x_i)$ is (i) right continuous on $\mathbb{R}^+$, and (ii) it is continuously differentiable to the right on $(0, \infty)$.*

Assumption A3: *(i) There exists a **threshold** $T > 0$ s.t. $a(T) = T$ and $a(x_i) < x_i$ for all $x_i > T$.*
(ii) $a(x_i)$ is continuously differentiable in an open neighborhood of T, with $a'(T) < 1$ for $i = 1, ..., n$.

A2 rules out that prizes can approach infinity for efforts approaching zero. As to A3, actions above the threshold are dominated. We apply the notation $a'(x_i)$ to denote the right derivative even if no left derivative exists at x_i. We call prize functions satisfying A2 and A3 *admissible.* The left panel in Figure 1 depicts examples. A3(i) implies that a player's set of undominated strategies has a maximum, which is required for existence of an MSE.[15] A3(ii) rules out degenerate cases where the prize function touches the cost function from below at T (as in the right panel of Figure 1).

Definition 1 *A (symmetric)* ***all-pay auction with endogenous prize (AEP)*** *is a simultaneous game with players $i \in \mathcal{I} \equiv \{1, 2, ..., n\}$, action spaces $X_i = \mathbb{R}^+$ and payoffs functions $\pi_i(x_i, \mathbf{x}_{-i})$ given by (2), where $K(x_i)$ satisfies A1 and $a(x_i)$ satisfies A2 and A3.*

With endogenous prizes, A1 is not a strong restriction.[16] As pure strategies often do not exist in AEP, we focus on mixed strategies.

[14] If several players exert the same highest effort, each of them wins with the same probability.

[15] This is the "finite reach" assumption of Siegel (2009, 2010) for the symmetric setting; similarly in Kaplan et al. (2003).

[16] It is straightforward to show that an AEP with some arbitrary effort measure x_i and a corresponding monotone increasing (potentially non-linear) cost function can always be transformed into an equivalent all-pay auction satisfying A1.

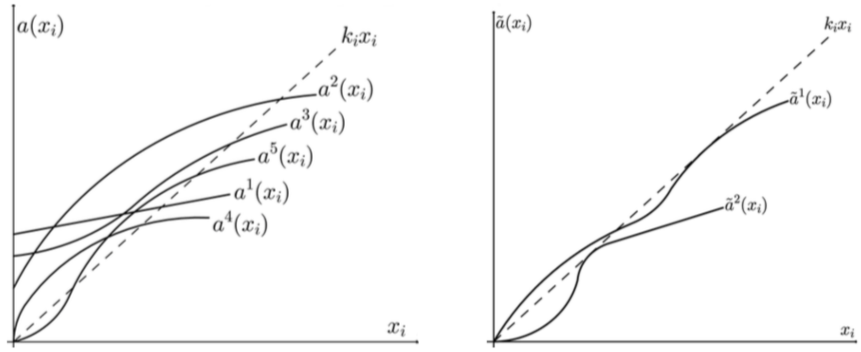

Figure 1: Admissible and non-admissible prize functions

Definition 2 *(i) A* ***mixed strategy*** *of player i in an AEP is given by a cumulative distribution function (CDF) $F_i : \mathbb{R} \to [0,1]$ s.t. $F_i(x_i) = 0$ for $x_i < 0$.*[17]
(ii) The ***support*** *$\mathcal{S}_i$ of F_i is the set $\{x_i \in X_i \,|F_i\,(x_i + \varepsilon) - F_i\,(x_i - \varepsilon) > 0 \forall \varepsilon > 0\,\}$. We write $\mathcal{S} = \mathcal{S}_i$ when $\mathcal{S}_i$ is independent of i.*

The following definition clarifies the meaning of implementation.

Definition 3 *An admissible prize function $a(.)$* ***implements a profile $\mathbf{F}$ of CDFs*** *if $\mathbf{F}$ is an MSE for the AEP given by $a(.)$. When $a(.)$ implements a symmetric $\mathbf{F} = (F, ..., F)$, we also say that $a(.)$* ***implements the CDF*** *F.*

The following definitions will play an important role.

Definition 4 *(i) $a(x_i)$ satisfies* ***decreasing (average) effort productivity (DEP)*** *if $\frac{a(x_i)}{x_i}$ is strictly decreasing on $(0, T]$.*
(ii) For given prize function a, the ***effort hurdle*** *H_a is the minimal $\underline{x_i}$ such that $\frac{a(x_i)}{x_i}$ is strictly decreasing on $\left(\underline{x_i}, T\right]$.*

H_a exists by A3(ii). If DEP holds, $H_a = 0$. DEP implies that $a(x_i) > 0$ for $x_i \in (0, T]$, because $\frac{a(T)}{T} = 1 > 0$. If $a(x_i)$ is differentiable, H_a is the minimal $\underline{x}_i < T$ such that $a'(x_i) < \frac{a(x_i)}{x_i}$ for all $x_i \in (\underline{x}_i, T]$.

We will focus mainly on the following types of equilibria.

Definition 5 *(i) A symmetric MSE with CDF F is an* ***interval equilibrium without atoms*** *if it admits a density f which is positive only on $\mathcal{S} = [0, T]$.*

[17] Equivalently, one can define a mixed strategy by a probability measure P on X_i; the corresponding F is given by $F = P\,[0, x_i]$. When a mixed strategy is given by F, the corresponding P is induced by $P\left(x_i^1, x_i^2\right] = F(x_i^2) - F(x_i^1)$.

*(ii) A symmetric MSE with CDF F is an **interval equilibrium with atoms** if it has support $[0, T]$, with an atom at 0.*
*(iii) A symmetric MSE with CDF F is a **hurdle equilibrium** if $\mathcal{S} = \{0\} \cup [H, T]$ for some $H > 0$, the **hurdle**.*

In the extensions, we will also allow for equilibria with supports consisting of unions of compact intervals.

4 Results

The first main results for symmetric players, Propositions 1 and 2, deal with implementation of interval equilibria; Proposition 3 addresses optimal design. Proposition 4 extends the analysis to hurdle equilibria.

4.1 Existence, Characterization and Uniqueness

As a preparation for our main results, we characterize the MSE for symmetric prize functions under DEP in Lemma 1. This characterization is closely related to Siegel (2009, 2010).[18] However, for our design problem, we need to allow for prize functions that Siegel does not treat. We first focus on interval equilibria; Section 4.3.2 deals with hurdle equilibria. We distinguish between three mutually exclusive conditions, C1-C3. These conditions all include DEP, but they require different properties of the prize function.

(C1) (i) $a(0) > 0$ and (ii) DEP holds.

Siegel (2010) uses C1(i), and instead of C1(ii) he assumes decreasing net prizes $a(x_i) - x_i$, as for $a^1(x_i)$ in the left panel of Figure 1.[19] DEP is more general than decreasing net prizes.[20] C1 also holds for strictly concave functions that do not satisfy decreasing net prizes, such as $a^2(x_i)$ in Figure 1. Some non-concave functions such as $a^3(x_i)$ also satisfy C1. The following alternative condition allows for prizes that are zero when there is no effort.

(C2) (i) $a(0) = 0$, (ii) $a'(0) \equiv \lim_{x_i \to 0} a'(x_i) = \infty$ and (iii) DEP holds.

[18]Siegel considers asymmetric all-pay auctions as well (that we discuss in Section 6.2).
[19]Siegel (2009) uses similar assumptions.
[20]For differentiable $a(x_i)$, declining net prizes imply $a'(x_i) < 1$ and thus $a'(x_i) < \frac{a(x_i)}{x_i}$ for $a(x_i) \geq x_i$, so that DEP holds.

This obviously excludes declining net prizes, but it includes concave prize functions such as $a^4(x_i)$ in Figure 1.[21] Finally, we allow for prize functions with $a(0) = 0$ and finite slope at zero.

(C3) (i) $a(0) = 0$, (ii) $a'(0) < \infty$ and (iii) DEP holds.

We can now characterize the MSE.

Lemma 1 *(i) Suppose DEP holds.*
(a) An interval equilibrium exists for which

$$[F(x_i)]^{n-1} = \frac{x_i}{a(x_i)} \text{ for } 0 < x_i \leq T \tag{3}$$

(b) If C1 or C2 holds, the MSE described in (a) is atomless; if C3 holds, there is an atom at 0.
(c) There are no MSE other than those described in (a) or (b).
(ii) If DEP does not hold, the game does not have an interval equilibrium.

Together (C1)-(C3) cover all functions satisfying DEP. Hence, Lemma 1 implies that an AEP has an interval equilibrium (with or without atoms) if and only if DEP holds. In Section 6.1, we shall provide characterization results for equilibria when DEP does not hold.

4.2 Implementation Results

The first main results deal with implementation. Proposition 6 in Appendix 2 characterizes the CDFs that can be implemented as interval equilibria of a suitable AEP for arbitrary $n \geq 2$. The following result implies a particularly strong positive statement for the two-player case:

Proposition 1 *If $n = 2$, any CDF F with a bounded and strictly positive density on $[0, C]$ for $C > 0$ can be implemented as interval equilibrium without atoms for an admissible prize function $a(x_i)$.*

Thus, the scope for implementation is substantial for $n = 2$.[22] In Section 4.3 we show that implementation is often possible for $n = 2$ even when the density is not bounded. For $n > 2$, we provide necessary and sufficient implementation conditions in Proposition 6 in Appendix 2, implying a simple negative result:

[21] For instance, the class $a(x) = \alpha x^\gamma$ with $\alpha > 0$ and $\gamma < 1$ satisfies C2.

[22] However, for many distributions, implementation relies on potentially implausible prize functions: Unless $f(x_i)/F(x_i) < 1/x_i$, that is, everywhere smaller than for a uniform distribution, implementation requires (at least locally) downward-sloping prize functions, which may not be available to the designer. For instance, in the innovation policy example, successful high-quality innovators would obtain lower profits than successful low-quality innovators.

Proposition 2 *Let $n > 2$. A CDF F with density f and support on an interval $[0, C]$ is not implementable if* $\lim_{x_i \to 0} f(x_i) < \infty$.

Intuitively, the candidate prize function would approach infinity as the effort approaches zero from above, thus violating Assumption A2(i). The negative result contrasts with the case $n = 2$, in which any bounded positive density is implementable. For unbounded densities, Section 4.3.1 below shows that implementation is possible for $n > 2$, but becomes more difficult with more players.

4.3 Optimal Design

We now analyze the optimal design of the prize function for different objective functions. We take the view that the designer does not want to maximize expected efforts. In the innovation policy example of Section 2, this is immediately plausible, as expected efforts are social costs. Even a designer who does not care directly about these costs, like a buyer who is procuring an innovation, may have to compensate the contestants for their efforts, so that they expect to break even. We thus ask how the designer can maximize alternative objectives while keeping average efforts fixed. Motivated by the innovation example, we focus mainly on the objective of maximizing the expected highest effort (first-order statistic) for fixed expected average efforts. Of course, a designer may also care about the impact of the institutional setting on the average effort level; but this can easily be controlled by the choice of the expected prize sum.

4.3.1 Interval Equilibria

Preliminary Remarks We say that two AEP are *prize equivalent* if the expected prize in the MSE is the same. In any MSE of a symmetric AEP with 0 in the support, players must receive expected prizes that exactly compensate their effort costs. Thus, the following result is immediate from A1.

Remark 1 *Two AEP are prize equivalent if and only if their MSE yield the same expected effort.*

In this subsection, we mainly focus on the case $n = 2$. In this case, maximizing the expected highest effort within a class of prize equivalent AEPs amounts to maximizing the ratio of the expected highest and expected average effort,

$$\rho_2 = \frac{2\int_0^C x_i f(x_i) F(x_i) dx_i}{\int_0^C x_i f(x_i) dx_i}.$$

Because $F(x_i) \leq 1$, $\rho_2 \leq 2$. To understand how close ρ_2 can be to this bound, we focus on a rich, but tractable class of examples. The bound can be approximated within this class of examples. We consider the prize functions

$$a(x_i) = \alpha x_i^{\gamma} \text{ for } \alpha > 0 \text{ and } \gamma < 1 \tag{4}$$

Without loss of generality, we suppose $\gamma \geq 0$, so that $a(x_i)$ is non-decreasing.[23] Then an increase in α unambiguously increases the prize for all positive efforts. For $\gamma = 0$, (4) corresponds to a fixed prize all-pay auction. The limit case $\gamma = 1$ is a linear prize function. A higher γ corresponds to a higher ratio between the prizes for winners with high and low effort; it increases the prize only for effort levels above 1.

Equilibrium Characterization: As (4) satisfies A2-A3 and C2, it defines an AEP. Thus, Lemma 1 implies the following result.

Remark 2 *For the AEP corresponding to (4), the unique symmetric MSE is*

$$F(x_i) = P^{\alpha;\gamma}(x_i) \equiv \begin{cases} \frac{1}{\alpha} x_i^{1-\gamma} & \textit{for} \quad 0 < x_i \leq \alpha^{\frac{1}{1-\gamma}} \\ 1 & \textit{for} \quad x_i \geq \alpha^{\frac{1}{1-\gamma}} \end{cases} . \tag{5}$$

Implementation: We first characterize prize equivalent AEPs for $n = 2$.

