Voting with Public Information

Shuo Liu *

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Abstract

We study the effect of public information on collective decision-making in committees, where members can have both common and conflicting interests. We show that in the presence of public information, the information aggregated via voting can be extremely limited under the commonly-used simultaneous voting rules, due to the non-existence of a simple and intuitive efficient equilibrium and the existence of a simple and intuitive but inefficient equilibrium. We propose a voting procedure that takes into account the content of the public information, and show that it can facilitate information aggregation by restoring the efficient equilibrium. Our voting procedure also has additional advantages when there is a concern for strategic disclosure of public information.

Keywords: strategic voting, collective decision-making, public information, committee design, optimal voting rule, information disclosure.

JEL classification: D72, D82.

*Department of Economics, University of Zurich, Schönberggasse 1, CH-8001 Zurich, Switzerland. Email: shuo.liu@econ.uzh.ch. I am indebted to my supervisor Nick Netzer for his valuable and continuous feedback on this project. For useful comments and discussion, I thank Pedro Dal Bó, Olga Chiappinelli, Lachlan Deer, Christian Ewerhart, Simon Fuchs, Hans Gersbach, Alex Gershkov, Bård Harstad, Andreas Hefi, Navin Kartik, Igor Letina, Philippos Louis, Kohei Kawamura, Lydia Mechtenberg, Georg Nöldeke, Christian Oertel, Harry Di Pei, Javier Rivas, Yuval Salant, Armin Schmutzler and seminar participants at the Universities of Basel (SSES 2015), Ghent (SMYE 2015), Groningen (EPCS 2015) and Zürich, ETH Zürich (YSEM 2015) and Universitat Pompeu Farbra (BGSE Political Economy Summer School 2014). A special thank goes to Jean-Michel Benkert for reading through the earliest version of this paper and providing numerous helpful suggestions.
1 Introduction

A common argument for voting mechanisms is that they help aggregate the information that agents in a committee privately hold, and thus lead to better decisions compared to the case of a single decision-maker. Indeed, in a setting of collective decision-making where agents have only common interests, the celebrated Condorcet Jury Theorem (CJT) suggests that the simple majority rule can lead to the first-best outcome if agents truthfully convey their private information through their votes (Condorcet, [1785] 1994). However, Kawamura and Vlaseros (2015) (henceforth KV) make the interesting observation that, as long as there exists a public signal that can be commonly observed by all agents and that is superior to each of their private signals, a vote-your-private-information strategy profile will not constitute an equilibrium under the simple majority rule, even though this would have been the case if the public signal were absent. What’s worse, the presence of public information opens the possibility for agents to coordinate on an equilibrium in which everyone just votes according to whatever the public signal suggests. Clearly, in such an equilibrium, the private information of the committee members is completely disregarded. This can be very inefficient since public information is rarely perfect and the total private information possessed by the committee is often more valuable in determining the optimal collective decision. Experimentally, KV find that a large proportion of subjects in the laboratory behave quite consistently with what the inefficient equilibrium would predict. Consequently, the outcome of the collective decision almost always coincides with that in the inefficient equilibrium.

This observation is highly relevant, because it should be clear that the access to both private and public information for the voters is the rule rather than the exception: in business, members of the board of directors receive (or even ask) advice from the advisory board of the company; in a court, a witness states his/her testimony in front of all members of the jury; the Central Committee of the Communist Party of China, which has only seven members, often invites renowned scholars in the relevant fields to give short presentations when important decisions that affect the well-being of more than 1.3 billion people are needed to be made. If in the end only the public information counts, why should we bother to use the voting mechanism in the first place? This issue is even more alarming if we take into account that in reality, the party that provides the relevant
public information is often strategic and self-interested as well.

With these practical concerns in mind, we first take KV’s observation one step further in this paper. We study the effect of public information in a richer setting where agents have both common and conflicting interests: while agents share the common goal of making a collective decision that will match the state, they may have different payoffs from the different types of decision errors that could occur. We show that the presence of public information can have a profound impact on the agents’ voting behavior. In particular, it significantly limits the existence of the informative voting equilibrium, in which every agent simply casts her vote in accordance with her private information. More specifically, we show that if the public information is superior to each agent’s private information and the voting threshold is fixed (which is the case for the simple majority rule), the informative voting equilibrium does not exist for any preference profile of the agents. In addition, even if the public information is less accurate than the private information, the set of preference profiles that allow for informative voting under some voting rule with a fixed threshold is strictly smaller than it would be in the absence of the public information: for example, if the public information is just slightly less precise than each agent’s private information, under the simple majority rule the informative voting equilibrium only exists if all agents are sufficiently unbiased ex ante. To make things worse, the presence of public information introduces the intuitive but inefficient obedient voting equilibrium, which robustly exists under different voting rules. In the obedient voting equilibrium, agents always support the alternative suggested by the public information and, hence, the public information is the only determinant of the final decision outcome. We later show that a self-interested party who controls the provision of public information may exploit its influential effect by strategically disclosing (withholding) good (bad) news about his favored alternative.

We then propose a simple voting procedure to tackle the potential harmful effect of public information. Specifically, we introduce a class of more flexible voting rules that incorporates the effect of the public information. We call such voting rules contingent k-voting rules. By allowing the public information to directly affect the voting threshold, the contingent k-voting rules reduce its direct effect on agents’ voting behavior. Especially, under a contingent k-voting rule, the number of votes required for the committee to select an alternative will depend on the content of the public information: for example, if
a job candidate is supported by an exceptionally strong recommendation letter, the committee should then require less votes to approve the hire of this candidate. Given this seemingly counter-intuitive voting procedure and conditional on the others are voting informatively, an agent is decisive only if the private information of the others is collectively more against the decision suggested by the public information. We thus reduce the agents’ incentives to deviate from the informative voting equilibrium. For any preference profile of the agents, we provide a simple algorithm to find the contingent $k$-voting rules that can be used to restore informative voting. Importantly, we prove that the informative voting equilibria restored by the contingent $k$-voting rules are asymptotically efficient, in the sense that the ex ante probability of the collective decision being matched to the state becomes arbitrary close to 1 as the size of the committee increases. In other words, we obtain a version of the CJT in a voting environment with both private and public information. This result implies that any equilibrium that is asymptotically inefficient (e.g., the obedient voting equilibrium) will be outperformed by the informative voting equilibria under the contingent $k$-voting rules, provided that the size of the committee is sufficiently large.

Within a setting where agents have only common interests, which is mostly studied in the literature, we demonstrate that the first-best informational efficiency can always be achieved by using a specific contingent $k$-voting rule, the contingent majority rule, under which the informative voting equilibrium is guaranteed to exist. In particular, we show that given all the information that is available to the committee, the probability of the collective decision being matched to the state is maximized in the informative voting equilibrium sustained by the contingent majority rule. In other words, the contingent majority rule aggregates both the private and the public information efficiently. As we will argue, this result implies that if agents have only common interests, the social optimum can also be implemented by a simple two-stage voting mechanism, in which agents first vote about which voting threshold value to use and then vote about which collective decision to take.

Finally, we show, perhaps to one’s surprise, that the contingent $k$-voting rules can actually have additional advantages when there is a concern for strategic disclosure of public information. Intuitively, the use of the contingent $k$-voting rules makes it possible for the agents to rationally commit to informative voting, independent of the disclosure
policy of the public information. Thus, even a self-interested party may find it optimal to always publicly communicate the information it receives to the agents, given that its message will not directly affect the agents’ voting behavior and will indirectly increase the accuracy of the collective decision.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the model. In Section 4 we discuss how the presence of public information may lead to inefficient information aggregation. We introduce in Section 5 a class of voting rules that are contingent on the public information, and show how the use of such voting rules can improve the collective decision outcomes. In Section 6, we extend our analysis to the cases where the provision of public information is strategically determined by a self-interested information controller. Finally, Section 7 concludes. All proofs are contained in the Appendix.

2 Related Literature

There is an extensive literature on strategic voting starting with the seminal paper of Austen-Smith and Banks (1996). Many of the papers in this line of literature study how informational efficiency of various voting mechanisms is affected by the agents’ strategic behavior (see, for example, Feddersen and Pesendorfer (1997) on simultaneous voting rules and Dekel and Piccione (2000) on sequential voting rules). Among all of them, the most closely related paper besides KV is actually Austen-Smith and Banks (1996). Specifically, they notice that whenever the voters do not have an extremely biased prior, the informative voting equilibrium will exist under some simultaneous voting rule with a fixed voting threshold value (p. 38, Lemma 2). However, our paper shows that if we explicitly take into account how agents’ prior is shaped by public information, then the simultaneous voting rules commonly used in practice may no longer suffice to incentivize agents to truthfully reveal their private information via their votes. As another connection to our paper, Section 2 of Austen-Smith and Banks (1996) extends their analysis to a case where agents have access to both private and (exogenous) public information. They conclude that in such a setting, sincere voting, which is equivalent to obedient voting in our model whenever the public information is more precise than each agent’s private information, cannot be both informative and rational (p. 42, Theorem 3). In contrast, we
address the related but distinct question of whether informative voting can be rational under some simultaneous voting rule when it is not required to be sincere. Our model and focus are also quite different from the few other papers that study the effect of public information in a voting environment (e.g., Gersbach 2000, Taylor and Yildirim 2010, Tanner 2014).

Several papers study the effect of pre-voting deliberation (e.g., Coughlan 2000, Austen-Smith and Feddersen 2006, Gerardi and Yariv 2007). In these models, agents can communicate their private information before the vote actually takes place, thus public information endogenously arises. Our model differs from them in at least two main aspects. Firstly, in the models with deliberation, conflicts between an agent’s private information and the public information usually do not matter because the former has already been incorporated in the latter. In the current paper, however, such conflicts have a direct and profound effect on agents’ provision of private information, which can lead to a severe loss of informational efficiency. Secondly, unlike in the obedient voting equilibrium in the current paper, in these models it is actually socially efficient for the agents to always follow the public information, conditional on their private information being credibly revealed in the deliberation stage.

Finally, there is a third strand of literature on committee design and optimal voting rules. For example, Persico (2004) studies the optimal size and threshold value for simultaneous voting rules when agents’ private information is endogenous. Subsequently, Gershkov and Szentes (2009) show that when information is costly, the optimal direct mechanism can actually be implemented by a random, sequential reporting/voting scheme, which suggests in general that the use of more flexible voting rules can be welfare-enhancing. This insight is also shared by Gersbach (2004, 2009), who shows that allowing the voting rule to depend on the proposal to be determined may yield efficient outcomes for classic social choice problems such as provision of public projects and division of limited resources among agents. More recently, Gershkov et al. (2015) show

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1 Buechel and Mechtenberg (2016) is a recent exception that shows pre-voting communication can actually impede efficient information aggregation within a common-interest setting. They consider a network model in which agents are heterogeneously informed, and each informed agent can privately make a voting recommendation to the uninformed agents that are connected to her. They show that if the network structure is too centralized around a few informed agents, majority voting may lead to inefficient information aggregation. Compared to their paper, we focus on the public communication between a (strategic or non-strategic) information controller and a group of homogeneously informed agents.
that in an environment where agents have single-crossing preferences, a successive voting rule with a descending threshold achieves the highest utilitarian efficiency among all anonymous, unanimous and dominant strategy incentive-compatible mechanisms. Our paper contributes to this line of literature by showing that when relevant public information is salient in the environment being considered, the voting rules should also be more carefully and flexibly designed in order to achieve a more efficient outcome.

