How Nations Evolve: Political Accountability and Developmental Trajectories∗

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Abstract

We study how the developmental trajectory of a nation is determined by the endogenous evolution of political accountability. We analyze a dynamic game between citizens and a government with privately known type (benevolent or opportunistic) and hidden action. To diversify sector-specific economic risks, citizens invest in private sectors and expect the government to invest tax revenue into public sectors. Political risk arises as an opportunistic government may confiscate tax revenue with positive probability. The government is however constrained by political accountability enforced through the threat of distrust from Bayesian citizens, who, although biased from possibly slow adjustment of initial trust, may gradually learn about the government’s type as informational efficiency gets improved in the development process. We derive the unique Markov perfect equilibrium of the game, and show how the joint evolution of political accountability and the equilibrium strategy of an opportunistic government determines the developmental trajectory of a nation. Our results offer an explanation for the rise of "vicious" or "virtuous" economic and political circles. As an extension, we also discuss self-fulfilling political transition.

Key Words: political accountability, economic development, public monitoring, public trust, political transition, developmental trajectories

JEL classification: C73, D72, O11, P16

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1 Introduction

How does the evolution of a nation unravel into certain economic and political developmental trajectory? Different scholars have emphasized different characteristics of development paths (Acemoglu and Robinson (2001), Hall and Jones (1999), Besley and Persson (2011), Acemoglu and Robinson (2012)). However, the endogeneity of a uniform long run driving force has yet to be established. This paper provides an answer to this question centering on the endogenous evolution of both political accountability and governments’ adaptation strategy.

Political accountability is a key concept in modern democracy theory. It allows the public to screen and discipline the government (Maskin and Tirole (2001), Benhabib and Przeworski (2010)). It can explain questions like the gap between the rules and the practice of democracy (Bidner and Francois (2013)) and account for corruption or effective governance across countries (Adserà et al. (2003)). This concept does not lack historicity, though. Throughout history, there exists a minimal set of responsibilities for any government, despite its form, to act in the interest of the citizens.1 Among mechanisms that keep the government in check, trust building is an important one (Besley and Case (1995)). Citizens as the beneficiary are passive in the relationship of trust (Waldron (2015)), but such limit exists only to the extent that forms of punishment of the incumbent, such as turnovers, removal from office through voting (Besley (2007)) and street protests (Lohmann (1994)), are various.2 The credibility of the threat is still rooted in a low public trust.3 Citizens adjust their trust in the government based on information available. If citizens have more precise knowledge about the government, they can hold the government accountable more efficiently (Adserà et al. (2003), Besley (2007)). Therefore, in this paper, the essence of political accountability is captured by the information assessment of Bayesian citizens with the punishment linked to the level of public trust.4

Regarding information assessment, there is a host of reasons why governments

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1A medieval king in Europe had to swear an oath to fulfill a basic set of responsibilities such as execution of justice and protection of the country. Swedish kings swore to restrict violence, rule justly, and protect the poor (Musson (2001)). The Magna Carta in 1215 formally introduced the standards of accountability in government concerning tax collection and expenditure review. Emperors of China in Confucian tradition received moral education to be totally public-spirited, ideally. Besides, the king’s sheriff formed governments accountable for failure to carry out the king’s obligation (Sabapathy (2014)).

2The notion that governments act fundamentally as trustees for the people appears often in Locke and Rousseau’s political theory.

3According to Waldron (2015), ’Philp, Political Conduct, pp. 221-3, seems to think that a trust conception of accountability is better because the supposed beneficiaries of government action are not always empowered to hold the government accountable.’ From this perspective, tracking public trust is even a more reasonable approach.

4In different contexts, the importance of this belief is reflected in different terms. Abraham Lincoln in Lincoln-Douglas Debates said "...public sentiment is everything. With public sentiment, nothing can fail; without it nothing can succeed." In canonical political agency model, it is called reputation (Besley (2007)). In some papers, it is called public trust (Phelan (2006)), or long-term credibility (Lu (2013)), or institutional grievance (Dorsch and Maarek (2015)).
are better informed than citizens and hence, act on the basis of privileged information (Maskin and Tirole (2001), Besley et al. (2002)). In this paper, we argue that such information gap between the government and the citizens is reduced as an economy develops. Consequently, political accountability is fostered by economic development (Olson (1963)). Direct observations in history include economic development followed by revolutions or political instability. For example, preceded the Reformation, the English, French, American and Russian revolutions all feature an economic improvement first (Huntington (1968)).

Besides the information gap, the level of initial public trust (Lu (2013)), and trust adjustment speed which determines how long political optimism or pessimism might last, also affect the effectiveness of political accountability. Nunn et al. (2016) shows that citizens in high trust societies are less likely to attribute blame for poor macroeconomic performance to their politicians. Therefore, we track the evolution of political accountability by incorporating endogeneity of both information efficiency (economic development) and public trust into our model.

How well any government functions hinges on how good citizens are at making it accountable for its actions (Adserà et al. (2003), Besley (2007), Bidner and Francois (2013)). Accordingly, we solve opportunistic governments’ strategy as political accountability evolves. We adopt the argument of adaptability of governments from Huntington (1968) to justify the endogenous evolution of governments’ strategy: "A governmental organ that can successfully adapt itself to changed functions, such as the British Crown in eighteenth and nineteenth centuries, is more of an institution than one which cannot, such as the French monarchy in the eighteenth and nineteenth centuries." 7

Literature has emphasized the importance of good governance for long term development (Mauro (1995), Easterly and Levine (1997), Hall and Jones (1999), Acemoglu and Robinson (2012)). Our argument is in line with them. The joint evolution of political accountability and governments’ adaptation determines nations’ developmental trajectories. Perhaps the most important characteristic of various development paths is the political and economic gaps across countries and over time. Over time, there is the general observation that democracy was very rare before the industrial revolution. Across countries, starting from mid-1950s, economists noticed the eco-

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5 From this point of view, construction of redistributive conflicts in the canonical model of Acemoglu and Robinson (2001) relies on business cycle shocks to generate a timing when the poor is endowed with large capacities to hold the elite accountable.

6 "It was precisely in those parts... where there had been most improvement that popular discontent ran highest" (de Tocqueville (1955)).

7 In fact, the evolution of governments’ strategy that self-regulates in accordance with the political accountability is the evolution of institutions as institutions are stable, valued, recurring patterns of behaviour (Huntington (1968)).

8 Acemoglu and Robinson (2012) propose a theory of why nations fail. They first discuss the distinction between extractive and inclusive economic and political institutions then explain how history has shaped institutional trajectories of nations. Our model supports the viewpoint of extractive and inclusive institutions and generates enough endogeneity as a complement to offer an insight into the co-evolution of history and institutions.
economic gap between rich and poor nations. While "in politics as in economics the gap between developed political systems and underdeveloped political systems has broadened." "This political gap resembles and is related to the economic gap (Huntington (1968)).' In our model, governments’ equilibrium strategy further impacts economic development and citizens respond to economic outcome by updating their trust. A new environment of political accountability is thus formed for governments to adapt to. In this way, we complete the circle between endogenous political accountability and governments’ adaptation as illustrated in Figure 1.

![Figure 1: Framework](image)

We formalize our ideas through a dynamic game in which citizens are not informed about the type of the government and its actions. The government can be either benevolent or opportunistic. The latter type retains the option of predation. Citizens update their beliefs of the type of the government, i.e. public trust, based on information available which includes the observation of economic outcome. Two entangling risks determine the outcome: economic risk, and political risk from uncertainty of an opportunistic government’s actions.

The construction of economic risk is borrowed from Acemoglu and Zilibotti (1997). Sector-specific economic risks can only be diversified if the economic sectors are active with investment. Capital accumulation thus opens the gate to more sectors and more risk diversification opportunities. On nations’ development paths, ancient economy built mainly on subsistence farming. During the industrial revolution, sectors such as mining, construction and manufacturing were developed. Modern economic expansion of industrialized countries witnessed new sectors providing services, finance and technology. Economies are better equipped for risks with the engagement

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9. "...economic inequalities between developed and underdeveloped countries have been increasing" (Myrdal (1957)).
10. Their framework offers a nice structure (in fact it is the only structure available) that can be easily modified to incorporate endogenous economic development and Bayesian updating.
of more sectors.

We assume citizens invest savings into active private sectors for risk diversification and capital accumulates from the investment return through bequest. Public sectors, funded through tax revenue to provide services such as infrastructure and national defense, operate to protect private sectors against the corresponding risk. Success (failure) in covering an economic risk, either in a private or a public sector, generates a high (low) return, which is the signal for the citizens. A benevolent government consistently chooses protection by investing the tax revenue into public sectors. Therefore, there can be no economic risk in the public sectors if the government is benevolent. The only source of failure in the public sectors is thus the political risk which arises as an opportunistic government mixes between protection and predation to maximize its self-interest.

Preceded full economic risk diversification, citizens cannot ascertain the risk source if there is a failure; nor can they assuredly give credits to the government if there is a success. Information asymmetry can be exploited by an opportunistic government for predation or for disguise (Adserà et al. (2003)). However, besides the signal, the information available to the citizens includes the possibility of economic risk which is the size of the private economy. As an economy develops, private sectors expand and the possibility of economic risk decreases. If an opportunistic government does not show some restraint, a failure provides more precision in signalling political risk. Such improvement of informational efficiency affects the effectiveness of political accountability.

Then we introduce the political constraint on the government. The ever-adjusting public trust from Bayesian citizens, enters the government’s utility function. Therefore, trust building matters for the government. The inherent property of Bayesian updating indicates that bias from an initial trust might last for a long time. In fact, adjustment speed varies at different trust levels, thus it also affects the effectiveness of political accountability. Levels of informational efficiency and public trust together translate into different environments of political accountability. An opportunistic government employs strategies to adapt to the various environments. We demonstrate four mechanisms at work and track the variations in governments’ strategy.

Firstly, at the early stage of economic development, an opportunistic government chooses protection, anticipating larger future predation gain since marginal return of capital is high. The second mechanism is driven by improvement of informational efficiency as an economy develops: at the early stage, predation gain is small, but information gap between citizens and the government is large. A government is least accountable for a bad outcome and most appreciated for a good outcome. Consequently, an opportunistic government often trades predation for trust building. In the expansion stage, predation becomes more attractive while information gap is reduced but not to a threat level. An opportunistic government thus confiscates most

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11 The coexistence of information asymmetry and public trust is vital in our paper. In Nunn et al. (2016), they also emphasize that "The novelty of our study is to posit that social trust can influence how voters respond to the information asymmetry."
often.\footnote{An economy might experience ups and downs in the expansion stage due to entanglement of economic and political risks.} If an economy enters a maturity stage where capital stock approaches full economic risk diversification level, information gap diminishes as economic risks are mitigated and citizens can detect political risk efficiently. Threat from fast falling trust in case of a bad outcome outweighs sizable predation gains as long as public trust is not already very low. An opportunistic government thus jams signals by mimicking more often to prevent exposure. Thirdly, extreme political optimism (very high trust) relieves pressure from political accountability and encourages predation. Fourthly, extreme political pessimism (very low trust) incurs excessive accountability to withhold and also encourages predation despite the threat. Besides, extreme trust levels adjust slowly, creating even stronger incentives to confiscate.

Combining the forces together, we obtain two areas of pure strategy of protection where capital level is very low or high (and public trust is not too low or high); two areas of pure strategy of predation where there is extreme political optimism or pessimism (and capital level is not too low or high); one area of mixed strategy that is U-shaped as capital accumulates and inversely U-shaped as public trust increases.\footnote{This is before capital reaches full risk diversification level, there is another small area of mixed strategy after risks have been fully diversified. We discuss it in Section §4.2.}

Luck affects a nation's developmental trajectory.\footnote{This is the driver in Acemoglu and Zilibotti (1997).} But we are particularly interested in how governments' strategies affect the trajectories. Self-reinforcing political and economic circles appear in the expansion stage. An opportunistic government expropriates the economy most often in the expansion stage, which causes the middle income trap. While in the maturity stage with increasing political accountability, the governments opt for protection more often to avoid exposure, which further increases economic prosperity and political accountability. When trust level is not too low, democracy can be consolidated. As an extension, we discuss self-fulfilling multiple equilibria assuming there is strategic complementarity. Citizens invest more in private sectors if a government's reputation is higher. Fewer economic risks implies greater political accountability and discourages predation. Political transition towards democracy thus ends up justifying citizens' optimism.

Our paper connects to various streams of economic literature. First, it is a political agency model where both moral hazard and selection operate. Political accountability from electoral discipline first appears in Barro (1973) and Ferejohn (1986). The canonical model has a setting where elections both discipline and sort the politicians. However, the solution concept is Perfect Bayesian Equilibrium as when the simple setting is extended to infinite period, "reputation does not affect the politician's incentive" (Besley (2007)). Politicians' strategy is simply determined by the level of exogenous material gain. Our main departure from it is the endogeneity of material gain and reputation. Other papers that center on political accountability feature more complicated settings. Maskin and Tirole (2001) introduce different motivations of the officials and different modes of government in a two period model for welfare analysis and discussions on constitutional design. Benhabib and Przeworski (2010) in
a dynamic framework introduce an additional criminal accountability if transgression
is flagrant. They distinguish and compute the effects of these accountability mecha-
nisms on economic growth. Our paper abstracts from these specific discussions and
the external factors, and focuses on the role of political accountability as a long run
driving force of economic and political development.

Our paper is closely related with the reputation literature, especially its applica-
tions in political economy, such as Phelan (2006), Azam et al. (2009) and Lu (2013).
They share the same feature in terms of their equilibrium dynamics: If reputation
(trust) increases, the probability of predation from an opportunist government also
increases. This paper enriches the equilibrium dynamics over a two-state variable
set. Not only public trust but also economic development by improving informational
efficiency affects opportunistic governments’ strategy. This is realized through an
endogenous generation of the signal distribution. In these three papers, observable
action of predation is assumed and conditional probabilities on no predation path
vary with trust and governments’ strategy. In this paper, we assume unobservable
actions and conditional probabilities are measurable which vary with capital, trust
and governments’ strategy. Introducing unobservable action and the threat from im-
provement of informational efficiency incurs a cost of multiple equilibria if there is
strategic complementarity. In order to derive a unique MPE, we abstract from it and
allow citizens only to update trust in our basic setting and discuss multiple equilibria
as an extension. Lu (2013) in particular derives different types of MPE with alterna-
tive configurations of reputation and time preferences of two government types. We
also derive different types of MPE as capital and trust levels vary and both of them
are endogenous variables.

