Inducing Variety: A Theory of Innovation Contests

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Abstract

This paper analyzes the design of innovation contests when the quality of an innovation depends on the research approach, but the best approach is unknown. Inducing a variety of research approaches is desirable because it generates an option value. We show that suitable contests can induce such variety. The optimal contest is a bonus tournament, where suppliers can choose only between a low bid and a high bid. We then compare the optimal contest to other commonly studied institutions, such as scoring auctions and fixed-prize tournaments.

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1 Introduction

The use of contests to procure innovations has a long history, and it is becoming ever more popular. Recently, private buyers have awarded the Netflix Prize, the Ansari X Prize, and the InnoCentive prizes. Public agencies have organized, for instance, the DARPA Grand Challenges, the Lunar Lander Challenge and the EU Vaccine Prize.[1] Reflecting the increasing importance of these prizes, a literature on contest design has developed. This literature focuses almost exclusively on how incentives for costly innovation effort can best be provided. However, effort is by no means the only important requirement for a successful innovation. A case in point is the 2012 EU Vaccine Prize to improve what is known as the cold-chain vaccine technology. The ultimate goal of the prize was to prevent vaccines from spoiling at higher temperatures, which is particularly challenging in developing countries. The rules of the competition contain the following statement:

> "It is important to note that approaches to be taken by the participants in the competition are not prescribed and may include alternate formulations, novel packaging and/or transportation techniques, or significant improvements over existing technologies, amongst others."[2]

This statement explicitly recognizes the fundamental uncertainty of the innovation process: Even when the buyer communicates a well-specified objective (such as finding a way to prevent vaccine spoilage), neither she nor the suppliers will necessarily know the best approach to achieving this goal. This uncertainty about the quality of innovation resulting from a particular approach will only be resolved by the act of innovation itself. The innovator will therefore have to choose between several conceivable approaches without being sure whether they lead to the goal. If innovators pursue different approaches, chances are higher that the best of these approaches yields a particularly valuable (high-quality) innovation. Thus, even if variety of research approaches has no intrinsic value, it has an option value. Our first question is therefore: Can innovation contests be used to incentivize suppliers to diversify their research approaches so as to generate a high expected value of the innovation?

In addition to efficiency (the expected value of the innovation), contest design may also affect distribution. A contest that induces diversity may yield a high expected value of the innovation and thereby foster efficiency, but at the same time leave high rents to the suppliers. Thus, the main question of our paper will be: Which contests are optimal for the buyers, when the expected value (reflecting the induced variety of approaches) as well as the expected payments to the suppliers are taken into account?

The diversity of potential approaches, which is highlighted in the guidelines of the Vaccine Prize cited above, played an important role in many other examples of innovation procurement. First, the often cited Longitude Prize of 1714 for a method to determine a ship's longitude at sea featured two competing approaches.[3] The lunar method was an attempt to use the position of the moon to calculate the position of the ship. The alternative, ultimately successful, approach relied on a clock which accurately kept Greenwich time at sea, thus allowing estimation of longitude by comparison with the local time (measured by the position of the sun). Second, when the Yom Kippur War in 1973 revealed the vulnerability of

[1] See "Innovation: And the winner is...", *The Economist.* Aug 5, 2010.

[2] European Comission (2012), "Prize Competition Rules." August 28, 2012 (accessed on April 3, 2015). http://ec.europa.eu/research/health/pdf/prize-competition-rules_en.pdf

[3] See, e.g., Che and Gale (2003) for a discussion of the Longitude Prize.

US aircraft to Soviet-made radar-guided missiles, General Dynamics sought to resolve the issue through electronic countermeasures, while McDonnell Douglas, Northrop, and eventually Lockheed, attempted to build planes with small radar cross-section.[4] Third, the announcement of the 2015 Horizon Prize for better use of antibiotics contains a similar statement as the announcement of the vaccine prize.[5]

Architectural contests share some important properties with innovation contests. A buyer who thinks about procuring a new building usually does not know what exactly the ideal building would look like, but once she examines the submitted plans, she can choose the one she prefers. Guidelines for architectural competitions explicitly recognize the need for diversity. For example, the Royal Institute of British Architects states: "Competitions enable a wide variety of approaches to be explored simultaneously with a number of designers."[6]

Motivated by this long list of examples, we focus in the following on the design of contests for innovation, with a view towards the induced variety of research approaches. As we will sketch below, however, our analysis also has interesting implications for the case that suppliers on anonymous markets decide on the introduction of new products.

In line with the examples, we consider innovation contests in a setting where both the buyer (the contest designer) and the suppliers (contestants) are aware that there are multiple conceivable approaches to innovation. Furthermore, none of the participants knows the best approach beforehand. However, after the suppliers have followed a particular approach, it is often possible to assess the quality of innovations, for instance, by looking at prototypes or detailed descriptions of research projects. In such settings, can buyers design contests in such a way that suppliers have incentives to provide variety? And will they benefit from doing so?

The existing literature on innovation contests mainly focuses on the problem of providing incentives for costly innovation effort.[7] To our knowledge, we are the first to analyze the optimal design of innovation contests with multiple conceivable research approaches. Our baseline model is chosen to isolate this design problem in a particularly stark way. We assume that there are two homogeneous suppliers who have to decide whether to exert costly research effort and which research approach to choose. In the baseline, we equip the buyer with strong instruments to induce effort: We assume that, once a supplier joins the contest, he cannot shirk. This enables the buyer to use subsidies to ensure that effort is exerted. This assumption, which we will relax later on, allows us to focus on the effects of contest design on the choice of approaches rather than on effort choice.[8]

We model the research approach as a point on the unit interval. We assume all approaches are equally costly, so as to exclude trivial reasons for buyers to prefer some approaches over others. Crucially, the quality of an innovation and thus the value to the buyer depends inversely on the distance between the chosen research approach and an ideal approach that is unknown to all parties. The suppliers and the buyer agree about the distribution of this

[4] See Paul Crickmore (2003), *Nighthawk F-117: Stealth Fighter.* Airlife Publishing Ltd.

[5] "The rules of the contest specify the targets that need to be met but do not prescribe the methodology or any technical details of the test, thereby giving applicants total freedom to come up with the most promising and effective solution, be it from an established scientist in the field or from an innovative newcomer." European Commision (2015), "Better use of antibiotics." March 24, 2015 (accessed on April 3, 2015). http://ec.europa.eu/research/horizonprize/index.cfm?prize=better-use-antibiotics

[6] See Royal Institute of British Architects (2013), "Design competitions guidance for clients." (accessed on Apr 3, 2015); http://competitions.architecture.com/requestform.aspx.

[7] Section 6 discusses this literature.

[8] If the suppliers can shirk, subsidies cannot be used to induce participation, as the suppliers can always collect subsidies and then shirk. We study this case in Section 5.1.

ideal approach, which has a strictly positive, symmetric and single-peaked density. If different suppliers try different approaches, this creates an option value for the buyer who can choose the preferred innovation once uncertainty is resolved.

In line with the literature on innovation contests, we assume that neither research inputs (approaches) nor research outputs (qualities) are verifiable, because they are both difficult to evaluate and the relation between them is stochastic. The lack of verifiability of research activity precludes any kind of contract that conditions on research inputs or outputs, and it motivates the focus on contests.[9] The notion of contest design that we use was suggested by Che and Gale (2003). The buyer prescribes a possible set of prices and commits herself to paying the price chosen by the supplier from which the innovation is procured. The class of such contests is very rich.[10] Examples include fixed-prize tournaments (when the price set is a singleton) as well as scoring auctions (when the price set is the set of non-negative real numbers).[11] Contest design in this setting is the choice of the allowable price set and the subsidies.[12]

The sequence of moves in our model is as follows: After the buyer has communicated the rules of the game (and, in particular, the price set), the suppliers choose whether to enter and, if so, which approach to pursue. Then qualities become common knowledge. After having observed qualities, suppliers choose bids from the price set. Finally, the buyer selects the preferred supplier.

We show that the optimal contest for the buyer is what we call a bonus tournament. In a bonus tournament, the price set consists of two elements — a low price and a high ("bonus") price. After qualities have been realized, the suppliers thus can only choose whether to ask for the high price or the low price. The selected supplier will be paid his bid. Anticipating this, the suppliers diversify in the hope that their quality advantage over the competitor will be sufficiently high that they can bid the bonus price and win even so. It will turn out that the amount of diversity implemented in a bonus tournament is determined by the difference between the bonus price and the low price. We show that, with a bonus tournament, the buyer can implement essentially any level of diversity. In particular, a bonus tournament with suitably chosen prices (and possibly a subsidy) implements the socially optimal diversity. However, full rent extraction is not always possible, and the buyer must trade off efficiency against rent extraction. Bonus tournaments are still optimal for the buyer: They induce any desired level of diversity while minimizing rent extraction. This will not lead to the socially optimal level of diversity, except when research costs are very high. Thus the buyer resolves the trade-off between efficiency and rent extraction in favor of the latter.

The existing literature on innovation contests has put particular emphasis on fixed-prize tournaments and on scoring auctions (henceforth auctions for brevity). We therefore also analyze how these institutions perform in our setting and why they fail to be optimal for the buyer. Unrestricted auctions induce the social optimum, while auctions with price ceilings induce less variety. The price ceiling determines the amount of variety. While auctions can in general implement the same diversity as the optimal bonus tournaments, they always generate higher revenues for the suppliers. Thus the buyer prefers bonus tournaments to auctions. Fixed-prize tournaments do not induce any diversity and are therefore less efficient than auctions and optimal bonus tournaments. Nevertheless, for low research costs, the buyer

[9] For an extensive discussion see Che and Gale (2003) and Taylor (1995).
[10] See Che and Gale (2003) for a detailed discussion.
[11] The term "auction", though common in the literature, is imprecise (see Section 2 for a discussion).
[12] We exclude participation fees in the baseline model (reflecting for instance, limited liability).

prefers the inefficient fixed-prize tournaments to the socially efficient unrestricted auctions.[13]

We then extend the analysis by showing that, with some caveats, bonus tournaments perform well even in more general environments. In particular, we extend the model by allowing the suppliers to shirk. The contests now not only need to incentivize the suppliers to choose the appropriate research approach, but also to avoid shirking. We show that, as long as the cost of effort is not too high, bonus tournaments are still optimal contests. However, as effort costs increase, it becomes more difficult to induce diversity, and for high enough costs, no contest can implement any diversity. In this case, the optimal contest for the buyer is a fixed-prize tournament. In addition, we study contests with more than two suppliers, and contests with more general distributions and quality functions. We also discuss heterogeneous suppliers, multiple prizes and multiple approaches per supplier. Under very general conditions, suitable bonus tournaments still induce the social optimum and for large parameter regions, they still are optimal contests. Moreover, the buyer may benefit from inviting a large number of suppliers, which is a straightforward implication of the option value provided by additional suppliers.

As we discuss in more detail in the conclusion, our analysis has potential applications beyond innovation contests organized by a single buyer. Our model can be applied to situations when suppliers in a new market choose products in the face of uncertain demand by a potentially large number of homogeneous buyers. If we interpret the prize as the expected product market profit of a successful innovator, contest design then corresponds to the choice of alternative regulatory frameworks for the new market. Our approach shows that unregulated markets provide incentives for suppliers to choose the socially optimal products, but at the cost of endowing them with ex-post market power. As a result, regulation may yield higher expected consumer surplus, even though it does not induce the optimal expected product quality.

While our main application is to the design of innovation contests, the model is not limited to innovation settings. Our results have important implication for any contest where contestants can choose some measure of correlation of outcomes. In particular, prize rules that award winners based on the margin by which they outperform the second-best contestant (like auctions or bonus tournaments) will incentivize the contestants to choose less correlated outcomes. Alternatively, fixed-prize contests will cause contestants to choose too correlated outcomes.

In Section 2, we introduce the model. Section 3 deals with the design of optimal contests for the buyer. Section 4 compares several commonly used contests, such as fixed-prize tournaments and auctions with and without price ceilings. Section 5 presents extensions of the model. Section 6 discusses the relation of our paper to the literature. Section 7 concludes, pointing in particular to the above-mentioned re-interpretation of our model for a world with many buyers. Proofs are in the Appendix.

2 The Baseline Model

A risk-neutral buyer B needs an innovation that two risk-neutral suppliers ($i \in \{1, 2\}$) can provide. Each supplier simultaneously chooses whether to carry out costly research and which approach $v_i \in [0, 1]$ to pursue. Without loss of generality, we assume that $v_1 \leq v_2$; if the

[13] When, contrary to the assumptions of our main model, sufficiently high participation fees are possible, the buyer implements the social optimum and appropriates the surplus with the participation fees. Whether he uses a bonus tournament or an auction for implementation is immaterial.

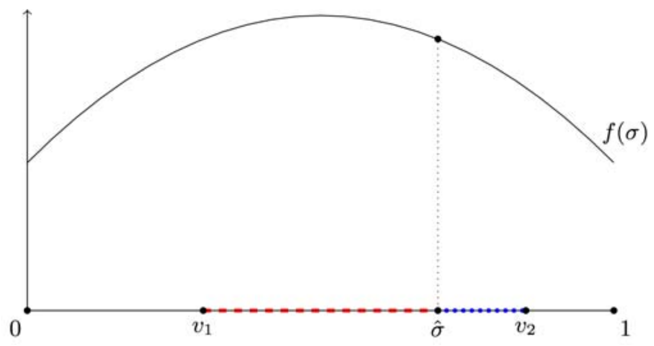

Figure 1: Illustration of an outcome given v_1 and v_2 where the ideal approach is $\hat{\sigma}$.

ordering of approaches does not matter, we use the generic notation v_i (and v_j, $j \neq i$). The cost of approach v_i is $C(v_i) \equiv C \geq 0$. Thus all approaches are equally costly, so that, once a supplier has decided to participate in the contest, he cannot influence the research cost anymore. This assumption allows us to study the effects of contest design on the choice of research approaches in isolation and to develop clear intuition for the results. In Section 5.1 we will analyze a model that allows for shirking in the contest, and we will show that the results from the main model are robust.

The quality q_i of the resulting innovation and thus the value to the buyer depends on a state $\sigma \in [0,1]$, which is distributed with density $f(\sigma)$, and corresponds to an (ex-post) ideal approach. Unless specified otherwise, we maintain two assumptions on q_i and σ:

Assumption (A1) $q_i = q(v_i, \sigma) \equiv \Psi - b|v_i - \sigma|$ with $b \in (0, \Psi]$.[14]

Thus, the quality difference between the ideal approach $\hat{\sigma}$ and v_i is proportional to the distance between v_i and $\hat{\sigma}$ (the dashed line for supplier 1, and the horizontal dotted line for supplier 2 in Figure 1). By (A1), the quality is bounded below by $\Psi - b$ and bounded above by Ψ.

We restrict the distribution of the ideal state as follows.

Assumption (A2) *The density function $f(\sigma)$ is (i) symmetric: $f(1/2 - \varepsilon) = f(1/2 + \varepsilon)$ $\forall \varepsilon \in [0, 1/2]$, (ii) single-peaked: $f(\sigma) \leq f(\sigma')$ $\forall \sigma < \sigma' < 1/2$, (iii) has full support: $f(\sigma) > 0$ $\forall \sigma \in [0,1]$ and (iv) satisfies $f'(x) < 2f(0)$ for all $x \in [0, 1/2]$.*

A wide class of distributions (including the one in Figure 1) satisfies (A2). For each of these distributions, there is an approach which has the highest expected quality ex ante, namely the median. Furthermore, single-peakedness makes it more difficult to induce diversity: As there is less mass on approaches that are further away from the median, contestants will not choose them without additional incentives. Part (iv) excludes the possibility that some states are much less probable than others; in this sense, it requires that the amount of uncertainty about the ideal approach is sufficiently high.

In this setting, the buyer chooses an *innovation contest* determining the procedure for choosing and remunerating suppliers. These contests are closely related to those analyzed

[14] Ψ needs to be large enough so that it is worthwhile for the buyer to hold a contest. A simple sufficient condition is $\Psi > b + 2C$. This assumption is innocuous as none of our results depend on Ψ.

by Che and Gale (2003), where suppliers choose efforts rather than approaches. In line with these authors, we assume that neither v_i nor q_i is contractible.[15] The environment (b, Ψ, C) of a contest consists of the utility and cost parameters. The buyer chooses a set $\mathcal{P}$ of allowable prices (bids), where $\mathcal{P}$ is an arbitrary finite union of closed subintervals of $\mathbb{R}^+$.[16] We denote the minimum of P as $\underline{P}$ and the maximum, if it exists, as $\overline{P}$. Moreover, we allow that the buyer can offer subsidies $t \geq 0$ to the suppliers.[17]

To sum up, an innovation contest is the extensive-form game between the buyer and the suppliers given by the buyer's choice of $\{\mathcal{P}, t\}$ and the following rules:

Period 1:	Suppliers simultaneously choose whether to engage in research and they select approaches $v_i \in [0, 1]$.
Period 2:	The state is realized. All players observe qualities q_1 and q_2.
Period 3:	Suppliers simultaneously choose prices $p_i \in \mathcal{P}$.
Period 4:	The buyer observes prices; then she chooses a supplier $i \in \{1, 2\}$. She pays $p_i + t$ to the chosen supplier and t to the other supplier.

Importantly, the suppliers potentially receive two types of payments, namely the revenue from the contest (that is paid only to the successful supplier) and the subsidies paid to both suppliers. For ease of exposition, we sharpen the requirement that qualities are observable by assuming that all players observe v_i and σ, as this allows us to apply the subgame perfect equilibrium (SPE). It will be obvious that, as long as qualities are observable, the observability of v_i and σ plays no role; as these variables are payoff-relevant only inasmuch as they affect qualities. As long as all players can observe qualities, all results still hold with the SPE replaced by a Perfect Bayesian Equilibrium with suitably specified beliefs.[18]
Moreover, we provide an extensive discussion of the case that not even quality is observable in the working paper (Letina and Schmutzler 2015); we summarize the discussion briefly in Section 5.3.2.
The following are examples of innovation contests:

1. $\mathcal{P} = \mathbb{R}^+$: an *auction without a price ceiling.*
2. $\mathcal{P} = \left[0, \overline{P}\right]$: an *auction with a price ceiling* $\overline{P}$.
3. $\mathcal{P} = \{A\}$, where $A \geq 0$: a *fixed-prize tournament* (FPT).
4. $\mathcal{P} = \{A, a\}$, where $A > a \geq 0$: a *bonus tournament.*

As to the first two examples, the (common) auction terminology is slightly misleading. While suppliers choose bids as in a standard auction, the rules do not commit the buyer to selecting the supplier with the lowest bid. Instead, the buyer's choice is fully discretionary. As the result, the buyer behaves as if she had committed to a scoring rule, which weighs prices

[15]For example, Che and Gale (2003) and Taylor (1995) assume that neither inputs nor outputs of innovative activity are verifiable. As an example of the verifiability problem, Che and Gale (2003) point to the protracted battle between John Harrison, the inventor of the marine chronometer, and the Board of Longitude, over whether his invention met the requirements of the 1714 Longitude Prize. See also references in Taylor (1995).