3 The Model

3.1 Players, actions and payoffs

Consider a committee of $n$ members (agents) indexed by $i \in I = \{1, \ldots, n\}$. We assume $n$ is odd and $n \geq 3$. Agents need to make a collective decision $d \in D = \{0, 1\}$ over a binary set of alternatives. For concreteness, one could think of a setting in which a board of directors is choosing between two business proposals.

Each agent can cast a vote to support one of the alternatives. We denote $v_i = 1$ if agent $i$ votes in favor of the decision $d = 1$, and $v_i = 0$ otherwise. We further denote $V_i = \{0, 1\}$ as agent $i$’s action set and $\mathbf{v} = (v_1, \ldots, v_n) \in V = \prod_{i=1}^{n} V_i$ as the agents’ voting profile. A collective decision rule is a function, $g : V \rightarrow D$, that assigns every voting profile to a collective decision. For the moment, we restrict our attention to a class of collective decision rules called $k$-voting rules, which are arguably most commonly used in practice. Formally, if we set the alternative associated with $d = 0$ as the default option, under a given $k$-voting rule the alternative associated with $d = 1$ will be chosen if and only if there are at least $k \in \{1, \ldots, n\}$ votes in favor of it:

$$g(\mathbf{v}) = \begin{cases} 
1 & \text{if } \sum_{i=1}^{n} v_i \geq k, \\
0 & \text{otherwise}.
\end{cases}$$

Each $k$-voting rule is uniquely characterized by its threshold value $k$. If $k = (n + 1)/2$, we are in a setting of majority voting, while $k > (n + 1)/2$ corresponds to some super-majority rule.
The state of the world $\theta$ is drawn from a binary set $\Theta = \{0, 1\}$ with equal probability.\(^2\) In the context of the board of directors and business proposals, one could think of $\theta$ as the uncertain (relative) quality of the two proposals, where $\theta = 1$ means the proposal associated with $d = 1$ is of higher prospective revenue, while the other is better if $\theta = 0$. We assume agent $i$’s utility function $u_i : \mathcal{D} \times \Theta \to \mathbb{R}$ takes the following form (see also Coughlan, 2000; Kojima and Takagi, 2010; Iaryczower and Shum, 2012):

$$u_i(d, \theta) = \begin{cases} 
0 & \text{if } d = \theta, \\
-q_i & \text{if } d = 1, \theta = 0, \\
-(1 - q_i) & \text{if } d = 0, \theta = 1,
\end{cases}$$

where $q_i \in [0, 1]$. In words, we assume the agents in the committee have a common interest in matching the collective decision to the state (i.e., choosing the proposal of higher quality), and we normalize the payoff of successfully choosing $d = \theta$ to zero. However, we allow the agents’ payoffs to differ when committing different types of decision errors. We also allow these differences to be heterogeneous across agents. Note that each agent’s utility function is uniquely characterized by the parameter $q_i$, which is a measure of how biased agent $i$ is towards the default option ex ante: if $q_i = 1/2$, agent $i$ is unbiased and indifferent between the two alternatives; if $q_i < 1/2$, agent $i$ is inclined to choose $d = 1$; similarly, $q_i > 1/2$ implies that agent $i$ would prefer $d = 0$ if there is no further information to be revealed. In addition, if $q_i \neq q_j$, the two agents $i$ and $j$ may strictly prefer different alternatives even when they have exactly the same information. Hence, we interpret $q_i \neq q_j$ as a conflict of interest between the two agents. We will call the vector $q = (q_i)_{i \in I}$ the preference profile of the agents.\(^3\) In addition, we refer to the case where $q_i = 1/2 \forall i \in I$ as the setting where agents have only common interests.

Note that, given the above specification of payoffs, if agent $i$ assigns a posterior probability $\pi \in [0, 1]$ to the event $\theta = 1$, she would prefer $d = 1$ over $d = 0$ if and only if $\pi \geq q_i$, that is, whenever the evidence of the state being 1 is sufficiently strong.

\(^2\)The assumptions that the prior probability of $\theta$ is uniform and that the accuracy of the agents’ private signals is state-independent (see Section 3.2) are mainly made for the convenience of exposition. To show how our analysis can be extended beyond the current setting, results in Section 4 are proved more generally in the Appendix without these two assumptions.

\(^3\)Our analysis do not depend on whether $q$ is common knowledge among the agents or not.
3.2 Information structure and timing

Before casting their votes, each agent privately receives a binary signal \( s_i \in S_i = \{0, 1\} \). The private signals are i.i.d. across agents and are drawn according to the conditional probability distribution \( \Pr(s_i = 1|\theta = 1) = \Pr(s_i = 0|\theta = 0) = \alpha \in (1/2, 1) \). In addition, all agents commonly observe a public signal \( s_p \in S_p = \{0, 1\} \). In the context of the board of directors and business proposals, one can think of the public signal as the opinion expressed by the advisory board to all directors before the vote takes place. We assume for the moment that the public signal is exogenously and independently drawn according to the conditional probability distribution \( \Pr(s_p = 1|\theta = 1) = \Pr(s_p = 0|\theta = 0) = \beta \in [1/2, 1) \). We will relax this assumption and study strategic disclosure of public information in Section 6.

The timing of the voting game is as follows. First, Nature draws \( \theta \). After that, each agent observes her private signal and, in addition, the public signal. Agents then cast their votes, and the collective decision \( d \) is determined according to the voting profile and the voting rule. Finally, the state is revealed and agents collect their payoffs.

3.3 Strategies and equilibrium

In the voting game, a pure strategy of agent \( i \) is a function \( v_i : S_i \times S_p \rightarrow V_i \) that maps from the Cartesian product of the private and public signal spaces to the action space. We are particularly interested in the following two types of voting strategies (see also KV):

**Definition 1** A strategy is **informative** if \( v_i(s_i, s_p) = s_i, \forall s_i \in S_i, s_p \in S_p \).

**Definition 2** A strategy is **obedient** if \( v_i(s_i, s_p) = s_p, \forall s_i \in S_i, s_p \in S_p \).

We call a Bayes-Nash equilibrium in which all agents play the informative strategy an **informative voting equilibrium**. Similarly, a Bayes-Nash equilibrium in which all agents play the obedient strategy will be called an **obedient voting equilibrium**. For a given preference profile \( q \), if there exists a \( k \)-voting rule under which the informative voting equilibrium exists, we say that such a preference profile allows for the existence of the informative voting equilibrium or simply allows for informative voting.

In the absence of public information, if \( q_i \in [1 - \alpha, \alpha] \ \forall i \in I \), it is easy to check that under the simple majority rule (i.e., \( k = (n + 1)/2 \)) the informative voting equilibrium...
exists and the CJT holds. If all agents are highly biased towards one of the alternatives, a
threshold value $k \neq (n+1)/2$ may need to be adopted in order to sustain informative voting
as an equilibrium. For example, if $q_i \in [\alpha, \alpha^3/(\alpha^3 + (1 - \alpha)^3)] \forall i \in I$, one can show that the
informative voting equilibrium still exists in a voting game with the super-majority rule
$k = (n + 3)/2$, and the CJT continues to hold as $n$ becomes sufficiently large (Laslier and
Weibull [2013]). However, as we will see in the next section, the set of preferences that
allow for informative voting may shrink drastically if a public signal is introduced to the
voting game.

4 Inefficient Information Aggregation

To see how the presence of a public signal could affect the equilibrium outcome of the
voting game, we first provide a necessary and sufficient condition for the existence of the
informative voting equilibrium under any given $k$-voting rule:

**Proposition 1** Given a $k$-voting rule, the informative voting equilibrium exists if and only if

$$\forall i \in I, q_i \in \left[ \frac{1}{1 + \left( \frac{1 - \alpha}{\alpha} \right)^{2k-n-2}}, \frac{1}{1 + \left( \frac{1 - \alpha}{\alpha} \right)^{2k-n}} \right].$$

In the Appendix, we prove a more general version of Proposition 1 which allows the
prior probability of the state to be non-uniform and the accuracy of the private signals to
be state-dependent. By doing so, we substantially generalize the similar result that Wit
(1998) obtains for common-interest voting games with majority rule.

To understand Proposition 1, first note that under a given $k$-voting rule, an agent is
only pivotal when there are exactly $k - 1$ other agents who vote in favor of the decision
$d = 1$, while the remaining $n - k$ agents choose to support the decision $d = 0$. Second, also
note that if agent $i$ prefers to vote according to her private signal even when it conflicts
with the public signal, she will also prefer to do so when the two signals agree. Assuming
all other agents $j \neq i$ follow the informative voting strategy, for a given $k$-voting rule, the
left (right) endpoint of the interval in (4.1) is the posterior probability that a Bayesian
agent $i$ will assign to the event $\theta = 1$ conditional on $s_i = 0, s_p = 1$ ($s_i = 1, s_p = 0$) and being
pivotal. Since a rational agent cares only about the case in which she is decisive about the
final voting outcome, we can conclude that all $q_i$ lying between the above two posterior probabilities is a necessary and sufficient condition for the existence of the informative voting equilibrium under the given $k$-voting rule.

KV observe that if the public signal is more accurate than each of the private signals ($\beta > \alpha$), informative voting for agents who have only common interests cannot constitute an equilibrium under the majority rule. The next two corollaries, which follow Proposition 1 immediately, generalize this important observation to arbitrary precision of the public signal, the whole class of $k$-voting rules, and a much larger set of preferences.

**Corollary 1** Suppose $\beta > \alpha$. For any threshold value $k$ and any preference profile $(q_i)_{i \in I}$, the informative voting equilibrium does not exist.

**Corollary 2** Suppose $\beta \leq \alpha$. The informative voting equilibrium does not exist under any $k$-voting rule if there exist $i, j \in \{1, \ldots, n\}$ such that

$$q_i < \frac{1}{1 + \frac{\alpha}{1 - \alpha} \frac{1}{\beta}} , \quad q_j > \frac{1}{1 + \frac{1 - \alpha}{\alpha} \frac{1}{1 - \beta}}.$$

In words, Corollary 1 confirms that whenever the public signal is strictly more precise than each of the private signals, it is impossible to obtain the informative voting equilibrium under any $k$-voting rule. Meanwhile, Corollary 2 implies that even if the public signal is less accurate, it is still hard to guarantee the existence of the informative voting equilibrium as long as there are two or more agents who are biased (even just slightly) toward different alternatives ex ante.

The intuition behind both corollaries can be understood via the following simple example of three agents with heterogeneous preferences, such that $q_1 = 1 - \alpha$, $q_2 = 1/2$ and $q_3 = \alpha$. Assume that the collective decision is made according to the majority rule ($k = 2$). In the absence of public information, one can check that informative voting constitutes an equilibrium, even though the first and third agents are biased toward different alternatives ex ante. Suppose now agents also observe a public signal that is more informative than each of their private signals. If the unbiased agent 2 assumes that the other two agents will vote informatively, she could infer that the only situation in which she is pivotal is when agent 1 and 3 receive conflicting signals, but this implies that the others’ private signals are collectively uninformative about the state. Hence, in this case, agent
Figure 1: The graphs of the correspondences $Q^{a,k}(\beta)$ given $n = 3, \alpha = 0.75$.

