Appendix B to "Intellectual Property Rights in a Quality-Ladder Model with Persistent Leadership"

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This appendix provides a formal proof of Lemma 2 (Section B1) and a section analyzing under which conditions there is persistent leadership under forward protection (B2). In further sections, the case of intermediate R&D inputs is discussed (B3) and a welfare analysis is provided (B4).

B1: Proof of Lemma 2

Proof. Given the incumbent bears the fraction β of the total R&D costs in an industry, the value of being an incumbent with a two step lead supplying vintage k of the good is defined by the following arbitrage condition

$$rV_2(k) = \pi_m g^k - \beta c g^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_2(k) + \beta \phi^* V_2(k+1)$$

The first term on the right hand side indicates per period profits, the second term the R&D costs of the incumbent, the third term accounts for the probability of losing the current market due to an innovation and the fourth term for the probability of achieving the next innovation. Replacing $V_2(k+1) = gV_2(k)$ and solving for $V_2(k)$ gives

$$V_2(k) = \frac{\pi_m g^k - \beta c g^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(\beta g - 1\right)}$$

In the following, it is assumed that $r > \phi^*(g-1)$ holds (Condition B) so that $V_2(k)$ is bounded. Incumbents want to increase their share in the total R&D expenditures,

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β, if $\frac{\partial V_2(k)}{\partial \beta} > 0$. We obtain

$$sign\left\{\frac{\partial V_2(k)}{\partial \beta}\right\} = sign\left\{g\pi_m - c\left(\phi^*\right)^{\epsilon}\left(r+\phi^*\right)\right\}$$

Inserting $(\phi^*)^{\epsilon} = \frac{V_1(k+1)}{cg^k} = \frac{V_1(k)}{cg^{k-1}}$ from the free entry condition (equation 8), $\frac{\partial V_2(k)}{\partial \beta} > 0$ therefore holds if $V_1(k) < \frac{g^k \pi_m}{r+\phi^*}$ (**Condition 10**). We know that

$$V_1(k) < V_2(k) = \frac{\pi_m g^k - \beta c g^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^*\left(\beta g - 1\right)}$$

Inserting $(\phi^*)^{\epsilon} = \frac{V_1(k)}{cg^{k-1}}$ and rearranging again gives Condition 10. Therefore, the inequality $V_1(k) < V_2(k)$ implies that $\frac{\partial V_2(k)}{\partial \beta} > 0$ holds, so that incumbents with a two step lead find it profitable to increase β to one and to undertake all the R&D.

The value of being an incumbent with a one step lead supplying vintage k of the good is defined by the arbitrage condition

$$rV_1(k) = \pi_1 g^k - \beta_1 c g^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_1(k) + \beta_1 \phi^* V_2(k+1)$$

with β_1 denoting the fraction of the total R&D undertaken by the incumbent. This arbitrage condition can be solved for

$$V_1(k) = \frac{\pi_1 g^k - \beta_1 c g^k \left(\phi^*\right)^{1+\epsilon} + \beta_1 \phi^* V_2(k+1)}{r+\phi^*}$$

Taking into account that $V_2(k+1)$ is independent of β_1, we get

$$\frac{\partial V_1(k)}{\partial \beta_1} = \frac{\phi^*}{r+\phi^*}\left[V_2(k+1) - cg^k\left(\phi^*\right)^{\epsilon}\right]$$

Inserting $(\phi^*)^{\epsilon} = \frac{V_1(k)}{cg^{k-1}}$ from equation 8 and replacing $V_2(k+1) = gV_2(k)$, we obtain

$$\frac{\partial V_1(k)}{\partial \beta_1} = \frac{g\phi^*}{r+\phi^*}\left[V_2(k) - V_1(k)\right] > 0$$

Therefore, also incumbents with a one step lead find it profitable to set $\beta_1 = 1$ and to do all the R&D.

As incumbents do all the R&D, there is persistence in leadership.

