The archive QuEST.zip contains the QuEST package.

QuEST is the acronym for "Quantized Eigenvalues Sampling Transform".

Purpose: estimate the covariance matrix and its eigenvalues in high dimension

Reference: "Spectrum Estimation: A Unified Framework for Covariance Matrix Estimation and PCA in Large Dimensions" by Olivier Ledoit and Michael Wolf, *Journal of Multivariate Analysis* (July 2015), volume 139, pages 260-384.

CONTENTS OF THE PACKAGE

QuESTreadme : this text document detailing the contents of the QuEST package

QuESTdemo: Matlab routine that demonstrates how to use the other functions in the package to estimate the covariance matrix and its eigenvalues

QuESTimate: main estimation function, called by QuESTdemo, uses MATLAB optimizer
inputs: sample eigenvalues and sample size
outputs: estimates of the population eigenvalues and the nonlinear shrinkage formula

QuESTimates: same as QuESTimate but using TOMLAB/SNOPT nonlinear optimizer

QuESTmse: objective function for the optimization problem solved by optimizer

QuESTdmse: gradient for the optimization problem solved by nonlinear optimizer

QuEST: the QuEST function, which is called by QuESTmse
to see help, type: QuEST('help')

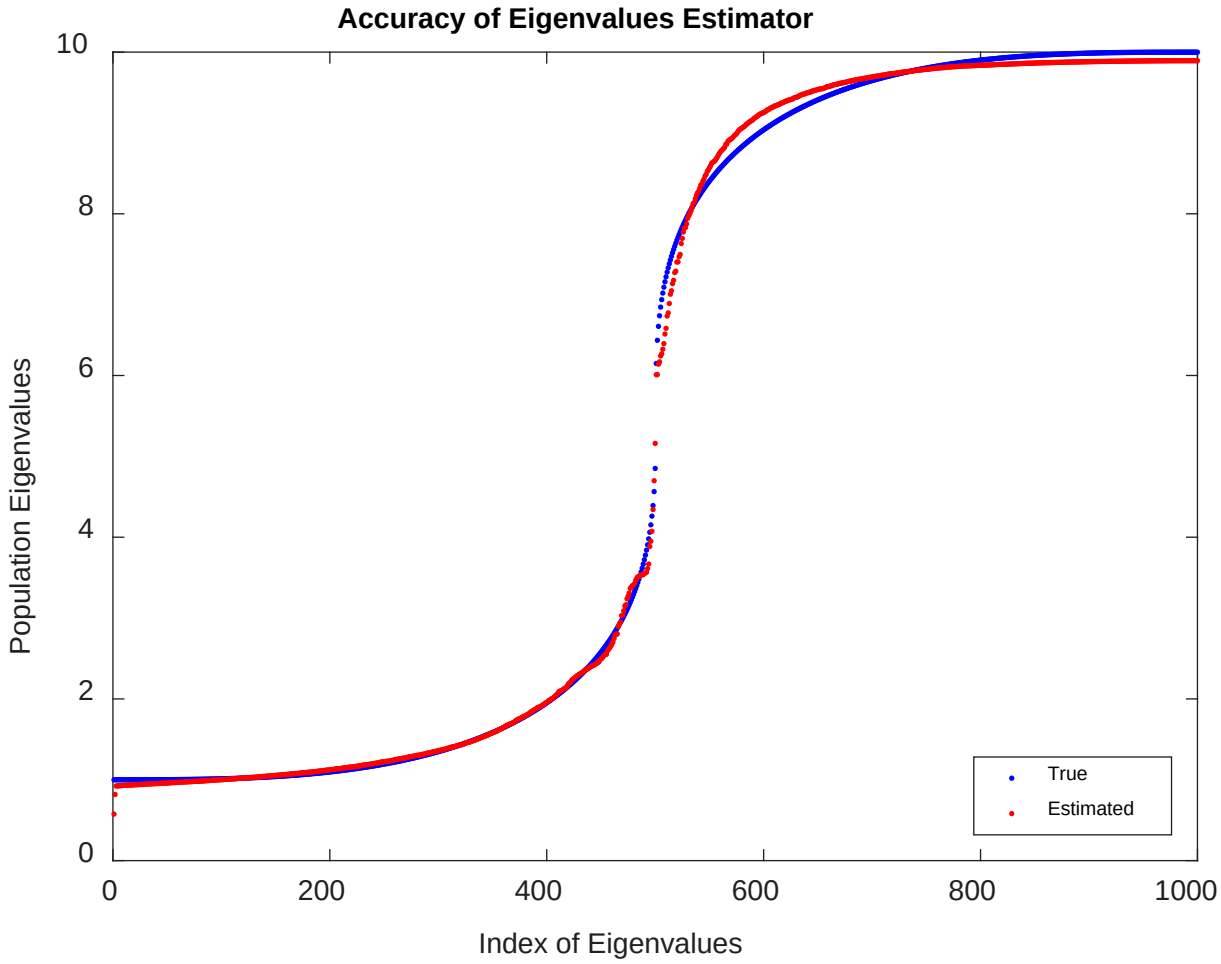
QuESTgrad: the gradient of the QuEST function, called by QuESTdmse
to see help, type: QuESTgrad('help')