2 would make her voting decision by comparing the observed public signal and her own private signal, and simply follows the public one because of its higher precision. Conversely, suppose the public signal is less informative than the private signals. While it is now rational for the unbiased agent 2 to vote informatively (assuming the other two agents do so as well), this is not the case for the two biased agents. For example, agent 1 will still strictly prefer to choose $v_1 = 1$ if $s_1 = 0$ and $s_p = 1$, even when she assumes that the other two agents are voting informatively. This is because the public signal, albeit less informative, is still in favor of her preferred alternative. Moreover, this problem cannot be resolved by using the unilateral ($k = 1$) or unanimity rule ($k = 3$) instead. For example, suppose all three agents are unbiased and the public signal is just slightly more informative than the private signals. While adopting the unanimity rule can successfully encourage agents to vote informatively whenever $s_p = 0$, it provides even stronger incentives for the agents to disregard their private information whenever $s_p = 1$.

Figure 1 interprets the above results graphically. Suppose for a given $k$-voting rule, an agent $i$ with $q_i$ will find it optimal to play the informative voting strategy when assuming that all other agents $j \neq i$ are voting informatively. Let $Q^{a,k}(\beta) \subseteq [0,1]$ denote the set of all such $q_i$, for given $k$, $\alpha$ and $\beta$. Clearly, for a preference profile $q$, the informative voting equilibrium exists under a given $k$-voting rule if and only if $q_i \in Q^{a,k}(\beta)$, $\forall i \in \mathcal{I}$. Note that, for a given $\alpha$, $Q^{a,k}(1/2)$ corresponds to the set of preferences that allow for informative voting under the given $k$-voting rule when the public signal is absent. For
fixed parameter values $n = 3$ and $\alpha = 0.75$, the top, middle, and bottom part of the gray area in Figure 1 corresponds to the graph of $Q^{a,3}(\beta)$, $Q^{a,2}(\beta)$ and $Q^{a,1}(\beta)$, respectively. As the precision of the public signal increases, the measure of each $Q^{a,k}(\beta)$ decreases. In particular, when $\beta > \alpha$, $Q^{a,k}(\beta) = \emptyset, \forall k = 1, 2, 3$.

Besides shrinking the set of preference profiles that allow for informative voting, the presence of the public signal also introduces the obedient voting equilibrium, which robustly exists under different $k$-voting rules:

**Proposition 2** Given a $k$-voting rule, the obedient voting equilibrium exists if any of the following three conditions is satisfied:

1. $1 < k < n$.
2. $k = 1$ and $\forall i \in I, q_i \geq 1/\left(1 + \frac{1-a}{a} \frac{\beta}{1-p}\right)$.
3. $k = n$ and $\forall i \in I, q_i \leq 1/\left(1 + \frac{a}{1-a} \frac{1-\beta}{p}\right)$.

Clearly, the obedient voting equilibrium can be highly inefficient, especially when the public signal is less accurate or just moderately more accurate than each of the private signals. In other words, unlike endowing its members with better private signals, introducing a public signal may actually lead to a worse performance of the committee. This is similar to one of the most striking findings in the global game literature, namely the heterogeneous effect of public and private information. For instance, in a highly influential paper, [Morris and Shin (2002)] show that in a setting where agents’ actions are strategic complements, additional public information can have negative social value. Although agents in the current setting have no intrinsic motive of coordination, our results suggest similarly that the conventional wisdom that additional information is always beneficial for decision-makers should be carefully examined.

One might question why agents would coordinate on such an inefficient equilibrium. Intuitively, depending on the preference profile, the voting game may also have a symmetric and/or an asymmetric mixed-strategy equilibrium. For some parameter configuration, asymmetric pure-strategy equilibria in which only a small subset of the agents vote obediently may also exist. However, compared to these alternative equilibrium candidates, the obedient voting equilibrium requires conceivably less sophisticated coordi-
nation from the agents. This may be an important reason for the obedient voting equilibrium to be an appealing focal point, especially when the informative voting equilibrium does not exist. For the important special case of unbiased agents, KV experimentally show that a large proportion of subjects tend to follow the public signal instead of their private signals much more frequently than either a symmetric mixed-strategy equilibrium or an asymmetric pure-strategy equilibrium would predict. Consequently, the collective decisions coincided with what the public signal suggested most of the time. This confirms empirically that the presence of a public signal can indeed lead to a significant welfare loss.

We are therefore interested in the question whether there exist more complex voting mechanisms that can help restore the informative voting equilibrium, which arguably requires even less coordination than the obedient voting equilibrium. In the next section, we will show that the answer to the this question is yes if we allow the voting rule to incorporate the information contained in the public signal.

5 Efficient Voting Mechanisms

We introduce a new class of voting rules that we call contingent $k$-voting rules, which can be obtained by adjusting the standard $k$-voting rules in an intuitive way. In particular, the threshold values in such voting rules will be no longer fixed but a function of the realization of the public signal:

$$k(s_p) = \begin{cases} 
  k_0 & \text{if } s_p = 0, \\
  k_1 & \text{if } s_p = 1,
\end{cases}$$

(5.1)

where $k_0, k_1 \in \{1, \ldots, n\}$. Any standard $k$-voting rule amounts to a special case of our contingent $k$-voting rules. In what follows, we will show how this more flexible voting procedure can facilitate information aggregation by restoring the informative voting equilibrium. For clarity of exposition, we will maintain the assumption that the public signal is exogenous throughout this section. We will show in Section 6 that our new voting procedure has also additional advantages when strategic information disclosure is a non-negligible concern.
5.1 Restoring informative voting

We first state the following counterpart to Proposition 1:

**Proposition 3** Given a contingent $k$-voting rule, the informative voting equilibrium exists if and only if

$$\forall i \in I, q_i \in \left[\max\{\pi^0_0, \pi^1_0\}, \min\{\pi^0_1, \pi^1_1\}\right], \quad (5.2)$$

where

$$\pi^0_0 = \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_0-n-2} \frac{\beta}{1-\beta}}, \quad \pi^0_1 = \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_0-n} \frac{\beta}{1-\beta}},$$

$$\pi^1_0 = \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_1-n-2} \frac{1-\beta}{\beta}}, \quad \pi^1_1 = \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_1-n} \frac{1-\beta}{\beta}}.$$

The intuition behind Proposition 3 is very similar to that of Proposition 1. The main difference is that now in the informative voting equilibrium, the information about the other agents’ private signals that an agent can infer from her pivotality depends on the realization of the public signal. In particular, depending on the choices of $k_0$ and $k_1$, it is no longer necessarily the case that an agent will have stronger incentives to deviate from the informative voting equilibrium when her private signal differs from the public signal than when the two signals agree. This is why the interval in (5.2) involves taking the maximum and minimum of the posterior probabilities assigned to the event $\theta = 1$ in different cases.

To see how the contingent $k$-voting rules can help restore the informative voting equilibrium, consider a simple example with $n = 5$ and $q_i = \frac{1}{2}, \forall i \in I$. If there is no public signal, the informative voting equilibrium exists under the standard majority rule. Now let us introduce a public signal that is as informative as two independent private signals.

By Corollary 1, this implies that the informative voting equilibrium no longer exists under any $k$-voting rule. However, consider the contingent $k$-voting rule with the threshold values $k_0 = 4$ and $k_1 = 2$. Suppose all other agents $j \neq i$ are voting informatively. If $s_p = 1$, agent $i$ is only pivotal when three of the other agents draw $s_j = 0$ and the remaining one draws $s_j = 1$. Given the above assumption on the informativeness of the public signal,

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4A formal measure of the relative informativeness of the public signal will be defined in Section 5.2.
these private signals are collectively uninformative about the state when they are combined with the realization of the public signal. Thus, voting according to her own private signal is a best response for agent $i$. Similarly, if $s_p = 0$, agent $i$ is only pivotal under the contingent $k$-voting rule when there are three private signals in favor of $d = 1$ and one in favor of $d = 0$ among all others’ private signals. Again, the collective informational effect of all $s_j, j \neq i$, will be exactly counterbalanced by the fact that $s_p = 0$, which makes it optimal for agent $i$ to simply follow her own signal.

Intuitively, the public signal introduces a common shock to all agents’ priors, which has a very similar effect as a common shock on the preference profile in our model. As mentioned in Section 3.3, when agents are commonly biased towards one of the alternatives, the informative voting equilibrium can still exist if the threshold value $k$ is appropriately chosen. However, since the shock due to the public signal is random, the direction of the correction of the threshold value must depend on the realization of the public signal. In particular, the threshold value should be adjusted in such a way that the alternative favored by the public signal is more likely to be chosen. At first glance, this might be counter-intuitive because the negative welfare impact of the public signal, as discussed in Section 4, is exactly due to the problem that agents are tempted to obey the public signal and disregard their valuable private signals. However, what we are doing here is to change the information that agents can infer from pivotality: under the contingent $k$-voting rule chosen in the above example, an agent is pivotal only if the private signals of the other agents are collectively more against the alternative favored by the public signal. This restores the incentive for agents to vote according to their own signals.

5.2 The choice of voting thresholds

We are now interested in the question of what would be suitable choices of $k_0$ and $k_1$ in order to restore the informative voting equilibrium by using a contingent $k$-voting rule that takes the form of (5.1). Similar to $Q^{\alpha,k} (\beta)$, let $Q^{\alpha,k_0,k_1} (\beta) \subseteq [0,1]$ denote the set of $q_i$ such that given the precision of the signals $\alpha$ and $\beta$ and a contingent $k$-voting rule with the threshold values $k_0$ and $k_1$, an agent $i$ with $q_i$ would find it optimal to play the informative voting strategy if all other agents $j \neq i$ are voting informatively. We first ask
what choices of \( k_0 \) and \( k_1 \) would lead to a non-empty \( Q^{\alpha,k_0,k_1}(\beta) \) for given \( \alpha \) and \( \beta \). To answer this question, we introduce the following measure of (relative) informativeness of the public signal:

\[
    r = \frac{\ln \beta - \ln(1 - \beta)}{\ln \alpha - \ln(1 - \alpha)}.
\]  

(5.3)

For given \( \alpha \) and \( \beta \) the value of \( r \) is uniquely determined, and we will say that the public signal is \( r \)-times as informative as a private signal. For example, if \( \alpha = 0.6 \), then \( \beta = 0.55, 0.69, 0.77 \) correspond to the cases where the public signal is 0.5-, 2- and 3-times as informative as a private signal, respectively. Intuitively, the measure \( r \) tells us how many private signals of opposite realization would counter-balance the informational effect of the public signal. We are now ready to state the next corollary, which follows from Proposition 3 and relates non-emptiness of the set \( Q^{\alpha,k_0,k_1}(\beta) \) to the relative informativeness of the public signal.

**Corollary 3** \( Q^{\alpha,k_0,k_1}(\beta) \neq \emptyset \) if and only if \( r - 1 \leq k_0 - k_1 \leq r + 1 \).

Given the result of Corollary 3, we will restrict our attention to the contingent \( k \)-voting rules with \( k_0 \) and \( k_1 \) such that the inequality \( r - 1 \leq k_0 - k_1 \leq r + 1 \) is satisfied. If \( r \leq 1 \), any pair of integers \( k_0 \) and \( k_1 \) such that \( k_0 = k_1 \) or \( k_0 = k_1 + 1 \) satisfies the above inequality. If \( 1 < r \leq n \), it is easy to see that there still exists at least one pair of integers \( k_0 \) and \( k_1 \) that satisfies the inequality and leads to a non-empty \( Q^{\alpha,k_0,k_1}(\beta) \). However, for the case \( r > 1 \), we know by Corollary 1 that \( Q^{\alpha,k}(\beta) = \emptyset \) \( \forall k \in \{1,..,n\} \). Hence, introducing the contingent \( k \)-voting rules can strictly enlarge the set of preferences that allow for informative voting, as long as the public signal is not extremely precise \( (r \leq n) \).

