International trade in a model with heterogeneous trade costs, parallel imports and non-homothetic preferences*

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Abstract
I incorporate heterogeneous trade costs into the model of Foellmi, Hepenstrick, and Zweimüller (2013) to analyze another reason for export zeros, additional to arbitrage constraints resulting from per capita income differences. In my model, high trade costs may be responsible for export zeros. The level of tariffs and transportation costs are crucial for patterns of international trade. I find that trade between a rich and a poor country is only happening, when trade costs lie within a certain range - i.e. high enough to prevent losses due to arbitrage but sufficiently low to still let exporting be profitable. Furthermore, the model suggests that with a sufficiently large per capita income gap between trading partners, some poor-country firms find it optimal to exclusively export their products to the rich county but not to offer them at home. These new features allow to explain a large share of the empirically observed regional exclusions. I illustrate with a numerical example that, while rich countries always benefit from a trade liberalization, poor countries may benefit or lose. This is due to the effect that trade costs have on the arbitrage constraint. My model also helps to explain the very low trade relations between poor countries. Some intuitive examples help to illustrate the predictions of the model.

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Keywords: Trade costs, non-homothetic preferences, parallel imports, arbitrage, extensive margin, export zeros, north-south trade

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1 Introduction

I present a model of international trade in which the determinants of the extensive margin of trade are the importer’s per capita income and trade costs. In a simple two-country setting with non-homothetic preferences, I show how heterogeneous trade costs cause regional exclusion of certain products.

Recent research has emphasized the role of differences in per capita income between trading partners in explaining export zeros. The typical international trade relation is between a rich and a significantly poorer country. Empirically, there exists a strong positive correlation between export probability and destination’s per capita income (Foellmi, Hepenstrick, and Zweimüller 2013, Bernard, Jensen, and Schott 2009).

I add heterogeneous trade costs to the Foellmi, Hepenstrick, and Zweimüller (2013) 0-1 preferences model. As in their setting, the threat of international arbitrage can be responsible for regional exclusion. New in the model presented here is, that export zeros can also be caused by prohibitively high trade costs. If variable production costs plus trade costs exceed consumers’ willingness to pay in a country, the product is not offered in this market.

The key contribution of my paper is that a product’s trade costs have to lie within a certain range in order for this product to be sold internationally. On the one hand, trade costs have to be high enough to prevent arbitrageurs from shipping northern goods, which have been exported to the poor South and offered there for a low price, back to the rich North and underbidding market prices there. On the other hand, trade costs need to be low enough to not prohibit profitable exporting. These findings allow to explain many empirically observed export zeros.

An example for a typical rich-county, low trade costs product, that might not be offered in poor countries because of arbitrage threats, is the service of a call center. Imagine a USA based service hotline that offers technical support for IT issues to European and North-American customers. Given an attractive contract with phone network providers could be negotiated, offering this service across the globe has extremely low trade costs. Because of the high willingness to pay for their service in the North, they are able to charge a quite high price. Assume now, that this U.S. call center has overcapacity and is therefore wondering if it should expand its business and also serve customers in India. Indian customers have a low willingness to pay, so that the firm could only charge a moderate price there. What could happen, is that an arbitrageur opens a company in India, which offers identical IT support to rich-country customers provided by U.S. specialists but for a marginally lower price. The arbitrageur earns a risk-free profit. The way he achieves this is by letting his calls be answered by the original call center in the USA. All he has to do is reconnecting the incoming calls in India to the U.S. hotline. Thus, the U.S. call center ends up providing support to rich-county customers for the much lower poor-country price. To avoid this arbitrage opportunity, which conquers its profitability, the U.S. firm may decide to not serve Indian customers.

The same point can be made for subscriptions to online media, such as newspapers, video-on-demand or music streaming. These goods have marginal production and trade costs near zero. If an arbitrageur succeeds in pretending a rich-county client lives in a poor country, by using a forged IP

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1 Hereinafter referred to as “FHZ”.

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address for example, firms may prefer to exclusively serve the rich North. Another classic and at the same time tragic example for rich-country goods that are not exported to poor markets because of arbitrage reasons is medication.  

An example for a typical rich-country product with high trade costs, which is not exported to the poor countries due to prohibitively high costs, is fresh food. Think of ready-for-consumption salad for a concrete example. Perishability causes such products to be transportable only at very high costs. Poor-country households’ willingness to pay falls short of the price that would be required to profitably serve them with such products.

I find that with a sufficiently large per capita income gap between trading partners, some poor-country firms find it optimal to exclusively export their products to the rich country but not offering them at home. For firms, the threat of international arbitrage creates a trade-off between market size and price. A producer can either sell his products on the large international market for a low price, corresponding to the poor countries’ moderate willingness to pay, or he can sell his products exclusively in rich countries for a high price, made possible by their high willingness to pay. The first strategy is known as eliminating arbitrage incentives and the second as eliminating arbitrage opportunities.

Trade costs play a key role in the no-arbitrage constraint. Consider a simple two country setting consisting of one country with high per capita income and a second one with low per capita income. Irrespective of location, a firm which supplies the international market can in the rich country charge a maximum price equal to the product’s price in the poor country plus trade costs, that would arise from shipping it from the poor to the rich country. Higher trade costs, therefore, loosen the arbitrage constraint and allow for a higher price in the rich country. For rich-country producers, who choose not to export under low trade costs initially, it becomes more attractive to go international when the arbitrage constraint relaxes. For poor-country firms, which exclusively served the rich country, it simultaneously becomes profitable to also serve the home market. Following this reasoning, higher trade costs can be desirable for poor countries because they are then supplied with a larger variety of goods, which increases consumers’ utility. This leads to another main result of my paper. While a symmetric trade liberalization always increases a rich country’s welfare, poor countries may benefit or lose.

When using homothetic preferences of the standard CES type, there is no separate impact of per capita income on patterns of trade. In such a framework only aggregate GDP matters, while its composition out of population and per capita endowment is irrelevant. In contrast, non-homothetic preferences embody this distinction. My model is an extension of the FHZ model, which is based on a standard, supply-side-emphasizing Krugman (1980) framework but uses 0-1 preferences to shift the focus on the demand side instead. In a 0-1 framework, goods are assumed to be indivisible. A household chooses between purchasing either one unit of a product or not purchasing it at all. I combine the analysis of supply and demand side by incorporating heterogeneous firms as well as non-homothetic preferences.

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2There is a wide strand of literature on pharmaceutical arbitrage. For a good overview see Outterson (2005).
3CES stands for constant elasticity of substitution.
Furthermore, my model supports the claim made by Coulibaly and Fontagné (2005) that the low trade interaction between poor countries is caused by prohibitively high trade costs. Chargeable prices in these markets are low and trade costs are potentially high because of poor infrastructure, high tariffs and inefficient bureaucracy. Southern firms do not ship their goods to another poor country if revenues from exporting fall short of costs. However, they do export to rich countries, since the higher prices chargeable there allow covering the incurring trade costs.

My focus on trade costs is motivated by the discussion of the impacts of falling trade costs and trade liberalizations in recent trade literature. Hummels (2007) discusses the decline in transportation costs in the second area of globalization. Glaeser and Kohlhase (2004) document that transportation costs have declined by over 90% in real terms over the twentieth century. Yi (2003) describes the effects of falling tariff barriers on growth. De Sousa, Mayer and Zignago (2012) discuss the role of non-tariff barriers in the fall of the border effect.

International trade costs include all transport and border-related costs from the foreign producer’s factory gate to passing the destination country’s border. Anderson and van Wincoop (2004) estimate international trade costs to be as large as a 74 percent ad-valorem tax equivalent for a representative rich country. My own analysis of product-level data on merchandise imports to the USA shows substantial heterogeneity in trade costs. Ranking the around 17’000 reported products for recent years by trade costs, the 10th percentile features ad-valorem trade costs of only 1 percent, while the 90th percentile is as high as 30 percent. The median trade costs level in this sample amounts to 8 percent. Trade costs do matter for international patterns of trade and so does their heterogeneity.

This paper is connected to the literature on trade costs, pricing-to-market, parallel trade as well as demand- and market-size effects and heterogeneous firm models. The literature on trade costs was shaped by Anderson and van Wincoop (2004). They introduced a gravity model of trade with iceberg transportation costs to study the effects of trade barriers, the border effect, geographical distance and county size.

Pricing-to-market means, firms set their prices equal to households’ willingness to pay in the local currency of the respective market. Atkeson and Bursein (2008) present a pricing-to-market model that uses imperfect competition, generated through variable mark-ups and marginal international trade costs, to explain deviations from perfect purchasing power parity. They focus on the role of international trade costs and various features of market structure in a standard CES demand system. Their model predicts that due to trade costs the majority of the domestic production of tradable goods is also consumed domestically.

4A good overview is given in the WTO’s World Trade Report 2008.
5The World Trade Organization (2008) broadly defines trade costs as all costs arising from policy barriers (tariffs and non-tariff barriers), transportation (freight and time costs), communication, contract enforcement, exchange rate risks, insurance, local distribution as well as legal and regulatory expenses. I do not include local distribution costs as trade costs in my model and only focus on international components.
6U.S. Imports of Merchandise data between 2005 and 2014 purchased from the U.S Census Bureau are used for this estimation. I calculated trade costs as reported duties and charges relative to the reported value of imports for consumption. Table 1 in the appendix provides summary statistics of the data.
7Other key papers that discuss the impact of trade costs include Limão and Venables (2001) and Hummels (1999).
An overview of the literature on parallel trade is given in Ganslandt and Maskus (2007). They define parallel imports as non-authorized international arbitrage trade of goods with intellectual property rights protection caused by price differences.

This paper is also connected to Melitz (2003) and the literature building on his heterogeneous firm model for analysis of the supply side. They use data based on U.S. industry-level tariff and transportation rates. Melitz (2003) focuses on intra-industry effects of exposure to international trade. Prior to investing fixed market entry costs, firms face uncertainty regarding their individual productivity levels. Having learned their productivity, they have to decide whether or not entering the export market, which requires another irreversible investment. The heterogeneous trade costs used in my model are very similar to heterogeneity in productivity. His dynamic model shows that only high productive firms enter the export market, while less productive firms only produce for the domestic market and the least productive firms are forced to exit. This triggers a resource reallocation process, which increases aggregate industry productivity and improves welfare for all economies involved. Bernard, Jensen and Schott (2006) provide empirical evidence for the predicted reallocation of economic activity towards high-productivity firms as trade costs fall. Arbitrage threats do not arise in the Melitz setting because he sticks to homothetic preferences.

Demand- and market-size effects such as the home market effect, developed in Krugman (1980), have been studied further. The home market effect claims that due to economies of scale a good is exported by those countries with the largest domestic markets for this product. Arkolakis (2011) develops a trade model with marketing costs and heterogeneity in firm productivity. A firm, that entered into a foreign market, faces increasing marginal penetration costs to access additional consumers. Such marketing costs make population size relevant for export decisions.

The remainder of this paper is organized as follows. In the next section, I present the basic assumptions and solve for the autarky equilibrium. In section 3, I apply the framework to two identical countries that are open for trade and study the possible equilibria depending on the level of trade costs. The analysis is extended to an asymmetric two-country model with differences in per capita labor endowment and population size in section 4. Results and comparative statics are presented in section 5. Finally, section 6 concludes.