[16]Formally, $\mathcal{P}$ is chosen from $\mathcal{I}(\mathbb{R}^+) \equiv \{\mathcal{P} \subseteq \mathbb{R}^+ : \mathcal{P} = \cup_{k=1}^{\bar{k}}[a_k, b_k]$ or $\mathcal{P} = \cup_{k=1}^{\bar{k}}[a_k, b_k] \cup \left[a_{\overline{k}+1}, \infty\right)$ for $a_k \leq b_k \in \mathbb{R}^+, \bar{k} \in \mathbb{N}\}$.

[17]In the current setting the buyer cannot benefit from using individualized subsidies; this is different with more than two suppliers (see Section 5.2).

[18]Proof available on request.

and (the monetary value of) quality in the same way. The last example differs from an FPT in that the supplier has to specify whether she accepts a low price a if chosen, or asks for the higher "bonus" price A instead. The bonus tournament will turn out to be the optimal contest for the buyer.

To finish the description of the contests, we require several further conventions. First, we apply the following tie-breaking rules, which can be interpreted as second-order lexicographic preference for winning and for higher quality.

(T1) (Preference for quality) If suppliers offer the same surplus, the buyer prefers the higher quality one. If both have the same quality, the tie is randomly broken.

(T2) (Preference for winning) Given equal monetary payoffs, the suppliers prefer to participate in the contest rather than to stay out and to win the contest rather than not.

(T1) and (T2) guarantee that the outcomes are robust to infinitesimal changes in the reward structure.

Second, we assume that, in cases where only one supplier decides to participate, the contest is called off and players obtain zero overall payoff.

Third, we will confine our analysis to the case of pure-strategy equilibria for simplicity.

3 The Optimal Contest for the Buyer

In this section, we characterize the optimal contest for the buyer.[19] We start with some auxiliary results. These results characterize the social optimum, and they deal with the pricing subgames.

3.1 Auxiliary Results

We introduce the following terminology which applies when both suppliers participate. For $(v_1, v_2) \in [0,1]\times[0,1]$, the (expected) *total surplus* is $S_T(v_1, v_2) \equiv E_\sigma [\max\{q(v_1,\sigma), q(v_2,\sigma)\}] - 2C$. The social optimum is $(v_1^*, v_2^*) \equiv \arg\max_{(v_1,v_2)\in[0,1]^2} S_T(v_1, v_2)$. Thus, we are focusing here on the optimal choice of approaches for a given number of suppliers (two). In Section 5.2, we deal with the optimal number of suppliers.

For (v_1, v_2), implemented as an equilibrium of a contest $(\mathcal{P}, t)$, $S_i^{(\mathcal{P},t)}(v_1, v_2)$, the (expected) *surplus of supplier i* in an equilibrium, is the sum of the expected revenue and the subsidies, net of research costs. The (expected) *buyer surplus*, $S_B^{(\mathcal{P},t)}(v_1, v_2)$, is expected maximal quality minus the expected revenues and subsidies of the suppliers. We usually drop the superscript $(\mathcal{P}, t)$ when there is no danger of confusion. For precise definitions of $S_B^{(\mathcal{P},t)}(v_1, v_2)$ and $S_i^{(\mathcal{P},t)}(v_1, v_2)$, we refer the reader to Appendix 8.1.1.

As the costs of each approach are the same, the social optimum (v_1^*, v_2^*) maximizes the expected maximal quality $E_\sigma[\max\{q(v_1,\sigma), q(v_2,\sigma)\}]$ or, equivalently, minimizes the expected

[19] An attentive reader might conjecture that the buyer could implement arbitrary outcomes with a mechanism where he just pays unconditional transfers $t = C$ and sets a singleton prize set $\mathcal{P} = \{0\}$. The suppliers are then indifferent between entering and not entering, and, *in monetary terms,* between all approaches. However, our "preference for winning" assumption (T2) would ensure that such a mechanism would have a unique equilibrium with $v_1 = v_2 = 1/2$. Even if we dispensed with assumption (T2), the equilibrium structure of such a mechanism would not be robust to small changes in the cost of different approaches or to assuming that duplicating an approach is less costly than developing an original one.

minimal distance to the ideal approach, $E_\sigma\left[\min\left\{\left|v_1-\sigma\right|,\left|v_2-\sigma\right|\right\}\right]$. With only one potential supplier i, the optimal approach would correspond to $v_i = 1/2$, as this maximizes the expected quality. With two suppliers, the optimization needs to take into account the option value generated by having different choices once qualities have been observed. It is always socially optimal to have at least some diversification. This simple but important observation holds without the restrictions on distributions coming from (A2), as long as there is any uncertainty about the ideal approach. The intuition is simple: Starting from a situation with identical approaches, suppose one of the suppliers chooses an arbitrary alternative approach, whereas the other supplier continues to choose the same one. After this modification, the minimal distance decreases for a set of ideal states with positive measure. There can be no σ for which the expected minimal distance to the best approach increases, as the initial approach is still available. The following result provides a sharper characterization of the social optimum:[20]

Lemma 1 *The unique social optimum with $v_1^* \leq v_2^*$ satisfies $F\left(v_1^*\right) = 1/4$ and $F\left(v_2^*\right) = 3/4$ and thus $v_2^* = 1 - v_1^*$.*

Hence v_1^* and v_2^* are symmetric around 1/2. The result relies on (A2(iv)), which states that the ideal state distribution is sufficiently dispersed.[21] The socially optimal location of approaches is fully determined by the distribution F, whereas the level of research costs has no influence on the optimal diversity. We now characterize the equilibria of the pricing subgames, using the following notation.

Notation 1 $\overline{p}\left(\sigma\right) = \overline{p}\left(\sigma; v_1, v_2\right) \equiv \max\left\{p \in \mathcal{P} \middle| \, p \leq \left|q(v_1,\sigma) - q(v_2,\sigma)\right| + \underline{P}\right\}$.

In words, for any realization of σ, $\overline{p}\left(\sigma\right)$ is the maximal allowed price which guarantees that the supplier with higher quality wins the contest, irrespective of the price chosen by the supplier with the lower quality. The following result is closely related to the familiar "asymmetric Bertrand" logic that inefficient firms choose minimal prices, whereas efficient firms translate their efficiency advantage into a price differential.[22]

Lemma 2 *The subgame of an innovation contest corresponding to (q_i, q_j) has an equilibrium such that $p_i\left(q_i, q_j\right) = \overline{p}\left(\sigma\right)$ if $q_i \geq q_j$ and $p_i\left(q_i, q_j\right) = \underline{P}$ if $q_i < q_j$. In any equilibrium of any contest, $p_i\left(q_i, q_j\right) = \overline{p}\left(\sigma\right)$ if $q_i \geq q_j$.*

Lemma 2 sharpens the Bertrand logic to account for bounded and/or non-convex price sets: The price differential will only fully reflect the quality differential when the corresponding

[20] The result is similar to the familiar finding that, in a Hotelling model (with uniformly distributed consumers and without price competition), firms should optimally spread equally.

[21] The condition guarantees that the expected quality is a strictly concave function of the approaches. It is thus more restrictive than necessary. A simple *necessary* condition for the optimum to satisfy $F\left(v_1^*\right) = 1/4$ and $F\left(v_2^*\right) = 3/4$ is $f(1/2) < 2f(v_1^*)$; otherwise the objective function is not even locally concave. Moreover, this condition turns out to be necessary for the existence of a social optimum with $v_2^* = 1 - v_1^*$. It is simple to provide examples where $f(1/2) < 2f(v_1^*)$ is violated.

[22] The adequacy of pure-strategy equilibria in asymmetric Bertrand games has received some attention, in particular, but not only, because they tend to involve weakly dominated strategies (see Blume 2003 and Kartik 2011). In our setting, these issues are resolved by the appeal to the "preference for quality" (T1) and "preference for winning" (T2). In some of our contests (in particular, in auctions with and without price ceilings), constructions as in Blume (2003) and Kartik (2011) exist, where the low-quality firm mixes over a small interval of prices.

bid of the high-quality supplier is in the price set $\mathcal{P}$. In many cases, the equilibrium described in Lemma 2 is unique.[23] We need further notation:[24]

Notation 2 $\Delta q(v_i, v_j) \equiv |q(v_i, v_i) - q(v_j, v_i)|$ *is the maximum quality difference given* (v_i, v_j).

To understand why $\Delta q(v_i, v_j)$ is the maximum quality difference, note that, for $\sigma \in [0, v_1] \cup [v_2, 1]$ the quality difference between the two approaches is equal to $|q(v_i, v_i) - q(v_j, v_i)|$ and thus constant; whereas it is smaller for $\sigma \in (v_1, v_2)$. By Lemma 2, in any subgame the successful supplier chooses the highest available price not exceeding the sum of the quality differential and the minimum bid. We now sharpen this result for subgames following equilibrium choices (v_1, v_2).

Lemma 3 *Let* $v_1 \leq v_2$. *(i) If a contest implements* (v_1, v_2), *then* $\Delta q(v_1, v_2) + \underline{P} \in \mathcal{P}$. *(ii) If* $\sigma \in [0, v_1] \cup [v_2, 1]$, *the successful supplier bids* $p_i(q_i, q_j) = \Delta q(v_i, v_j) + \underline{P}$.

Lemma 3 is a key result. It implies that the amount of diversity that any contest can implement is limited by the highest price that the contest allows. Intuitively, (i) states that, if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, suppliers could increase their chances of winning by small moves towards the approach of the other party, without reducing the price in those cases where they win. (ii) shows that in all states outside the interval (v_1, v_2) the buyer pays a constant price, reflecting the (maximal) quality difference between the two suppliers. Therefore, to implement any (v_1, v_2), a buyer has to pay at least $\Delta q(v_1, v_2)(F(v_1) + 1 - F(v_2))$ in expectation to the suppliers.

3.2 Characterizing the Optimum

We now turn to our main results. Before identifying the optimal contest for the buyer, we first show that bonus tournaments can implement a wide range of allocations.

Proposition 1 *Any* (v_1, v_2) *such that* $0 < v_1 \leq 1/2 \leq v_2 < 1$ *can be implemented by a bonus tournament with sufficiently high subsidies. In particular, the social optimum can be implemented.*

Thus, the buyer can implement any desired diversity in a bonus tournament. The proof shows that implementation works with $\mathcal{P} = \{A, 0\}$ and $A = \Delta q(v_1, v_2)$, so that A is the corresponding maximal quality difference. For instance, to induce the social optimum, the buyer has to set $A = \Delta q(v_i^*, v_j^*)$. The equilibrium pricing strategies turn out to be $p_1(), p_2()$ such that $p_i(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise.[25] The supplier only asks for the bonus A when his quality advantage is maximal ($\sigma \in [0, v_1] \cup [v_2, 1]$); otherwise he accepts

[23] If $\mathcal{P}$ is convex and $\sup \mathcal{P} > \overline{p}(\sigma)$ for all σ, then $p_i(q_i, q_j) = \underline{P}$ for $q_i < q_j$ in every equilibrium. To see this, note that, according to Lemma 2, $p_j = \overline{p}(\sigma) = \underline{P} + q(v_j, \sigma) - q(v_i, \sigma)$ in any equilibrium for the high-quality supplier j. If $p_i > \underline{P}$, then j can choose a slightly higher prize, and he still wins. Hence, this is a profitable deviation.

[24] Here and in the following, $q(v_i, v_i) = q(v_i, \sigma)|_{\sigma = v_i}$, etc.

[25] Implementation is not unique, and a bonus tournament will generally admit many equilibria. In particular, if $v_i^* < v_j^*$ is an equilibrium of a bonus tournament, then any v_i, v_j such that $|v_i - v_j| = |v_i^* - v_j^*|$ and $v_i \leq 1/2 \leq v_j$ are equilibria of the same bonus tournament. The natural equilibrium to focus on is the equilibrium which is symmetric around the mean, which will be the equilibrium that the principal will want to implement.

the low price. Therefore, the buyer pays the lowest price compatible with Lemma 3 for $\sigma \in [0, v_1] \cup [v_2, 1]$. Clearly, the price 0 is also minimal on (v_1, v_2). The bonus tournament is thus a flexible instrument with which the buyer can fine-tune diversity with low supplier revenues. This suggests that the optimal contest is in this class. However, this intuition is incomplete, as it does not account for subsidies. We now show that it is nevertheless always optimal for the buyer to use bonus tournaments. However, she will not always implement the social optimum.

Theorem 1 *(i) The buyer optimum can be implemented with a suitable bonus tournament* $(\{A, 0\}, t)$ *where the suppliers obtain an expected surplus of zero.*
(ii) If $C \geq F(v_1^*)\,\Delta q(v_i^*, v_j^*)$*, the optimal contest for the buyer is a bonus tournament that implements the social optimum, with subsidies used to ensure break even.*
(iii) If $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$*, the optimal contest for the buyer is a bonus tournament that implements suboptimal diversity.*

Whereas (i) states the optimality of bonus tournaments, (ii) and (iii) specify the details for the two different parameter regions. In both cases, the suppliers earn zero surplus. When research costs are high enough and quality differences in the social optimum are low (ii), the buyer implements the social optimum. When research costs are low, this is no longer true. Intuitively, diversity yields high quality for the buyer, but gives ex-post monopoly power to the supplier.[26] As a result, whenever there is a tradeoff between rent extraction and efficiency, the buyer resolves it in favor of rent extraction by distorting variety relative to the social optimum.

To understand the desirable properties of bonus tournaments, recall from Lemma 3 that in any contest implementing (v_1, v_2), the price $\Delta q(v_1, v_2) + \underline{P}$ has to be in the price set. This fixes the price that the buyer has to pay in any state of the world when the quality difference is maximal. What contest design can achieve, then, is to reduce prices paid in those states of the world when $\sigma \in (v_1, v_2)$, implying that the quality difference is not maximal. With a bonus tournament (A, a), the buyer commits herself not to pay prices between a and A in these states: Even when the quality difference is greater than a, she only pays a. Setting $a = 0$ clearly minimizes the revenues of the suppliers. The only remaining question is how much diversity the buyer optimally induces. Through the option value it generates, diversity can increase efficiency. However, it is costly for the buyer to induce. As mentioned before, the theorem shows that whenever there is a tradeoff between efficiency and rent extraction, the buyer sacrifices efficiency.

Participation fees: We now briefly discuss what would happen if buyers could charge participation fees $e > 0$. She would do this only if $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, in which case the optimal fee e^* satisfies $C + e^* = F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, so that she achieves the first-best.[27] With or without participation fees, the buyer thus designs the contest so that the suppliers exactly break even on expectation. Moreover, the bonus tournament is still optimal with participation fees. However, contrary to the case without participation fees, its incentive properties are used for efficiency reasons rather than for rent extraction.

[26] Recall that, to implement (v_1, v_2), the buyer sets $A = \Delta q(v_1, v_2)$. Thus, for small diversity the bonus price is small.
[27] If the buyer is limited to setting fees below e^*, she will charge the maximum allowable fee.

4 Auctions and Fixed Prize Tournaments

In Section 3.2, we characterized the optimal contest. We now study two other types of contests that are discussed in the literature, namely auctions and fixed prize tournaments.

Auctions generally have good incentive properties; for example, auctions are the optimal contest in the setting of Che and Gale (2003). On the other hand, fixed prize tournaments are very common innovation contests. Next, we examine how these contests perform in our setting, where the choice of research approaches is important.

Proposition 2 *(i) For any t such that the suppliers' participation constraints are met, the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implements the social optimum. (ii) For any $A \geq 2C$, the unique equilibrium of an FPT ($\mathcal{P} = \{A\}$) implements $(v_1, v_2) = (1/2, 1/2)$. (iii) Whenever $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, the buyer prefers the inefficient FPT to the efficient auction.*

Proposition 2(i) states that the auction induces the efficient amount of diversity. It is intuitively clear that an auction implements some diversity: With identical approaches, no supplier will earn a positive revenue. Any move away from the other supplier will lead to quality advantages in a measurable set of states and thereby to positive expected revenues. Auctions implement the socially efficient outcome because they align the externalities of the choice of an approach v_i with the private benefits. For example, fix some v_2 and consider a marginal change of v_1. Such a change generates externalities only in the states of the world for which the quality of supplier 2 is greater than the quality of supplier 1. Furthermore, the size of the externality is exactly the change in the quality difference. Since supplier 1 wins the auction only when his quality is higher and he bids exactly the quality difference, the private incentives and the externalities are aligned. While we prove Proposition 2(i) directly in Appendix 8.4, an analogous result also applies for more general state distributions and quality functions and for arbitrary numbers of suppliers. It also holds when suppliers are heterogeneous. This result extends beyond auctions to any type of institution that gives the chosen supplier a positive share of the quality difference to the next-best alternative.

Proposition 2(ii) states that an FPT induces no diversity at all. The intuition for the absence of diversity is straightforward and well-known; it corresponds to the principal of minimum differentiation in the standard model of locational competition with fixed prices (Hotelling, 1929) and to the median voter theorem (Downs, 1957).[28] As the size of the prize is independent of quality differences in an FPT, the suppliers care only about maximizing the expected winning probability. By (A2), this requires moving to the center. In particular, there is no diversity.

As to (iii), even though an auction implements the social optimum, it leaves rents to the successful supplier whenever research costs are low enough. Because it avoids such rents, the buyer may prefer to use a suitable FPT. A bonus tournament combines the advantages of FPTs and auctions: It can achieve the same efficiency as an auction, while reducing supplier rents, as the Figure 2 illustrates. If the realized state of the world is $\sigma \in [0, v_1] \cup [v_2, 1]$, the payment is the same in the auctions and in the bonus tournament by Lemma 3. However, for $\sigma \in (v_1, v_2)$ the winning supplier captures the entire quality difference in the auction, while in the bonus tournament the winning supplier receives only the low price a.

[28] However, the voting literature has also discussed why parties might differentiate by choosing "polarized platforms" (as in Wittman, 1977, 1983). On a broadly related note, the relative weight on accuracy and publicity of forecasts determines whether or not experts want to cluster on the most likely outcome (Laster et al., 1999).

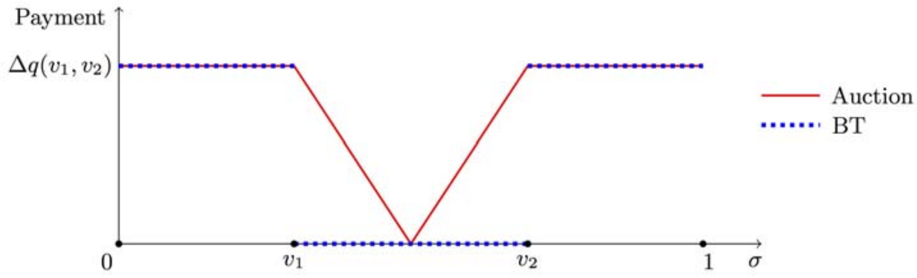

Figure 2: Comparison of payments in an auction and a bonus tournament.

The trade-off between efficiency and rent extraction also shows up when analyzing price ceilings in auctions.