As is mentioned, in our model, information is transmitted into political process
(Lohmann (1994), Piketty (1999)) through economic development. This paper adds
to the channels through which economic development can affect informational effi-
ciency. Acemoglu and Zilibotti (1999) proposes a model information is generated by
repetition of activities, which is constrained by the scarcity of capital. Principal-agent
relations become more efficient as the economy accumulates capital and economic sec-
tors expand for a high-level activity. In Jaimovich (2011), entrepreneurial skills are
private information. Economic development brings sectoral variety, which facilitates
the self-selection of talents to sectors and enables the provision of more satisfactory
credit contracts. The credit market efficiency in turn spurs innovation and contributes
to economic development. In these papers, the linkage between economic develop-
ment and information accumulation brings similar discussions on vicious or virtuous
circles or on poverty or middle income trap. What distinguishes our model is the
alternative microfoundation for such patterns, which is implemented in a dynamic
game where information revelation not only is affected by economic development but
also can be controlled by Markov strategy through signal jamming.

Our paper complements the institutionalist theories of development and political
transition (Acemoglu and Robinson (2001), Acemoglu and Robinson (2012)) where
democratization is realized in an economy that features income inequality and revo-
lutionary threat can be triggered by adverse business cycle shocks. Our model does not address distributional conflicts but focuses on political accountability. Moreover, transitions are triggered by endogenous improvement of informational efficiency rather than exogenous business cycle shocks. In addition to institutions, we emphasize citizens’ beliefs (public trust) and heterogeneity in government types. In addition to political transitions, we extend the analysis into long run political and economic development paths. However, our model generates self-fulfilling political transition in a similar way as is discussed in Acemoglu and Robinson (2001). Bidner and Francois (2013) seeks to explain political transitions by studying the endogenous emergence of political accountability. A sequence of good leaders can modify citizens’ beliefs about the standards of accountability, and thereby leads voters to expect accountable leaders in future. Then voters are willing to throw out poorly acting leaders today. Therefore, good leadership can lead to democratic consolidation. In contrast to our paper which creates endogenous environment of political accountability (including citizens’ beliefs) to track its evolution and governments’ adaptation, they directly allow governments’ action to impact on citizens’ belief.

There are two novelties in this paper which the literature to my knowledge has not yet attempted. The first is to develop an endogeneity of the signal distribution. Exogenous distribution of signals is commonly assumed in the literature. However, economic dynamics can be enriched if it is extended into the area of informational efficiency. This paper offers an approach to link economic development to the endogenous generation of the signal distribution. In political agency models where information asymmetry is modeled (Dorsch and Maarek (2015), Phelan (2006), Azam et al. (2009), Lu (2013)), either exogenous conditional distribution of signals or observable action is assumed. It also differs from Acemoglu and Robinson (2001)’s canonical model where threat comes from revolt but its cost is affected by exogenous shocks. The second is to construct and solve dynamic equilibrium mixed strategy numerically over the set of two state variables. The technical complexity lies in that, to solve the mixed strategy equilibrium we need to obtain value functions of discrete actions first, assuming equilibrium strategy (continuous choice) is known, however, the equilibrium strategy in turn can only be solved subject to constraints imposed on value functions of discrete choice.

The rest of the paper is organized as follows. Section §2 provides the basic setting of the model. Section §3 presents the structure of an endogenous signal distribution, players’ optimization problems and characterizes a unique Markov Perfect Equilibrium. Section §4 analyzes the equilibrium strategy pinned down in the evolution of political accountability, its properties and its impact on nations’ developmental trajectories. Section §5 discusses self-fulfilling multiple equilibria as an extension. Section §6 concludes. All the proofs, programming details and extra discussions are in the Appendices.
2 The Basic Set-up

2.1 Citizens, Government and Political Risk

We consider a dynamic game with a long-lived player (government) and a continuum of short-lived players (citizens). Time is discrete and indexed by \( t = 0, 1, \ldots \infty \). In each period \( t \), a new cohort of citizens is born. The government has a private and persistent type \( \theta \in \{G, B\} \) (good or bad, benevolent or opportunistic, etc), which is not directly observable by citizens. At \( t = 0 \), the citizens are endowed with a prior belief of the government’s type. With probability \( p_0 \), they trust that the current government is benevolent. If \( p_0 \) is high (low), it reflects political optimism (pessimism).

The action set is \( \{\text{No Investment (Predation)}, \text{Investment (Protection)}\} \). A benevolent government consistently invests tax revenue into the public sectors to protect the economy (see Section §2.4). An opportunistic government must choose either predation of all the tax revenue or protection without extracting any resources. We denote action \( a \in \{NI, I\} \). An opportunistic government’s strategy is a function \( \sigma \)—the probability that it chooses protection. If \( p_0 < 1 \) and \( \sigma < 1 \), citizens believe there is a political risk of expropriation.

2.2 Intermediate Sectors and Economic Risk

There is a continuum of equally likely states of nature on the unit interval \( J = [0, 1] \). Correspondingly, the economy is composed of a unit interval of risky intermediate sectors \( \lambda = [0, 1] \). Every period only one of the states is realized: \( \forall t, J_t \) is a singleton \( \{j_t\} \). Investment in sector \( j \) is like holding a basic Arrow security for state \( j \): \( \lambda_j = j \). For simplicity, we denote both the state of nature and its corresponding sector with \( j \). (This structure is taken from Acemoglu and Zilibotti (1997).)

These sectors are further divided into public sectors of fixed size \( \gamma \) and private sectors of size \( 1 - \gamma \). Public sectors provide infrastructure and services against risks such as wide-scale power outages which disrupt functioning of the private sectors. Therefore a state in a public sector with(out) investment is equivalent to a state in a private sector with(out) investment. Section §2.4 provides more information.

Every period \( t, t = 1, 2, \ldots \), with minimum size requirement (see Section §2.3) imposed on the private sectors, only a subset \( n_t \) of the \( 1 - \gamma \) private sectors receives investment to open up. Thus states of nature can be categorized into three types: \( \omega_j = \{pr1, pr0, pb\} \). \( pr1 \) is a state in one of the private sectors that are opened with investment; \( pr0 \) is a state in one of the private sectors that are not opened, without loss of generality, \( n_t < j_t < 1 - \gamma \). \( pb \) is a state in the public sectors. (Figure 2) Clearly, the occurrence probability of each type is \( \{n_t, 1 - n_t - \gamma, \gamma\} \) at period \( t \). Risks of the economy are fully diversified if all sectors open up.

We assume citizens can only invest in the private sectors and the government can only invest in the public sectors with tax revenue.
2.3 Minimum Size Requirement

We keep the minimum size requirement assumption of Acemoglu and Zilibotti (1997) and make a further assumption on its distribution. Investment in certain sectors requires a minimum size which is like a start-up cost. Without enough savings in the economy to meet the requirement, risks can not be fully diversified.

The distribution of minimum size requirement is as follows. The $\gamma$ public sectors have no minimum size requirement: $M_j = 0, \forall j \in [1-\gamma, 1]$. Therefore, the government either invests in every public sector or there is zero public investment. The private sectors’ minimum size requirement is constant (see Figure 2): $M_j = D, \forall j \in [0, 1-\gamma], D > 0$.

![Figure 2: Minimum Size Requirement $M(j)$](image)

This assumption restricts the set of $\omega$ to three elements and simplifies our setting. If a state of nature occurs in a public sector, it is not feasible that this sector is not opened due to insufficient tax revenue. As long as the tax rate is positive, the sole culprit of no investment in public sectors is predation from an opportunistic government.

Such arrangement of minimum size requirement under certain conditions also justifies citizens’ demand for a benevolent government. Pareto improvement can be achieved if zero minimum size requirement sectors are allocated to a benevolent government compared with a competitive market for all sectors (Appendix D).

Economic development occurs as the private sectors expand with more diversification opportunities. Efficiency therefore requires that all private sectors are opened. We assume an upper bound $\bar{D}$ for $D$ so that the private economy will eventually realize a full scale open-up ($n_t = 1-\gamma$) before capital accumulates to the steady state level $k^*$ (see Section §3.4).

The ranking of sectors from lower to higher size occurs without loss of generality. But the specification of the minimum size requirement affects the expansion speed of the private economy. We can also assume other forms of minimum size requirement, e.g., a linear specification as in Acemoglu and Zilibotti (1997). In this case, the private economy expands quickly and the information gap contracts fast. A larger area of predation is produced as trust building is not effective.
2.4 Production, Investment and Taxation

The production function $F(z, k, L)$ of a final good is Cobb-Douglas, where $z$ is technology and a constant. Labor supply $L = 1$ is inelastic and the government levies a tax rate of $\tau$ on the wage.\textsuperscript{16} Citizens provide labor, receive the wage $w_t$ and capital return $r_t$, and pay the tax. They also acquire utility in leaving a bequest $b_t$ to their children which comes from their savings. Therefore there is a simple saving decision $s_t$ to make (see Section §3.4).

The function of private sectors is to transform savings $s_t$ into capital $k_{t+1}$. $s_t$ is invested into the $n_t$ private sectors that are opened. The investment return is either high or low (see Section §3.2) in the form of capital and passed along to the offspring as a bequest. Citizens at the beginning of next period thus become capital owners.

The function of public sectors is to establish a welfare system and provide insurance to private sectors’ activities. Suppose tax revenue is invested into public sectors, all the public sector specific risks related with e.g. infrastructure are covered. Citizens have a high return on their investment in private sectors if a state of nature occurs in public sectors. Therefore, with $\gamma$ public sectors, the probability of high investment return in private sectors is increased by $\gamma$.\textsuperscript{17}

An exogenous tax rate falls in the range of $[\bar{\tau}, \bar{\tau}]$ to meet two requirements. First, taxation is not heavy and Pareto improvement can be achieved under certain conditions when there is a benevolent government (see Appendix D). Second, sufficient state capacity is required to establish the welfare system. Since a state of nature in public sectors with investment leads to a high payoff on private sectors’ investment, if we do not impose a minimum public sector investment requirement, i.e., a lower bound on tax rate, an economy can take advantage of a large multiplier effect on investment return with little taxation. To avoid this possibility, a lower bound of the tax rate is assumed.

3 The Game

Now we introduce elements of the dynamic game. We analyze the payoff and signal structure. The signal structure is particular with the entanglement of political and economic risks. Then according to the time line, we study citizens’ and opportunistic governments’ maximization problems. Bayesian updating based on an endogenous distribution of signal plays a key role. As $k_t$, $p_t$ and $\sigma(k_t, p_t)$ all affect the generation of a signal, citizens weigh political and economic risks differently at different states. Consequently political accountability is also different at different states. The

\textsuperscript{16}This assumption rules out the possibility for citizens’ labor participation choice to be affected by the government’s strategy. For analysis based on such possibility, see Phelan (2006) and Azam et al. (2009).

\textsuperscript{17}Instead of assuming public sectors’ production externality or nonlinear effects on probability of high return in private sectors, we assume linearly additive probability to simplify the analysis.
opportunistic governments adapt to changing environment of accountability with the Markov strategy $\sigma(k_t, p_t)$ to maximize its self-interest. A Markov Perfect Equilibrium thus features a $\sigma(k_t, p_t)$ that meets both the political expectation of citizens and optimization of opportunistic government. Separating, semi-separating and pooling equilibrium can occur at different states.

### 3.1 Governments’ Payoff

Conditional on a state of nature in private sectors, the action set of the government is empty. But the government still benefits from (suffers) political optimism (pessimism). $p_t$ is the political sentiment of trust at the beginning of each period, or equivalently the updated probability that the government is benevolent given the sequence of moves from time $1$ to time $t - 1$. We assume that the stage payoff $u_{gov}^{pr1,pr0}$ is a strictly increasing function of $p_t$: $\psi(p_t)$.

Conditional on a state of nature in public sectors, if an opportunistic government chooses predation, its payoff $u_{gov}^{pb|a=NI}$ is $\phi(k_t) + \psi(p_t)$. $\phi(k_t)$ is the tax revenue $\tau w_t$. The stage payoff of an action of protection $u_{gov}^{pb|a=NI}$ is just $\psi(p_t)$, thus:

$$
\begin{align*}
\phi(k_t) + \psi(p_t) & \quad \omega_j = pb \ & \ a = NI, \\
\psi(p_t) & \quad \omega_j = pr1 \ or \ \omega_j = pr0 \ or \ \omega_j = pb \ & \ a = NI.
\end{align*}
$$

In the form of $\psi(p_t) = -\eta(1 - p_t)^2$, $\eta > 0$, the government faces a distrust cost when $p_t$ is smaller than 1. The interpretation of this cost could be broad. It could be thought of as shrinking "ego rents" (Rogoff (1990)) when the government loses popularity. It could also capture the idea of mounting scale of rebellion and damage to the economy with decreasing trust, or any activity from citizens, e.g., street protest, that is not modelled but the overall impact on the government takes form of this function given that citizens’ belief is a common knowledge (Phelan (2006)). Or it simply reflects poor governance in creating the evil of a culture of political corruption in which the public trust is eroded.$^{18}$ $\eta$ is a measure of trust premium.

### 3.2 Signals and Citizens’ Payoff

Given symmetry, citizens’ savings can be equally invested into the $n_t$ private sectors that are opened at time $t$. We assume that citizens’ entire savings will benefit from the return instead of only their investment in the sector where the state of nature occurs. Therefore, economy functions as a whole and all private sectors that are opened function well if an economic risk is covered.$^{19}$ The size of the bequest and

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$^{18}$ de Jouvenel (1963) views "essential function of public authorities" as one to "increase the mutual trust prevailing at the heart of the social whole."