2 Model setup

The basic setup of the model is identical to the one presented in Foellmi, Hepenstrick, and Zweimüller (2013). A country consists of a population of identical households $P$. Each household is endowed with $L$ units of labor, which is the only factor of production. Labor is perfectly mobile within a country but immobile across countries. The wage paid on a country’s competitive labor market is $W$. I use a representative agent per county. Hence, there is no income distribution within countries.

Every firm produces one individual product variety, which is different from the other products on the market. Firms draw their individual product out of a set of inventions. For this they have to invest fixed set-up costs $F$, measured in units of labor, and can then start production. These fixed set-
up costs capture the effort of inventing the product and establishing the firm. Each firm has variable production costs of \(1/a\) units of labor per unit of output, with \(a\) capturing technology.\(^8\) The production function is identical for all firms. Hence, total labor input required to produce \(q(n)\) units of good \(n\) is \(F + q(n)/a\). Total monetary production costs are simply total labor input times the market wage.

Households spend all their income on consumption of a continuum of differentiated goods and have non-homothetic preferences. Only the first unit of a indivisible good yields a positive utility. This results in a binary choice between buying one unit or not buying the product at all. Let \(z(n)\) denote an indicator that takes value 1 if good \(n\) is purchased and value 0 if not. Then utility takes the simple form

\[
U = \int_0^\infty z(n)dn, \quad \text{where } z(j) \in 0, 1.
\] (1)

Goods enter symmetrically into the utility function, which is additively separable. Households have a preference for diversity and maximize the variety of indivisible goods they consume subject to their budget constraint. The marginal utility of income is denoted by \(\lambda\). A household purchases a good \(n\) if the price \(p(n)\) does not exceed his willingness to pay \(1/\lambda\). As a result of symmetrical goods, the household’s willingness to pay is the same for all products and is itself determined by the household’s income and product prices.

There are interesting differences resulting from using 0-1 preferences instead of the standard constant elasticity of substitution preferences. An agent with homothetic preferences always consumes every product available in positive quantity and only responds along the intensive margin. Due to the Inada condition of CES preferences the agents have infinite marginal utility out of widening their set of consumed varieties and therefore infinite willingness to pay for a good that they do not consume yet. Under non-homothetic preferences consumers only respond along the extensive margin. The quantity of each good consumed is one and agents choose how many different product varieties they consume. All goods are identical, so that these with the lowest price provide the highest marginal utility of income and are purchased first. Consumers then proceed with purchasing higher and higher priced products as long as feasible, given their budget. Utility maximization implies that they deplete their full disposable income for consumption.

\section*{2.1 Autarky equilibrium}

To start with, I solve the model for a single country in autarky. The autarky equilibrium is identical to the one presented in Foellmi, Hepenstrick, and Zweimüller (2013) since international trade costs do not matter if there is no trade between countries.

In equilibrium, there is a single price \(p = 1/\lambda\) in all markets and all goods are purchased by all consumers.\(^9\) Equal profits for all firms must result in identical prices for all goods. The mass of prospective entrants is unbounded but due to zero profits only a finite number of firms \(N\) will produce.

\(^8\)In this setting a higher technology parameter is equivalent to higher productivity as it reduces the variable labor input required.

\(^9\)This is true as long as the household’s willingness to pay exceeds marginal costs. For the proof see FHZ’s Lemma 1.
All \( N \) goods supplied are purchased by all consumers for \( p = 1/\lambda \).\(^{10}\) It is profit maximizing for firms to set this price, as long as it exceeds marginal costs.

The autarky equilibrium is characterized by two conditions. The first is the zero-profit condition of the firms,

\[
\Pi = pP - W(F + \frac{P}{a}) = 0,
\]

which guarantees that set-up costs \( F \) are covered but no further entry happens, so that we have a stationary equilibrium. Without loss of generality, wage is set to be the numéraire, \( W = 1 \). The equilibrium price obtained from the zero-profit condition is

\[
p = \frac{aF + P}{aP}.
\]

This implies a mark-up \( \mu \), calculated as the ratio of price over marginal cost \( 1/a \), of

\[
\mu = \frac{aF + P}{P}.
\]

The mark-up is determined by technology parameters and market size. We will see that the variable mark-ups arising with 0-1 preferences are crucial for the patterns of trade.

The second equilibrium condition is the resource constraint,

\[
PL = N(F + \frac{P}{a}),
\]

which ensures full employment. All resources available must be used in production.

In equilibrium a household’s utility is simply given by the number of consumed goods \( N \). Product diversity in both production and consumption and therefore also welfare is given by

\[
N = \frac{aP}{aF + P}L.
\]

It is important to notice that the price does not depend on the per capita labor endowment. Higher \( L \) increases product variety and welfare but lets price and mark-up unchanged.\(^{11}\)

\(^{10}\)Notice that the integral in equation (1) runs from zero to infinity. While a household’s preferences are defined over an infinitely large measure of potential goods, the number of goods actually supplied is limited by firm entry, i.e. only a subset of potential goods are produced and purchased.

\(^{11}\)This autarky section is an outline of the one presented in Foellmi, Hepenstrick, and Zweimüller (2013). For a more comprehensive coverage consult the original.
3 Trade between identical countries

Let us now consider a two-country world with these two countries $i$ and $j$, being completely identical and trading with each other. I introduce additional fixed costs of $C$ for entering the export market, which are identical for all firms. This is in line with the standard heterogeneous firms models building on Melitz (2003). Moreover, exporting a good to another country causes per unit trade costs. Variable trade costs are denoted as ad-valorem costs of the standard iceberg type introduced by Samuelson (1954). For each unit of a good to arrive in the destination country, $\tau > 1$ units have to be shipped and $\tau - 1$ units are “melted” away during transportation. Hence, trade costs enter multiplicatively into variable production costs. New in my model is, that trade costs are heterogeneous and product-specific, which will drive the main results of this paper. Firms learn their individual level of trade costs corresponding to the product they drew, after having irreversibly invested the fixed export entry costs of $C$.$^{12}$ Still every firm has to pay fixed set-up costs of $F$ to invent a product and start production. After learning their individual trade costs firm decide which markets they want to serve.

There are various possible probability distributions to model heterogeneous trade costs. I will study a Bernoulli and a Pareto distribution. The former because it is the simplest possible form and has a straightforward intuition.$^{13}$ The latter because it fits the real world data well and is state of the art.

When using a Bernoulli distribution to model the firms’ drawing of their individual product and corresponding trade costs, there are only two possible trade costs levels. A fraction $x$ of the firms has low trade costs of $\tau_L$, while the other $(1-x)$ have high trade costs of $\tau_H$.

Alternatively, the Pareto distribution can be used to model continuously distributed trade costs and endogenize the cutoff level. Pareto distributions are popular in the recent trade literature on heterogeneous firm models. Multiple variations and extensions of Melitz (2003) use Pareto distributions to model firm-level productivity.$^{14}$

A Pareto Type 1 distribution with scale parameter equal to one and shape parameter$^{15} \alpha = 7$ fits the trade costs distribution observed in the real world data closely, as I show in figure 13.$^{16}$ I overlaid a broad sample of U.S. Imports trade costs data with this specification of the Pareto distribution in figure 13. The Pareto distribution’s key property of a heavy left side with a long but slim right tail captures the empirical pattern fairly well. Most likely are trade costs slightly above one. The higher $\tau$ gets the less frequent such values are observed.

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$^{12}$In general, heterogeneous ad-valorem trade costs are very similar to heterogeneous productivity. A high trade costs firm has to produce relatively more output for a given amount of abroad sales, which is comparable to producing with an inferior technology. However, in firms’ status of information my model deviates from the setting Melitz (2003) uses. In his model, firms’ already know their productivity before they consider to invest the export entry costs, while here they have uncertainty about their trade costs. This difference roots from the different kinds of firm heterogeneity. Uncertainty about productivity can already be observed when producing for the domestic market, while uncertainty about trade costs is only observable in the export market.

$^{13}$The Bernoulli distribution allows to derive a full trade equilibrium, which is not possible using a Pareto distribution. Additional Bernoulli trade costs equilibria are presented in the appendices A.2 and A.4.

$^{14}$A good overview and discussion on recent trade models is provided in Arkolakis, Costinot, and Rodríguez-Clare (2012).

$^{15} \alpha > 1$ is necessary for a finite mean and choosing $\alpha > 2$ guarantees that the distribution has a finite variance.

$^{16}$I used U.S. Census Imports of Merchandise data between 2005 and 2014 to derive this rough quantitative approximation.
The cumulative distribution function (CDF) of a Pareto Type 1 distribution is

\[ F(\tau) = \begin{cases} 1 - \left(\frac{1}{\tau}\right)^{\alpha} & \tau \geq 1 \\ 0 & \tau < 1 \end{cases} \]

and the corresponding probability density function is

\[ f(\tau) = \begin{cases} \frac{\alpha}{\tau^{\alpha+1}} & \tau \geq 1 \\ 0 & \tau < 1 \end{cases}. \]

The existence of fixed entry costs \( C \) and fixed set-up costs \( F \) generates increasing returns to scale. When variable trade costs surpass a certain upper threshold \( \tau_u \), trade is no longer profitable since prices chargeable per unit do not cover variable costs anymore. Firms with trade costs above this threshold choose to be active exclusively on the home market. I am referring to this threshold value \( \tau_u \) as “cutoff trade costs”. In a symmetric country model, all firms with trade costs below \( \tau_u \) sell their product worldwide. Firms with trade costs above \( \tau_u \) only serve the domestic market since this gives them a higher payoff than selling internationally. As long as countries are identical, there are no arbitrage incentives.

### 3.1 Full trade equilibrium

Using a Bernoulli distribution, if trade costs are sufficiently low, all goods will be traded internationally in a so-called “full trade equilibrium”. In such an equilibrium optimal prices set by firms are equal to the willingness to pay, hence \( p = 1/\lambda \). Firms’ profit maximization implies that the prices of imported and home produced goods are identical. If exporting firms tried to pass through trade costs to consumers, the price of imported products would be above the price of domestically produced products. Domestic producers would respond with price increases on their part until prices are equal again. As both countries are absolutely identical, equilibrium prices must be equal. In a full trade equilibrium all firms prefer to export. The expected profit from selling internationally is higher than from staying domestic. Hence, each firm invests the fixed export entry costs \( C \) to learn its individual trade costs.

The resource constraint of a country \( i \), in such an equilibrium is given by

\[ L^i P^i = N^i(F + C + \frac{P^i}{a}) + xN^i \frac{\tau L^j P^j}{a} + (1 - x)N^i \frac{\tau H^j P^j}{a}, \]

where \( j \) denotes the second country.\(^{17}\) Each of the households \( P^i \) inelastically supplies \( L^i \) units of labor. All available resources are employed to set up \( N^i \) firms and produce one consumable unit of each good for every household worldwide. Setting up and entering the export market uses \( N^i(F + C) \) units of labor, \( N^i \frac{P^i}{a} \) units of labor are employed to serve the home market, \( xN^i \frac{\tau L^j P^j}{a} \) units of labor are employed to produce products with low trade costs for export and \( (1 - x)N^i \frac{\tau H^j P^j}{a} \) to produce products.