When the size of the committee is large, there might be many contingent \( k \)-voting rules whose threshold values satisfy the inequality in Corollary 3. Therefore, for a given preference profile \( q \) it can be tedious to go through every pair of these threshold values,  

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5The measure \( r \) can be related to the conditional entropy of \( \theta \). To see this, note that the conditional entropy of \( \theta \) given the private signal \( s_i \) is \( H(\theta|s_i) = 1 - \alpha \ln \alpha - (1 - \alpha) \ln(1 - \alpha) \), and similarly the conditional entropy of \( \theta \) given the public signal \( s_p \) is \( H(\theta|s_p) = 1 - \beta \ln \beta - (1 - \beta) \ln(1 - \beta) \). Hence, we have

\[
\frac{dH(\theta|s_p)/d\beta}{dH(\theta|s_i)/d\alpha} = \frac{\ln \beta - \ln(1 - \beta)}{\ln \alpha - \ln(1 - \alpha)}.
\]

This gives a formal justification for using \( r \) as a measure of relative informativeness of the public signal. The same entropy formula \( \ln \alpha - \ln(1 - \alpha) \) is also used by Nitzan and Paroush (1982) and Ben-Yashar and Nitzan (2014) to construct the optimal decisive decision rule in settings of non-strategic voting.
and see with which the condition in Proposition 3 will be satisfied. Fortunately, we can rely on the following algorithm. First, define \( q = \min_{i \in I} q_i \) and \( \bar{q} = \max_{i \in I} q_i \) for the given preference profile. Second, invert the functions \( \pi_0^0 \) and \( \pi_1^0 \) of \( k_0 \) and the functions \( \pi_0^1 \) and \( \pi_1^1 \) of \( k_1 \) that are defined in Proposition 3. Note that this is feasible because all these are strictly increasing functions. Third, apply the inverse functions \((\pi_0^0)^{-1}\) and \((\pi_1^0)^{-1}\) to \( \bar{q} \) and \((\pi_0^1)^{-1}\) and \((\pi_1^1)^{-1}\) to \( q \). Finally, pick an integer that lies in the interval \([(\pi_1^0)^{-1}(\bar{q}),(\pi_0^0)^{-1}(q)]\) and an integer that lies in the interval \([(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(q)]\). We then obtain a pair of voting threshold values \( k_0 \) and \( k_1 \) that can be used to sustain informative voting as an equilibrium for the given preference profile.

Note that both \((\pi_0^0)^{-1}\) and \((\pi_1^0)^{-1}\) are strictly increasing in \( \bar{q} \), while both \((\pi_0^1)^{-1}\) and \((\pi_1^1)^{-1}\) are strictly increasing in \( q \). Hence, it is possible that both of the above-mentioned intervals contain no integer if \( \bar{q} \) is sufficiently larger than \( q \). Intuitively, if the degree of conflict of interest between the agents is too large, it is very difficult to find a voting rule that ensures the incentive for all agents to vote informatively, even if we allow the voting threshold value to be flexibly contingent on the public signal. One might therefore expect that it would be easier to find a contingent \( k \)-voting rule that helps restore the existence if the agents’ preferences are more aligned. This is not true in general because it also depends on the exact values of \( q \) and \( \bar{q} \). Nevertheless, for the important limiting cases where agents’ preferences are perfectly aligned (e.g., Feddersen and Pesendorfer, 1998; Persico, 2004; Koriyama and Szentes, 2009), we are able to prove that there always exists a contingent \( k \)-voting rule under which the informative voting equilibrium exists, as long as the size of the committee is sufficiently large:

**Proposition 4** Suppose \( \forall i \in I, q_i = q \). There exists \( \bar{n}(q) \), such that for each \( n \geq \bar{n}(q) \), there exists a contingent \( k \)-voting rule that can sustain informative voting as an equilibrium.

Figure 2 illustrates the main findings of this section graphically. Consider the same parametric example that we discussed in Section 4, where \( n = 3 \) and \( \alpha = 0.75 \). Besides the three standard \( k \)-voting rules, we know from Corollary 3 that there are three other contingent \( k \)-voting rules that may lead to a non-empty \( Q^{\alpha, k_0, k_1}(\beta) \) (depending on the exact value of \( r \)), with the threshold values \( k(s_p) = 1 + 1_{s_p=0} \), \( k(s_p) = 2 + 1_{s_p=0} - 1_{s_p=1} \) and \( k(s_p) = 2 + 1_{s_p=0} \), respectively. Figure 2 above plots the graphs of the correspondences \( Q^{\alpha, k_0, k_1}(\beta) \) for all these contingent \( k \)-voting rules. As in Figure 1, the top, middle and
bottom part of the light gray area corresponds to the graph of \( Q^{a,3}(\beta) \), \( Q^{a,2}(\beta) \) and \( Q^{a,1}(\beta) \), respectively. In addition, the graphs of the correspondences \( Q^{a,2,1}(\beta) \) and \( Q^{a,3,2}(\beta) \) are represented respectively by the top and the bottom part of the deep gray area. Finally, the dark area corresponds to the graph of \( Q^{a,3,1}(\beta) \). Given a preference profile \( q \) and the precision of the public signal, the informative voting equilibrium can be sustained by some contingent \( k \)-voting rule if and only if all \( q_i \) lie on a vertical line that is entirely contained in one of the different segments in the figure. The algorithm that we described before is just an analytical way for us to find a segment that contains all \( q_i \). Comparing Figure 1 and Figure 2, it is clear that our new voting procedure can help restore the informative voting equilibrium in many circumstances.

5.3 Welfare analysis

In this section, we investigate the welfare implications of the contingent \( k \)-voting rules. This is not a trivial task, because the threshold value in a contingent \( k \)-voting rule depends on the realization of the public signal, and it is well-known that different thresh-

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\(^6\)For some preference profiles, one can actually induce the informative voting equilibrium by introducing a public signal of certain accuracy and using some contingent-\( k \) voting rule, even though this would not be possible by using any standard \( k \)-voting rule in the absence of the public signal. For example, suppose \( n = 3 \) and the agents’ preferences correspond to the three dots that lie on the vertical axis in Figure 2. If there is no public signal, the informative voting equilibrium cannot be sustained by any standard \( k \)-voting rule, since not all \( q_i \) are located in the same segment when \( \beta = 1/2 \). However, it is clear from the graph that when \( \beta \) is sufficiently close to \( \alpha \), all \( q_i \) lie on a line that is entirely contained in the top deep gray area, thus the informative voting equilibrium exists if we adopt the threshold value \( k(s_p) = 2 + \mathbb{1}_{s_p=0} \).
old values can lead to different probabilities of admitting different types of decision errors (Feddersen and Pesendorfer, 1998; Duggan and Martinelli, 2001).

While the mere fact that the contingent $k$-voting rules can restore the informative voting equilibrium does not guarantee that it leads to the first-best outcome, it seems intuitive that its efficiency should be increasing in the size of the committee. However, by adding new members to the committee, both the amount of private information and the degree of conflict of interest may be increased at the same time. The latter effect is problematic because it may destroy the existence of a contingent $k$-voting rule that can incentivize the agents to vote informatively, even if this is not a problem at all for the initial committee. To avoid this complication, we consider only the type of expansions of the committee that do not exaggerate the initial degree of conflict of interest. Formally, let $\mathbf{q} = (q_1, \ldots, q_n)$ be a preference profile with $\bar{q} = \max_{i \in I} q_i$ and $\underline{q} = \min_{i \in I} q_i$. We say a sequence of preference profiles \{q^s = (\hat{q}_1, \ldots, \hat{q}_{n+s})\}_{s \in \mathbb{N}} preserves the conflict of $\mathbf{q}$ if $\forall \mathbf{q}^s$, $\max_{j \in [1, \ldots, n+s]} \hat{q}_j \leq \bar{q}$ and $\min_{j \in [1, \ldots, n+s]} \hat{q}_j \geq \underline{q}$. For this type of expansions of the committee, the following proposition states that the informative voting equilibria under the contingent $k$-voting rules asymptotically achieve the first-best:

**Proposition 5** Suppose, for a given preference profile $\mathbf{q}$ with $\underline{q}, \bar{q} \in (0, 1)$, that there exists a contingent $k$-voting rule that can sustain informative voting as an equilibrium. Then, for any sequence of preference profiles \{q^s\}_{s \in \mathbb{N}} that preserves the conflict of $\mathbf{q}$:

1. $\forall \mathbf{q}^s$, there exists a contingent $k$-voting rule that can sustain informative voting as an equilibrium.

2. As $s \to \infty$, the ex ante probability of the collective decision being matched to the state in the informative voting equilibrium under the corresponding contingent $k$-voting rule becomes arbitrarily close to 1.

We thus obtain a version of the Condorcet Jury Theorem for the contingent $k$-voting rules in a general voting environment with both private and public information. As an implication, any equilibrium that is not asymptotically efficient (e.g., the obedient voting equilibrium) will be outperformed by the informative voting equilibria under the contingent $k$-voting rules, provided that the size of the committee is sufficiently large.
In the setting where agents have only common interests, KV show that if the public signal is $r$-times as informative as a private signal, where $r \leq (n - 1)/2$, then under the simple majority rule there exists an asymmetric equilibrium in which $r^\ast = \mathbb{N} \cap (r-1, r]$ agents obey the public signal, while the remaining $n - r^\ast$ agents vote according to their private signals. This $r^\ast$-asymmetric equilibrium is shown to be more efficient than both the obedient voting equilibrium and the symmetric mixed-strategy equilibrium, as well as all other asymmetric pure-strategy equilibria in the same voting game. In the following, we will show in the same setting that one can always construct a contingent $k$-voting rule that not only ensures the existence of the informative voting equilibrium, but also leads to strictly higher efficiency than the $r^\ast$-asymmetric equilibrium.

Specifically, consider a contingent $k$-voting rule with the following threshold value:

$$k(s_p) = \begin{cases} \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ & \text{if } s_p = 0, \\ \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ & \text{if } s_p = 1, \end{cases}$$

where $\left[\frac{(r-1)/2}{2}\right]^+$ denotes the smallest integer that is larger or equal to $(r-1)/2$. For convenience, we will call this rule the contingent majority rule. Note that the contingent majority rule is well-defined whenever $r \leq n$. The following corollary of Proposition 3 justifies our focus on this particular contingent $k$-voting rule:

**Corollary 4** The informative voting equilibrium exists under the contingent majority rule if and only if

$$\forall i \in \mathcal{I}, \ q_i \in \left[ \frac{1}{1 + \left( \frac{1-a}{a} \right)^{|r-2((r-1)/2)|-1}}, \frac{1}{1 + \left( \frac{1-a}{a} \right)^{-|r-2((r-1)/2)|+1}} \right]. \quad (5.4)$$

Note that $1/2$ always belongs to the above interval, since $|r-2((r-1)/2)| \in [0, 1]$ for all $r \geq 0$. In other words, for the special case where all agents are unbiased, which is mostly studied in the literature and, in particular, in KV, one can always use the contingent majority rule to ensure the existence of the informative voting equilibrium. In fact, for this case the contingent majority rule is essentially the unique contingent $k$-voting rule that restores the existence.$^7$ The next proposition states that the informative voting equilib-
rium under the contingent majority rule achieves the first-best informational efficiency.