QuESTpaper: working paper that details the mathematics behind the code of the QuEST.m and QuESTgrad.m functions

Given the article, this package should be pretty self-explanatory.

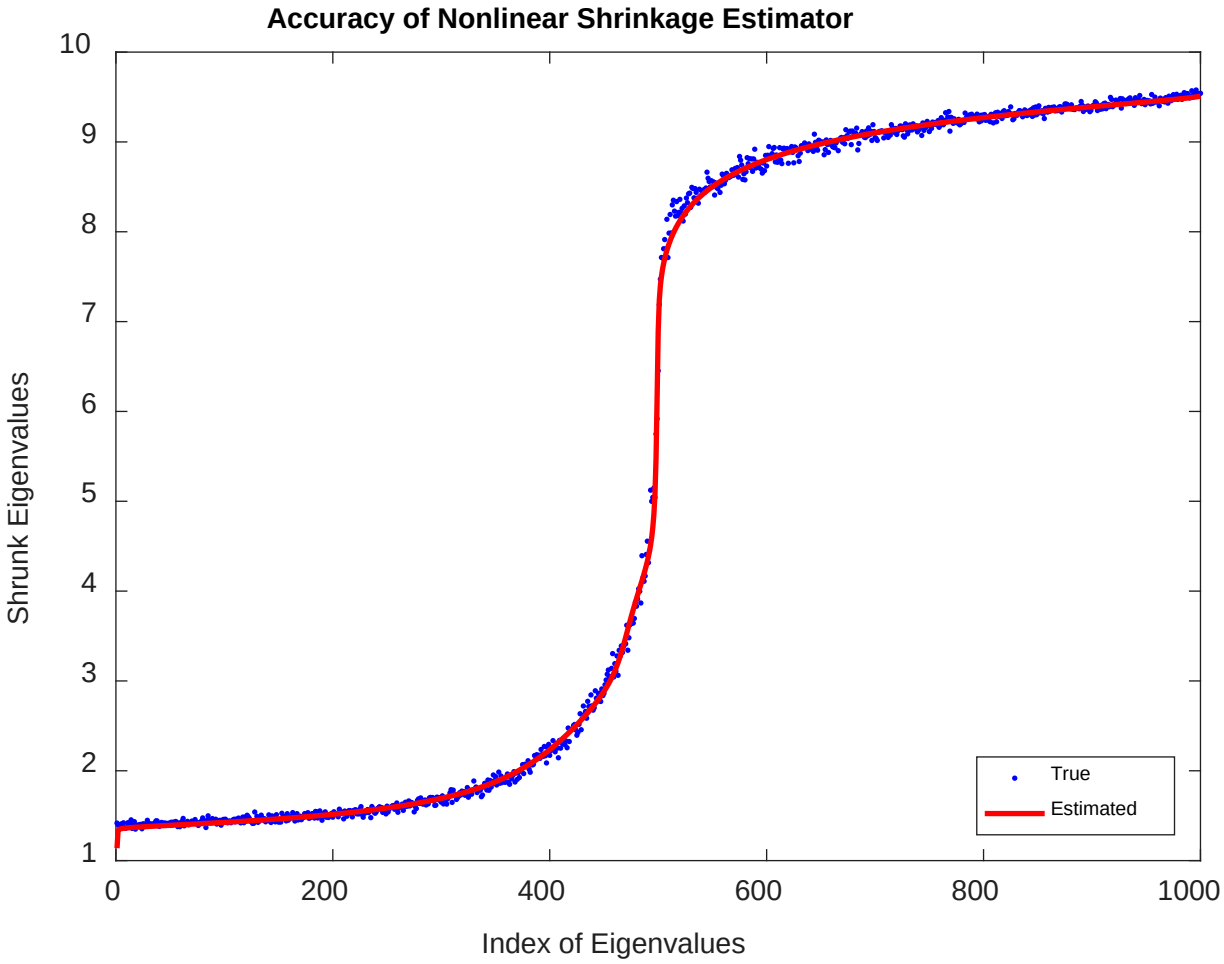
Most of the time you will use QuESTimate for your estimation problems.
Run QuESTdemo to see how to call it.
QuESTdemo runs in 22 seconds on a 4GHz Windows desktop running Matlab v9.0 and MATLAB Optimization Toolbox v7.4.
QuESTdemo is configured by default to use the function QuESTimate.m that calls fmincon.m from the Matlab Optimization toolbox, but the comments make it clear how to switch over to QuESTimates.m that calls TOMLAB/SNOPT.
Note that TOMLAB/SNOPT has another function called fmincon.m with incompatible syntax, so it needs to be disabled for QuESTimate.m to run properly.