\(^{17}\)When countries are identical variables could be written without identifying superscripts. However, I present a more general notation here, that can be used to derive a full trade equilibrium for asymmetric countries.
with high trade costs for export. Similarly, for the other county. Solving equation (2) for the produced variety determines the number of active firms per county

\[ N^i = \frac{aP^i L^i}{a(F + C) + P^i + [\tau_H - x(\tau_H - \tau_L)]P^j}. \]  

Given the unbounded mass of prospective entrants, the expected value of entry must be zero to find a stationary equilibrium. The free entry condition for country \( i \), which ensures zero profits in aggregate, is given by

\[ x\Pi^i_1(\tau_L) + (1 - x)\Pi^i_1(\tau_H) - W^i C = 0, \]  

where \( \Pi^i_1(\tau_L) \) indicates the profit of an internationally active firm with low trade costs. In a full trade equilibrium, the profit from selling internationally with high trade costs must be higher than from selling exclusively at home, after having invested the export entry costs. Using the free entry conditions, given in equation (4), for both countries allows to express relative wages as

\[
\omega = \frac{W^j}{W^i} = \frac{a(F + C) + P^i + [\tau_H - x(\tau_H - \tau_L)]P^j}{a(F + C) + P^j + [\tau_H - x(\tau_H - \tau_L)]P^i}.
\]  

The representative household’s budget constraint in country \( i \) is

\[ W^i L^i = p^i(N^i + N^j). \]

Rearranging the budget constraint and plugging in the expression for the product varieties from equation (3), prices in country \( i \) can be expressed as

\[ p^i = W^i L^i \frac{a(F + C) + P^i + [\tau_H - x(\tau_H - \tau_L)]P^j}{aP^i L^i + a\omega P^j L^j}, \]

where the expression for relatives wages \( \omega \) from the free entry condition (5) is used. In contrast to the autarky equilibrium, prices do now depend on per capita labor endowment. The relative price is defined as \( p^j / p^i = \omega L^j / L^i \) and is equal to one as long as countries are identical.

A full trade equilibrium is summarized by product varieties, prices, and mark-ups. The mark-up of a country-\( i \) firm with trade costs \( \tau_k \) is

\[ \mu = \frac{p^i}{\tau_k} > 1, \quad where \ k \in L, H. \]

In a full trade equilibrium, mark-ups are larger than one. Thus, firms generate a positive marginal return form exporting. Low trade costs firms make a positive profit because they generate a higher mark-up than the average firm, while high trade costs firms, due to their lower mark-up, have a loss. However, given positive marginal returns, selling internationally with high trade costs is still more attractive than staying exclusively domestic. A firm, which stays domestic, could save on trade costs.

\[ ^{18} \text{For the written out form of equation (4) see appendix A.1.} \]
but the lost revenue from a 50 percent reduction in market size would predominate this savings. The economies of scale in production outbalance trade costs.

In a full trade equilibrium, all firms sell to all households worldwide, so consumption and welfare are equalized across countries. The country that gains more from opening to trade is the one with lower product variety in autarky which was

\[ N^i = \frac{aP^i L^i}{aF + P^i}. \]

Product variety and therefore welfare in autarky is increasing in population and in per capita labor endowment. Consequently, the country with fewer inhabitants and lower per capita income benefits more from trade.\(^19\)

Under a Pareto distribution in contrast, a full trade equilibrium can never exist since the highest trade costs drawn are potentially infinite.\(^20\)

### 3.2 Pareto trade costs equilibrium

When using a more realistic Pareto distribution to model trade costs, both countries are in a partial trade equilibrium. The trade costs cutoff level is now endogenous. Again firms learn their trade costs level only after having invested the fixed export entry costs \(C\). Some firms have so high trade costs that variable costs exceed the chargeable price in the other country. Both countries are in a “Pareto trade costs equilibrium”, where a fraction of firms export, while those above the trade costs cutoff stay domestic due to prohibitive high trade costs. The arbitrage constraint is not binding since identical countries have identical prices. Because of perfect symmetry of the two countries, I abstain from using country-identifying superscripts in this section. In equilibrium, the expected value of entering the market with unknown trade costs must be zero. The free entry condition, identical for both countries, is given by

\[ v^e = \int_1^{\tau_u} \Pi_I(\tau) f(\tau) d\tau + \left[ 1 - F(\tau_u) \right] \Pi_D - C = 0, \quad (6) \]

where \(\Pi_D\) indicates the profit of a firm selling exclusively domestic and is set equal to zero. \(\Pi_I(\tau)\) denotes the profit of a internationally active firm as a function of trade costs.\(^21\) This is derived from the wage setting condition, which requires that profits of a exclusively domestically active firm are zero,

\[ \Pi_D = 0. \quad (7) \]

Wages in equilibrium are such that exclusively domestically active firms can exactly cover their set-up costs of \(F\) but have a net loss of \(C\). This implicates that they do not exit, despite of suffering a net

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\(^{19}\) However, this is only true if the result from opening up to trade is a full trade equilibrium, which requires the per capita income gap between the two countries to be sufficiently small. Otherwise, arbitrage threats become relevant and poor country’s benefits from opening deteriorate.

\(^{20}\) Unless a bounded Pareto distribution is used, so that the long right tail is truncated at some maximal value.

\(^{21}\) For the written out and integrated form of equation (6) see appendix A.3. Notice, that fixed export entry costs \(F_e\) are not yet included in this notation of profits.
loss. The aggregate profit from exporting is positive, while the aggregate profit from staying at home is negative.

I assume that all firms initially invest the fixed export entry costs to learn their trade costs. This is reasonable since the expected profit from drawing a product, learning trade costs and having the opportunity to go international is equal to the profit from not investing the fixed export entry costs and staying domestic - i.e. both are zero. Hence, the assumption made is that firms who are indifferent between the two strategies all choose being internationally active. Introduction of fixed export entry costs is not necessary for the existence of a Pareto trade costs equilibrium but is a very realistic feature. The trade costs cutoff condition, which pins down the marginal internationally active firm, is

$$\Pi_I(\tau_u) = \Pi_D.$$ (8)

The marginal firm must be indifferent between selling internationally and staying exclusively domestic. Combining the conditions (7) and (8) allows to write prices as

$$p = \frac{\tau_u W}{a}$$ (9)

and wages, which are chosen to be the numéraire, as

$$W = \frac{aP}{aF + P} = 1.$$ (10)

Solving equation (10) for prices yields

$$p = \frac{aF + P}{aP},$$ (11)

which is equivalent to the autarky price. Setting the two expressions for the price - (9) and (11) - equal, yields a cutoff of

$$\tau_u = \frac{aF + P}{P}.$$ (12)

The cutoff trade costs level is pinned down by exogenous parameters only. Therefore, this is the equilibrium expression for the trade costs cutoff. Taking partial derivatives, we see that the critical trade costs value increases in the fixed entry costs and technology but decreases in market size of the countries. This is in line with intuition. When fixed costs become larger, exporting is more attractive due to economies of scale in production. If countries are very large, export is less attractive since they have already a large home market to serve. Higher technology lowers marginal costs and enables exports to be worthwhile even at higher trade costs. Notice that the equilibrium cutoff trade costs are identical to the expression for the mark-up in autarky. This is intuitive since a firm is indifferent between exporting and staying domestic if the mark-up is just sufficient to finance trade costs. At the cutoff, marginal costs of exporting equal marginal revenue, \(p = W \tau_u/a\).

\[22\text{This wage setting condition is sufficient but not necessary to prevent exit. Firms would stay in the market until their net loss exceeds } F + C. \text{ The wage setting condition is introduced since dropping the second summand of the free entry condition allows solving for a convenient closed form solution.}\]
The resource constraint under this regime is

\[ LP = N(F + C) + N \frac{P}{a} + N \frac{\phi(\tau_u)P}{a}, \]  

where \( \phi(\tau_u) \) denotes total arising trade costs as a function of the cutoff trade costs. It can be calculated as

\[ \phi(\tau_u) = \int_{1}^{\tau_u} f(\tau) \tau d\tau = \frac{\alpha}{\tau_{a+1}} \tau \frac{\tau_{d\tau}}{\tau_{a+1}} = \frac{\alpha}{1 - \alpha} (\tau_u^{1-\alpha} - 1). \]

Equation (12) lets us obtain the number of equilibrium product varieties,

\[ N = \frac{aPL}{a(F + C) + P[1 + \phi(\tau_u)]}. \]

This version of the model with two identical countries and Pareto distributed trade costs has a convenient closed form solution, which can be obtained by plugging all the functional forms into the variables’ equilibrium expressions. The advantage of introducing fixed export entry costs and setting the marginal domestic profit equal to zero is, that it crucially simplifies the free entry condition, which allows solving for this easily traceable closed form equilibrium values. More interesting is the discussion of asymmetric countries. So I will not analyze identical countries further since that can be derived easily as a special case of the asymmetric country framework later.\(^{23}\)

4 Trade between a rich and a poor country

Imagine now a two-country world with a rich and a poor country. Variables of the rich country are denoted with superscript \( R \) and those of the poor with superscript \( P \). The two countries are of equal population size but the rich country has a higher per capita labor endowment than the poor country, \( L_R > L_P \). Trade costs are still obtained the same way, either from a Bernoulli or a Pareto distribution.\(^{24}\)

4.1 Full trade equilibrium revised

Lets revise the full trade equilibrium presented in section 3.1. Assume product specific trade costs are drawn from a Bernoulli distribution. If the income gap between the rich and the poor country is small enough, we still are in a full trade equilibrium with all goods being traded internationally. In such an equilibrium optimal prices set by firms worldwide are equal to the local willingness to pay, hence \( p_R = 1/\lambda R \) and \( p_P = 1/\lambda P \). Since country \( R \) is richer than country \( P \), we have \( p_R > p_P \).\(^{25}\) The prices of all imported and home-produced goods are again equal due to firms’ profit maximization. If imported products were offered at a higher price and find demand, the domestic producers would also increase their prices to the same level and make an additional profit. If prices of the domestically

\(^{23}\)Bernoulli trade costs equilibria with non-traded goods for identical countries are briefly discussed in appendix A.2.

\(^{24}\)The assumption of equal population can be relaxed later. A brief verbal analysis is made in section 4.4.

\(^{25}\)Remember that relative prices in a full trade equilibrium are given by \( p_P/p_R = \omega L_P/L_R \).
produced goods were higher, consumers would prefer imported products and buy them first. Given household's limited budgets, potentially not all domestic producers could sell their products to the entire market. Thus, either domestic producers would reduce their prices to be competitive and able to serve both markets completely or otherwise the other country producers would increase their prices to match the domestic firms in the manner described before.

Resource constraints, free entry conditions and budget constraints for both countries analogous to those presented in section 3.1 can be used to write down this equilibrium regime. Furthermore, an asymmetric two-country world requires a trade balance condition. To keep trade balanced, the value of exports of one country must equal the value of imports of the other and vice versa,

\[ N^R p^P P^P = N^P p^R P^R. \]

As argued, prices for all products available within a country are identical, regardless where they are produced. As a consequence, for a rich-country firm, exported goods generate a lower mark-up than domestic sales since they cannot pass through trade costs to consumers.

A full trade equilibrium under a Pareto distribution does never exist, as argued before. A discussion of Bernoulli trade costs equilibria with non-traded goods for asymmetric countries is provided in appendix A.4.