Remark 3 *The effort distributions that can be implemented by AEP with prize function (4) and expected prize sum μ are given by the $P^{\alpha;\gamma}(x_i)$ with $\gamma \in [0, 1)$ and $\alpha = \left(\mu \frac{2-\gamma}{1-\gamma}\right)^{1-\gamma}$.*

Remark 3 follows from Remark 1, as the expected effort for an AEP with prize function (4) is

$$\mu = \alpha^{\frac{1}{1-\gamma}} \frac{1-\gamma}{2-\gamma}. \tag{6}$$

Figure 2 depicts prize-equivalent combinations of γ and α for different levels of expected prizes.[24] For very low expected prizes/efforts ($\mu = 0.5$), iso-effort lines are non-monotone; for high levels ($\mu = 1$ or 1.5), they are decreasing.[25]

[23] Decreasing prize functions ($\gamma < 0$) lead to lower expected highest effort levels than increasing prize functions ($\gamma \in (0, 1)$). This is worth pointing out, because Cohen et al. (2008) have shown that decreasing prize functions can increase efforts when there is uncertainty about player types (See Section 1). However, even in their case, this requires the number of players to be sufficiently large.

[24] Figure A1 in Appendix 5 depicts members of a class of prize equivalent AEP for $\mu = 1$ and $\gamma = 0$, 0.5 and 0.8.

[25] This reflects the fact that increasing γ has positive effects on the prize only when efforts are high enough (above 1). For high efforts, therefore, γ and α both increase expected efforts, whereas this is not the case for lower values.

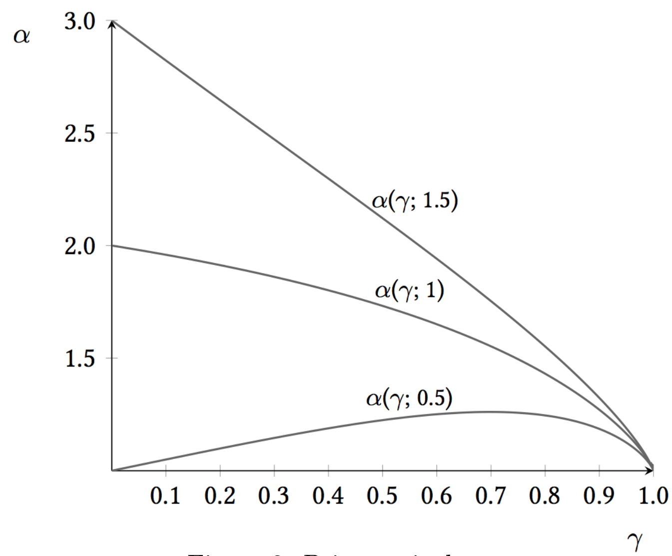

Figure 2: Prize equivalence

Recall that for $n = 2$, implementation is possible for all distributions with bounded density, whereas it is impossible for $n > 2$. On a related note, with unbounded distributions implementation is potentially possible for $n = 2$, but it becomes more difficult with more players: Proposition 6 shows that power distributions given by x^γ for $\gamma \in [0, 1)$ can be implemented if and only if $\gamma \leq \frac{1}{n-1}$. This condition is harder to fulfil for increasing n.

Maximizing expected winning efforts: We now state the main result of this section.

Proposition 3 *Let $n = 2$. In each class of prize-equivalent prize functions $a(x_i) = \alpha x_i^\gamma$, the expected highest effort level increases in γ; it approaches the upper bound of twice the expected average effort as γ approaches 1.*

The intuition for the result is that, with higher γ, higher efforts are rewarded relatively more than low efforts, as discussed above.[26]

Figure 3 illustrates the result. The left panel displays expected highest and average efforts as decreasing functions of γ for $\alpha = 1$. As an increase in γ is not compensated by a change in α, the AEP for different values of γ are not prize equivalent. In the right panel, as γ increases, α increases to guarantee prize equivalence. The expected average effort line ($\mu_\varnothing$) is thus horizontal. The

[26]The proof relies on the fact that the first-order statistic of F is $2\int_0^T F(x)f(x)xdx$.

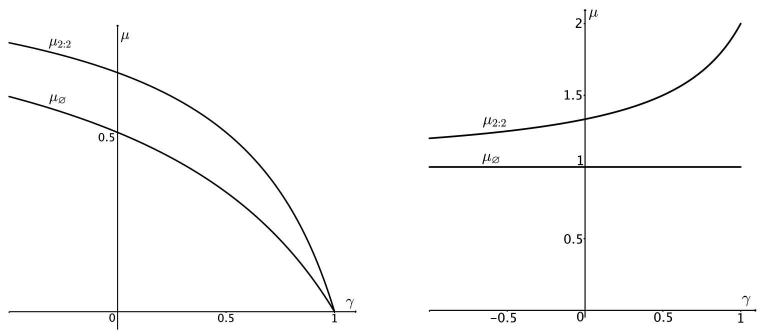

Figure 3: Maximizing expected highest effort

upward-sloping curve in the right panel in Figure 3 shows the expected winning effort $\mu_{2:2}$ as a function of γ within a class of prize equivalent AEP: In line with Proposition 3, the expected winning effort approaches the bound of twice the expected average effort as the prize function approaches linearity.

To understand the result better, note that, by (5), as γ approaches 1, the equilibrium density is more concentrated "close to the axes": The density is very high near zero, but there is also an increasing chance of very high efforts. This clearly illustrates the trade-offs involved in the design problem, and it indicates why, for instance, risk considerations will make such extreme prize functions less attractive.

Maximizing expected lowest efforts: Sometimes, a designer may want to avoid very low positive efforts. For instance, with perfect effort complementarities, the designer should maximize the expected lowest rather than highest effort. For $n = 2$ and fixed expected average efforts, the expected minimal effort is highest when the expected maximal effort is lowest. This is true for $\gamma = 0$, that is, for the all-pay auction with fixed prize.

4.3.2 Hurdle Equilibria

We now deal with hurdle equilibria. We first provide an existence condition, which, holds, e.g., for a^5 in Figure 1, and we characterize the equilibrium.

Lemma 2 *Let* $n = 2$*. If* $\frac{a(x_i)}{x_i}$ *has an interior global maximum* $H_a > 0$ *on*

$[0,T]$*, the AEP has a hurdle equilibrium with support* $\mathcal{S} = \{0\} \cup [H_a, T]$ *and*

$$[F(x_i)]^{n-1} = \frac{x_i}{a(x_i)} \textit{ for } H_a \leq x_i \leq T. \tag{7}$$

$$\textit{There is an atom at } 0 \textit{ with mass } \left(\frac{H_a}{a(H_a)}\right)^{\frac{1}{n-1}}. \tag{8}$$

Intuitively, above the hurdle, the equilibrium is analogous to the interval equilibrium of Lemma 1: Condition (7) guarantees indifference between all efforts in $[H_a, T]$. The size of the hurdle at 0 is such that the total mass is one.[27]

Implementation Lemma 2 leads to an implementation result for distributions without small positive efforts.

Proposition 4 *Consider a strictly increasing CDF F with a density f such that $\{x_i | f(x_i) > 0\} = [0, C]$ for an arbitrary $C > 0$. For $H \in (0, C)$, let*

$$F^H(x_i) = \begin{cases} F(x_i) & \textit{for} \quad x_i > H \\ F(H) & \textit{for} \quad x_i \leq H \end{cases}.$$

For all $H \in (0, C)$, there exists an AEP which implements $F^H(x_i)$ as a hurdle equilibrium.

The prize function implementing $F^H(x_i)$ has to induce contestants to mix between choosing zero effort and efforts above the hurdle. A suitable prize function is $a(x_i) = 0$ for $x_i \leq H$ and $a(x_i) = \frac{x_i}{F(x_i)}$ for $x_i \in (H, T]$.[28]

Optimality A designer can use hurdles to influence the expected highest effort. When a designer can choose α and γ freely in the class of power functions αx_i^γ, hurdles are not necessary, as the upper bound can be approached with suitable (α, γ).

Thus, we fix γ at specific values and vary the hurdle. We write $H = hT$ for the threshold $T = \alpha^{\frac{1}{1-\gamma}}$ and $h \in (0, 1)$. Proceeding as in Section 4.3.1, we show that combinations (h, α) that induce the same expected average effort lie on an upward sloping level curve (see Figure A2 in the appendix). If α is reduced, a reduction of h is necessary to keep average efforts at the same level.

[27] A more precise derivation of the result follows from Proposition 5 in Section 6.1 below that characterizes MSE when DEP is violated.

[28] Implementation of a CDF as a hurdle equilibrium is unique only on the interval $[H, T]$. For values below the hurdle, any prize function that provides sufficiently small rewards for small positive efforts guarantees that such efforts will not be chosen. Above T, the prize function can be extended by any function that satisfies $a(x_i) \leq x_i$.

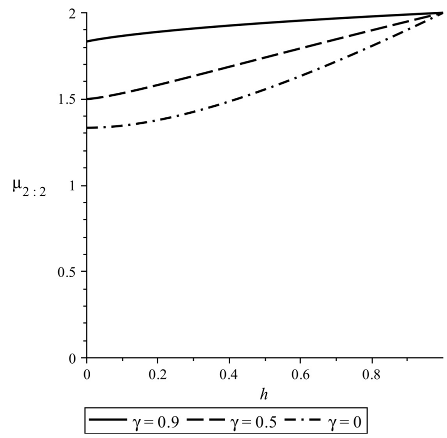

Figure 4: Expected highest effort for prize-equivalent AEP with hurdle equilibria

For high hurdles, the atom at zero becomes very large, so that the average effort decreases; however, the positive efforts are higher on average. Hence, the ratio of expected winning to expected average effort increases with the hurdle. Figure 4 depicts expected highest efforts as a function of h with $\alpha = \alpha(h)$ so that expected average efforts are fixed at one. Thus, a simultaneous increase in the prize level parameter α and the hurdle that keeps expected average efforts fixed increases expected highest efforts.

5 Applications

We now discuss more concrete applications.

5.1 Innovation

We apply our results to study innovation incentives. A limitation of the framework is that the prize always goes to the player who exerted the highest effort. This rules out situations where luck plays a role in the selection of the winner. The analysis is consistent with innovations that are relatively straightforward, so that a successful innovation merely requires diligence rather than luck. We think of this as a useful polar case to illustrate how the design of competitive innovation systems can influence innovation effort.[29]

[29] Letina and Schmutzler (2017) treat contest design in the other polar case that the uncertainty is so large that not even the right approach to innovation is clear ex ante.

5.1.1 Innovation Procurement

Buyers often use fixed-prize tournaments to procure innovations. Our analysis suggests that they would do better if they could commit to quality-dependent (effort-dependent) prize functions. In particular, with almost-linear prize functions they could approximate the highest possible level of expected quality for given expected efforts of the suppliers. However, it is not obvious how buyers could commit to quality-dependent prize functions. Standard justifications for using fixed-prize tournaments rather than more general mechanisms rely on the idea that the payment of an exogenous prize sum to the participants is easily verifiable. However, commitment to an AEP is more demanding. The prize depends on the effort of the successful player. This can be implausible if effort (quality) is not verifiable. For instance, if the buyer has announced an upward-sloping quality-dependent prize function, she will want to claim that the quality of a good is low ex post, so as to lower the payment. Thus, the initial prize function is not credible unless quality is verifiable. With verifiable quality, however, it is not clear why to use a contest in the first place. Instead, the designer could simply write a contract with one party. This would solve the effort duplication problem and allow to fine-tune quality as desired.

Reputation is one reason why commitment to quality-dependent prize functions might be possible: As long as there is a possibility that potential future suppliers will learn about opportunistic behavior, buyers might shy away from the temptation to avoid high payments by claiming low quality. Nevertheless, it is interesting to ask whether the buyer could rely on contests with lower commitment requirements. For instance, the buyer could combine a fixed-prize tournament with a hurdle (which is the case of Section 4.3.2 with $\gamma = 0$). Verifying that efforts are above such a hurdle is simpler than verifying the exact quality. Moreover, the buyer will not want to claim incorrectly that the quality is below the hurdle, because she would then not have access to the good. However, if all participants have failed to reach the hurdle, the buyer might still have an incentive to buy the best of these goods, so that the commitment to the hurdle may not be credible.[30] Nevertheless, the hurdle approach would appear like a useful alternative to more general quality-dependent prize functions. The analysis of Section 4.3.2 (for $\gamma = 0$) shows that this approach is promising: Within a class of combinations of the prize and the hurdle that induce the same expected effort, the expected highest effort increases towards the theoretical

[30] A partial solution to the problem would be to delegate the rating of quality to a disinterested jury. However, it would still not be clear why the buyer would have to invite more than one agent in this setting.

maximum as the hurdle approaches the threshold. The buyer thus uses a high prize, but only pays it out under very restrictive conditions. The downside of the approach is obvious – the buyer may often end up without any product at all. This would be particularly troublesome for risk-averse buyers – that we have ruled out by assumption.