It generates figures that should look like this:

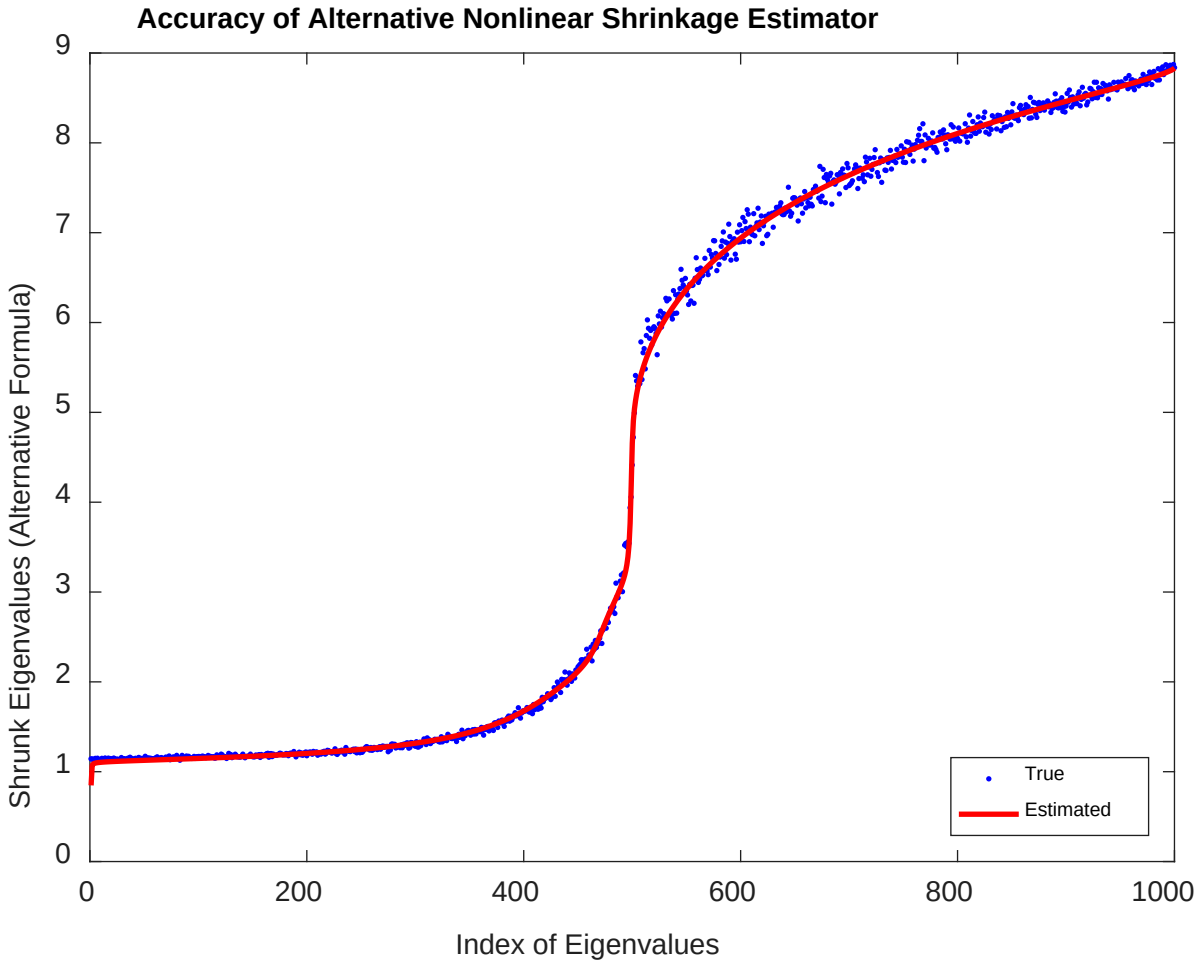


The cumulative distribution function of population eigenvalues chosen here is challenging because it is highly nonlinear.

This figure shows that numerically inverting the QuEST approach identifies the population eigenvalues with a high degree of accuracy by inverting the Fundamental Equation of Random Matrix Theory.



Here we show that the *bona fide* nonlinear shrinkage transformation of covariance matrix eigenvalues proposed in the paper approximates very closely the finite-sample optimal transformation (which is unfeasible because it would require knowledge of the population covariance matrix).



This is similar except that we use an alternative nonlinear shrinkage formula which is optimal with respect to other loss functions, namely Stein's loss and the Frobenius-norm error of the precision matrix, as specified in Theorem 6.2 of "Optimal Estimation of a Large-Dimensional Covariance Matrix under Stein's Loss" by Olivier Ledoit and Michael Wolf, forthcoming in *Bernoulli.*

The mathematics behind the code of the QuEST.m function are detailed in the working paper QuESTpaper.pdf. The sub-function supfun07 corresponds to Section 5, netfun02: Section 6, solfun02: Section 7, denfun03: Section 8, disfun04: Section 9, and intfun03: Section 10. The paper also explains the mathematics behind the code of the function QuESTgrad.m, and the organization is similar.

Version information

v001: - dated Thursday 11 April 2013
- based on a version without number whose most recent file was QuESTimate.m, dated 07 March 2013 at 9:40pm
- change: in QuEST.m and QuESTgrad.m, increased tolerance for negative population eigenvalues (tau) in subfunction checkinputs01 from 10^(-16) to 10^(-14) in order to prevent error messages from breaking the estimation of the covariance matrix in SNOPT

v002: - dated Friday 16 August 2013
- the function QuESTimate.m now returns, in addition to sigmahat (estimator of the covariance matrix) and tauhat (estimator of the population eigenvalues), dhat (the eigenvalues of sigmahat) for people interested in Principal Component Analysis (PCA)