### 4.2 Pareto trade costs/arbitrage equilibrium

Until now, the two countries' equilibrium prices were similar enough to prevent arbitrage incentives. In this section, I present the most complete version of my model, where both reasons for regional exclusion - prohibitively high trade costs and arbitrage threats - are at work.

When using a Pareto distribution to model continuously distributed trade costs, differences in per capita labor endowment between countries will also lead to different cutoff trade costs across countries. The trade costs cutoff for the rich and the poor country are denoted by \( \tau^R_u \) and \( \tau^P_u \), respectively.

Under a Pareto distribution, not only the rich country but also the poor country ceases to export all its products. The endogenous trade costs cutoff level in the poor country \( \tau^P_u \) is expected to be higher than in the rich county. This is because relatively higher willingness to pay in the rich country allows poor-country firms to at least partially pass through trade costs to consumers, whereas rich-country firms have to bear full trade costs. But also in the poor country, some firms have trade costs that are so high that total costs exceed the chargeable price in the rich country. In both countries, only a fraction of firms export, while those above the trade costs cutoff stay domestic due to their prohibitively high trade costs. This is the first reason for regional exclusion, which stops firms with too high trade costs from exporting.

The second reason for regional exclusion is caused by too low trade costs and thereby triggered arbitrage threats. Under a Pareto distribution, the arbitrage constraint kicks in as soon as prices in the

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26Trade balance was fulfilled by construction for identical countries.

27Poor-country producers can partially pass through trade costs but not fully, as this would trigger arbitrage. This will be discussed in detail in section 4.2.
two countries are not identical anymore. Prices across countries differ if population, labor endowment or technology differs. Internationally operating rich-country firms with the lowest trade costs are then in any event confronted with the threat of arbitrage. If a rich-country producer would try to set a high price for his product at home and charge a low one abroad, arbitrageurs would purchase the product in the poor country, ship it back to the rich country and offer it there for slightly less than the market price. With this underbidding, the arbitrageur captures the whole domestic market share, the producer used to hold. Notice that this arbitrage mechanism is a consequence of the product specific trade costs and the pricing-to-market in the poor country. Rich-country firms with low trade costs cannot set higher prices in any market by pretending to have high trade costs since shipping the product back to the North costs arbitrageurs only the low product specific trade costs.

In equilibrium, northern firms anticipate this and adjust their pricing and exporting behavior accordingly. They can either reduce home prices sufficiently to eliminate arbitrage incentives or they can sell their product exclusively domestic for a high price to eliminate arbitrage opportunities. Hence, northern firms are confronted with a trade-off between market size and price. The “rich-country arbitrage cutoff”, below which trade costs are so low that rich-county firms find staying exclusively domestic the more attractive strategy, will be denoted by $\tau R$.

Similar argumentation can also be made for poor county firms. They have to choose between offering their product internationally for a low price, that allows selling at home, or exclusively export to the rich country and benefit from the northern consumers’ high willingness to pay. If a poor-country firm attempts to sell cheap at home but for a price that is higher than $\tau P$ in the rich country, arbitrageurs will purchase the product in the poor country and parallel import it to the rich market. The “poor-country arbitrage cutoff”, below which trade costs are so low that poor-county firms prefer to exclusively export to the rich county and not offer in the home market, is denoted by $\tau P$. It is obvious, that always only one of the two causes for regional exclusion can be responsible. If high trade costs prevent trade, arbitrage would not be a threat and vice versa.

By solving the model presented below, I show under which conditions such poor firm behavior exists in equilibrium. For exclusive exporting to take place, price differences between countries need to be large enough. The arbitrage constraint for rich- and poor-country firms is given by

$$p^R_I \leq \tau P.$$

In words, the price for internationally traded goods in the rich country $p^R_I$ may at most be the poor-county price times ad-valorem trade costs. This implies that the arbitrage constraint is not binding anymore for trade costs above $p^R/p^P$.28 There is now a continuum of prices in the rich country caused by the trade costs distribution.

Assuming both reasons for regional exclusion - prohibitively high trade costs and arbitrage threat - are at work, firms are now divided into four groups in each country, depending on their trade costs level. This division is illustrated in figure 1. The groups collect firms with the same sales strategy. In the rich country, the first group of firms, with the lowest trade costs, chooses not to export for arbitrage

28$p^I$ without specifying subscript always denotes the price for exclusively domestically offered products.
reasons. They are better off when they sell exclusively domestic and set prices equal to rich-country consumers’ willingness to pay, \( p^R = 1/\lambda^R \). The second group, with trade costs above the arbitrage cutoff level, offers its products internationally and prices them according to the binding arbitrage constraint.\(^{29}\) The third group’s trade costs are so high that the arbitrage constraint is not binding anymore and firms can, at home, charge a price equal to the rich-country households’ willingness to pay, \( p^R = 1/\lambda^R \). Finally, the fourth group has trade costs above the trade costs cutoff level and only offers at home, also pricing-to-market.

The same structure exists in the poor country. With the important difference that the first group with trade costs below the arbitrage cutoff level chooses to exclusively export its product to the rich country, again pricing-to-market.

Figure 1: Pareto distributed trade costs divided in four groups

\[
\begin{align*}
\text{arb. prev. serving poor} & \quad \text{arb. binding} & \quad \text{arb. not binding} & \quad \text{too high trade costs} \\
1 & \quad \tau_l & \quad p^R/p^P & \quad \tau_u \\
\end{align*}
\]

Notes: The four groups collect firms with the same market strategy. In the rich country, the first group, with the lowest trade costs, chooses not to export for arbitrage reasons. The second group, with trade costs above the arbitrage cutoff level \( \tau_l \), offers its products internationally and prices them according to the binding arbitrage constraint. The third group is internationally active with trade costs high enough for arbitrage to not be binding. Finally, the fourth group has trade costs above the trade costs cutoff level \( \tau_u \) and only offers at home. The same structure exists in the poor country, with the important difference that the first group chooses to exclusively export its product to the rich country.

Given this division, the “Pareto trade costs/arbitrage equilibrium” is determined by 9 conditions, when \( W^R \) is again set to be the numéraire. I use the same notation as before, so that \( F(\tau) = 1 - \left( \frac{1}{\tau} \right)^\alpha \) is the fraction of firms with trade costs below \( \tau \).\(^{30}\) For simplicity’s sake, fixed export entry costs \( C \) are set equal to zero in this section.

\(^{29}\)This second group acts corresponding to the behavior of internationally active firms described in FHZ’s Lemma 2. They set \( p^P = 1/\lambda^P \) in country \( P \), \( p^R = \tau p^P \) in country \( R \) and sell their product to all households in both countries.

\(^{30}\)For the calculation of expected trade costs \( \phi(\tau_l, \tau_u) \) see appendix A.5.
The free entry condition, consisting of the four groups profits in the rich country, is

\[ v^e = F(\tau^R_l) \Pi^R_D + \int_{\tau^R_l}^{p_R/p^P} \Pi^R_I(\tau) f(\tau) d\tau + \int_{p_R/p^P}^{\tau^R_u} \Pi^R_P(\tau) f(\tau) d\tau + \left( 1 - F(\tau^R_u) \right) \Pi^R_D = 0. \]  

(13)

and for the poor country

\[ v^e = \int_{1}^{\tau^P_l} \Pi^P_E(\tau) + \int_{\tau^P_l}^{p_R/p^P} \Pi^P_I(\tau) f(\tau) d\tau + \int_{p_R/p^P}^{\tau^P_u} \Pi^P_P(\tau) f(\tau) d\tau + \left( 1 - F(\tau^P_u) \right) \Pi^P_D = 0. \]  

(14)

The arbitrage cutoff level for the rich country, pinned down by setting marginal firms’ profits equal, is

\[ \Pi^R_I(\tau^R_l) = \Pi^R_D \]  

(15)

and for the poor country

\[ \Pi^P_I(\tau^P_l) = \Pi^P_D(\tau^P_l), \]  

(16)

where \( \Pi^P_E(\tau^P_l) \) stands for the profit of a poor-country firms exclusively exporting to the rich country, evaluated at the poor-country arbitrage cutoff. At the rich-country arbitrage cutoff firms must be indifferent between selling exclusively at home and being internationally active. At the poor-country arbitrage cutoff firms must be indifferent between being internationally active and exclusive exporting to the rich North.

Furthermore, the trade costs cutoff level for the rich country is defined by

\[ \Pi^R_I(\tau^R_u) = \Pi^R_D \]  

(17)

and analogous for the poor country,\(^{31}\)

\[ \Pi^P_I(\tau^P_u) = \Pi^P_D. \]  

(18)

Solving equation (17) for the cutoff trade costs yields \( \tau^R_u = p^P a \). When, the rich-county wage and technology parameters are all set to unity, the rich-country trade costs cutoff level is simply equal to the poor-country price. This implies that the marginal costs from exporting are equal to marginal revenue from exporting at the cutoff. The same principle applies to the poor country, where \( \tau^P_u = p^R a \frac{a}{W^r}. \)

\(^{31}\)For written out forms of all profits see appendix A.5.
The resource constraint for the rich country is

\[ L^R P^R = N^R \left( F + \frac{P^R}{a} + \frac{\phi(\tau_{1l}^R, \tau_{1u}^R) P^P}{a} \right) \] (19)

and for the poor country

\[ L^P P^P = N^P \left( F + (1 - F(\tau_{1l}^P)) \frac{P^P}{a} + \frac{\phi(1, \tau_{1u}^P) P^R}{a} \right). \] (20)

Finally, the trade balance condition in this regime, which sets the value of poor-country exports equal to the value of rich-country exports is

\[ \left( F(\tau_{1u}^P) - F(\frac{P^R}{P^P}) \right) N^P p^R P^R + N^P \phi(\tau_{1l}^P, \frac{P^R}{P^P}) p^P P^R + F(\tau_{1l}^P) N^P p^R P^R = \left( F(\tau_{1u}^R) - F(\tau_{1l}^R) \right) N^R P^P P^P. \] (21)

The first term on the left hand side represents the fraction of poor-country products exported by firms with non-binding arbitrage constraint. The second term denotes the fraction of the poor-country products exported by firms with binding arbitrage constraint. The third term represents the share of poor-country products exclusively exported to the rich country. On the right hand side stands the fraction of rich-country products exported to the poor country.

By solving this system of non-linear equations numerically, values for prices, product varieties, wages and cutoff levels in both countries can be obtained. A couple of requirements have to be met for a Pareto trade costs/arbitrage equilibrium to exist. The per capita labor endowment gap between countries needs to be large enough to generate sufficient price differences, so that arbitrage cutoff levels in both countries cause firms to eliminate arbitrage opportunities. To ensure \( \tau_{1l}^R > 1 \), it is required that \( p^R > 2p^P - 1 \). To ensure \( \tau_{1l}^P > 1 \), it is required that \( p^R > 2p^P - W^P \), which is the more restrictive condition since \( W^P < 1 \), whenever countries differ in labor endowment. Furthermore, the technology parameter \( a \) may not be too low. Otherwise, no poor-country firm finds it attractive to exclusively export. The Pareto distribution’s shape parameter \( \alpha \) needs to be sufficiently high in order to have \( \tau_{1l}^P > 1 \).\(^{32}\) Finally, fixed costs may not be too high because otherwise profitable production is impossible.