**Proposition 6**  Given all the information that is available to the committee, the probability of the collective decision being matched to the state is maximized in the informative voting equilibrium sustained by the contingent majority rule.

To gain some intuition, consider a simple example of $n = 5$ and $r = 2$. Assume all agents are unbiased. Imagine that we introduce two additional phantom agents on top of the existing five real agents. These phantom agents are programmed so that they simply vote in line with the public signal. Suppose now the simple majority rule is used to decide which alternative will be chosen. One can easily show that (1) all real agents voting informatively constitutes an equilibrium in this game (despite that the public signal observed by the agents is more precise than each of their private ones), (2) the equilibrium outcome is identical to that of the informative voting equilibrium under the contingent majority rule without the phantom agents, and (3) the equilibrium outcome maximizes the probability of the decision being matched to the state, given all the available information. Intuitively, by allowing the threshold value to be dependent on the public signal and by encouraging agents to vote informatively, the contingent majority rule aggregates both the private and the public information efficiently.

On the contrary, in the $r^*$-asymmetric equilibrium, inefficiency still prevails because there are $r^*$ agents who always disregard their valuable private information. To see this issue more clearly, consider again the above example. Since in this case we have $r^* = \mathbb{N} \cap (r - 1, r] = 2 = (n - 1)/2$, under the simple majority rule there exists an asymmetric equilibrium in which two agents play the obedient strategy, while the remaining three agents vote informatively. Without loss of generality, assume the first two agents are the obedient voters. Consider the signal profile $s = (1, 1, 0, 1, 1)$ and $s_p = 0$. In equilibrium, such a realization of signals will lead to a collective decision $d = 0$. However, from a benevolent social planner’s point of view, given all the available information, the welfare maximizing decision should be $d = 1$. Therefore, the $r^*$-asymmetric equilibrium is strictly less efficient than the first-best. 

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*can be used to restore the informative voting equilibrium (if exists) is unique if and only if either of the following two conditions is met: (a) $\bar{q} < \bar{q}$; (b) both $(\pi_0)^{-1}(\bar{q})$ and $(\pi_1)^{-1}(\bar{q})$ are not integers. If $\bar{q} = \bar{q} = 1/2$, both $(\pi_0)^{-1}(1/2) = (n + r)/2$ and $(\pi_1)^{-1}(1/2) = (n - r)/2$ are not integers unless $r$ is an odd integer that is strictly less than $n$.  

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We close this section with an important remark about the implementation of the contingent $k$-voting rules, which is an intuitive yet non-obvious implication of Proposition 6. In practice, it might be difficult to implement a voting rule that is contingent on the public information, especially when the source of the relevant public information is ambiguous ex ante. However, within a setting where agents have only common-interests, we may consider instead implementing a simple two-stage voting mechanism, in which, after observing the private and the public signals, the agents first vote about which voting threshold value $k \in \{1, ..., n\}$ to use and then vote about which collective decision to take. Independent of the specific procedure of the first-stage voting rule and the off-equilibrium beliefs and strategies of the agents, in the two-stage voting game there must exist a Perfect Bayesian Nash equilibrium that is outcome-equivalent to the informative voting equilibrium under the contingent majority rule, i.e., an equilibrium in which agents first collectively vote to agree on the threshold value that would be chosen by the contingent majority rule, and then they vote informatively in the second stage. The reason being is that, according to Proposition 6, the expected social welfare is maximized when such a voting threshold value is in use. Since each agent’s interest is perfectly aligned with the social welfare, any deviation in the first stage will only yield a lower expected payoff to an agent.

6 Strategic Information Disclosure

In this section, we drop the assumption that the disclosure of public information is exogenous, but consider it to be strategically determined by a possibly biased information controller (e.g., an external expert). As illustrated in Section 4, public information can have a huge influence on the committee’s decision when the standard simultaneous voting rules are in use. Taking this into account, a biased controller may only publicly reveal his information to the agents when its content is in support of his favored alternative. For example, an advisory board member who has private interests in the targeted firm may withhold unfavorable information from the directory board when the acquisition decision is being made. In what follows, we will formalize this intuition by extending our baseline model in Section 3 and, in addition, argue that using a contingent voting rule adapted from the ones constructed in Section 5 can mitigate the controller’s incentive for
strategic disclosure and his influence on the voting outcome, which in turn improves the efficiency of the collective decision.

Suppose now the signal $s_p$ described in Section 3.2 is no longer public by default but can only be observed by an information controller with some probability. Specifically, with probability $\lambda \in (0, 1)$, the controller is uninformed and can only send a public message $m = \emptyset$ (remains silent) to the agents. With the complementary probability $1 - \lambda$, the controller observes the signal and can decide whether to publicly communicate its content to the agents or not. While we allow the controller to withhold his information, we assume that the signal is hard information and hence cannot be faked. In other words, in the latter case the public message $m$ can be only chosen from the set $\{s_p, \emptyset\}$.

Assume for simplicity that $q_i = 1/2 \forall i \in I$ and that the collective decision is made according to the simple majority rule. Note again that in this case, the informative voting equilibrium exists if no additional public information is available to the committee members. Assume also that the controller has the same form of utility function as the agents, and his bias parameter is given by $q_c \in [0, 1]$. Let $\hat{\lambda} = \max\{0, (\beta - \alpha)/ (\beta - (1 - \alpha))\}$. The following proposition establishes that if agents update their beliefs sufficiently little upon observing silence (i.e., $\lambda$ is large enough), a biased information controller may indeed exploit the publicity of his message and reveal his information selectively.

**Proposition 7** Suppose $\lambda \geq \hat{\lambda}$. There exists $\hat{\beta} \in [1 - \beta, 1/2]$ such that if $q_c \leq \hat{\beta}$ ($q_c \geq 1 - \hat{\beta}$), then there exists a sequential equilibrium in which the controller sends $m = s_p$ if and only if he observes $s_p = 1$ ($s_p = 0$), and the agents vote obediently if $m = s_p$ and informatively if $m = \emptyset$.

As a numerical example, if $n = 3$, $\alpha = 0.65$ and $\beta = 0.7$, the threshold values are given by $\hat{\lambda} \approx 0.14$ and $\hat{\beta} \approx 0.48$, respectively. Depending on the relative precision of the signals, the informational efficiency of the committee’s decision could be improved if there were more or less information disclosure than that in the equilibrium. For instance, the decision will be more accurate in the above numerical example if the controller always keeps silent and just let the agents credibly coordinate on informative voting in the voting stage.

Some recent papers look at the question how an information controller can optimally persuade uninformed agents by designing the informational content of a public signal (e.g.,

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3With the general results in Sections 4 and 5, our analysis in the current section can be straightforwardly extended to the case of general preference profiles and voting rules.
In our model, voters are privately informed and the controller has control over the disclosure of the public signal only. Hence, our environment is notably less favorable for the controller. Nevertheless, Proposition 7 suggests that the strategic incentive of the controller and his impact on the committee’s decision still cannot be ignored.

Fortunately, this concern can be mitigated by instead using a contingent voting rule with the following threshold value:

\[ k(m) = \begin{cases} 
\frac{n+1}{2} & \text{if } m = \emptyset, \\
\frac{n+1}{2} + \left\lceil \frac{r-1}{2} \right\rceil^+ & \text{if } m = 0, \\
\frac{n+1}{2} - \left\lfloor \frac{r-1}{2} \right\rfloor^+ & \text{if } m = 1.
\end{cases} \]

**Proposition 8** Suppose \( r \leq n \) and the proposed contingent voting rule is used. There exists \( q^* \in [0, 1 - \beta] \) such that

1. If \( q_c \in [q^*, 1 - q^*] \), there exists a sequential equilibrium in which the controller sends \( m = s_p \) whenever he is informed, and the agents always vote informatively.

2. If \( q_c \leq q^* (q_c \geq 1 - q^*) \) and \( \lambda \geq \hat{\lambda} \), there exists a sequential equilibrium in which the controller sends \( m = s_p \) if and only if he observes \( s_p = 1 (s_p = 0) \), and the agents always vote informatively.

By comparing Propositions 7 and 8, we can see that the contingent voting rule has two main advantages over the simple majority rule. First, the contingent voting rule incorporates the informational content in the controller’s message appropriately and makes it credible for the agents to coordinate on informative voting in the voting stage. Hence, by the same reasoning as in Proposition 6, the decision selected by the contingent voting rule is most likely to match the state, given all the information that is available to the committee, independent of the controller’s disclosure policy and the relative precision of the signals. Second, under the contingent voting rule the controller also has a higher incentive to share his information unconditionally, since he anticipates that his message will not have a direct impact on the agents’ voting behavior and will always help increase the accuracy of the committee’s decision. Indeed, for the previous numerical example \( (n = 3, \alpha = 0.65 \text{ and } \beta = 0.7) \), we have \( q^* \approx 0.23 \), which is substantially smaller than \( \hat{q} \).
7 Conclusion

This paper makes two main contributions. First, we show in a general setting of collective decision-making that the provision of public information can have a detrimental effect on the efficiency of the committee decision. In particular, the presence of public information may significantly limit the existence of the informative voting equilibrium, and it introduces the inefficient obedient voting equilibrium. We believe these results to be of high policy relevance, especially since the immense influence of public information may be exploited by a strategic information controller. Second, we show that by appropriately incorporating public information into the voting procedure, the informative voting equilibrium can be restored and the efficiency of the committee decision can be enhanced. By reducing the direct effect of public information on the agents’ voting behavior, the proposed voting procedure also mitigates the concern of strategic information disclosure.

In general, our results suggest that in a voting environment where both private and public information is present, the voting procedure matters and the optimal voting rule should reflect the content of the public information. For example, if the advisory board indicates that one of the business proposals is more promising than the other, it might be desirable for the board of directors to set up a voting rule that is more in favor of the acceptance of that proposal. The design of optimal decision rules in more general social choice environments with public information remains an open and important research question.
Appendix

8.1 Proofs of results in Section 4

We prove the main results in Section 4 (i.e., Proposition 1 and 2) more generally by allowing the prior probability of the state to be non-uniform and the accuracy of the private signals to be state-dependent. Specifically, we assume $Pr(\theta = 0) = 1 - Pr(\theta = 1) = \pi \in (0, 1)$, and each of the private signals is independently drawn according to the conditional distribution characterized by $Pr(s_i = 0|\theta = 0) = \alpha_0$ and $Pr(s_i = 1|\theta = 1) = \alpha_1$, where $\alpha_0, \alpha_1 \in (1/2, 1)$. The results in the main text will then follow by letting $\pi = 1/2$ and $\alpha_0 = \alpha_1 = \alpha$.