5.1.2 Innovation Policy

In the innovation policy example of Section 2, commitment is less of an issue. Recall that in this example the prize function is determined by law (intellectual property law, competition policy, taxation rules) and the market environment (demand and technology). The design problem exclusively concerns the legal environment. Importantly, the legal system has to deal with many different issues; innovation incentives are only one concern out of many. Here we focus exclusively on the effects on innovation. The legal environment is quite rigid, applying to a wide range of innovation decisions within an economy. This rigidity is a source of commitment power: Potential innovators form their decisions based on the expected future legal environment. They do not expect any adaptations of the framework as a reaction to the quality of their product. Hence, the innovator can rely on the profit expectations corresponding to the given legal system, and we think of our analysis as investigating how the design of this system affects specific innovation decisions. If patent length increases or novelty standards (hurdles) become more stringent, how does this affect innovation behavior and product quality?

We specify demand as $D(p;x_i) = A\left((x_i)^{\beta/2} - p\right)$, where $A > 0$ and $\beta \in (0,1)$. Then the positive demand effects of higher R&D effort decline so fast that per-period monopoly profits $\Pi^m(x_i) = \frac{1}{4}(x_i)^{\beta}$ are concave. We specify the redistribution function as $g(\Pi^m) = (\Pi^m)^{1-\kappa}$ where $\kappa \in (0,1)$. Thus, a higher κ corresponds to more redistribution, as it takes away more profits from firms that earn a lot. With $\gamma \equiv \beta(1-\kappa)$ and $\alpha \equiv A\frac{1-\delta^{T+1}}{4(1-\delta)}$, (1) simplifies to $a^{PC}(x_i) = \alpha x_i^{\gamma}$, so that the analysis of Section 4.3 is immediately applicable. Policy only partly determines α and γ: α is increasing in the discount factor δ and the patent length T, whereas γ is increasing in the demand parameter β and decreasing in the redistribution policy parameter κ.

The designer chooses the patent length $T \in (0,\infty)$ and the redistribution policy $\kappa \in (0,1)$. For fixed parameter values, these choices correspond one-to-one to $\alpha \in \left(0, \frac{A}{4(1-\delta)}\right)$ and $\gamma \in (0,\beta)$, respectively.[31] According to Proposition

[31] In principle, a designer could choose policies that violate these constraints (e.g., by subsi-

3, among all (α, γ) inducing the same level of expected average effort, the expected highest effort increases in γ. Thus, γ should be chosen as high as possible given the restrictions on α and γ. For high expected efforts (corresponding to downward-sloping prize-equivalence lines in Figure 2), this amounts to refraining from redistribution (so that $\gamma = \beta$) and choosing the minimal patent length that induces the targeted expected efforts. For low expected efforts (corresponding to the non-monotone prize-equivalence line in Figure 2), this choice may, however, be excluded by the restriction $\alpha < \frac{A}{4(1-\delta)}$ which reflects the maximal discounted expected profits. Then the expected highest efforts are maximized within the class by guaranteeing an infinite patent length, adjusting the redistribution policy so that the desired expected efforts are reached.

Our results on hurdles have an interesting application in this context: As the effort level corresponds one-to-one to product quality, a hurdle would correspond to a minimum product quality that suffices to obtain a patent. The analysis of Section 4.3.2 thus shows that demanding novelty standards can help to increase expected product quality (while adjusting patent length so as to keep expected innovation costs fixed) – at the risk of ending up without a new product at all.

5.2 Contests in Organizations

Firms and other organizations often use contests to induce effort, for instance, using bonus payments. The size of these payments is typically not determined in an explicit contract. Nevertheless, a firm might be able to establish a reputation for how bonus payments depend on performance. In particular, it can refuse to pay out a bonus if the best performance is below a bar. Without modeling the reputation formation explicitly, we ask whether a firm that can commit to such an instrument should make use of this possibility or pay a fixed prize independent of the winning effort. We suppose that the firm uses a tournament with prize A, but can employ an effort hurdle H. The design problem then consists of the choice of A and $h = H/A$.

Section 4.3.2 shows that the expected average effort is increasing in A and decreasing in $h = H/A$, reflecting the upward-sloping iso-effort lines in Figure A2. Along such a line, the expected highest effort is increasing in the size of the hurdle and decreasing in the size of the prize (see the case $\gamma = 0$ in Figure 4). Thus, an organization that cares only about the highest effort level of all employees can benefit from using hurdles. This is appropriate, for instance, if

dizing the profits of innovators or engaging in a progressive redistribution policy). We abstract from this possibility here.

different employees are supposed to come up with solutions to a single concrete problem, so that only the best solution matters. At the other extreme, the tasks could be highly complementary, so that maximizing the expected minimal effort would be a more plausible objective. Then the organization should not use hurdles. This may seem counter-intuitive as a hurdle would appear to foster high minimal efforts. However, hurdles only increase the minimal *positive* effort – zero efforts become more likely with high hurdles.

6 Extensions

6.1 Equilibria with Non-Convex Support

For completeness, we now consider implementation of effort distributions with non-convex support. Most importantly, Proposition 5 below implies the previously stated result on hurdle equilibria (Lemma 2).

By Lemma 1 a non-convex support requires that the prize function violates DEP. We therefore start by characterizing the equilibrium in this case. We sketch the argument; for details see Appendix 3. We use the following mild restrictions on $a(x_i)$.

(C4) $\frac{a(x_i)}{x_i}$ is non-constant on any open interval in $[0, T]$.

(C5) $\frac{a(x_i)}{x_i}$ has finitely many local maxima on $[0, T]$.

Proposition 5 *Suppose (C4) and (C5) hold.*
(i) If there exists $\varepsilon > 0$ *such that* $\frac{a(\widetilde{x}_i)}{\widetilde{x}_i} > \max_{x_i > \widetilde{x}_i} \frac{a(x_i)}{x_i}$ $\forall \widetilde{x}_i \in (0, \varepsilon]$*, an MSE without atoms exists for which* $\mathcal{S}$ *is a finite union of non-degenerate compact intervals, and* $[F(x_i)]^{n-1} a(x_i) = x_i$ *on* $\mathcal{S}$*. One of these intervals contains* 0*.*
(ii) If $\max_{x_i \epsilon (0,T)} \frac{a(x_i)}{x_i}$ *exists, the AEP has an MSE for which* $\mathcal{S}$ *is the union of* $\{0\}$ *and a finite union of non-degenerate compact intervals. There is an atom at* 0*, and* $[F(x_i)]^{n-1} a(x_i) = x_i$ *on* $\mathcal{S} \setminus \{0\}$*.*
(iii) There exist no other equilibria than those described in (i) and (ii).

The condition in (i) generalizes DEP: It requires only that $\frac{a(x_i)}{x_i}$ takes the highest values near zero, not that $\frac{a(x_i)}{x_i}$ is decreasing globally.[32] The support still contains a non-degenerate interval including zero (See Figure 5).[33] When (ii) applies, low positive values are not attained. In both cases, the support consists of intervals on which DEP holds locally. The equilibrium structure in

[32] DEP corresponds to the special case that $\mathcal{S}$ only consists of one interval.
[33] For a definition of the points on the horizontal axis, see Section 10.3 in the appendix.

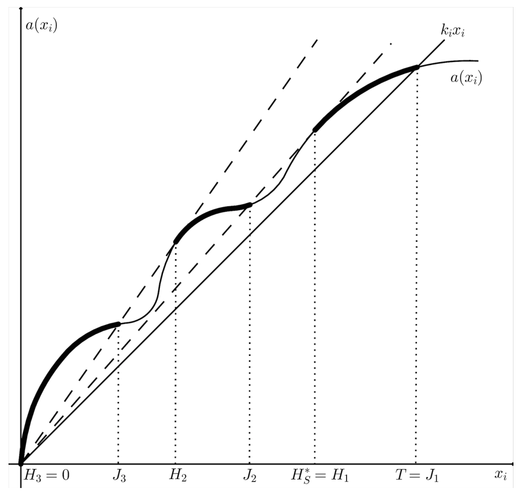

Figure 5: Equilibria with non- convex support

this general setting is thus potentially very complicated. An exception is the special case of hurdle equilibria. Proposition 7 in the Appendix implies that, in this case, the support only contains one non-degenerate interval.

6.2 Asymmetric Cost Functions

We sketch the case of asymmetric costs functions, confining ourselves to $n = 2$. The details of the set-up and exact statements of the results are in Appendix 4. The main new assumption is that one player has effort costs $k_i x_i$ *with* $k_1 \equiv k < 1 = k_2$. In Lemma 10, we characterize the MSE for prize functions that are somewhat more general than in Siegel (2010). The equilibrium for the strong player is the same as in Lemma 1, except that the support does not contain zero when asymmetries are large; see Figure A3 in Appendix 5. Moreover, the high-cost player must have an atom at zero, whereas the low-cost player has an atom at his minimal effort level. Finally, while the weak player earns zero expected payoff, the strong player expects a positive rent.

We also provide implementation results (Propositions 8 and 9), asking which pairs of CDFs (F_1, F_2) can be obtained as MSE for suitable prize functions. The effort distributions of the strong player that can be implemented in an MSE are essentially the same as in the symmetric case, except that the lower tail of the distribution is replaced by an atom. However, the weak player's CDF is completely determined by the choice of the first player's CDF. Thus, compared with the symmetric case the scope for implementation is reduced.

Instead of addressing the design problem as in Section 4.3, we note that the expected prizes and expected average efforts differ with asymmetric players, reflecting the positive expected rents of the strong players. Unlike in the symmetric case, it thus becomes meaningful to ask how to obtain high expected efforts (or, equivalently, low expected rents of the strong player) for given expected prizes. For instance, this is relevant if the designer is a buyer. For convenience, we instead address the dual problem: For fixed rent of the strong player, what is the maximal expected effort that can be achieved?

As an illustration, we use the same class of prize functions as in the symmetric case, and we focus on $k = 0.95$. Figure A4 in Appendix 5 plots the ratio of expected total efforts (as well as individual efforts) to rents as a function of γ, where, as before, α is adjusted so that the expected rent is the same for all (γ, α). The figure shows that the ratio of expected total efforts to rents is particularly high for extreme values of γ: However, as γ approaches 1, the ratio increases slightly above the value for $\gamma = 0$. Hence, suitably designed AEP again do better than fixed-prize tournaments, but many AEP are also worse than fixed-prize tournaments.

6.3 Negative prize externalities

In an AEP as defined in Section 3 the effort of the loser has no effect on the prize of the winner. In some applications, a negative effect is natural. For instance, consider two firms that can invest in cost reductions (without patents and spillovers) before interacting in (potentially asymmetric) homogeneous price competition. Then only the firm that invested more earns a positive profit. This profit is not only increasing in its own investment, but also decreasing in the investment of the other firm.[34] Similarly, consider a procurement situation where suppliers invest (for instance, by building prototypes or making detailed plans) to convince a buyer of the quality of their product before they negotiate about the supply price. As an increase in the quality of the other supplier improves the outside options of the buyer, it is natural to assume that it adversely affects the price that a successful supplier can ask for. We capture this crucial aspect of the strategic interaction by prize functions $v(x_i, x_j)$ that are decreasing in $x_j \neq x_i$. One might think that allowing for such prize functions as well increases the set of implementable effort distributions. The analysis of Section 4.2 suggests that this is not necessarily true. For instance, in the two-player case, all

[34] This follows from standard results on asymmetric Bertrand competition.

bounded distributions with finite supports of the form $[0, C]$ for $C > 0$ can be implemented with prize functions $a(x_i)$. This suggests that there may be no gains from using more general prize functions with negative externalities.

To go beyond distributions with bounded density, consider a symmetric all-pay auction with additively separable prize function $v(x_i, x_j) = \widetilde{\alpha}\sqrt{x_i} - \beta\sqrt{x_j}$ where $\widetilde{\alpha} > \beta > 0$. Such a game has a symmetric MSE with distribution function $F(x_i) = \frac{2\sqrt{x_i}}{2\widetilde{\alpha}-\beta}$ on the support $\left[0, \left(\frac{2\widetilde{\alpha}-\beta}{2}\right)^2\right]$.[35] Thus, for every such combination of $\widetilde{\alpha}$ and β, there exists an $\alpha > 0$ such that the AEP with prize function $\alpha\sqrt{x_i}$ has the same symmetric MSE, namely $\alpha = 0.5\,(2\widetilde{\alpha} - \beta)\sqrt{x_i}$. In this sense, implementation does not improve when considering prize functions $\widetilde{\alpha}\sqrt{x_i} - \beta\sqrt{x_j}$ rather than only $\alpha\sqrt{x_i}$.