Corollary 1 *An outcome (v_i, v_j) that is implemented in an auction with price ceiling $\bar{P}$ satisfies $\Delta q(v_i, v_j) \leq \bar{P}$. Thus diversity is bounded by the price ceiling.*

If the maximal quality difference between the two suppliers were above the maximum feasible bid, the supplier could not charge the buyer for this quality difference. He could thus choose an approach slightly closer to the competitor to increase his chances of winning without reducing the price.

Corollary 1 embeds the auction without price ceiling and the FPT as polar cases. In an auction without price ceiling, suppliers are free to choose the bid and thus capture the benefits of diversification. This results in optimal diversity. By Corollary 1, price ceilings limit this possibility: They determine an upper bound on equilibrium diversity. A reduction in the price ceiling leads to lower equilibrium diversity. Thus, the choice of the price ceiling involves a trade-off between efficiency-increasing diversity and market power for the suppliers. Consistent with the logic of Theorem 1(iii) and Proposition 2(ii), the following result shows that the buyer never resolves the trade-off in favor of efficiency when costs are low.

Corollary 2 *Let $C = 0$. Among all contests where $\mathcal{P}$ is convex, the buyer's surplus is maximal in an FPT with $A = 0$.*

The proof of Corollary 2 relies heavily on the fact that higher quality suppliers bid the sum of the quality differential and the minimum $\underline{P}$ whenever available (Lemma 2). Thus the buyer surplus, as the difference between the expected maximal quality and the expected payment, is the difference between the expectation of the minimum quality and the minimum bid. The buyer's best choice is an FPT with $A = 0$, because this maximizes the minimum quality and minimizes the minimum bid.

Remember that the price set in a bonus tournament is of the form $\{A, a\}$. The last result thus clearly underlines the role of non-convex price sets in a bonus tournament for the buyer optimum. A buyer who is confined to the class of contests with convex price sets (including auctions and FPTs) cannot profitably induce diversity.

5 Extensions

In this section, we extend the model in several directions and study the robustness of our main results. When suppliers have the option to shirk, the analysis is more subtle, because the buyer can no longer rely on subsidies. Nevertheless, we show that as long as the cost of effort is not too high, bonus tournaments are still optimal. Moreover, we show that, even with more than two suppliers, bonus tournaments still have desirable properties and are optimal contests under some circumstances. Finally, we study more general distributions and quality functions and several other extensions.

5.1 Inducing Effort and Diversity

The baseline model assumes that once a supplier joins a contest, his only choice is which research approach to pursue. Importantly, all approaches are equally costly, so that the supplier cannot shirk by reducing effort. Because of this property of the baseline model, it was possible to focus on implementing diversity in contests, while shutting down the effects of contest design on the incentives to induce effort. In this section, we consider contest design when both the effort choice and the research approach choice affect the final quality of the innovation. We show that our main result is robust as long as the cost of effort is not too high. In this case, a bonus tournament is an optimal contest for the buyer. However, for sufficiently high costs, we show that no contest can implement any diversity. Consequently, the optimal contest is an FPT.

In the baseline model, a supplier i was forced to choose an approach $v_i \in [0,1]$ and all approaches were equally costly. In this section, we allow the suppliers to shirk, by choosing an approach $\{-1\}$. That is, each supplier i chooses an approach $v_i \in \{-1\} \cup [0,1]$. The cost of shirking is zero, that is $C(-1) = 0$, while all other approaches are equally costly, i.e. $C(v) = C > 0$ for all $v \in [0,1]$. Shirking produces zero quality, irrespective of σ, while the quality of other approaches is determined as before. As in the main model, we assume that the value of the innovation Ψ is large enough, so that $q(v,\sigma) > q(-1,\sigma)$ for any $v \in [0,1]$ and all σ. One major difference to the baseline model is that subsidies cannot be used to induce agents to choose an approach $v \in [0,1]$, as a supplier could simply collect the subsidy and then shirk. Hence, the buyer will not use subsidies, and has to rely on expected revenues from the contest to induce the suppliers to exert effort.

The next result examines the optimal contests which induce both diversity and effort.

Proposition 3 *Suppose (A1) and (A2) hold and shirking is possible. Denote the social optimum with v_1^*, v_2^*.*
(i) If $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$, the optimal contest for the buyer is a bonus tournament with prizes $\mathcal{P} = \{A, 0\}$, which implements less diversity than socially optimal.
(ii) If $F(v_1^*)\,\Delta q(v_i^*, v_j^*) \le C \le F(v_2^*)\,\Delta q(v_i^*, v_j^*)$, the optimal contest for the buyer is a bonus tournament that implements the socially optimal diversity. The prizes are $\mathcal{P} = \{A, a\}$, where $A = 2C + \Delta q(v_i^*, v_j^*)/2$ and $a = 2C - \Delta q(v_i^*, v_j^*)/2$.
(iii) There exists $\bar{C}$ such that if $C > \bar{C}$, no contest can implement strict diversity and the optimal contest for the buyer is an FPT.

Part (i) of Proposition 3 is a direct implication of Theorem 1(iii), which shows that, even when subsidies are available, a bonus tournament without subsides is optimal if $C < F(v_1^*)\,\Delta q(v_i^*, v_j^*)$. Thus, a fortiori, when this condition holds, the buyer also optimally uses a bonus tournament when subsidies are not available. However, the buyer implements suboptimal diversity, and in equilibrium the expected revenues of the suppliers are equal to C, so that the suppliers just break even. In the baseline model of Section 2, when $C \ge F(v_1^*)\,\Delta q(v_i^*, v_j^*)$ the buyer implements the socially optimal diversity, and uses subsidies to make sure that the suppliers break even. However, in the current setting subsidies are not feasible. Proposition 3(ii) shows that the buyer can actually use the low price in the bonus tournament as an (imperfect) substitute for the subsidies. A positive low price a acts as a subsidy because it increases expected revenues of the suppliers. It is imperfect because, as we know from Lemma 3, $A - a = \Delta q(v_i^*, v_j^*)$ in any bonus tournament that implements the social optimum. Thus as a grows, the relative difference between the bonus and the low price $((A-a)/A)$ decreases; as a result, it becomes more attractive to deviate and increase the probability of winning the low price (at the cost of never winning the bonus price). This can be shown to imply that there is a bound to the revenue that can be given to the suppliers in a bonus tournament, while implementing the socially optimal diversity — which is given by $F(v_2^*)\,\Delta q(v_i^*, v_j^*)$. More generally, as the proof of 3(iii) shows, there is a bound on the revenue that can be given to suppliers in *any* contest implementing any (strict) diversity of research approaches. If the costs C are sufficiently high, then there does not exist any contest which implements any diversity of approaches when subsidies are not available, and the best the buyer can do is to use an FPT.

In the following, we simplify the analysis by assuming that the distribution of σ is uniform.

Assumption (A2)' $f(\sigma) = 1 \; \forall \sigma \in [0, 1]$.

In this case, Proposition 3 has a particularly simple interpretation.

Corollary 3 *Suppose research costs are* $C > 0$ *and that (A1) and (A2)' hold.*
(i) If $C < \frac{b}{8}$*, the optimal contest for the buyer is a bonus tournament with prizes* $\mathcal{P} = \{A, 0\}$*, which implements less diversity than socially optimal.*
(ii) If $\frac{b}{8} \le C \le \frac{3b}{8}$*, the optimal contest for the buyer is a bonus tournament that implements the socially optimal diversity. The prizes are* $\mathcal{P} = \{A, a\}$*, where* $A = 2C + b/4$ *and* $a = 2C - b/4$*.*
(iii) If $C \ge \frac{9b}{16}$*, the optimal contest for the buyer is an FPT.*

When the state distribution is uniform, we can explicitly derive the value of a bound above which no contest can implement diversity. Moreover, Corollary 3 reveals an alternative intuition for when the bonus tournament is optimal. For a fixed C an increase in b will eventually lead to a situation where a bonus tournament is optimal. Remember that b captures the cost (quality loss) resulting from an approach to innovation that is not ideal. Thus, when b is high, the suppliers know that the buyer is willing to offer high bonus prizes for diversity, which will in turn make it easier to induce suppliers to exert effort.

The main message of this extension is that bonus tournaments remain optimal contests for inducing diversity, as long as the effort costs are not too high. When the effort costs are high, we have shown that no contest can induce variety, leading to the optimality of FPTs.

5.2 The Number of Suppliers

In innovation contests there are usually more than two suppliers. For example, there were 49 registered competitors in the EU Vaccine Prize, 12 of which submitted final designs for evaluation.[29] We therefore now deal with the possibility that there are many suppliers. For simplicity, we also assume that the distribution of ideal states is uniform. Otherwise the model corresponds to the baseline case of Section 2. In this framework, we can characterize the social optimum and the equilibria of the main contests previously discussed. Though most

[29] European Commision (2014), "German company has won the EU's € 2 million vaccine prize." March 10, 2014 (accessed on April 3, 2015). http://ec.europa.eu/research/health/vaccine-prize_en.html

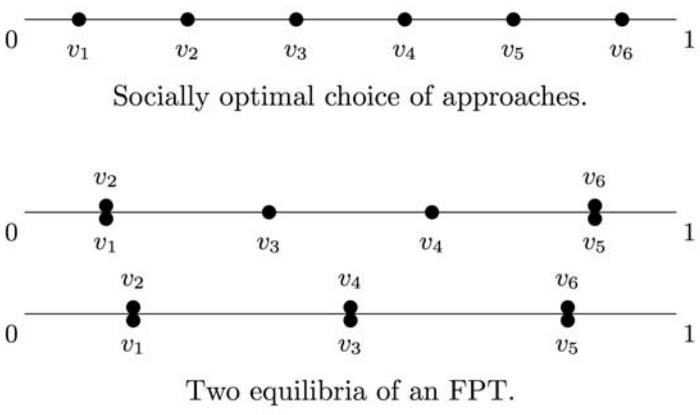

Figure 3: Equilibria when $n = 6$.

results also apply to the case $n = 3$, an FPT does not have a pure strategy equilibrium in this case.[30] To allow for simple formulations, we confine ourselves to $n > 3$. Finally, in this extension as well as in the case of more general distributions of the ideal state σ (Section 5.3.1), some suppliers will ex ante be in a more favorable position than others. Thus, the minimum subsidy needed to satisfy the participation constraints will generally be different across suppliers. To focus on the incentives of suppliers to diversify, we will allow the buyer to offer different subsidies to different suppliers. That is, for each supplier i, the buyer will offer a subsidy t_i. We consider the case of individualized subsidies here; we briefly sketch the main arguments for common subsidies ($t_1 = \cdots = t_n$) below.

Lemma 4 *Suppose there are $n > 3$ suppliers and (A1) and (A2)' hold.*
(i) The social optimum is $(v_1^*, ..., v_n^*) = (1/2n, 3/2n, 5/2n, ..., (2n-1)/2n)$.
(ii) The social optimum can be implemented with a suitable bonus tournament or with an auction.
(iii) In any equilibrium of an FPT with n suppliers, there is duplication, and the amount of diversity (defined as the distance between the highest and lowest approach) is inefficiently low. As n increases, the difference between the socially optimal diversity and the minimal diversity in any FPT equilibrium converges to zero.

Figure 3 illustrates the result for $n = 6$. In line with Lemma 4(i), there is no duplication in the social optimum, and the approaches are evenly spread. The buyer can implement the social optimum with a bonus tournament or an auction. The two other constellations describing the equilibria of the FPT highlight implications of Lemma 4(iii). First, the two most extreme approaches are not as far apart as the most extreme approaches of the social optimum; in this sense, there is less than optimal diversity. Second, there is duplication.[31]

Part (iii) of Lemma 4 is very closely related to familiar results for locational competition (Eaton and Lipsey, 1975). We nevertheless state it here for completeness because we are interested in the comparison between the different institutions.

[30] This has been observed for the equivalent Hotelling model with fixed prizes by Eaton and Lipsey (1975); see Shaked (1982) for a calculation of the mixed-strategy equilibrium.

[31] The remaining features of the depicted FPT hold in a class of FPT equilibria given in Lemma 10 in Appendix 8.6: The two most extreme approaches are always chosen by two suppliers. Moreover, depending on the specific equilibrium, there may be additional duplication for intermediate approaches.

Proposition 4 *Suppose there are $n > 3$ suppliers and (A1) and (A2)' hold.*
(i) If $C \geq b/2n^2$, a suitable bonus tournament is an optimal contest for the buyer.
(ii) If $C < b/2n^2$ the buyer strictly prefers to implement the social optimum with a bonus tournament rather than with an auction.
(iii) The buyer prefers to implement the social optimum with a bonus tournament over any outcome of an FPT whenever $C \geq b(n-4)/4n^2(n-2)$.

For the case $n = 2$, we know that a bonus tournament is an optimal contest for the buyer for any C. Furthermore, we know that when C is high, the buyer will implement the social optimum and extract the entire surplus. On the other hand, as C becomes lower, the buyer trades off efficiency for surplus extraction and implements less diversity than socially optimal. Proposition 4(i) is analogous to the first of these results for $n = 2$: Namely, when C is sufficiently high, the buyer can still use bonus tournaments to implement the social optimum and extract the surplus from the suppliers. For the case of lower C, we cannot characterize the optimal contest, as the tradeoff between surplus extraction and efficiency becomes more complicated. We therefore confine ourselves to the comparison of auctions and FPTs with a bonus tournament which implements the social optimum.

Proposition 4(ii) shows that the buyer still prefers to implement the social optimum with a bonus tournament rather than with an auction even when C is arbitrarily low. The intuition is the same as for $n = 2$, as a suitable bonus tournament requires lower ex post payments than an auction.

Proposition 4(iii) compares the bonus tournament implementing the social optimum with the (inefficient) FPTs. It says that when $C \in [b(n-4)/4n^2(n-2), b/2n^2)$, the buyer prefers bonus tournaments to FPTs, without claiming that bonus tournaments are optimal contests. For $C < b(n-4)/4n^2(n-2)$, an FPT can outperform a bonus tournament implementing the social optimum. Intuitively, to implement the social optimum the buyer has to give positive surplus to the suppliers, while an appropriately designed FPT can extract all surplus from the suppliers. However, $b(n-4)/4n^2(n-2)$ is equal to zero for $n = 4$ and it approaches zero as $n \to \infty$, so the parameter range in which FPTs outperform bonus tournaments is restricted.

With uniform subsidies ($t_1 = \cdots = t_n$), Lemma 4 still applies. Nevertheless, the analysis becomes more complicated. This reflects the fact that in certain contests the supplier revenues are not symmetric. When this is the case, in order to satisfy the participation constraint of suppliers with low revenues, uniform (as opposed to individual) subsidies must leave rents to suppliers with high revenues. Since the revenues in auctions and FPTs are more symmetric then those in bonus tournaments, the rents left to the suppliers will be lower. This makes it more likely that the buyer will prefer auctions and FPTs to bonus tournaments.

Lemma 4 has another simple but important implication: It may be socially optimal to invite a large number of suppliers. This differs from the case of contests that merely influence the suppliers' efforts: Several papers show that, in those settings, the optimal number of participants is typically two.

Corollary 4 *Suppose research costs are $C > 0$ and that (A1) and (A2)' hold. Define $n_-(C) = \max\left\{ n \in \mathbb{N} \middle| 2 \leq n \leq \sqrt{b} \middle/ 2\sqrt{C} \right\}$ and $n_+(C) = n_-(C) + 1$. Auctions or bonus tournament with $n_-(C)$ or $n_+(C)$ suppliers maximize total surplus in the set of all contests with an arbitrary number of suppliers.*

With straightfoward additional arguments, Corollary 4 is implied by the previous results. Lemma 4(i) characterizes the socially optimal allocation for given n, and auctions and bonus tournaments implement this allocation. Corollary 4 describes the number of suppliers that optimally balances the gains from higher expected quality against the losses from higher research costs. The result implies that the optimal number of suppliers increases in b and decreases in C. While the corollary is stated for the socially optimal contest, it is simple to show that the buyer can also often benefit from inviting more than two suppliers and that the comparative statics are similar. In particular, in a bonus tournament an increase in n leads not only to an increase in the expected quality (reflecting higher option value), but also to a reduction in rents that suppliers 1 and n obtain (reflecting an increase in competition).

5.3 Other Extensions

We now discuss several other extensions of the baseline model of Section 2. We deal with heterogeneous suppliers, multiple prizes and multiple research approaches of each supplier. In particular, the first issue is treated in much more detail in the working paper (Letina and Schmutzler 2015).

5.3.1 Generalized distributions and quality functions.

In this subsection, we assume that there are two suppliers, but we generalize the assumptions as follows:
Assumption (A1)' $q_i(v_i, \sigma) = \Psi - \delta(|v_i - \sigma|)$, where $\delta(|v_i - \sigma|)$ *is increasing and continuous.*
Assumption (A2)" *The density function $f(\sigma)$ is (i) symmetric and (ii) has full support: $f(\sigma) > 0\ \forall \sigma \in [0, 1]$.*

Thus, we relax the requirements that the distribution be single-peaked and relatively flat and that the distance function be linear. As in Section 5.2, we allow for individual subsidies t_1 and t_2.

Lemmas 2, 3 and Proposition 1 also hold under the relaxed assumptions (A1)' and (A2)". The proofs are analogous and are therefore omitted here. As a result, the main contests that we previously dealt with have the same properties as before:

Corollary 5 *Modify the baseline model by assuming that (A1)' and (A2)" hold. Then, (i) the bonus tournament ($\mathcal{P} = \{\Delta q(v_1^*, v_2^*), 0\}$) and the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implement the social optimum. Moreover, (ii) in any FPT ($\mathcal{P} = \{A\}$ for $A \geq 2C$), the unique equilibrium is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

We are not able to prove the optimality of the bonus tournaments in general when assumptions on distributions and quality functions are relaxed. However, as the next result shows, when C is high enough the bonus tournament is an optimal mechanism. Furthermore, we can compare bonus tournaments to auctions and FPTs and we can show that bonus tournaments generally perform better (from the buyer's perspective) then the other two institutions.

Proposition 5 *Modify the baseline model by assuming that (A1)' and (A2)" hold and denote with (v_1^*, v_2^*) the social optimum. (i) If $C \geq \max\{F(v_1^*)\,\Delta q(v_1^*, v_2^*),\ (1 - F(v_2^*))\,\Delta q(v_1^*, v_2^*)\}$ then a suitable bonus tournament is an optimal contest for the buyer. (ii) The buyer strictly prefers a suitable bonus tournament to the FPT whenever $C > 0$. (iii) The buyer prefers to implement the social optimum with a bonus tournament rather than with an auction. The preference is strict for low enough C.*

According to (ii), a suitable bonus tournament is still always preferable to an FPT in the more general set-up. On the other hand, (iii) shows that when implementing a social optimum, it is better for the buyer to use a bonus tournament than an auction. The intuition is similar to that in Section 5.2.

5.3.2 Heterogeneous Suppliers

The assumption of homogeneous suppliers simplifies the analysis. In many contexts, it is nevertheless natural to allow for exogenous heterogeneity: Suppliers may differ with respect to expertise or research capabilities. Architects may have different and essentially fixed styles. In Letina and Schmutzler (2015), we extend the model to allow for such exogenous heterogeneity. To this end, we consider a two-dimensional state space $[0, 1]^2$ to capture both exogenous and endogenous heterogeneity. A firm's choice of approach is still one-dimensional, however, corresponding to the first argument of a point in $[0, 1]^2$. The second argument is fixed by the identity of the firm, reflecting exogenous heterogeneity.[32] We focus on uniform state distributions and the case $C = 0$.