Notice that a household’s budget constraint is not used here since it only contains redundant information, which is already implied by the free entry condition. Households receive income from providing labor and spend it entirely on consumption, which is the aggregate demand firms face. Therefore, the aggregate revenue of firms is equal to the aggregate labor income, which corresponds to aggregate production costs. Thus, firms make zero profits on average.

For some firms to make positive profits, their revenue must exceed their labor costs. To generate the demand to allow this profits, there must be firms where labor costs exceed revenue. The net losses of

\(^{32}\)The reasoning behind this is explained in section 5.3.
this unprofitable firms are financed by the net profits of the profitable group. I use the same ownership structure as Chaney (2005), who extends the model of Melitz (2003). At the time they entered, firms got their fixed costs investments financed out of a mutual fund, whose shares are equally owned by all workers of a country. This mutual fund contains all the firms in an economy. In equilibrium, these funds - one for each country - absorb all profits and losses. Thus, the mutual fund breaks even, so that workers do not have additional income from dividends and the free entry condition holds.33

4.3 Other equilibrium regimes

If price differences between the two countries are not large enough to cause arbitrage cutoffs, the equilibrium conditions (15) and (16) will not hold with equality. In this case, solving the model numerically yields arbitrage cutoff levels that lie below one, which is not possible. The respective arbitrage cutoff condition must then be dropped and $\tau_i^l$ set equal to 1, in order to guarantee correct calculation of the other conditions. In this case, trade costs are the only reason for regional exclusion. The economies are then in a Pareto trade costs equilibrium similar to the one presented for identical countries in section 3.2. The important difference is that firms with low trade costs are still confronted with a binding arbitrage constraint. However, the negative effect of deteriorated prices (due to arbitrage) on profits is smaller than the positive effect of increased market size. Thus, low trade costs producers choose to serve the international market.

When price differences between countries are only intermediate, it is possible that low trade costs, rich-country firms do not serve the poor market for arbitrage reasons, while all poor-country firms below the trade costs cutoff offer their products internationally. Thus, rich-country firms exclude the poor region from their goods for both the trade costs and the arbitrage reason, while poor-county firms only exclude because of the former. The rich-country arbitrage cutoff still exists for lower price differences than the arbitrage cutoff in the poor country. Remember that the poor country always has a lower arbitrage cutoff level since compared to the rich-country firms who stay domestic, poor-country firms who exclusively export already carry trade costs. Given the same decay in rich-country price from start selling internationally, it soon becomes attractive for poor-country firms to also sell at home, when trade costs increase. Rich-country firms, however, prefer to stay exclusively at home longer since exporting newly introduces trade costs.

A pure Pareto trade costs equilibrium with non-traded goods, where prohibitively high trade costs are the only reason for regional exclusion, can exist as long as prices in the two countries are similar. However, a pure arbitrage equilibrium, where the only reason of exclusion is arbitrage threat, can never exist with Pareto distributed trade costs for the same reason as full trade equilibrium is not possible. The highest trade costs a firm may have under a Pareto distribution are infinite.

33 This argumentation is more intuitive in a dynamic model as Chaney (2005) presents it. In his model, some incumbent firms randomly die and the new entrants’ fixed costs are financed by the accumulated profits of incumbents in the workers’ mutual fund.
4.4 Asymmetric population size

Until now I assumed the population size of the two countries to be identical. In contrast to using CES preferences, population and labor endowment have separate roles in a non-homothetic preferences framework. While an increase in labor endowment per household means higher income per capita and therefore a larger variety of products produced and consumed, higher population mainly results in the existing products being produced and consumed in higher number. Due to economies of scale, an increase in population size also increases the number of product varieties, but the effect is weaker than that of a corresponding increase in labor endowment.

Allowing market sizes to be different, has important effects on the prevailing equilibrium regime. A Pareto trade costs/arbitrage equilibrium is more likely to exist, when the rich market is relatively larger. To see this, assume that the poor county has a relatively higher population. It is then less attractive for poor-country firms to exclusively export to the rich country because of the large loss of market size this strategy implies. At some point the poor county’s population advantage is so large, that the market size effect dominates the price effect. Also for the rich-country firms regional exclusion due to arbitrage threat is more likely to happen when the home market is relatively larger.

5 Results

The main results of this paper are:

a. There are two possible reasons for regional exclusion of certain product varieties - prohibitively high trade costs and threat of arbitrage.

b. Trade between a rich and a poor country is only happening, when trade costs lie within a certain range - i.e. high enough to prevent losses due to arbitrage but sufficiently low to still let exporting be profitable.

c. Given a sufficiently large per capita income gap between trading partners, some poor-country firms find it optimal to exclusively export their products to the rich county but not to offer them at home. This strategy is even more attractive if the rich country market is relatively larger.

d. While rich countries always benefit from a trade liberalization, poor countries may benefit or lose. This result is derived in section 5.3.

e. The low trade interaction between poor countries may be caused by prohibitively high trade costs. The reasoning behind this is provided in section 5.4.

f. In a Pareto trade costs/arbitrage equilibrium, poor-county producers, who exclusively export to the rich North, earn higher profits than rich-country producers, who exclusively sell at home.

g. Poor-county firms, who prefer to stay domestic due to prohibitively high trade costs, suffer higher losses than rich-county firms following the same strategy.

The last two results are explained in more detail in the next section.
5.1 Firm’s profits

A comparison of a rich- and a poor-country firm’s profit as a function of trade costs in the trade costs/arbitrage regime is illustrated in figure 2. At first sight, it is surprising that poor-country firms earn distinctively higher profits than rich-country firms for all trade costs below the rich countries trade costs cutoff level. This is caused by the lower wages that poor-country firms have to pay their workers. However, when the trade costs drawn are large, poor-country firms make huge losses because of their low domestic price level. Rich-country firms can limit their losses to a significantly lower level, thanks to their high domestic price. This change of ranking in productivity allows expected profits for new entrants to be zero in both countries, which is necessary to maintain a stationary equilibrium.

As for the Pareto trade costs equilibrium for identical countries, fixed export entry costs are feasible but not necessary for a Pareto trade costs/arbitrage equilibrium. No matter the type and combination of fix costs used, exclusively domestically active firms, which is the group with lowest profits in both countries, have to bear a net loss out of running production. However, this net loss is still smaller than the total fixed costs they invested. Producing allows them to amortize this investment partially, so that shutting down is not reasonable for any firm.

As predicted, the trade costs cutoff level is distinctively higher in the poor country. First because the rich-country price is relatively higher and second because the poor-country wage is relatively lower. Both properties cause exporting to be worthwhile until a higher level of trade costs.

Relatively lower wages in the poor country enhance the profitability of exclusive exporting to the rich North. Due to their labor costs advantage, southern firms with trade costs close to 1 are more profitable in serving the rich-country market than local rich-country producers who stay domestic. This is reminiscent of the familiar argument to outsource labor intensive production for the rich markets to low wage countries in a globalized world economy. Exclusive production for the rich market ceases to be the best strategy if a poor-country firm’s good has trade costs above the poor-county arbitrage cutoff $\tau^P_l$. It is then better to increase the market size by also offering the product at home and accept selling for a lower, parallel-import-preventing price in the North. The arbitrage cutoff of rich-country producers $\tau^R_l$, where they switch from exclusive home selling to international activity, lies at a higher trade costs level than for the poor country. This is because for the rich-country firms higher trade costs make it less attractive to produce for the world market from a costs perspective, while for the poor-country producers, deciding whether to also sell at home, this is not the case. Thus, the poor-country firms have a cost advantage in serving the poor country market, while the revenue effects are identical for all firms irrespective of their location.

In both countries, the most profitable firms are those having trade costs equal to relative prices. At this point, where the arbitrage constraint stops binding because the rich country’s price ceiling is reached, firms realize the highest mark-ups. Internationally active rich-country producers can set their home prices equal to the prices of non-traded goods in the rich country and equal to poor consumers’ willingness to pay abroad, without suffering losses from arbitrage-exploiting reflows. Internationally

\[34\text{Even beyond } \tau^R_u \text{, poor-country firms earn higher profits. The intersection after which rich-country firms are more profitable than poor-country firms lies somewhere between } \tau^R_u \text{ and } \tau^P_u.\]
active poor-country producers can do pricing-to-market at home and sell for the rich country’s domestic price abroad, while still completely passing through trade costs and not suffering losses from parallel imports. Trade costs above relative prices have no pricing benefit anymore but only mean higher costs, so profits decrease until the trade costs cutoff is reached and firms prefer to stay domestic.

This is a Nash equilibrium since given the other firms’ actions no firm has an incentive to deviate from its strategy.

Figure 2: Rich- and poor-country firm’s profit as function of trade costs under the trade costs/arbitrage regime

Notes: Poor-country firms earn higher profits than rich-country firms for low trade costs. This is caused by lower wages. Poor-country firms with high trade costs make huge losses because of their low domestic price level. Rich-country firms can limit their losses to a significantly lower level, thanks to a high domestic price. The trade costs cutoff level \( \tau_u \) is distinctively higher in the poor country. First because the rich-country price is relatively higher and second because the poor-country wage is relatively lower. The arbitrage cutoff of rich-country producers \( \tau^R_l \) lies at a higher trade costs level than for the poor country. This is because for the rich-country firms higher trade costs make it less attractive to produce for the world market from a costs perspective, while for the poor-country producers, deciding whether to also sell at home, this is not the case. Thus, the poor-country firms have a cost advantage in serving the poor country market, while the revenue effects are identical for all firms irrespective of their location. In both countries, the most profitable firms are those having trade costs equal to relative prices. Trade costs above relative prices have no pricing benefit anymore but only mean higher costs, so profits decrease until firms prefer to stay domestic. Shown in this figure is the setting, where the rich country is endowed with three-times as much labor as the poor country, the Pareto distribution’s shape parameter is \( \alpha = 3 \) and all other input parameters are equal to 1.
The Pareto trade costs/arbitrage equilibrium is the most interesting and complete version of my model. I will proceed by analyzing the sensitivity of prices, product varieties, wages and cutoffs with respect to the exogenous parameters labor endowment and trade costs. It turned out to be more convenient and insightful to use a Pareto distribution rather than a Bernoulli distribution because continuously distributed trade costs endogenize the cutoff levels, on which lies the main focus of this discussion.

5.2 Comparative statics in labor endowment

The resulting equilibrium mainly depends on the relative labor endowment gap between the two countries. A certain level of inequality is required to construct the most interesting equilibrium regime in which all four cutoff levels occur. Figure 3 shows how the cutoffs develop when the labor endowment gap between the two countries decreases.\footnote{For the sake of better illustration, a shape parameter of $\alpha = 3$ is used for all figures in this section. The qualitative results remain the same as with the more realistic value $\alpha = 7$.} When $L^P$ is increased towards one, while $L^R$ is held constant at one,

- the trade costs cutoff in the rich country $\tau^R_u$ increases weakly as long as the arbitrage cutoff in the rich country exists and then increases more strongly. This can be explained by the behavior of prices. As long as the rich country’s arbitrage cutoff constraint (15) is still binding, the poor market becomes only moderately more attractive due to the moderate poor-country price increase and non-traded, rich-country products’ price decrease. The development of prices is shown in figure 4. As soon as the rich country’s arbitrage cutoff is not binding anymore, the growth rates of the poor-country price increase and rich-country price decrease rises and so does attractiveness for rich-country firms to go international.