Nevertheless, there are important differences between an AEP with prize externalities and the corresponding AEP without prize externalities. First, while the AEP without prize externalities never has a pure-strategy equilibrium, AEP with negative prize externalities may have asymmetric PSE even for symmetric players.[36] Second, while in an AEP without prize externalities efforts outside the support are above the threshold and thus strictly dominated, in an AEP with negative prize externalities actions outside the support may be best responses to positive efforts of the opponent. Thus there are interesting strategic similarities and differences between AEP with and without prize externalities. Among others, the experimental analysis in Section 7 will therefore investigate whether the behavior in the two cases is indeed similar.

7 Experimental Evidence

7.1 Introduction

The coordination requirements in the MSEs of AEPs are substantial. It is thus unclear whether the MSEs have any predictive power. We therefore briefly summarize some experimental results.[37] We ask whether average efforts are really the same in prize equivalent symmetric AEPs, as implied by Remark 1. Moreover, we test whether the curvature of the prize functions influences the expected highest effort as predicted.

[35] Details available on request.

[36] For the above example, (multiple) PSE exist if the negative prize externality is strong enough ($\beta > \widetilde{\alpha}/2$).

[37] See Jönsson (2013) for more detail. More generally, Dechenaux et al. (2015) provide a comprehensive overview of the experimental literature on contests.

We consider three treatments.[38] In *NoExt*, we use the prize function

$$a(x_i) = 2\sqrt{x_i}. \tag{9}$$

The second treatment, *NegExt*, has the prize function

$$v(x_i, x_j) = 3\sqrt{x_i} - 3\sqrt{x_j}, \tag{10}$$

These two prize functions are prize equivalent and thus have the same MSE. Finally, we consider treatment *PrEqv* with prize function

$$\tilde{a}(x_i) = 2.41\, x_i^{0.2}. \tag{11}$$

This prize function induces (approximately) the same expected effort as the two other ones (1.33), but the average winning effort is lower (1.42 rather than 1.54).

7.2 Experimental Design

For each treatment, one experimental session was conducted at the University of Zurich. Each session had 20 rounds. 32 subjects participated in *NoExt* and *NegExt*, respectively, while 28 participants attended *PrEqv*. In each treatment, the subjects remained in a matching group of four for the entire 20 rounds. In each round, the subjects were randomly paired in groups of two. Each subject chose an effort between 0 and 5 in increments of 0.01. At the end of each round, subjects were informed about the effort level of both competitors, their gain or loss, and their endowment net of accumulated gains and losses. Each subject received an initial endowment such that bankruptcy or a negative profit was impossible. The net gain in each round was added to the endowment. After the 20 rounds were over, each subject participated in two simple lottery tasks to elicit risk and loss aversion. Thereafter, the subjects received payments capturing endowments, payoffs from the AEP and from the lottery tasks.

7.3 Results

Figure 6 depicts the average efforts in each round in all three treatments throughout the 20 rounds. On average, subjects exert most effort in *NegExt* followed by *NoExt* and *PrEqv*. Average efforts decrease slightly over time. They are significantly higher than in the MSE in all treatments, except in the last 10 rounds of *PrEqv*.[39]

[38] For the instructions, see Appendix 6.

[39] Here and elsewhere, we cluster with respect to matching groups.

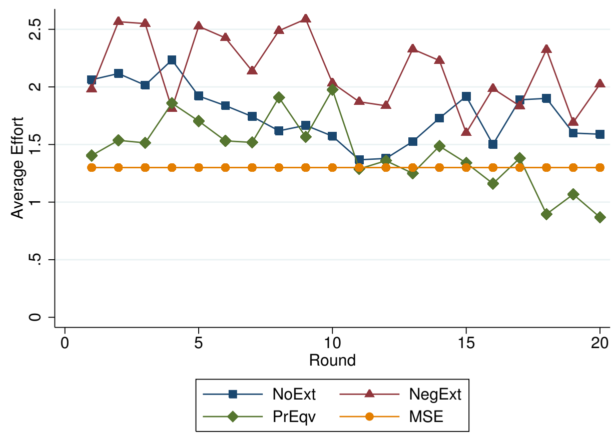

Figure 6: Average efforts in each round in *NoExt*, *NegExt*, and *PrEqv*.

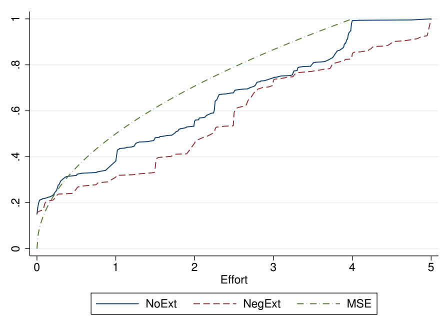

Figure 7: CDFs in *NoExt* and *NegExt*

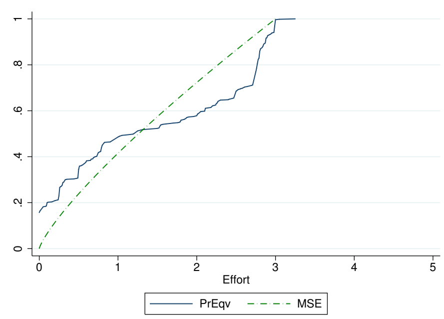

Figure 8: CDF in *PrEqv*

In spite of the apparent differences across treatments, two-tailed Mann-Whitney-U tests do not reject the null-hypothesis of no differences in average efforts between *NoExt* and *PrEqv*; similarly for the comparison between *NoExt* and *NegExt*.[40] We thus look more carefully at the effort distributions (Figure 7 and 8). We can reject the null hypothesis that the samples in *NoExt* and *NegExt* are drawn from the same distribution.[41] Thus, even though the two MSE are the same, the observed behavior differs. Consistent with the discussion in Section 6.3, the distribution for *NegExt* first-order stochastically dominates the one

[40]The p-valus are $p = 0.6434$ and $p = 0.1722$, respectively. There is no statistical difference for the last ten periods; with $p = 0.7237$ and $p = 0.2207$, respectively.

[41]$p = 0.000$ in a Kolmogorov-Smirnov test.

for *NoExt*, even though the MSE are the same. In particular, for *NegExt* observations outside the support occur frequently, whereas they are rare for *NoExt* (and *PrEqv*). Moreover, the effort distribution of *PrEqv* differs considerably from the two other cases. *PrEqv* has a bimodal effort distribution such that very high and very low values occur more frequently than predicted.[42] The CDFs in *NoExt* and *NegExt*, however, lie almost everywhere beneath the MSE.

Finally, we consider average winning efforts. Table 1 reports the average winning efforts in *NoExt* and in *PrEqv*.

	NoExt			*NegExt*			*PrEqv*		
	$t \geq 1$	$t \geq 11$	MSE	$t \geq 1$	$t \geq 11$	MSE	$t \geq 1$	$t \geq 11$	MSE
Effort	2.591	2.400	2.00	3.071	3.0187	2.00	2.048	1.850	1.85
	(1.203)	(1.191)	(1.155)	(1.355)	(1.437)	(1.155)	(.9761)	(1.031)	(1.506)
N	320	160		320	160		280	140	

Standard errors in parentheses.

Table 1: Average winning efforts in *NoExt*, *NegExt*, and *PrEqv*.

The result that average winning efforts are lowest for *PrEqv* confirms the theoretical prediction. However, Proposition 3 also requires that an analogous result holds for the ratio between the average winning effort and the average effort. The theoretical prediction for the ratio is 1.54 in *NoExt* and *NegExt* and 1.42 in *PrEqv*. The observed ratios are 1.47 in *NoExt* and 1.43 in *PrEqv*. Though the observations are quite close to the predictions, the difference between the treatments is thus smaller than predicted. Indeed, it is not significant.

We sum up our observations as follows.[43]

Observation 1 *(i) Efforts are higher than predicted for NoExt and NegExt; in PrEqv this is only true in the first ten rounds.*
(ii) Contrary to the predictions, the distributions for NoExt and NegExt differ; in particular, average winning efforts are higher for NegExt.
(iii) Average and highest winning efforts are lower for PrEqv than for the two other treatments. However, while the ratio of average winning efforts to average efforts is close to the theoretical predictions, the differences between PrEqv and NoExt are not significant.

[42] This is similar to the observations of Ernst and Thöni (2013) for the fixed-prize case, which is closest to *PrEqv*, which has a relatively flat prize function.

[43] Jönsson (2013) also discusses in some detail how the observed behavior depends on behavioral traits. Consistent with existing literature, we find that overconfident subjects tend to exert higher effort, whereas loss averse subjects exert lower efforts.

In particular, we find weak support for the notion that the highest expected effort in an AEP increases with the concavity parameter γ. Contrary to the prediction, AEP with negative prize externalities generate higher expected efforts than AEP without negative prize externalities with the same MSE. Apparently, the direct adverse effect of the other player's effort on the prize creates a perception of competitive pressure that enhances efforts.

8 Conclusion

Many economic institutions have features of all-pay auctions with endogenous prizes. This paper has shown that very general distributions can be obtained as MSE of symmetric two-player AEP. Implementation is more difficult if players are asymmetric or the number of players increases. For two players, compared to all-pay auctions with fixed prizes, AEPs yield high expected winning effort while avoiding excessive losing efforts, in particular, when the prize function is approximately linear. Effort hurdles increase the expected winning efforts. Our experimental evidence provides weak support for the predicted relation between the curvature of the prize function and expected maximal efforts. However, AEP with negative prize externalities tend to induce higher efforts than AEP without such externalities with the same MSE.

The paper can be extended in various directions. Most importantly, principals could maximize other objectives than expected highest or minimal effort. For instance, with risk-averse players our simple constructive approach is no longer applicable, and it is an open issue how the design results would change.

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10 APPENDIX

10.1 Appendix 1: Proofs of Characterization Results

10.1.1 Proof of Lemma 1

We appeal to the standard characterization result for MSE with continuous action spaces (adapted from Osborne 2004, Proposition 142.2).

Lemma 3 *F is an MSE if and only if (i) there is no action which, given the opponents' behavior, yields higher expected payoff than F and (ii) F assigns zero probability to the set of actions for which, given the opponents' behavior, the expected payoff is less than her expected equilibrium payoff.*

We now prove Part(i) of Lemma 1.
(a) DEP implies that $F(x_i)$ as in (3) is strictly increasing on $(0, T]$. Moreover $F(T) = 1$. If $a(0) > 0$ (C1) or $a'(0) = \infty$ (C2), then $\lim_{x_i \to 0} F(x_i) = 0$ and F is a CDF without atoms, with support $[0, T]$.
(b) If C3 holds, then $\lim_{x_i \to 0} (F(x_i))^{n-1} = \frac{1}{\lim_{x_i \to 0} a'(x_i)} > 0$. Hence $\lim_{x_i \to 0} F(x_i) > 0$. Adding an atom with mass $\lim_{x_i \to 0} F(x_i)$ at 0, F is a CDF with support $[0, T]$. (3) implies that expected payoffs are zero on $[0, T]$. Because $a(x_i) < x_i$ for all $x_i > T$, F corresponds to an MSE by Lemma 3.
(c): The proof follows from Lemmas 4-9 below; which will also be useful in the proof of Part (ii).

Lemma 4 *Any symmetric MSE must satisfy (i) or (ii):*
(i) It has no atoms, and $M \equiv min\ \mathcal{S} = 0$.
(ii) It has an atom at 0.
Case (ii) requires that $a(0) = 0$ and $q_n a'(0) \leq 1$ where $q_n = (F(0))^n$.

Proof. The proof has three steps:

Step 1: $M \equiv min\ \mathcal{S} = 0$ for every symmetric MSE without atoms.

Step 2: A symmetric MSE with an atom at $\widetilde{x} > 0$ does not exist.

Step 3: A symmetric MSE with atom at 0 requires $a(0) = 0$ and $q_n a'(0) > 1$.

We now prove each step in turn.

Step 1: Suppose $M > 0$. For any sequence $x_n \to M$ with $x_n \in \mathcal{S}$, the probability of winning approaches zero. By continuity, $\lim_{x_n \to M} a(x_n) = a(M) < \infty$. The expected net payoff thus falls below zero as $x_n \to M$, so that $M \notin \mathcal{S}$.

Step 2: Suppose $\widetilde{x} > 0$ is an atom. This requires $a(\widetilde{x}) \geq \widetilde{x}$. In the proposed MSE, for each player who chooses $\widetilde{x}$ and each $m \in \{1, ..., n\}$, there is a probability $q_m \geq 0$ that he is among the m players with the highest effort, where strict inequality holds for $m = n$. The expected net gain for a player from shifting the atom to the right by ε is approximately $\sum_{m=2}^{n} q_m \frac{m-1}{m} a(\widetilde{x}) + \sum_{m=1}^{n} \varepsilon q_m a'(\widetilde{x}) - \varepsilon$, which is positive if $a(\widetilde{x}) > 0$ and ε is sufficiently small. Thus, such a shift by some sufficiently small ε increases expected payoffs, a contradiction.