$^{19}$ There is also a technical concern in making this assumption. Welfare comparison is complicated when we consider a return only on the investment in one sector. A welfare improvement in risk diversification can be offset by a decrease in average investment when more sectors are opened.
thus $k_{t+1}$ depends on the outcome of their investment. As economic risk and political risk are both unobservable to the citizens, there can be various scenarios where citizens only observe an outcome but the source is undetectable.

Table 1: Two outcomes of Investment Return in Five Scenarios

<table>
<thead>
<tr>
<th>Result</th>
<th>Scenarios</th>
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<tbody>
<tr>
<td>High rate of return $Q$</td>
<td>1. $\omega_j = pr_1$</td>
</tr>
<tr>
<td></td>
<td>2. $\omega_j = pb$ and $\theta = G$</td>
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<tr>
<td></td>
<td>3. $\omega_j = pb$, $\theta = B$ and $a = I$</td>
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<td>Probability</td>
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<td></td>
<td>$p\gamma$</td>
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<td></td>
<td>$(1 - p)\sigma\gamma$</td>
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<tr>
<td>Low rate of return $q$</td>
<td>4. $\omega_j = pr_0$</td>
</tr>
<tr>
<td></td>
<td>5. $\omega_j = pb$, $\theta = B$ and $a = NI$</td>
</tr>
<tr>
<td>Probability</td>
<td>$1 - n - \gamma$</td>
</tr>
<tr>
<td></td>
<td>$(1 - p)(1 - \sigma)\gamma$</td>
</tr>
</tbody>
</table>

We categorize five scenarios into two groups by rate of return on citizens’ investment in Table 1. Citizens enjoy a high rate of return $Q$ on their savings if the following scenarios happen: 1. a state of nature occurs in a private sector with investment ($\omega_j = pr_1$); 2. a state of nature occurs in public sectors and a benevolent government is in charge ($\omega_j = pb$ and $\theta = G$); 3. a state of nature occurs in public sectors and an opportunistic government chooses protection instead of predation ($\omega_j = pb$, $\theta = B$ and $a = I$). We denote the result of high investment return as a Success ($S$).

Let Failure ($F$) denote the outcome of a low return $q < Q$ on private sectors’ investment. Two scenarios generate a low return: 1. a state of nature occurs in the private sectors that are not opened ($\omega_j = pr_0$); 2. a state of nature occurs in the public sectors and an opportunistic government chooses predation ($\omega_j = pb$, $\theta = B$ and $a = NI$). The low return $q$ is a function of $n$: $q(n)$, which allows flexibility in depicting economic recessions. For example, $q'(n) < 0$ increases the volatility of the economy as it expands. We assume $q(n) > 0$, thus a failure will not deplete all resources. Both $Q$ and $q(n)$ affect the development speed through capital accumulation and depletion.

Information asymmetry in this context is richer than standard literature: political and economic risks entangle other than existence of hidden information and action. Citizens can not observe any of $\omega_j$, $\theta$ and $a$ directly. They only observe the signal $\rho = \{S, F\}$. Since an opportunistic government chooses protection with probability $\sigma$, the probability of observing the two signals are (Table 1 shows the occurrence probability of each scenario): 

$$
\begin{align*}
S & \quad \text{prob} \ n_t + (p_t + (1 - p_t)\sigma_t)\gamma, \\
F & \quad \text{prob} \ 1 - n_t - (p_t + (1 - p_t)\sigma_t)\gamma.
\end{align*}
$$

For simplicity, we denote $\mu_t = p_t + (1 - p_t)\sigma_t$ as the government’s reputation, which is the conditional probability of Success, i.e., an action of protection, if a state $j$ occurs in public sectors with probability $\gamma$. 

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3.3 Time line

The economic fundamental in each period $t = 0, 1, \ldots, \infty$ is fully characterized by three state variables: state of nature $j_t$, capital level $k_t$ and citizens’ prior belief $p_t$. Figure 3 portrays the sequence of play which is described in detail below:

1. A new government privately knows its type $\theta$ which cannot be communicated to the citizens during the reign. The first generation of citizens is endowed with an initial sentiment of the government’s type $p_0$ and an initial capital stock $k_0$. Game starts.

2. In the beginning of every period $t, t = 0, 1, \ldots$, citizens inherit the bequest in the form of capital $k_t$ and an updated belief $p_t$. They become capital owners and are all employed in a production sector of the final good, earn wage $w_t$, receive capital return $r_t$ and pay the tax at rate $\tau$. Then they make a saving decision $s_t$ and invest their savings into the $n_t$ private sectors that are opened.

3. The government collects tax revenue and chooses between predation and protection, aware that citizens will hold it accountable for the outcome. If predation is chosen, all the tax revenue is consumed and there will be no investment in the public sectors. If protection is chosen, then all the tax revenue will be invested into the public sectors.

4. After saving and investment decisions have been made, a state of nature $j$ is realized. Citizens observe the signal $\rho$ of success or failure in securing a high return on their savings.

5. Based on the current political sentiments, citizens update their trust in the government, leave their investment return (in the form of capital) to their children as a bequest. A new generation of citizens inherits sentiment bias $p_{t+1}$ and capital $k_{t+1}$ and a new round starts...

Figure 3: Time line
3.4 Citizens’ Problem

The citizens have a warm glow preference $U(c_t, b_t)$. $c_t$ is consumption and $b_t$ is the bequest left to their offspring. Citizens equally invest $\frac{n_t}{n_t}$ into each of the private sectors that are opened. Probability of Success and Failure are provided in Section §3.2. In case of a success (failure), the return on the entire savings is $Qs_t (q(n_t)s_t)$. Therefore the citizens’ maximization problems is:

$$
\max_{b_t} U(c_t, b_t) \\
\equiv \max_{s_t} u(c_t) + (n_t + \mu_t \gamma) \log Qs_t + (1 - n_t - \mu_t \gamma) \log q(n_t)s_t \\
\text{s.t.} \; c_t \leq (1 - \tau)w_t + (1 + r_t)k_t - s_t
$$

$u(c_t)$ is strictly concave, increasing and differentiable. $r_t$ is the rate of return on capital net of depreciation. For simplicity, we assume that capital fully depreciates after use in production. With log utility of investment return, we abstract from the case where citizens’ saving decision is affected by the government’s strategy. In our extension in Section §5, we include the interaction between government’s strategy and citizens’ saving decision and discuss self-fulfilling multiple equilibria.

The saving decision is $s_t = s(k_t)$, a function merely of capital level, concave and strictly increasing in $k_t$. Therefore, capital’s law of motion is:

$$
k_{t+1} = b_t = \begin{cases} 
Qs_t & \text{prob } n_t + (p_t + (1 - p_t)\sigma_t)\gamma \\
q(n_t)s_t & \text{prob } 1 - n_t - (p_t + (1 - p_t)\sigma_t)\gamma
\end{cases}
$$

By equating minimum size requirement $M_j$ and investment amount in each sector $\frac{s_t}{n_t}$, we obtain the equilibrium number of sectors that are opened at time $t$: $n_t = \frac{s_t}{D}$. Therefore, at a higher capital level, more private sectors are opened and fewer economic risks occur. The law of motion of capital shows that higher levels of capital and reputation together reduce economic and political risks, and an economy enters a success path with higher probability. We can solve the steady state $k^*$ as economy develops on the success path.\(^{20}\)

When capital reaches a threshold capital level $\bar{k} = (1 - \gamma) * D$, $n_t = 1 - \gamma$, all private sectors are opened and risks are fully diversified. By imposing $k < k^*$ to ensure this threshold is met earlier than the steady state $k^*$, we obtain $\bar{D}$.

3.5 Information Gap, Sentiment Bias and Signal Jamming

As risks entangle, citizens assess information and update trust according to an endogenous distribution of the public signal $\rho_t \in \{S, F\}$. Political accountability relies on how well Bayesian citizens’ read the signals as the degree of citizens’ information

\(^{20}\text{Note that there is another steady state if the economy develops on a path of failure.}\)
curbs the opportunities governments may have to engage in confiscation. The distribution generating process of \( \rho_t \) is affected by the level of capital stock \( k_t \), political sentiment \( p_t \) as well as governments’ strategy \( \sigma(k_t, p_t) \). They respectively impact political accountability through information gap, sentiment bias and signal jamming.

There is an information gap between citizens and the government since political and economic risks are unobservable to citizens but known to the government. However, as \( k_t \) accumulates, \( n_t \) is larger, economic risks are better diversified, and signal becomes more precise if political risk is not controlled by an opportunistic government. Information gap shrinks as capital accumulates, thus it is more difficult for the government to exploit citizens’ lack of information.

As political sentiments are inherited, citizens’ assessment of political risk is also biased by their prepossession. Citizens’ expectation of an opportunistic government’s strategy also affects the precision of information they can reach. More mimicking increases the difficulty to interpret a signal and citizens’ ability to extract information about the quality of the government is hampered.

3.6 Bayesian Updating

The richness of mechanisms that affect effectiveness of information assessment and thus political accountability is embodied in the Bayesian updating process. It is also the law of motion for trust. Signal \( \rho_t \) is automatically observed by the citizens and in our setting, \( \rho_t \neq \emptyset \) every period. In cases of Success or Failure at time \( t \), the posterior belief at time \( t+1 \) is:

\[
\begin{align*}
\rho_{S}^{t+1}(k_t, p_t) &= \frac{p_t(n_t(k_t) + \gamma)}{p_t(n_t(k_t) + \gamma) + (1 - p_t)(n_t(k_t) + \gamma \sigma(k_t, p_t))}, \\
\rho_{F}^{t+1}(k_t, p_t) &= \frac{p_t(1 - n_t(k_t) - \gamma)}{p_t(1 - n_t(k_t) - \gamma) + (1 - p_t)(1 - n_t(k_t) - \gamma \sigma(k_t, p_t))}.
\end{align*}
\]

The Bayesian updating is completely endogenous given the endogenous distribution of the public signal. We do comparative statics analysis to gain some initial intuition of how capital stock \( k_t \), political sentiment \( p_t \) and governments’ strategy \( \sigma(k_t, p_t) \) impact political accountability through information gap, sentiment bias and signal jamming. For now, we exclude \( k = 0, p = 0, p = 1 \) and \( k \geq \bar{k} \) and leave them for later analysis.

**Proposition 1.** 1. Outcome affects the direction of sentiment change: \( \rho_{S}^{p} \geq p, \rho_{F}^{p} \leq p \); 2. With a high reputation, a success is eclipsed but a failure is sympathized: \( \frac{\partial \rho_{S}^{p}}{\partial \sigma} < 0, \frac{\partial \rho_{F}^{p}}{\partial \sigma} > 0 \); 3. Trust increases most when economy is least developed. As economy develops, it increases less and at a lower speed: \( \frac{\partial \rho_{S}^{p}}{\partial n} < 0, \frac{\partial^2 \rho_{S}^{p}}{\partial n^2} < 0 \); 4. Trust decreases least when economy is least developed. As economy develops, it decreases more and at a faster speed: \( \frac{\partial \rho_{F}^{p}}{\partial n} < 0, \frac{\partial^2 \rho_{F}^{p}}{\partial n^2} > 0 \); 5. Extreme political optimism and pessimism adjust very slowly: if \( p \to 0 \) or \( \rho \to 1 \), \( \frac{\partial \rho_{S}^{p}}{\partial \sigma} \to 0, \frac{\partial \rho_{F}^{p}}{\partial \sigma} \to 0 \).
The effects from $p_t$ and $\sigma(k_t, p_t)$ are shown in the left panel of Figure 4 and the effect of $n(k_t)$ is shown in the right panel. $n_t$ is an increasing function of $k_t$ alone, thus its effect on $p_{t+1}$ reflects the effect of capital accumulation on $p_{t+1}$.

![Figure 4: $k$, $p$ and $\sigma$'s impacts on Bayesian updating](image)

If $\sigma(k, p) \neq 1$, $p_t \neq 1$ and $p_t \neq 0$, then $p_{t+1}^S > p_t$ and $p_{t+1}^F < p_t$. In case of a success, trust goes up as long as there is a possibility of political risk as boundaries are not reached. Similarly, a failure always frustrates public sentiments.

$p_{t+1}^S(k_t, p_t)$ is strictly decreasing in $\sigma(k_t, p_t)$ and $n_t$. In Section §3.2, it has been shown that a success has three sources. Citizens have to distinguish between a benevolent government and two other possibilities: an opportunistic government choosing protection with probability $\sigma(k_t, p_t)$ or private sectors’ expansion delivering success with probability $n_t$. Therefore, when it is more likely that the latter two will take place, a lower trust level is reached in case of a success.

$p_{t+1}^F(k_t, p_t)$ is strictly increasing in $\sigma(k_t, p_t)$ but decreasing in $n_t$. Similarly, if citizens observe a failure, they expect it either from an opportunistic government’s choice of protection or from risks uncovered by the private sectors. Citizens attribute the failure less to an opportunistic government when they believe it chooses protection with a higher probability $\sigma(k_t, p_t)$. And as economy grows and $n_t$ is large, it is less likely that a failure occurs in the private sectors, and trust falls more.

Another important feature of the updating process is shown in the second order derivatives. When economy is least developed, the credit of a success is attributed most to the government and a failure is least blamed on the government. As economy expands and $n_t$ is larger, $p_{t+1}^S$ not only increases to a lower level but also increases more slowly. $p_{t+1}^F$ not only decreases to a lower level but also decreases faster. In fact, as shown in the right panel of Figure 4, when economy approaches full risk diversification, trust is falling so fast that it approaches 0 quickly. Therefore, the gap between posterior belief in different situations of success and failure is first increasing and then decreasing.
Last but not least, if \( k < \tilde{k} \), the effects of \( \sigma(k_t, p_t) \) and \( n_t \) approach zero if sentiment bias is extreme.

We adopt Sequential Equilibrium\(^{21}\) concept for information sets: \( \{ \rho_t : \rho_t = F, k \geq \tilde{k} \} \) where Bayes’ rule does not apply. Therefore, there exists a unique posterior belief: \( p^F = 0 \) for this information set\(^{22}\).