Denote the signal profile of the agents by $s = (s_1, ..., s_n)$ and let $m_s = \sum_{i=1}^n s_i$. As an auxiliary result, note that conditional on observing the whole profile of private signals and the public signal, the posterior probability that a Bayesian agent would assign to the event $\theta = 1$ is given as follows:

$$Pr(\theta = 1|s, s_p) = \frac{Pr(\theta = 1, s, s_p)}{Pr(s, s_p)}$$

$$= \frac{Pr(s, s_p|\theta = 1) Pr(\theta = 1)}{Pr(s, s_p|\theta = 1) Pr(\theta = 1) + Pr(s, s_p|\theta = 0) Pr(\theta = 0)}$$

$$= \frac{\alpha_1^{m_s} (1 - \alpha_1)^{n - m_s} \beta^{\sum_{p=1} m_s} (1 - \beta)^{\sum_{p=0} (1 - \pi)}}{\alpha_1^{m_s} (1 - \alpha_1)^{n - m_s} \beta^{\sum_{p=1} m_s} (1 - \beta)^{\sum_{p=0} (1 - \pi)} + (1 - \alpha_0)^{m_s} \alpha_0^{n - m_s} (1 - \beta)^{\sum_{p=1} m_s} \beta^{\sum_{p=0} (1 - \pi)}}$$

where the first equality follows from Bayes rule and the third equality follow from the independence assumption of the signals.
8.1.1 Proof of Proposition 1

We will show that given a $k$-voting rule, the informative voting equilibrium exists if and only if

$$\forall i \in I, \quad q_i \in \left[ \frac{1}{1 + \left( \frac{1 - \alpha_0}{\alpha_1} \right)^k \left( \frac{\alpha_0}{1 - \alpha_1} \right)^{n-k} \left( \frac{1 - \beta}{\beta} \right) \left( \frac{\pi}{1 - \pi} \right)}, \frac{1}{1 + \left( \frac{1 - \alpha_0}{\alpha_1} \right)^{k-1} \left( \frac{\alpha_0}{1 - \alpha_1} \right)^{n-k+1} \left( \frac{1 - \beta}{\beta} \right) \left( \frac{\pi}{1 - \pi} \right)} \right].$$

Suppose all agents $j \neq i$ play $v_j(s_j, s_p) = s_j$. Firstly, note that if $v_i(1,0) = 1$ is rational for agent $i$, so is $v_i(1,1) = 1$; similarly, if $v_i(0,1) = 0$ is rational for agent $i$, so is $v_i(0,0) = 0$. Hence, we only need to consider the optimality of the informative voting strategy in the cases where $s_i \neq s_p$.

Secondly, note that agent $i$ is only decisive when and only when there are $k - 1$ agents who observe a positive signal ($s_j = 1$) and each of the remaining $n - k$ agents observes an opposite signal ($s_j = 0$). Hence, given $s_i = 1, s_p = 0$ and being pivotal, the posterior probability that agent $i$ assigns to the event $\theta = 1$ is:

$$\pi_1^0 = \frac{1}{1 + \left( \frac{1 - \alpha_0}{\alpha_1} \right)^k \left( \frac{\alpha_0}{1 - \alpha_1} \right)^{n-k} \left( \frac{1 - \beta}{\beta} \right) \left( \frac{\pi}{1 - \pi} \right)}.$$

Similarly, given $s_i = 0, s_p = 1$ and being pivotal, the posterior probability that agent $i$ assigns to the event $\theta = 1$ is:

$$\pi_0^1 = \frac{1}{1 + \left( \frac{1 - \alpha_0}{\alpha_1} \right)^{k-1} \left( \frac{\alpha_0}{1 - \alpha_1} \right)^{n-k+1} \left( \frac{1 - \beta}{\beta} \right) \left( \frac{\pi}{1 - \pi} \right)}.$$

Hence, to have informative voting as an equilibrium, it is both necessary and sufficient to have $\forall i \in I, \quad q_i \in [\pi_0^1, \pi_1^0]$.

Finally, by letting $\pi = 1/2$ and $\alpha_0 = \alpha_1 = \alpha$, we immediately obtain condition (4.1). \(\square\)
8.1.2 Proof of Corollary 1

Note that the interval \([\pi_0^1, \pi_0^0]\) as defined in the proof of Proposition 1 is non-empty if and only if

\[
\left(\frac{1 - \alpha_0}{\alpha_1}\right)^{k-1} \left(\frac{\alpha_0}{1 - \alpha_1}\right)^{n-k+1} \left(1 - \frac{1 - \beta}{\beta}\right) \geq \left(\frac{1 - \alpha_0}{\alpha_1}\right)^k \left(\frac{\beta}{1 - \beta}\right) \left(\frac{\pi}{1 - \pi}\right)
\]

which is equivalent to

\[
\left(\frac{\alpha_0}{1 - \alpha_0}\right) \left(\frac{\alpha_1}{1 - \alpha_1}\right) \geq \left(\frac{\beta}{1 - \beta}\right)^2. \tag{A.1}
\]

If the accuracy of the private signals is state-independent, i.e., \(\alpha_0 = \alpha_1 = \alpha\), \((A.1)\) is further equivalent to \(\alpha \geq \beta\). \qed

8.1.3 Proof of Corollary 2

Suppose \(\pi = 1/2\), \(\alpha_0 = \alpha_1 = \alpha\) and there exists a \(k\)-voting rule under which the informative voting equilibrium exists. According to the proof of Proposition 1, the preferences of agents \(i\) and \(j\) must satisfy

\[
q_i, q_j \in \left[\frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{2k-n-2} \frac{1 - \beta}{\beta}}, \frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{2k-n} \frac{1 - \beta}{\beta}}\right]. \tag{A.2}
\]

Moreover, \((A.2)\) and \(q_i < \frac{1}{1 + \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta}}\) implies

\[
\frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{2k-n-2} \frac{1 - \beta}{\beta}} < \frac{1}{1 + \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta}} \iff k < \frac{n + 1}{2}. \tag{A.3}
\]

Similarly, \((A.2)\) and \(q_j > \frac{1}{1 + \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta}}\) implies

\[
\frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{2k-n} \frac{1 - \beta}{\beta}} > \frac{1}{1 + \frac{\alpha}{1 - \alpha} \frac{1 - \beta}{\beta}} \iff k > \frac{n + 1}{2}. \tag{A.4}
\]
Clearly, (A.3) and (A.4) are mutually exclusive. Hence, we can conclude that the informative voting equilibrium does not exist under any \( k \)-voting rule.

### 8.1.4 Proof of Proposition 2

If \( 1 < k < n \), given all other agents \( j \neq i \) are simply obeying the public signal, agent \( i \) will never be pivotal, thus playing \( v(s_i, s_p) = s_p \) for all \( (s_i, s_p) \) is clearly a best response.

If \( k = 1 \), assuming all other agents \( j \neq i \) are voting obediently, agent \( i \) will be pivotal if and only if \( s_p = 0 \). However, agent \( i \) cannot infer anything about the others’ private signals conditional on her pivotality, because they are just obeying the public signal. Thus, agent \( i \) will simply vote according to her posterior about the state, and it is a best response for her to always follow the public signal if she is not too inclined to choose \( d = 1 \) even when she receives \( s_i = 1 \). Hence, within the current more general setting, the obedient voting equilibrium exists if

\[
\forall i \in I, q_i \geq \frac{1}{1 + \frac{1 - \alpha_0}{\alpha_1} \frac{\beta}{1 - \beta} \frac{\pi}{1 - \pi}}.
\]

If \( k = n \), assuming all other agents \( j \neq i \) are voting obediently, agent \( i \) will be pivotal whenever \( s_p = 1 \). Hence, analogously, the obedient voting equilibrium exists if

\[
\forall i \in I, q_i \leq \frac{1}{1 + \frac{\alpha_0}{1 - \alpha_1} \frac{1 - \beta}{\beta} \frac{\pi}{1 - \pi}}.
\]

Finally, by letting \( \pi = 1/2 \) and \( \alpha_0 = \alpha_1 = \alpha \), the statement in Proposition 2 immediately follows.

### 8.2 Proofs of results in Section 5

#### 8.2.1 Proof of Proposition 3

Suppose \( s_p = 1 \). Under the stated contingent \( k \)-voting rule, the threshold value for choosing \( d = 1 \) is \( k_1 \). Assume all agents \( j \neq i \) are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent \( i \) would assign to the event
θ = 1 if \( s_i = 0 \) or \( s_i = 1 \) are, respectively:

\[
\pi_0^1 = \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{k_1-1} \left( \frac{\alpha}{1-\alpha} \right)^{n-k_1+1} 1 - \frac{\beta}{1-\beta}} \quad \text{and} \quad \pi_1^1 = \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{k_1} \left( \frac{\alpha}{1-\alpha} \right)^{n-k_1} 1 - \frac{\beta}{1-\beta}}
\]

Now suppose \( s_p = 0 \). Under the stated contingent \( k \)-voting rule, the threshold value for choosing the decision \( d = 1 \) is \( k_0 \). Assume all agents \( j \neq i \) are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent \( i \) would assign to the event \( \theta = 1 \) if \( s_i = 0 \) or \( s_i = 1 \) are, respectively:

\[
\pi_0^0 = \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{2k_1-n-2} 1 - \frac{\beta}{1-\beta}} \quad \text{and} \quad \pi_1^0 = \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{2k_0-n} 1 - \frac{\beta}{1-\beta}}
\]

Hence, the informative voting equilibrium exists if and only if \( \forall i \in I, q_i \geq \max\{\pi_0^0, \pi_1^0\} \) and \( q_i \leq \min\{\pi_1^0, \pi_1^1\} \).

### 8.2.2 Proof of Corollary 3

For a contingent \( k \)-voting rule with the threshold value \( k(s_p) = \mathbb{1}_{s_p=1} k_1 + \mathbb{1}_{s_p=0} k_0 \), it is clear that \( Q_{\alpha,k_0,k_1}(\beta) \) is non-empty if and only if the interval in (5.2) is non-empty, i.e., \( \max\{\pi_0^0, \pi_1^1\} \leq \min\{\pi_1^0, \pi_1^1\} \). Note that

\[
\max\{\pi_0^0, \pi_1^1\} = \pi_0^1 \iff \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{2k_1-n-2} 1 - \frac{\beta}{1-\beta}} \geq \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{2k_0-n} 1 - \frac{\beta}{1-\beta}}
\]

\[
\iff \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{2k_1-n-2} 1 - \frac{\beta}{1-\beta}} \geq \frac{1}{1 + \left( \frac{1-\alpha}{\alpha} \right)^{2k_0-n} 1 - \frac{\beta}{1-\beta}}
\]

\[
\iff \left( \frac{1-\alpha}{\alpha} \right)^{2k_1-n-2+r} \leq \left( \frac{1-\alpha}{\alpha} \right)^{2k_0-n-2}
\]

\[
\iff k_0 - k_1 \leq r
\]
\[
\min\{\pi_0^0, \pi_1^1\} = \pi_1^0 \iff \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_0-n}} \leq \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_1-n}}
\]

\[
\iff \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_0-n} \left(\frac{a}{1-a}\right)^{r}} \leq \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_1-n} \left(\frac{a}{1-a}\right)^{r}}
\]

\[
\iff k_0 - k_1 \leq r.
\]

Therefore, for the case \(k_0 - k_1 \leq r\), the interval in (5.2) is non-empty if and only if

\[
\pi_1^0 \leq \pi_0^0 \iff \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_1-n-2} \left(\frac{a}{1-a}\right)^{r}} \leq \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_0-n} \left(\frac{a}{1-a}\right)^{r}}
\]

\[
\iff k_0 - k_1 \geq r - 1.
\]