As a next step, it is shown that the fact that incremental profits fall in a firm's lead (Lemma 1) indeed implies that incumbents with a one or two step lead have lower stand-alone innovation incentives than entrants (with a zero step lead) and therefore only want to do as much R&D as needed to preempt entry. The value of a firm with an l step lead that undertakes all the R&D in its industry and innovates at rate ϕ_l is

given by the arbitrage condition

$$rV_l(k) = \pi_l g^k - cg^k (\phi_l)^{1+\epsilon} + \phi_l [V_{l+1}(k+1) - V_l(k)]$$

Therefore, the firm wants to increase the innovation rate if $\frac{\partial(V_l)}{\partial \phi_l} > 0$, which holds if

$$[V_{l+1}(k+1) - V_l(k)] - cg^k (1+\epsilon) \phi_l^\epsilon > 0$$

(**Condition 11**). Suppose that $V_{l+1}(k+1) - V_l(k)$ falls in l so that it is maximal for $l = 0$. Then, Condition 11 cannot hold if the incumbent already innovates at the entry preempting rate ϕ^* as $\left.\frac{\partial(V_l)}{\partial \phi_l}\right|_{\phi_l=\phi^*} < 0$ holds due to the free entry condition (equation 8), which implies that $V_o(k) = 0$ and that $V_1(k+1) = cg^k (\phi^*)^\epsilon$.[1] Therefore, a sufficient condition that guarantees that incumbents never want to increase the innovation rate above the level ϕ^* is that $V_{l+1}(k+1) - V_l(k)$ falls in l (**Condition 12**) if $\phi = \phi^*$. It is now shown that Condition 12 is satisfied for any innovation rate $\tilde{\phi} > 0$ if incumbents are the only innovators and if they choose this rate for any lead size. In the case of quasi-drastic innovations ($s = 2$) Condition 12 then consists of the following two inequalities

$$V_2(k+1, \tilde{\phi}) - V_2(k, \tilde{\phi}) < V_2(k+1, \tilde{\phi}) - V_1(k, \tilde{\phi})$$

$$V_2(k+1, \tilde{\phi}) - V_1(k, \tilde{\phi}) < V_1(k+1, \tilde{\phi}) - V_0(k, \tilde{\phi})$$

The first inequality is satisfied because $V_1(k, \tilde{\phi}) < V_2(k, \tilde{\phi})$ holds due to the fact that $\pi_1 < \pi_2 = \pi_m$. Inserting the corresponding values into the second inequality[2], simplifying and rearranging terms leads to the inequality $r(g\pi_2 - g\pi_1 - \pi_1) < \phi\pi_1$. This inequality is satisfied for any value $\tilde{\phi} \geq 0$ if $g\pi_1 > g\pi_2 - \pi_1$. The latter inequality (implying that incremental profits are larger for entrants than for incumbents with a one step lead) holds as due to Lemma 1. Therefore, $V_{l+1}(k+1, \tilde{\phi}) - V_l(k, \tilde{\phi})$ falls in l, implying that $\left.\frac{\partial(V_l)}{\partial \phi_l}\right|_{\phi_l=\phi^*} < 0$ holds. Consequently, incumbents never want to increase the innovation rate above the entry-preempting level ϕ^*. □

B2: Persistent Leadership under Forward Protection

While there is clearly persistent leadership under forward protection if the latter eliminates all entry pressure, this is less clear in the remaining cases. A preemption equi-

[1] The free entry condition equates the value of an innovation to the average R&D costs while, absent free entry, a firm with a zero-step lead would want to only equate it to the marginal R&D costs and therefore to undertake less R&D.

[2] $V_2(\tilde{\phi})$ and $V_1(\tilde{\phi})$ are the same as in the main part of the text with ϕ^* replaced by $\tilde{\phi}$. Note that $V_0(k, \tilde{\phi}) = -cg^k \tilde{\phi}^{1+\epsilon} + \frac{\tilde{\phi}}{(r+\tilde{\phi})} V_1(k+1, \tilde{\phi})$ need not be equal to zero for an arbitrary $\tilde{\phi}$.

librium results when the incumbent values not being replaced and obtaining the next innovation herself more than an entrant values entry. This holds if entry leads to a reduction in the total surplus (efficiency effect). The latter is always the case if $1 \leq R < s - 1$ holds, as profits are then reduced below the unconstrained monopoly level once the entrant obtains the lead $l = R + 1 < s$ at which he is not infringing the IPRs of the previous incumbent anymore (implying that collusion is inhibited) and at which he has not yet reached unconstrained monopoly power himself.