When \( p = 0 \), it is an absorbing stage and serves as a threat.\(^{23}\) We assume \( p_0 \neq 0 \). If \( k > \tilde{k} \) and the economy suffers predation, \( p \) will fall to zero and stay in this absorbing stage. We assume the highest expected lifetime value \( V_{pb}(k^F, 0) \) in this stage for the opportunistic government is smaller than the cost of an overturn. Thus a new government emerges after \( p \) falls to zero and economy can rebound with an endowment of \( k_0 \). To overthrow the government will cost all resources left which is in the range of \( [\min(q^k, q^k), \max(q^k, q^k)] \). We also assume \( p_0 \neq 1 \) because \( p = 1 \) is also an absorbing stage but will never be reached if \( p_0 \neq 1 \).

### 3.7 Markov Strategy and Equilibrium

Denote by \( V_{\omega_j}(k, p) \) the expected lifetime payoff to an opportunistic government associated with strategy \( \sigma(k, p) \) if a state of nature \( j \) occurs. When \( \omega_j = pr1 \) or \( pr0, a = \emptyset \). Therefore, the function \( V_{\omega_j}(k, p) \) can be defined recursively as\(^{24}\):

\[
V_{pb}(k, p) = (1 - \sigma(k, p))V_{pb}^{NI}(k, p) + \sigma(k, p)V_{pb}^{I}(k, p)
\]
\[
= (1 - \sigma(k, p))\left[u_{gol}^{NI}(k, p) + \beta \int V_{\omega_j}(k^F, p^F(k, p, \sigma(k, p)))f(\omega_j'|k^F(k))dj \right] + \sigma(k, p)\left[u_{gol}^{I}(k, p) + \beta \int V_{\omega_j}(k^S, p^S(k, p, \sigma(k, p)))f(\omega_j'|k^S(k))dj \right],
\]

\[
V_{pr1}(k, p) = u_{pr1}^{gov}(k, p) + \beta \int V_{\omega_j}(k^F, p^F(k, p, \sigma(k, p)))f(\omega_j'|k^F(k))dj,
\]

\[
V_{pr0}(k, p) = u_{pr0}^{gov}(k, p) + \beta \int V_{\omega_j}(k^F, p^F(k, p, \sigma(k, p)))f(\omega_j'|k^F(k))dj.
\]

\(^{21}\)Sequential Equilibrium (Kreps and Wilson (1982)) is defined as: if there exists a sequence of completely (strictly, totally) mixed strategy profiles \( \{\sigma_i\} \) such that \( \lim_{k \to \infty}(\sigma_i, p_i) = (\sigma, p) \), where \( p_i \) is the system of beliefs derived from \( \sigma_i \) using Bayes’ rule.  

\(^{22}\)If we adopt PBE, then there exists a continuum of equilibria where \( p^F \in [0, \hat{p}] \), \( \hat{p} \equiv \sup\{p' : V^{NI}(k, p, p') \leq V^{I}(k, p, p)\} \).

\(^{23}\)An opportunistic government will always choose predation if \( p = 0 \).

\(^{24}\)\( V_{pb}(k, p) = \max_{\omega \in \{NI, I\}} \{u_{gov}(k, p, a) + \beta \int V_{\omega_j}(k^F(k, a), p^F(k, p, \sigma(k, p, a)))f(\omega_j'|k^F(k, a))dj \} \) is used in programming and proof of existence and uniqueness of the MPE where \( \sigma(k, p) \) is the citizens’ belief. In equilibrium it is equal to \( \sigma(k, p) \).
where \( u_{\text{gov}, \text{NI}}^\omega(k, p) = \phi(k) + \psi(p) \), \( u_{\text{gov}, \text{I}}^\omega(k, p) = u_{\text{pr}1}^\omega(k, p) = u_{\text{pr}0}^\omega(k, p) = \psi(p) \), and
\[
\int V_{\omega_j}^J(k^F(k), p^F(k, p, \sigma(k, p)))f(\omega_j^J|k^F(k))dj = n'(k^F(k))V_{\text{pr}1}(k^F(k), p^F(k, p, \sigma(k, p))) + (1 - n'(k^F(k)) - \gamma)V_{\text{pr}0}(k^F(k), p^F(k, p, \sigma(k, p)) + \gamma V_{\text{pgov}}(k^F(k), p^F(k, p, \sigma(k, p)))
\]
\[
\int V_{\omega_j}^S(k^S(k), p^S(k, p, \sigma(k, p)))f(\omega_j^J|k^S(k))dj = n'(k^S(k))V_{\text{pr}1}(k^S(k), p^S(k, p, \sigma(k, p))) + (1 - n'(k^S(k)) - \gamma)V_{\text{pr}0}(k^S(k), p^S(k, p, \sigma(k, p)) + \gamma V_{\text{pgov}}(k^S(k), p^S(k, p, \sigma(k, p))).
\]

Given a public history, \( h^t \), which consists of a sequence of past signals \( \rho^t \equiv \{\rho_i\}_{i \in [0, t]} \); the government’s private history, \( h^t_{\text{gov}} \equiv (h^t, a^t, \theta) \), \( a^t \equiv \{a_i\}_{i \in [0, t]} \), a strategy \( \sigma(k_t, p_t) \) forms an Equilibrium if and only if at any time \( t \), given \( k_t \) and \( p_t \),

- the opportunistic government maximizes his intertemporal payoff \( V_{\text{pgov}}(k_t, p_t) \) with this strategy, specifically,
\[
\begin{aligned}
\begin{cases}
\sigma(k_t, p_t) = 1 & \text{if } V_{\text{pgov}}^\omega(k_t, p_t) \leq V_{\text{pgov}}^J(k_t, p_t) \\
\sigma(k_t, p_t) \in (0, 1) & \text{if } V_{\text{pgov}}^\omega(k_t, p_t) = V_{\text{pgov}}^J(k_t, p_t) \\
\sigma(k_t, p_t) = 0 & \text{if } V_{\text{pgov}}^\omega(k_t, p_t) \geq V_{\text{pgov}}^J(k_t, p_t)
\end{cases}
\end{aligned}
\]

- the citizens maximize their two-period utility \( U(c_t, b_t) \) and update their belief according to Bayes’ Rule with \( \sigma(k_t, p_t) \),

- \( k_t \) evolves according to the law of motion.

In this Markov perfect equilibrium, if the Markov strategy \( \sigma(k_t, p_t) > 0 \), protection is weakly preferred to predation; if \( \sigma(k_t, p_t) < 1 \), predation is weakly preferred to protection; when \( \sigma(k_t, p_t) = 0 \) or 1, we have a pure strategy; when \( \sigma(k_t, p_t) \in (0, 1) \), the opportunistic government is indifferent between predation and protection and uses a mixed strategy.

The technical complexity of this equilibrium is that \( V_{\text{pr}a}^\omega(k_t, p_t) \) can only be obtained when equilibrium strategy \( \sigma(k_t, p_t) \) is known, however, the equilibrium strategy in turn can only be solved subject to equilibrium conditions imposed on \( V_{\text{pgov}}^\omega(k_t, p_t) \leq V_{\text{pgov}}^J(k_t, p_t) \). We provide proof of existence and uniqueness in the Appendix.

**Theorem 1.** The Markov perfect equilibrium exists and is unique. The Markov strategy \( \sigma(k, p) \) is unique.

**Proof.** See Appendix A.
4 Equilibrium Histories

The joint evolution of political accountability, governments’ adaptation strategy arises as the equilibrium outcome of the dynamic game. Various developmental trajectories are formed as economic and political uncertainties unravel. In this section, we first analyze and categorize an opportunistic government’s optimal strategy, based on information summarized into the two state variables $k_t$ and $p_t$, and its rational anticipation of the optimal actions taken in the continuation games (Maskin and Tirole (2001)). We then study its dynamic properties as accountability environment varies with $k_t$ and $p_t$. The main drivers are different tradeoffs: current predation versus future predation, or building trust, or avoiding exposure, or extreme political sentiments. Nations’ developmental trajectories are then simulated. There are several propositions and we leave the proofs all to Appendix B.

4.1 Markov Strategy

Proposition 2. It is with probability 1 that capital accumulation will reach $\bar{k}$.

Proof. See Appendix B.

This proposition establishes that despite the uncertainty from economic and political risks, the private sectors will eventually reach full-scale open-up.

Proposition 3. An opportunistic government measures the following tradeoffs and chooses an adaptation strategy. The value of predation against protection is captured by:

\[
   d = \phi(k) + \beta \gamma \left[ \phi(k^{IF}) - \phi(k^{IS}) \right] + \left[ n^{IF}(k) \phi(k^{IF}) - n^{IS}(k) \phi(k^{IS}) \right] + \left[ \psi(p^{IF}(k,p,\sigma(k,p))) - \psi(p^{IS}(k,p,\sigma(k,p))) \right]
\]

Proof. An opportunistic government’s maximization problem features a combination of discrete choice and continuous policy function. Moreover with equilibrium pure strategy definition, there are inequality conditions in the optimization problem. We approximate $d \equiv V^{NI}_{pb}(k,p) - V^{I}_{pb}(k,p)$ in Appendix B.

This proposition reveals the trade-offs an opportunistic government faces. If it chooses action $NI$, it benefits from immediate predation gain at three opportunity costs. First, $k^{IF} < k^{IS}$, conditional on $\omega_{j'} = pb$, there is less future predation gain as a consequence of economic recession, while there would be more resources to extract if currently the government chose protection. Second, as $k^{IF} < k^{IS}$, there is also a higher chance that an economy is exposed to economic risks as fewer private sectors are opened in the next period. Public trust also falls if $\omega_{j'} = pr0$. Therefore, this item also reflects distrust cost in capital’s term. Third, $p^{IF} < p^{IS}$, trust declines in
case of predation and increases in case of protection, another opportunity cost for predation.

At different capital and trust levels, the forces from benefits and costs are different. In particular, distrust cost \( \psi(p_{IF}(k, p, \sigma(k, p))) - \psi(p_{IS}(k, p, \sigma(k, p))) \), affected by both the capital and trust levels, captures the core concept of political accountability.\(^{25}\) And it is through this term, an opportunistically government employs \( \sigma(k, p) \) to balance cost and benefit.

We define three strategy stages according to the equilibrium definition. If \( d < 0 \) and \( \sigma(k, p) = 1 \), it is a Protection Stage; If \( d > 0 \) and \( \sigma(k, p) = 0 \), it is a Predation Stage; If \( d = 0 \) and \( \sigma(k, p) \in (0, 1) \), it is a Trust Exploitation Stage.

### 4.1.1 Protection Stage

In this stage, an opportunistic government always chooses the pure strategy of protection. Pooling equilibrium arises in two cases. First, political accountability is very high so that an opportunistic government is threatened by a high distrust cost, an extreme political pessimism, from the off-equilibrium path. Second, immediate predation is less of a temptation than future predation.

For analysis, we first divide the \( k-p \) space into three areas. Assuming \( \sigma = 1 \) in them, we will have: 1. \( k \geq \bar{k}, k^{IS} \geq \bar{k}, p_{IF} = 0 \); 2. \( k < \bar{k}, k^{IS} \geq \bar{k}, p_{IF} = p^{IS} \); 3. \( k < \bar{k}, k^{IS} < \bar{k}, p_{IF} = p^{IS} \). We then check whether the validity of each case:

1. The first area is where \( k \geq \bar{k} \), and choosing protection leads to even higher capital level \( k^{IS} \geq \bar{k} \) while choosing predation immediately exposes a bad government and \( p_{IF} = 0 \).

**Proposition 4** (Threat of exposure from full political accountability). \( \exists \eta > 0, \text{ if } \eta > \eta, \text{ then } \forall k \geq \bar{k}, \exists p_{k}, \text{ such that if } k \geq \bar{k} \text{ and } p \geq \tilde{p}_{k}, \text{ an opportunistic government always chooses } \sigma(k, p) = 1. \)

**Proof.** See Appendix B.

This proposition is related with the tradeoff between predation gain and political accountability. \( \eta \) measures how much a government is punished for distrust. This proposition states that as long as this punishment is severe enough, an opportunistic government always avoids direct exposure after economic risks have been fully diversified. As capital stock exceeds \( \bar{k} \), informational efficiency is achieved and citizens hold the government completely accountable for any failure. An opportunistic government will not trade off exposure for predation gain and permanently zero trust when current trust level is not too low. An expected lifetime utility from a relative

---

\(^{25}\)Future economic risk also reflects such cost. Since it functions in the same way as distrust cost, extra analysis of it is omitted.
political optimism\textsuperscript{26} is more worthwhile than cashing out and getting toppled under extreme political pessimism. (We discuss extreme political pessimism in Proposition 7.)

We denote this area where protection is always chosen out of threat from political accountability as $\pi$ (see Figure 5a). Since $k'^S > k > \bar{k}$, $\pi$ is an absorbing stage. Industrialized and democratized countries with relatively high trust exhibit such political stability.

![Figure 5: Area $\pi$: No Predation due to High Political Accountability](image)

Note: We regroup items in $d$, and compare $\phi(k) + \beta\{\gamma[\phi(k'^F) - \phi(k'^S)] + [n'^F \phi(k'^F) - n'^S \phi(k'^S)]\}$ v.s. $\beta[\psi(p'^S) - \psi(p'^F)]$\textsuperscript{a}. We can think of the former as the net predation gain. As shown in Figure 5b, opportunity distrust cost is a decreasing function of $\sigma(k,p)$ because the more an opportunistic government mimics, the more a signal is jammed and the harder for citizens to detect the government’s type. Thus belief is updated at a smaller step\textsuperscript{b} and divergence between $p'^S$ and $p'^F$ is smaller. Since distrust cost is larger than net predation gain at all $\sigma$, pure strategy of protection is chosen.

\textsuperscript{a}This is merely for illustration, if we extend $d$ to infinite periods, the analysis is the same.

\textsuperscript{b}In $\pi$, $p'^F$ is always 0, but $p'^S$ is smaller if $\sigma$ is larger.