v003: - dated Monday 18 November 2013
- the functions QuEST.m and QuESTgrad.m stop running after one year

v004: - dated Tuesday 26 November 2013
- the functions QuEST.m and QuESTgrad.m do not run unless the machine's hostname is on the authorized list

v005: - dated Wednesday 27 November 2013
- changed objective function scaling coefficient in QuESTimate.m
- changed distribution of population eigenvalues in demo script
- updated results in readme file

v006: - dated Wednesday 1st January 2014
- readme file converted from plain text to PDF to include pictures

v007: - dated Thursday 30 January 2014
- allow optimizer to switch the ordering of population eigenvalues
- change optimizer termination criteria to speed up estimation

v008: - dated Monday 3 February 2014
- bug fixes

v009: - dated Monday 17 February 2014
- return alternative shrinkage function based on Frobenius norm of inverse

v010: - dated Tuesday 22 April 2014
- added optional parameter in QuESTimate.m to handle demeaning of data

v011: - dated Friday 23 May 2014
- new permissions

v012: - dated Monday 20 October 2014
- new permissions

v013: - dated Monday 10 November 2014
- function QuESTimate outputs additional information:
  - computational speed
  - observed sample eigenvalues
  - fitted sample eigenvalues

v014: - dated Friday 6 February 2015
- function QuESTimate outputs additional information: optimizer exit flag, number of iterations, and optimizer starting point based on linear shrinkage
- function QuESTimate takes optional input parameter to control optimizer speed
- use global variable instead of TOMLAB structure in order to pass user-defined parameters to the objective function and to the gradient function

v015: - dated Wednesday 25 February 2015
- use optimization function fmincon.m from the MATLAB Optimization Toolbox with interior point algorithm instead of TOMLAB/SNOPT
- reduce limit on maximum number of iterations

v016: - dated Friday 20 March 2015
- changed output of QuEST('help')

v017: - dated Wednesday 15 July 2015
- refreshed licensing data

v018: - dated Friday 24 July 2015
- new permissions

v019: - dated Tuesday 15 September 2015
- updated references of papers cited in the help function

v020: - dated Saturday 3 October 2015
- updated reference of paper cited in the ReadMe document
- check that sample size is an integer if it is strictly less than matrix dimension
- introduce the notion of effective sample size in QuESTimate.m

v021: - dated Tuesday 17 November 2015
- minor bug fixes and improvements

v022: - dated Sunday 20 December 2015
- add QuESTimates.m to give choice between Matlab optimizer and TOMLAB/SNOPT

v023: - dated Wednesday 23 December 2015
- renew licenses

v024: - dated Monday 18 January 2016
- remove permissioning restrictions

v025: - dated Thursday 21 January 2016
- updated references to Spectrum paper in QuESTimate.m and QuESTimates.m
- changed limit on number of iterations to limit of number of function evaluations

v026: - dated Wednesday 26 July 2017
- changed the pictures in QuESTreadme.pdf
- added to the QuEST package the working paper QuESTpaper.pdf, which details the mathematics behind the code of the QuEST.m and QuESTgrad.m functions
- fixed a bug in the function QuEST.m when some population eigenvalues are equal to zero and the sample size is equal to the matrix dimension, initially highlighted by Wen Jun from the Department of Statistics and Applied Probability at the National University of Singapore, and resolved jointly with Zhao Zhao from the Department of Economics at Huazhong University of Science and Technology in China
- fixed a separate bug in the function QuESTgrad.m when some population eigenvalues are equal to zero and the sample size is equal to the number of nonzero eigenvalues, also initially highlighted by Wen Jun and resolved jointly with by Zhao Zhao

v027: - dated Tuesday 07 November 2017
- updated references to Stein paper in QuESTimate.m and QuESTimates.m
- reverted back to *interior-point* instead of *sqp* algorithm in call to fmincon.m