But even if $R \geq s - 1$, so that collusion allows to jointly earn unconstrained monopoly profits for any lead size l, $\phi_l > \phi^*$ might hold for some lead sizes $l < s$, implying that joint net profits are also reduced when compared to the case without entry where the previous incumbent would have kept innovating at the lower (and less expensive) entry-preempting rate ϕ^*. Therefore, a new efficiency effect arises in such a case and the incumbent has again incentives to do all the R&D and to preempt entry, implying that there is persistent leadership[3]. The condition $\phi_l > \phi^*$ can hold if a (recent) entrant with an l step lead values obtaining the next innovation sufficiently more than entrants with a zero step lead do[4]. This condition is most likely satisfied for step size $l = R$, as the next innovation then "frees" the innovator from paying royalties to the previous incumbent. From Lemma 4 i) it can be inferred that $\phi_1 > \phi^*$ indeed holds if $R = 1$, $s = 2$, $\epsilon \to 0$, $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_1 g}{r}$ (Condition C) and if $\frac{g^2 \pi_m}{g+1} > rc$[5].

It, however, cannot be ruled out that in certain cases the efficiency effect disappears under forward protection, implying that incumbents are indifferent about their share in total R&D so that there might not be persistent leadership anymore.

B3: Intermediate R&D Inputs

This section studies the case where two R&D stages have to be completed in order to improve the quality of an intermediate good by one step. In the first stage, an intermediate R&D input (which might be thought of as an idea) has to be invented and

[3]It should be noted that the result that forward protection reduces growth was derived under the assumption that forward protection only allows the entrant and the incumbent to collude in prices but not to coordinate their joint R&D expenditures. If forward protection also allows for the latter and if under forward protection $\phi_l > \phi^*$ holds for some l, the incumbent is willing to compensate the entrant if he reduces the innovation rate from ϕ_l to ϕ^*. In a previous version of the paper that studies a simplified version of the model it is shown that forward protection might even increase entry pressure and growth in such a case if entrants have all the bargaining power.

[4]In order to induce a firm with an l step lead to increase ϕ_l above the level ϕ^* that is pinned down by the free entry condition, $V_{l+1}(k+1) - V_l(k)$ needs to be sufficiently larger than $V_1(k+1)$ as such a firm wants to equate $V_{l+1}(k+1) - V_l(k)$ to the marginal R&D costs while entrants with a zero step lead "over-invest" in R&D and equate $V_1(k+1)$ to the average R&D costs.

[5]This can be shown by verifying that the inequalities defining case i) in Lemma 4 are satisfied if $\gamma_1 = \gamma_2 = 0$ (no IPR expiration), if $\pi_1 = \tilde{\pi}_1^{max} = \pi_m(g-1)$, i.e. if the entrant obtains the maximal possible profits under forward protection if $l = 1$, and if $\frac{g^2 \pi_m}{g+1} > rc$.

this input is used in the second stage to develop an improved version of the intermediate good. The two stages can also be interpreted as a research and a development stage. The R&D technology is again stochastic and given by equation 3 for both stages that are needed for the invention of a vintage $k+1$. Therefore, preemption is possible at both stages. It is assumed that there is full IP protection against imitation in the intermediate goods markets ($\gamma = 0$). In the case where IP protection is granted on an intermediate R&D input, it allows to prevent other firms from using this input, and therefore from developing a better version of the corresponding intermediate good.

Looking again at the limit case of constant returns to R&D ($\epsilon \to 0$) and assuming that Condition C ($\frac{\pi_2\left(g^{k+1}-g^k\right)}{r} < cg^k < \frac{\pi_1 g^{k+1}}{r}$) holds, three cases are considered:

1): *No IP protection is granted on intermediate R&D inputs.*

In this case, there is no growth as no firm has incentives to invent such an input. Once the input is invented, there is free entry into the second stage development race, so that expected profits for entrants are zero in this race. Expecting this, no entrant has incentives to spend money on inventing such an intermediate input in the first place. But neither does an incumbent who has already obtained a two step lead and, due to Condition C, does not find it profitable to continue innovating if there is no threat of entry.