We show that the social optimum only involves diversification if exogenous heterogeneity is not too strong. As in the case of homogeneous suppliers with low research costs, however, fixed-prize tournaments do not induce any diversification, but buyers prefer them to auctions.

The framework with heterogeneous suppliers has an additional advantage: For sufficiently heterogeneous buyers, the modified framework allows us to use the alternative informational assumption that suppliers cannot observe qualities when they submit bids, which is intractable for homogeneous suppliers. We show that there is no diversification in this equilibrium for an auction.

5.3.3 Fixed-Prize Tournaments with Multiple Prizes

The US military research agency DARPA carried out various contests to foster the development of unmanned vehicles capable of navigating in rugged terrain. In the 2005 DARPA Grand Challenge, only the winner of the contest was eligible for the prize ($2 million), while the other contestants received nothing. This corresponds to an FPT as introduced above. However, in the subsequent DARPA contest, known as the 2007 Urban Challenge, rules specified that not only would the winner receive a prize (which was again $2 million), but the next two participants would also receive prizes ($1 million and $0.5 million).[33] While a full analysis is beyond the scope of this paper, we can show that a buyer is worse off in an FPT with two prizes than with a single prize.[34] The following result shows that the buyer has nothing to gain from using multiple prizes.

Lemma 5 *In the model with $n > 3$ players of Section 5.2, suppose that t is sufficiently large. For any equilibrium in an FPT with two prizes $A_1 > A_2 > 0$, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off.*

Clearly, when there are only two suppliers, a second prize has no effect, as the suppliers would consider it as a pure subsidy, and the effective prize would be the difference between

[32] Specifically, firm 1 (2) lives on the lower (upper) edge of $[0, 1]^2$.

[33] See Section 1.4 of the DARPA Urban Challenge Rules (2007) (accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge/docs/Urban_Challenge_Rules_102707.pdf

[34] The results can be extended to more than two prizes.

the first and the second prize. The proof of Lemma 5 shows that any equilibrium of an FPT with two prizes involves more duplication than the chosen equilibrium of an FPT with a single prize, which leads to a lower buyer surplus. This result suggests that multiple prizes do not improve diversity.[35]

5.3.4 Multiple Designs by the Same Supplier

We have assumed so far that each supplier can only develop a single approach. However, in the 2005 DARPA Grand Challenge, vehicles designed by the Red Team from Carnegie Mellon University took the second and third place. By developing multiple designs, a supplier internalizes some of the resulting option value. It is thus natural to allow for multiple approaches of different suppliers. The modified model is analytically intractable, but a numerical analysis suggest that our main results are robust. We study the cases with $n \in \{2, ..., 5\}$ suppliers, each of which can develop $m = 2$ approaches, and the case with $n = 2$ suppliers, each of which can develop $m = 3$ approaches. We assume that (A1)' and (A2) hold and that $C = 0$. We also fix values of Ψ and b.[36]

Numerical Result: Modify the framework of Section 5.2 with $n > 3$ suppliers by assuming that each of them develops m approaches, then: (i) Both a bon*us tournament and an auction implement the socially optimum described in Lemma 4(i), with n replaced by $n \cdot m$. (ii) In an FPT, there exists an equilibrium which is identical to the maximally duplicative equilibrium of an FPT with $n \cdot m$ suppliers, each of which develops one approach.*

The notion of a maximally duplicative equilibrium is made precise in Lemma 10 in Appendix 8.6: There, we consider a class of equilibria where maximal duplication occurs when each active research approach is chosen by two suppliers. While the analysis is clearly incomplete, the numerical result suggests that the case where n suppliers each develop m approaches can be analyzed using the framework where $n \cdot m$ suppliers each develop one approach (see Section 5.2).

6 Relation to the Literature

This paper contributes to the literature on optimal contest design, especially the design of innovation contests. The existing design literature focuses exclusively on effort incentives. In models of fixed-prize tournaments, Taylor (1995) shows that free entry is undesirable, and Fullerton and McAfee (1999) show that the optimal number of participants is two. Fullerton et al. (2002) find that buyers are better off with auctions rather than fixed-prize tournaments. In a very general framework, Che and Gale (2003) show that an auction with two suppliers is the optimal contest. Contrary to the previous literature, our paper focuses on the suppliers' choice of research approaches rather than on effort levels. We characterize the optimal contests in such settings, highlighting in particular the useful role of bonus tournaments.

Letina (2016) also studies the diversity of approaches to innovation, but the objects of analysis and the employed models are very different. He focuses on a market context with

[35] Of course, there may be reasons outside of the model which would make multiple prizes a desirable choice for a contest designer. For example, if suppliers are risk averse, providing multiple prizes may be a way of increasing their expected utility.

[36] For details and the code used to obtain numerical results, see Supplementary Material, available at https://sites.google.com/site/iletina/research.

anonymous buyers, and he deals with comparative statics rather than optimal design. In particular, the paper finds that a merger decreases the diversity of approaches to innovation.

While we are not aware of any other paper that considers optimal contest design when diversity plays a role, some authors compare contests in related, but different settings. In Ganuza and Hauk (2006), suppliers choose both an approach to innovation and a costly effort.[37] However, these authors focus exclusively on fixed-prize tournaments, while we study the optimal contest design. Erat and Krishnan (2012) analyze a fixed-prize tournament where suppliers can choose from a discrete set of approaches.[38] The authors find that suppliers cluster on approaches delivering the highest quality. This result is related to our result that there is duplication of approaches in the equilibria of fixed-prize tournaments. In addition to allowing for alternative contests, our model also considers correlated rather than independent qualities; it is thus meaningful to speak of similar approaches.[39] Schöttner (2008) considers two contestants who influence quality stochastically by exerting effort. She finds that, for large random shocks, the buyer prefers to hold a fixed-prize tournament rather than an auction to avoid the market power of a lucky seller in an auction. This resembles the trade-off underlying our Proposition 2. However, her analysis does not speak to optimal design and the role of bonus tournaments. It also does not address the setting with $n > 3$ suppliers.[40]

Gretschko and Wambach (2016) analyze the design of mechanisms for public procurement when exogenously differentiated suppliers offer different specifications, and the buyer does not know her preferences. The modelling of buyer utility is similar to ours, except that the authors use a Salop circle rather than a Hotelling line. Contrary to our study, however, the paper does not deal with the question of inducing variety. Instead the authors ask whether intransparent negotiations or transparent auctions yield higher social surplus.

Our paper is also related to the literature on innovation contests with exponential-bandit experimentation (see Halac, Kartik and Liu (2016) and references therein). In these models, it is uncertain whether the innovation is feasible. Suppliers participating in the contest expend costly effort to learn the state, and they also learn from the experimentation of their opponents. The goal of the contest is to induce experimentation. However, each supplier experiments in the same way. In our model, experimentation arises at the industry level for suitable contests, as the heterogeneity of approaches allows the buyer to pick the best available choice.

More broadly, our paper is related to the literature on policy experimentation. For instance, Callander and Harstad (2015) show that decentralized policy experimentation yields too much diversity. In their model, there are two heterogeneous political districts which choose whether or not to experiment with policy. In addition, they choose which policy to experiment with. A policy experiment is successful with some probability and a successful experiment

[37] In Ganuza and Pechlivanos (2000), Ganuza (2007) and Kaplan (2012), the buyer has to choose the design or alternatively can reveal information about the preferred design.

[38] See also Terwiesch and Xu (2008) for the effect of number of suppliers when exogeneous random shocks are large. For empirical evidence see Boudreau, Lacetera and Lakhani (2011).

[39] See also Konrad (2014) for a variant of Erat and Krishnan's model where first best is restored if the tie-breaking is decided via costly competition (for example lobbying) as opposed to randomly.

[40] More broadly related is Bajari and Tadelis (2001) who do not deal with innovations, but with construction projects. The issue of the right approach to the problem arises in such settings as well. The supplier obtains new information during the period when the contract is being executed, which allows him to adapt the original approach at some cost. Since the relationship is between a buyer and only one supplier, the question of diversity of approaches does not arise. This is also true for the related work by Arve and Martimort (2016) who study risk-sharing considerations in the design of contracts with ex-post adaptation. Additionally, Ding and Wolfstetter (2011) consider a case where a supplier can choose to bypass the contest and negotiate with the buyer directly in an environment where innovation quality is obtained by expending costly effort.

increases the value of that policy (for all districts) by a fixed amount. Since experimentation is costly, there is a free riding problem, which is especially severe when the districts want to experiment with similar policies. To reduce the free riding problem, a district will choose to experiment with a policy which is not desirable from the perspective of the other district. Hence, in equilibrium the policy experiments will be inefficiently diverse. Next, they show that centralization of political power can improve the outcome by reducing diversity. Contrary to our model, Callander and Harstad (2015) assume that the success probabilities of different experiments are independent, no matter how similar the policies are. This assumption removes the option value of having different experiments, which is central to our model. If there existed an ideal policy (in terms of quality) as in our model, then the option value would have to be traded off against the benefits of convergence emphasized by Callander and Harstad (2015). It would be interesting to see whether and how centralization would help to resolve this trade off.

In a related paper, Bonatti and Rantakari (2016) consider a setting where two agents choose which project to develop. To successfully develop a project, an agent exerts effort until a success occurs. For a successful project to be adopted (and yield a positive payoff) both agents have to consent to the adoption. By assumption, the agents have opposite preferences over the set of projects. The agents have an incentive to pursue extreme projects (which they like the most) but the veto power of the other agent forces them to compromise. As in Callander and Harstad (2015) the success of one approach is unrelated to the success of any other approach. This removes the option value of diversity that we identify in our paper.

7 Conclusions and Discussion

The ideal approach to solving an innovation problem is usually unknown to suppliers and buyers. Our paper investigates the implications of this uncertainty for contest design. Under very general conditions, it is socially optimal for suppliers to take diverse research approaches, and the social optimum can be obtained with both bonus tournaments and auction mechanisms. Inducing diversity of approaches to innovation is costly for the buyer. To reduce supplier rents, she may therefore want to induce suboptimal diversification. Bonus tournaments are in the set of optimal contests under quite general conditions. The difference between the bonus and the low price provides incentives for suppliers to diversify, which allows the buyer to fine-tune the amount of diversity induced. At the same time, bonus tournaments minimize the power of suppliers to exploit their quality advantage. The non-convexity of the price set is decisive for this feature.

Our results have practical implications for the design of innovation contests. While today most innovation contests feature fixed prizes, our results suggest that a better outcome could be achieved if an additional bonus prize was paid whenever the winner outperformed the second-best contestant by a sufficient margin. Such bonus prizes would be easy to implement and would not make the innovation tournaments significantly more complicated then they are today. Bonus prizes would give incentives to contestants to not only win the contest, but to win with a large margin. Our model suggests that this incentive would lead to an increase in the diversity of approaches to innovation.

Beyond innovation contests, our model can be used to analyze how institutions affect the incentives for experimentation when the optimal approach to solving a given problem is not known. We can think of our model as capturing product choice in markets with a unit mass of homogeneous buyers, each of which has unit demand. We can then interpret the uncertainty about the ideal state in two ways. First, it may reflect uncertainty about the buyers' taste. Second, it may capture an "engineering uncertainty" where the suppliers know what the buyers would like, but are uncertain about how to achieve this. Our results imply that an unregulated market maximizes expected total surplus.[41] The unregulated market gives incentives for firms to optimally diversify, but leaves them with market power. The trade-off resembles the one between ex-ante incentives and ex-post monopoly power in the innovation literature. In our case, however, the higher expected quality from the unregulated market does not result from higher innovation incentives at the individual firm level, but rather from the higher diversification incentives at the market level. Our results point to a novel source of potential inefficiency stemming from price regulation: Firms in industries with regulated competition will be less likely to sufficiently experiment by introducing diverse products. At the same time, this result also points to the importance of vigorous competition. The incentive to diversify would be diminished if firms colluded or divided the market.

[41] An unregulated market has analogous properties to an unrestricted auction in our contest setting.

8 Appendix

8.1 Basics

In the following, we introduce some notation that we use throughout the Appendix. We also formulate the restrictions implied by subgame perfection.

8.1.1 Notation

We consistently use subscripts B for buyers, $i = 1, 2$ for suppliers and T for "total" (buyers plus suppliers). Superscripts such as fpt for fixed-price tournament, bt for bonus tournament or a for auction refer to the contest $\mathcal{P}$ under consideration. We will drop these superscripts whenever there is no danger of confusion.

1. $p_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b,\Psi]^2}$ is a *price strategy function.*[42]

2. $\pi_i(p_i, p_j|\, q_i, q_j)$ is the realized revenue that supplier i earns with prices p_1 and p_2, conditional on qualities q_1 and q_2, assuming that the buyer chooses the i sequentially rationally, i.e., the i that maximizes $q_i - p_i$ in contest $\mathcal{P}$.[43]

3. $\widehat{\Pi}_i(v_i, v_j, p_i, p_j)$ is the expectation over $\pi_i(p_i, p_j|\, q_i, q_j)$ when suppliers choose v_1,v_2, $p_1()$ and $p_2()$, where the expectation is taken over all pairs of quality realizations for given (v_1, v_2).

4. $\Pi_i^{\mathcal{P}}(v_i, v_j) = \widehat{\Pi}_i(v_i, v_j, p_i, p_j)$, where $p_i()$ and $p_j()$ are the subgame equilibria for the contest $\mathcal{P}$ as in Lemma 2, is the *(expected) revenue* of supplier i.

5. $S_i^{\mathcal{P}}(v_i, v_j) = \Pi_i^{\mathcal{P}}(v_i, v_j) + t - C$ is the *(expected) surplus* of supplier i.

6. $S_B^{\mathcal{P}}(v_i, v_j) = E_\sigma[\max\{q(v_1, \sigma), q(v_2, \sigma)\}] - \Pi_1^{\mathcal{P}}(v_i, v_j) - \Pi_2^{\mathcal{P}}(v_i, v_j) - 2t$ is the *(expected) surplus* of the buyer.

8.1.2 Subgame-Perfect Equilibrium

A subgame-perfect equilibrium of the innovation contest given by $\mathcal{P}$ consists of supplier strategies $s_i = (v_i, p_i) \in [0,1] \times \mathcal{P}^{[\Psi-b,\Psi]^2}$ and buyer strategies $\nu \in \{v_1, v_2\}^{(\mathcal{P}\times[\Psi-b,\Psi])^2}$ such that:

(DC1) ν_1 and v_2 are sequentially rational.

(DC2) $\pi_i(p_i(q_i, q_j), p_j(q_j, q_i)|\, q_i, q_j) \geq \pi_i(p_i', p_j(q_j, q_i)\,|q_i, q_j)$ for all $p_i' \in \mathcal{P}$,$(q_i, q_j) \in [\Psi - b, \Psi]^2$ (sequential rationality of supplier i)

(DC3) $\widehat{\Pi}_i(v_i, v_j, p_i(q_i, q_j), p_j(q_j, q_i)) \geq \widehat{\Pi}_i(v_i', v_j, \widetilde{p}_i(q_i, q_j), p_j(q_j, q_i))$ for all $v_i' \in [0,1]$ and all $\widetilde{p}_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b,\Psi]\times[\Psi-b,\Psi]}$ (best-response condition for supplier i).

[42] For sets X and Y, Y^X is the set of all mappings from X to Y.

[43] When $q_1 - p_1 = q_2 - p_2$, we appeal to tie-breaking rule (T1) below.

8.2 Proofs of Auxiliary Results (Section 3.1)

8.2.1 Proof of Lemma 1

Suppose, without loss of generality, that $v_1 \leq v_2$. The total surplus is

$$S_T(v_1, v_2) - 2C = \int_0^1 \max\{q(v_1,\sigma), q(v_2,\sigma)\} dF(\sigma) - 2C =$$

$$\Psi - b \begin{pmatrix} \int\limits_0^{v_1} (v_1 - \sigma)\, dF(\sigma) + \int\limits_{v_1}^{(v_1+v_2)/2} (\sigma - v_1)\, dF(\sigma) + \\ \int\limits_{(v_1+v_2)/2}^{v_2} (v_2 - \sigma)\, dF(\sigma) + \int\limits_{v_2}^{1} (\sigma - v_2)\, dF(\sigma) \end{pmatrix} - 2C.$$

This is a continuous function with a compact domain, hence it attains the maximum. Note that

$$\frac{\partial S_T(v_1, v_2)}{\partial v_1} = b\left(-2F(v_1) + F((v_1 + v_2)/2)\right) \tag{1}$$

$$\frac{\partial S_T(v_1, v_2)}{\partial v_2} = b\left(1 - 2F(v_2) + F((v_1 + v_2)/2)\right). \tag{2}$$

(1) and (2) imply that there are no boundary optima. To see this, first note that $\frac{\partial S_T(0,v_2)}{\partial v_1} > 0 \forall v_2 > 0$ and $\frac{\partial S_T(v_1,1)}{\partial v_2} < 0 \forall v_1 < 1$. Moreover $(v_1, v_2) = (0,0)$ and $(1,1)$ are both dominated by $(1/2, 1/2)$. Thus, the optimum must satisfy

$$-2F(v_1) + F((v_1 + v_2)/2) = 0 \tag{3}$$

$$1 - 2F(v_2) + F((v_1 + v_2)/2) = 0. \tag{4}$$

Together these conditions imply $F(v_2^*) = 1/2 + F(v_1^*)$.

For $v_1 \in [0, 1/2]$, let $g(v_1) = F^{-1}\left(F(v_1) + \frac{1}{2}\right)$. F^{-1} is well-defined because of (A2)(iii). Inserting $v_2 = g(v_1)$ in (3) and (4), the first-order conditions hold for $(v_1, v_2) = (v_1, g(v_1))$ if

$$v_1 = F^{-1}\left(\frac{F((v_1 + g(v_1))/2)}{2}\right). \tag{5}$$

(5) has at least one solution $v_1^* \in (0, 1/2)$. This holds because both sides of (5) are strictly increasing, and the r.h.s. is positive for $v_1 = 0$ and strictly less than 1/2 for $v_1 = 1/2$. Now consider $(v_1^*, v_2^*) = (v_1^*, g(v_1^*))$ such that $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Thus $F(v_2^*) = F(v_1^*) + 1/2$. Moreover, symmetry implies $v_1^* + v_2^* = 1$ and thus the r.h.s. of (5) is $F^{-1}\left(\frac{1}{4}\right)$, so that the first-order condition holds for (v_1^*, v_2^*).
Before proceedingm, we prove one intermediate step.