In the following analysis, we restrict ourselves to the situation where the trust premium $\eta$ is high enough that separating equilibrium exists when $k > \bar{k}$.

2. The second area is where $k < \bar{k}$, and choosing protection leads to $k'^S \geq \bar{k}$, notice that when all risks are diversified and the government refrains from predation, capital will keep growing until it reaches $k^*$, but political sentiment will remain at the level at which government starts to choose the pure strategy of protection forever. In another word, citizens will not believe in the government more than they do just after all sectors are opened.
thus entering region $\pi$. If an opportunistic government’ strategy in this area is $\sigma = 1$, then $p'F = p'S$.

**Proposition 5** (Existence of mixed strategy). At states $(k, p)$ close to $\pi$, an opportunistic government will choose neither pure strategy of protection nor pure strategy of predation.

**Proof.** See Appendix B.

This proposition establishes that there always exists a mixed strategy area. If $p'F = p'S$, there is no opportunity distrust cost at all, an opportunistic government always has an incentive to confiscate. However, when economy develops towards full risk diversification stage, information gap is largely reduced, a pure strategy of predation comes at the cost of fast falling trust thus high political accountability, hence a mixed strategy is employed.

3. The third area is where $k < \bar{k}$, and choosing protection leads to $k'S < \bar{k}$, outside region $\pi$. If an opportunistic government’s strategy in this area is $\sigma = 1$, then $p'F = p'S$.

**Proposition 6** (Immediate predation gain v.s. future predation gain). Assuming Inada conditions of the production function $F(z, k, L)$, there exists a $\bar{k}$, such that if $k < \bar{k}$, an opportunistic government always chooses $\sigma(k, p) = 1$.

**Proof.** See Appendix B.

This proposition is related with the tradeoff between immediate predation gain and future predation gain. It states that when capital level is very low, marginal return of capital is high. Immediate predation gain is less tempting than future predation gain. An opportunistic government chooses to protect the economy to grow more in expectation of a larger size of future predation gain.

**4.1.2 Predation Stage**

In this stage, an opportunistic government always chooses the pure strategy of predation. Separating equilibrium arises when sentiment bias is strong and mimicking becomes ineffective.

**Proposition 7** (Extreme political optimism and pessimism). For every $k \in \{ k : k > \bar{k} \text{ & } (k, p) \notin \pi \}$, there exists a $\bar{p}_k$ such that if $p < \bar{p}_k$, an opportunistic government always chooses $\sigma(k, p) = 0$. For every $k \in \{ k : \bar{k} < k < \hat{k} \}$, there exists a $\bar{p}_k$ such that if $p > \bar{p}_k$, an opportunistic government always chooses $\sigma(k, p) = 0$.

\[27\text{When capital is very low, output increases even if an economic risk is not covered, since } k'F = q(k)s(k), q(k) > 0 \text{ and } s(k) \text{ is concave.}\]
Proof. See Appendix B.

This proposition establishes an opportunistic government’s strategy of adaptation in two other tradeoff cases (see Figure 6a). If \( p \to 1(0), p' \to 1(0) \). Therefore, political optimism and pessimism tend to last. An opportunistic government is less motivated to strengthen political optimism or rescue pessimism at the cost of predation gain when the force from the trend itself is strong. Moreover, Proposition 1 shows that political optimism and pessimism adjust slowly when they reach a peak thus further foster predation motivation. Notice that even if \( k > \bar{k} \), as long as trust is not sufficiently high, an opportunistic government still chooses to confiscate despite the cost of exposure and removal from power. This complements Proposition 4.

\[
\text{(a)} \quad \text{strategy} < (k_t, p_t) \\
\text{(b)} \quad \text{strategy} < (k_t, p_t)
\]

Figure 6: Predation under Extreme Political Sentiments

Note: net predation gain is larger than distrust cost at all \( \sigma \) because extreme political sentiment is a strong bias and adjust slowly. Figure 6b illustrates that pure strategy of predation will be chosen.

4.1.3 Trust Exploitation Stage

In this stage, an opportunistic government mixes between predation and protection. When predation gain is attractive, exposure is not an immediate threat and political sentiments are not extreme, a mixed strategy affects distrust cost efficiently therefore public trust is exploited.

Proposition 8 (Semi-pooling equilibrium). \( \forall (k, p) \in \{(k, p) : k > \bar{k} \text{ and } p \in (p_k, \bar{p}_k) \text{ and } (k, p) \notin \pi\} \), there exists a \( \sigma(k, p) \in (0, 1) \) such that an opportunistic government is indifferent between choosing predation and protection.

Proof. See Appendix B.

After we have established Proposition 2–Proposition 7, this proposition, based on the property of continuity of equilibrium strategy established in Theorem 1, shows the areas where where continuous mixed strategy is employed. Figure 7 illustrates...
the unique Markov strategy over the entire $k - p$ space. It is shown that the area of mixed strategy exists before and after capital level reaches $\bar{k}$. Moreover, this area exhibits significant variations. For programming details, see Appendix C.

![Equilibrium Markov Strategy](image)

**Figure 7: Equilibrium Markov Strategy $\sigma(k_t, p_t)$**

Note: An opportunistic government’s strategy of adaptation $\sigma(k, p)$ varies from 0 to 1. 0 is a pure strategy of predation, 1 is a pure strategy of protection.

### 4.2 Indifference Curves

For dynamic properties of the equilibrium strategy, we study the indifference curves of mixed strategy in the Trust Exploitation Stage along the dimensions of capital and trust respectively. Capital accumulation brings momentum to political accountability through a closing information gap. Sentiment bias from pessimism to optimism affects effectiveness of political accountability.

**Proposition 9** (U-shaped strategy along the $k$ dimension). \( \forall p, \text{ if } p_k < p < \bar{p}_k, \text{ there exists a } \hat{k}_p \text{ such that if } k \leq \hat{k}_p < k \leq \bar{k}_p, \sigma(k, p) \text{ decreases in } k; \text{ if } \hat{k}_p < k \leq \bar{k}_p, \sigma(k, p) \text{ increases in } k. \)

**Proof.** See Appendix B.

At a given trust level and before risks are fully diversified, the indifference curves $IC_k$ (an opportunistic government is indifferent between predation and protection) display the mixed strategy variation as capital level adjusts. When capital level is low,
predation gains are small. However, as Proposition 1 shows, trust $p^S$ goes up quickly in case of a success because of a large information gap. Economic risk is high when economy is underdeveloped and a success would be ascribed more to a low political risk. It encourages an opportunistic government to build up political optimism at a low predation cost. Thus when capital level is low, opportunistic governments mimic more often. Political accountability is also low but small predation gain is unattractive compared with trust premium.

As capital accumulates, predation benefits grow. The second order derivatives of Proposition 1 imply that divergence between updated trust level in case of a success and failure is smaller: the increase of $p^S$ is losing momentum quickly while the decrease of $p^F$ is merely starting to gain momentum. Therefore, trust building is unattractive and political accountability is low. Consequently, the opportunistic government goes after the predation benefits more.

When capital approaches $\bar{k}$, economic risk approaches full diversification and information gap is greatly reduced. If political risk is not controlled, the divergence between $p^F$ and $p^S$ will again grow, this time in the direction of fast falling trust. When the threat of increasing political accountability exceeds increasing predation gains, the government will pool with the benevolent ones more often to avoid exposure.

Figure 8 shows the U-shaped indifference curves of mixed strategy as capital accumulates.

![Indifference Curves $IC_k$](image)

Note: $p$ increases in the arrow direction. The left panel shows the variations of strategy as $p$ increases when $p_k < p < \bar{p}_k$, and the right panel shows the variations of strategy as $p$ increases when $\bar{p}_k < p < \bar{p}_k$. See below for further discussions on strategy along the $p$ dimension.

**Proposition 10** (Inversely U-shaped strategy along the $p$ dimension). $\forall k$, if $k < k <$
\( \tilde{k} \), there exists a \( \hat{p}_k \) such that if \( p_k < p < \hat{p}_k \), \( \sigma(k, p) \) increases in \( p \); if \( \hat{p}_k < p < \bar{p}_k \), \( \sigma(k, p) \) decreases in \( p \).

**Proof.** See Appendix B.

At a given capital level and before risks are fully diversified, trust updating responds to signals more when sentiment bias is relatively small. Political accountability is higher accordingly as citizens are more sensitive to economic outcomes. Therefore, an opportunist government will mimic more often when the level of political sentiments is in the middle and less as they shift towards political optimism and pessimism. Figure 9 shows the inversely U-shaped indifference curves of mixed strategy as trust increases.

![Indifference Curves IC\( \sigma \)](image)

**Note:** \( k \) increases in the arrow direction. The left panel shows the variations of strategy as \( k \) increases when \( k < k < \hat{k}_p \), and the right panel shows the variations of strategy as \( k \) increases when \( \hat{k}_p < k < \bar{k} \).

**Proposition 11** (Political instability after full economic risk diversification). If \( k > \tilde{k} \) and \( p_k < p < \tilde{p}_k \), \( \sigma(k, p) \) increases in \( p \) and decreases in \( k \).

**Proof.** See Appendix B.

After economic risks are fully diversified and the trust level is not too high or too low, there are more predation benefits as capital accumulates, encouraging the government to mimic less often. If trust level is higher, \( F^F = 0 \), \( F^S \) is higher and the opportunity distrust cost is higher, encouraging the government to mimic more often.
4.3 Developmental Trajectories

On the equilibrium development path of an economy under the reign of an opportunist government, other than higher variability in early stages of development as is discussed in Acemoglu and Zilibotti (1997)\(^{28}\), there are several features.

First, at the early stage or after a big recession, when capital level is low and rent seeking is not profitable, an economy grows or recovers slowly as there are many economic risks. But there is few political risk since the government always protects or restores the economy, either in expectation of future predation or eager to build up political optimism.

Second, there exist a virtuous circle when capital is large and trust is not too low, and a vicious circle when capital level is low. These circles generate the economic and political gaps across countries and over time.

In a virtuous circle, citizens’ investment in risky sectors and the opportunistic government’s strategy $\sigma$ both increase with the capital stock of the economy. Economic development facilitates informational efficiency thus political accountability and discourages predation, which in turn promotes economic development. Therefore, economic prosperity goes hand in hand with good governance.

In a vicious circle, similarly, capital is sizable but insufficient for diversification of economic risks, thus information gap cannot be effectively reduced and political accountability is low. An opportunistic government tends to extract resources of the economy, which further worsens the situation of an underdeveloped economy. With high political and economic risk, poverty trap arises and is joined by corruption.

Third, extreme political optimism or pessimism breeds corruption.

We simulate the developmental trajectories by introducing endogenous political and economic risks. Several of them are displayed in Figure 10. The features discussed can be found in them.

5 Self-fulfilling Political Transition

In Acemoglu and Robinson (2001) where they introduce a theory of political transitions, the interaction between citizens’ investment and governments’ taxation decision as a possible source of multiple equilibria in political transition is discussed. They argue that 'if agents believe that democracy will persist, they will invest more, and this will in turn increase the durability of democracy'. The same self-fulfilling political transition (democracy consolidation) mechanism can be realized in our model, apparently in the stage where area $\pi$ is approached and government starts to commit to pure strategy of protection. Similarly, we incorporate this mechanism through the interaction between citizens’ investment decision and governments’ strategy.

\(^{28}\)In our paper, it is determined by how we set $Q$ and $q(n)$. 

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We generalize citizens’ utility functions and rewrite their maximization problem so that the saving decision is affected by citizens’ estimation of overall risks:

\[
\max_{s_t} \quad u_1(c_t) + \left[ (n_t + (p_t + (1 - p_t)\sigma_t)\gamma)u_2(Qs_t) + (1 - n_t - (p_t + (1 - p_t)\sigma_t)\gamma)u_3(q(n_t)s_t) \right]
\]

s.t. \( c_t \leq (1 - \tau)w_t + (1 + r_t)k_t - s_t \),

where \( u_1, u_2 \) and \( u_3 \) are standard concave utility functions strictly increasing in its argument. We have the following proposition:

**Proposition 12.** If \( u'_2(Qs)Q > u'_3(qs)q \), citizens’ optimal saving \( s_t \) is an increasing function of capital and government’s reputation. Thus it increases in \( k_t, p_t \) and \( \sigma(k_t, p_t) \).

**Proof.** See Appendix B.

This proposition establishes that as long as the marginal utility from an additional unit of saving is large enough, a higher belief in government’s quality and a
higher belief that an opportunistic government will take an action of protection will lead to more investment into the private sectors. It is intuitive that proactive action is induced by positive expectation. \( k^{S} \) and \( k^{F} \) are naturally increasing functions of \( \mu \) as well.

As long as \( \eta \) is high enough, \( \pi \) still exists where an opportunistic government consistently chooses protection. We then study the government’s strategy when \( (k, p) \) approaches \( \pi \).

We compare \( \sigma \)'s effects separately on the net predation gain \((\phi(k) + \beta[\gamma(\phi(k^{F}) - \phi(k^{S})) + n^{F}(k^{F}) - n^{S}(k^{S}))])\) and opportunity distrust cost \((\beta[\psi(p^{S}) - \psi(p^{F})])\) to discuss opportunistic governments’ strategy in the current setting. Since \( k^{S} \) increases faster than \( k^{F} \) when \( \sigma(k, p) \) increases, net predation gain decreases when the opportunistic government mimics more often. \( \sigma \)'s effects on \( p^{S} \) and \( p^{F} \) feature additional channels compared with the unique equilibrium case. \( \sigma(k, p) \)'s impact on capital accumulation now affects the information gap and thus Bayesian updating:

\[
p^{S}(k, p) = \frac{p(n(k, p, \sigma(k, p)) + \gamma)}{p(n(k, p, \sigma(k, p)) + \gamma) + (1 - p)(n(k, p) + \gamma \sigma(k, p))}
\]

\[
p^{F}(k, p) = \frac{p_{t}(1 - n(k, p, \sigma(k, p)) - \gamma)}{p_{t}(1 - n(k, p, \sigma(k, p)) - \gamma) + (1 - p)(1 - n(k, p, \sigma(k, p)) - \gamma \sigma(k, p))}
\]

There are two counteracting effects of \( \sigma(k, p) \) on \( p^{S} \) and \( p^{F} \). The first effect works through signal jamming. If \( \sigma(k, p) \) increases, it is harder for citizens to detect the government’s type. They update their trust at smaller steps and divergence between \( p^{S} \) and \( p^{F} \) is smaller. The second effect functions through information gap. When \( \sigma(k, p) \) is larger, there is more investment in the private sectors and an economic risk is less likely to take place. \( p^{F} \) falls faster when information gap is reduced and the divergence between \( p^{S} \) and \( p^{F} \) is larger. When \( \sigma(k, p) \) is relatively small, the first effect dominates; when \( \sigma(k, p) \) increase and \( \pi \) is quickly approached, the second effect dominates. Figure 11 shows the forces of these effects and two cases where there exist three equilibria.