Similarly, for the case \(k_0 - k_1 > r\), the interval in (5.2) is non-empty if and only if

\[
\pi_0^0 \leq \pi_1^1 \iff \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_1-n-2} \left(\frac{a}{1-a}\right)^{r}} \leq \frac{1}{1 + \left(\frac{1-a}{a}\right)^{2k_0-n} \left(\frac{a}{1-a}\right)^{r}}
\]

\[
\iff k_0 - k_1 \leq r + 1.
\]

We can now conclude that \(Q^{a,k_0,k_1}(\beta)\) is non-empty if and only if \(r - 1 \leq k_0 - k_1 \leq r + 1\). □

8.2.3 Proof of Proposition 4

From the algorithm that we described in Section 5.2, we know that for a given preference profile \(q\), there exists a contingent \(k\)-voting rule that can sustain informative voting as an equilibrium if and only if there exists a pair of integers \(k_0, k_1 \in \{1, ..., n\}\) that satisfies the inequalities \((\pi_0^0)^{-1}(\bar{q}) \leq k_0 \leq (\pi_0^0)^{-1}(q)\) and \((\pi_1^1)^{-1}(\bar{q}) \leq k_1 \leq (\pi_1^1)^{-1}(q)\). These four inverse function are given as follows:

\[
(\pi_0^0)^{-1}(q) = \frac{1}{2} \left(\ln \left(\frac{1-q}{\bar{q}}\right) + n + 2 + r\right), \quad (\pi_0^0)^{-1}(\bar{q}) = \frac{1}{2} \left(\ln \left(\frac{1-q}{\bar{q}}\right) + n + r\right),
\]

\[
(\pi_0^1)^{-1}(q) = \frac{1}{2} \left(\ln \left(\frac{1-q}{\bar{q}}\right) + n + 2 - r\right), \quad (\pi_1^1)^{-1}(\bar{q}) = \frac{1}{2} \left(\ln \left(\frac{1-q}{\bar{q}}\right) + n - r\right).
\]
As a result, we have

\[
\frac{(\pi_0^0)^{-1}(q) - (\pi_1^0)^{-1}(\bar{q})}{2} = \frac{\ln\left(\frac{1-q}{q}\right) - \ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2\ln\left(\frac{1-a}{a}\right)} + 1,
\]

\[
\frac{(\pi_0^0)^{-1}(q) - (\pi_1^1)^{-1}(\bar{q})}{2} = \frac{\ln\left(\frac{1-q}{q}\right) - \ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2\ln\left(\frac{1-a}{a}\right)} + 1.
\]

If \( q < \bar{q} \), it is easy to check that both \((\pi_0^0)^{-1}(q) - (\pi_1^0)^{-1}(\bar{q})\) and \((\pi_0^0)^{-1}(q) - (\pi_1^1)^{-1}(\bar{q})\) are strictly less than one, which implies that both the interval \([(\pi_0^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(q)]\) and the interval \([(\pi_1^1)^{-1}(\bar{q}), (\pi_0^0)^{-1}(q)]\) can contain at most one integer. If \( q = \bar{q} = q \), then \((\pi_0^0)^{-1}(q) - (\pi_1^1)^{-1}(\bar{q}) = (\pi_0^0)^{-1}(q) - (\pi_1^1)^{-1}(\bar{q}) = 1\). Thus, in this case both the interval \([(\pi_0^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(q)]\) and the interval \([(\pi_1^1)^{-1}(\bar{q}), (\pi_0^0)^{-1}(q)]\) will contain at least one integer.\(^9\) Note that these results are independent of the size of the committee. Hence, it remains to be shown that if \( \forall i \in I, q_i = q \), then for each of these two intervals, at least one of the integers that are contained in it must belong to the set \( \{1, \ldots, n\} \) when \( n \) is sufficiently large, so that the corresponding contingent \( k \)-voting rule is well-defined. For this, it suffices to have

\[
(\pi_1^1)^{-1}(q) \geq 0 \iff \frac{1}{2}\left(\frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-a}{a}\right)} + n - r\right) \geq 0 \iff n \geq r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-a}{a}\right)}.
\]

and

\[
(\pi_0^0)^{-1}(q) \leq n + 1 \iff \frac{1}{2}\left(\frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-a}{a}\right)} + n + 2 + r\right) \leq n + 1 \iff n \geq r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-a}{a}\right)},
\]

since \((\pi_0^1)^{-1}(q) \leq (\pi_0^0)^{-1}(q)\) and \((\pi_1^1)^{-1}(q) \leq (\pi_1^0)^{-1}(q)\) for all \( q \in [0, 1] \) and \( r \geq 0 \). Let

\[
\bar{n}(q) = \left\lceil \max\left\{r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-a}{a}\right)}, r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-a}{a}\right)}\right\} \right\rceil,
\]

where \([x]^{+}\) denotes the smallest integer that is larger or equal to the real number \( x \). We can now conclude that when agents’ preference are perfectly aligned, there exists a threshold

\(^9\)The interval \([(\pi_1^1)^{-1}(\bar{q}), (\pi_0^0)^{-1}(q)]\) will contain exactly two integers if and only if \((\pi_1^1)^{-1}(q)\) is an integer. Similarly, there will be two integers in the interval \([(\pi_1^1)^{-1}(\bar{q}), (\pi_0^0)^{-1}(q)]\) if and only if \((\pi_1^1)^{-1}(q)\) is an integer.
value \( \hat{n}(q) \), such that for each \( n \geq \hat{n}(q) \), there exists a contingent \( k \)-voting rule under which the informative voting equilibrium exists.

8.2.4 Proof of Proposition 5

Pick any \( q^s \) from the sequence and let \( \bar{q}^s = \max_{j \in \{1, \ldots, n+s\}} \hat{q}_j \) and \( q^s = \min_{j \in \{1, \ldots, n+s\}} \hat{q}_j \). From the algorithm that we described in Section 5.2, we know that for such a preference profile, there exists a contingent \( k \)-voting rule that can sustain informative voting as an equilibrium if and only if there exists a pair of integers \( k_0^s, k_1^s \in \{1, \ldots, n+s\} \) that satisfies the inequalities \( (\pi_0^0)^{-1}(\bar{q}^s) \leq k_0^s \leq (\pi_0^0)^{-1}(q^s) \) and \( (\pi_1^0)^{-1}(\bar{q}^s) \leq k_1^s \leq (\pi_1^0)^{-1}(q^s) \), where

\[
(\pi_0^0)^{-1}(\bar{q}^s) = \frac{1}{2} \left( \frac{\ln \left( \frac{1-q^s}{q^s} \right)}{\ln \left( \frac{1-a}{a} \right)} + n + s + 2 + r \right), \quad (\pi_1^0)^{-1}(\bar{q}^s) = \frac{1}{2} \left( \frac{\ln \left( \frac{1-q^s}{q^s} \right)}{\ln \left( \frac{1-a}{a} \right)} + n + s + r \right),
\]

\[
(\pi_0^1)^{-1}(\bar{q}^s) = \frac{1}{2} \left( \frac{\ln \left( \frac{1-q^s}{q^s} \right)}{\ln \left( \frac{1-a}{a} \right)} + n + s + 2 - r \right), \quad (\pi_1^1)^{-1}(\bar{q}^s) = \frac{1}{2} \left( \frac{\ln \left( \frac{1-q^s}{q^s} \right)}{\ln \left( \frac{1-a}{a} \right)} + n + s - r \right).
\]

Because \( \ln \left( \frac{1-q}{q} \right) / \ln \left( \frac{1-a}{a} \right) \) is increasing in \( q \) and \( \bar{q} \geq \bar{q}^s \) and \( q \leq q^s \), comparing the above four functions to the four functions \( (\pi_0^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\bar{q}), (\pi_1^0)^{-1}(\bar{q}), (\pi_1^0)^{-1}(\bar{q}) \) and \( (\pi_0^1)^{-1}(\bar{q}), (\pi_1^1)^{-1}(\bar{q}) \) in the proof of Proposition 4, it is clear that if there is a pair of integers \( k_0, k_1 \in \{1, \ldots, n\} \) that are respectively contained in the intervals \( [(\pi_0^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\bar{q})] \) and \( [(\pi_1^0)^{-1}(\bar{q}), (\pi_1^0)^{-1}(\bar{q})] \), i.e., there exists a contingent \( k \)-voting rule that can sustain informative voting as an equilibrium with the preference profile \( q \), there will be also a pair of integers \( k_0^s, k_1^s \in \{1, \ldots, n+s\} \) that are respectively contained in the intervals \( [(\pi_0^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\bar{q})] \) and \( [(\pi_1^0)^{-1}(\bar{q}), (\pi_1^0)^{-1}(\bar{q})] \).

For asymptotic efficiency, note that \( \forall q \in (0, 1) \), all \( (\pi_0^0)^{-1}(\bar{q})/\left( n + s \right) \), \( (\pi_0^0)^{-1}(\bar{q})/\left( n + s \right) \), \( (\pi_1^0)^{-1}(\bar{q})/\left( n + s \right) \), and \( (\pi_1^0)^{-1}(\bar{q})/\left( n + s \right) \) converge to 1/2 as \( s \to \infty \). Hence, after adding sufficiently many members to the committee, the probability that the collective decision made in the informative voting equilibria under the corresponding contingent \( k \)-voting rules coincide with that in the informative voting equilibrium under the standard majority rule becomes arbitrarily close to 1. Since the informative voting equilibrium under the standard majority rule is asymptotically efficient if the agents’ private signals are informative \( (\alpha > 1/2) \), so are the informative voting equilibria under the contingent \( k \)-voting rules.
8.2.5 Proof of Corollary 4

Plugging \( k_0 = (n + 1)/2 + [(r - 1)/2]^+ \) in the formulas of \( \pi_0^0 \) and \( \pi_1^0 \), one can easily verify that for all \( r \geq 0 \),

\[
\max\{\pi_0^0, \pi_1^0\} = \frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{-|2(r-1)/2|^+ + 1}}.
\]

Similarly, with \( k_1 = (n + 1)/2 - [(r - 1)/2]^+ \), we have

\[
\min\{\pi_1^0, \pi_0^1\} = \frac{1}{1 + \left(\frac{1 - \alpha}{\alpha}\right)^{-|2(r-1)/2|^+ + 1}}
\]

for all \( r \geq 0 \). The result of the corollary thus immediately follows Proposition 3.