Step 3: Suppose there is an atom at 0. Then in the proposed MSE, all players tie with some probability $q_n > 0$. The expected net gain from shifting the atom to the right by ε is approximately $q_n \frac{n-1}{n} a(0) + \varepsilon q_n a'(0) - \varepsilon$, which is positive for sufficiently small ε if $a(0) > 0$ or $q_n a'(0) > 1$. Thus, unless $a(0) = 0$ and $q_n a'(0) \leq 1$, shifting the atom to the right by some sufficiently small ε increases expected payoffs, a contradiction. ■

Lemma 5 *$F(x_i)^{n-1} a(x_i) = x_i$ for all $x_i \in \mathcal{S}$.*[44]

Proof. By Lemma 4, for every symmetric MSE and all $\varepsilon > 0$, there exist $x_i < \varepsilon$ such that x_i is played. Also, there is no atom at 0 unless $a(0) = 0$. Thus, expected payoffs approach zero as x_i does. By Lemma 3 (i), in an MSE there exists no $x_i \in X_i$ for which $F(x_i)^{n-1} a(x_i) - x_i > 0$. Moreover, as $0 \in \mathcal{S}$, F assigns zero probability to the set of x_i for which $F(x_i)^{n-1} a(x_i) - x_i < 0$ by Lemma 3(ii). Right continuity of F and a thus imply $F(x_i)^{n-1} a(x_i) - x_i = 0$ $\forall x_i \in \mathcal{S}$. ■

Lemma 6 *Suppose $x^1, x^2 \in \mathcal{S}$ and $0 < x^1 < x^2$. Then $\frac{a(x^2)}{x^2} \leq \frac{a(x^1)}{x^1}$.*

Proof. (3) implies $\frac{a(x^1)}{x^1} = \frac{1}{[F(x^1)]^{n-1}}$ and $\frac{a(x^2)}{x^2} = \frac{1}{[F(x^2)]^{n-1}}$. Monotonicity of F implies $\frac{a(x^2)}{x^2} \leq \frac{a(x^1)}{x^1}$. ■

Lemma 7 *If DEP holds, there can be no $x^1 < x^2$ such that $x^1 \in \mathcal{S}$, $x^2 \in \mathcal{S}$, $x^1 > 0$ and $(x^1, x^2) \cap \mathcal{S} = \varnothing$.*

Proof. There are no atoms at x^2 by Lemma 4. $(x^1, x^2) \cap \mathcal{S} = \varnothing$ would imply $F(x^1) = F(x^2)$. Thus, by Lemma 5, $\frac{a(x^2)}{x^2} = \frac{a(x^1)}{x^1}$, violating DEP. ■

Lemma 8 *If DEP holds, $\mathcal{S} = \{0\} \cup [L, T]$ for some $L \geq 0$.*

Proof. By Lemma 4, $\min \mathcal{S} = 0$. By A3(i), $\mathcal{S} \subset [0, T]$. Next, for all $\varepsilon > 0$, $\mathcal{S} \cap (T - \varepsilon, T] \neq \varnothing$, because there exists a left neighborhood of T in which $a(x_i) - x_i > 0$ holds everywhere by A3(ii). Hence, $\max \mathcal{S} < T$ would imply that a player could guarantee himself a positive payoff by choosing $x_i \in (\max \mathcal{S}, T)$. By Lemma 7, $\mathcal{S} \cap (0, \infty)$ is an interval. ■

[44] For a similar, which is applicable if, e.g., C1 holds, see Siegel 2009, Corollary 3.

Lemma 9 $[H_a, T] \subset \mathcal{S}$.

Proof. Suppose that $[H_a, T] \subsetneqq \mathcal{S}$. Arguments in the proof of Lemma 8 that do not depend on DEP show that $T \in \mathcal{S}$. Thus, there exists an $L \in (H_a, T] \cap \mathcal{S}$ and $\varepsilon > 0$ such that $(L - \varepsilon, L) \cap \mathcal{S} = \varnothing$. Choose $x_i \in (L - \varepsilon, L)$. By Lemma 5, the mass of players choosing $x_i \leq L$ is $\left(\frac{L}{a(L)}\right)^{\frac{1}{n-1}}$ and there is no atom at L by Lemma 4. A player who deviates from L to x_i earns expected net payoffs $\frac{L}{a(L)} a\left(x_i\right) - x_i$. This is positive if $\frac{a(x_i)}{x_i} > \frac{a(L)}{L}$, which is true for $L > x_i > H_a$, a contradiction. Thus, $L = H_a$. ∎

We now prove Part (c) of Lemma 1(i): From Lemmas 8 and 9, if DEP holds, $\mathcal{S} = \{0\} \cup [L, T]$, where $L \leq H_a$. DEP implies $H_a = 0$. Thus, $L = 0$ and $F(x_i)^{n-1} a(x_i) = x_i$ on $[0, T]$ by Lemma 5. Thus the MSE is as described in Proposition 1.

Finally, we prove Part (ii) of Lemma 1: Suppose DEP does not hold, that is, $H_a > 0$. By Lemma 6, which does not rely on DEP, $[0, T] \subsetneq \mathcal{S}$. However, by Lemma 9, $[H_a, T] \subset \mathcal{S}$. As $\mathcal{S} \subset [0, T]$, there exists no $x_i > 0$ such that $\mathcal{S} = [0, x_i]$.

10.2 Appendix 2: Proofs of Implementation Results

We now provide a complete characterization of the CDFs that can be implemented as interval equilibria of a suitable AEP. The result immediately implies Propositions 1 and 2.

10.2.1 Proof of Proposition 6

By Lemma 1, any candidate prize function with MSE F and support $[0, T]$ must satisfy

$$\begin{array}{rll} a^F(x_i) = \frac{x_i}{[F(x_i)]^{n-1}} & \text{for} & 0 < x_i \leq T \\ a^F(0) = \lim_{x_i \to 0} \frac{x_i}{[F(x_i)]^{n-1}} & \text{for} & x_i = 0 \end{array}. \tag{12}$$

Also by Lemma 1, a prize function satisfying (12) is admissible and yields the equilibrium CDF F without atoms if and only if A2, A3 and, in addition, C1 or C2 hold. This leads to the following result.

Proposition 6 *Suppose a CDF F has a density f such that $\{x_i | f(x_i) > 0\} = [0, C]$ for $C > 0$.*
(i) F can be implemented as interval equilibrium without atoms if and only if

the following conditions both hold:

$$\lim_{x_i \to 0} F(x_i)^{n-2} f(x_i) \quad > \quad 0. \tag{13}$$

$$\lim_{x_i \to 0} F(x_i)^{n-2} f(x_i) \quad < \quad \infty \text{ or } \lim_{x_i \to 0} \frac{F(x_i) - x_i\,(n-1)\,f(x_i)}{F(x_i)^n} > 0 \tag{14}$$

(ii) If (13) holds, but (14) does not, then F can be implemented as interval equilibrium with atom at zero.

Proof. The proof shows that (13) guarantees that a^F is admissible.[45] (14) is equivalent with the requirement that a^F satisfies C1 or C2. Hence, $\mathbf{F} = (F, ..., F)$ really is an interval equilibrium without atoms in case (i) and with atoms in case (ii).
(i) Consider a CDF F that admits a density f such that $\{x_i | \, f(x_i) > 0\} = [0, C]$ for $C > 0$. Then, a^F defined by (12) on $[0, C]$ satisfies differentiability on $(0, C]$, as required by A2(ii). A2(i) (continuity at 0) holds if and only if $\lim_{x_i \to 0} a^F(x_i)$ is finite, that is, (13) holds. In this case, a^F is also right differentiable at 0. Further, a^F can always be extended to the right in a continuously differentiable way such that $a^F(x_i) < x_i$ on $[C, \infty)$ and therefore A3(i) holds. A3(ii) holds because $\left(a^F\right)'(C) < 1$ for $f(C) > 0$. It remains to be shown that a^F satisfies C1 or C2 if and only if (14) holds. First, consider C1(ii) and C2(iii), that is, DEP. This requires that

$$\frac{F(x_i) - x_i\,(n-1)\,f(x_i)}{F(x_i)^n} < \frac{1}{F(x_i)^{(n-1)}} \tag{15}$$

for all $x_i > 0$. This follows from $f(x_i) > 0$. (14) is equivalent with the requirement that $\lim_{x_i \to 0} a^F(x_i) > 0$ or $\lim_{x_i \to 0} \left(a^F\right)'(x_i) = \infty$ if $\lim_{x_i \to 0} a^F(x_i) = 0$, that is, with C1(i) or C2(i) and C2(ii).
(ii) The preceding analysis shows that A2, A3 and C3 hold in this case. ■

10.2.2 Proof of Propositions 1 and 2

As to Proposition 1, implementability follows immediately from Proposition 6: For $n = 2$, (13) holds if $f(x_i) > 0$ and (14) holds if $f(x_i)$ is bounded. The statement on the slope follows immediately from the form of the candidate prize function a^F. Proposition 2 follows from Proposition 6 because for $lim_{x_i \to 0} f(x_i) < \infty$, (13 is violated for $n > 2$).

[45] If (13) does not hold, the candidate function does not converge to a finite value as efforts approach zero, so that A2 is violated.

10.3 Appendix 3: Equilibria with non-convex Support

Proof of Proposition 5 We first provide a general characterization of equilibria when DEP is violated. In particular, this analysis implies our results on hurdle equilibria (Lemma 2 and Proposition 4).

We construct non-degenerate intervals $\left[H^1, J^1\right]$, $\left[H^2, J^2\right]$,...,$\left[H^K, J^K\right]$ such that $H^1 = H_a$, $J^1 = T$ and $H^k > J^{k+1}$ so that one of the following two cases arises: (i) The MSE has support $\mathcal{H}^k \equiv \left[H^1, J^1\right] \cup \left[H^2, J^2\right] \cup ... \cup \left[H^K, J^K\right]$, where $H^K = 0$ (Figure 5); (ii) the MSE has support $\{0\} \cup \mathcal{H}^k$ (not depicted). In both cases, $\frac{a(x_i)}{x_i}$ monotone decreasing on each of the constituent intervals.

We use the following recursive definition.

Definition 6 *Suppose C4 holds. Let $J^1 = T$ and $H^1 = H_a$. Moreover, for all k where the corresponding quantities are well-defined, let*

$$
\begin{aligned}
J^k &= \max\left\{ x_i < H^{k-1} \;\middle|\; \begin{array}{l} \frac{a(x_i)}{x_i} = \frac{a\left(H^{k-1}\right)}{H^{k-1}} \text{ and } \exists \varepsilon > 0 \text{ such that} \\ \frac{a(x_i)}{x_i} \text{ is strictly decreasing on } (x_i - \varepsilon, x_i) \end{array} \right\} \\
H^k &= \max\left\{ x_i < J^k \;\middle|\; \frac{a(x_i)}{x_i} \text{ is strictly decreasing on } \left[x_i, J^k\right] \right\}
\end{aligned}
$$

Proposition 5 follows immediately from the following stronger statement.

Proposition 7 *Suppose SYM, (C4) and (C5) hold*
(i) If there exists $\varepsilon > 0$ such that $\frac{a(\widetilde{x}_i)}{\widetilde{x}_i} > \max_{x_i > \widetilde{x}_i} \frac{a(x_i)}{x_i}$ $\forall \widetilde{x}_i \in (0, \varepsilon]$, an MSE without atoms exists for which $\mathcal{S} = \mathcal{H}^K$ and $[F(x_i)]^{n-1} a(x_i) = x_i$ on $\mathcal{H}^K$.
(ii) If $\max_{x_i \epsilon (0,T)} \frac{a(x_i)}{x_i}$ exists, an MSE exists for which $\mathcal{S} = \{0\} \cup \mathcal{H}^K$. There is an atom at 0 with size $\left(\frac{H^K}{a(H^K)}\right)^{\frac{1}{n-1}}$. Also, $[F(x_i)]^{n-1} a(x_i) = x_i$ on $\mathcal{H}^K$.
(iii) There exist no other equilibria than those described in (i) and (ii).