Lemma 6 *If A2 is satisfied, then $f(x) < 2f(y)$ for all $x, y \in [0,1]$.*

Proof: By the second fundamental of calculus, $f(1/2) = \int_0^{1/2} f'(x)\, dx + f(0)$. Since by (A2)(iv) $f'(x) < 2f(0)$ for all $x \in [0, 1/2]$, it follows that $f(1/2) < \int_0^{1/2} 2f(0)\, dx + f(0) <$

$2f(0)$. By (A2)(ii) $f(x) \leq f(1/2)$ and $f(0) \leq f(y)$ for all $x, y \in [0,1]$, the statement in the Lemma follows.□

Finally, consider the Hessian matrix

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial^2 S_T}{\partial v_1^2} & \frac{\partial^2 S_T}{\partial v_1 \partial v_2} \\ \frac{\partial^2 S_T}{\partial v_1 \partial v_2} & \frac{\partial^2 S_T}{\partial v_2^2} \end{bmatrix} \\ &= \begin{bmatrix} -2f(v_1) + \frac{1}{2} f((v_1+v_2)/2) & \frac{1}{2} f((v_1+v_2)/2) \\ \frac{1}{2} f((v_1+v_2)/2) & -2f(v_2) + \frac{1}{2} f((v_1+v_2)/2) \end{bmatrix}. \end{aligned}$$

First, H is negative definite at (v_1^*, v_2^*) if and only if $f(1/2) < 2f(v_1^*)$. To see this, note that $f(v_1^*) = f(v_2^*)$ and $f((v_1^* + v_2^*)/2) = f(1/2)$. Hence,

$$-2f(v_1^*) + \frac{1}{2} f((v_1^* + v_2^*)/2) = -2f(v_1^*) + \frac{1}{2} f(1/2) < 0 \Leftrightarrow f(1/2) < 4f(v_1^*).$$

In addition,

$$|H| = 4f(v_1^*) f(v_2^*) - (f(v_1^*) + f(v_2^*)) f((v_1^* + v_2^*)/2) = 4f(v_1^*)^2 - 2f(v_1^*) f(1/2).$$

This condition holds if and only if $f(1/2) < 2f(v_1^*)$, which holds by Lemma 6.

Second, H is negative definite $\forall (v_1, v_2)$ if $f(1/2) < 2f(0)$. To see this, note that $f(v)$ is minimized at $v = 0$ and maximized at $v = 1/2$. Hence, $f(1/2) < 2f(0) < 4f(0)$ implies

$$-2f(v_i) + \frac{1}{2} f\left(\frac{v_1 + v_2}{2}\right) \leq -2f(0) + \frac{1}{2} f\left(\frac{1}{2}\right) < 0 \; \forall i \in \{1, 2\}.$$

and

$$|H| = f(v_1)\left(2f(v_2) - f\left(\frac{v_1+v_2}{2}\right)\right) + f(v_2)\left(2f(v_1) - f\left(\frac{v_1+v_2}{2}\right)\right) > 0.$$

Therefore, $f(1/2) < 2f(0)$, which holds by Lemma 6, is a sufficient condition for (v_1^*, v_2^*) to be the unique global optimum.

8.2.2 Proof of Lemma 2

Step 1: Pricing subgame for $q_1 = q_2$.

Consider the equilibrium for the subgame defined by (v_1, v_2, σ) and the resulting quality vector (q_1, q_2). If $q_1 = q_2$, the standard Bertrand logic implies that $(\overline{p}(\sigma), \overline{p}(\sigma)) = (\underline{P}, \underline{P})$ is the unique equilibrium.

Step 2: Pricing subgame for $q_i > q_j$

Clearly, if $q_i > q_j$. the suggested strategy profile is a subgame equilibrium. To see that i must bid $\overline{p}(\sigma)$ in equilibrium, first suppose $p_i > \overline{p}(\sigma)$. If $p_i > p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier j wins. By setting $p_i = \overline{p}(\sigma) \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i can ensure that he wins, which is a profitable deviation by (T2). If $p_i > \overline{p}(\sigma)$ and $p_i \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i wins. By setting $p_j = \underline{P}$, supplier j can profitably deviate. If $p_i < \overline{p}(\sigma)$, supplier i can deviate upwards to $\overline{p}(\sigma)$. He then still wins by (T1), and revenues are higher.

8.2.3 Proof of Lemma 3

(i) The result is trivial for $v_1 = v_2$. For $v_1 < v_2$, we show that supplier 1 can profitably deviate to some $v_1' > v_1$ if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$. This immediately follows from the following two steps:
Step 1: If $v_1 < v_2$ and $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, there exists a deviation to $v_1' \in (v_1, v_2]$ such that the set of states in which supplier 1 wins after the deviation is a strict superset of the set of states in which the supplier wins before the deviation.
Before the deviation, by Lemma 2, if $\sigma \in [0, v_1]$, supplier 1 wins and $\overline{p}(\sigma) < \Delta q(v_1, v_2) + \underline{P}$. By continuity, $\exists\, v_1' \in (v_1, v_2]$ such that $\overline{p}(\sigma) < \Delta q(v_1', v_2) + \underline{P} < \Delta q(v_1, v_2) + \underline{P}$. By deviating to v_1', supplier 1 wins whenever $\sigma < (v_1' + v_2)/2$ rather than when $\sigma < (v_1 + v_2)/2$. Step 1 thus follows.
Step 2: After this deviation, the buyer pays a weakly higher price than before.
For $\sigma \in [0, v_1]$, the price is unaffected. For $\sigma \in (v_1, (v_1' + v_2)/2]$, the price is at least as high as before the deviation. Thus, v_1' is a profitable deviation by (T2).
(ii) follows directly from Lemmas 2 and 3 (i).

8.3 Proofs of Main Optimality Results (Section 3.2)

8.3.1 Proof of Proposition 1

Let $A = \Delta q(v_1, v_2)$ for some (v_1, v_2). We will show that, in the bonus tournament with $P = \{A, 0\}$ and sufficiently high subsidies, the strategy profiles $(v_1, v_2, p_1(), p_2())$ such that $p_i(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise, form an equilibrium.

Sequential rationality of $p_i()$ follows from Lemma 2. We now show that $(v_1, p_1())$is a best response of supplier 1 to $(v_2, p_2())$; the argument for supplier 2 is analogous. For $A = 0$, only $(v_1, v_2) = (1/2, 1/2)$ satisfies the above conditions. Thus, the statement for $A = 0$ will follow from Proposition 2(ii). If $v_1 < v_2$, $\Delta q(v_1, v_2) > 0$, and the probability that supplier 1 wins with a positive prize is $F(v_1)$. Deviating to $v_1' < v_1$ is not profitable, because the winning probability falls to $F(\widehat{v}_1)$, with $\widehat{v}_1 < v_1$ implicitly defined by $q(v_1', \widehat{v}_1) - q(v_2, \widehat{v}_1) = \Delta q(v_1, v_2)$, and the prize does not rise. It is not profitable to deviate to $v_1'' \in (v_1, \widetilde{v})$, where $\widetilde{v} = \min(2v_2 - v_1, 1) \geq 1/2$: For such deviations, $\Delta q(v_1'', v_2) < \Delta q(\widetilde{v}, v_2) \leq \Delta q(v_1, v_2)\, \forall \sigma$, so that the probability of winning a positive prize is 0. Finally, if $\widetilde{v} < 1$, deviating to $v_1''' \in [\widetilde{v}, 1]$ is not profitable, because $\widetilde{v} \geq 1/2 + v_2 - v_1$ implies $1 - \widetilde{v} \leq 1/2 - (v_2 - v_1) \leq v_2 - (v_2 - v_1) = v_1$ and therefore, by symmetry of the state distribution, $1 - F(v_1''') \leq 1 - F(\widetilde{v}) \leq F(v_1)$. By analogous arguments, there are no profitable deviations for supplier 2.

By Lemma 1, the social optimal satisfies $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Clearly, it must be that $0 < v_1^* \leq 1/2 \leq v_2^* < 1$, and the social optimum can be implemented.

8.3.2 Proof of Theorem 1

The buyer optimally chooses $(v_1, v_2, p_1, p_2, \mathcal{P}, t) \in [0,1]^2 \times \mathcal{P}^{[\Psi - b, \Psi]^2} \times \mathcal{I}(\mathbb{R}^+) \times [0, +\infty)$ so as to maximize

$$S_T(v_1, v_2) - \widehat{\Pi}_1(v_1, v_2, p_1, p_2) - \widehat{\Pi}_2(v_1, v_2, p_1, p_2) - 2t$$

such that, for all $i \in \{1, 2\}$ and $j \neq i$, (DC1)-(DC3) hold and

$$\widehat{\Pi}_i(v_i, v_j, p_i, p_j) + t - C \geq 0 \text{ for all } i, j \in \{1, 2\} \text{ and } i \neq j. \tag{6}$$

(i) The statement follows from three lemmas. Lemma 7 shows that allocations maximizing buyer surplus satisfy the conditions of Proposition 1 and can thus be implemented by a bonus tournament. Lemma 8 shows that implementation requires lower expected transfer than any alternative; hence buyer surplus is maximal. Finally, Lemma 9 shows that the suppliers optimally break even on expectation.

Lemma 7 *If $\left(v_1^B, v_2^B, p_1, p_2\right)$ is an equilibrium of a contest that maximizes buyer surplus, then* $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$.

We prove this lemma in two steps.

Step 1: *If $\left(v_1^B, v_2^B, p_1^B, p_2^B\right)$ is an equilibrium where w.l.o.g. $v_1^B \leq v_2^B$, then $v_1^B \leq 1/2 \leq v_2^B$.*

Proof: We will show that $v_1 \leq 1/2 \leq v_2$ must hold in any contest equilibrium. Suppose, to the contrary, that $v_1 \leq v_2 < 1/2$. The case that $1/2 < v_1 \leq v_2$ follows analogously. Let p_1, p_2 be the associated pricing strategies. Then, the expected revenue of supplier 1 is $\Pi_1(v_1, v_2) = \int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. Consider the deviation $v_1' = 2v_2 - v_1 < 1$ with the same pricing function. Supplier 1 now wins whenever $\sigma > (v_2 + v_1')/2$. We can write the expected revenue as $\Pi_1(v_1', v_2) = \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma) + \int_{2v_2}^{1} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. Clearly, $(v_1 + v_2)/2 = 2v_2 - \frac{v_1'+v_2}{2}$. Moreover, there exists a bijective mapping $[0, (v_1 + v_2)/2] \to [(v_1' + v_2)/2, 2v_2]$; $\sigma' \mapsto \sigma''$ such that $q(v_1, \sigma') - q(v_2, \sigma') = q(v_1', \sigma'') - q(v_2, \sigma'')$ and $f(\sigma') \leq f(\sigma'')$. Thus $\int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma) \leq \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$. As a result, $\Pi_1(v_1, v_2) \leq \Pi_1(v_1', v_2)$ and v_1' leads to strictly higher probability of winning, hence v_1' is a profitable deviation.[44] Thus, $v_1 \leq 1/2 \leq v_2$ must hold in any equilibrium; in particular, therefore $v_1^B \leq 1/2 \leq v_2^B$.

Step 2: *If $\left(v_1^B, v_2^B, p_1^B, p_2^B\right)$ is an equilibrium maximizing buyer surplus, then $0 < v_i^B < 1$ for $i \in \{1, 2\}$.*

Proof: By Step 1, we know that $v_1 \leq 1/2 \leq v_2$. Suppose $v_1^B = 0$ and $v_2^B = 1$. We will distinguish two cases, $C = 0$ and $C > 0$. First suppose $C = 0$. By single-peakedness (A2), $v_1 = v_2 = 1/2$ results in weakly higher total surplus than $\left(v_1^B, v_2^B\right)$. As the allocation $(v_1, v_2) = (1/2, 1/2)$ can be implemented with an FPT and $A = 2C$ by Proposition 2(ii), the buyer would be strictly better off than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$ where the suppliers earn positive surplus. Finally, observe that $v_1^B = 0$ and $v_2^B = 1$ cannot be implemented so that the suppliers earn zero surplus, as the suppliers could increase their probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. Next suppose $C > 0$. There exists some small ε such that $S_T\left(v_1^B = 0, v_2^B = 1\right) < S_T(\varepsilon, 1 - \varepsilon)$ and $F(\varepsilon)\Delta q(\varepsilon, 1 - \varepsilon) < C$. But then a bonus tournament with subsidy $t' = C - F(\varepsilon)\Delta q(\varepsilon, 1 - \varepsilon)$, and $\mathcal{P} = \{\Delta q(\varepsilon, 1 - \varepsilon), 0\}$ implements $(\varepsilon, 1 - \varepsilon)$, achieves higher total surplus, and the supplier surplus not higher than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$. Hence, the buyer surplus is higher, which is a contradiction.

Next suppose $v_1 = 0$ and $v_2 < 1$ (the case that $v_1 > 0$ and $v_2 = 1$ follows analogously). By Lemma 2, the revenue is $\Pi_1(0, v_2) = \int_0^{\frac{v_2}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma)$ for supplier 1 and $\Pi_2(v_2, 0) = \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma))\, dF(\sigma) + \int_{v_2}^{1} \bar{p}(q_2(\sigma), q_1(\sigma))\, dF(\sigma)$ for supplier 2. Moreover, $\Pi_1(0, v_2) > 0$, because otherwise supplier 1 could increase his probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. By single-peakedness (A2) it holds $\int_0^{\frac{v_2}{2}} \bar{p}(q_1(\sigma), q_2(\sigma))\, dF(\sigma) \leq \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma))\, dF(\sigma)$. Suppose that this equilibrium is implemented with transfers t such that $t + \Pi_1(0, v_2) \geq C$. This implies

[44] Given the tie-breaking rule T2, this is even true for $p = 0$.

$t+\Pi_2(v_2,0)>C$. Further, using (1), $dS_T\left(v_1^B,v_2^B\right)/dv_1^B\big|_{v_1^B=0}=bF(v_2/2)>0$, so that there exists some $\bar{\varepsilon}>0$ such that $S_T\left(\varepsilon,v_2^B\right)>S_T\left(0,v_2^B\right)$ for every $\varepsilon\in(0,\bar{\varepsilon})$. Fix ε such that $F(\varepsilon)\Delta q(\varepsilon,v_2)\leq\Pi_1(0,v_2)$ and $F(\varepsilon)<1-F(v_2)$. Let $t'=t+\Pi_1(0,v_2)-F(\varepsilon)\Delta q(\varepsilon,v_2)$. Now consider a bonus tournament with subsidy t' and $\mathcal{P}=\{\Delta q(\varepsilon,v_2),0\}$. By Proposition 1, this bonus tournament will implement (ε,v_2) if the participation constraint is met. This condition holds for both suppliers, because $t'+(1-F(v_2))\Delta q(\varepsilon,v_2)>t'+F(\varepsilon)\Delta q(\varepsilon,v_2)\geq C$. Compared to the original situation with $v_1=0$ and $v_2<1$, the rent of supplier 1 is unchanged, but the rent of supplier 2 decreases since $\int_{\frac{v_2}{2}}^{v_2}\bar{p}(q_2(\sigma),q_1(\sigma))\,dF(\sigma)+t>t'$ and $\int_{v_2}^{1}\bar{p}(q_2(\sigma),q_1(\sigma))\,dF(\sigma)>(1-F(v_2))\Delta q(\varepsilon,v_2)$. Since the total surplus increases and the suppliers' surplus decreases, the buyer's surplus must increase. Therefore, the bonus tournament that implements (ε,v_2) increases the buyer surplus, which is a contradiction.$\Box$

Lemma 8 *If $\left(v_1^B,v_2^B,p_1^B,p_2^B\right)$ is an equilibrium of a contest maximizing buyer surplus, then it can be implemented by a contest with $\mathcal{P}=\{A,0\}$.*

Proof: From Proposition 1 and Lemma 7, we know that the bonus tournament with $A=\Delta q\left(v_1^B,v_2^B\right)$ implements $\left(v_1^B,v_2^B\right)$. It remains to be shown that the buyer cannot implement $\left(v_1^B,v_2^B\right)$ with lower expected total transfers with any other contest. First, suppose that $v_1^B+v_2^B=1$. By Lemmas 2 and 3, in any contest that implements $\left(v_1^B,v_2^B\right)$ the price paid by the buyer is exactly $\Delta q(v_1^B,v_2^B)+\underline{P}$ if $\sigma\in[0,v_1^B]\cup[v_2^B,1]$ and it is at least 0 if $\sigma\in\left(v_1^B,v_2^B\right)$. Thus, if $\Delta q(v_1^B,v_2^B)F(v_1^B)>C$, a bonus tournament implements $\left(v_1^B,v_2^B\right)$ with the lowest possible expected total transfers. If $\Delta q(v_1^B,v_2^B)F(v_1^B)\leq C$, a bonus tournament with an appropriate t implements $\left(v_1^B,v_2^B\right)$ with zero expected supplier surplus. Next, consider an arbitrary contest implementing $\left(v_1^B,v_2^B\right)$ with $v_1^B+v_2^B<1$ with subsidy t (the case $v_1^B+v_2^B>1$ is analogous). The surplus of supplier 1 is then $S_1=\Delta q(v_1^B,v_2^B)F(v_1^B)+\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}}\bar{p}(q_1(\sigma),q_2(\sigma))\,dF(\sigma)+t-C$, and for supplier 2 it is $S_2=\Delta q(v_1^B,v_2^B)(1-F(v_2^B))+\int_{\frac{v_1^B+v_2^B}{2}}^{v_2}\bar{p}(q_1(\sigma),q_2(\sigma))\,dF(\sigma)+t-C$. By similar arguments as in Lemma 7, $\Delta q(v_1^B,v_2^B)F(v_1^B)<\Delta q(v_1^B,v_2^B)(1-F(v_2^B))$ and $\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}}\bar{p}(q_1(\sigma),q_2(\sigma))\,dF(\sigma)\leq\int_{\frac{v_1^B+v_2^B}{2}}^{v_2}\bar{p}(q_1(\sigma),q_2(\sigma))\,dF(\sigma)$. Now consider a bonus tournament with $\mathcal{P}=\{\Delta q\left(v_1^B,v_2^B\right),0\}$ and $t'=\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}}\bar{p}(q_1(\sigma),q_2(\sigma))\,dF(\sigma)+t$. The surplus of supplier 1 now becomes $S_1'=S_1$ by construction. On the other hand, $S_2'\leq S_2$, but $S_2'>S_1'$. Thus, the proposed bonus tournament implements $\left(v_1^B,v_2^B\right)$ with lowest possible net supplier surplus, which implies that the buyer surplus is maximized.$\Box$

Lemma 9 *In the buyer optimum, the suppliers obtain zero expected surplus.*

Proof: The proof follows from the three steps below.
Step 1: *In an optimal contest $v_1^B+v_2^B=1$.*