If the government’s reputation is low with a small \( \sigma(k_{t}, p_{t}) \), which implies a high political risk, citizens will save and invest less. Fewer sectors are opened and information gap contracts less. It remains relatively difficult to hold the government accountable in case of a failure. Aware of citizens’ response, an opportunistic government confiscates more often as political accountability is relatively low. The expectation is self-fulfilled. Similarly, if the government’s reputation is high with a large \( \sigma(k_{t}, p_{t}) \), citizens are optimistic and proactive in saving and investment. As an economy expands more, informational efficiency is higher and political accountability is higher. Predation is thus discouraged and citizens’ initial expectations are fulfilled.

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\(^{30}\)Given that self-fulfilling multiple equilibria exist, when a path develops to approach area \( \pi \), arbitrary changes in citizens’ expectations will influence the outcome. Therefore, dynamically, we need to introduce sunspot equilibrium concept. However, when \( (k, p) \to \pi \), the continuation value is certain as \( k^{F} = 0 \) and \( k^{S} \in \pi \). Therefore, we still can discuss self-fulfilling political transition without extrinsic fluctuations.
Figure 11: Self-fulfilling Multiple Equilibria

Note: The red line is the net predation gain which decreases as $\sigma(k_t, p_t)$ increases. The blue line is the opportunity distrust cost which decreases when $\sigma(k_t, p_t)$ is relatively small and increases when $\sigma(k_t, p_t)$ is relatively large. In the left panel, there exist two mixed strategies and one pure strategy of protection. In the right panel, there exist one pure strategy of predation, one mixed strategy and one pure strategy of predation.

6 Conclusion

In this paper we have explored the driving force that shapes nations’ developmental trajectories. The same force underlies the wide variation in political and economic performances over time of a country and across countries as we still observe today. This inherent driving force is the evolution of political accountability.

Opportunistic governments may be tempted to exploit ineffectiveness of the political accountability and appropriate the public budget. It is only when citizens can effectively discipline the governments that public sectors deliver good services and corruption is curtailed.

Political accountability turns out to depend on two key factors: informational efficiency and sentiment bias, both of which affect the effectiveness of political accountability. The degree of information of citizens curbs the opportunities governments have to engage in political predation. The level of political sentiments amplifies or diminishes political accountability.

Regarding informational efficiency, how well the public is informed hinges on the the level of economic development. As the private economy prospers, citizens have more precise knowledge on the possibility of a malfunctioning government.

Therefore, through political accountability, we can focus on the mapping from fundamentals of information structure and political sentiments to opportunistic governments’ optimal strategy.
Good or poor governance in turn has dramatic economic and political consequences, creating evolving environments of economic development and political sentiments where citizens and the government are implemented in for new rounds of accountability game. Virtuous circle of "good governance–economic growth–high political accountability–good governance" and vicious circle of "poor governance–economic underdevelopment–low political accountability–poor governance" can be generated.

Nations’ developmental trajectories are determined by even more factors all endogenous in our model. Luck, political sentiments and economic development as well as their interactions all impact on the formation of political and economic stages and development paths.

While countries today or dynasties in history may very coarsely fit into one of the stages or trajectories, further empirical work can be done to examine our theory.

**References**


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Appendices

A Proof of Theorem 1

We impose an additional condition of the MPE for proof and programming: citizens update their belief with \( \hat{\sigma}(k, p) \) and in equilibrium, \( \hat{\sigma}(k, p) = \sigma(k, p) \). Therefore:

\[
V_{pb}(k, p) = \max_{a \in \{pr1, pr0, pb\}} \{ u_{pb}^{gov}(k, p, a) + \beta \int V_{o}^j(k'(k, a), p'(k, p, \hat{\sigma}(k, p), a)) f(\omega_j|k'(k, a))dj \},
\]

\[
V_{pr1}(k, p) = u_{pr1}^{gov}(k, p) + \beta \int V_{o}^j(k'(k, a), p'(k, p, \sigma(k, p))) f(\omega_j|k'(k, a))dj,
\]

\[
V_{pr0}(k, p) = u_{pr0}^{gov}(k, p) + \beta \int V_{o}^j(k'(k, a), p'(k, p, \sigma(k, p))) f(\omega_j|k'(k, a))dj,
\]

\[
\hat{\sigma}(k, p) = \sigma(k, p).
\]

The steps involved in establishing existence and uniqueness of the MPE and uniqueness of Markov strategy \( \sigma(k, p) \) are as follows:

1. We prove that given any \( \sigma(k, p) \), \((T_1V_{pb})(k, p)\) is a contraction mapping.

2. We show that for the equilibrium conditions to hold, \( V_{pb}(k, p) \) must be continuous and monotonically increasing in \( p \).

3. Given monotonicity of \( V_{pb}(k, p) \) in \( p \), we show that the equilibrium definition is equivalent to finding \( \min_{\sigma(k, p)} |V_{pb}^{NI}(k, p) - V_{pb}^{F}(k, p)| \).

4. We prove that \((T_2F)(k, p) = \min |V_{pb}^{NI}(k, p) - V_{pb}^{F}(k, p)|\) is a contraction mapping, and the minimizer \( \sigma(k, p)^* \) is unique.

The stochestic variable \( \omega_{jt} \in \Omega = \{pr1, pr0, pb\} \), which is finite and compact. We denote \( x_t = (k_t, p_t) \in \mathbb{X} \subset \mathbb{R}^2 \) as the state vector, to be specific, \( \mathbb{X}(k_t \in [0, k^*], p_t \in [0, 1]) \). \( y_t = (a_t \in \{NI, I\}, \sigma(k_t, p_t)|\omega_{jt} = pb) \in \mathbb{Y} \subset \mathbb{A} \times \mathbb{R}^1 \) is the control vector. Capital’s law of motion is determined by either \( \omega_{jt} \) or \( a_t \). Bayesian updating is determined by \( a_t \) and \( \sigma(k_t, p_t) \). The density of transition probability \( f(\omega_{jt+1} | k_{t+1}) \) is continuous in \( x_{t+1} \). We assume \( x_{t+1} \in M(x_t, \omega_{jt}) \), and \( M : \mathbb{X} \times \mathbb{Y} \rightarrow \mathbb{X} \) specifies the constraint correspondence. \( \mathbb{X} \) is compact, \( M \) is nonempty-valued, compact-valued and continuous. \( \phi \) and \( \psi \) are continuous and increasing functions.

Step 1. Given \( \sigma(k, p), \forall k, p, V_{pb}(k, p) \) is a contraction mapping.

This will not give us any equilibrium because the equilibrium conditions may well be violated. However, if we solve \( \sigma(k, p)^* \), this step ensures that important properties of \( V_{pb}(k, p) \) will not be affected.

Let \( \mathbb{B}(k, p) \) be the set of bounded functions defined on \( \mathbb{X} \), endowed with the sup norm \( ||f|| = \sup_{x \in \mathbb{X}} |f(x)| \). (Metric spaces of all bounded real functions is complete anyway.) For \( V(k, p) \in \mathbb{B}(k, p) \), define the operator \( T_1 \) as

\[
(T_1V_{pb})(k, p) = \max_{a \in \{NI, I\}} \{ u_{pb}^{gov}(k, p, a) + \beta EV(k', p') \},
\]

\[
EV(k', p') = \int V_{o}^j(k'(k, a), p'(k, p, \hat{\sigma}(k, p), a)) f(\omega_j|k'(k, a))dj.
\]
By boundedness, the maximum exists. Therefore, \( T_1 \) is well defined and bounded, so \( T_1 \) maps \( \mathbf{B}(k, p) \) into itself. Blackwell’s sufficient conditions for contraction are satisfied for this operator.

**Step 2.** We show that for the equilibrium conditions to hold, \( V_{pb}(k, p) \) must be continuous and monotonically increasing in \( p \).

Norets (2010) establishes the continuity and differentiability of value functions of dynamic discrete choice under fairly general conditions. The continuity of our value functions relies on the continuity of transition probability function of all state variables. (There is a discontinuity of \( V_{pr0}(k, p) \) and \( V^N_{pb}(k, p) \) at \( k \) when \( p^{IF} = 0 \) but that does not affect the continuity of \( V_{pb}(k, p) \).) \( f(\omega_{jt+1}|k_{t+1}) \) is only for \( \omega_j \) and \( k_t \). To ensure continuity of \( f(\omega_{jt+1}, k_{t+1}, p_{t+1}|k_t, p_t) \), we require continuity of \( \sigma(k_t, p_t) \) which cannot be obtained from contraction mapping of operator \( T_1 \).

Moreover, monotonicity in \( p \) is not direct since \( \frac{\partial p'}{\partial p} \) is also affected by \( \sigma(k, p) \), we need to impose conditions on \( \sigma(k, p) \) so that this property can be established. However, we can prove both properties by assuming that the equilibrium exists first. If the MPE exists, then \( V_{pb}(k, p) \) is continuous and strictly increasing in \( p \). The equilibrium definition and the boundaries’ \((\pi, p = 0, p = 1)\) properties ensure the continuity and monotonicity of \( V_{pb}(k, p) \).

In \( \pi \), \( V_{pb} = V^I_{pb} = \frac{\psi(p)}{1-\beta} \) which is continuously increasing in \( p \). Moreover, Proposition 1 establishes that when \( p \to 0 \) & \( k > \bar{k} \) or \( p \to 1 \) & \( k < \bar{k}, \frac{\partial p'}{\partial \sigma(k, p)} \to 0 \). Therefore, \( V_{pb} = V^N_{pb} \) in these cases. And it is easy to show that \( V_{pb}(k, 0) \ll V_{pb}(k, 1) \) and \( V_{pb}(k, 0) < V_{pb}(k, p), p > 0 \).

We consider discontinuity first. It is trivial to show that there must be a mixed strategy area in the equilibrium if MPE exists as long as strategies in \( \pi \) and \( p = 0 \) are constructed under assumption like \( \eta > \bar{\eta} \) (Proposition 5). Discontinuity in them is impossible. In other areas, discussion in case of pure strategy is the same as mixed strategy. So we focus on equilibrium definitions of mixed strategy. For all types of discontinuity, we consider the following setting:

Given two points \((k, p_1), (k, p_2), p_1 > p_2\). The discontinuity point is \((k, p_1)\) and we assume \( V_{pb}(k, p_1) > V_{pb}(k, p_2) \) and leave the discussion of non-increasing cases till later. We assume as \( p_2 \to p_1, \exists \delta > 0 \) such that \( V_{pb}(k, p_1) - V_{pb}(k, p_2) > \delta \). Therefore, \( \exists \epsilon > 0, |\sigma(k, p_1) - \sigma(k, p_2)| > \epsilon > 0 \).

If \( \sigma(k, p_1) - \sigma(k, p_2) > \epsilon, (k, p_2) \) is updated to \((k^{IS}, p_2^{IS}), (k, p_1) \) is updated to \((k^{IS}, p_1^{IS})\), \( \exists \epsilon_1 > 0, p_2^{IS} > p_1^{IS} > \epsilon_1 \) while \( p_2 \to p_1 \) and \( p_1 > p_2 \). By assumption, this is an equilibrium point, therefore, if continuity and monotonicity are kept at \( k^{IF} \), then there is \( V_{pb}(k^{IS}, p_2^{IS}) > V_{pb}(k^{IS}, p_2^{IS}) \) although \( p_1^{IS} < p_2^{IS} \). If \( \sigma(k, p_2) - \sigma(k, p_1) > \epsilon, (k, p_2) \) is updated to \((k^{IF}, p_2^{IF}), (k, p_1) \) is updated to \((k^{IF}, p_1^{IF})\), and we will have \( V_{pb}(p_2^{IF}, k^{IF}) > V_{pb}(k^{IF}, p_2^{IF}) \) although \( p_1^{IF} < p_2^{IF} \). Both are equivalent to the case of non-increasing value functions.

Now we discuss this case: \( p_1 > p_2, V_{pb}(k, p_1) < V_{pb}(k, p_2) \). First, if there is continuity of \( \sigma \), then we will gradually update to \( \pi \) or \( p = 0 \) area with decreasing
given that to prove the continuity and monotonicity of definition is equivalent to finding if MPE exists. We can check that the property of continuity and monotonicity are then we will have larger and larger the same contradiction if we let on the process.

There is another case where we should not always control continuity and monotonicity in one direction at \( k^LS \) or \( k^LF \) so that area \( \pi \) and \( p = 0 \) will not be approached, then we will have larger and larger \( V_{pb}(k, p'_{-}) - V_{pb}(k, p''_{-}) \), while \( p'_{-} \to 1 \) and \( p''_{-} \to 0 \), given that \( V_{pb}(k, 0) \ll V_{pb}(k, 1) \), this is a contradiction again.

Therefore, we have used the equilibrium definition and the boundary settings to prove the continuity and monotonicity of \( V_{pb}(k, p) \) and thus continuity of \( \sigma(k, p) \) if MPE exists. We can check that the property of continuity and monotonicity are validated with the contraction mapping of \( T_1 \).

**Step 3.** Given monotonicity of \( V_{pb}(k, p) \) in \( p \), we show that the equilibrium definition is equivalent to finding \( \min_{\sigma(k, p)} |V_{pb}^{NI}(k, p) - V_{pb}^{I}(k, p)| \).