- the trade costs cutoff in the poor country $\tau^P_u$ decreases weakly as long as the arbitrage cutoff in the rich country exists and then decreases concavely. This works analogously to the argumentation for the rich country. As long as the rich country’s arbitrage cutoff is still binding, the rich market becomes only moderately less attractive due to the moderate poor-country price increase and rich-country price decrease. As soon as the rich country’s arbitrage cutoff reaches unity, the growth rates of the poor-country price increase and rich-country price decrease rises and so does attractiveness for poor-country firms to stay domestic. When prices rise at home and fall abroad, poor-country firms, which chose to export before the change in prices despite of relatively high trade costs, now prefer to stay at home.

- the arbitrage cutoff in the rich country $\tau^R_l$ only exists above a certain amount of inequality and decreases concavely. Sufficient inequality in labor endowment is necessary to create price differences, which cause a sufficient threat of arbitrage for rich-country firms, so that they prefer to stay domestic. The fraction of rich-county producers, who stay domestic due to arbitrage reasons, decreases as the price differences become smaller.

- the arbitrage cutoff in the poor country $\tau^P_l$ only exists above a even higher amount of inequality and also decreases concavely. Even higher inequality in labor endowment is necessary to create...
price differences large enough to cause a threat of arbitrage for poor-country firms, so that they prefer to exclusively export to the rich country despite of their trade costs disadvantage relative to rich-country producers.

The number of total varieties produced in the poor country strongly increases in poor-country labor endowment, as illustrated in figure 6. \( N_i^T \) denotes the number of goods produced in county \( i \), which are traded internationally. \( N_i^D \) analogously denotes the non-traded goods of county \( i \), which are offered exclusively domestic. The poor country goods, which are exclusively exported to the North are represented by \( N_i^P \).

Higher labor endowment means more resources available for production. Due to the Pareto probability distribution, most resources are used to produce internationally traded products but also the number of non-traded products increases. Figure 4 shows that wages in the poor county catch up to the rich-country level, as the labor endowment differences decrease. Higher wages partly absorb the higher profits earned by new, internationally active poor-country firms. As seen in figure 5, the number of rich-country internationally traded (exclusive domestic) products increases (decreases), as the poor market becomes more attractive.

Note that the total number of varieties produced in the rich country decreases. The reason for this is the rich country’s resource constraint. The higher rich-country trade costs cutoff, resulting from increased prices in the poor country, causes more rich-country firms to export their goods, which uses up more resources to pay trade costs. The higher aggregate trade costs reduce the total number of varieties produced in the rich country. Altogether however, poor country’s caching up is beneficial to the welfare of both countries, as shown in figure 7. In the rich country, the increasing number of imported poor-country varieties overcompensates the effect of the reduction in home produced varieties. For the poor-country consumers welfare implications are unambiguous since they produce more varieties on their own, exclusive exports decrease and also the rich country is providing them with more imports. As the poor-country labor endowment reaches one, the two countries are perfectly identical and all equilibrium variables are equal. Equally endowed countries correspond to the highest total level of welfare achievable.\(^{36}\)

The equilibrium is scalable in product varieties by labor endowment. A multiplication of both countries endowments by a certain factor, scales product varieties by the same factor. Cutoffs, prices and wages only depend on the relative endowment gap \( L_P^T / L^R \) but not on absolute endowment and therefore remain unchanged, when resources increase symmetrically.

\(^{36}\)This can be shown by using mean-preservation of labor endowment in the model presented. Hence, only the allocation of labor endowment between the two countries changes as the poor country catches up but the total sum of resources remains constant. The development of total welfare depending on mean-preserving resource allocation is illustrated in figure 14 in the appendix.
Notes: Area (1) contains poor-country firms that choose to stay domestic due to too high trade costs. Areas (1) + (2) contain rich-country firms that choose to stay domestic due to too high trade costs. Area (3) contains rich-country firms that are internationally active. Areas (2) + (3) + (4) contain poor-country firms that are internationally active. Areas (4) + (5) contain rich-country firms that stay domestic because of arbitrage reasons. Area (5) contains poor-country firms that exclusively export to the rich North because of arbitrage reasons. The Pareto distribution’s shape parameter used is $\alpha = 3$. All other exogenous parameters are equal to 1.
Figure 4: Development of prices and wages as labor endowment gap decreases

Notes: Wages in the poor county catch up to the rich-country level, as the labor endowment difference decreases. The rich-country (poor-country) price decreases (increases), when labor endowment becomes more similar. The Pareto distribution’s shape parameter used is $\alpha = 3$. All other exogenous parameters are equal to 1.
Figure 5: Development of total, internationally traded and non-traded rich-country product varieties as labor endowment gap decreases.

Notes: The number of rich-country internationally traded (exclusive domestic) products increases (decreases), as the poor market becomes more attractive. Note that the total number of varieties produced in the rich country decreases. The Pareto distribution’s shape parameter used is $\alpha = 3$. All other exogenous parameters are equal to 1.
Figure 6: Development of total, internationally traded, non-traded and exclusively exported poor-country product varieties as labor endowment gap decreases

Notes: The total number of varieties produced in the poor country strongly increases in poor-country labor endowment and at some point poor-country firms cease to exclusively export to the rich North. The Pareto distribution’s shape parameter used is $\alpha = 3$. All other exogenous parameters are equal to 1.
Figure 7: Development of welfare as labor endowment gap decreases

Notes: The caching up of the poor county is beneficial to the welfare of both countries. Rich-country welfare is given by the product variety produced at home $N_R$ plus the imported products $N_P$. Poor-country welfare is given by the product variety produced at home $N_P$ minus the exclusively exported products $N_P^E$ plus the imported products $N_R^I$. The Pareto distribution’s shape parameter used is $\alpha = 3$. All other exogenous parameters are equal to 1.
5.3 Comparative statics in trade costs parameters

A Pareto Type 1 distribution is determined by a scale parameter and a shape parameter. Varying the shape parameter $\alpha$ changes the probabilities assigned to a certain trade costs level by the density function. As $\alpha$ increases, more weight is put on low trade costs and the expected value of trade costs decreases. Hence, a low $\alpha$ goes along with a high dispersion of trade costs across products. An $\alpha > 1$ is required for a finite mean, such that the expression for expected trade costs is defined. Varying the scale parameter makes limited sense since starting the distribution at a minimum possible value of one seems very natural. Ad-valorem trade costs of below one are impossible, as this would imply negative trade costs. Setting the minimum possible value significantly above one is also not reasonable since as observed in the data some products do have very low trade costs. The Pareto Type 1 distribution with a scale parameter of one and different values of $\alpha$ is illustrated in figure 15 in the appendix.

Consider an increase in $\alpha$, which means a decrease in trade cost heterogeneity, due to an innovation in transportation technology. The global average trade costs decrease because there are now relatively more firms with low trade costs products. Shown in all figures in this section is the setting, where the rich country is endowed with three-times as much labor as the poor country and all other input parameters are equal to 1.

In figure 8 the cutoffs are plotted against $\alpha$. The increase of $\alpha$ has a strong effect on the poor country’s trade costs cutoff level. The few poor-country firms who still have relatively high trade costs prefer to stop exporting earlier since poor-country wages clearly increased and the rich-country price slightly declined. The poor-country wages increased to absorb the higher aggregate profitability of the poor-country firms due to the higher mass of firms having trade costs in the profitable section. In the rich country this role is taken by the domestic price since rich-country wages are held constant. The change required is much less pronounced, as high additional profits are prevented in the rich county because some rich-country firms, who used to be be internationally active, stay exclusively domestic due to arbitrage reasons after their trade costs decreased. Also in the poor country the price is falling for low initial values of $\alpha$, which helps limiting profits and preventing new entry. The rich-country trade costs cutoff behaves identical to the poor country price as this is its only determinant. The connection is intuitive. A marginally lower poor country prices implies that rich firms at the margin choose to not export anymore and stay domestic instead.

The rich-country arbitrage cutoff kicks in only at a certain level of $\alpha$. It requires that poor-country prices decreased and poor-country wages increased sufficiently to make exclusive exporting to the rich country attractive. This strategy allows charging the high rich-country price and saving the labor expenses out of producing for the low paying poor market. The course of the rich-country arbitrage cutoff level is determined by an interaction between rich- and poor-country prices. When $p^R$ increases relatively to the poor country price, $\tau_l^R$ increases as more rich-country firms prefer to stay exclusively domestic and vice versa.

Figure 10 shows the development of product varieties. In line with intuition, at least for low

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37 Recall from section 5.1, that poor-country firms make strictly positive profits for relatively low trade costs.

38 The variation in price is small and barely visible in figure 9.
initial values of $\alpha$, the number of internationally traded (excursive domestic) products clearly increase (decrease) in both countries as expected trade costs decline. However, for low initial levels of $\alpha$, the total number of rich and poor varieties produced decreases. This is caused by the change of shape of the Pareto distribution. Even though, the unconditional expected value of total trade costs decreases, aggregate trade costs conditional on exporting actually increase, since there are now more firms exporting. In other words, the area under the probability density function that lies below the trade costs cutoff increases.\(^{39}\) Paying this higher aggregate trade costs conditional on exporting uses resources, so that the total number of varieties produced decreases. This effect reverses when trade costs decline further. Firms then save so much labor, thanks to lower trade costs, that the aggregate number of products produced in each country increases again. Confronted with lower trade costs, firms have to produce less output for the same quantity of goods consumed. Resulting spare resources can be used to establish new firms, offering new product varieties.

To complete the analysis of this regime let us consider some welfare arguments. Until a certain degree of trade costs decline, not only the relative fraction but also the absolute number of internationally active firms rises.\(^{40}\) This leads to an improvement in welfare for both countries, as can be seen in figure 12. For low initial values of $\alpha$, the increased number of imported products available overcompensates the decline in home produced varieties.

Interesting is the monotonous increase in poor-country products, which are exclusively exported to the rich market. Lower trade costs make this strategy more attractive and at some point, this becomes harmful to the poor country’s welfare. Tightening the arbitrage constraint increases the number of firms, who exclude the poor market. When a large mass of poor-county firms has trade costs below the arbitrage cutoff level, so that they all prefer to only serve the rich market, the number of products available in the poor country declines. Rich-country welfare strictly improves, when trade costs decline, because they benefit from strictly increasing poor-country exports, which more than compensates the occasional decline in home produced varieties. Thus, while poor-country consumers might lose from lowering trade barriers, rich-country consumers always gain. Rich-country consumers gain from lower trade costs directly through lower prices for internationally traded products, which takes pressure off their budget. Thus, they can afford purchasing the higher product variety, which arose from the spare resources. Poor-country consumers also benefit, as they can purchase new varieties thanks to their increased wages, unless the decline in trade costs is so strong that they are not served anymore.

This is in line with what Foellmi, Hepenstrick and Zweimüller (2013) find for their arbitrage equilibrium in a model with homogenous trade costs. It can be argued that to some extent high trade costs are beneficial to a poor country, since it loosens the arbitrage constraint and therefore increases the available product variety in the poor country.\(^{41}\)

However, there is another trade-off at work here. For a poor country, generally higher trade costs also mean less product varieties available through import from other poor countries. As a consequence, poor countries may find it a good idea to only asymmetrically increase trade barriers, for example

\(^{39}\)Notice that trade costs for rich-country firms below the arbitrage cutoff do not arise since these firms do not export.