Consider any (v_1,v_2) such that $v_1+v_2<1$. We show that $(v_1,v_2)\neq\left(v_1^B,v_2^B\right)$; the case $v_1+v_2>1$ follows analogously. By Step 1, the optimal outcome can be implemented by some $\mathcal{P}=\{A,0\}$ and $t\geq0$. The equilibrium values of p_i in this contest are zero if and only if $\sigma\in(v_1,v_2)$. Hence, the participation constraint for supplier 1 implies that $F(v_1)A+t\geq C$; thus $v_1+v_2<1$ implies $(1-F(v_2))A+t>C$. Now suppose the buyer implements $(v_1+\varepsilon, v_2+\varepsilon)$, where ε is sufficiently small. We know that $(v_1+\varepsilon, v_2+\varepsilon)$ can also be implemented with $\mathcal{P}=\{A,0\}$. Thus, we can write the buyer surplus as

$$S_B(\varepsilon) = S_T(v_1+\varepsilon, v_2+\varepsilon) - F(v_1+\varepsilon)A - (1-F(v_2+\varepsilon))A - 2t$$

for $\varepsilon \geq 0$. Thus

$$\frac{dS_B(\varepsilon)}{d\varepsilon} = dS_T(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon - Af(v_1+\varepsilon) + Af(v_2+\varepsilon).$$

Since $v_1+v_2<1$, single-peakedness and symmetry (A2) imply $f(v_1+\varepsilon) < f(v_2+\varepsilon)$. Thus $dS_B(\varepsilon)/d\varepsilon > dS_T(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon$. We will show that $dS_T(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon > 0$; because $F(v_1+\varepsilon)A + t > C$ and (for sufficiently small ε) $(1-F(v_2))A + t \geq C$, the buyer will thus be better off implementing $(v_1+\varepsilon, v_2+\varepsilon)$ than (v_1, v_2). Maximizing total surplus is equivalent to minimizing the expected distance

$$\begin{aligned} D(v_1+\varepsilon, v_2+\varepsilon) = & \int_0^{v_1+\varepsilon} (v_1+\varepsilon-\sigma) f(\sigma)d\sigma + \int_{v_1+\varepsilon}^{\frac{v_1+v_2}{2}+\varepsilon} (\sigma - v_1 - \varepsilon) f(\sigma)d\sigma \\ & + \int_{\frac{v_1+v_2}{2}+\varepsilon}^{v_2+\varepsilon} (v_2+\varepsilon-\sigma) f(\sigma)d\sigma + \int_{v_2+\varepsilon}^{1} (\sigma - v_2 - \varepsilon) f(\sigma)d\sigma. \end{aligned}$$

From this we obtain

$$\begin{aligned} \frac{dD(v_1+\varepsilon, v_2+\varepsilon)}{d\varepsilon} &= \int_0^{v_1+\varepsilon} f(\sigma)d\sigma - \int_{v_1+\varepsilon}^{\frac{v_1+v_2}{2}+\varepsilon} f(\sigma)d\sigma + \int_{\frac{v_1+v_2}{2}+\varepsilon}^{v_2+\varepsilon} f(\sigma)d\sigma - \int_{v_2+\varepsilon}^{1} f(\sigma)d\sigma \\ &= 2F(v_1+\varepsilon) + 2(F(v_2+\varepsilon)) - 2F\left(\frac{v_1+v_2}{2}+\varepsilon\right) - 1. \end{aligned}$$

We will show that this expression is negative for $v_1+v_2<1$ and sufficiently small ε. To see this, fix any v_2 such that $1/2 \leq v_2 < 1$. Note that $h(v_1,v_2) \equiv \left.\frac{dD(v_1+\varepsilon, v_2+\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = 0$ for $v_1 = 1-v_2$. Furthermore

$$\frac{\partial h}{\partial v_1} = 2f(v_1) - f\left(\frac{v_1+v_2}{2}\right) > 0,$$

were the last inequality follows by Lemma 6. Thus, $v_1+v_2<1$ implies $2F(v_1) + 2(F(v_2)) - 2F((v_1+v_2)/2) - 1 < 0$ and thus $dD(v_1+\varepsilon, v_2+\varepsilon)/d\varepsilon < 0$ for small enough ε. This in turn implies that $S_T(v_1+\varepsilon, v_2+\varepsilon)$ increases in ε so that buyer surplus also increases in ε.

Step 2: *The buyer surplus when implementing any $(v_1, 1-v_1)$ with fixed t is strictly convex in v_1.*

Arguing as in the proof of Proposition 1, the buyer surplus when implementing any $(v_1, 1-v_1)$ with fixed t can be expressed as

$$S_B(v_1, 1-v_1) = 2\left[\int_0^{v_1} (\Psi - b(1-v_1-\sigma))\, dF(\sigma) + \int_{v_1}^{1/2} (\Psi - b(\sigma - v_1))\, dF(\sigma)\right].$$

Straightforward calculations show that $\frac{\partial^2 S_B(v_1, 1-v_1)}{\partial v_1^2} = 2f(v_1) + 2v_1 f'(v_1) - f'(v_1) \geq 2f(v_1) - f'(v_1) > 0$, where the last inequality follows from (A2)(iv).

Step 3: *In the buyer optimum, suppliers earn zero expected surplus.*

From Proposition 1 and Step 1 we know that the buyer optimum can be implemented by a suitable bonus contest $\mathcal{P} = \{\Delta q\left(v_1^B, 1-v_1^B\right), 0\}$ and some transfer t. This implies that the suppliers have symmetric payoffs. Suppose, in contradiction to the statement above, that the suppliers do not break even on expectation. If $t > 0$, the buyer can increase her surplus by marginally reducing t. Hence, it must be that $t = 0$. Then, the supplier payoff is $F\left(v_1^B\right)\Delta q\left(v_1^B, 1-v_1^B\right) - C > 0$ and thus $\Delta q\left(v_1^B, 1-v_1^B\right) > 0$. Thus $v_1^B < 1/2 < 1 - v_1^B$. By Step 2 of Lemma 7 we know that $v_1^B > 0$. Hence, $v_1^B \in (0, 1/2)$. Since $F\left(v_1^B\right)\Delta q\left(v_1^B, 1-v_1^B\right) - C$ is a continuous function, then there exists $\varepsilon > 0$, such that $F\left(v_1^B+\varepsilon\right)\Delta q\left(v_1^B+\varepsilon, 1-v_1^B-\varepsilon\right) - C \geq 0$ and $F\left(v_1^B-\varepsilon\right)\Delta q\left(v_1^B-\varepsilon, 1-v_1^B+\varepsilon\right) - C \geq 0$. But since $S_B\left(v_1, 1-v_1\right)$ is strictly convex, than either $S_B\left(v_1^B, 1-v_1^B\right) < S_B\left(v_1^B+\varepsilon, 1-v_1^B-\varepsilon\right)$ or $S_B\left(v_1^B, 1-v_1^B\right) < S_B\left(v_1^B-\varepsilon, 1-v_1^B+\varepsilon\right)$, a contradiction. Hence, suppliers earn zero expected surplus.□

(ii) Suppose $C \geq F\left(v_1^*\right)\Delta q\left(v_1^*, v_2^*\right)$. From Proposition 1 we know that for the proposed $\mathcal{P} = \{A, 0\}$, $\left(v_1^*, v_2^*\right)$ emerges in equilibrium; and the result also gives the pricing strategies p_1 and p_2. For $t = C - F\left(v_1^*\right)\Delta q\left(v_1^*, v_2^*\right)$, the buyer surplus in the proposed equilibrium is

$$\begin{aligned}&S_T\left(v_1^*, v_2^*\right) - \Pi_1\left(v_1^*, v_2^*\right) - \Pi_2\left(v_1^*, v_2^*\right) + 2t \\ =\ &S_T\left(v_1^*, v_2^*\right) - 2F\left(v_1^*\right)\Delta q\left(v_1^*, v_2^*\right) + 2\left(F\left(v_1^*\right)\Delta q\left(v_1^*, v_2^*\right) - C\right) = S_T\left(v_1^*, v_2^*\right) - 2C\end{aligned} \tag{7}$$

This is the highest surplus that the buyer can achieve without violating the suppliers' participation constraints.

(iii) Suppose $C < F\left(v_1^*\right)\Delta q\left(v_1^*, v_2^*\right)$. By Lemma 3, the minimum supplier revenue in any contest implementing $\left(v_1^*, v_2^*\right)$ is $F\left(v_1^*\right)\Delta q\left(v_1^*, v_2^*\right)$. Thus, in any such contest the suppliers would earn a positive expected surplus. By Part (i) this is suboptimal.

8.4 Proofs on Auctions and Tournaments (Section 4)

8.4.1 Proof of Proposition 2

(i) By Lemma 2, the unique equilibrium of the pricing subgame induced by q_1 and q_2 is $p_i = \max\{q_i - q_j, 0\}$ for $i, j \in \{1, 2\}$; $j \neq i$. Suppose that an auction does not implement the social optimum $\left(v_1^*, v_2^*\right)$. Then, for some i, there exists $\bar{v}_i \neq v_i^*$ such that $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$. Let $\Theta_i\left(v_i, v_j\right) = \{\sigma \in [0,1] |\, q\left(v_i, \sigma\right) \geq q\left(v_j, \sigma\right)\}$ and $\Theta_{-i}\left(v_i, v_j\right) = [0,1] \setminus \Theta_i\left(v_i, v_j\right)$. Thus $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$ if and only if

$$\int_{\Theta_i\left(\bar{v}_i, v_j^*\right)} \left(q\left(\bar{v}_i, \sigma\right) - q\left(v_j^*, \sigma\right)\right) dF\left(\sigma\right) > \int_{\Theta_i\left(v_i^*, v_j^*\right)} \left(q\left(v_i^*, \sigma\right) - q\left(v_j^*, \sigma\right)\right) dF\left(\sigma\right),$$

or equivalently

$$\begin{aligned}&\int_{\Theta_i\left(\bar{v}_i, v_j^*\right)} \left(q\left(\bar{v}_i, \sigma\right) - q\left(v_j^*, \sigma\right)\right) dF\left(\sigma\right) + \int_0^1 q\left(v_j^*, \sigma\right) dF\left(\sigma\right) > \\ &\int_{\Theta_i\left(v_i^*, v_j^*\right)} \left(q\left(v_i^*, \sigma\right) - q\left(v_j^*, \sigma\right)\right) dF\left(\sigma\right) + \int_0^1 q\left(v_j^*, \sigma\right) dF\left(\sigma\right)\end{aligned}$$

Splitting $[0,1]$ into $\Theta_i\left(\bar{v}_i, v_j^*\right)$ and $\Theta_{-i}\left(\bar{v}_i, v_j^*\right)$ in the first line and into $\Theta_i\left(v_i^*, v_j^*\right)$ and $\Theta_{-i}\left(v_i^*, v_j^*\right)$ in the second line and simplifying, this is equivalent with

$$\int_{\Theta_i\left(\bar{v}_i,v_j^*\right)} q\left(\bar{v}_i,\sigma\right) dF\left(\sigma\right) + \int_{\Theta_{-i}\left(\bar{v}_i,v_j^*\right)} q\left(v_j^*,\sigma\right) dF\left(\sigma\right) >$$
$$\int_{\Theta_i\left(v_i^*,v_j^*\right)} q\left(v_i^*,\sigma\right) dF\left(\sigma\right) + \int_{\Theta_{-i}\left(v_i^*,v_j^*\right)} q\left(v_j^*,\sigma\right) dF\left(\sigma\right).$$

and thus

$$\int_0^1 \max\{q(\bar{v}_i,\sigma), q(v_j^*,\sigma)\} dF\left(\sigma\right) > \int_0^1 \max\{q(v_i^*,\sigma), q(v_j^*,\sigma)\} dF\left(\sigma\right),$$

contradicting optimality of (v_1^*, v_2^*).

(ii) This follows from the more general statement in Corollary 5(ii) below.

(iii) Using Proposition 2(ii), any FPT such that the supplier breaks even has a unique equilibrium with $(v_1, v_2) = (1/2, 1/2)$. For $A = 2C$ and $t = 0$, the participation constraint of the suppliers binds. Hence, buyer surplus is maximized in the class of FPTs. It is

$$\begin{aligned} S_B^{fpt} &= \int_0^{1/2} \left(\Psi - b\left(\frac{1}{2} - \sigma\right)\right) f\left(\sigma\right) d\sigma + \int_{1/2}^1 \left(\Psi - b\left(\sigma - \frac{1}{2}\right)\right) f\left(\sigma\right) d\sigma - 2C \\ &= \Psi + \int_0^{1/2} b\sigma f\left(\sigma\right) d\sigma - \int_{1/2}^1 b\sigma f\left(\sigma\right) d\sigma - 2C \end{aligned}$$

The surplus of supplier 1 (supplier 2 follows by symmetry) is

$$\begin{aligned} S_1^a &= F\left(v_1^*\right) \Delta q\left(v_1^*, v_2^*\right) + \int_{v_1^*}^{1/2} \left(q\left(v_1^*,\sigma\right) - q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma \\ &= \frac{b\left(v_2^* - v_1^*\right)}{4} + \int_{v_1^*}^{1/2} \left(q\left(v_1^*,\sigma\right) - q\left(v_2^*,\sigma\right)\right) f\left(\sigma\right) d\sigma. \end{aligned}$$

Thus whenever $C < b\left(v_2^* - v_1^*\right)/4$, the participation constraint of the suppliers is satisfied even with $t = 0$. By Lemma 2, in an auction the winning supplier bids exactly the quality difference. This implies that the value the buyer receives, in any state of the world, is equal to the quality of the losing supplier. Then, the buyer surplus in an auction with $t = 0$ is

$$\begin{aligned} S_B^a &= \int_0^{1/2} \left(\Psi - b\left(v_2^* - \sigma\right)\right) f\left(\sigma\right) d\sigma + \int_{1/2}^1 \left(\Psi - b\left(\sigma - v_1^*\right)\right) f\left(\sigma\right) d\sigma \\ &= \Psi + \int_0^{1/2} b\sigma f\left(\sigma\right) d\sigma - \int_{1/2}^1 b\sigma f\left(\sigma\right) d\sigma - \frac{bv_2^*}{2} + \frac{bv_1^*}{2} \end{aligned}$$

The buyer prefers FPT to the auction if $S_B^{fpt} - S_B^a > 0$, which holds whenever $\frac{bv_2^*}{2} - \frac{bv_1^*}{2} - 2C > 0$ or equivalently $\frac{b\left(v_2^* - v_1^*\right)}{4} > C$.

When $\frac{b\left(v_2^* - v_1^*\right)}{4} < C$, the participation constraints require positive subsidies. In this case, the buyer implements the social optimum by using an auction with $t = C - \Pi_1^a$ with zero supplier surplus. Obviously this outperforms the inefficient FPT.

8.4.2 Proof of Corollary 2

Denote the minimum allowable price with $\underline{P}$. If $v_1 \neq v_2$ in equilibrium, by Proposition 2(ii), the contest is not an FPT. Suppose that $v_1 < v_2$. By Lemmas 2 and 3, the buyer pays $q_i - q_j + \underline{P}$ to the supplier with $q_i \geq q_j$ in equilibrium. Thus, for any σ, the buyer surplus is $\min\{q_1, q_2\} - \underline{P}$. Hence, the surplus of a buyer who induces $v_1 < v_2$ with $\underline{P}$ is

$$\begin{aligned} S_B(v_1, v_2; \underline{P}) &= \int_0^1 \min\{q_i(v_i, \sigma), q_j(v_j, \sigma)\} dF(\sigma) - \underline{P} \\ &= \int_0^{\frac{v_1+v_2}{2}} q_2(v_2, \sigma) dF(\sigma) + \int_{\frac{v_1+v_2}{2}}^1 q_1(v_1, \sigma) dF(\sigma) - \underline{P} \end{aligned}$$

Thus

$$\frac{dS_B}{dv_1} = \int_{\frac{v_1+v_2}{2}}^1 \frac{\partial q_1}{\partial v_1} dF(\sigma) > 0; \; \frac{dS_B}{dv_2} = \int_0^{\frac{v_1+v_2}{2}} \frac{\partial q_2}{\partial v_2} dF(\sigma) < 0.$$

Thus, the buyer surplus is maximal for $v_1 = v_2$ and $\underline{P} = 0$. Given $v_1 = v_2$, the buyer surplus is maximal for $v_1 = v_2 = 1/2$, the unique equilibrium of an FPT with A arbitrarily close to 0. Given (T2), it is an equilibrium for $A = 0$.

8.5 Extensions: Inducing Effort and Diversity (Section 5.1)

8.5.1 Proof of Proposition 3

Proof. (i) Follows directly from Theorem 1(iii).
(ii) As $F(v_1^*) \Delta q(v_i^*, v_j^*) \leq C$ by assumption, both prizes are weakly positive. For supplier 1, the expected profit of following the candidate equilibrium is $\Pi_1(v_1^*, v_2^*) = F(v_1^*) A + (1/2 - F(v_1^*)) a - C$. Inserting the values of A and a and $F(v_1^*) = 1/4$, $\Pi_1(v_1^*, v_2^*) = 0$. By symmetry, both suppliers just break even on expectation. Thus, the suggested allocation maximizes total surplus, with full rent appropriation by the buyer. It thus suffices to show that $\{A, a\}$ implements (v_1^*, v_2^*). Consider supplier 1. First, any deviation $v_1 = v_2^* + \varepsilon$ is dominated by $v_1' = v_2^* - \varepsilon$. Next, a deviation to $v_1' < v_1^*$ cannot increase expected supplier profit, as the probability of winning decreases and the price charged in any state of the world does not increase. Thus, the only remaining case is a deviation to $v_1' \in (v_1^*, v_2^*]$. The expected gross profit can be written as $\Pi_1(v_1', v_2^*) = aF((v_1' + v_2^*)/2)$. This is clearly increasing in v_1' and the profit of supplier 1 is at most $\Pi_1(v_1', v_2^*) = aF(v_2^*)$. The expected profit of following the candidate equilibrium is $\Pi_1(v_1^*, v_2^*) = F(v_1^*) A + (1/2 - F(v_1^*)) a$. Thus there is no profitable deviation to values just below v_2^* if and only if $F(v_1^*) A + (1/2 - F(v_1^*)) a \geq aF(v_2^*)$. Inserting the values of A and a and $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ shows that (v_1^*, v_2^*) is an equilibrium.
(iii) The proof proceeds in two steps. Step 1 shows that the expected revenues in any contest implementing $v_1 \neq v_2$ are bounded. Thus, for high enough costs, no contest can implement any $v_1 \neq v_2$. In this case, the best the principal can do is implement no diversity. Step 2 shows that this can be optimally done with a FPT.
Step 1: Suppose that a contest implements some $v_1 \neq v_2$, where $v_1, v_2 \in [0, 1]$. Denote with $\Pi_i(v_i, v_j)$ the expected revenue of supplier i. Suppose (w.l.o.g.) that $\Pi_1(v_1, v_2) \leq \Pi_2(v_2, v_1)$. By Lemma 2 of the buyer, supplier 1 wins the contest if $q_1(v_1, \sigma) \geq q_2(v_2, \sigma)$ and obtains the

price $p_1(v_1, v_2, \sigma) \leq q_1(v_1, \sigma) - q_2(v_2, \sigma) + \underline{P}$. But then, in any contest implementing v_1, v_2, we have

$$\Pi_1(v_1, v_2) \leq F(v_1)(\Delta q(v_1, v_2) + \underline{P}) + \int_{v_1}^{\frac{v_1+v_2}{2}} (q_1(v_1, \sigma) - q_2(v_2, \sigma) + \underline{P}) f(\sigma)\, d\sigma \tag{8}$$
$$\leq F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma + F\left(\frac{v_1+v_2}{2}\right)\underline{P} \equiv \overline{\Pi}_1(v_1, v_2, \underline{P})$$

where $\overline{\Pi}_1(v_1, v_2, \underline{P})$ is the upper bound on the supplier's revenue, given that a contest implements v_1, v_2 and the lowest feasible price is $\underline{P}$. Observe that by deviating to $v_2 - \varepsilon$, supplier 1 can obtain an expected payoff which is arbitrarily close to $F(v_2)\underline{P}$. Since the contest implements v_1, v_2 there can be no profitable deviations, so that $\Pi_1(v_1, v_2) \geq F(v_2)\underline{P}$. Therefore

$$\begin{aligned} F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma + F\left(\frac{v_1+v_2}{2}\right)\underline{P} &\geq F(v_2)\underline{P} \Leftrightarrow \\ \frac{F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} &\geq \underline{P} \end{aligned}$$

Using this upper bound for $\underline{P}$, we obtain that for any $\underline{P}$

$$\begin{aligned} \overline{\Pi}_1(v_1, v_2, \underline{P}) &\leq F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma \\ &\quad + F\left(\frac{v_1+v_2}{2}\right) \frac{F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} \\ &\leq \left(F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma\right) \frac{F(v_2)}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} \end{aligned}$$

To show that the RHS of the expression is finite for any v_1 and v_2, it suffices to show that the last expression converges to a finite value as $v_1 \to v_2$, in which case the denominator $F(v_2) - F\left(\frac{v_1+v_2}{2}\right)$ approaches zero. As the numerator also converges to 0 as $v_1 \to v_2$, Both are differentiable with respect to v_1 on the interval $(0, v_2)$ and $\frac{d}{dv_1}\left(F(v_2) - F\left(\frac{v_1+v_2}{2}\right)\right) \neq 0$ for $v_1 < v_2$. Hence, we can use L'Hôpitals Rule to evaluate the RHS as $v_1 \to v_2$. Standard calculations show that

$$\lim_{v_1 \to v_2} \left(F(v_1)b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma\right) \frac{F(v_2)}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} = \frac{bF(v_2)^2}{f(v_2)\frac{1}{2}}.$$

which is finite for any v_2. Thus $\Pi_1(v_1, v_2)$ is bounded. Then there exists $\bar{C}$ such that $\bar{C} > \Pi_1(v_1, v_2)$ for any contest and any $v_1 \neq v_2$, where $v_1, v_2 \in [0, 1]$, so that no supplier can break even in expectation in a contest without subsidies.