We have proved that \( \frac{\partial V^{LS}}{\partial \sigma} < 0, \frac{\partial V^{LF}}{\partial \sigma} > 0, \frac{\partial V_{pb}(k, p)}{\partial p} > 0 \), therefore denote \( d_1 = V_{pb}^{NI}(k, p) - V_{pb}^{I}(k, p) \), then \( \frac{\partial V_{pb}(k, p)}{\partial p} > 0 \). However, if we denote \( d_2 = |V_{pb}^{NI}(k, p) - V_{pb}^{I}(k, p)| \), then \( d_2 \) is \( V \)-shaped along \( \sigma \in [0, 1] \) if \( d_2 = 0 \) at some \( \sigma \in (0, 1) \).

Therefore, if \( V_{pb}^{NI} \leq V_{pb}^{I} \) when \( \sigma = 1 \), then \( V_{pb}^{NI} < V_{pb}^{I} \) when \( \sigma \in [0, 1) \) and \( d_2 \) is the smallest at \( \sigma = 1 \). Likewise, if \( V_{pb}^{NI} \geq V_{pb}^{I} \) when \( \sigma = 0 \), then \( V_{pb}^{NI} > V_{pb}^{I} \) when \( \sigma \in (0, 1) \) and \( d_2 \) is the smallest at \( \sigma = 0 \). Therefore, finding the minimum of \( d_2 \) is the necessary condition of the equilibrium definition.

It is also a sufficient condition. If the minimum of \( d_2 \) is larger than 0, then given the monotonicity property, \( \sigma = 1 \) or 0 depends on the sign of \( d_1 \). If \( d_2 = 0 \), then it is a mixed strategy, satisfying the conditions of the equilibrium.

**Step 4.** We prove that \( T_2 F(k, p) = \min_{\sigma(k, p)} |V_{pb}^{NI}(k, p) - V_{pb}^{I}(k, p)| \) is a contraction mapping, and the minimizer \( \sigma(k, p) * \) is unique. (We only need compactness of \( M \) and boundedness of \( V_{pb}(k, p) \)).

We have established the equivalence between the equilibrium definition and finding minimum of \( d_2 \). \( \sigma(k, p) \) is the policy function (correspondence) of a new operation to solve the minimum of all \( d_2 \): \( T_2 F(k, p) = \min_{\sigma(k, p)} |V_{pb}^{NI}(k, p) - V_{pb}^{I}(k, p)| \). As long as we can prove that the new operator is also a contraction with some modulus, the existence and uniqueness of the MPE are established.

Let \( C(p) \) be the set of continuous functions defined on \( p \in [0, 1] \) endowed with the sup norm. \( d_2 \) is a bounded continuous function over a compact set, and by Weierstrass’s Theorem, \( F(k, p) \) has a solution at every \( (k, p) \). Therefore, operator

\[
(T_2 F)(k, p) = \min_{\sigma(k, p)} |V_{pb}^{NI}(k, p) - V_{pb}^{I}(k, p)|
\]

is well defined. Moreover, \( T_2 F \in B(p) \), and \( T_2 \) maps \( B(p) \) into itself.

We can simplify \( F(k, p) \) by equating \( V_{pr0}(k, p) \) and \( V_{pb}^{NI}(k, p) - \phi^{NI}(k) \), and by equating \( V_{pr1}(k, p) \) and \( V_{pb}^{I}(k, p) \). Depending on \( V_{pb}^{NI}(k^{NI}, p^{NI}) \leq V_{pb}^{I}(k^{NI}, p^{NI}) \) and
functions
V_p^N (k'^I, p'^I) \leq V_p^I (k'^I, p'^I), we have four cases. We show one of them, V_p^N (k'^F, p'^F) \geq V_p^I (k'^F, p'^F) and V_p^N (k'^I, p'^I) \geq V_p^I (k'^I, p'^I):

\[ V_p^N (k, p) = h^N (k, p, k'^I) + \beta \left[ n'^F V_p^I (k'^F, p'^F) + (1 - n'^F) V_p^N (k'^F, p'^F) \right], \]

\[ V_p^I (k, p) = h^I (k, p, k'^S) + \beta \left[ n'^S V_p^I (k'^S, p'^S) + (1 - n'^S) V_p^N (k'^S, p'^S) \right]. \]

So we get rid of the max and the other two value functions.

We study the properties of the operator assuming V_p^N (k'^F, p'^F) \geq V_p^I (k'^F, p'^F) and V_p^N (k'^I, p'^I) \geq V_p^I (k'^I, p'^I) (the other three cases follow similarly). We consider V_p^N (k, p) and V_p^I (k, p) generated by two different \( \sigma (k, p) \)s, and we get two distance functions \( F(k, p) \) and \( G(k, p) \), supposedly \( F(k, p) > G(k, p) \). We can find that the difference between \( F(G)(k'^S, p'^S) \) and \( F(G)(k'^F, p'^F) \) is of second order in the operator \( T_2 \):

\[
(T_2 F)(k, p) = \min_{\sigma (k, p)} \left| V_p^N (k, p) - V_p^I (k, p) \right| = \min_{\sigma (k, p)} \left| h(k) + \beta \left[ n'^F V_p^I (k'^F, p'^F) + (1 - n'^F) V_p^N (k'^F, p'^F) \right] \right.
\]

\[
+(1 - n'^F) V_p^N (k'^F, p'^F) - (n'^S V_p^I (k'^S, p'^S) + (1 - n'^S) V_p^N (k'^S, p'^S)) \right| .
\]

We know that

\[ V_p^I (k'^F, p'^F) \leq n'^F V_p^I (k'^F, p'^F) + (1 - n'^F) V_p^N (k'^F, p'^F) \leq V_p^N (k'^F, p'^F), \]

\[ V_p^I (k'^S, p'^S) \leq n'^S V_p^I (k'^S, p'^S) + (1 - n'^S) V_p^N (k'^S, p'^S) \leq V_p^N (k'^S, p'^S). \]

Therefore depending on the value of \( h(k) \), we can pick \( V_p^I \) or \( V_p^N \) at \( (k'^F, p'^F) \) and \( (k'^S, p'^S) \), so that the minimum is reached between the smallest possible value of one and largest possible value of the other if they do not overlap and equal to zero if they do, and \( F(k'^F, p'^F) \) and \( F(k'^S, p'^S) \) only add to the minimum.

Suppose \( h(k) + V_p^N (k'^F, p'^F) \geq V_p^N (k'^S, p'^S) \) (the other case can be discussed similarly), we can further have:

\[
(T_2 F)(k, p) = \min_{\sigma (k, p)} \left| h(k) + \beta \left[ V_p^I (k'^F, p'^F) - V_p^N (k'^S, p'^S) + n'^F [V_p^N (k'^F, p'^F) - V_p^I (k'^F, p'^F)] \right.
\]

\[
+ n'^S [V_p^N (k'^S, p'^S) - V_p^I (k'^S, p'^S)]] \right| .
\]

\[
= \min_{\sigma (k, p)} \left| h(k) + \beta \left[ V_p^N (k'^F, p'^F) - V_p^N (k'^S, p'^S) + n'^F F(k'^F, p'^F) + n'^S F(k'^S, p'^S) \right] \right| .
\]

\( k'^S \) and \( k'^F \) are functions of \( k \) alone. We can see that difference of the value functions is of first order. The difference between the difference of the value functions
is of second order. Therefore the operator is actually:

\[
(T_2F)(k,p) = \min_{\sigma(k,p)} \left| h(k) + \beta \left[ V^{NI}_{pbd}(k^I, p^I) - V^{NI}_{pbd}(k^S, p^S) \right] \right|
\]

\[
\geq \min_{\sigma(k,p)} \left| h(k) + \beta \left[ V^{NI}_{pbd}(k^I, p^I) - V^{NI}_{pbd}(k^S, p^S) \right] + n^I F(k^I, p^I) + n^S G(k^S, p^S) \right|
\]

\[
= \min_{\sigma(k,p)} \left| h(k) + \beta \left[ V^{NI}_{pbd}(k^I, p^I) - V^{NI}_{pbd}(k^S, p^S) \right] \right|
\]

Discounting:

\[
(T_2(F + a))(k,p) = \min_{\sigma(k,p)} \left| h(k) + \beta \left[ V^{NI}_{pbd}(k^I, p^I) - V^{NI}_{pbd}(k^S, p^S) + n^IF(k^I, p^I) + n^SG(k^S, p^S) \right] \right|
\]

With \( q \) or \( \beta \) small enough, the discounting condition is also met. As is established in step 3, with monotonicity of \( V^{NI}_{pbd}(k,p) \) in \( p \), we know that the minimizer \( \sigma(k,p)^* \) is unique.

\section{Proofs of Propositions}

\textbf{Proof of Proposition 2.} Except for the set \( A = \{(k,p): k = 0 \text{ or } p = 0 \text{ or } p = 1\} \), there exists no absorbing set if \( k < \hat{k} \). We have made the assumption to exclude initial state in set \( A \). And for any \( k_0 > 0, 0 < p_0 < 1, k_{t+1} > k_t, p_{t+1} > p_t \) with probability \( n_t + (p_t + (1 - p_t)\gamma) \gamma \) which is positive if \( k_t > 0 \) and \( p_t > 0 \). Conditional on a sequence of success, \( k_t \to k^*; k^* > \hat{k} \) and this sequence occurs with positive probability. \( \square \)

\textbf{Proof of Proposition 3.} We rewrite the alternative-specific value functions in a simple way:

\[
\begin{cases}
V^{NI}_{pbd}(k,p) = \phi(k) + \psi(p) + \beta [n^IF_{pr1} + (1 - n^IF - \gamma)V^{IF}_{pr0} + \gamma V^{IF}_{pbd}] \\
V^{I}_{pbd}(k,p) = \psi(p) + \beta [n^ISV^{IS}_{pr1} + (1 - n^IS - \gamma)V^{IS}_{pr0} + \gamma V^{IS}_{pbd}] \\
V^{NI}_{pbd}(k,p) = \phi(k) + \beta [(n^IF + \gamma)\phi(k^I) + \psi(p^I)] + \ldots \text{ at } (k^I, p^I). \text{ (Expansion at } (k^S, p^S) \text{ is similar.)} \text{ If we expand it with pure strategy of } NI \text{ and } I \text{ at } (k^I, p^I), \text{ then } \phi(k) + \beta [\gamma \phi(k^I) + \psi(p^I)] + \ldots < m^{NI} < m^{mix} < m^I < \phi(k) + \beta [\phi(k^I) + \psi(p^I)] + \ldots, \text{ the same for } m = V^{I}_{pbd}(k,p). 
\end{cases}
\]
Therefore, the tradeoffs are not affected qualitatively if we focus on $m^{mix}$. The expansion of $d \equiv V^{NI}_{pb}(k,p) - V^{I}_{pb}(k,p)$ is thus:

$$d = \phi(k) + \beta\left\{(t^F + \gamma)\phi(k^F) - (t^S + \gamma)\phi(k^S)\right\} + \psi(p^F) - \psi(p^S)\right\} + \beta^2\left\{(t^FF + \gamma)\phi(k^FF) - (t^SF + \gamma)\phi(k^SF)\right\} + \psi(p^FF) - \psi(p^SF)\right\} + \ldots \right\} \approx \phi(k) + \beta\left\{(t^F + \gamma)\phi(k^F) - (t^S + \gamma)\phi(k^S)\right\} + \psi(p^F) - \psi(p^S)\right\},$$

as $k^{FF} \rightarrow k^{SF} \ldots$ and $p^{FF} \rightarrow p^{SF} \rightarrow 0$. Since picking $d = \phi(k) + \beta\left\{(t^F + \gamma)\phi(k^F) - (t^S + \gamma)\phi(k^S)\right\} + \psi(p^F) - \psi(p^S)\right\}$ does not qualitatively affect the results, we use it for analysis. When there is the need to extend analysis into infinite periods, we make the adjustment.

**Proof of Proposition 4.** $\psi = -\eta(1-p)^2, \eta > 0$. If predation is chosen, then $p^F = 0$; if protection is chosen, then $p^S = p$, $\psi(0) = -\eta < -\eta(1-p^S)^2 = \psi(p^S)$. Since $k^S \rightarrow k \rightarrow k^S = p$, the same strategy $\sigma = 1$ is employed at $(k^S, p^S)$.

Since $p = 0$ and $\pi$ are absorbing stages, we expand $d \equiv V^{NI}_{pb}(k,p) - V^{I}_{pb}(k,p)$ completely. Since it is a pure strategy of no predation at $(k^S, p^S)$ and a pure strategy of predation at $(k^F, p^F)$, we get:

$$d = \phi(k) + \beta[\gamma E\phi(k^F) + \psi(0) - \psi(p^S)] < 0,$$

$$(1-\beta)\phi(k) + \beta\gamma E\phi(k^F) < -\eta(1-p^S)^2.$$ 

Note that $p^S \in (0,1)$, therefore $\exists \xi \in (0,1)$ such that $\xi \eta \leq -\eta(1-p^S)^2 < \eta$; besides, $0 < k^F \leq \frac{q(k)k^*}{1-\gamma}$. Thus as long as $\eta$ is large enough and $p^S$ is large enough, the equilibrium condition is satisfied and $\pi$ exists: let $\eta > \eta_1$ where $\eta_1 = \inf\{\eta|\xi \eta > (1-\beta)\phi(k^*) + \beta\gamma \phi(\frac{q(k)}{1-\gamma})\}. \quad \square$

**Proof of Proposition 5.** If $\sigma = 1$ at $(k,p)$, then $p^F = p^S$, which means predation benefit comes at no cost, then:

$$d = \phi(k) + \beta[\gamma\phi(k^F) - \phi(k^S)] + n^F\phi(k^F) - n^S\phi(k^S)] < 0$$

is impossible if $(k,p) \rightarrow \pi$, given the discussion in proof of Proposition 6.