Proof. (i) By (C4) and (C5), the sequences H_k and J_k are well-defined and stop after finitely many iterations. F defines a distribution: It is increasing on each $\left[H^k, J^k\right]$, satifies $F\left(J^{k+1}\right) = F\left(H^k\right)$, $F(0) \geq 0$ and $F\left(J^1\right) = F(T) = 1$. All $x_i \in \mathcal{H}^K$ yield zero expected payoffs. For $x_i \notin \mathcal{H}^K$, expected payoffs are negative if $x_i > J^1 = T$. If $x_i \in \left(J^{k+1}, H^k\right)$ for $k \in 1, ..., k-1$, then $\frac{a(x_i)}{x_i} \leq \frac{a\left(H^k\right)}{H^k}$. Expected payoffs $a\left(x_i\right) \frac{H^k}{a(H^k)} - x_i$ are therefore non-positive. (ii) is analogous. (iii) By Lemma 5, $[F(x_i)]^{n-1} a(x_i) = x_i$ on $\mathcal{S}$. Second, we show that $\mathcal{H}^K \subset \mathcal{S}$. From Lemma 9, $\left[H^1, J^1\right] \subset \mathcal{S}$. Next suppose $\left[H^k, J^k\right] \subset \mathcal{S}$, but $\exists x_i \in \left[H^{k+1}, J^{k+1}\right] \notin \mathcal{S}$: Clearly, for every $\widetilde{x}_i \in \left(H^{k+1}, J^k\right]$ and all $\varepsilon > 0$ $\exists x_i \in \left(\widetilde{x}_i - \varepsilon, \widetilde{x}_i\right)$ such that $a\left(x_i\right) \frac{H^k}{a(H^k)} - x_i > 0$, a contradiction. Thus, $\left[H^{k+1}, J^{k+1}\right] \subset \mathcal{S}$ and, by induction, $\mathcal{H}^K \subset \mathcal{S}$. Finally, we show that $\mathcal{S} \subset$

$\{0\} \cup \mathcal{H}^K$. For $x_i > J^1 = T$, expected payoffs are negative. Moreover, by Lemma 6, if $x_L \in \mathcal{S}$ and $x_H \in \mathcal{S}$, $\frac{a(x_H)}{x_H} \leq \frac{a(x_L)}{x_L}$. Thus, as $\left[H^k, J^k\right] \subset \mathcal{S}$, $\left(J^{k+1}, H^k\right) \cap \mathcal{S} = \varnothing$. If the conditions of Lemma 5(i) hold, $H^K = 0$ and thus $\mathcal{S} \subset \{0\} \cup \mathcal{H}^K = \mathcal{H}^K$. If the conditions of Lemma 5(ii) hold, $H^K > 0$ and $F\left(H^k\right) = \frac{H^k}{a(H^k)} > 0$. Hence, there is an atom at 0 or H^k. The latter possibility violates Lemma 4. ■

10.4 Appendix 4: Asymmetric cost functions

We now consider asymmetric costs functions. We confine ourselves to the two-player case.

Assumption A1': *Players have cost functions $k_i x_i$ with $k_1 \equiv k < 1 = k_2$.*

For simplicity, we use a more restrictive version of A2 from now on.

Assumption A2': *$a(x_i)$ is (i) continuous, and (ii) continuously differentiable on* $(0, \infty)$.

Moreover, we adapt A3 to fit the asymmetric case.

Assumption A3': (i)$\forall i \in \mathcal{I} \exists r_i > 0$ *s.t.* $a(r_i) = k_i r_i$ *and* $a(x_i) < k_i x_i$ *for* $x_i > r_i$.[46]

(ii) *$a(x_i)$ is continuously differentiable in an open neighborhood of r_i, with $a'(r_i) < k_i$ for $i = 1, ..., n$.*

We use the notation $T = r_2$ for the threshold of the game, above which all efforts are dominated for the weak player.

10.4.1 Characterization of MSE

We first give conditions under which we can characterize MSE.[47] C1' replaces C1 with the condition from Siegel (2010).[48]

(C1') $a(0) > 0$ and $a(x_i) - kx_i$ is strictly decreasing.

The alternative condition C2 is replaced as follows:

(C2') $a(x_i)$ is strictly concave.

Recall that C2 does not require concavity. However, contrary to C2, C2' does not restrict $a(x_i)$ to be positive near zero. The following definition is useful to describe the support.

[46] r_i is called the "reach" of player i (Siegel 2009, 2010); points above r_i are dominated.

[47] It is simple to show that pure-strategy equilibria may exist for large cost asymmetries that are ruled out by Assumption A3'(ii).

[48] To repeat, however, the framework of Siegel is more general in other dimensions.

Definition 7 *For asymmetric cost functions, define the* ***effort hurdle****,* H_A *as* $\min\{x_i \geq 0 | ka(x_i) - a'(x_i)(w_1 + kx_i) \geq 0\}$*, where* $w_1 \equiv (1-k)T$*.*

This generalizes Definition 4, as $w_1 = 0$ for symmetric cost functions.

Lemma 10 *(a) If C1' or C2' holds, an MSE* (F_1^*, F_2^*) *exists such that:*
(i) F_1^* *has support* $\mathcal{S}_1 = [H_a, T]$ *and*

$$F_1^*(x_1) = \frac{x_1}{a(x_1)} \text{ for } H_a \leq x_1 \leq T. \tag{16}$$

(ii) F_2^* *has support* $\mathcal{S}_2 = \{0\} \cup [H_a, T]$ *and*

$$F_2^*(x_2) = \begin{cases} \frac{w_1 + kH_a}{a(H_a)} & \text{for} \quad x_2 = 0 \\ \frac{w_1 + kx_2}{a(x_2)} & \text{for} \quad H_a \leq x_2 \leq T \end{cases}. \tag{17}$$

(b) $H_a = 0$ *if and only if*

$$a'(0)w_1 - ka(0) \leq 0. \tag{18}$$

(c) Expected payoffs are zero for player 2 *and for player* 1*.*
(d) If C1' or C2' holds, any MSE must be of the form described in (a)-(c).

Proof of Lemma 10(a)-(c) The proof relies on Lemmas 11 to 15 below.

Lemma 11 *Suppose the players choose strategies* F_1^* *and* F_2^* *according to (16) and (17).*
(i) Player 2 *earns expected payoffs of zero on* $\{0\} \cup [H_a, T]$*.*
(ii) Player 1 *earns expected payoffs* $w_1 = (1-k)T$ *on* $[H_a, T]$*.*

Proof. (i) Choosing $x_2 = 0$ yields expected payoffs of 0 if $a(0) = 0$ or if $H_a > 0$. If $a(0) > 0$ and $H_a = 0$, then $\lim_{x_1 \to 0} F_1(x_1) = 0$, so that expected payoffs of player 2 are still zero if he chooses 0. (16) implies that net expected payoffs are zero for $x_2 > 0$ as well. (ii) By (17), player 1 obtains expected payoff w_1 for all $x_1 \in [H_a, T]$. ■

Lemma 12 *Deviations of player* $i = 1, 2$ *to* $x_i \notin \mathcal{S}_i$ *are non-profitable if*

$$H_a = 0 \text{ or } ka(H_a) - a'(H_a)(w_1 + kH_a) \leq 0. \tag{19}$$

Proof. By definition of T, deviations of player 2 to $x_2 > T$ are non-profitable. As C1' and C2'+A3'(ii) each imply $a'(x_1) < k$ for $x_1 > T$, player 1 does not benefit from deviating to $x_1 > T$. Thus deviations to $x_i \notin \mathcal{S}_i$ are unprofitable if $H_a = 0$ and, in particular, under C1'. If C2' holds and $H_a > 0$, player 2 cannot deviate profitably to $x_2 \in (0, H_a)$, as this would involve positive efforts without ever obtaining the prize. Player 1 cannot profitably deviate to $x_1 \in (0, H_a)$ if $a'(x_1)F_2(0) - k \geq 0 \; \forall \; x_1 \in (0, H_a]$. As C2' requires concavity of $a(x_i)$, this holds if $a'(H_a)F_2(0) - k \geq 0$. Inserting $F_2(0)$ gives $a'(H_a)\frac{w_1+kH_a}{a(H_a)} - k \geq 0$, that is, $ka(H_a) - a'(H_a)(w_1 + kH_a) \leq 0$. ■

Lemma 13 *F_1^* is a CDF. F_2^* is a CDF if and only if*

$$ka(H_a) - a'(H_a)(w_1 + kH_a) \geq 0. \tag{20}$$

Proof. By definition of T, $F_1^*(T) = 1$. By C1' or C2', $F_1^*(x_1)$ is weakly increasing on $[0, T]$. Thus, (16) defines a CDF; it has an atom at H_a with mass $F_1^*(H_a) = \frac{H_a}{a(H_a)}$. For F_2^* to be a CDF, it has to be increasing, which requires

$$ka(x_2) - a'(x_2)(w_1 + kx_2) \geq 0 \; \forall x_2 \in [H_a, T]. \tag{21}$$

C1' implies (21) because $a'(x_2) < k$ and $w_1 = a(T) - T < a(x_2) - x_2$. If C2' holds, the left-hand side of (21) is increasing in x_2. Thus, (21) holds on $[H_a, T]$ if and only if $ka(H_a) - a'(H_a)(w_1 + kH_a) \geq 0$. ■

Lemma 14 *Suppose*

$$ka(0) - a'(0)w_1 \geq 0. \tag{22}$$

Then (19) and (20) both hold for $H_a = 0$.

Proof. $H_a = 0$ implies (19). (22) is (20) for $H_a = 0$. ■

Lemma 15 *If C2' holds and*

$$ka(0) - a'(0)w_1 < 0, \tag{23}$$

$H_a \in (0, T)$ is the only effort level that satisfies both (19) and (20).

Proof. For $x_1 > 0$, (19) and (20) hold if and only if

$$ka(x_1) - a'(x_1)(w_1 + kx_1) = 0 \tag{24}$$

By (23), $H_a > 0$. Thus (24) holds for $x_1 = H_a$. C2' implies that the left hand side of (24) is increasing, so that the solution is unique. ■

We now derive Parts (a)-(c) of Lemma 10. If C1' holds, $a(0) > a(T) - kT = w_1$ and $k > a'(0)$. Thus (22) holds. Hence, by Lemma 14, (19) and (20) hold for $H_a = 0$. Thus, Lemmas 11 - 13 imply that Part (a) of Lemma 10 holds if C1' does. If C2' holds and (22) does, the argument is as above. If instead (23) holds, Lemma 15 implies that $H_a > 0$ satisfies (19) and (20); thus F_1 and F_2 are distributions by Lemma 13, and they correspond to an MSE by Lemma 12; so that Part (a) of Lemma 10 also holds in this case. Part (b) of Lemma 10 follows from the Definition of H_a. Part (c) follows from Lemma 11.

Proof of Lemma 10(d) As the result for C1' has been shown by Siegel (2009, 2010), we confine ourselves to C2'. The proof follows from Lemmas 16-25 below. With few exceptions, these results are so general that they do not require C2' (or C1').

Lemma 16 *$\mathcal{S}_i \subset [0,T]$ for any MSE and $i = 1,2$.*

Proof. $x_2 > T$ is not a best response for player 2 because $a(x_2) < x_2$. Thus $\mathcal{S}_2 \subset [0,T]$. If player 1 chooses $x_1 > T$, his net payoff is thus $a(x_1) - kx_1$. As $a'(x_i) < k$ for all $x_i \geq T$ by C1' or C2'+A3'(ii), there exists an $\widetilde{x}_1$ in (T, x_1) such that the net payoff is $a(\widetilde{x}_1) - k\widetilde{x}_1 > a(x_1) - kx_1$. Thus $x_1 > T$ is not a best response. ■

Lemma 17 *If player i has an atom in x^*, then for $j \neq i$, there exists an $\varepsilon > 0$ such that there are no best responses in $[x^* - \varepsilon, x^*]$.*

Proof. Suppose player i has an atom in x^* with mass $p(x^*)$. Suppose for all $\varepsilon > 0$ there exists a best response $x_j \leq x^*$ such that $|x_j - x^*| < \varepsilon$. Let $\varepsilon \to 0$. By deviating to $x_j + \varepsilon$, player j would increase his expected prize by at least approximately $p(x^*)\frac{a(x^*)}{2}$; his costs would increase by ε. The increase in the expected prize is higher than the increase in expected costs, so that x_j is not a best response. ■

Lemma 18 *In any MSE, $0 \in \mathcal{S}_2$ and the expected net payoff of player 2 is zero, that is, $F_1(x_2)\, a(x_2) = x_2$ for all best responses x_2.*

Proof. Player 2 obtains a payoff of at least zero by choosing 0. Thus, for any best response and, by continuity, for any $x_2 \in \mathcal{S}_2$, $F_1(x_2)\, a(x_2) \geq x_2$ follows. To show that Player 2 does not obtain an expected payoff above zero, it suffices to show that there exists a best response x_2 for which he wins with probability arbitrarily close to zero. Let $\underline{x} \equiv \inf \mathcal{S}_1 \cup \mathcal{S}_2$. First, suppose no player has an atom at $\underline{x}$. By definition of $\underline{x}$ one can find a sequence x_n converging to $\underline{x}$ such that x_n is a best response for at least one player i. As there is no atom at $\underline{x}$, the probability of winning and thus the expected payoff converges to zero as $x_n \to \underline{x}$. By Lemma 16, player 1 obtains a payoff of approximately w_1 by choosing x_1 just above T. Thus, x_n cannot consist of best responses for player 1. Hence, the x_n are best responses of player 2 who therefore obtains a payoff of zero in the MSE. Second, suppose exactly one player has an atom at $\underline{x}$. Then this player obtains zero payoffs at $\underline{x}$; and it must therefore be player 2. Third, by Lemma 17, it is impossible that both players have atoms at $\underline{x}$. Finally, $\underline{x} = 0$ and thus $\underline{x} \in \mathcal{S}_2$: Because player 2 wins with probability zero, his net payoffs would be negative if $\underline{x} > 0$. ■

Lemma 19 $\max \mathcal{S}_i = T$ *for* $i = 1, 2$.