Step 2: By Step 1, diversity cannot be implemented if $C > \bar{C}$. Therefore, in an optimal contest both suppliers must choose the same research approach, or one supplier must shirk ($v_j = -1$). One supplier shirking generates higher surplus than both suppliers exerting effort and choosing the same approach, regardless of which approach is chosen. Furthermore, when only one supplier exerts effort, the surplus is maximized if that supplier chooses the approach 1/2. Consider an FPT with $A = C$. Then $v_i = 1/2$, $v_j = -1$ is clearly an equilibrium and the suppliers receive no rent. Thus, the FPT with $A = C$ is an optimal contest for the buyer. ■

8.5.2 Proof of Corollary 3

Proof. (i) and (ii) follow directly from Proposition 3(i) and (ii). For (iii), from the proof of 3(iii) above we have

$$\begin{aligned}
\overline{\Pi}_1 (v_1, v_2, \underline{P}) &\leq \left(F(v_1) b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) f(\sigma)\, d\sigma \right) \frac{F(v_2)}{F(v_2) - F\left(\frac{v_1+v_2}{2}\right)} \\
&\leq \left(v_1 b(v_2 - v_1) + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) d\sigma \right) 2 \frac{v_2}{v_2 - v_1}
\end{aligned}$$

While in the proof of Proposition 3, we showed that a bound exists, in this proof we need to find the bound. Let the RHS be $\overline{\Pi}_1 (v_1, v_2)$, the bound on the revenues for any $\underline{P}$. Assume (w.l.o.g.) that $v_1 + v_2 \leq 1$. Observe that $\overline{\Pi}_1 (v_1 + \varepsilon, v_2 + \varepsilon, \underline{P}) + \overline{\Pi}_2 (v_2 + \varepsilon, v_1 + \varepsilon, \underline{P})$ is constant in ε as long as $v_1 + \varepsilon, v_2 + \varepsilon \in [0, 1]$. In order to incentivize both suppliers to exert costly effort, the revenues of the supplier who is worse off will be binding. Since the suppliers are splitting a constant sum, the revenue of the worse off supplier is maximized when the payoffs are symmetric. That is,

$$\min \left\{ \overline{\Pi}_1 (v_1, v_2), \overline{\Pi}_2 (v_2, v_1) \right\} \leq \min \left\{ \overline{\Pi}_1 (v_1 + \bar{\varepsilon}, v_2 + \bar{\varepsilon}), \overline{\Pi}_2 (v_2 + \bar{\varepsilon}, v_1 + \bar{\varepsilon}) \right\}$$

for $\bar{\varepsilon} = (1 - v_1 - v_2)/2$. Thus, the upper bound on supplier revenue is determined by $v_1 + v_2 = 1$, which is assumed from now on. Substituting $v_2 = 1 - v_1$ into the expression above we obtain

$$\begin{aligned}
\overline{\Pi}_1 (v_1, v_2, \underline{P}) &\leq \left(v_1 b(1 - 2v_1) + \int_{v_1}^{\frac{1}{2}} b(1 - 2\sigma) d\sigma \right) 2 \frac{1 - v_1}{1 - 2v_1} \\
&\leq \frac{1}{2} b (2v_1 + 1)(1 - v_1)
\end{aligned}$$

The expression on the RHS is maximized for $v_1 = 1/4$ and is equal to $b(9/16)$, which represents the upper bound on supplier revenues in any contest in which $v_1 \neq v_2$. ■

8.6 Extensions: n>3 (Section 5.2)

8.6.1 Proof of Lemma 4

(i) Arguing as for two suppliers, $v_i^* \neq v_j^*$ for all $i \neq j \in \{1, ..., n\}$. Thus

$$S_T(\mathbf{v}) = \int_0^{\frac{v_1+v_2}{2}} q_1(v_1, \sigma)\, d\sigma + \sum_{k=2}^{n-1} \int_{\frac{v_{k-1}+v_k}{2}}^{\frac{v_k+v_{k+1}}{2}} q_k(v_k, \sigma)\, d\sigma + \int_{\frac{v_{n-1}+v_n}{2}}^{1} q_n(v_n, \sigma)\, d\sigma$$

The maximum of this function exists and it obviously does not involve corner solutions. Hence, it is given by the first order conditions

$$\frac{\partial S_T(\mathbf{v})}{\partial v_1} = -bv_1 + b\frac{v_2 - v_1}{2} = 0 \tag{9}$$

$$\frac{\partial S_T(\mathbf{v})}{\partial v_k} = -b\frac{v_k - v_{k-1}}{2} + b\frac{v_{k+1} - v_k}{2} = 0 \tag{10}$$

$$\text{for } k \in \{2, ..., n-1\}$$

$$\frac{\partial S_T(\mathbf{v})}{\partial v_n} = -b\frac{v_n - v_{n-1}}{2} + b(1 - v_n) = 0 \tag{11}$$

(10) can be rearranged to give $v_k - v_{k-1} = v_{k+1} - v_k \equiv \Delta^v$ for $k = 2, ..., n-1$. (9) and (11) give $v_1 = 1 - v_n = \Delta^v/2$. Inserting these equations into $v_1 + (v_2 - v_1) + ... + (v_n - v_{n-1}) + (1 - v_n) = 1$ gives $\Delta^v = \frac{1}{n}$. Thus, $v_1 = \frac{1}{2n}$ and $v_k = \frac{1}{2n} + \frac{k-1}{n} = \frac{2k-1}{2n}$ for $k \in \{2, ..., n\}$.
(ii) To ensure participation set $t_i = C$ for all i. The proof of the result on auctions is analogous to the proof of Proposition 2(i) above. Consider the bonus tournament. If suppliers $1, ..., n$ choose $v_1^*, v_2^*, ..., v_n^*$, then suppliers $2, .., n-1$ receive no revenues, but they break even because of the subsidy. There are no feasible deviations for which they can earn a positive price. Consider supplier 1 (supplier n is analogous): His surplus is $\frac{1}{2n}\left(\frac{b}{n}\right) + C - C = \frac{b}{2n^2}$. Deviating to $v_1 < v_1^*$ would reduce the probability of winning the prize, with no compensating benefits. Deviating to $v_1 > v_1^*$ would mean that supplier 1 would only win the low prize 0. This is clearly not profitable.
(iii) Let $\mathbf{v} = [v_1, ..., v_n]$ be the vector of approaches, ordered so that $v_1 \leq ... \leq v_n$. In Step 1-5, we show that diversity is less than socially optimal in the FPT. In Step 6, we consider the effect of increasing n.

Step 1: *In any equilibrium of the FPT, $v_1 = v_2$ and $v_{n-1} = v_n$. This implies that there are at most $n-2$ active approaches.*

Suppose $v_1 < v_2$. Then the revenue of supplier 1 is $A\frac{v_1+v_2}{2}$. For $v_1' = v_1 + \varepsilon$, $\varepsilon > 0$, such that $v_1' < v_2$, the revenue is $A\frac{v_1'+v_2}{2} > A\frac{v_1+v_2}{2}$. A similar argument holds for $v_{n-1} < v_n$.

We prove the second claim (that there is an inefficiently low amount of diversity) in several steps. For any supplier i, let $P^i_{\sigma<v_i}$ ($P^i_{\sigma>v_i}$) be the probability that supplier i wins and, in addition, $\sigma < v_i$ ($\sigma > v_i$). Let $P^i = P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$ be the total probability that supplier i wins.

Step 2: *If for suppliers i and j there exist $k \neq i$* and *$l \neq j$* such that *$v_i = v_k$* and *$v_j = v_l$*, then *$P^i_{\sigma<v_i} = P^i_{\sigma>v_i} = P^j_{\sigma<v_j} = P^j_{\sigma>v_j}$ in any equilibrium.*

Suppose first that $P^i_{\sigma<v_i} \neq P^i_{\sigma>v_i}$ for some supplier i using the same approach as another one. Suppose that $P^i_{\sigma<v_i} > P^i_{\sigma>v_i}$ (the opposite case is analogous). Then, a deviation to $v_i - \varepsilon$ for some sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P^i_{\sigma<v_i} > P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$,[45] which is a profitable deviation. Next, suppose that $P^i_{\sigma>v_i} < P^j_{\sigma<v_j}$(the opposite case is analogous). Then, a deviation of supplier i to $v_j - \varepsilon$ for sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P^j_{\sigma<v_j} > P^i_{\sigma<v_i} + P^i_{\sigma>v_i}$,[46] which is a profitable deviation.

Step 3: *In any equilibrium of an FPT with n suppliers, $\overline{P} \equiv P^1 = P^2 = P^{n-1} = P^n \geq \frac{1}{2(n-2)}$.*

By Step 2, all extreme approaches are duplicate. The three equalities thus follow from Step 1. Suppose that the inequality does not hold. Then $P^1 + P^2 + P^{n-1} + P^n < \frac{2}{n-2}$ which in turn implies that $\sum_{j=3}^{n-2} P^j \geq \frac{n-4}{n-2}$. But then there exist at least one $k \in \{3, ..., n-2\}$ such that $P^k \geq \frac{1}{n-2}$. By deviating to v_k, each supplier $1, 2, n-1$ or n would win with a probability of at least $\frac{1}{2(n-2)}$, which would be a profitable deviation.

Step 4: *Any equilibrium of an FPT with n suppliers satisfies* $\max_i v_i^T - \min_i v_i^T \leq \frac{n-3}{n-2}$.

Suppose not. As $\frac{2(n-2)-1}{2(n-2)} - \frac{1}{2(n-2)} = \frac{n-3}{n-2}$, there exists an equilibrium of an FPT such that either $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$ or $\min_i v_i^T < \frac{1}{2(n-2)}$ or both. If $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$, then Steps 1 and 2 imply $P^n < \frac{1}{2(n-2)}$, which is impossible by Step 3. If $\min_i v_i^T < \frac{2-1}{2(n-2)}$, then $P^1 < \frac{1}{2(n-2)}$ by Steps 1 and 2, which is again impossible by Step 3.

Step 5: *The diversity in an FPT is lower than socially optimal.*

[45] The winning probability is approximately $2P^i_{\sigma<v_i}$ if $v_i = \min\{v_1, ..., v_n\}$.
[46] The winning probability is approximately $2P^i_{\sigma<v_j}$ if $v_j = \min\{v_1, ..., v_n\}$.

By (i), the socially optimal diversity is $\frac{n-1}{n}$. By Step 4, the diversity in an FPT is at most $\frac{n-3}{n-2} < \frac{n-1}{n}$.

Step 6: *The difference between the FPT and the social optimum converges to zero as the number of suppliers increases.*

By Step 3, we know that each supplier $1, 2, n-1, n$ wins with probability $\overline{P}$. Then in any equilibrium of an FPT, there exists a supplier j such that $P^j \leq \frac{1-4\overline{P}}{n-4}$. A deviation to $v_1 - \varepsilon$ would result in a probability of winning approximately $\overline{P}$. Then, a necessary condition for an equilibrium is that $\overline{P} \leq \frac{1-4\overline{P}}{n-4}$, which implies that $\overline{P} \leq 1/n$ and consequently $v_1 \leq 1/n$ and $v_n \geq (n-1)/n$. Then, $\max_i v_i^T - \min_i v_i^T \geq \frac{n-2}{n}$ in any equilibrium of an FPT. By (i), the socially optimal diversity is $(n-1)/n$, so the difference between the socially optimal diversity and diversity in any equilibrium of an FPT is at most $\frac{n-1}{n} - \frac{n-2}{n} = 1/n$. Thus, the difference converges to zero as n increases.

8.6.2 Sufficient Conditions for FPT equilibria

We now provide sufficient conditions for equilibria in the FPT. These conditions hold in the equilibria described in Figure 3.

Lemma 10 *An outcome with k active approaches $(r_1, ..., r_k)$ can be supported in an equilibrium if the following conditions both hold:*
(a) $k \in \{\underline{k}, ..., \bar{k}\}$, where $\bar{k} = n-2$ and $\underline{k} = n/2$ if n is even and $\underline{k} = (n+1)/2$ if n is odd;
(b) $(r_1, ..., r_k) = (1/2k, 3/2k, 5/2k, ..., (2k-1)/2k)$.
Two suppliers choose the extreme approaches r_1 and r_k; each of the intermediate approaches $r_2, ..., r_{k-1}$ is chosen by one or two suppliers.

Proof. Step 1: *Suppose n is even and $k = n/2$. Then any choice of $r_1, ..., r_k$ as stated in part (b) of the lemma can be supported as an equilibrium.*
In the suggested equilibria, the active approaches are equidistant. Also, $r_1 = 1/n$ and $r_{n/2} = 1 - 1/n$. For any $1 < m < n/2$, $r_m - r_{m-1} = 2/n$, any of the active approaches offers the highest quality with probability $1/k = 2/n$. Now suppose each approach $r_1, ..., r_k$ is chosen by exactly two suppliers. Then each supplier has a revenue of $\Pi_i = A/n$. Deviating to any other active approach leads to payoff of $2A/3n$; hence it is not profitable. A deviation to $[0, r_1)$ or $(r_{n/2}, 1]$ results in a winning probability strictly lower than $1/n$, so this is not a profitable deviation either. Finally, consider a deviation to $v \in (r_{m-1}, r_m)$, $m \in \{2, ..., n/2\}$. The deviating supplier wins if and only if σ is in the set $[\frac{v+r_{m-1}}{2}, \frac{v+r_m}{2}]$, so that the winning probability is $1/n$ and this is also not a profitable deviation.
Step 2: *Now suppose n is even or odd and $k > n/2$. Then any choice of $r_1, ..., r_k$ as stated in part (b) of the lemma is an equilibrium.*
Arguing as in Step 1, any of the active approaches offers the highest quality with probability $1/k$. Suppose two suppliers choose r_1 and r_k, respectively. Moreover, suppose that each of the approaches $r_2, ..., r_{k-1}$ is chosen by one or two suppliers. Thus, if there are two suppliers using an approach, each of them wins with probability $1/2k$, and if there is only one supplier using this approach, he wins with probability $1/k$. Consider a supplier who wins with probability $1/2k$. By the same argument as in Step 1, if he deviates to $[0, r_1)$ or $(r_k, 1]$, he wins with probability strictly lower than $1/2k$. Deviating to any approach in some interval (r_l, r_{l+1}); $l \in \{1, ..., k-1\}$, he wins with probability of at most $1/2k$; hence such a deviation is not profitable either. If he deviates to any active approach, he wins with a probability of at most

$1/2k$. Thus, such suppliers do not have profitable deviations. Finally consider a deviation by a supplier who is the only one to choose some r_m, where $1 < m < k$. Any deviation to $[0, r_{m-1}]$ or $[r_{m+1}, 1]$ leads to strictly lower revenues, by the same argument as above. For any approach $v \in (r_{m-1}, r_{m+1})$, he wins whenever $\sigma \in [\frac{v+r_{m-1}}{2}, \frac{v+r_{m+1}}{2}]$, so that the winning probability is $\frac{v+r_{m+1}}{2} - \frac{v+r_{m-1}}{2} = \frac{r_{m+1}-r_{m-1}}{2} = 1/k$. Hence, this is not a profitable deviation either. ■

8.6.3 Proof of Proposition 4

(i) Arguing as in Proposition 1, the bonus tournament $(b/n, 0)$ implements the social optimum. Thus, to prove the optimality of the bonus tournament it is sufficient to show that the buyer can extract all surplus from the suppliers. Let $t_2 = \cdots = t_{n-1} = C$ and $t_1 = t_n = C - b/(2n^2)$. Suppliers $i \in \{2, \ldots n-2\}$ win the bonus price with probability zero, hence their revenue is zero, and their participation constraint binds. Suppliers 1 and n win the bonus price with probability $1/2n$, and their expected revenue is $\Pi_1 = \Pi_n = (1/2n)(b/n) = b/2n^2$. Thus, their participation constraint binds as well, and hence the bonus tournament implements the optimum for the buyer.

(ii) In an auction, the conditional transfers to suppliers 1 and n differ from those for the remaining suppliers. The revenue of supplier 1 is

$$\Pi_1 = \frac{b}{2n^2} + \int_{1/2n}^{2/2n} \left(\Psi - b\left(\sigma - \frac{1}{2n}\right) - \left(\Psi - b\left(\frac{3}{2n} - \sigma\right)\right)\right) d\sigma = \frac{3b}{4n^2}$$

For supplier 2 it is

$$\Pi_2 = 2\int_{2/2n}^{3/2n} \left(\Psi - b\left(\frac{3}{2n} - \sigma\right) - \left(\Psi - b\left(\sigma - \frac{1}{2n}\right)\right)\right) d\sigma = \frac{2b}{4n^2}$$

By symmetry, $\Pi_1 = \Pi_n$ and $\Pi_2 = \Pi_i$ for all $i \in \{2, ...n-1\}$. As $\Pi_1 > \Pi_2 = b/(2n^2) > C$, the participation constraint of all suppliers is satisfied without any subsidies. Then, the buyer optimally sets $t_i = 0$ for all i in the auction. The total transfers of the buyer to the suppliers are thus $\Sigma_{i=1}^{n}\Pi_i = (n-2)\Pi_2 + 2\Pi_1 = (n-2)\frac{b}{2n^2} + \frac{3b}{2n^2} = (n+1)\frac{b}{2n^2}$.