2): *Both entrants and incumbents can obtain IP protection on intermediate R&D inputs.*

Innovation and growth also come to a halt in this case as incumbents always use the possibility of obtaining IP protection on the newest inputs in order to block entry and to eliminate the threat of being replaced by an entrant. Given that an incumbent has obtained a two step lead in an industry and has also obtained an IPR on the R&D input which is needed to develop the next vintage of the good, she uses it to block follow-on R&D by entrants. If, instead, the entrant has IP protection on the newest version of the input, he finds it profitable to license it to the incumbent as he values doing follow-on R&D and entering the market with an improved vintage of the good less than the leader values blocking follow-on R&D. Even if entrants are not allowed to license to incumbents, the incumbent can prevent an entrant from obtaining IP protection on the intermediate R&D input by undertaking a sufficient amount of research effort in the race for the input.

3): *Only entrants can obtain IP protection on intermediate R&D inputs, but are not allowed to license to incumbents.*

In this case, sustained innovation and growth is possible, as an incumbent has incentives to preempt R&D of entrants at each stage, without ever being able to block future entry completely. If an input is invented by an incumbent and freely accessible to entrants, the incumbent has incentives to preempt entry by exerting a large enough

effort in the race for the second R&D stage. Expecting this, she finds inventing the intermediate input worthwhile (even if she does not obtain IP protection on it), as it prevents entrants from inventing and obtaining IP protection on it, which would allow them to replace her in the future. Once the second R&D stage is completed and the incumbent has developed the next vintage of an intermediate good, the whole process starts again.

B4: Welfare

Suppose that that IP protection does not expire and that incumbents with a two-step lead can maximally charge the price $\bar{p} < \frac{1}{\alpha}$, implying that their profits π_2 rise in $\bar{p}$. Once incumbents in all industries have obtained the maximal lead and charge price $\bar{p}$, the innovation rate is the same in all sectors ($\phi_\omega = \phi^*$) and consumption in period t is given by

$$c(t) = y(t) - \int_{\omega=0}^{1} x\left(k, \omega, t\right) d\omega - \int_{\omega=0}^{1} c g^{k(\omega,t)} \left(\phi^*\right)^{1+\epsilon} d\omega$$

Inserting $y(t)$ from equation 2 and $x(k, \omega, t)$ from equation 5 gives

$$c(t) = \left(\alpha^{\frac{\alpha}{1-\alpha}} \bar{p}^{\left(-\frac{\alpha}{1-\alpha}\right)} - \alpha^{\frac{1}{1-\alpha}} \bar{p}^{\left(-\frac{1}{1-\alpha}\right)} - c\left(\phi^*\right)^{1+\epsilon}\right) \int_{\omega=0}^{1} g^{k(\omega,t)} d\omega$$

For $\bar{p} > 1$, $c(t)$ falls in $\bar{p}$.

Along a balanced growth path, $c(t)$ grows at the rate $G^* = (g-1)\phi^*$ at which the aggregate quality index $\int_{\omega=0}^{1} g^{k(\omega,t)} d\omega$ grows (see Appendix A2) and falls in π_m and therefore in π_2 and in $\bar{p}$ (see Proposition 1). Intertemporal utility (welfare) is then given by

$$U(\tau) = \int_{t=\tau}^{\infty} c(t) e^{-r(t-\tau)} dt = \frac{c\left(\tau\right)}{r - G^*}$$

and increases in $c(\tau)$ and in G^*. If a certain innovation rate ϕ^* and growth rate G^* can be attained with either a high $\bar{p}$ and low entry pressure (e.g. a low π_1 resulting from forward protection) or high entry pressure and a low level of $\bar{p}$, the latter option is therefore preferable from a welfare perspective as it increases consumption (for a given rate of growth).