Proof. By Lemma 16, it suffices to show that $\max \mathcal{S}_i \geq T$. If $\max \mathcal{S}_1 < T$, then by A3(ii), given the equilibrium strategy of player 1, player 2 could obtain positive payoffs by choosing $x_2 \in (\max \mathcal{S}_1, T)$, contradicting Lemma 18. If $\max \mathcal{S}_2 < T$, $a'(T) - k < 0$ implies that player 1 could profitably deviate downwards from T. Therefore $\max \mathcal{S}_2 = T$. ■

Lemma 20 *If DEP holds, there can be no atom of player* 2 *at any* $x^* > 0$.

Proof. If player 2 has an atom at $x^* > 0$, then by Lemma 17 there exists an $\varepsilon > 0$ such that there are no best responses of player 1 in $(x^* - \varepsilon, x^*] = \varnothing$. Thus, by choosing $x_2 \in (x^* - \varepsilon, x^*)$, player 2 would obtain profits of $F_1(x^*)a(x_2) - x_2$. By Lemma 18, these profits are $\frac{x^*}{a(x^*)}a(x_2) - x_2$. Optimality of x^* thus requires $\frac{x^*}{a(x^*)}a'(x^*) \geq 1$, violating DEP. ■

Lemma 21 *The expected payoff of player* 1 *is* w_1 *in any MSE.*

Proof. By Lemma 19, given the equilibrium strategy of player 2, player 1 can guarantee himself a payoff of arbitrarily close to $w_1 > 0$ with certainty by choosing x_1 just above T. By Lemma 20, player 2 cannot have an atom at T. Therefore, player 1 must obtain an expected payoff of exactly w_1 at T. ■

Lemma 22 *Let* $H_1 \equiv \min \mathcal{S}_1$. *Then* $(0, H_1) \cap \mathcal{S}_2 = \varnothing$.

Proof. Player 2's expected payoffs for $x_2 \in (0, H_1)$ are negative. ■

Lemma 23 *(i) If C2' holds, then* $\nexists\ \underline{x}, \overline{x} \in \mathcal{S}_1$ *with* $\underline{x} < \overline{x}$ *such that* $(\underline{x}, \overline{x}) \cap \mathcal{S}_2 = \varnothing$.
(ii) If C2' holds, then $\nexists\ \underline{x}, \overline{x} \in \mathcal{S}_2 \cap (0, \infty)$ *with* $\underline{x} < \overline{x}$ *such that* $(\underline{x}, \overline{x}) \cap \mathcal{S}_1 = \varnothing$.

Proof. (i) Suppose $\exists\ \underline{x}, \overline{x} \in \mathcal{S}_1$ with $\underline{x} < \overline{x}$ such that $(\underline{x}, \overline{x}) \cap \mathcal{S}_2 = \varnothing$. If so, then, by choosing $x_1 \in (\underline{x}, \overline{x})$, player 1 would obtain profits of $F_2(\underline{x})a(x_1) - x_1$, and $F_2(\overline{x})a(x_1) - x_1$ by Lemma 20. $\overline{x} \in \mathcal{S}_1$ requires $F_2(\underline{x})a(x_1) - x_1 \leq min\,\{F_2(\underline{x})a(x_1) - x_1, F_2(\overline{x})a(x_1) - x_1\}$. Using $F_2(\overline{x}) = F_2(\underline{x})$, these two conditions together violate C2'.

(ii) Suppose $\exists\ \underline{x}, \overline{x} \in \mathcal{S}_2 \cap (0, \infty)$ with $\underline{x} < \overline{x}$ such that $(\underline{x}, \overline{x}) \cap \mathcal{S}_1 = \varnothing$. Then $\underline{x} \in \mathcal{S}_1$. To see this, note that Player 2 has no atoms at any $x_2 > 0$ by Lemma 20. Thus $\forall \varepsilon > 0 \exists \delta \in (0, \varepsilon)$ such that $F_2(\underline{x} - \delta) - F_2(\underline{x} - \varepsilon) > 0$. If $\underline{x} \notin \mathcal{S}_1$, ε can be chosen so that $F_1(x_2)\, a(x_2) = F_1(\underline{x} - \varepsilon)\, a(x_2)$ on $[\underline{x} - \varepsilon, \underline{x} - \delta]$. Thus $F_1(\underline{x} - \varepsilon)\, a'(x_2) \equiv 1$ on this interval, violating C2'. By analogous arguments, $\overline{x} \in \mathcal{S}_1$. Thus, $\underline{x}, \overline{x} \in \mathcal{S}_1$ and (i) shows that $(\underline{x}, \overline{x}) \cap \mathcal{S}_2 \neq \varnothing$. As player 2 has no atom, this would require that there exists an open subinterval of $(\underline{x}, \overline{x})$ in $\mathcal{S}_2$. Because $(\underline{x}, \overline{x}) \cap \mathcal{S}_1 = \varnothing$, this would imply that $F_1(\underline{x})a(x_2) - x_2$ is constant on $(\underline{x}, \overline{x})$, which is incompatible with C2'. ■

Lemma 24 *If C2' holds, $\exists H > 0$ such that $\mathcal{S}_1 = [H, T]$ and $\mathcal{S}_2 = \{0\} \cup [H, T]$.*

Proof. Let $H_2 = \min(\mathcal{S}_2 \cap (0, \infty))$. By Lemma 22, $H_1 \leq H_2$. We show that $H_1 < H_2$ is impossible. First, suppose $(H_1, H_2) \cap \mathcal{S}_1 = \varnothing$. Thus, H_1 must be an atom of player 1. As H_2 is not an atom of player 2 by Lemma 20, $F_2(H_1) = F_2(H_2) = F_2(0)$ and $H_2 \in \mathcal{S}_1$ by Lemma 23. Thus, $F_2(0)a'(H_1) \leq 1$ and $F_2(0)a'(H_2) \geq 1$. These conditions together violate C2'. Second, suppose there exists $x_1 \in (H_1, H_2) \cap \mathcal{S}_1$. If x_1 is an atom, the preceding argument applies. Otherwise, $F_2(0)a(x_1) - x_1$ on a continuum, violating C2'.
Thus $H_1 = H_2$. Therefore, if $H_1 = 0$, then $H_2 = 0$ and, by Lemma 19 , $\mathcal{S}_i \subset [0, T]$ $(i = 1, 2)$. Suppose $\mathcal{S}_i \subsetneq [0, T]$. Then there exist $\overline{x} > \underline{x} > 0$ such that $(\underline{x}, \overline{x}) \cap \mathcal{S}_i = \varnothing$. Hence, by Lemma 23, $\underline{x} \notin \mathcal{S}_j$ or $\overline{x} \notin \mathcal{S}_j$ $(j \neq i)$. Thus, there exists a subinterval of $(\underline{x}, \overline{x})$ which has empty intersection with $\mathcal{S}_1$ and $\mathcal{S}_2$. Choose the interval such that $\underline{x}$ is minimal and $\overline{x}$ is maximal. Then by Lemma 20, there can be no atom of player 2 in $\underline{x}$ or $\overline{x}$. Thus either there is an atom of player 1 at these effort levels or $\mathcal{S}_2$ must contain intervals of the form $(\underline{x} - \varepsilon, \underline{x})$ or $(\overline{x}, \overline{x} + \varepsilon) > 0$. Because of C2', the latter possibility can only arise if $\underline{x} \in \mathcal{S}_1$ $(\overline{x} \in \mathcal{S}_1)$. In any event, $\underline{x} \in \mathcal{S}_1$, $\overline{x} \in \mathcal{S}_1$ and $(\underline{x}, \overline{x}) \cap \mathcal{S}_2 = \varnothing$, which is inconsistent with C2' by Lemma 23. If $H_1 = H_2 \equiv H > 0$, analogous arguments show that $\mathcal{S}_1 = [H, T]$ and $\mathcal{S}_2 \cap (0, \infty) = [H, T]$ by Lemma 23. ■

Lemma 25 *Suppose C2' holds and $H \neq H_a$. Then there can be no MSE with $\mathcal{S}_1 = [H, T]$ and $\mathcal{S}_2 = \{0\} \cup [H, T]$.*

Proof. This follows from Lemma 15. ■

Lemma 10(d) now follows immediately: Lemmas 24 and 25 imply that $\mathcal{S}_1 = [H_a, T]$ and $\mathcal{S}_2 = \{0\} \cup [H_a, T]$. Lemmas 18 and 21 imply that the distribution must satisfy (16) and (17).

10.4.2 Implementation

Next, we present implementation results. Suppose given $H \geq 0$, $C > H_A$ and CDFs F_1 and F_2 with supports $[H, C]$ and $\{0\} \cup [H, C]$, respectively. We ask under which conditions a prize function with MSE (F_1, F_2) exists. Lemma 10 implies that the equilibrium CDF F_2 must satisfy

$$F_2(x_2) = \begin{cases} \left(\frac{(1-k)C}{x_2} + k\right) F_1(x_2) & \text{for} \quad H \leq x_2 < C \\ \left(\frac{(1-k)C}{x_2} + k\right) F_1(H) & \text{for} \quad 0 \leq x_2 < H \end{cases}. \tag{25}$$

Thus, once the CDF of the strong player is fixed, the CDF of the weak player is determined by (25). The only candidate prize function that can implement F_1 on $[H, C]$ is $a^{F_1}(x_1) = \frac{x_1}{F_1(x_1)}$. We start with implementation by a decreasing net prize function, so that $a'(0) < k$. By Lemma 10(b), this requires that the MSE has support $[0, C]$ for both players.

Proposition 8 *Suppose a CDF F_1 has density f_1 and $\{x_1 | f_1(x_1) > 0\} = [0, C]$ for $C > 0$. Let F_2 be given by (25). Then (F_1, F_2) can be implemented as the MSE of an AEP with a decreasing net prize function if and only if*

$$F_1(x_1) - x_1 f_1(x_1) < k \left(F_1(x_1)\right)^2. \tag{26}$$

Proof. We first show that (26) is sufficient for implementation by monotone net prize functions. By Proposition 10, this requires that $a(x_1)$ satisfies A2 and A3 and C1'. As to A2, the candidate function $a(x_1) = \frac{x_1}{F_1(x_1)}$ is continuously differentiable for $x_1 > 0$. Continuity at 0 requires $lim_{x_1 \to 0} a(x_1) = \frac{1}{f_1(0)} < \infty$, which is equivalent with $f_1(0) > 0$. A3 is clearly satisfied. C1' is equivalent with (26). Necessity follows because A2 is violated if (26) does not hold. ■

Next, we ask which CDFs can result as MSE for strictly concave $a(x_i)$.

Proposition 9 *Suppose a CDF F_1 has differentiable density f_1 and $\exists\, C > H \geq 0$ such that $\{x_1 | f_1(x_1) > 0\} = [H, C]$. If F_2 is given by (25), (F_1, F_2) can be implemented as the MSE of an AEP with a strictly concave $a(x_i)$ if and only if*

$$\frac{f_1'(x)}{f_1(x)} + \frac{2}{x} - 2\frac{f_1(x)}{F_1(x)} < 0. \tag{27}$$

Proof. We show that (27) is sufficient for implementation by a strictly concave prize function. By Lemma 10, this is true if the candidate prize function satisfies A2 and A3 and C2'. A2 follows as in the proof of Proposition 8. A3 is clearly satisfied. C2' is equivalent with (27). ■

10.4.3 Optimality

We now reconsider the design problem for the specific case $k = 0.95$. Figure A3 depicts equilibria for $\gamma = 0$ (*fixed prize*), $\gamma = 0.5$ (*intermediate case*) and $\gamma = 0.5$ (*almost linear case*).

In the fixed prize case ($\gamma = 0$), only the weak player has an atom at zero. As γ increases, atoms for both players emerge and the support becomes smaller.[49]

[49] For $k = 0.85$ (not depicted), the supports are smaller than for $k = 0.95$ and the atoms are larger; the MSE thus approaches an asymmetric PSE (which can easily be shown to arise when $k \leq 1 - \gamma$.

Tedious calculations show that the expected average efforts of the two players (with the threshold $T = \alpha^{\frac{1}{1-\gamma}}$) are $T\frac{1-\gamma}{2-\gamma}\left(\left(1+\frac{1}{1-\gamma}\left(\frac{\gamma}{19(1-\gamma)}\right)^{2-\gamma}\right)\right)$ for Player 1 and $T\left(\frac{1}{20(2-\gamma)(1-\gamma)}\left(\left(-40\gamma+20\gamma^2+19\right)+\gamma\left(\frac{\gamma}{19(1-\gamma)}\right)^{1-\gamma}\right)\right)$ for Player 2. As the expected rent is $0.05T$, all (α, γ) for which $\alpha^{\frac{1}{1-\gamma}}$ is identical generate the same expected rent. Total expected efforts within a class of AEP with identical rent is thus maximized by maximizing $\frac{1-\gamma}{2-\gamma}\left(\left(1+\frac{1}{1-\gamma}\left(\frac{\gamma}{19(1-\gamma)}\right)^{2-\gamma}\right)\right)+\frac{1}{20(2-\gamma)(1-\gamma)}\left(\left(-40\gamma+20\gamma^2+19\right)+\gamma\left(\frac{\gamma}{19(1-\gamma)}\right)^{1-\gamma}\right)$.