\(^{40}\)The development of the share of firms following the different market strategies is plotted in figure 16 in the appendix.

\(^{41}\)FHZ’s Proposition 4 states that in an arbitrage equilibrium a trade liberalization increases the welfare of rich-country households but decreases it for poor-country households.
through establishment of export taxes, in order to relax the arbitrage constraint of domestic and rich-country firms. This is in strong contrast to the classical Krugman (1980) model, where reducing trade barriers symmetrically has a beneficial effect on both countries welfare, since it makes goods relatively cheaper and therefore increases the intensive margin of consumption.

Figure 8: Development of cutoffs as trade costs heterogeneity decreases

Notes: Area (1) contains poor-county firms that choose to stay domestic due to too high trade costs. Areas (1) + (2) contain rich-country firms that choose to stay domestic due to too high trade costs. Area (3) contains rich-country firms that are internationally active. Areas (2) + (3) + (4) contain poor-country firms that are internationally active. Areas (4) + (5) contain rich-country firms that stay domestic because of arbitrage reasons. Area (5) contains poor-country firms that exclusively export to the rich North because of arbitrage reasons. Shown in this figure is the setting, where the rich country is endowed with three-times as much labor as the poor country and all other input parameters are equal to 1.
Figure 9: Development of prices and wages as trade costs heterogeneity decreases

Notes: Wages in the poor county catch up to the rich-country level, as the shape parameter of the Pareto distribution $\alpha$ increases ($\hat{\alpha}$ = trade costs heterogeneity decreases). Prices only move marginally. Shown in this figure is the setting, where the rich country is endowed with three-times as much labor as the poor country and all other input parameters are equal to 1.
Figure 10: Development of total, internationally traded and non-traded rich-country product varieties as trade costs heterogeneity decreases.

Notes: The number of rich-country internationally traded (exclusively domestic) products increases (decreases), as heterogeneity in trade costs decreases ($\alpha$ increases). Note that the total number of varieties produced in the rich country first decreases and then increases again. The number of rich-country exclusively domestic products is higher than internationally traded products because the rich country is endowed with three-times as much labor as the poor country, which makes the home market more attractive. All other input parameters are equal to 1.
Figure 11: Development of total, internationally traded, non-traded and exclusively exported poor-country product varieties as trade costs heterogeneity decreases.

Notes: The number of poor-country internationally traded products first increases and then decreases again, as heterogeneity in trade costs declines ($\bar{\alpha}$ increases). The number of non-traded products monotonously decreases and the number of exclusively exported products monotonously increases. Note that the total number of varieties produced in the poor country first decreases and then increases again. Shown in this figure is the setting, where the rich country is endowed with three-times as much labor as the poor country and all other input parameters are equal to 1.
Notes: Rich-country welfare strictly improves, when heterogeneity in trade costs declines (i.e. $\alpha$ increases), because they benefit from strictly increasing poor-country exports, which more than compensates the occasional decline in home produced varieties. Lower trade costs make exclusive exporting to the rich North more attractive for poor-country firms and at some point, this becomes harmful to the poor country’s welfare. Thus, poor-country welfare first increases but then flattens and at high values of $\alpha$ even decreases again. Shown in this figure is the setting, where the rich country is endowed with three-times as much labor as the poor country and all other input parameters are equal to 1.
5.4 South-South trade

My model can also be used to argue why South-South trade interaction is typically low. Looking at poor-country produced goods, the high price chargeable in the North enables profitable export of goods even with fairly high trade costs. Take the southern exporting of a large variety of tropical fruits to the North as an example. The local southern population also consumes many of these fruits but some varieties are often reserved for exclusive exporting to the North. Poor counties do typically not trade their goods - such as exotic fruits - among each other. Coulibaly and Fontagné (2005) discuss the low trade interaction between Sub-Saharan African countries. They argue that the reason for the modest South-South trade are very high trade costs due to being landlocked. This is consistent with the predictions of my model. For those poor-county varieties, that are also available domestically, arbitrage threats cannot be the reason for exclusion of other poor markets. Observing the home market being served but other poor countries not, suggests that the chargeable price in these countries would fall short of total costs of offering the product there. The high domestic price in the rich countries allow passing through trade costs to the consumers and hence, profitable exporting even with high trade costs. At some point, high trade costs prevent exports even to the richest countries.

It would be interesting to analyze real world data, such as U.S. Imports of Merchandise data from the U.S. Census Bureau or data from UN Comtrade and UNCTAD Trains, to identify typical high and low trade costs products and test the model predictions empirically.\footnote{Captured in this data are the two main components of trade costs: tariffs and freight costs. Other aspects of trade costs such as non-tariff and informational barriers are much more difficult to observe empirically. An attempt to quantify informational barriers in international trade is presented in Portes and Rey (2005). They use the volume of bilateral phone communications and the number of affiliates of domestic banks in the destination country as proxy measures. Rauch and Trindade (2002) study the impact of ethnic networks on international trade flows. They argue that networks reduce searching costs by helping to match sellers and buyers.}

6 Conclusions

This paper presents a model which features two reasons for regional exclusion of certain product varieties. The first is trade costs, which can make exporting unprofitable. The second is the threat of arbitrage, which can make it unattractive to serve countries with low per capita income. This implies that in order for consumers worldwide to be served with a product, its trade costs must lie within a certain range, namely low enough to allow profitable trading but sufficiently high to prevent loss-causing arbitrage. This two-country model is able to generate an endogenous arbitrage cutoff for both countries. Thus, there are firms producing in the poor country, who find it optimal to exclusively serve the rich market. A sufficiently large per capita income gap and a relatively large rich-country market in terms of population are the main requirements for such an equilibrium to exist.

The focus of this paper lies on the modeling of product specific trade costs and how they affect the patterns of trade. I discuss two different probability distributions to assign trade costs to products, a Bernoulli and a Pareto distribution. The latter turned out to be more convenient as it allows to endogenously determine cutoff levels. Non-homothetic preferences and consumption indivisibilities are used, which shifts the focus on the exogenous margin of trade. Incorporation asymmetric labor
endowment and population size, allows to generate various different equilibrium regimes. I analyzed the Pareto trade costs/arbitrage equilibrium in greater detail because as it allows to explain regional exclusion most comprehensive. Comparative statics in the exogenous parameters labor endowment and trade costs help to understand the effects on product varieties, prices, wages and especially cutoffs.

Interesting are the profit patterns in the Pareto trade costs/arbitrage equilibrium. Poor-county producers, who exclusively export to the rich market, earn higher profits than rich-country producers, who exclusively serve their home market. This is possible because the lower poor-country equilibrium wage overcompensates the trade costs disadvantage that these firms face. However, poor-county firms, who prefer to stay domestic due to prohibitively high trade costs, suffer higher losses than rich-county firms following the same strategy.

As discussed in Foellmi, Hepenstrick and Zweimüller (2013), I find that the welfare effects of a trade liberalization are ambiguous for a poor country. Constellations, where poor consumers are harmed by a decrease in trade costs, exist. The responsible mechanism is that lower trade costs tighten the arbitrage constraint, which causes more firms to exclude the poor market. This deteriorates a poor-country consumer’s welfare, which is simply given my the number of varieties he can consume. In contrast, the rich-country population always benefits from trade liberalizations. They benefit from the lower prices of all internationally available traded products, which allow them to purchase the new varieties produced from the spare resources.

Furthermore, my model suggests that the low trade interaction between poor countries may be caused by prohibitively high trade costs.

What is not captured in my model but would be interesting to study in the field of international trade are quality differentiation of the products, within-country inequality and cost-structure heterogeneity between firms. Heterogeneity in firms’ productivity and its effect on aggregate productivity when opening to trade is analyzed in Melitz (2003). Chaney (2005) extends this framework and studies transitional dynamics of opening to trade. Besides of trade costs and arbitrage arguments, quality differentiation is particularly relevant in explaining regional exclusion. Products that satisfy the same need in different qualities allow to moderate the threat of arbitrage by producing a customized product for each income group.
References


International trade in a model with heterogeneous trade costs, parallel imports and non-homothetic preferences


Appendix

A Calculations and Formulas

A.1 Full trade equilibrium

With profits plugged into equation (4) the free entry condition is

\[ x[p^i P^i + p^j P^j - W^i(F + C + \frac{P^i}{a} + \tau_L P^j)] + (1 - x)[p^j P^i + p^i P^j - W^i(F + C + \frac{P^i}{a} + \frac{\tau_H P^j}{a})] = 0. \]

A.2 Bernoulli trade costs equilibrium with non-traded goods

This section analyzes Bernoulli trade costs regimes with non-traded goods. There is a large number of possible constellations under a Bernoulli trade costs distribution, depending on the prevailing levels of trade costs.

One possible outcome is an equilibrium, in which all high trade costs firms do not export. If high trade costs exceed a certain threshold level, exporting such goods to the other country is no longer profitable. Firms which have low trade costs prefer to be internationally active. They are large and benefit from the increasing returns to scale. Those firms that drew high trade costs products choose to sell exclusively domestic. They are small and have higher average production costs.

Another possible equilibrium regime results if “low” trade costs \( \tau_L \) surpass the critical threshold level as well. Then, no trade will take place anymore and each country is in its autarky equilibrium again. A “Bernoulli trade costs equilibrium with non-traded goods” for identical countries will not be written out formally since it is only a special case of the rich/poor country version discussed in appendix A.4.

A.3 Pareto trade costs equilibrium

The free entry condition (6) written out is

\[ v^e = \int_1^{\tau_u} \left( 2pP - W(F + \frac{P(1 + \tau)}{a}) \right) f(\tau) d\tau \\
+ (1 - F(\tau_u)) \left( pP - W(F + \frac{P}{a}) \right) - C = 0 \]
and integrated
\begin{align*}
v^e &= F(\tau_u) \left( 2pP - W(F + \frac{P}{a}) + \phi(\tau_u)P \right) \\
&\quad + (1 - F(\tau_u)) \left( pP - W(F + \frac{P}{a}) \right) - C \\
&= 0
\end{align*}

The term $F(\tau)$ denotes the cumulative Pareto distribution function, thus $F(\tau_u) = 1 - \left(\frac{1}{\tau_u}\right)^\alpha$ is the fraction of firms with trade costs below the cutoff trade costs. The total arising trade costs as a function of the cutoff trade costs are denoted by $\phi(\tau_u)$. It can be calculated as

\begin{align*}
\phi(\tau_u) &= E(\tau | \tau < \tau_u) Pr(\tau < \tau_u) = \int_1^{\tau_u} f(\tau) d\tau = \int_1^{\tau_u} \frac{\alpha}{\tau^{\alpha+1}} d\tau = \frac{\alpha}{1 - \alpha} (\tau_u^{(1 - \alpha)} - 1).
\end{align*}

Note that total arising trade costs are identical to expected trade costs conditional on exporting times exporting probability - hence unconditional expected trade costs.