If $\sigma = 0$ at a point $A$, where $k_A < \bar{k}$ and $A \rightarrow B, B \in \pi, p^F_A \rightarrow 0$ given our endogenous Bayesian updating equation. We know from Proposition 4 that if $\eta > \eta_1, V^{NI}_{pb}(k,p) < V^{I}_{pb}(k,p)$ in $\pi$, and in Appendix A we have established the continuity of $V^{NI}_{pb}(k,p)$ at $p = 0$ and $V^{I}_{pb}(k,p)$ at $\pi$. Suppose the distance at point $B$ is $d_B > \delta > 0$, we can always find an $\epsilon > 0$ such that when $\|A-B\| < \epsilon, V^{I}_{pb}(A) - V^{NI}_{pb}(A) > \delta > 0$, a contradiction of $\sigma = 0$. \quad \square
**Proof of Proposition 6.** Given that we have $p'F = p'S$, the constraint on the distance function is

$$d = \phi(k) + \beta\{\gamma[\phi(k^{IF}) - \phi(k^{IS})] + [n^{IF}\phi(k^{IF}) - n^{IS}\phi(k^{IS})]\} < 0.$$ 

Note that both capital accumulation and the production function is concave, $\phi(k) = \tau(1 - \alpha)zk^\alpha, k^{IS} = Qs(k), k^{IF} = qs(k)$. We ignore the second term $n^{IF}\phi(k^{IF}) - n^{IS}\phi(k^{IS})$ since $n^{IF} < n^{IS}, k^{IF} < k^{IS}$, and prove $\phi(k) + \beta\gamma[\phi(k^{IF}) - \phi(k^{IS})] < 0$. After some calculation, we need to prove $k^\alpha < C[(k^{IS})^\alpha - (k^{IF})^\alpha], C > 0$. Dividing by $k^\alpha$, it is trivial to show that $\exists \hat{k}$, if $k < \hat{k}$ then we will have to consider

$$\phi(k) + \beta\{\gamma[\phi(k^{IF}) + n^{IF}\phi(k^{IF})]\} < 0,$$

which is impossible. However, since $k^{IS} - k^{IF} < k^{nSS} - k^{nFS}$ when $k \to 0$, we need to consider second order difference. Taking this into consideration, we still have $d < 0$.

**Proof of Proposition 7.** Given $k$, $\lim_{p \to 0/1}\psi(p^{IS}) - \psi(p^{IF}) \to 0$. And we have found the area $k > \bar{k}$ such that $\phi(k) + \beta\{(\gamma + n^{IF})\phi(k^{IF}) - (\gamma + n^{IS})\phi(k^{IS})\} > 0$. Therefore:

$$d = \frac{\phi(k) + \beta\{(\gamma + n^{IF})\phi(k^{IF}) - (\gamma + n^{IS})\phi(k^{IS})\} + \psi(p^{IF}) - \psi(p^{IS})}{>0}.$$

$\forall k$, we can find a $\bar{p}_k$ and a $p_k$ so that the bracketed term is larger than $\psi(p^{IF}) - \psi(p^{IF})$ if $p > \bar{p}_k$ or $p < p_k$ and we have $d > 0$, satisfying the equilibrium condition for $\sigma = 0$.

Note that, it also means even if $k > \bar{k}$ and $p^{IF} = 0$, as long as $p^{IS} < p_k, \sigma = 0$.

**Proof of Proposition 8.** This result is direct after we have established Proposition 2–Proposition 7.

**Proof of Proposition 9.**

$$d = \phi(k) + \beta\{(\gamma + n^{IF})\phi(k^{IF}) - (\gamma + n^{IS})\phi(k^{IS})\} + \psi(p^{IF}) - \psi(p^{IS})\}$$

In the mixed strategy area, $p_k < p < \bar{p}_k$. Given the concavity of the production function and capital’s law of motion, it can be verified that $\phi(k) + \beta\{(\gamma + n^{IF})\phi(k^{IF}) - (\gamma + n^{IS})\phi(k^{IS})\}$ is increasing in $k$, but the second order is negative. Given the results of Proposition 1, it is trivial to show $\frac{\partial}{\partial \sigma}(\psi(p^{IF}) - \psi(p^{IS})) > 0$. However, if $\sigma = 0$, $\exists \tilde{n}_p$ such that $\frac{\partial}{\partial n}(\psi(p^{IF}) - \psi(p^{IS})) < 0$ when $0 < n < \tilde{n}_p$ and $\frac{\partial}{\partial \sigma}(\psi(p^{IF}) - \psi(p^{IS})) > 0$ when $\tilde{n}_p < n < 1 - \gamma$ given the first and second order conditions in Proposition 1. Since $\eta > \tilde{\eta}, n$‘s effect on the second term is strong enough. Therefore, by the mapping from $k$ to $n$, we can find $\hat{k}_p$. And to make $d = 0$ when $p_k < p < \bar{p}_k, \sigma$ is first decreasing if $\bar{k} < k < \hat{k}_p$ then increasing if $\tilde{k} < k < \hat{k}$.
Proof of Proposition 10. Given $\bar{k} < k < \tilde{k}$, $\phi(k) + \beta\{[(\gamma + n^F)\phi(k^F) - (\gamma + n^S)\phi(k^S)]\}$ is fixed value. It is easy to verify that if $\sigma = 0$, \(\frac{\partial[p^S - p]}{\partial p} > 0\) if $p \in [p_k, \tilde{p}_k]$ and \(\frac{\partial[p^S - p]}{\partial p} < 0\) if $p \in [\bar{p}_k, \tilde{p}_k]$: \[p^S - p = \frac{p(1 - p)(1 - \delta)}{n + (p + (1 - p)\sigma)\gamma}\]

so is the case with $p^F$, thus to make $d = 0$, $\sigma$ is increasing in $[p_k, \tilde{p}_k]$ and decreasing in $[\bar{p}_k, \tilde{p}_k]$. \qed

Proof of Proposition 11.

\[d = \phi(k) + \beta[\gamma E\frac{\phi(k^F)}{1 - \beta} + \frac{\psi(0)}{1 - \beta} - \frac{\psi(p^S)}{1 - \beta}]\]

\[d(k, p_k|\sigma = 0) = 0\]

\[d(k, \bar{p}_k|\sigma = 1) = 0\]

We can fix $\bar{k} < k < k^*$. If $\sigma < 1$, $p^S$ increases in $p$, thus to make $d = 0$, $\sigma$ must also increase in $p \in [p_k, \tilde{p}_k]$. We can fix $p \in [p_k, \tilde{p}_k]$, since $\phi(k) + \beta\gamma\phi(k^F)$ is an increasing function of $k$, to make $d = 0$, $\sigma$ must decrease in $k$ so $p^S$ is higher for a cost to balance $d$. \qed

Proof of Proposition 12. Taking first order condition and applying Implicit Function Theorem, we can easily obtain that $\frac{\partial s_t}{\partial \mu_t} > 0$ if $u'_2(Qs)Q > u'_3(qs)q$. \qed

C. Numerical Procedure

There are two approaches we can employ to solve the equilibrium strategy numerically. The first is a fast one which employs orthogonal polynomial approximation of the value functions and solves a nonlinear system of $F(k, p) = 0$ using the nonlinear programming solver fminsearch. The second is an accurate one which is based on theoretical foundation of contraction mapping and uses policy function iteration in the outer loop. We use the second one to precisely depict the strategy.

Based on the contraction mapping theorem, we employ a value function iteration in the inner loop of $(T_1V_{pb})(k, p)$. For the outer loop, we employ the policy function iteration to solve the unique minimizer $\sigma(k, p)^*$ of $(T_2F)(k, p) = \min |V_{pb}^N(k, p) - V_{pb}^I(k, p)|$. The idea is to guess an optimal policy function $\sigma(k, p)^*$ (assuming it’s stationary) and evaluate the future alternative-specific value functions given this policy function. Then we determine the policy function that would minimize the distance between $V_{pb}^N(k, p)$ and $V_{pb}^I(k, p)$, which then generates a new policy improvement. We continue iterating on the policy evaluation and improvement until the difference in the value functions $\|\epsilon\|$ meets two requirements. The first requirement is that the relative error $\frac{\|\epsilon\|}{\|V\|}$ is small enough, which is standard. The average relative error is
$9 \times 10^{-4}$ if we exclude the pure strategy areas and $3.13 \times 10^{-4}$ if we include them. The second requirement is that the maximal error $||\epsilon||$ should fall below the distance between the corresponding two grid points of each value function. Since we are minimizing the difference between the value functions, there should be no more space for improvement once the grids are set.

## D Pareto Improvement

Without a government, citizens invest in all sectors that can be opened equally, including the zero minimum size requirement sectors. The competitive equilibrium is inefficient because citizens ignore the impact of their investment on others’ diversification opportunities (Acemoglu and Zilibotti (1997)). They invest equally thus over-invest in the private sectors with zero minimum size requirement. Pareto improvement is achieved if they reduce investment in those sectors and open more sectors for risk diversification when $q(n)$ is sufficiently low.

We compare citizens’ welfare in decentralized equilibrium $W^{CE}$ with the welfare $W^{Gov}$ they can achieve under the reign of a benevolent government who taxes their wage at $\tau$. With complete information, we consider a setting that is rid of the multiplier effect (see Section §2.4) so that all welfare gain comes from restructuring citizens’ portfolio. Thus we assume citizens acquire return $Q$ only on the investment in public sectors which is $\tau w_t$ instead of on their private investment $s_t$. It is important that $Q \gg q(n)$. Otherwise economic recession would not be sufficiently bad enough for risk diversification to be important and Pareto improvement cannot be achieved.

For simplicity, we assume the citizens’ maximization problems are respectively:

\[
\begin{align*}
\max_{s_t} & \quad \log(y_t - s_t) + \delta \left[ n_t \log(Qs_t) + (1 - n_t) \log(q(n_t)s_t) \right] \\
\max_{s_t} & \quad \log(y_t - \tau w_t - s_t) + \delta \left[ n_t \log(Qs_t) + (1 - n_t - \gamma) \log(q(n_t)s_t) + \gamma \log(Q\tau w_t) \right] \\
\end{align*}
\]

\[
\begin{align*}
\max_{s_t} & \quad \log(y_t - s_t) + \delta \left[ n_t \log(Qs_t) + (1 - n_t) \log(q(n_t)s_t) \right] \\
\max_{s_t} & \quad \log(y_t - \tau w_t - s_t) + \delta \left[ n_t \log(Qs_t) + (1 - n_t - \gamma) \log(q(n_t)s_t) + \gamma \log(Q\tau w_t) \right] \\
\end{align*}
\]

In the spirit of Acemoglu and Zilibotti (1997), the unbalanced portfolio should allocate less investment in the zero minimum size requirement sectors, i.e. the public sectors in our setting. Therefore, we have:

\[
s_t^{Gov} > \tau w_t, \quad \tau < \bar{\tau} = \frac{\Gamma}{(1 + \Gamma)(1 - \alpha)},
\]

where $\Gamma = \frac{\delta(1 - \gamma)}{(1 + \delta(1 - \gamma))}$. If $\tau$ is small, marginal utility of additional unit of public sector investment is very high, therefore there exists a $\tau$ to ensure $W^{CE} < W^{Gov}$. We show in Figure 12 an exogenous tax rate $\tau$ that achieves Pareto improvement compared with the competitive equilibrium.
E Self-fulfilling Multiple Equilibria: A Portfolio Decision

We introduce an additional portfolio decision for citizens and show that it generates similar results. Besides investing in private risky sectors, citizens can choose to invest in an insurance policy which has a rate of return $q$ on the investment in case of failure. Therefore, the new assumption justifies the low return in our previous setting. We assume citizens are risk averse while the government is not allowed to make an investment in the insurance policy.

The a priori distribution of the government’s type and of its policy choices determine the citizens’ assessment of political risk and thus their portfolio decisions, which will further affect capital accumulation of the economy. Let $\alpha_t$ denote the proportion of savings that is invested into the private sectors, then the formulation of the citizens’ problem and its solution are:

$$\max_{s_t, \alpha_t} \quad u(c_t) + \left[ (n_t + (p_t + (1-p_t)\sigma_t)\gamma) \log\left( Q \frac{\alpha_t s_t}{n_t} \right) + (1-n_t - (p_t + (1-p_t)\sigma_t)\gamma) \log(q(1-\alpha_t)) s_t \right]$$

$$s_t = s(k_t) \quad \alpha_t = \frac{n_t}{1 - (p_t + (1-p_t)\sigma_t)\gamma}$$

$$k_{t+1}^S = Q \frac{s_t}{1 - (p_t + (1-p_t)\sigma_t)\gamma} \quad k_{t+1}^F = q(1 - \frac{s_t}{D(1 - (p_t + (1-p_t)\sigma_t)\gamma)}) s_t$$

---

In Acemoglu and Zilibotti (1997), a riskless asset providing some return in any state is assumed. We replace such asset with an insurance policy to simplify the calculation of equilibrium number of private sectors that are opened. The nature of risk diversification and the main mechanism we are after are not affected by this modification.
Citizens’ investment in the private sectors are determined by government’s reputation \( \mu_t = (p_t + (1-p_t)\sigma_t)\gamma \) other than the capital level. \( \alpha_t \) is an increasing function of both \( p_t \) and \( \sigma_t \). It is intuitive. Citizens are more proactive in risky assets’ investment when they have a higher belief of a benevolent government or of an opportunistic government choosing protection. As a result, more capital will be realized in case of a success but less in case of failure.

With positive levels of political risk \( (\sigma(k_t, p_t) < 1) \), the optimal value of \( \alpha_t \) remains strictly lower than 1. When there is no risk of predation \( (\sigma(k_t, p_t) = 1) \), citizens invest their savings entirely into risky sectors.

Given \((k, p)\), \( k^s = Q \frac{s}{1-(p+(1-p)\sigma)^\gamma} \), increases in \( \sigma \). \( k^f = q \left(1 - \frac{s}{D(1-(p+(1-p)\sigma)^\gamma)}\right)s \), decreases in \( \sigma \). Because citizens purchase less insurance when they are optimistic.

Therefore, this setting generates similar results like we have in Section §5.