8.2.6 Proof of Proposition 6

Consider a social planner who observes the whole vector of private signals \( s = (s_1, \ldots, s_n) \) and the public signal \( s_p \). Suppose the public signal is \( r \)-times more informative than the private signal, where \( r \geq 0 \). Recall \( m_s = \sum_{i=1}^{n} s_i \). To maximize the probability that his decision will be matched to the state, the social planner would choose the following optimal decision rule:

\[
d^* = \begin{cases} 
1 & \text{if } m_s - (n - m_s) + r \mathbb{1}_{s_p=1} - r \mathbb{1}_{s_p=0} > 0, \\
\{0, 1\} & \text{if } m_s - (n - m_s) + r \mathbb{1}_{s_p=1} - r \mathbb{1}_{s_p=0} = 0, \\
0 & \text{if } m_s - (n - m_s) + r \mathbb{1}_{s_p=1} - r \mathbb{1}_{s_p=0} < 0.
\end{cases}
\]

Under the contingent majority rule, \( k(s_p) = \frac{n+1}{2} - \left\lceil \frac{r-1}{2} \right\rceil \) if \( s_p = 1 \) and \( k(s_p) = \frac{n+1}{2} + \left\lceil \frac{r-1}{2} \right\rceil \) if \( s_p = 0 \). Hence, when \( s_p = 1 \), in the informative voting equilibrium, \( d = 1 \) if and only if

\[
m_s \geq \frac{n+1}{2} - \left\lceil \frac{r-1}{2} \right\rceil \iff (n - m_s) - m_s \leq 2 \left\lceil \frac{r-1}{2} \right\rceil - 1 =: R_1,
\]

while when \( s_p = 0 \), \( d = 1 \) if and only if

\[
m_s \geq \frac{n+1}{2} + \left\lceil \frac{r-1}{2} \right\rceil \iff m_s - (n - m_s) \geq 2 \left\lceil \frac{r-1}{2} \right\rceil + 1 =: R_0.
\]
There are four possible scenarios for the relative locations of \( r, [r]^+, R_1 \) and \( R_0 \) on the real line:

1. \( r \) is an even integer,

   \[
   r - 2 \quad r - 1 \quad r \quad r + 1
   \]

2. \( r \) is an odd integer,

   \[
   r - 2 \quad r - 1 \quad r \quad r + 1
   \]

3. \( r \) is not an integer and \( [r]^+ \) is even,

   \[
   r - 2 \quad [r]^+ - 2 \quad r - 1 \quad [r]^+ - 1 \quad r \quad [r]^+ \quad r + 1 \quad [r]^+ + 1
   \]

4. \( r \) is not an integer and \( [r]^+ \) is odd,

   \[
   r - 2 \quad [r]^+ - 2 \quad r - 1 \quad [r]^+ - 1 \quad r \quad [r]^+ \quad r + 1 \quad [r]^+ + 1
   \]

Since \( |m_s - (n - m_s)| \) is odd, the above four figures jointly show that the collective decision achieved by the contingent majority rule always coincides with the social planner’s choice.
8.3 Proofs of results in Section 6

8.3.1 Proof of Proposition 7

Since the signal $s_p$ is only observed to the controller and the vote only takes place after the agents receive the message from the controller, we have a dynamic game of incomplete information. We look for sequential equilibria, which require the beliefs and the strategies of the players to be sequentially rational and consistent [Fudenberg and Tirole, 1991]. Note that since the biased of the controller is not (directly) payoff-relevant to the agents, we need to keep track of agents’ beliefs about the state only.

First, consider the scenario where $m = s_p$ is sent. By Proposition 2, no agent would have the incentive to deviate from obedient voting given all other agents are following the public signal revealed by the controller. This is always the case regardless of the relative precision of the signals.\(^{10}\)

Now consider the information set where $m = \emptyset$ is sent and the controller’s disclosure policy is to withhold her information if and only if she observes $s_p = 1$. Conditional all other agents are voting informatively, the informative voting strategy is optimal for agent $i$ if and only if

$$
\Pr(\theta = 1|s_i = 1, m = \emptyset) = \frac{\frac{1}{2} \alpha (\lambda + (1 - \lambda)(1 - \beta))}{\frac{1}{2} \alpha (\lambda + (1 - \lambda)(1 - \beta)) + \frac{1}{2} (1 - \alpha)(\lambda + (1 - \lambda))} \\
\geq \frac{\frac{1}{2} (1 - \alpha)(\lambda + (1 - \lambda))}{\frac{1}{2} \alpha (\lambda + (1 - \lambda)(1 - \beta)) + \frac{1}{2} (1 - \alpha)(\lambda + (1 - \lambda))} \\
= \Pr(\theta = 0|s_i = 1, m = \emptyset)
$$

and

$$
\Pr(\theta = 0|s_i = 0, m = \emptyset) = \frac{\frac{1}{2} \alpha (\lambda + (1 - \lambda)\beta)}{\frac{1}{2} \alpha (\lambda + (1 - \lambda)\beta) + \frac{1}{2} (1 - \alpha)(\lambda + (1 - \lambda)(1 - \beta))} \\
\geq \frac{\frac{1}{2} \alpha (\lambda + (1 - \lambda)\beta)}{\frac{1}{2} \alpha (\lambda + (1 - \lambda)(1 - \beta)) + \frac{1}{2} (1 - \alpha)(\lambda + (1 - \lambda)(1 - \beta))} \\
= \Pr(\theta = 1|s_i = 0, m = \emptyset).
$$

\(^{10}\)Note that by Corollary 1, informative voting does not constitute an equilibrium in these subgames whenever $\beta > \alpha$.  

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Since $\alpha, \beta \geq 1/2$ and $\lambda > 0$, the second inequality always holds. It can be also checked that the first inequality holds if and only if $(\beta - (1 - \alpha)) \lambda \geq \beta - \alpha$. Hence, whenever $\lambda \geq \hat{\lambda}$, the informative voting strategy profile and the beliefs that are formed according to Bayes rule are sequentially rational for the agents at the information set $m = \emptyset$. By the same token, if the controller only reveals $s_p = 0$ to the agents, no agent can profitably deviate from the proposed strategy profile as long as $\lambda \geq \hat{\lambda}$ and beliefs are formed according to Bayes rule.

Given the strategies of the agents, suppose the controller observes $s_p = 1$. By revealing this information to the agents, his expected payoff is given by $U_c^r(1) = -q_c(1 - \beta)$. On the other hand, withholding this information from the agents yields him an expected payoff of $U_c^{nr}(1) = -q_c(1 - \beta)P - (1 - q_c)\beta P$, where

$$P = \sum_{k=\frac{n+1}{2}}^{n} C_n^k (1 - \alpha)^k \alpha^{n-k}$$

is the probability that the committee reaches a wrong decision when all agents vote informatively. Similarly, by revealing $s_p = 0$ to the agents, the controller’s expected payoff is $U_c^r(0) = -(1 - q_c)(1 - \beta)$, while concealing yields him an expected payoff of $U_c^{nr}(0) = -q_c \beta P - (1 - q_c)(1 - \beta)P$. Hence, the controller would find it optimal to reveal $s_p = 1$ and withhold $s_p = 0$ if $U_c^r(1) \geq U_c^{nr}(1)$ and $U_c^{nr}(0) \geq U_c^r(0)$, which, after some rearrangements, are equivalent to

$$q_c \leq \hat{q} = \min \left\{ \frac{\beta P}{(1 - \beta)(1 - P) + \beta P}, \frac{(1 - \beta)(1 - P)}{(1 - \beta)(1 - P) + \beta P} \right\}.$$

Similarly, revealing $s_p = 0$ and withholding $s_p = 1$ is optimal for the controller if

$$q_c \geq 1 - \hat{q} = \max \left\{ \frac{\beta P}{(1 - \beta)(1 - P) + \beta P}, \frac{(1 - \beta)(1 - P)}{(1 - \beta)(1 - P) + \beta P} \right\}.$$

The threshold value $\hat{q}$ achieves its supremum at $P = 1 - \beta$, which equals to $1/2$. Also, since $\alpha > 1/2$, it is straightforward to check that $P < 1/2$ and, hence, $\hat{q} \geq \min\{\beta, 1 - \beta\} = 1 - \beta$.

In conclusion, if $\lambda \geq \hat{\lambda}$ and $q_c \leq \hat{q}$ ($q_c \geq 1 - \hat{q}$), the strategy profile stated in the proposition together with the beliefs formed according to Bayes rule constitute a Perfect Bayesian Equilibrium. Since all information sets can be reached with positive probability, it is also a sequential equilibrium.

$\Box$
8.3.2 Proof of Proposition 8

First, consider the scenario where \( m = s_p \) is sent. By the same reasoning as in Corollary 4, no agent would have the incentive to deviate from informative voting given all other agents are voting informatively.

Next, consider the information set where \( m = \emptyset \) is sent and suppose the strategy of controller is such that he never withholds information. In this case, the controller’s message is not informative at all and given that the agents are unbiased and the voting threshold corresponds to the simple majority rule, the informative voting strategy profile along with the beliefs formed according to Bayes rule are clearly sequentially rational for the agents. Now suppose the controller’s strategy is such that she will reveal her information to the agents if and only if \( s_p = 1 \) (or \( s_p = 0 \)). By Proposition 7, we know that in this case no agent can profitably deviate from informative voting provided that \( \lambda \geq \hat{\lambda} \).

Given that the agents will always vote informatively, suppose the controller observes \( s_p = 1 \). By withholding the signal, the controller obtains an expected payoff of 
\[
U_{ncr}^n(1) = -q_c(1 - \beta)P - (1 - q_c)\beta P,
\]
while revealing yields 
\[
U_{rcr}^n(1) = -q_c(1 - \beta)P' - (1 - q_c)\beta \tilde{P},
\]
where
\[
P' = \sum_{k=\lfloor \frac{n+1}{2} \rfloor}^{n} C_n^k (1 - \alpha)^k \alpha^{n-k}, \quad \tilde{P} = \sum_{k=\lfloor \frac{n+1}{2} \rfloor}^{n} C_n^k (1 - \alpha)^k \alpha^{n-k}.
\]

Similarly, by concealing \( s_p = 0 \) from the agents, the controller’s expected payoff is given by 
\[
U_{ncr}^n(0) = -q_c \beta P - (1 - q_c)(1 - \beta)P, \quad U_{rcr}^n(0) = -q_c \beta \tilde{P} - (1 - q_c)(1 - \beta)P'.
\]
Hence, the controller would find it optimal to always share his information with the agents if 
\[
U_{rcr}^n(1) \geq U_{rcr}^n(1) \quad \text{and} \quad U_{rcr}^n(0) \geq U_{rcr}^n(0).
\]
Note that these inequalities trivially hold for all \( q_c \in [0, 1] \) if \( [(r - 1)/2]^+ = 0 \) or, equivalently, \( \beta \leq \alpha \). Now suppose \( [(r - 1)/2]^+ \geq 1 \). Then, it is straightforward to check that these two inequalities are satisfied if and only if 
\[
q^* = \frac{(1 - \beta)(P' - P)}{\beta(P - \tilde{P}) + (1 - \beta)(P' - P)} \geq 0.
\]
Since $\alpha \geq 1/2$ and $C^k_n = C^{n-k}_n$, we have

$$P - \bar{P} = \frac{\sum_{k=\lceil \frac{n+1}{2} \rceil}^{\frac{n+1}{2} + \lceil \frac{n-1}{2} \rceil} C^k_n (1 - \alpha)^k \alpha^{n-k}}{\sum_{k=\lceil \frac{n+1}{2} \rceil}^{\frac{n+1}{2} - \lceil \frac{n-1}{2} \rceil} C^k_n (1 - \alpha)^k \alpha^{n-k}} \leq 1$$

and, hence, $q^* \leq 1 - \beta$. Clearly, together with the beliefs formed according to Bayes rule, the proposed strategies for the controller (always share his information) and the agents (always vote informatively) constitute a Perfect Bayesian equilibrium. It is also a sequential equilibrium since all information set can be reached with positive probability in equilibrium. We thus have proven the first part of the proposition. The proof of the second part of the proposition is analogous, and we omit it here to avoid repetition. \qed
References