Figure 10 depicts the ratios of expected efforts to expected rents.

10.5 Appendix 5: Figures

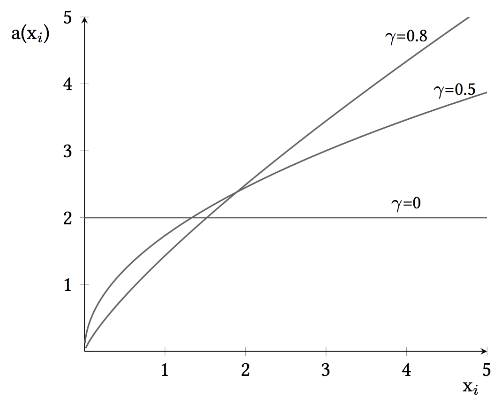

Figure A1: Prize-equivalent AEPs

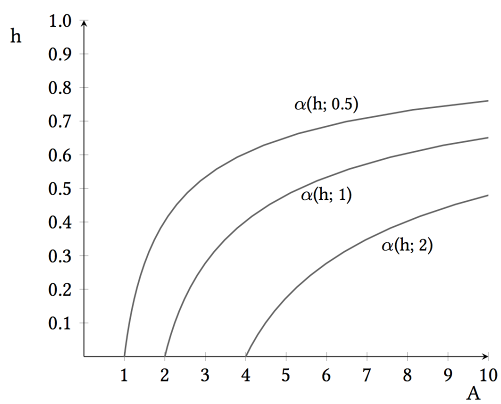

Figure A2: Prize-equivalent AEP with hurdle equilibria

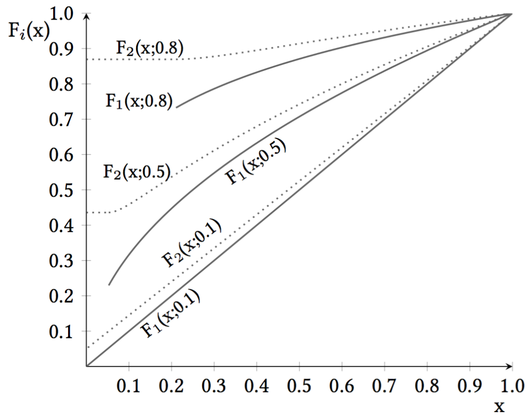

Figure A3: MSE for asymetric players (k=0.9)

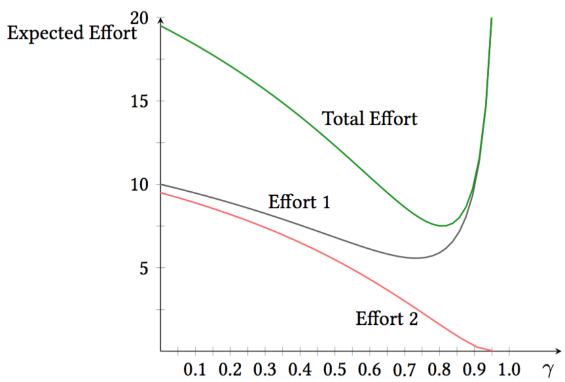

Figure A4: Ratio of expected efforts to rents for asymmetric players

Appendix 6: Experimental Instructions (Translation from German)

This appendix contains the instructions that the subjects received in *NoExt*. The instructions in *NegExt*, and *PrEqv* are identical except for the numerical examples.

General Explanation

Welcome to today's experiment! The experiment serves to analyse individual decision behaviour. Read carefully through the following instructions. Your decisions during the experiment influence if you earn more or less.

During the experiment your income is calculated in points. The following rule applies:

$$3 \text{ points} = \text{CHF } 1$$

At the end of the experiment your points will be transferred into Swiss Francs and you will be paid out this amount.

All information you receive is solely for you. It is prohibited to exchange any information or communicate otherwise with other participants of the experiment. It is important that you also do not communicate during the experiment. Raise your hand if you have any questions. We will answer your question individually.

This session will proceed as follows:

1. You and all other participants will read through these instructions. Subsequently you will answer questions. The sole purpose of these questions is to control whether you understand the instructions.

2. The experiment that is described in the instructions will be carried out one time.

3. After the experiment was carried out you will answer a questionnaire and participate in two small experiments, which are both independent of the experiment described in these instructions. Afterwards you will receive your payment.

Keep in mind that all numerical examples in the instructions are chosen randomly and do not constitute any hints or proposals for your decision!

The Experiment

You are participating in an experiment where you interact with an other participant in a group of two. The decision that you have to make is called *investment*.

The experiment consists of 20 rounds. After each round all participants are matched anonymously into new groups of two. No participant will know with whom she or he was paired in a group.

Your initial endowment at the beginning of the experiment is 150 points (CHF 50). In each of the 20 rounds you can earn a profit. Due to the investments this income can either be positive, negative, or zero. Your total profit is your initial endowment and the sum of your profits in each round. Hence,

$$\text{total profit} = 150 + \text{all profits}$$

Keep in mind that your total profit will never be negative after 20 rounds.

The subsequent two sections will explain the decision procedure within one round and the procedure throughout the 20 rounds.

Decision in one Round

In each round you have the possibility to invest. In each round you can invest at most 5 points. The lowest possible amount is 0 points. Choosing 0 points means that you do not invest.

a) Your profit in each round is determined by your investment and the investment of the other participant in your group of two. Your profit in each round revenue net of costs. Hence,

$$\text{profit} = \text{revenue} - \text{costs}$$

Your profit will be subtracted from your initial endowment if it is negative.

b) Investments greater than 0 cause costs. If you do not invest you do not incur costs. Your costs correspond to the amount you invest. For example, if you invest 3.58 points you will incur costs of 3.58 points.

c) Investments also yield revenue. You always yield positive revenue if you invest more points than the other participant in your group of two. Your revenue increases as you invest more points. Your revenue equals 0 if the other participant in your group of two invests more points than you. If you invest the same amount of points as the other participant in your group of two you receive positive revenue with probability 0.5 and a revenue equal to 0 with probability 0.5.

This can be made clear by looking at the revenue table. Note that the steps between the next higher investment are given by 0.25 in the table. However, when submitting your decision with the computer you can choose your investment in steps of 0.01 (for instance 1.83 or 2.94). The columns contain the investments that you can choose. The lines contain the investments that the other participant in your group of two can choose.

For instance, if you invest in one of the 20 rounds 3.25 points and the other participant in your group of two invests 1.75 points, then your revenue is 3.61 points. If however you invest in one of the 20 rounds 1.75 points and the other participant in your group of two invests 3.25 points, then your revenue is 0 points.

Decision Submission with the Computer

During the experiment you will submit 20 times your decision with the computer on the screens stated below.

In each of the 20 rounds you see the following decision screen (Figure A5). Enter in the central field the amount of points you would like to invest. To submit your decision you have to press the button “OK”. Before submitting your decision you have the possibility to calculate your hypothetical profit. In the lower part of the screen you can enter hypothetical own investments and hypothetical investments of the other participant in your group of two. By pressing the button “Calculate” you can see the hypothetical revenue resulting from these investments.

At the end of every round after each participant submits her or his investment the following information screen appears (Figure A6). This screen provides information about your investment, and about the investment of the other participant in your group of two. Further, your revenue, costs and profit in the respective round are displayed. The screen also displays your current total profit. The next round starts automatically after 30 seconds.

Before the first and the eleventh round you have to answer questions (Figure A7). Each time you have to forecast how often you will invest more points than the other participants in your group of two during the following 10 rounds. Further, you have to forecast how often the other participants in your group of two will invest between 0.00 and 1.00, 1.01 and 2.00, 2.01 and 3.00, 3.01 and 4.00, and 4.01 and 5.00 during the following 10 rounds. These numbers should sum up to 10. These forecasts have no influence on your profits and do not alter the experiment in any other way! You submit your forecasts by pressing the button “OK”.

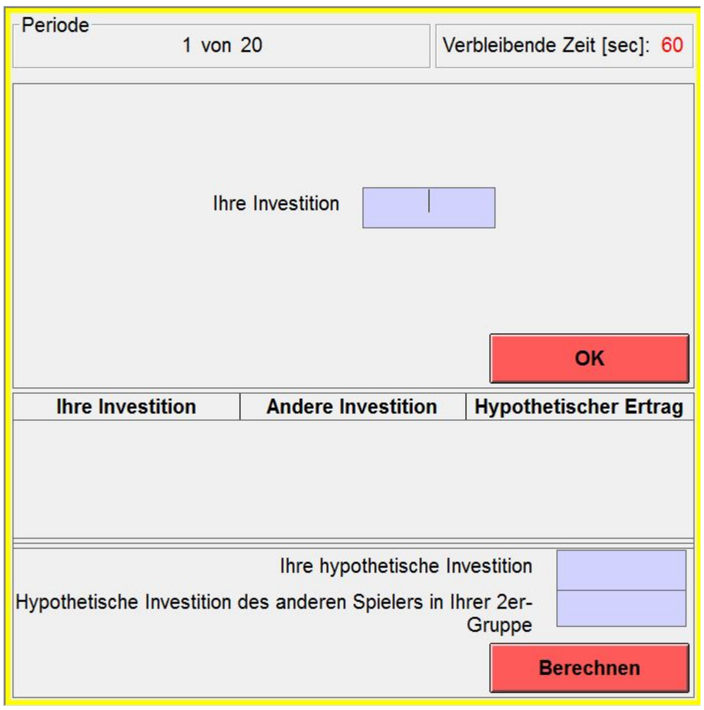

Figure A5: Decision screen.

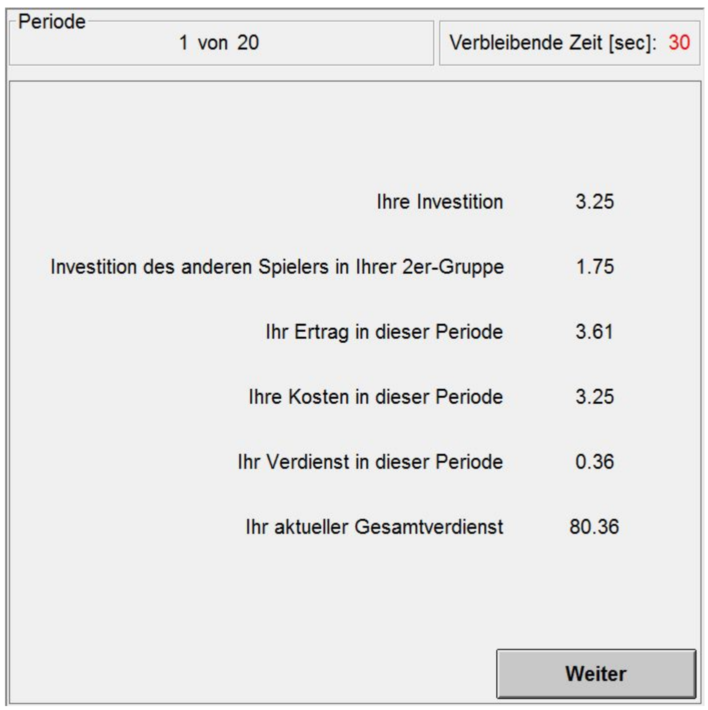

Figure A6: Information screen.

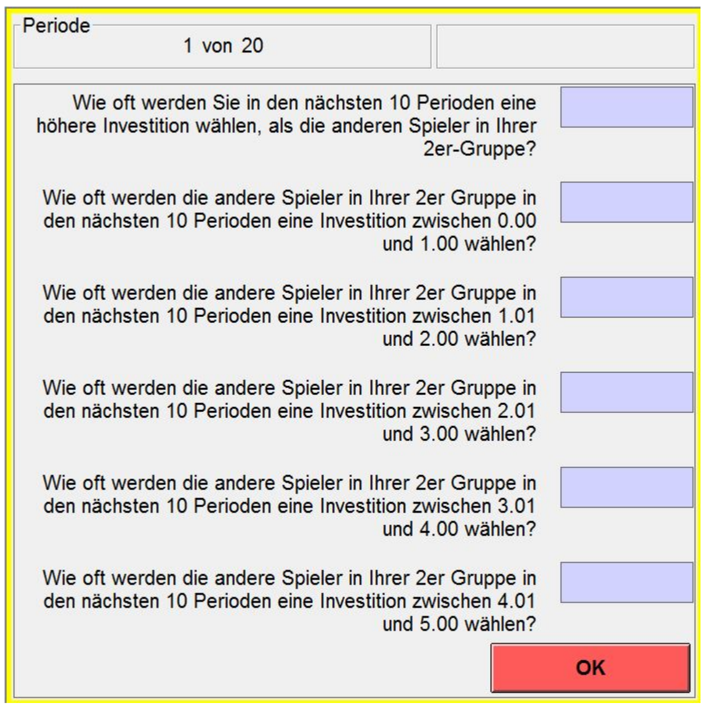

Figure A7: Forecast screen.