**A.4 Bernoulli trade costs equilibrium with non-traded goods revised**

For partial trade equilibria with Bernoulli trade costs, I only consider regimes, where all rich-country firms with high trade costs prefer to not serve the poor South. There exist other equilibria where only a fraction of firms with equal trade costs choose the same sales strategy. When high trade costs surpass a certain threshold, exporting such goods from the rich to the poor county is no longer profitable since total costs exceed the achievable price, $pP < \tau_H a$. Full trade then ceases to be an equilibrium. Rich-country firms that have high trade costs, decide not to export and sell their product only on the home market. Export zeros are now caused by prohibitively high trade costs. One version of a Bernoulli trade costs equilibrium with non-traded goods is a setting, where poor county firms still stick to full trade. They sell all their products internationally because the higher price in the rich country is large enough to cover even the high trade costs. A second version could be a setting, where both countries do not export high trade costs products. Only the former version will be formally derived in the remaining of this section.

Resource constraints, free entry conditions and budget constraints for both countries as well as a trade balance condition are required to characterize the Bernoulli trade costs equilibrium with non-traded goods. The resource constraint for the rich country is

\begin{equation}
P^R L^R = N^R (F + C) + N^R \frac{P^R}{a} + x N^R L P^P.
\end{equation}

Compared to the full trade regime, the last summand disappeared since high trade costs products are not produced for export anymore. For the poor country however, resource constraint and produced product variety remain identical to the full trade expressions.
Solving for the number of active firms in the rich country yields

\[ N^R = \frac{aP^R L^R}{a(F + C) + PR + x\tau L PP}. \]

The number of internationally traded varieties in the rich country can be calculated as \( N^I_R = x N^R \), since a fraction \((1 - x)\) of all northern firms has drawn high trade costs and does not export.

The free entry condition is such that the expected profit from entering and selling internationally with low trade costs, respectively from entering and staying exclusively domestic with high trade costs, is zero before learning individual trade costs. Setting the expected profit in the rich county equal to zero yields

\[ x\Pi^R_I(\tau_L) + (1 - x)\Pi^R_D - F_e = 0 \]

and written out

\[ x[p^R P^R + p^P P^P - W^R(F + C + \frac{PR}{a} + \frac{\tau L PP}{a})] + (1 - x)[p^R P^R - W^R(F + C + \frac{PR}{a})] = 0. \]

The second square bracket must in equilibrium be larger than

\[ \Pi^R_I(\tau_H) = p^R P^R + p^P P^P - W^R(F + C + \frac{PR}{a} + \frac{\tau H PP}{a}), \]

which is the profit of a firm selling internationally with high trade costs. For the poor country the zero-profit condition remains the same as under full trade. Using these zero-profit conditions lets us express relative wages as

\[ \omega = \frac{W^P}{W^R} = \frac{p^R P^R + p^P P^P}{aP^P PP + aP^R PR} \frac{x\tau L PP + a(F + C) + PR}{\tau H - x(\tau_H - \tau_L) + PR + a(F + C) + PP}. \]

The budget constraint for a household in country \( R \) is \( W^R L^R = p^R(N^R + N^P) \) and \( W^P L^P = p^P(x N^R + N^P) \) in country \( P \). Plugging the expressions for the product varieties obtained from the resource constraints into the budget constraints enables solving for prices. Prices in country \( R \) can be expressed as

\[ p^R = W^R L^R \left( \frac{aL^R P^R}{a(F + C) + PR + x\tau L PP} + \frac{aL^P P^P}{a(F + C) + PP + [\tau H - x(\tau_H - \tau_L)]PR} \right)^{-1} \]

and in country \( P \) as

\[ p^P = W^P L^P \left( \frac{x aL^R P^R}{a(F + C) + PR + x\tau L PP} + \frac{aL^P P^P}{a(F + C) + PP + [\tau H - x(\tau_H - \tau_L)]PR} \right)^{-1}. \]
Finally, the trade balance condition has to hold. It is
\[ x_N N^R p_p P^P = N^P p_R P^R. \]

In general, the existence of export zeros under a Bernoulli distribution depends on whether abroad prices are high enough to cover domestic firms variable production costs and trade costs, arising from shipping the product to the other country. When considering a sufficiently small per capita income gap, no arbitrage trades are possible. Hence, prices are equal to the local willingness to pay and identical for imported and home-produced goods.

### A.5 Pareto trade costs/arbitrage equilibrium

The expected trade costs are calculated as

\[ \phi(\tau_l, \tau_u) = E(\tau|\tau_l < \tau < \tau_u) Pr(\tau_l < \tau < \tau_u) = \int_{\tau_l}^{\tau_u} \frac{\alpha}{\tau^{\alpha+1}} \tau d\tau = \frac{\alpha}{1-\alpha} (\tau_u^{1-\alpha} - \tau_l^{1-\alpha}). \]

Rich country’s free entry condition (13) written out is

\[
v^c = F(\tau_l^R) \left( p_R^R P^R - \left( F + \frac{P_R}{a} \right) \right) \\
+ \int_{\tau_l^R}^{\tau_u^R} \left( p_R^P P^R + p_P^P P^P - \left( F + \frac{P_R}{a} + \frac{\tau P^P}{a} \right) \right) f(\tau) d\tau \\
+ \int_{p_R^R}^{p_P^P} \left( p_R^R P^R + p_P^P P^P - \left( F + \frac{P_R}{a} + \frac{\tau P^P}{a} \right) \right) f(\tau) d\tau \\
+ (1 - F(\tau_u^R)) \left( p_R^R P^R - \left( F + \frac{P_R}{a} \right) \right)
= 0
\]

and integrated

\[
v^c = [1 - F(\tau_l^R) + F(\tau_l^R)] \left( p_R^R P^R - \left( F + \frac{P_R}{a} \right) \right) \\
+ [F(\tau_l^R) - F(\tau_l^R)] \left( p_R^P P^P - \left( F + \frac{P_R}{a} \right) \right) + \left( p_P^P P^P - \frac{P_R}{a} \right) \phi(\tau_l^R, p_R^P), \tau_l^R \\
+ [F(\tau_u^R) - F(\tau_u^R)] \left( p_R^R P^R + p_P^P P^P - \left( F + \frac{P_R}{a} \right) \right) - \frac{P_P}{a} \phi(p_R^P, \tau_u^R) \\
= 0
\]
Poor country’s free entry condition (14) written out is

\[
v^e = \int_1^{\tau_P^R} \left( p^R p^R - W^P \left( F + \frac{\tau^R p^R}{a} \right) \right) f(\tau) d\tau \\
+ \int_{\tau_P^R}^{\tau_P^P} \left( \tau p^P p^R + p^P p^P - W^P \left( F + \frac{p^P}{a} + \frac{\tau^R p^R}{a} \right) \right) f(\tau) d\tau \\
+ \int_{p^P/\tau_P^P}^{\tau_P^P} \left( p^R p^R + p^P p^P - W^P \left( F + \frac{p^P}{a} + \frac{\tau^R p^R}{a} \right) \right) f(\tau) d\tau \\
+ (1 - F(\tau_P^P)) \left( p^P p^P - W^P \left( F + \frac{p^P}{a} \right) \right) \\
= 0
\]

and integrated

\[
v^e = F(\tau_P^P) \left( p^R p^R - W^P F \right) - W^P p^R a \phi(1, \tau_P^P) \\
+ \left[ F(\tau_P^P) - F(\tau_P^P) \left( p^P p^P - W^P \left( F + \frac{p^P}{a} \right) \right) \right] + \left( p^P p^R - W^P \frac{\tau^R p^R}{a} \right) \phi(\tau_P^P, \frac{p^R}{\tau_P^P}) \\
+ \left[ F(\tau_P^P) - F(\tau_P^P) \left( p^P p^R + p^P p^P - W^P \left( F + \frac{p^P}{a} \right) \right) \right] - W^P p^R a \phi(\frac{p^R}{\tau_P^P}, \tau_P^P) \\
+ [1 - F(\tau_P^P)] \left( p^P p^P - W^P \left( F + \frac{p^P}{a} \right) \right) \\
= 0
\]

Rich country’s arbitrage cutoff condition (15) written out is

\[
\tau_R p^P p^R + p^P p^P - (F + \frac{p^R}{a} + \tau_R p^P a) = p^R p^R - (F + \frac{p^R}{a}).
\]

Poor country’s arbitrage cutoff condition (16) written out is

\[
\tau_P p^P p^R + p^P p^P - W^P (F + \frac{p^P}{a} + \tau_P p^R a) = p^R p^R - W^P (F + \frac{\tau_P p^R}{a}).
\]

Rich country’s trade costs cutoff condition (17) written out is

\[
p^R p^R + p^P p^P - (F + \frac{p^R}{a} + \tau_R p^P a) = p^R p^R - (F + \frac{p^R}{a}).
\]

Poor country’s trade costs cutoff condition (18) written out is

\[
p^R p^R + p^P p^P - W^P (F + \frac{p^P}{a} + \tau_R p^R a) = p^R p^P - W^P (F + \frac{p^P}{a}).
\]
B Tables

Table 1: Summary statistics for U.S. Imports of Merchandise trade costs

<table>
<thead>
<tr>
<th>Summary statistics</th>
<th>Trade costs</th>
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<tbody>
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<td>Mean</td>
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<tr>
<td>Std. Dev.</td>
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<td>Min</td>
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<tr>
<td>Max</td>
<td>62.334</td>
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<tr>
<td>Obs</td>
<td>15,827,720</td>
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<tr>
<td>Percentiles</td>
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<tr>
<td>P1</td>
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<td>P99</td>
<td>2.077</td>
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</table>

Notes: This data was purchased from the U.S. Census Bureau. The sample includes all reported imports into the USA from 2005-2014.
C Figures

Figure 13: Fit of Pareto Type 1 distribution on real world trade costs from U.S. Imports of Merchandise

Notes: A theoretical Type 1 Pareto distribution with shape parameter $\alpha = 7$ fits the trade costs observed for U.S. Imports of Merchandise 2005-2014 closely.
Figure 14: Development of welfare as poor country’s resource share $\beta$ increases (mean-preserving)

Notes: Mean-preservation holds the total available resources constant and only lets the relative allocation vary. The variable $\beta$ denotes the poor country’s share of total labor endowment $L$. Even though rich-country welfare decreases because of relatively lower resources allocated, the total welfare increases as the two countries become more similar. The Pareto distribution’s shape parameter used is $\alpha = 3$. All other exogenous parameters are equal to 1.
Figure 15: Pareto Type 1 distribution for different values of shape parameter $\alpha$

Notes: Varying the shape parameter $\alpha$ of a Pareto Type 1 distribution changes the probabilities assigned to a certain trade costs level by the density function. As $\alpha$ increases, more weight is put on low trade costs and the expected value of trade costs decreases. Hence, a low $\alpha$ goes along with a high dispersion of trade costs across products.
Figure 16: Development of the shares of firms' market strategies as trade costs heterogeneity decreases

Notes: The variable $\delta_i^s$ denotes the share of a market strategy $s$ on the total number of firms in a country $i$. The share of rich-country firms which choose to sell internationally (exclusively domestic) increases (decreases), as heterogeneity in trade costs declines ($\alpha$ increases). The share of internationally active poor-country firms first increases and then decreases again. The share of exclusively domestically active (exclusively exporting) poor-country producers monotonously decreases (increases). Shown in this figure is the setting, where the rich country is endowed with three-times as much labor as the poor country and all other input parameters are equal to 1.