Now consider a bonus tournament with $(b/n, 0)$, $t_2 = \cdots = t_{n-1} = C$ and $t_1 = t_n = 0$. As argued before, this bonus tournament implements the social optimum if participation constraints are met. For suppliers 2 to $n-2$ the subsidy ensures participation. For suppliers 1 and n the expected revenue is $\Pi_1 = \Pi_n = b/(2n^2) > C$. Hence, participation is ensured. Thus the total transfers of the buyer to the suppliers are $\Sigma_{i=1}^{n}(\Pi_i + t_i) = \Pi_1 + \Pi_n + \Sigma_{i=2}^{n-1} t_i = \frac{b}{n^2} + (n-2)C$. The buyer will strictly prefer the bonus tournament to the auction if and only if $(n+1)\, b/\, 2n^2 > b/\, n^2 + (n-2)C$, or equivalently, $(n-1)\, b/\, 2n^2 > (n-2)C$. This always holds since $b/(2n^2) > C$.

(iii) According to the proof of Lemma 4(iii), an FPT can implement at most $n-2$ different approaches. By Lemma 10, an FPT implementing $n-2$ approaches exists. The FPT implementing maximum diversity (hence maximizing total surplus) thus implements $k = n-2$ with $A = 0$ and $t_i = C$ for all i. The participation constraint of all suppliers binds, so that this is the best outcome for the buyer. In the FPT, the buyer has expected costs from suboptimal quality of $\frac{b}{4(n-2)}$. Moreover, she pays subsidies nC. Now consider the bonus tournament implementing the socially optimal outcome, as above. If $C \geq b/(2n^2)$, the bonus tournament extracts the entire surplus like the FPT, but implements a strictly more efficient outcome. Hence, the buyer payoff is strictly higher. Next, suppose that $C < b/(2n^2)$, and let the subsidies be as in (ii). Then, the buyer has expected costs from suboptimal quality of $\frac{b}{4n}$, pays expected bonus prizes $\frac{b}{n^2}$ and subsidies $(n-2)C$; together these costs amount to $\frac{b}{4n} + \frac{b}{n^2} + (n-2)C$. Thus, the buyer is better of in the bonus tournament if $\frac{b}{4n} + \frac{b}{n^2} + (n-2)C < \frac{b}{4(n-2)} + nC$. This is equivalent with the condition in the proposition.

8.6.4 Proof of Corollary 4

According to Lemma 4(i), the social optimum is given by the choices $v_k^* = (2k-1)/2n$ ($k \in \{1, ..., n\}$). The average quality in the social optimum is thus $\Psi - b/4n$. Therefore the total surplus is $\Psi - b/4n - nC$. The maximum of this expression in $\mathbb{R}^+$ is $n = \sqrt{b}/2\sqrt{C}$. By concavity of the objective function, the optimal choice of $n \in \mathbb{N}$ is thus given by $n_-(C)$ or $n_+(C)$. According to Lemma 4(ii), the social optimum for any given number of suppliers can be implemented with an auction.

8.7 Other Extensions (Section 5.3)

8.7.1 Proof of Corollary 5

Proof. (i) The proof of the result on auctions is the same as the proof of Proposition 2(i) above. By the generalized Proposition 1, the social optimum can be implemented with a bonus tournament if $0 < v_1^* \leq 1/2 \leq v_2^* < 1$. Thus, we only need to show that the social optimum always satisfies these conditions. Therefore, first consider any $v_1 = 0$ ($v_2 = 1$ is analogous). Clearly, $\left.\frac{\partial S_T(v_1,v_2)}{\partial v_1}\right|_{v_1=0} > 0$. Hence, in the social optimum $v_1^* > 0$. Next, consider (v_1, v_2) such that $v_1 \leq v_2 < 1/2$ (the case $1/2 < v_1 \leq v_2$ is analogous). Supplier 2 offers higher quality than supplier 1 in the interval $\left[\frac{v_1+v_2}{2}, 1\right]$. We can write the total surplus from this interval as

$$\left.S_T(v_1,v_2)\right|_{\sigma \geq \frac{v_1+v_2}{2}} =$$

$$\Psi\left(1 - F\left(\frac{v_1+v_2}{2}\right)\right) - \int_{\frac{v_1+v_2}{2}}^{v_2} \delta(|v_2-\sigma|) f(\sigma)\, d\sigma - \int_{v_2}^{1/2} \delta(|v_2-\sigma|) f(\sigma)\, d\sigma$$

$$- \int_{1/2}^{1-v_2} \delta(|v_2-\sigma|) f(\sigma)\, d\sigma - \int_{1-v_2}^{1-\frac{v_1+v_2}{2}} \delta(|v_2-\sigma|) f(\sigma)\, d\sigma - \int_{1-\frac{v_1+v_2}{2}}^{1} \delta(|v_2-\sigma|) f(\sigma)\, d\sigma$$

Consider a deviation to $v_2' = 1 - v_2$. Symmetry of $f(\sigma)$ implies that $\int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}} \delta(|v_2 - \sigma|) f(\sigma)\, d\sigma = \int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}} \delta(|v_2' - \sigma|) f(\sigma)\, d\sigma$. As the highest available quality determines the total surplus, it follows $\left.S_T(v_1,v_2)\right|_{\frac{v_1+v_2}{2} \leq \sigma \leq 1-\frac{v_1+v_2}{2}} \leq \left.S_T(v_1,v_2')\right|_{\frac{v_1+v_2}{2} \leq \sigma \leq 1-\frac{v_1+v_2}{2}}$. Observe that $\int_{1-\frac{v_1+v_2}{2}}^{1} \delta(|v_2-\sigma|) f(\sigma)\, d\sigma < \int_{1-\frac{v_1+v_2}{2}}^{1} \delta(|v_2'-\sigma|) f(\sigma)\, d\sigma$, because δ is increasing. Thus $\left.S_T(v_1,v_2)\right|_{\sigma \geq \frac{v_1+v_2}{2}} < \left.S_T(v_1,v_2')\right|_{\sigma \geq \frac{v_1+v_2}{2}}$. For $\sigma < \frac{v_1+v_2}{2}$, the highest quality always comes from v_1. Thus $\left.S_T(v_1,v_2)\right|_{\sigma < \frac{v_1+v_2}{2}} = \left.S_T(v_1,v_2')\right|_{\sigma < \frac{v_1+v_2}{2}}$. Thus, we obtain $S_T(v_1,v_2) < S_T(v_1,v_2')$. Thus, there can be no social optimum with $v_1^* \leq v_2^* < 1/2$.
(ii) *The unique equilibrium in an FPT is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*
First, we show that the suggested (v_1, v_2) emerges as an equilibrium. Denote the prize with A. Let v_j be such that $F(v_j) = 1/2$. Since f is everywhere positive, such a v_j is unique.

Now if supplier $i \in \{1,2\}$ plays $v_i = v_j$, his revenue is $\Pi_i(v_i, v_j) = A/2$. For any $v_i < v_j$ the revenue is $\Pi_i(v_i, v_j) = AF((v_i + v_j)/2) < A/2$. Similarly, for any $v_i > v_j$ the revenue is $\Pi_i(v_i, v_j) = A(1 - F((v_i + v_j)/2)) < A/2$. Thus, $v_i = v_j$ is an equilibrium. Second, $v_i' = v_j'$ is an equilibrium only if $F(v_j') = 1/2$. Suppose not. Then, a supplier i can profitably deviate to v_i such that $F(v_i) = 1/2$, since his revenue will be $\Pi_i(v_i, v_j) > A/2$. Third, $v_i \neq v_j$ is never an equilibrium. Suppose it was. Let $v_1 < v_2$. Then, the revenue of supplier 1 is $\Pi_1(v_1|v_2) = AF((v_1 + v_2)/2)$, while deviating to $(v_1 + v_2)/2$ leads to a revenue of $AF((v_1 + 3v_2)/4) > AF((v_1 + v_2)/2)$. ■

8.7.2 Proof of Proposition 5

(i) By Corollary 5(i) the bonus tournament with $(\Delta q(v_1^*, v_2^*), 0)$ implements the social optimum with appropriate subsidies. Let the subsidies be $t_1 = C - F(v_1^*)\Delta q(v_1^*, v_2^*)$ and $t_2 = C - (1 - F(v_2^*))\Delta q(v_1^*, v_2^*)$. $C \geq \max\{F(v_1^*)\Delta q(v_1^*, v_2^*),\ (1 - F(v_2^*))\Delta q(v_1^*, v_2^*)\}$ implies $t_1, t_2 \geq 0$, and the participation constraint of both suppliers bind. Thus, the bonus tournament implements the social optimum and extracts all surplus from the suppliers and hence it is the optimal contest for the buyer.

(ii) By Corollary 5(ii), the FPT uniquely implements $v_1 = v_2 = 1/2$ and $F(v_i) = 1/2$ for $i = 1, 2$. Then there exists $\varepsilon > 0$, such that $F(1/2 - \varepsilon)\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon) = (1 - F(1/2 - \varepsilon))\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon) < C$. Then, by the generalized version of Proposition 1, a bonus tournament with prices $\mathcal{P} = \{\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon), 0\}$ and transfers $t_1 = t_2 = C - F(1/2 - \varepsilon)\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon)$ implements $(v_1, v_2) = (1/2 - \varepsilon, 1/2 + \varepsilon)$. This yields strictly greater total surplus, with weakly lower supplier surplus than any FPT. Hence, buyer surplus is strictly greater in such a bonus tournament than in any FPT.

(iii) By Corollary 5(i), both the auction and the bonus tournament implement the social optimum with appropriate subsidies. We can write the revenues for each supplier in an auction as

$$
\begin{aligned}
\Pi_1^a &= F(v_1^*)\Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma)\, d\sigma \\
\Pi_2^a &= (1 - F(v_2^*))\Delta q(v_1^*, v_2^*) + \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma)\, d\sigma
\end{aligned}
$$

while the revenues in the bonus tournament $(\Delta q(v_1^*, v_2^*), 0)$ are

$$
\begin{aligned}
\Pi_1^{bt} &= F(v_1^*)\Delta q(v_1^*, v_2^*) \\
\Pi_2^{bt} &= (1 - F(v_2^*))\Delta q(v_1^*, v_2^*).
\end{aligned}
$$

Let $t_1^a, t_2^a \geq 0$ be the minimum subsidies needed for the suppliers to be willing to participate in the auction. Then, given subsidies

$$
\begin{aligned}
t_1^{bt} &= t_1^a + \int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma)\, d\sigma \\
t_2^{bt} &= t_2^a + \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma)\, d\sigma
\end{aligned}
$$

both suppliers are willing to participate in the bonus tournament. Furthermore, since both the auction and the bonus tournament implement the social optimum and have the same total transfers to the suppliers ($\Pi_1^j + \Pi_1^j + t_1^j + t_2^j$, for $j = a, bt$), the buyer is indifferent between the auction and the bonus tournament. However, suppose $C < \max\{\Pi_1^a, \Pi_2^a\}$ and suppose w.l.o.g. that $\max\{\Pi_1^a, \Pi_2^a\} = \Pi_1^a$. Because $\Pi_1^a = \Pi_1^{bt} + t_1^{bt}$, there exists $\varepsilon > 0$ such that $\Pi_1^{bt} + t_1^{bt} - \varepsilon \geq C$. Then, a bonus tournament with $(\Delta q\,(v_1^*, v_2^*)\,, 0)$ with subsidies $\hat{t}_1^{bt} = t_1^{bt} - \varepsilon$ and $\hat{t}_2^{bt} = t_2^{bt}$ still implements the social optimum but with strictly lower total transfers than the auction with the minimum subsidies. Thus, the buyer strictly prefers this bonus tournament to the auction with any subsidies.

8.7.3 Proof of Lemma 5

This section provides the proof of Lemma 5 from Section 5.3.3. Suppose that there are n suppliers and that assumption (A2)' holds. Consider an FPT with two prizes $A_1 > A_2 > 0$, where the supplier with the highest quality receives A_1 and the supplier with the second-highest quality receives A_2.[47] For notational convenience, suppose that $v_1 \leq v_2 \leq \cdots \leq v_n$. We first provide an intermediate result.

Lemma 11 *If $v_1, v_2, \ldots, v_n$ is an equilibrium of an FPT with two prizes, then $v_1 = v_2 = v_3$ and $v_{n-2} = v_{n-1} = v_n$.*

Proof. We will prove that $v_1 = v_2 = v_3$. The other claim follows by an analogous argument.
Step 1: $v_1 = v_2$. Suppose not. Then $v_1 < v_2$. Thus, the revenue of supplier 1 is

$$\Pi_1\,(v_1, v_{-1}) = \frac{v_1 + v_2}{2} A_1 + \frac{v_3 - v_2}{2} A_2.$$

Therefore, a deviation to any $v_1' \in (v_1, v_2)$ increases the probability of winning the first prize, while not affecting the probability of winning the second prize. Hence, it is profitable.
Step 2: $v_1 = v_2 < v_3 = v_4$ cannot be an equilibrium. Denote with $P^{i,1}_{\sigma<v_i}$ the probability that supplier i wins the first prize when $\sigma < v_i$. Analogously define the probabilities of winning when the state is greater than the chosen approach and the probabilities of winning the second prize. By random tie breaking we have $P^{1,1}_{\sigma<v_1} = P^{2,1}_{\sigma<v_2} = P^{1,2}_{\sigma<v_1} = P^{2,2}_{\sigma<v_2}$ and $P^{1,1}_{\sigma>v_1} = P^{2,1}_{\sigma>v_2} = P^{1,2}_{\sigma>v_1} = P^{2,2}_{\sigma>v_2}$. We will show that $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$. Suppose that this was not true. First, suppose $P^{1,1}_{\sigma<v_1} > P^{1,1}_{\sigma>v_1}$. Then, there exist $\varepsilon, \varepsilon', \varepsilon'' > 0$ arbitrarily small such that a deviation $v_1' = v_1 - \varepsilon$ leads to revenues

$$\Pi_1\left(v_1', v_{-1}\right) = 2\left(P^{1,1}_{\sigma<v_1} - \varepsilon'\right) A_1 + 2\left(P^{1,1}_{\sigma>v_1} - \varepsilon''\right) A_2.$$

For sufficiently small ε this constitutes a profitable deviation. The case $P^{1,1}_{\sigma<v_1} < P^{1,1}_{\sigma>v_1}$ follows by an analogous argument, but the incentives to deviate are even stronger.

Now suppose that $P^{1,1}_{\sigma<v_1} = P^{1,1}_{\sigma>v_1}$ and $v_1 = v_2 < v_3 = v_4$. We will show that this cannot be an equilibrium. In the proposed equilibrium $P^{1,1}_{\sigma<v_1} = v_1/2$ and $P^{1,1}_{\sigma<v_1} + P^{1,1}_{\sigma>v_1} = P^{1,2}_{\sigma<v_1} + P^{1,2}_{\sigma>v_1} = v_1$. Hence, the expected revenue is

$$\Pi_1\,(v_1, v_{-1}) = v_1 A_1 + v_1 A_2.$$

[47] Ties are broken randomly, with equal chance of winning for each firm with the respective quality.

For any deviation $v_1' \in (v_2, v_3)$ the probability of winning the first prize is

$$\frac{v_1'+v_3}{2}-\frac{v_1'+v_2}{2}=\frac{v_3-v_2}{2}=v_1$$

where the last equality follows from $P^{1,1}_{\sigma<v_1}=P^{1,1}_{\sigma>v_1}$. Using $v_3=v_4$, the probability of winning the second prize is

$$\frac{v_2+v_1'}{2}>v_1$$

thus it follows that $\Pi_1(v_1', v_{-1}) > \Pi_1(v_1, v_{-1})$.
Step 3: $v_1 = v_2 < v_3 < v_4$ cannot be an equilibrium. The revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1}{2}A_1 + \frac{v_3-v_1}{4}A_1 + \frac{v_3+v_1}{4}A_2 + \frac{v_4-v_3}{4}A_2. \tag{12}$$

Consider a deviation to $v_1' \in (v_1, v_3)$. The revenue is

$$\Pi_1\left(v_1', v_{-1}\right) = \frac{v_3-v_1}{2}A_1 + \frac{v_1'+v_1}{2}A_2 + \frac{v_4-v_3}{2}A_2.$$

If $\Pi_1(v_1', v_{-1}) > \Pi_1(v_1, v_{-1})$, then this is a profitable deviation. If $\Pi_1(v_1', v_{-1}) \leq \Pi_1(v_1|v_{-1})$ is equivalent with

$$\frac{v_1}{2}A_1 - \frac{v_3-v_1}{4}A_1 + \frac{v_3-v_1}{4}A_2 - \frac{v_1'}{2}A_2 - \frac{v_4-v_3}{4}A_2 \geq 0 \tag{13}$$

But consider in that case a deviation to $v_1'' = v_1 - \varepsilon$ for small positive ε. The expected revenue is

$$\Pi_1\left(v_1'', v_{-1}\right) = \frac{v_1''+v_1}{2}A_1 + \frac{v_3-v_1}{2}A_2$$

and $\lim_{\varepsilon\to 0}\Pi_1(v_1'', v_{-1}) = v_1A_1 + \frac{v_3-v_1}{2}A_2$. Together with (12), this implies

$$\lim_{\varepsilon\to 0}\Pi_1\left(v_1'', v_{-1}\right) - \Pi_1(v_1, v_{-1}) = \frac{v_1}{2}A_1 + \frac{v_3-v_1}{4}A_2 - \frac{v_3-v_1}{4}A_1 - \frac{v_1}{2}A_2 - \frac{v_4-v_3}{4}A_2.$$

Since $v_1' > v_1$, (13) implies $\lim_{\varepsilon\to 0}\Pi_1(v_1'', v_{-1}) - \Pi_1(v_1, v_{-1}) > 0$. Hence, there always exists $\varepsilon > 0$ small enough such that $\Pi_1(v_1'', v_{-1}) - \Pi_1(v_1, v_{-1}) > 0$. ■

The lemma implies that the maximal number of active approaches in an FPT with two prizes is $n-4$. By Lemma 10 an FPT with a single prize implements an equilibrium with $n-2$ active approaches. By Lemma 4(ii), it is possible to implement the socially optimal allocation with $n-2$ approaches in an FPT with a single prize. Implementing this equilibrium in a single-prize FPT, where the prize size is the sum of the two prizes in an FPT with two prizes, strictly increases the total payoff. On the other hand, the payoff of the suppliers remains the same, as the total size of the fixed prize remains the same. Hence, the expected buyer payoff strictly increases.

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